1.

$$k(x,z) = \exp\left(-\frac{\|x-z\|^2}{2\delta^2}\right)$$

$$= \exp\left(-\frac{x^Tx}{2\delta^2}\right)\exp\left(-\frac{z^Tz}{2\delta^2}\right)\exp\left(\frac{x^Tz}{\delta^2}\right)$$

$$= \exp\left(-\frac{x^Tx}{2\delta^2}\right)\exp\left(-\frac{z^Tz}{2\delta^2}\right)\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{x^Tz}{\delta^2}\right)^j$$

$$= \exp\left(-\frac{x^Tx}{2\delta^2}\right)\exp\left(-\frac{z^Tz}{2\delta^2}\right)\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{x^Tz}{\delta^2}\right)^j$$

$$= \exp\left(-\frac{x^Tx}{2\delta^2}\right)\exp\left(-\frac{z^Tz}{2\delta^2}\right)\left[\sum_{j=0}^{\infty}\frac{1}{\delta^{2j}j!}\sum_{i_1=0}^{i}\cdots\sum_{i_j=0}^{i}(x_{i_1}\cdots x_{i_j})(z_{i_1}\cdots z_{i_j})\right]$$

$$= \sum_{j=0}^{\infty}\left[\frac{(x_{i_1}\cdots x_{i_j})}{\delta^j\sqrt{j!}}\exp\left(-\frac{x^Tx}{2\delta^2}\right)\right]\left[\frac{(z_{i_1}\cdots z_{i_j})}{\delta^j\sqrt{j!}}\exp\left(-\frac{z^Tz}{2\delta^2}\right)\right]$$

Where $\sum_{j=0}^{\infty}\frac{1}{\delta^{2j}j!}\sum_{i_1=0}^{j}\cdots\sum_{i_j=0}^{j}(s_{i_1}\cdots s_{i_j})=1$ if $j=0$. Therefore, computing the Gaussian kernel is equivalent to taking the inner product after mapping the input to an infinite dimensional feature space, where each element of the mapping $\phi(x)$ has the form

$$\frac{(x_{i_1}\cdots x_{i_j})}{\delta^j\sqrt{j!}}\exp\left(-\frac{x^Tx}{2\delta^2}\right)$$