Lecture 15: Applications of DFS: SCC

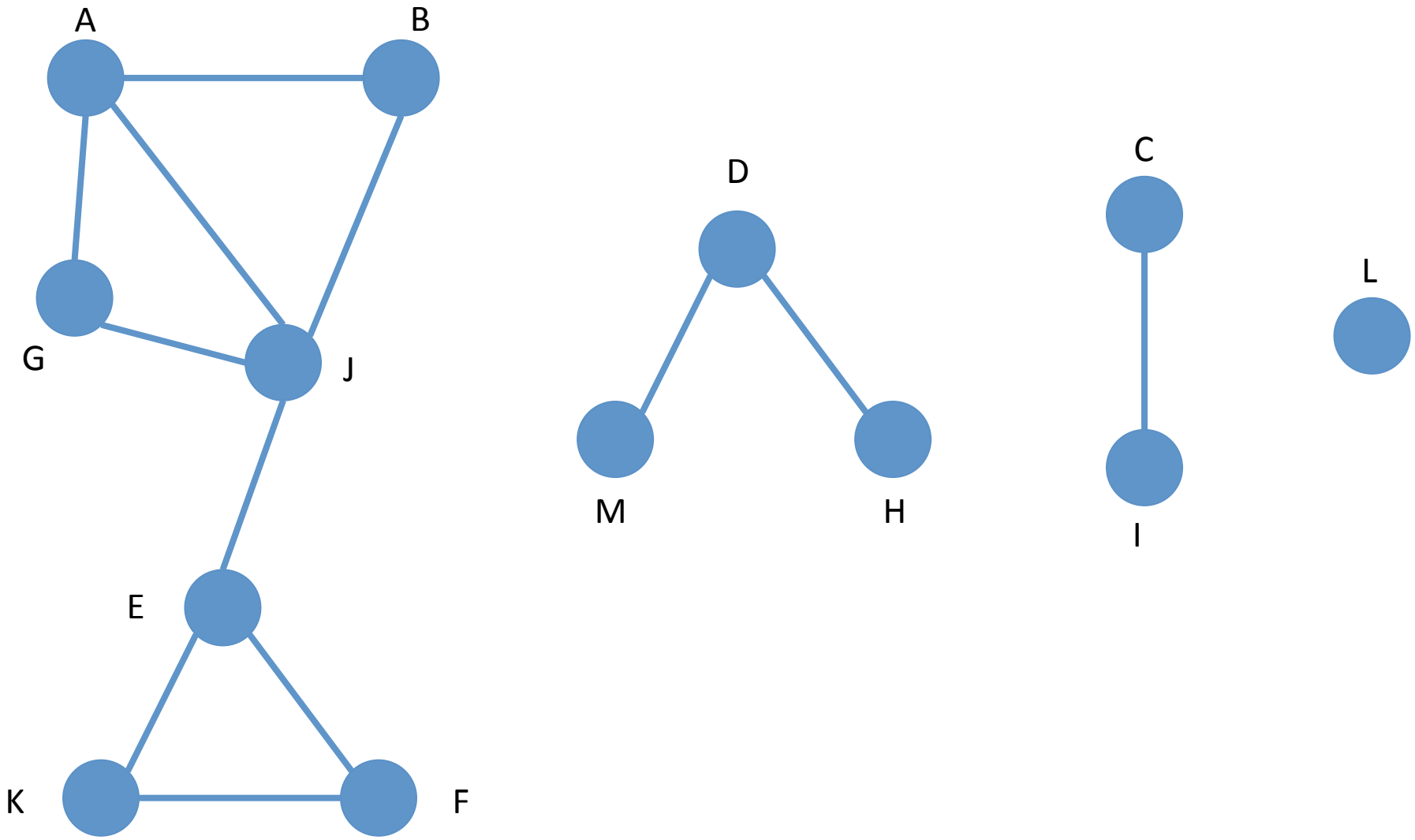
CS 341: Algorithms

Thursday, March 7th 2019

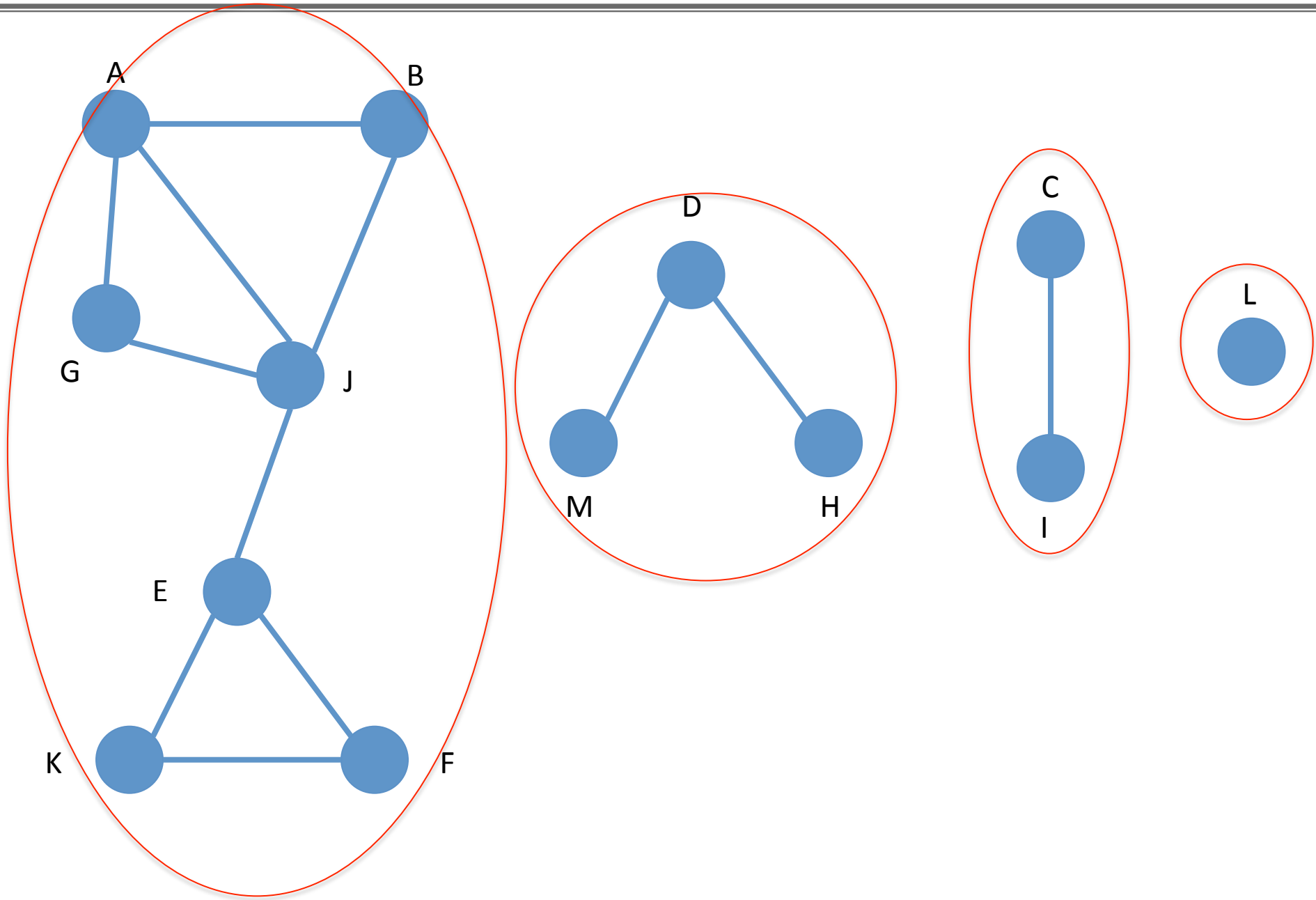
Outline For Today

1. Strongly Connected Components in Directed Graphs

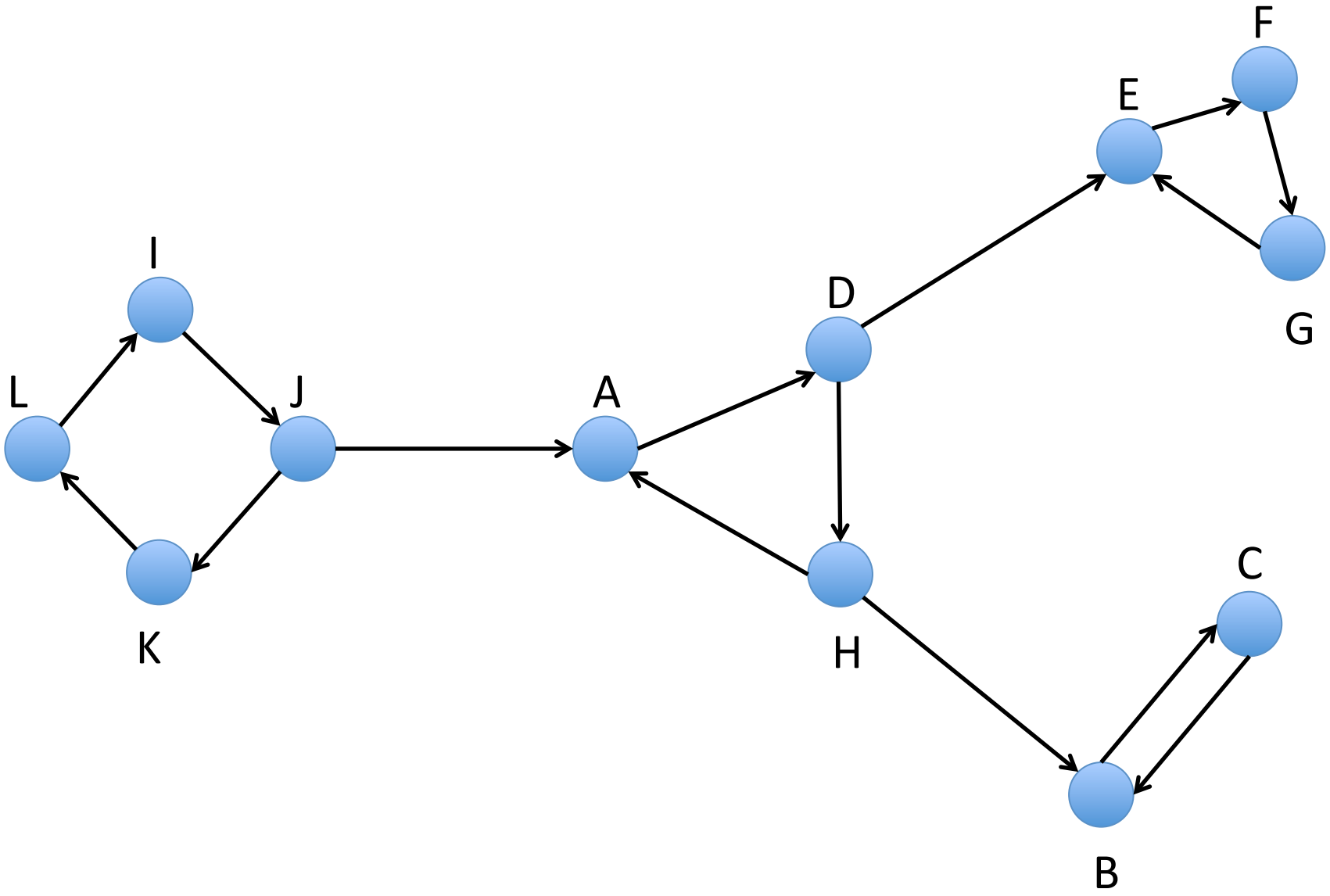
Recall: Connectivity in Undirected Graphs



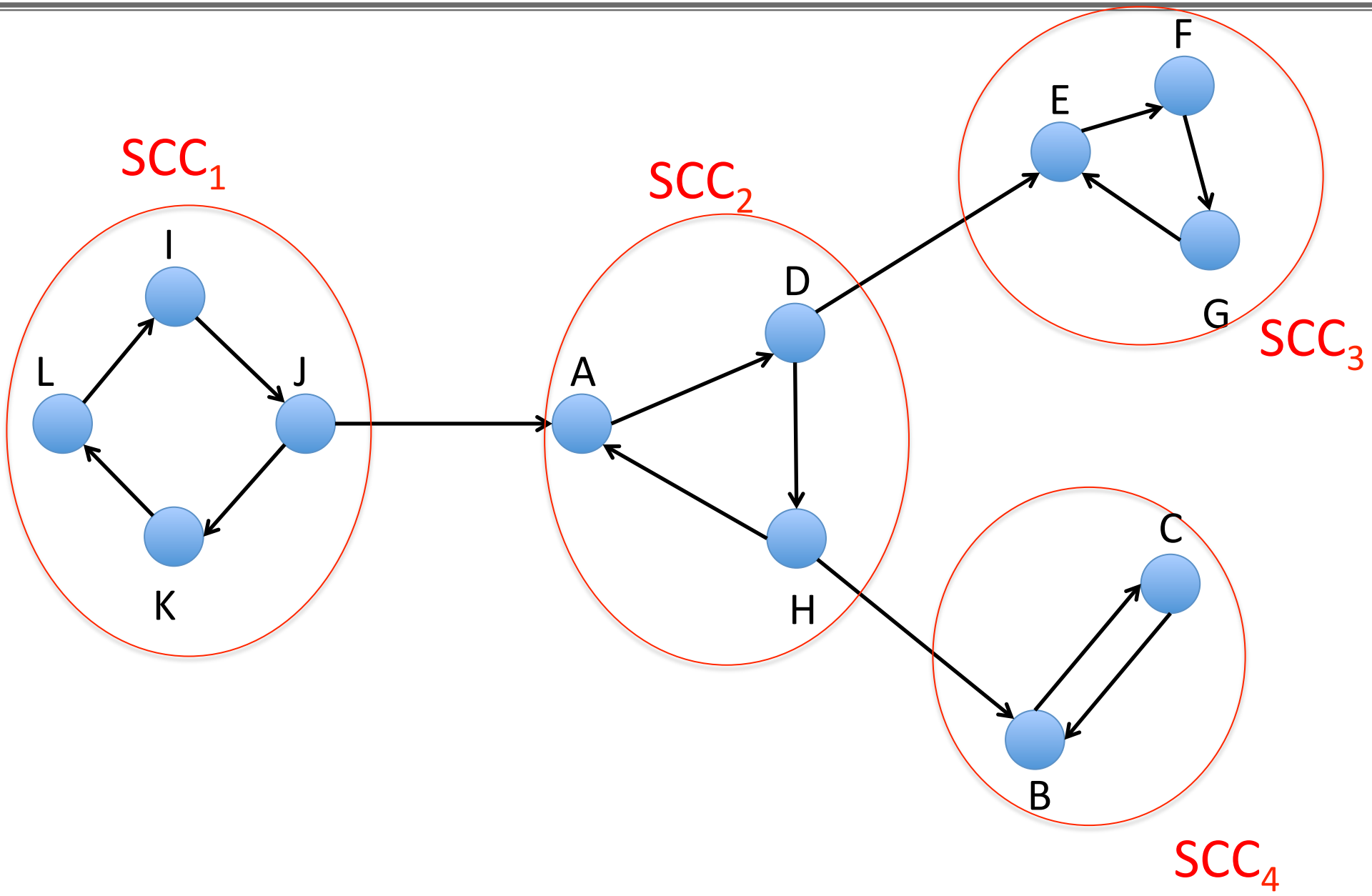
Recall: Connectivity in Undirected Graphs



Connectivity in Directed Graphs



“Strong Connectivity” in Directed Graphs



Formal Definition of SCCs

- Given a directed graph G(V, E)
- u and v are strongly connected iff:

