

SET-②

- Q1 Define time-space tradeoff with example :-
 time space tradeoff means using more memory (i.e. Space) to make an algorithm run in less time or using less memory but taking more time.
 Eg:- In mergesort we take extra space as we make a temp array but it works faster.

- Q2 Application of multidimensional array:-
- used to store data in table or matrix form.
 - used in mathematical operation like addition & multiplication
 - 2D arrays store pixel values of images.
 - 2D arrays represent grids or chess boards.

- Q3 1-D array index formula :-
 In memory, array elements are stored continuously.

$$\text{LOC}(A[i]) = \text{Base}(A) + (i \times w)$$

where $\text{Base}(A)$ = Base address, i = index, w = size in bytes

Example → $\text{Loc}[A(4)] = 1000 + (4 \times 4)$

$$\begin{aligned} \text{Base}(A) &= 1000 \\ w &= 4 \text{ bytes.} \end{aligned}$$

Significance:

- Enables random access to elements in $O(1)$ time
- Helps compilers & system locate elements quickly

Q4.

Algorithm (Linear Search)

- 1) Start from 1st element of the array.
- 2) Compare the target element with each element.
- 3) If match found, return index of that element.
- 4) If not found, even after checking all elements, return -1.

Pseudocode :-

```
linearSearch( int arr, target )
```

```
for ( i = 0 to length of arr - 1 ) {
```

```
    if ( arr[i] == target ) {
```

```
        return i;
```

```
}
```

```
return -1;
```

Advantages

- easy to understand & implement.
- Works on both sorted and unsorted.
- No extra memory required.

disadvantages

- slow for large dataset ($TC = O(n^2)$)
- inefficient when compared to Binary Search on Sorted array.

Q5.

Quick Sort Algorithm →

It's a divide and conquer algo that works & sorts an array by partitioning it into subarrays arrays around a pivot.

Steps :-

- 1) choose pivot element from the array
- 2) Partition the array so that,
 - all elements smaller than pivot go to its left.
 - all elements greater than pivot go to its right.
- 3) Recursively apply the same process to the left & right.
- 4) The process continues until each subarray has one element.
of partitioning.

Pseudocode:

```

int partition (int arr, int low, int high) {
    int pivot = arr[high]
    i = low - 1
    for (j=low → j < high) {
        if (arr[j] < pivot) {
            swap (arr[i], arr[j])
            i++
        }
    }
    swap (arr[i+1], arr[high])
    return (i+1);
}
  
```

Q6. Discuss different types of Recursion with examples :-

① Direct Recursion - A function calls itself directly.

eg) `int fact(int n)`

```

if (n==0) return 1;
return n * fact(n-1);
  
```

Ques. (2) Indirect Recursion - A function calls itself after calling another function first.

```
eg > void funcA (int n) {
    if (n > 0) funcB (n-1);
}

void funcB (int n) {
    if (n > 0) funcA (n/2);
}
```

(3) Tail Recursion - the recursion call is the last statement in the function.

```
eg > int sum (int n, int acc) {
    if (n == 0) return acc;
    return sum (n-1, acc+n); // last call.
}
```

(4) Non-Tail Recursion - Recursion call is not the last statement.

eg > factorial function in (1).

Ques. Double linked list (DLL)

each node contains

- data : value stored
- next : pointer to the next node.
- prev : pointer to the previous node.

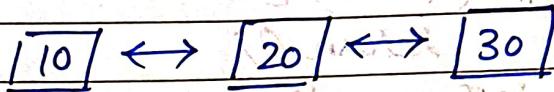
→ Allows traversal in both directions.

(a)

Insertion in DLL :-

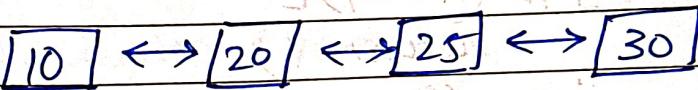
- 1) Create a new node.
- 2) Adjust the prev & next pointers of the node & its neighbouring nodes.

Before ins.



Insert
25 after
20.

After ins.



- 3) IF inserting at Beginning - update head pointer, set next to current head, set prev to NULL.