  $u \rightsquigarrow v$ *AND* $v \rightsquigarrow u$

- *An SCC of G is a set* $C \subseteq V$ *s.t*

  1. $\forall (u, v) \in C$, *u and v are strongly connected*
  2. *C is non-empty and maximal, i.e.* $\forall v`$ *in V-C,*

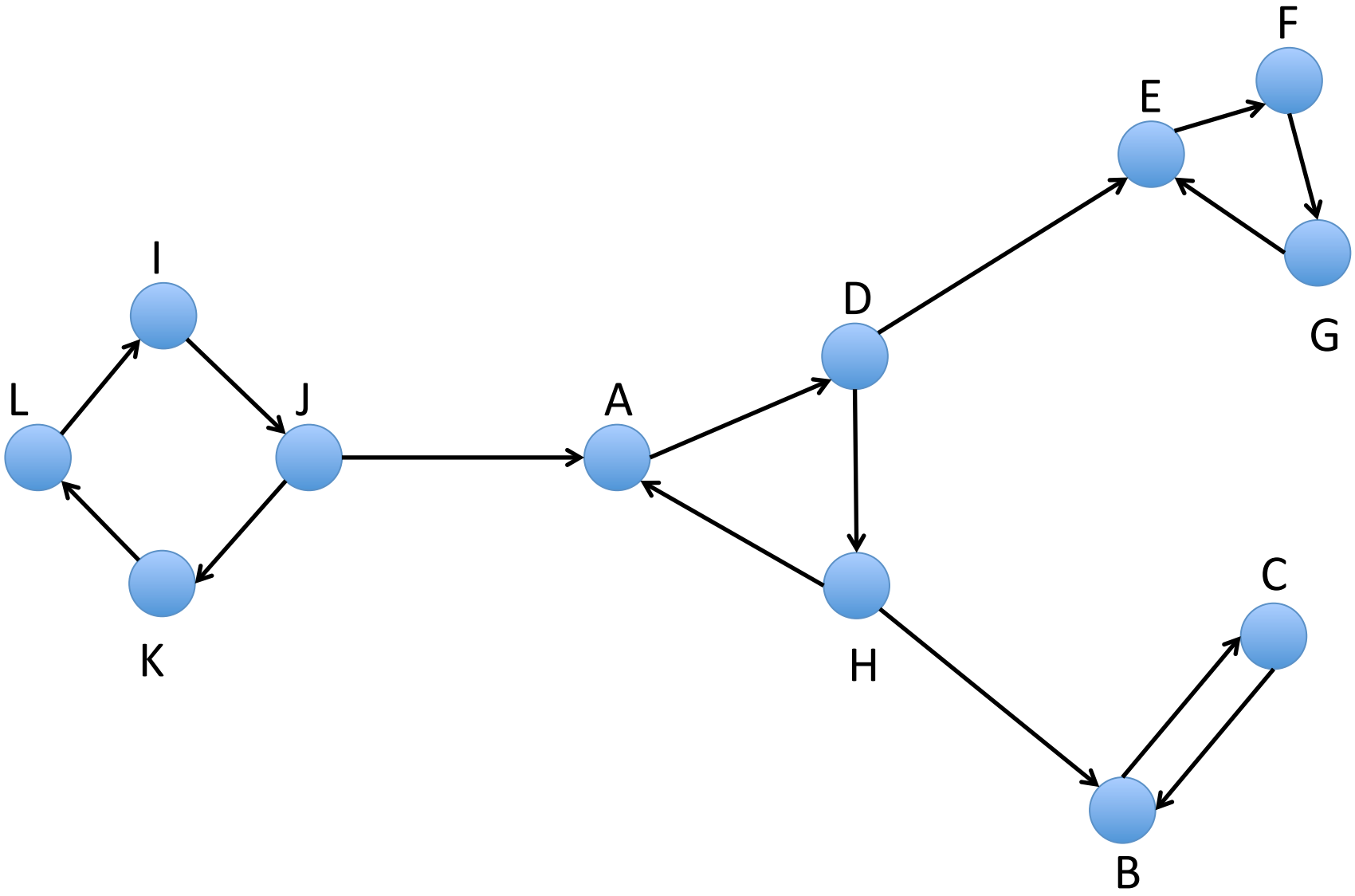
     v` is not strongly connected to any vertex in C

Two-tiered Structure

- Given a directed graph G(V, E)
- Consider G^{SCC}: the meta-graph of its SCCs
- We'll call this "SCC graph of G"
- *Claim: G^{SCC} is a DAG!*

Each directed graph has a second higher-level structure which is a DAG.

Example of Two-tiered Structure



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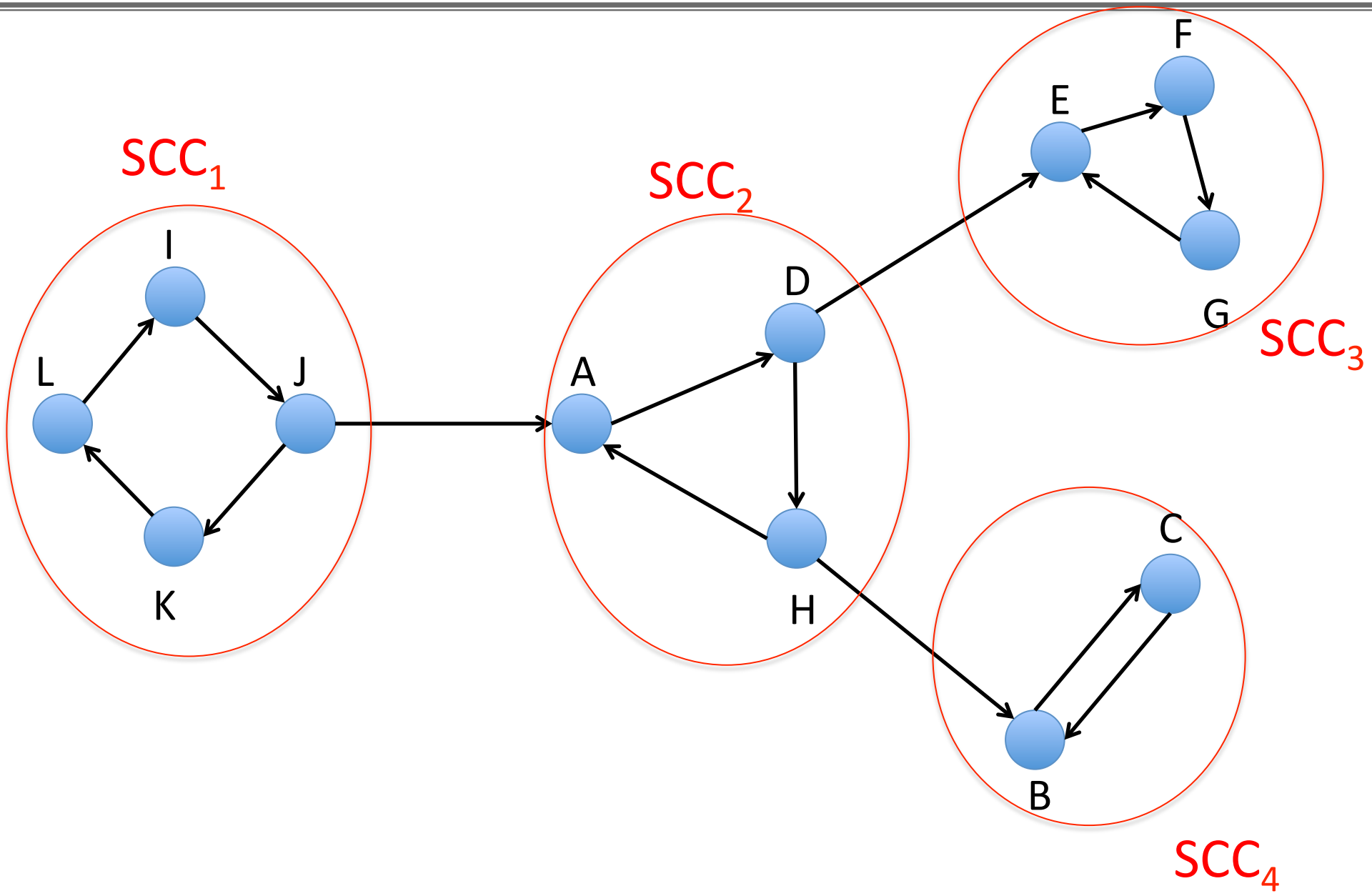

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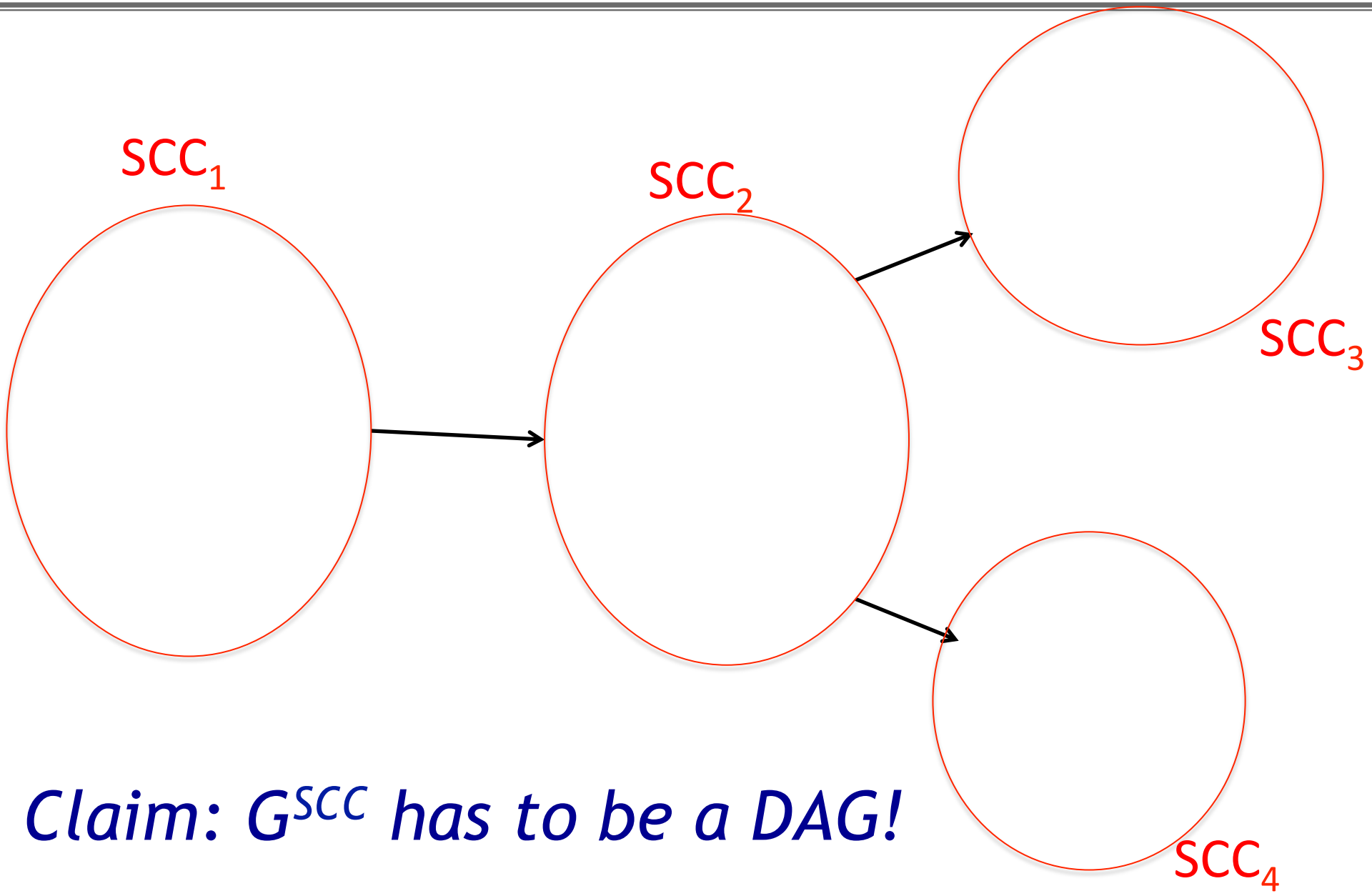
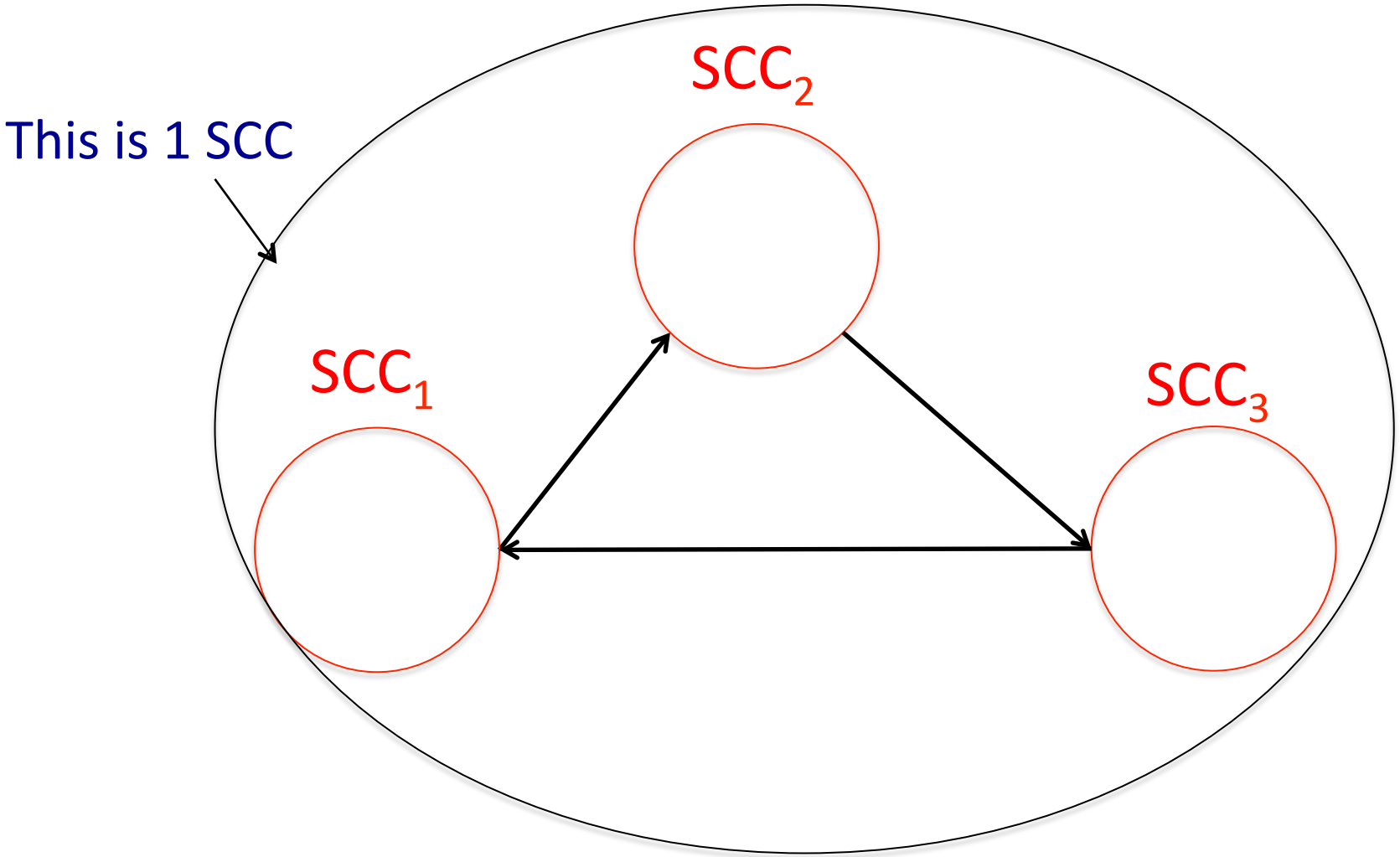


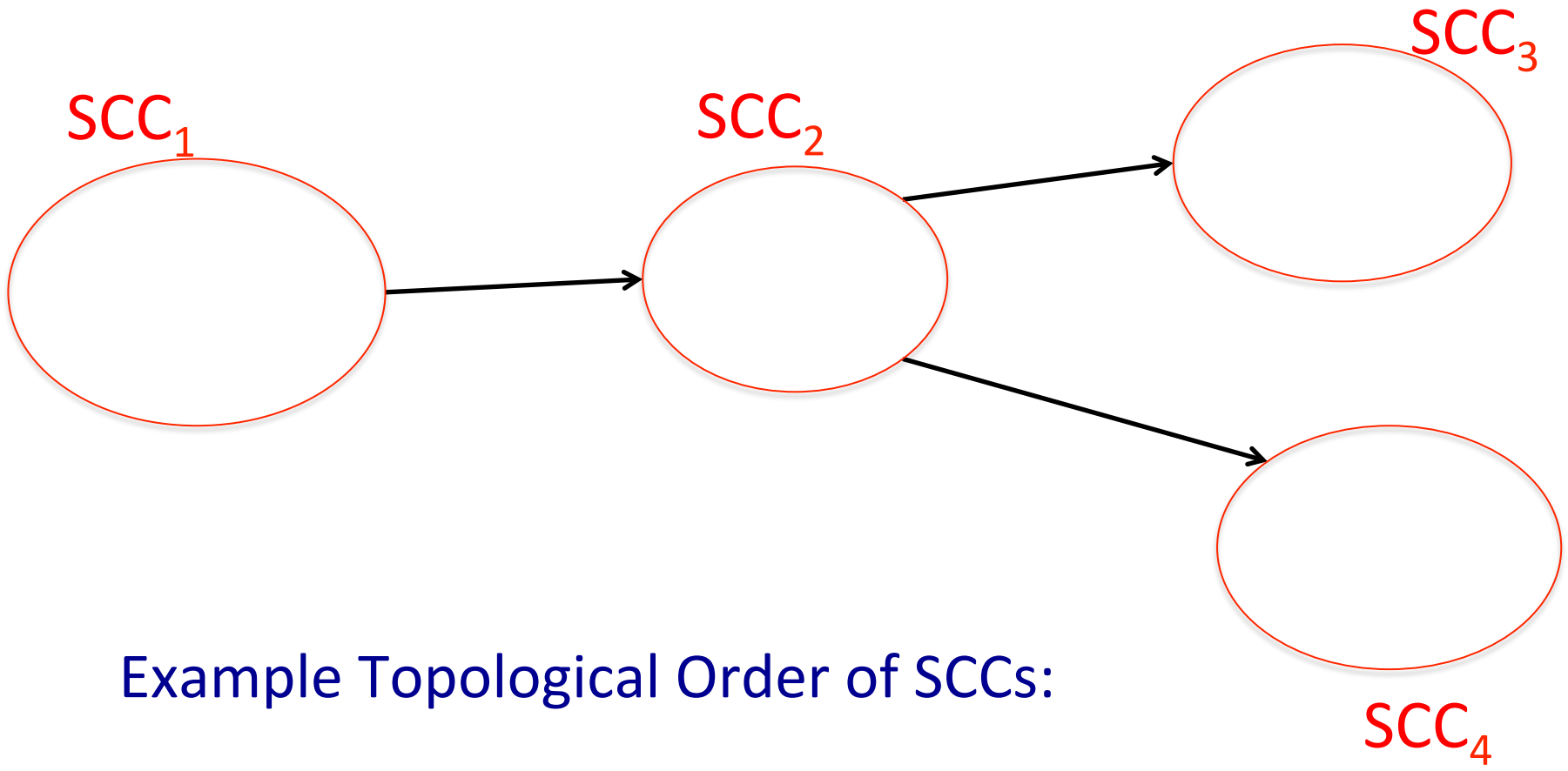
Claim: G^{SCC} has to be a DAG!

Proof Sketch of G^{SCC} being a DAG:



Therefore, can't have cycles in G^{SCC}

So We Can Topologically Order G^{SCC}

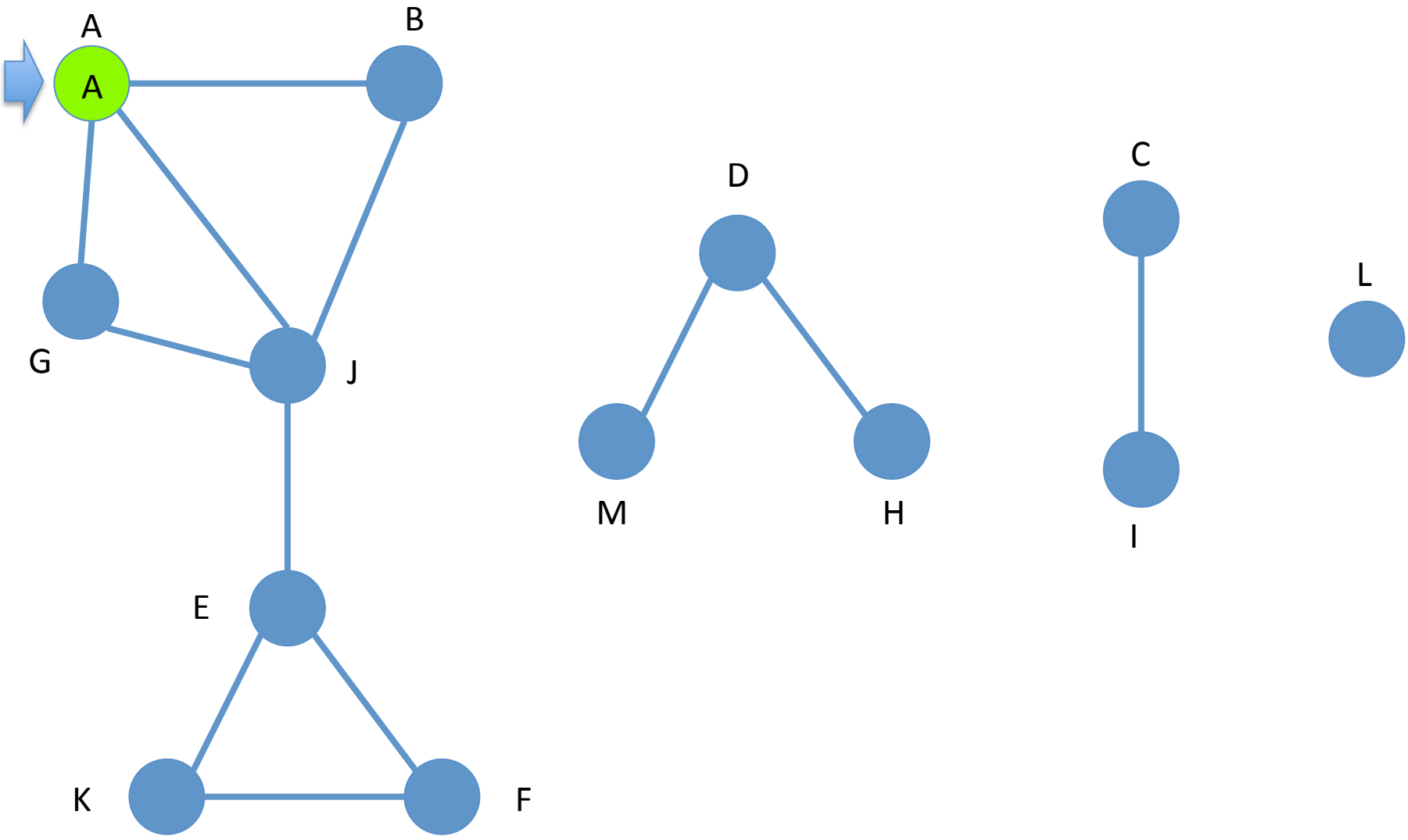


Example Topological Order of SCCs:

SCC_1 SCC_2 SCC_4 SCC_3

Will be important for Kosaraju's Algorithm!

Recall Algorithm for Undirected Graphs



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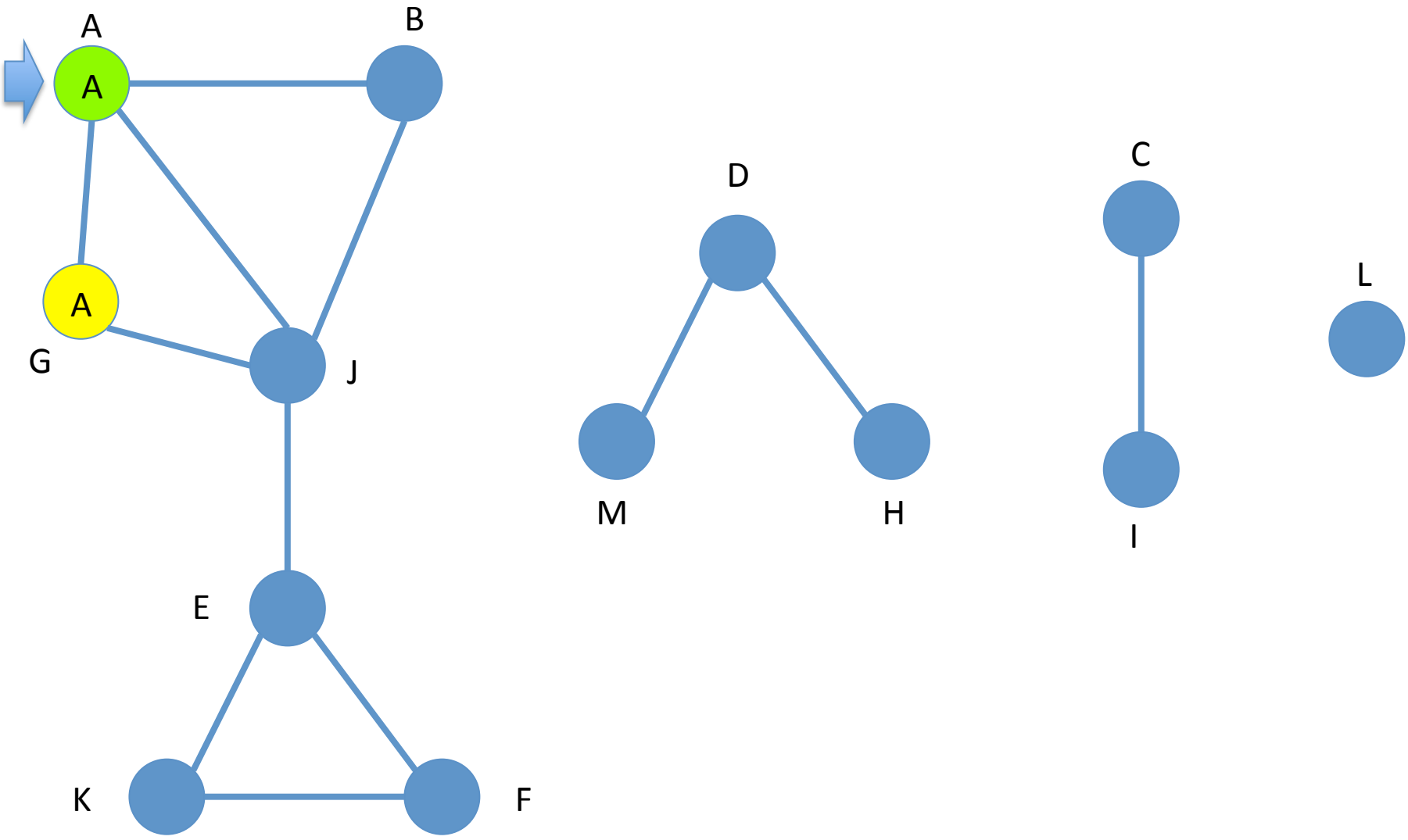

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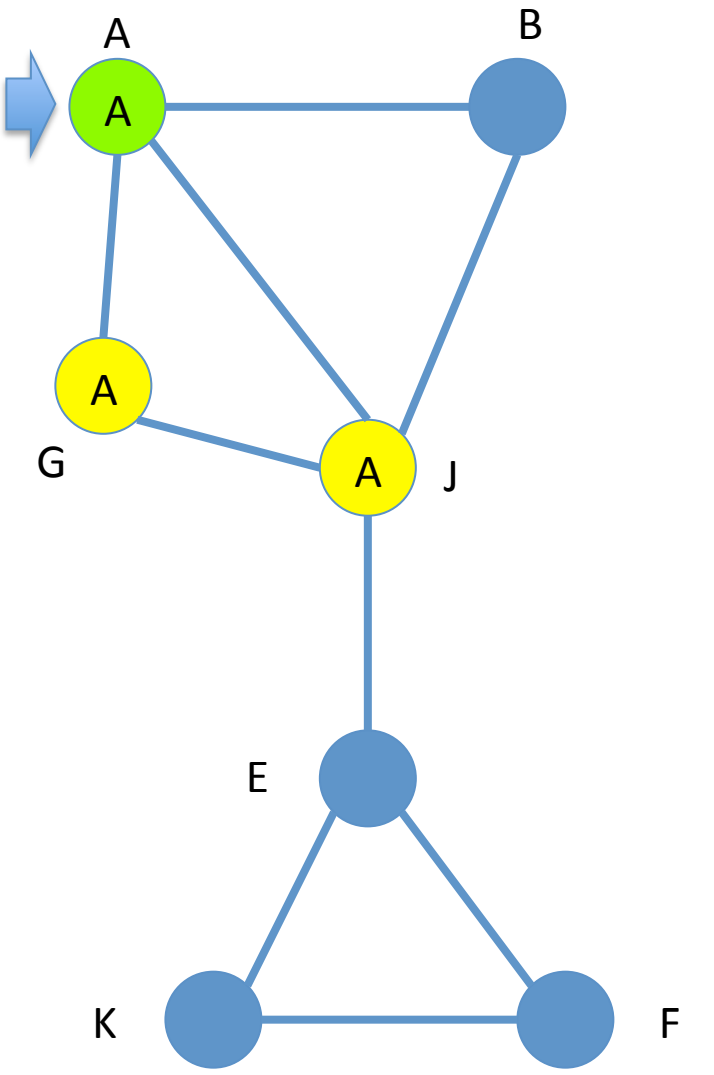