End - update last node's next pointer, set prev to last node, set next to last node. NULL

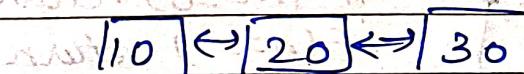
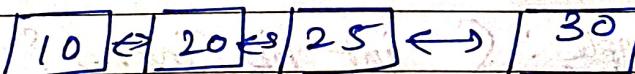
Middle - Update pointers of previous & next nodes accordingly.

(b)

Deletion in DLL :-

- 1) find the node to delete.
- 2) Adjust the next of the previous node & prev of the next node to bypass the node.
- 3) prev / delete the node.

Before delete



Q8.(a) Iterative solution for calculating Fibonacci numbers 0, 1, 2, 3, 5, 8, 13

```

int fib (int n) {
    if (n == 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    }
    int fib0 = 0, fib1 = 1, fibx = 0;
    for (int i = 2; i <= n; i++) {
        fibx = fib0 + fib1;
        fib0 = fib1;
        fib1 = fibx;
    }
    return fibx;
}

```

(b). Removal of Recursion

Recursion can be replaced by loops to save stack memory.

Recursive calls are converted to iterative constructs like 'for' or 'while'.

Example :

```

int fib (int n) {
    if (n == 0) return 0;
    if (n == 1) return 1;
    return fib(n-1) + fib(n-2);
}

```

$O(2^n)$

Iterative approach like above in part(a) avoids recursion overhead. $TC = O(n)$
 SC reduces from $O(n)$ stack space to $O(1)$.

Q9. Sparse Matrix Storage using LinkedList.

1. Sparse matrix

$$\begin{bmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 5$$

2. LinkedList Representation

each node contains

row, col, data, next

↳ pointer to the next non-zero element.

Code :-

```
class Node {
    int row, col, data;
    Node next;
}
Node (int r, int c, int d) {
    row = r; col = c; data = d;
    next = null;
}
```

} }

LL representation of the example :

Node (row = 0, col = 2, data = 3) →

Node (row = 2, col = 1, data = 7) →

Node (row = 3, col = 7, data = 1) → null.

[0, 2, 3] → [2, 1, 7] → [3, 4, 1] → null.

Advantages of LLR →

→ space efficient → Reduces wasted storage

→ Dynamic size → Easy traversal of non-zero elements.

Set 3

Q1. Explain the concept of algorithm efficiency and how it is measured.

A1. Algo efficiency refers to how well an algo uses time and memory resources to solve a problem. It helps determine which algo performs better for large inputs.

measured using :-

1) Time efficiency
2) Space efficiency

Efficiency is measured by Big O notation [$O(n)$] which shows how the algo scales with input n .

Q2.

Time Complexity

Amount of time taken by an algorithm.

Number of basic operations

Linear search $\rightarrow O(n)$

Minimize execution time

Space Complexity

Amount of memory taken or used during execution.

No. of variables/storage locations

using an extra array $\rightarrow O(n)$

Minimize memory usage

Q3. Define index formula for accessing elements in a 3D - array in column-major order.

A3. Let the array be $A [L_1 \dots U_1, L_2 \dots U_2, L_3 \dots U_3]$

at L_1, L_2, L_3 = lower bounds of dimensions

U_1, U_2, U_3 = upper bounds of dimensions

$$N_1 = U_1 - L_1 + 1 \quad N_2 = U_2 - L_2 + 1 \quad N_3 = U_3 - L_3 + 1$$

Base Address = Base(A)

$$\text{LOC}(A[i][j][k]) = \text{Base}(A) + (i - L_1) + (j - L_2) \times N_1 + (k - L_3) \times N_1 \times N_2$$

Example $\Rightarrow A[0 \dots 2][0 \dots 2][0 \dots 2]$ let $B(A) = 100$

$$\begin{aligned} \text{LOC}(A[1][1][1]) &= 100 + (1 - 0) + (1 - 0) \times (2 - 0 + 1) \\ &\quad + (1 - 0) \times (2 - 0 + 1) \times (2 - 0 + 1) \\ &= 100 + 1 + 1 \times 3 + 1 \times 3 \times 3 \\ &= 100 + 1 + 3 + 9 \end{aligned}$$

113 Ans

(Q4.) Write the Algorithm for Binary Search & explain its' steps:-

Binary Search Algorithm :-

works on sorted arrays & repeatedly divides the search interval in half to find the target element efficiently.