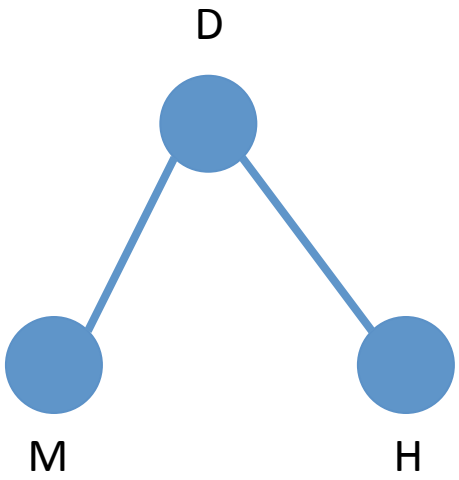

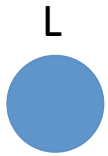

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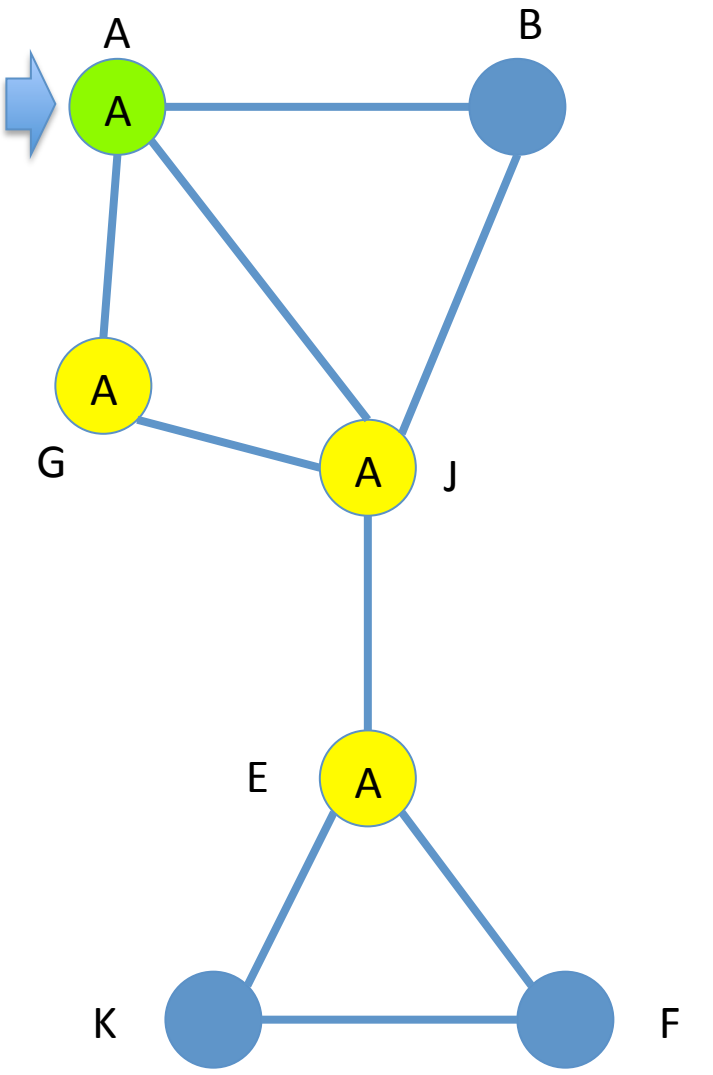

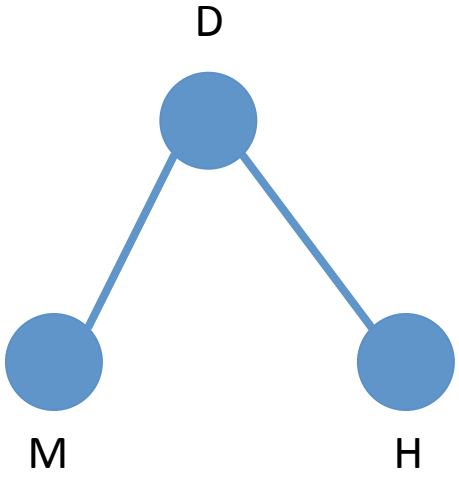

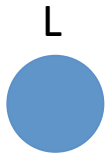

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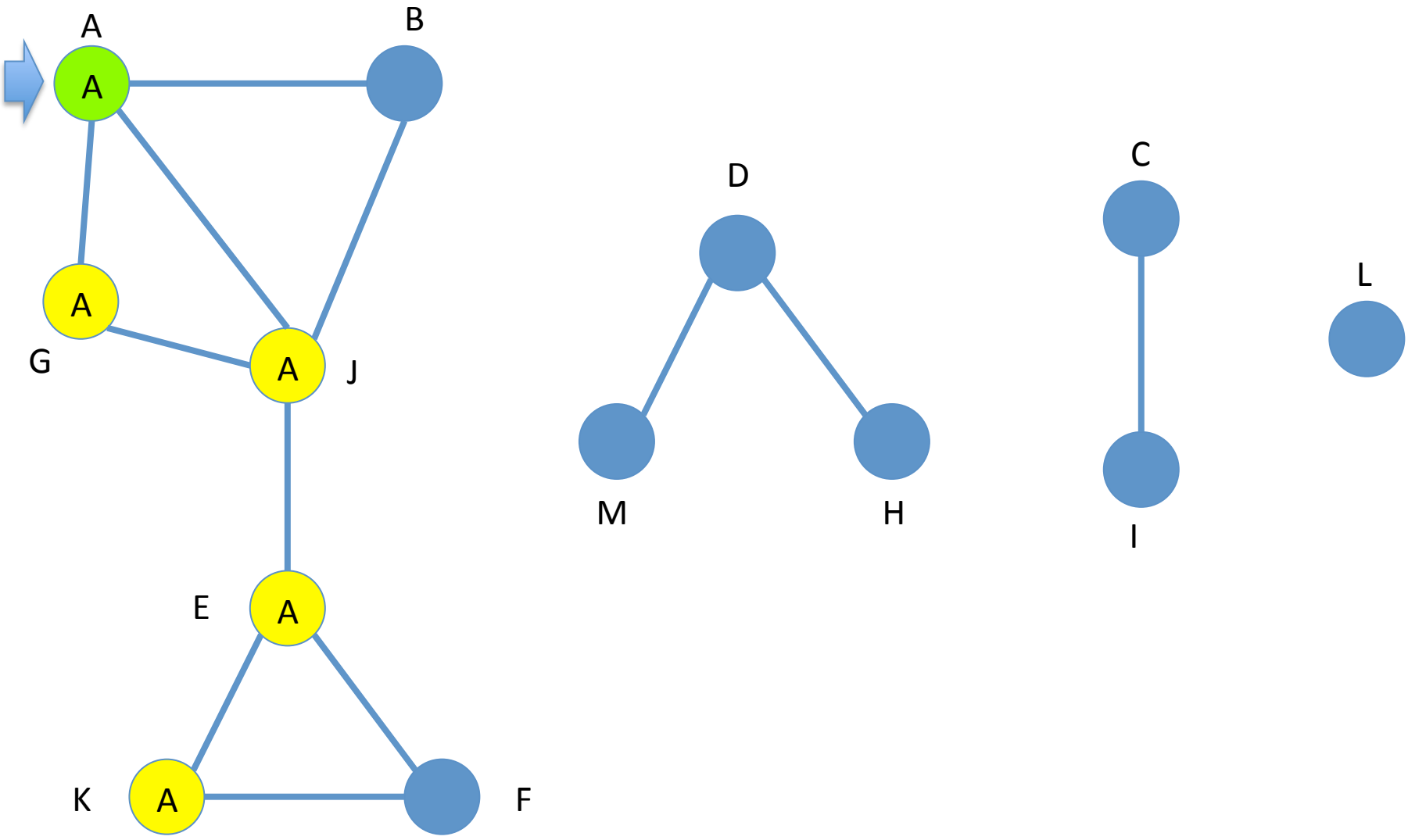

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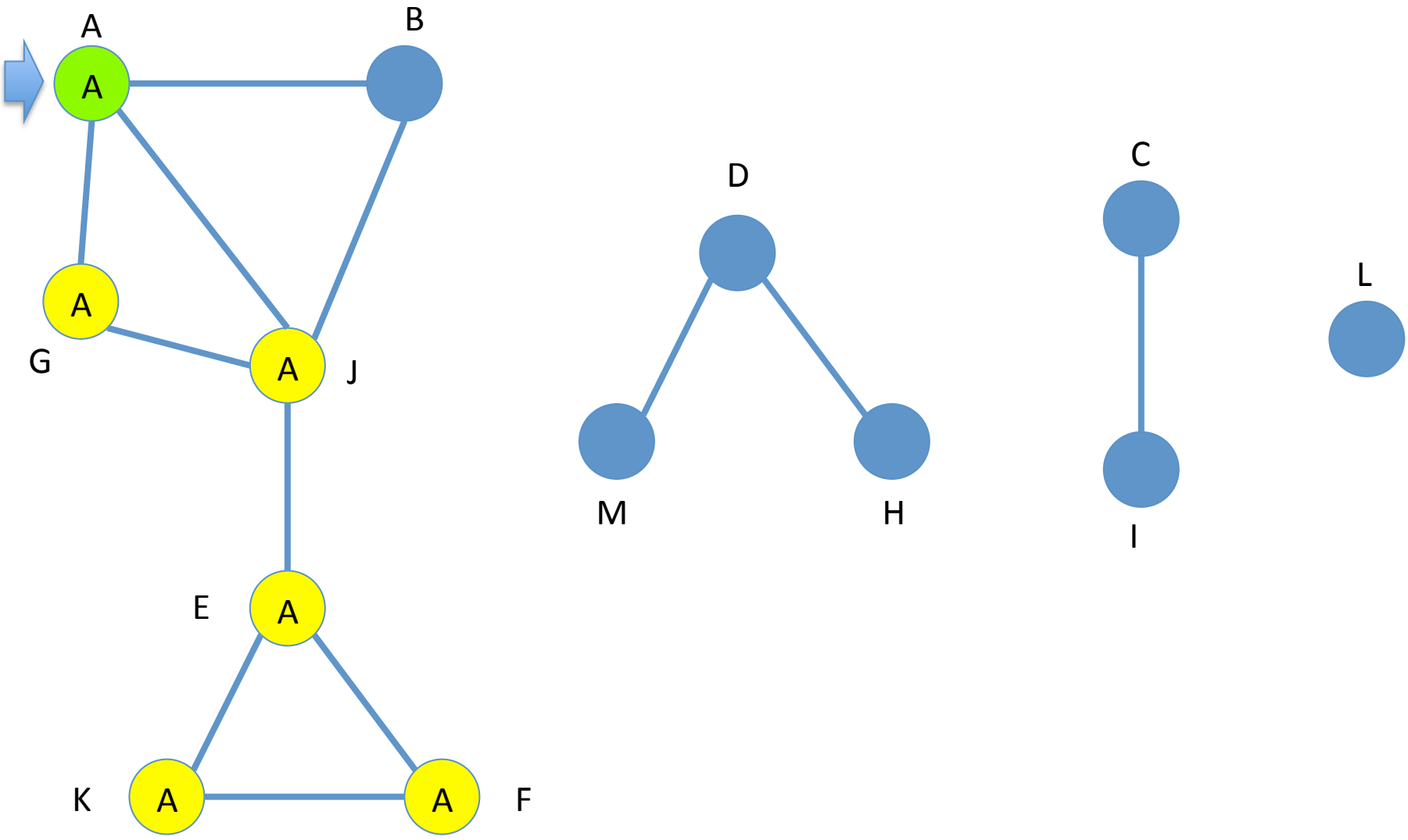

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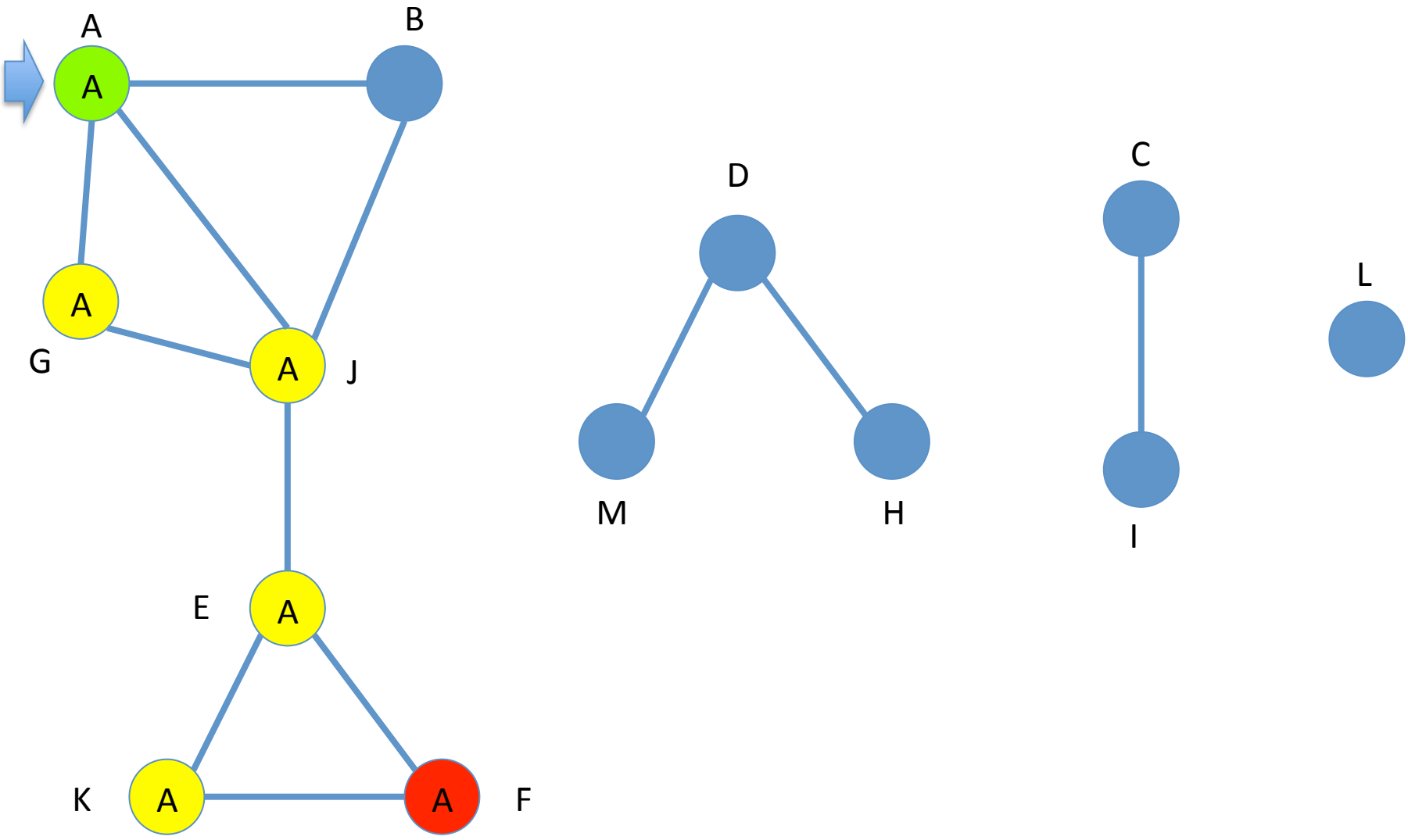

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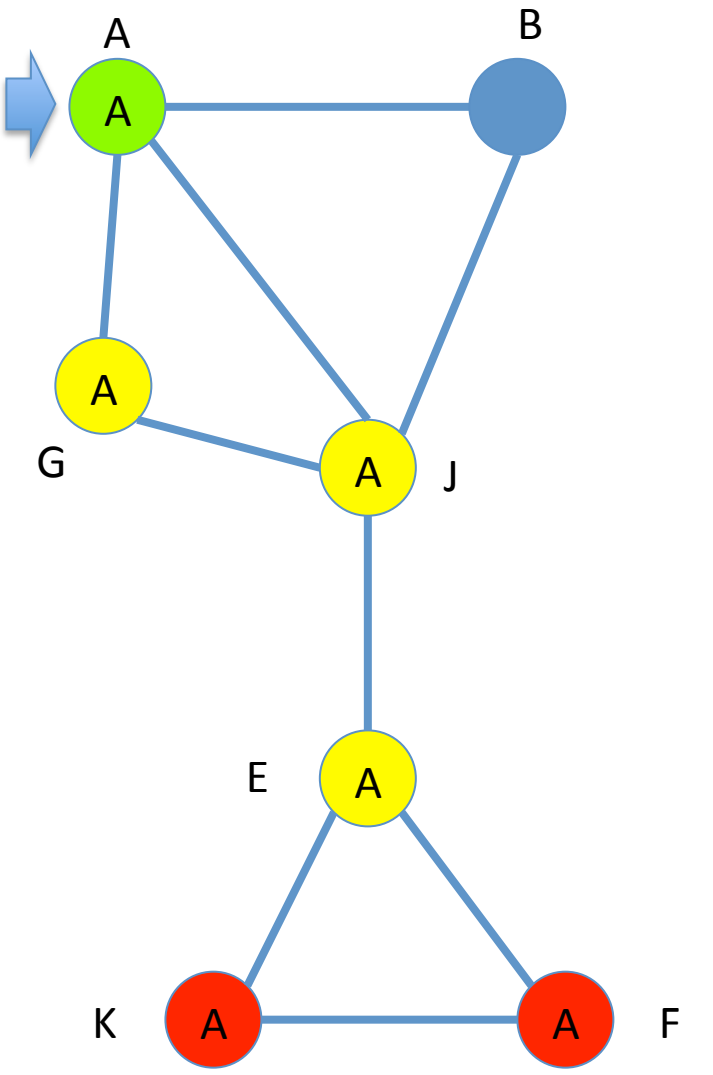

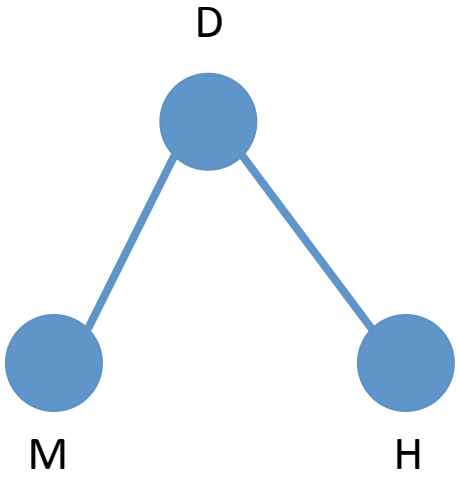

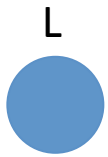

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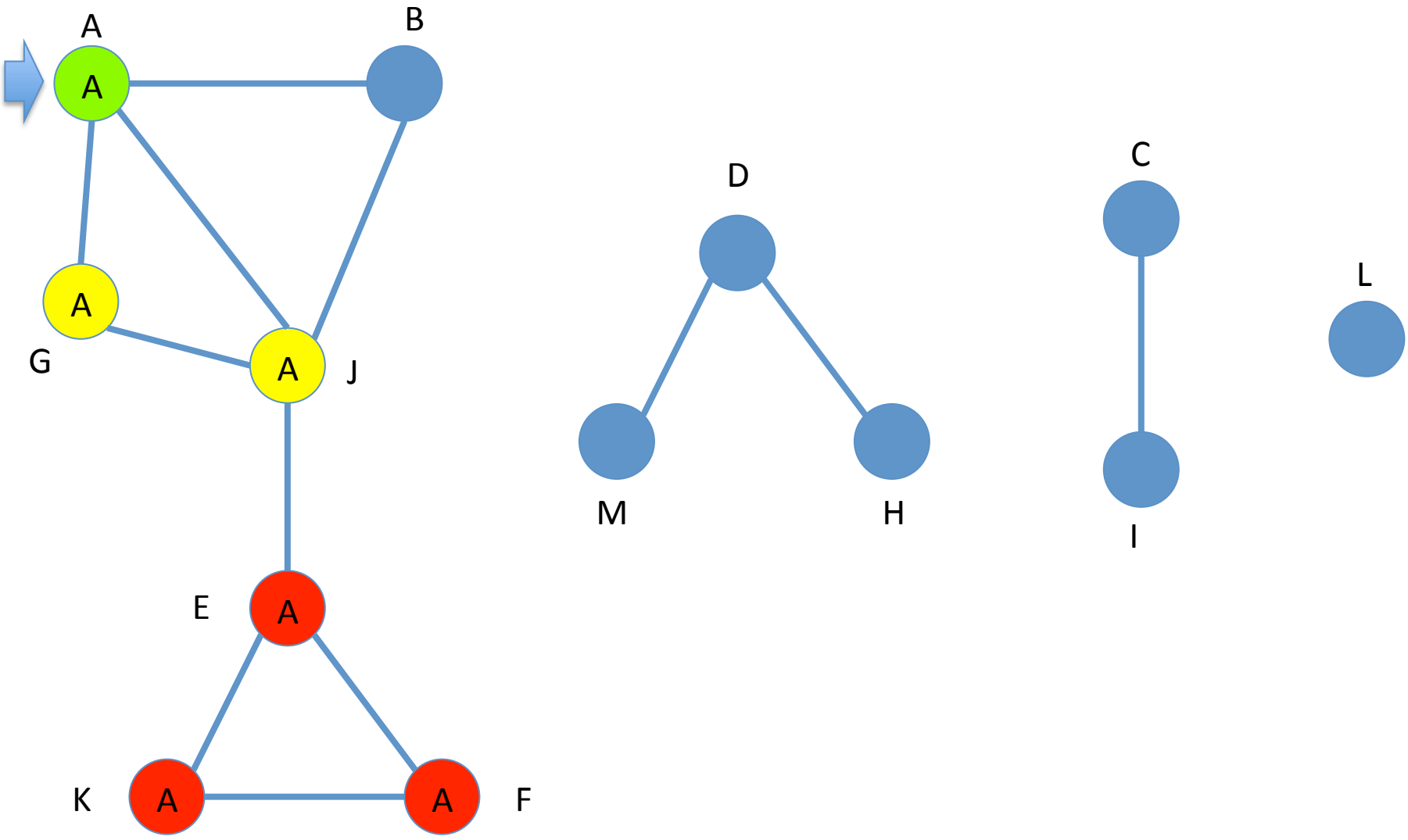

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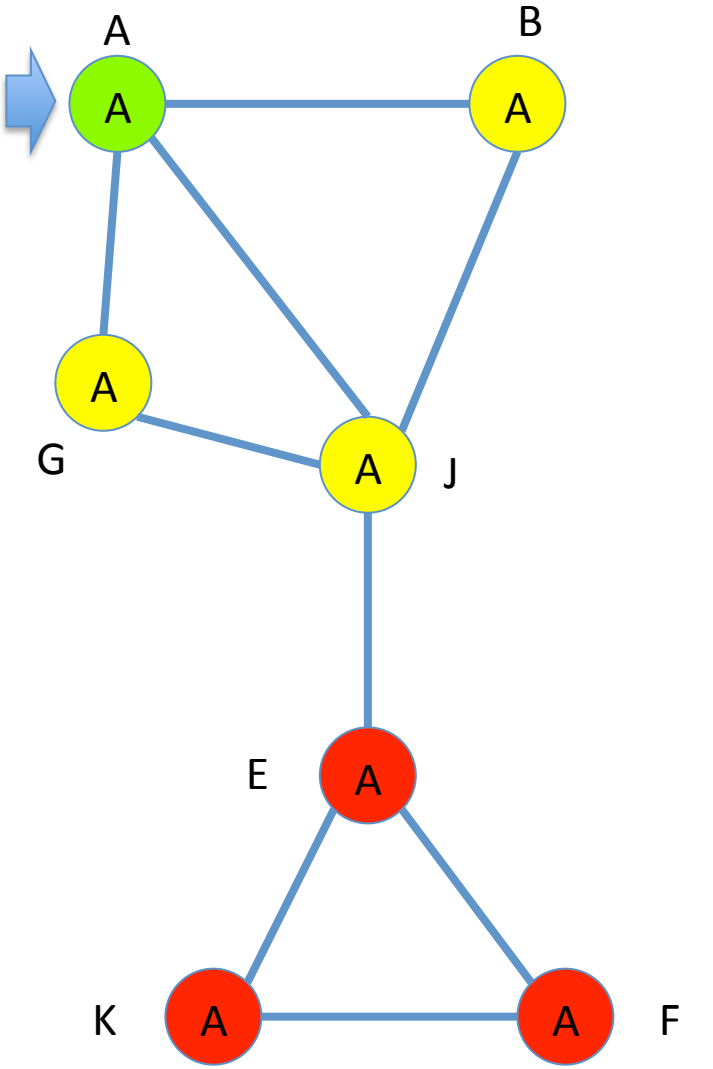

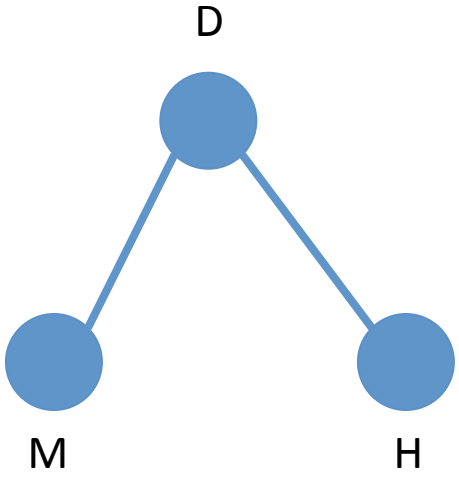

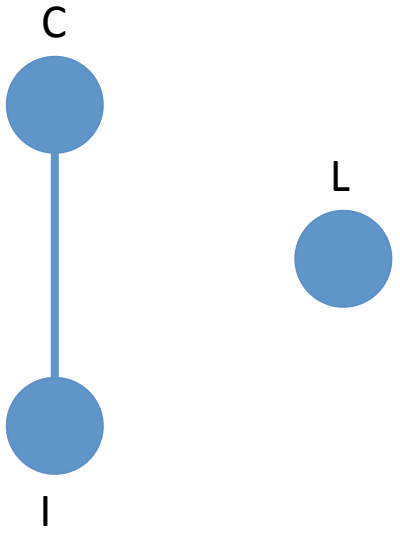

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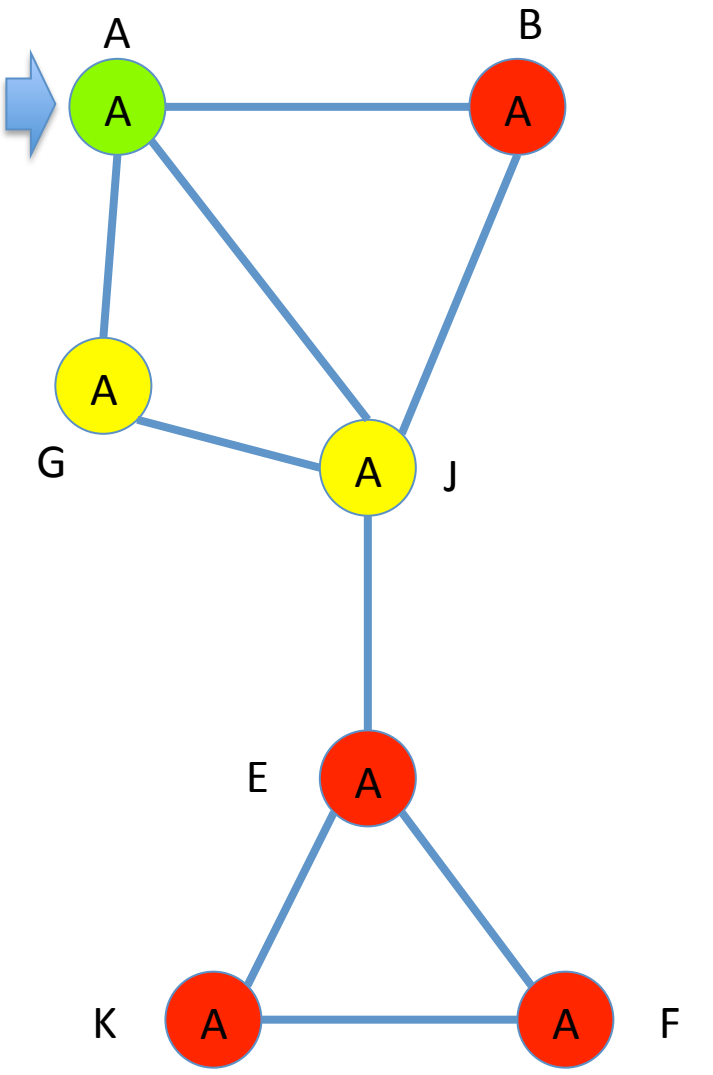

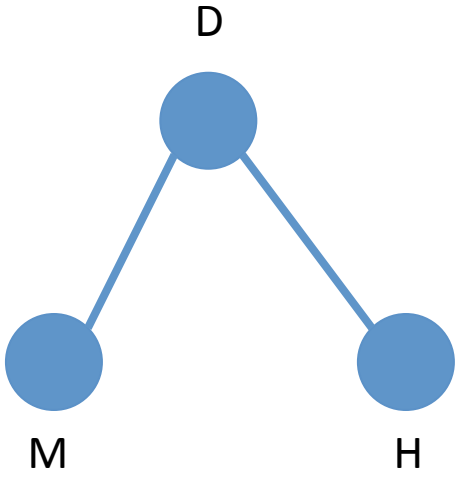

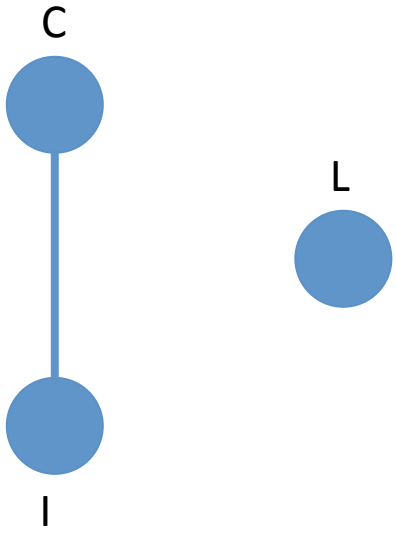

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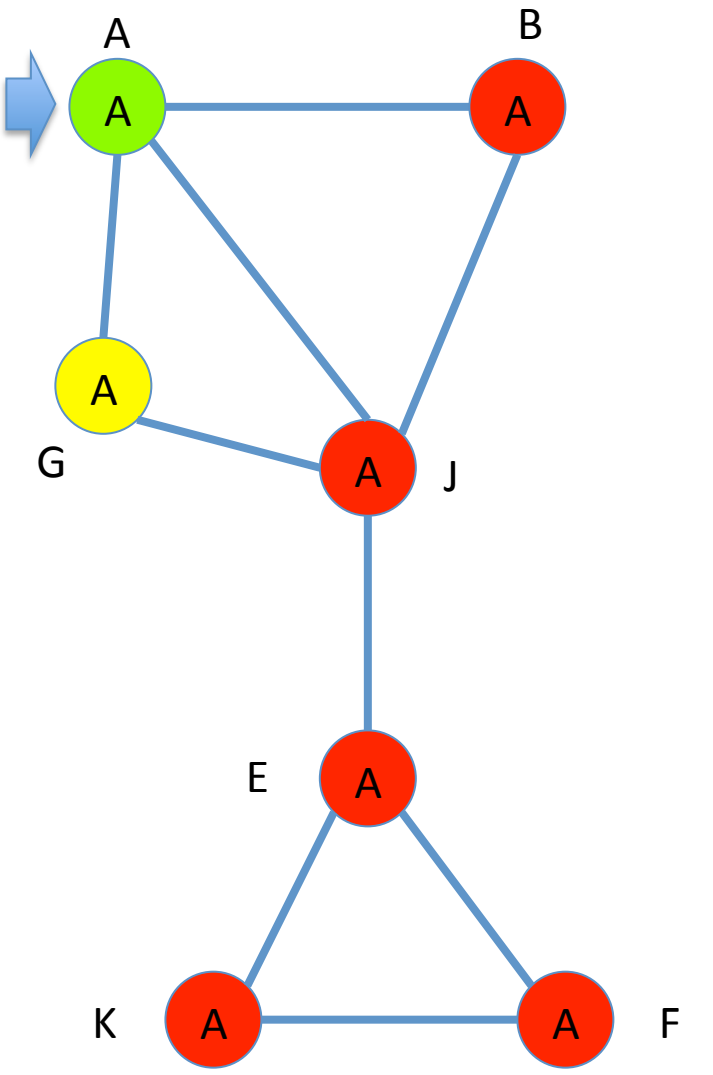