Algorithm :-

1) Start with low index = 0 & upper index high = n - 1

2) Find mid index,

$$\text{mid} = (\text{low} + \text{high}) / 2 \text{ or,}$$

$$(\text{optimized}) \Rightarrow \text{mid} = \text{low} + (\text{high} - \text{low}) / 2$$

3) Compare the mid element arr[mid] with target
 → arr[mid] == key, element found, return mid
 → arr[mid] > key, search left, high = mid - 1
 → arr[mid] < key, search right, low = mid + 1

4) Repeat step 2 and 3 until (low > high)

5) If key NOT found, return -1.

Time Complexity :-

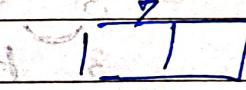
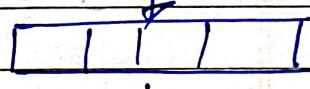
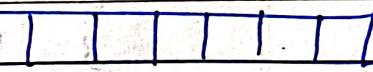
$$\frac{n}{2^0}, \frac{n}{2^1}, \frac{n}{2^2}, \dots, \frac{n}{2^k} = 1$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k \log 2$$

$$\log_2 n = k$$



T.C.: $O(\log n)$

Q5.

Describe Bubble Sort & Explain its working with example :-

Bubble Sort :-

Simple comparison-based sorting algorithm that repeatedly steps through the list, compares adjacent elements & swap them if they are in wrong order.

After each pass, the largest element bubbles up to its correct position.

Algorithm :-

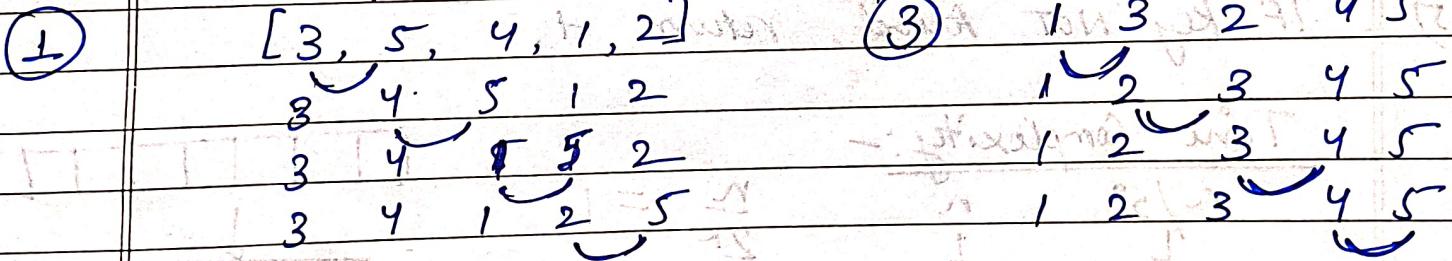
- 1) Start from first element and compare with next one.
- 2) If current element isn't in the correct order, swap them.
- 3) Continue this for entire array — one pass.
- 4) Repeat until no swaps are needed.

Sort the Array [5, 3, 4, 1, 2]

Example:-

Pass

Compare & Swap



(2)

[3, 4, 1, 2, 5]

⇒ [1, 2, 3, 4, 5]

Hence, sorted

Time Complexity :-Best : $O(n)$ Avg / Worst : $O(n^2)$ Space Complexity :-

Code :-

```

void bubbleSort (int arr[]) {
    int n = arr.length;
    for (int i=0; i<n-1; i++) {
        for (int j=0; j<n-i-1; j++) {
            if (arr[j] > arr[j+1]) {
                int temp = arr[j];
                arr[j] = arr[j+1];
                arr[j+1] = temp;
            }
        }
    }
}

```

Dry Run :- [2, 3, 1, 0] index
 $i = 0 \rightarrow 3$

Pass 1 \rightarrow 2 < 3 no swap $j = 0 \rightarrow 3 - i$

3 > 1 swap

3 > 0 swap. $\Rightarrow [2, 1, 0, 3]$

Pass 2 \rightarrow 2 > 1 swap

2 > 0 swap

2 < 3 NO swap $\Rightarrow [1, 0, 2, 3]$

Pass 3 \rightarrow 1 > 0 swap

1 < 2 NO swap

2 < 3 NO swap $\Rightarrow [0, 1, 2, 3]$

\hookrightarrow sorted

Q6. Write a recursive method for calculating factorial of a number and explain it.