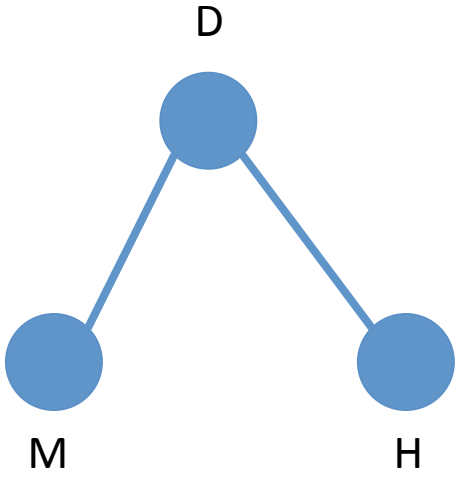

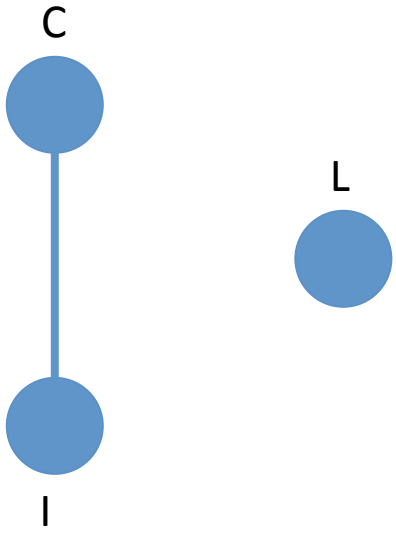

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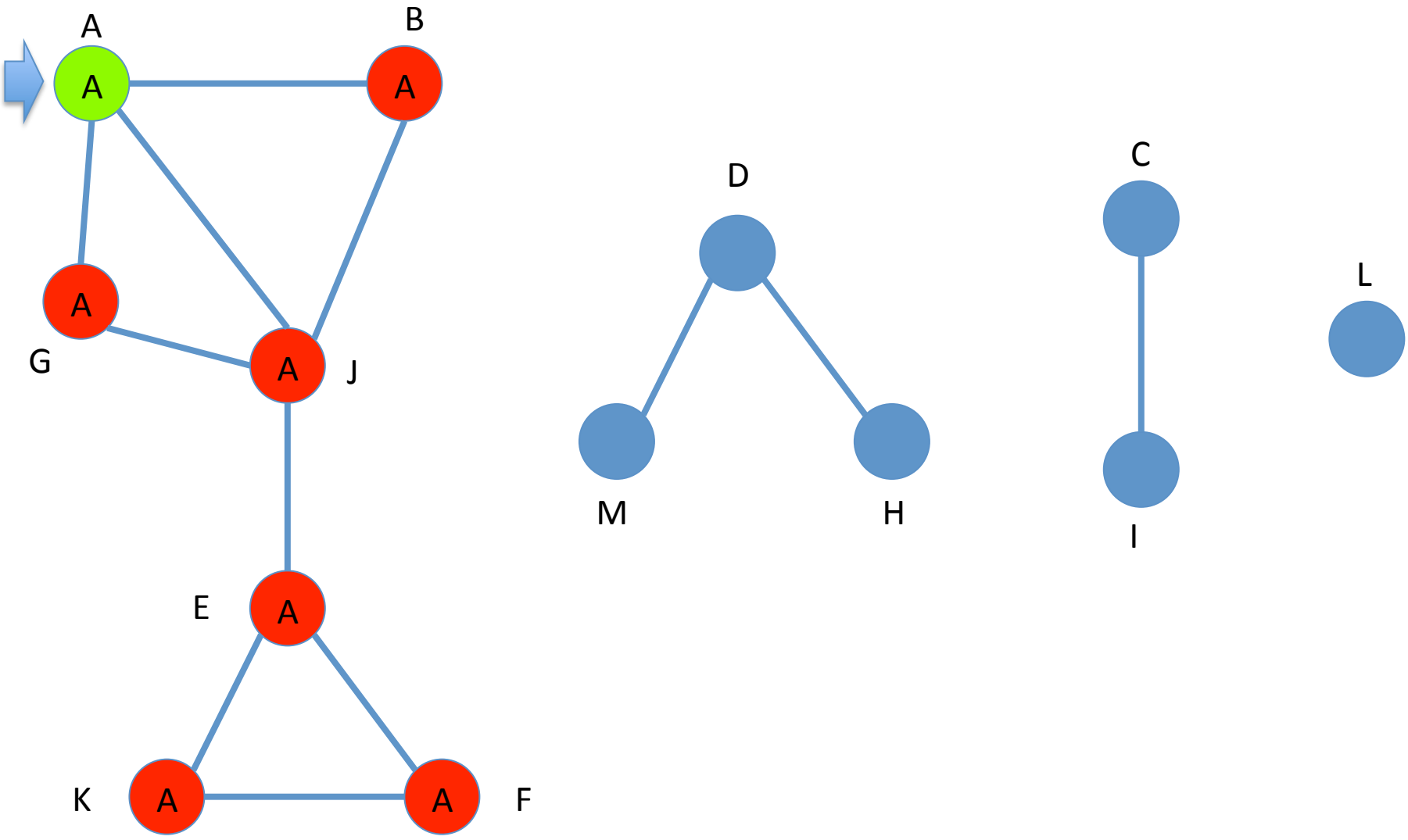

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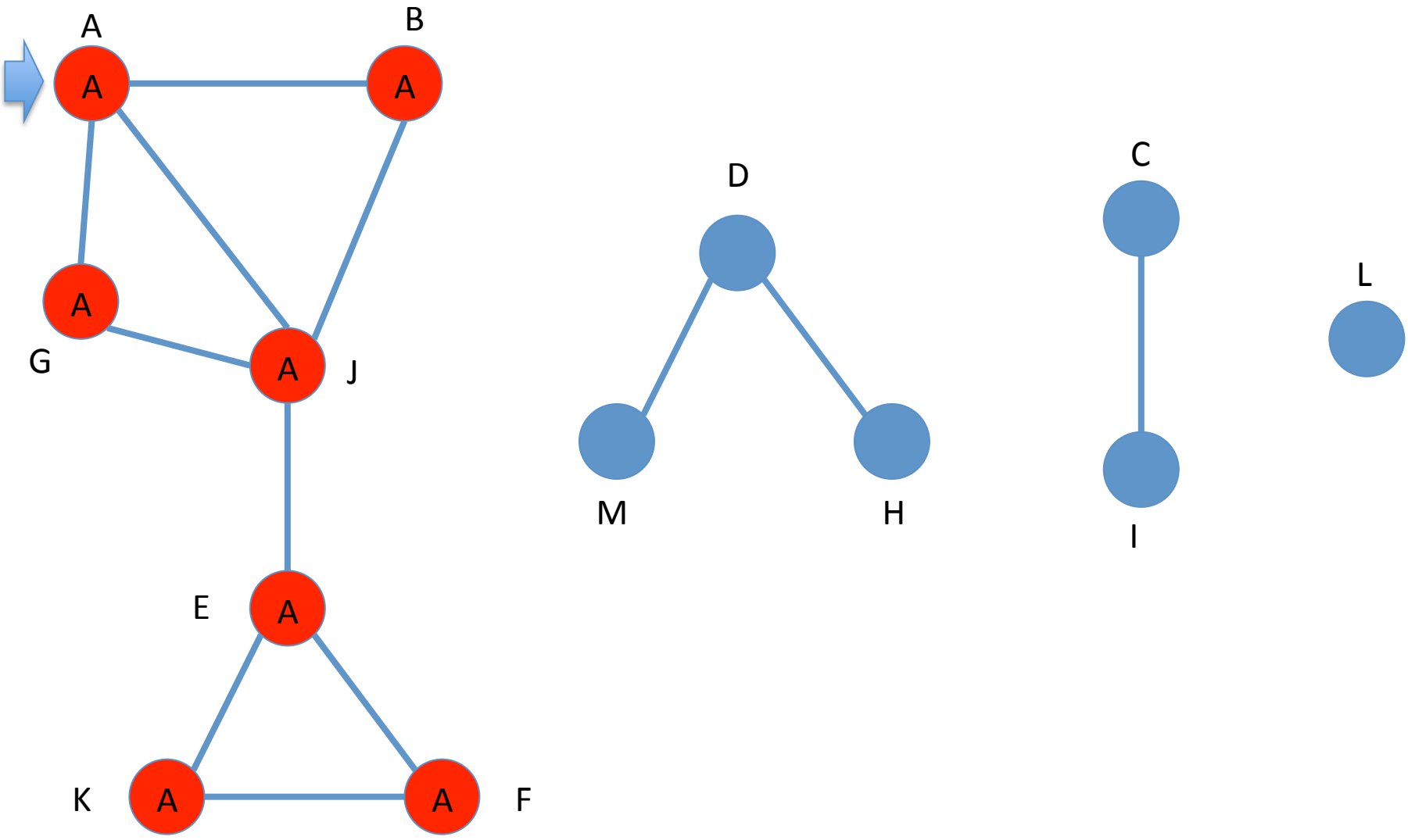

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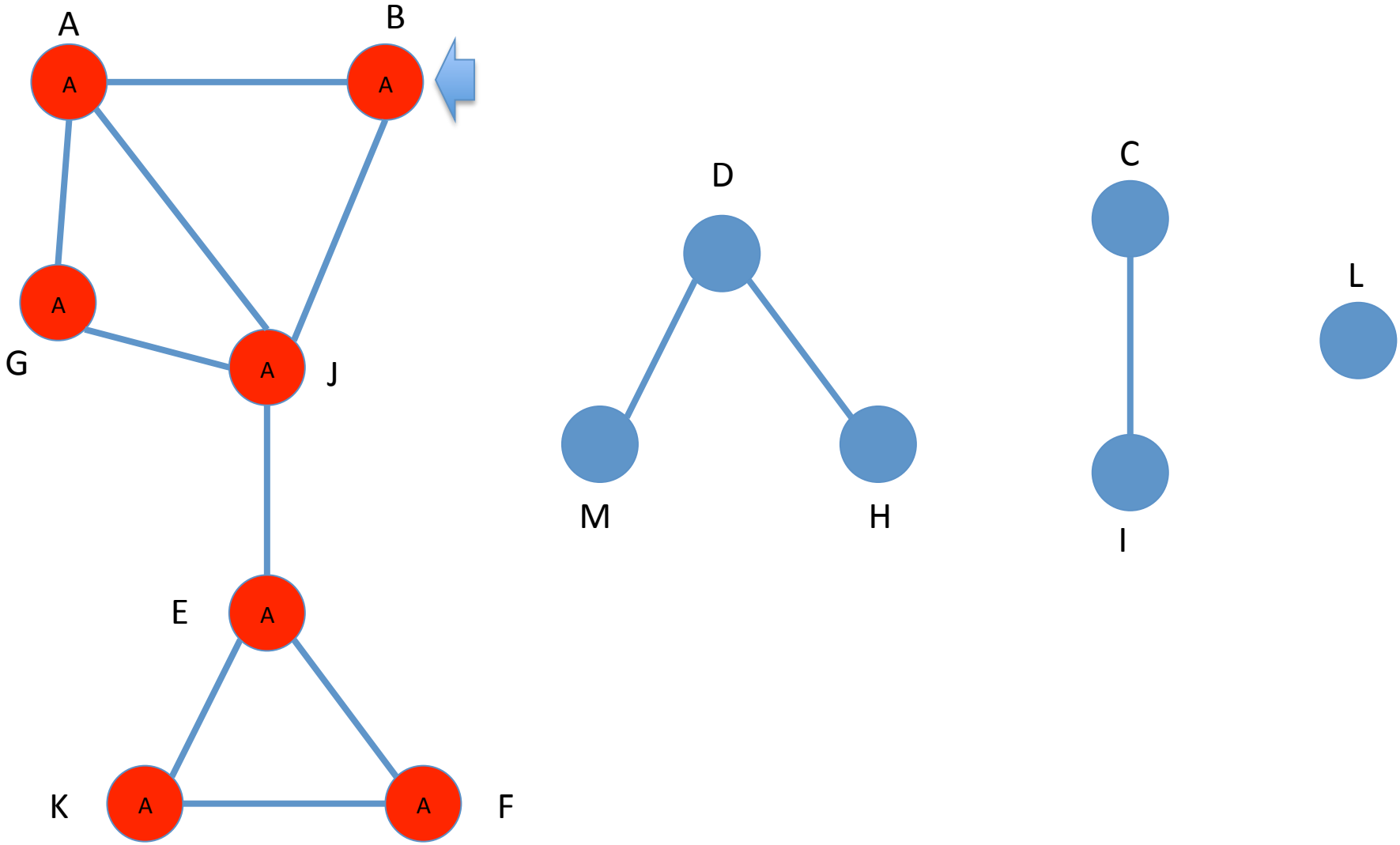

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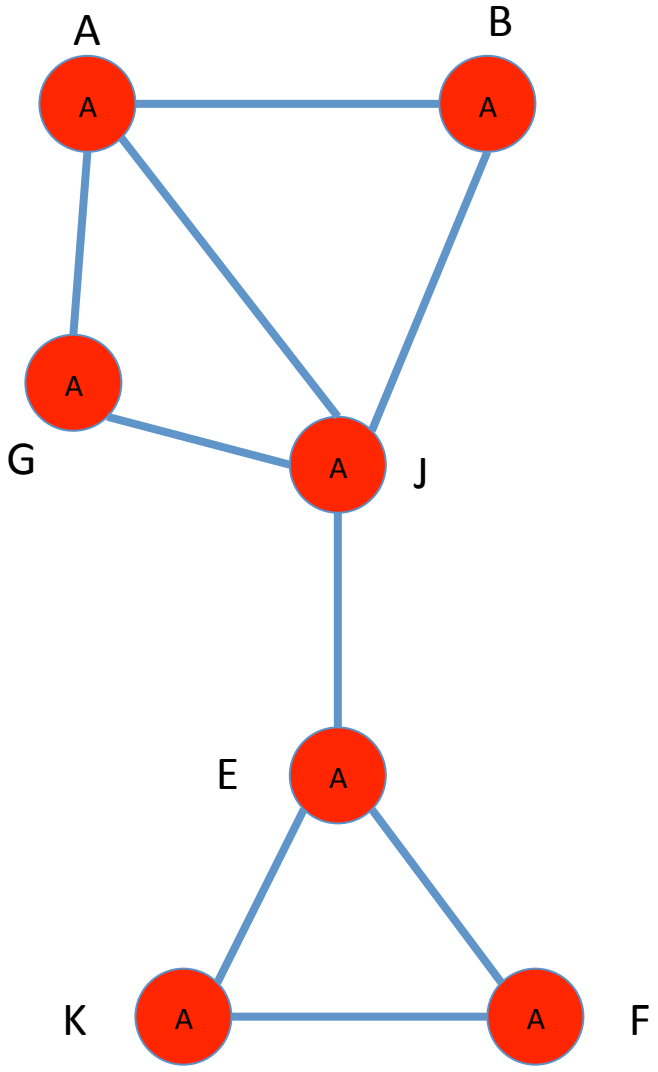