Ans 6.

Algorithm :-

- 1) Base case : if $n=0$ or 1 return 1 .
- 2) Recursive case : multiply n by factorial of $(n-1)$.

int factorial (int n) {

 if ($n==0$ || $n==1$) {

 return 1 ; }

 return $n * \text{factorial}(n-1)$;

fact(5)

fact(4)

fact(3)

fact(2)

fact(1)

T.C. & S.C. $\Theta(n)$.

Q7. Explain circular linked list & write the algorithm to traverse it.

Circular linked List:-

A variation of a linked list where the last node points back to the first node.

- Can be singly circular (last node → first node)
- or doubly circular (last node → first node and first node → last node).

Advantage over regular linked list →

- traverse the list starting from any node.
- Useful in buffered systems, round robin scheduling.

Structure (Singly CLL Node).

```
class Node {
```

```
    int data;
```

```
    Node next;
```

```
    Node(int data) {
```

```
        this.data = data;
```

```
        this.next = null;
```

Algorithm to traverse a CLL :- (Objective: Print all nodes in CLL)

- 1) Start with head node.
- 2) Use a temp pointer & move through the list
 → print temp.data
 → move temp = temp.next
- 3) Stop when temp becomes equal to head again.

Pseudocode :-

```

int data : Node * next;
void traverse( Node * head ) {
    if ( head == NULL ) return;
    Node * temp = head;
    do {
        cout << temp->data << " ";
        temp = temp->next;
    } while ( temp != head );
    cout << endl;
}

```

- Q8. (a) Discuss the trade-offs between recursion & iteration with reference to memory usage.

Recursion

Iteration

- * Each recursive call adds * Iterative loops use constant a new stack frame in memory $\rightarrow O(1)$ space memory $\rightarrow O(n)$ space (no stack overhead) for n calls.
- * Simpler & more intuitive * Complex to implement.
- * May cause stack overflow * Iteration is safer for for large inputs.

(b)

Write a recursive algorithm for the Fibonacci Series.

0, 1, 1, 2, 3, 5, 8, ...

Algorithm :-

Base case : if $n=0$, return 0

if $n=1$, return 1

Recursive case : Return $\text{fib}(n-1) + \text{fib}(n-2)$.

```
int fib(int n) {
    if (n==0) return 0;
    if (n==1) return 1;
    return fib(n-1) + fib(n-2);
}
```

(Q9.)

Merge Sort on an Unsorted Array:-

- divide and conquer sorting algorithm
- split the array into two halves
- recursively sort both halves
- merge those two sorted halves into a single array.

Time Complexity $\rightarrow O(n \log n)$ — even in worst case.