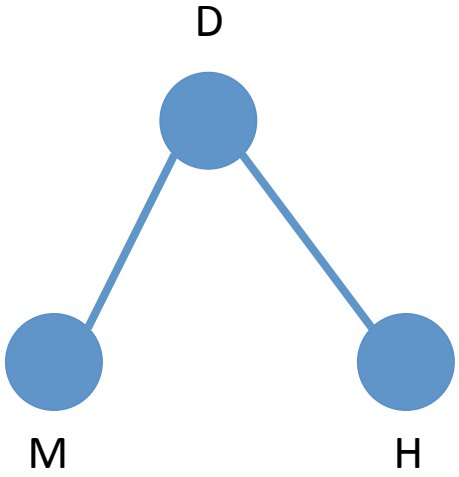

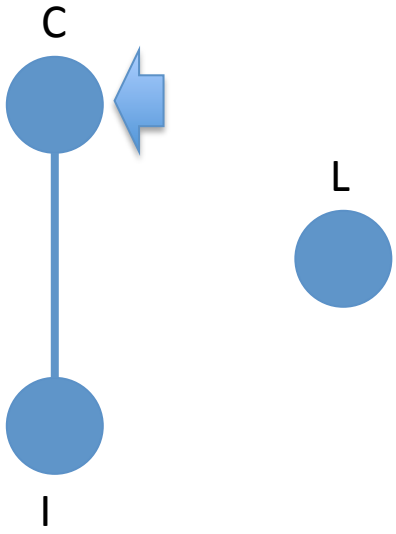

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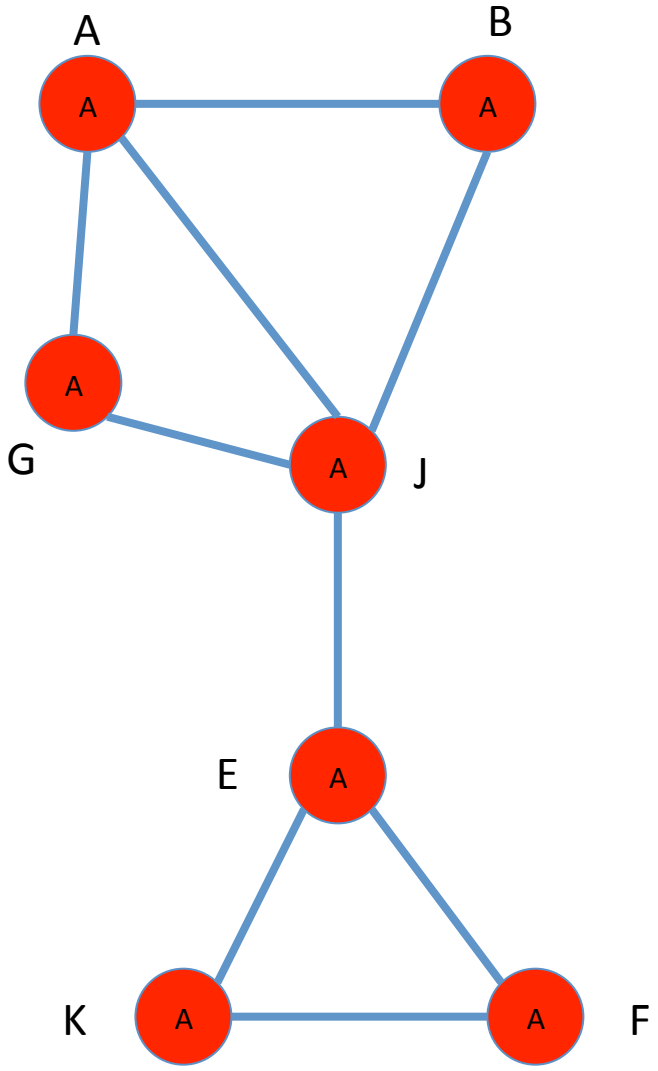

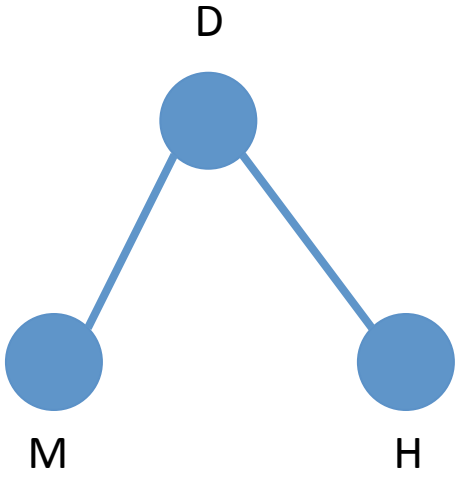

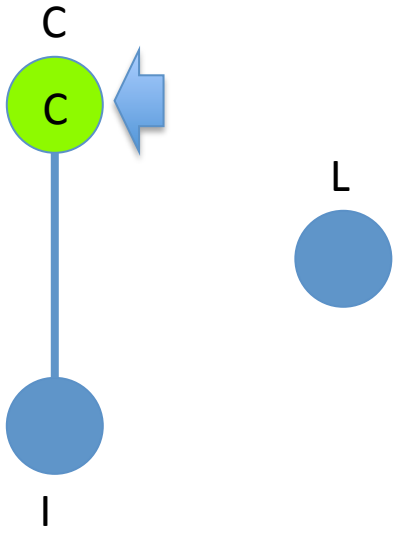

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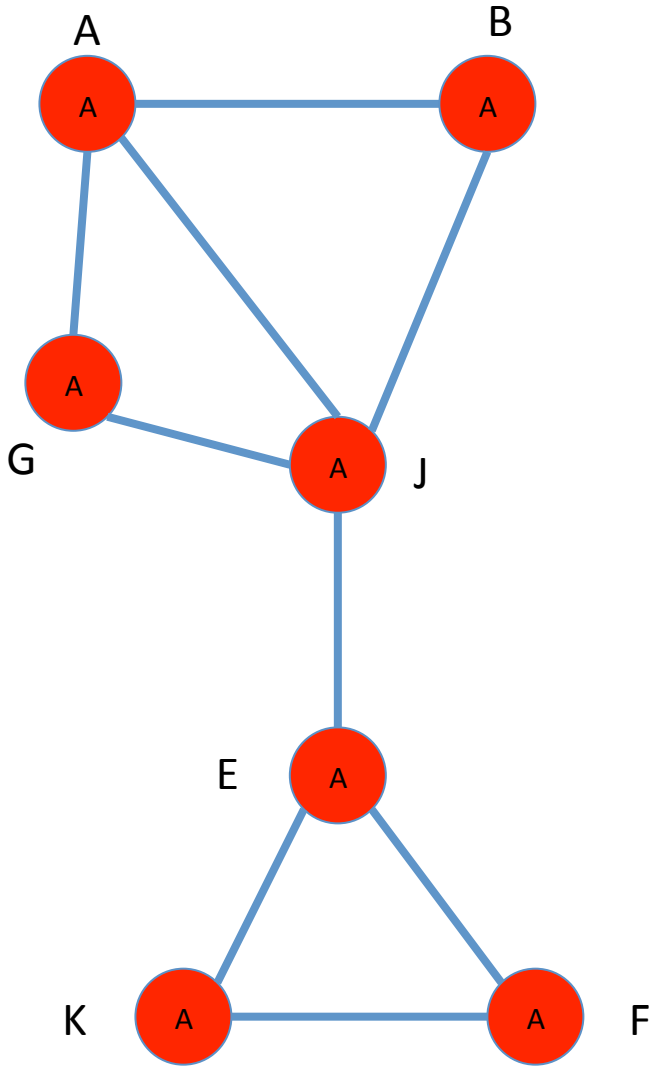

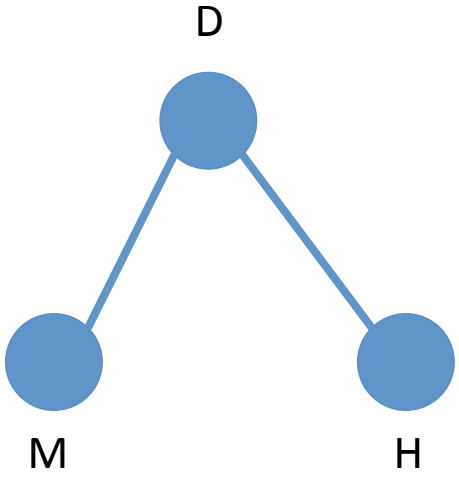

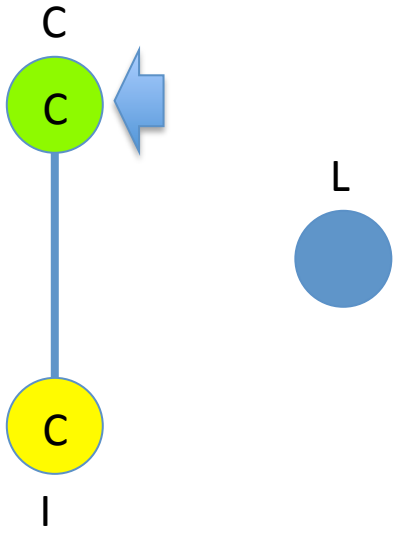

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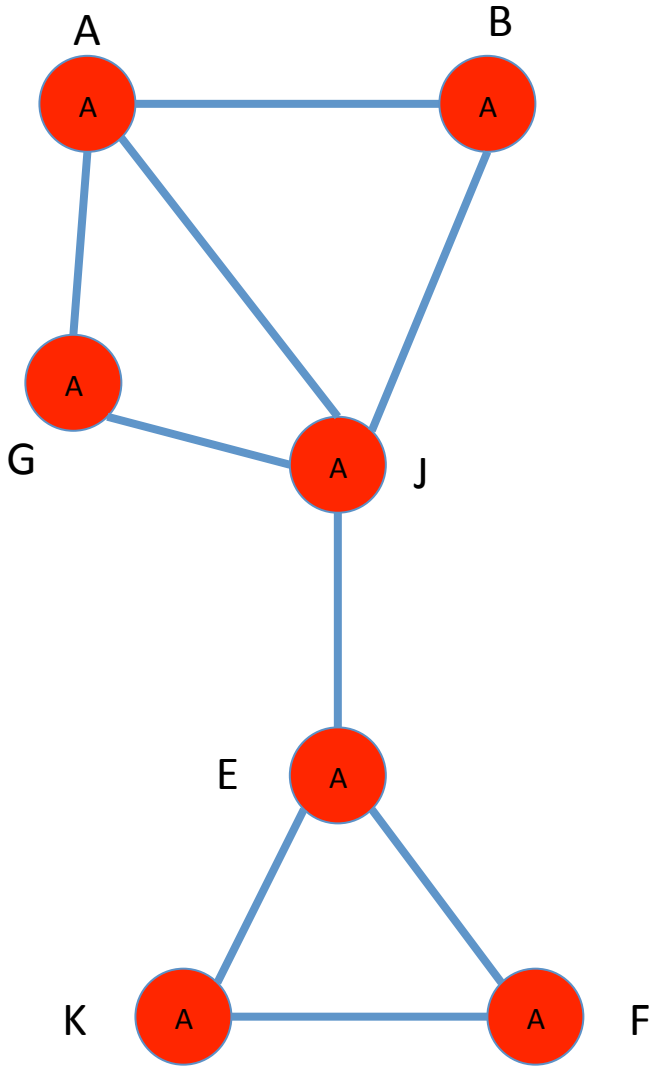