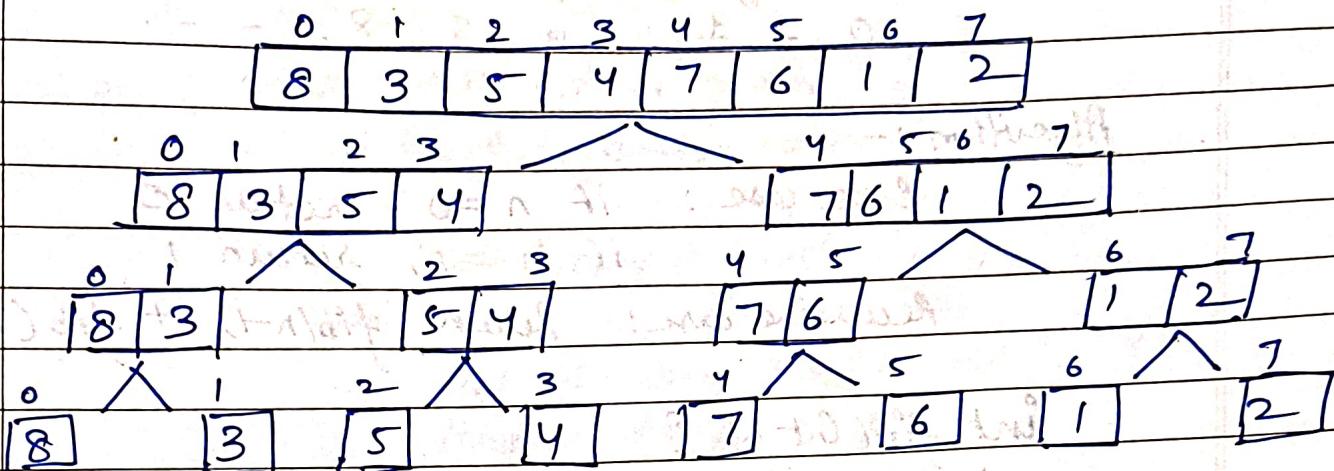
Space Complexity $\rightarrow O(n)$

Real World Applications \rightarrow

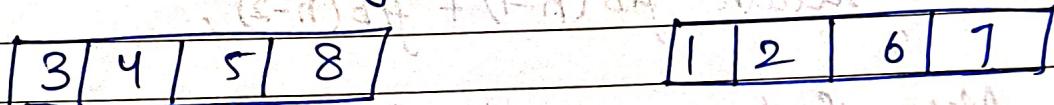
1. External sorting — sorting data too large to fit in memory
2. Large scale analytics — big data frameworks to sort massive datasets.
3. DBMS — sorting records for merge-joins & query optimizations.
4. Linked List sorting — works well as doesn't require random access.

Example →

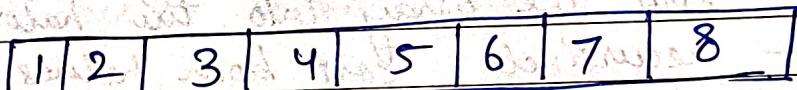
Given arr = [8, 3, 5, 4, 7, 6, 1, 2]



left Half after sorting right half after sorting



Merge both halves :-



Algorithm :-

mergesort (arr, left, right) {

 if (left < right) {

 mid = (left + right) / 2

 mergesort (arr, left, mid)

 merge sort (arr, mid+1, right)

 merge (arr, left, mid, right).

Set-4

1. Define algorithm and explain importance of efficiency in algorithm design.

Ans An Algorithm is a finite set of well-defined, step-by-step instructions designed to perform a specific task or solve a particular problem.

Importance of Efficiency in Algorithm Design:

- 1) Time Optimization - Reduce the execution time
- 2) Space Optimization - Algorithm must be memory-efficient
- 3) Scalability - Efficient algorithms can handle large datasets without a significant increase in computation time.
- 4) Cost and Energy Saving - Lower Computational Costs & energy consumption
- 5) LISTER Experience:

2. Explain row-major and column-major representation of arrays with suitable diagrams.

Ans Row-major representation-

All the elements of the first row are stored in consecutive memory locations, followed by the elements of the second row, & so-on.

Eg. Consider a 2D array $A[3][3]$

| Row/col | 0 | 1 | 2 | → Row-Major Storage Sequence: |
|---------|---|---|---|-------------------------------|
| 0 | 1 | 2 | 3 | → 1, 2, 3, 4, 5, 6, 7, 8, 9 |
| 1 | 4 | 5 | 6 | |
| 2 | 7 | 8 | 9 | |

Memory Mapping Formula (Row-major):

$$LOC(A[i][j]) = \text{Base}(A) + [i \times N + j] \times w$$

Column-Major Repn

All the elements of the first column are stored in consecutive memory location, followed by the elements of the second column & so-on.

Eg Array [3][3]:

| Row\Col | 0 | 1 | 2 | → Column-Major Storage Seq: |
|---------|---|---|---|---|
| 0 | 1 | 2 | 3 | → 1, 4, 7, 2, 5, 8, 3, 6, 9 |
| 1 | 4 | 5 | 6 | Memory Mapping formula: |
| 2 | 7 | 8 | 9 | $LOC(A[i][j]) = \text{Base}(A) + [j \times M + i] \times W$ |

? Q3] Derive the general index formula for multi-dimensional arrays?

Ans Let A be a k-dimensional array with dimension size.

$$D_1, D_2, \dots, D_k \quad (0 \leq i_j < D_j \text{ for } j=1 \dots k)$$

4] Compare insertion sort and selection sort with respect to their time complexity?

| | Insertion Sort | Selection Sort |
|-------------------|--|-------------------------------------|
| Best Case Time | $O(n)$ | $O(n^2)$ |
| Average Case Time | $O(n^2)$ | $O(n^2)$ |
| Worst Case Time | $O(n^2)$ | $O(n^2)$ |
| Space Complexity | $O(1)$ | $O(1)$ |
| Stability | Stable | Not-stable |
| Adaptiveness | Adaptive | Non-Adaptive |
| No. of Comparison | Varies with input order | Always fixed $\Rightarrow n(n-1)/2$ |
| No. of Swaps | Up to $O(n^2)$ | Almost $(n-1)$ |
| Practical | Efficient for small or nearly sorted data. | Used when memory bytes are costly. |

5] Explain merge sort algorithm with an example.

Ans Merge sort is a divide and conquer sorting algorithm that:

- 1) Divide the array into two halves.
- 2) Sorts each half recursively, and then
- 3) Merges the two sorted halves into one sorted array.

Algorithm steps

- 1) Divide - Split the array into two halves - left and right
- 2) Conquer - Recursively sort the two halves using merge sort.
- 3) Combine (Merge) - Merge the two sorted halves into a single sorted array.

- 6] Discuss Removal of recursion and write an iterative algorithm equivalent to the recursive factorial function.

Ans Drawback of Recursion-

- i) Extra memory usage - Each recursive call add a new stack-frame.
- ii) Functional call overhead. - (consume time during each call)
- iii) Risk of Stack overflow - Deep recursion can exceed the system's call stack limit.

Removal Of Recursion-

- Use loops (iteration) instead of recursive calls.
- Use an explicit stack or queue (data structure).

Recursive fn Pseudocode

```
int factorial (int n) {
    if (n==0) return 1;
    else return n*factorial (n-1);
}
```

Equivalent Iterative algorithm.

Algorithm Iterative_factorial(n)

1. fact $\leftarrow 1$
2. For $i \leftarrow 1$ to n do :
3. fact \leftarrow fact $\times i$
4. return fact

Ex Execution :

$$\begin{aligned}
 \text{fact} &= 1 \\
 i=1 &\longrightarrow \text{fact} = 1 \times 1 = 1 \\
 i=2 &\longrightarrow \text{fact} = 1 \times 2 = 2 \\
 i=3 &\longrightarrow \text{fact} = 2 \times 3 = 6 \\
 i=4 &\longrightarrow \text{fact} = 6 \times 4 = 24 \\
 i=5 &\longrightarrow \text{fact} = 24 \times 5 = 120
 \end{aligned}$$

Q7] Explain insertion operation in singly linked list with an example.

A singly linked list is a linear data structure where each element (called a node) consists two part -

- 1) Data - The value stored in the node.
- 2) Next Pointer - A reference (or address) to the next node in the list.

Types of Insertion Operations -

- a) Insertion at the beginning
- b) Insertion at the end
- c) Insertion after a given node (in the middle)

(a) Insertion at the beginning

Steps

- 1) Create a new node;
- 2) Set its next pointer to the current head.
- 3) Move the head pointer to the new node.

Pseudocode

Create newNode

```
newNode.data = value  
newNode.next = head  
head = newNode
```

(b) Insertion at the End

Steps

- 1) Create a new node & set its next to NULL.
- 2) Traverse the list to find the last node.
- 3) Set last node's next pointer to new node

Pseudocode

Create newNode

```
newNode.data = value
```

```
newNode.next = NULL
```

```
if head == NULL:
```

```
    head = newNode
```

```
else: temp = head
```

```
while temp.next != NULL
```

```
    temp = temp.next
```

```
temp.next = newNode
```

c) Insertion After a Given Node (Middle).

Steps:

- ① Transverse: find node after which you want to insert.
- ② Create a new node
- ③ Set `newNode.next = target.next`
- ④ Set `target.next = newNode`

Pseudocode:

Create `newNode`

`newNode.data = value`

`newNode.next = target.next`

`target.next = newNode`

Q9)

With an example, explain sparse matrix representation

Using a 2-D array & discuss its advantage & limitation

Ans

A sparse matrix is a matrix that contains mostly zero elements and only a few non-zero elements.

Eg)

Consider a 4×5 matrix

$$\begin{matrix} & & 4 \\ \begin{matrix} 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 & 0 \end{matrix} & : \end{matrix}$$

Limitation

- 1) complex Implementation
- 2) slower Random access
- 3) Overhead for Dense Matrices

Advantage:

- i) Memory Efficient
- ii) Faster Computation
- iii) Easy to store & Transfer
- iv) Useful for Large Data