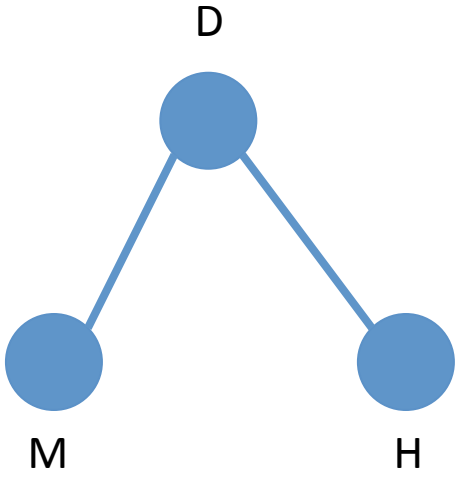

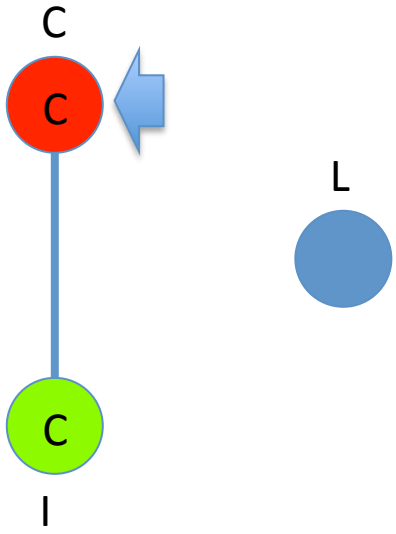

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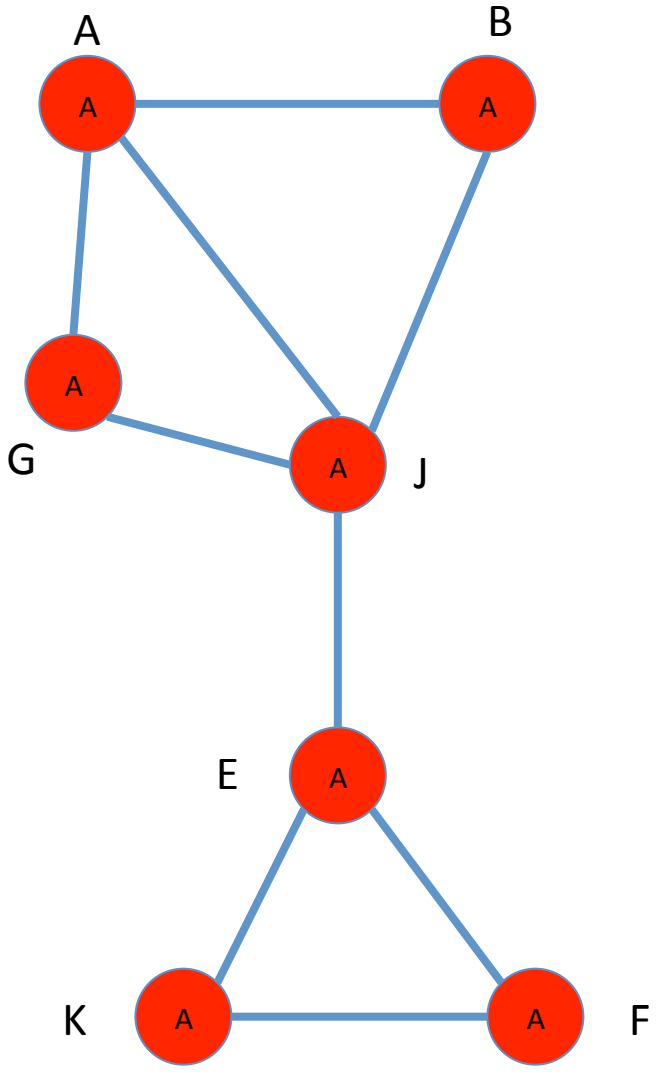
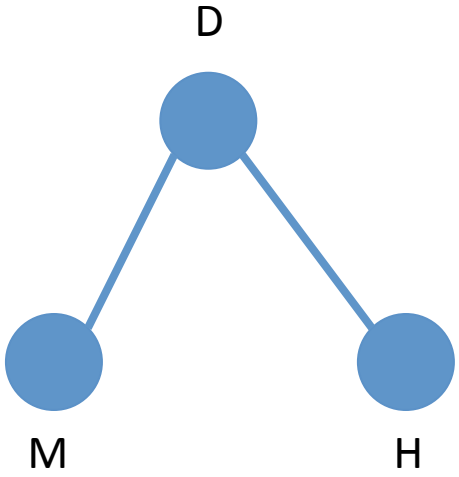
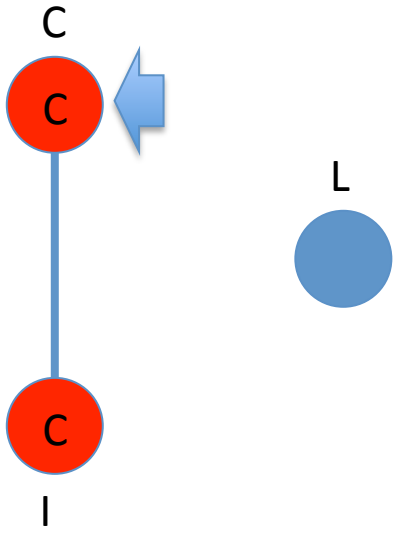
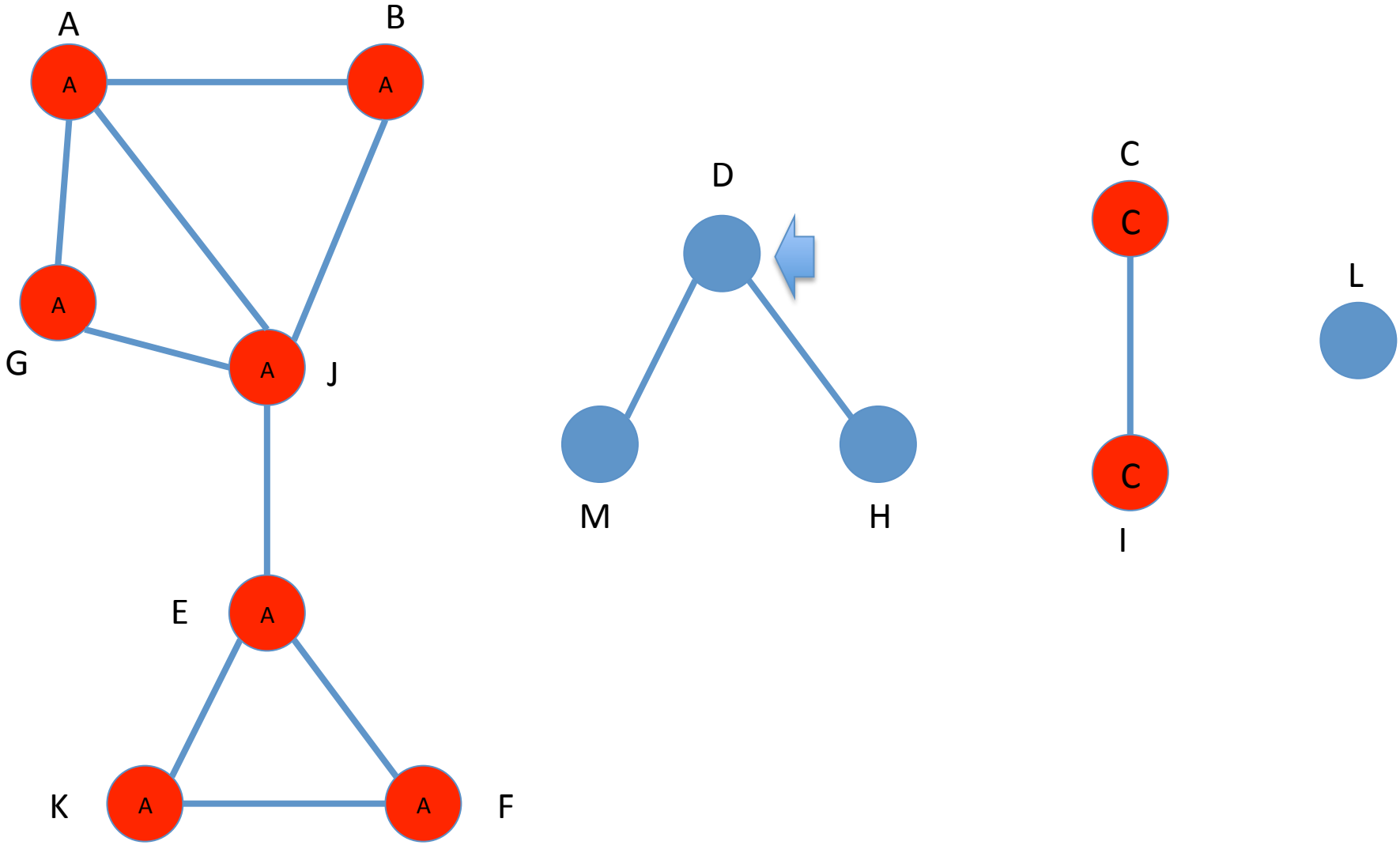






Recall Algorithm for Undirected Graphs



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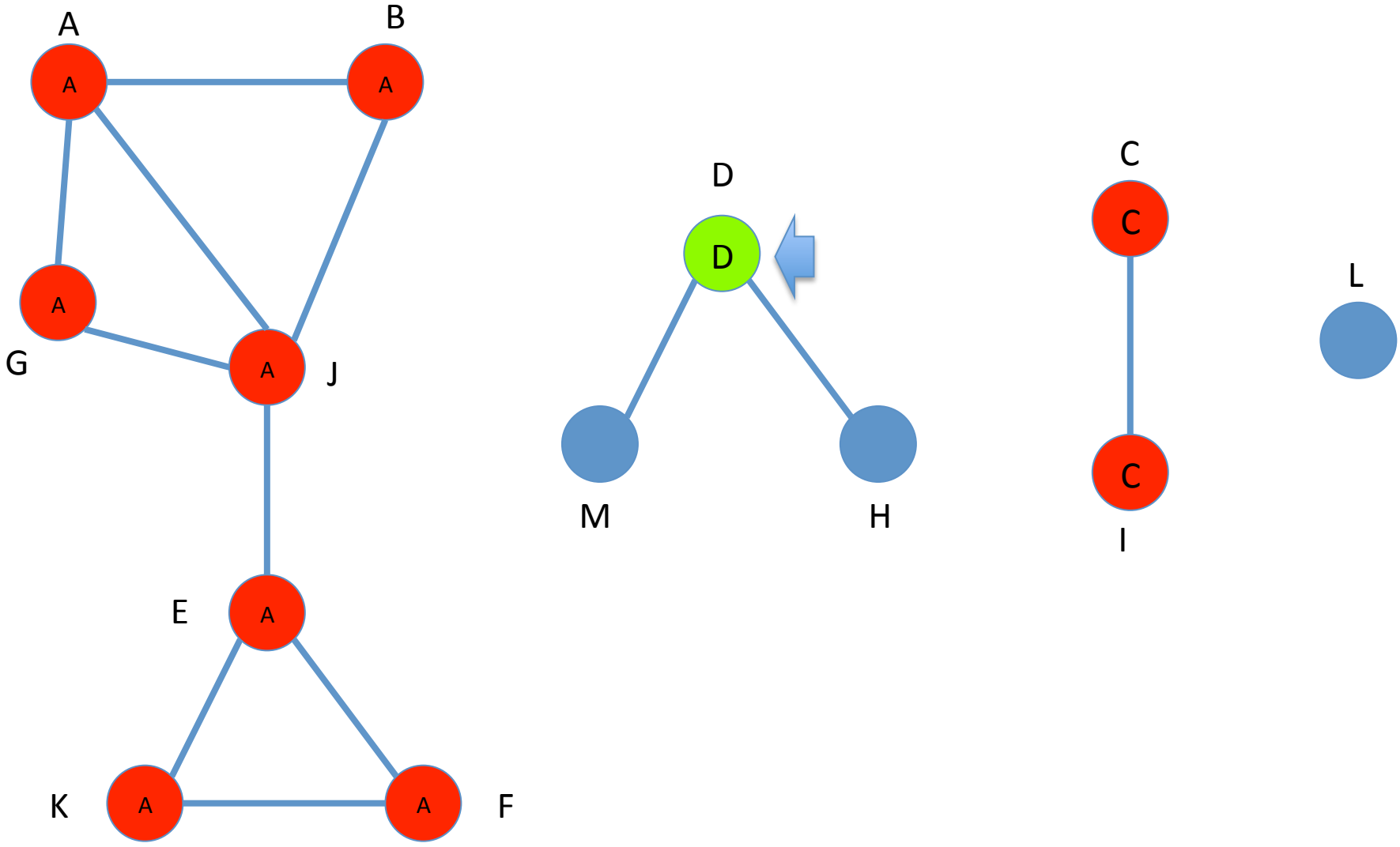

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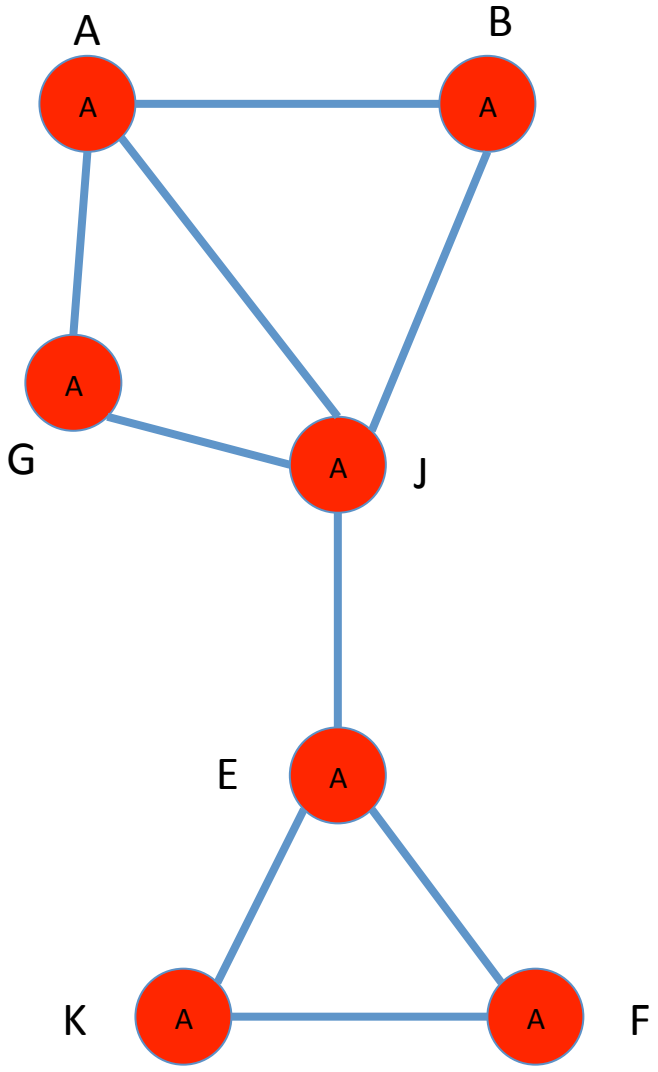

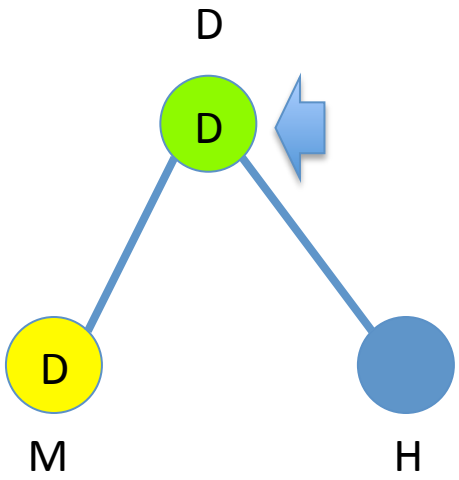

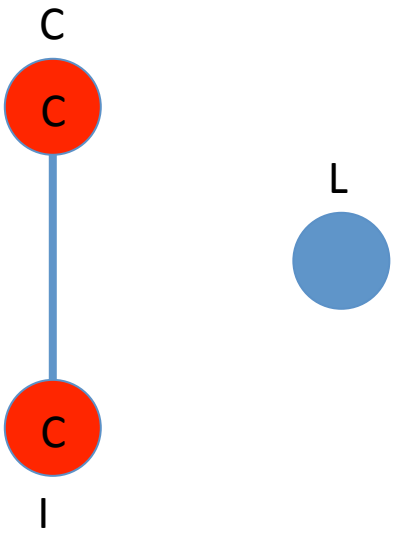

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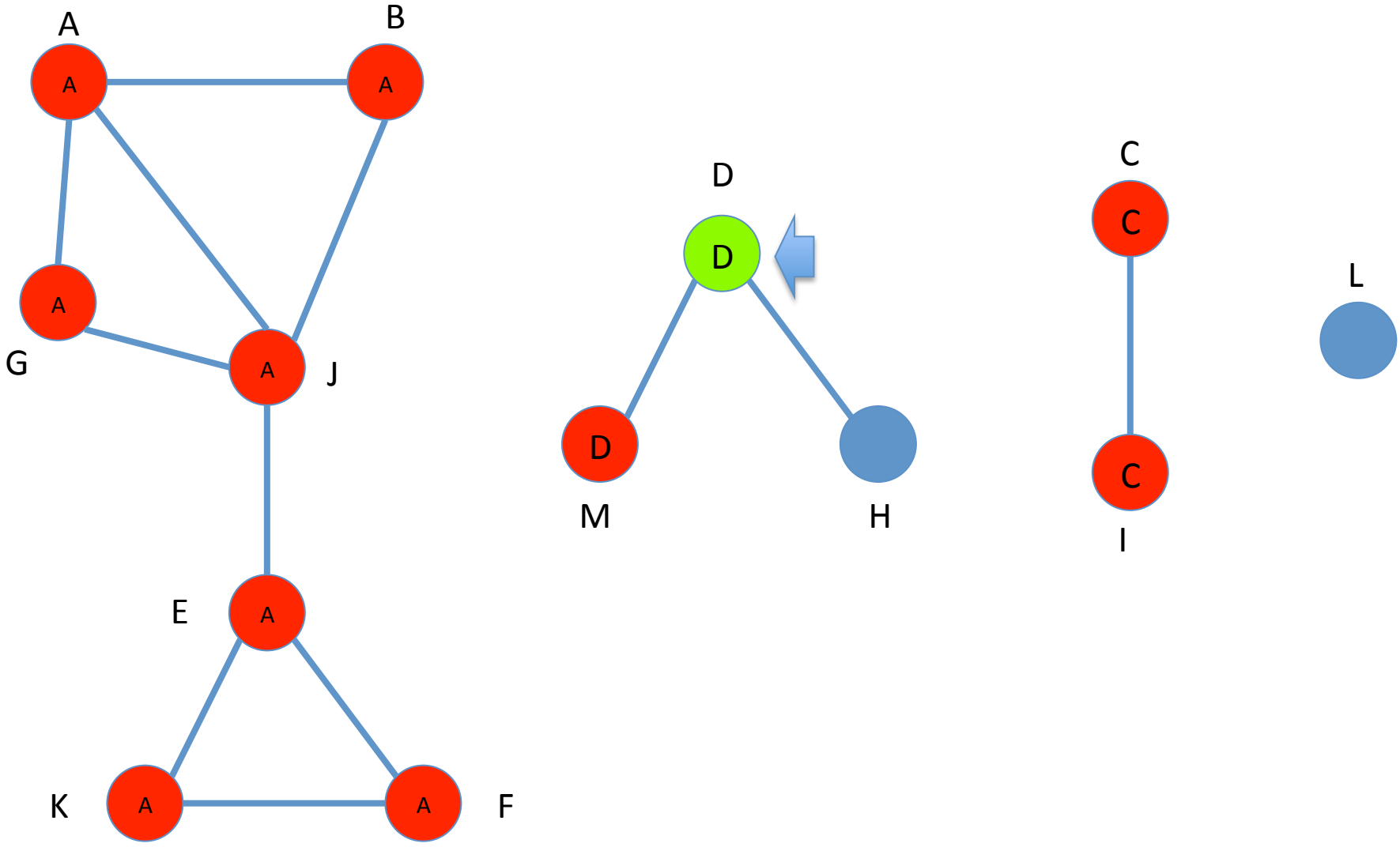

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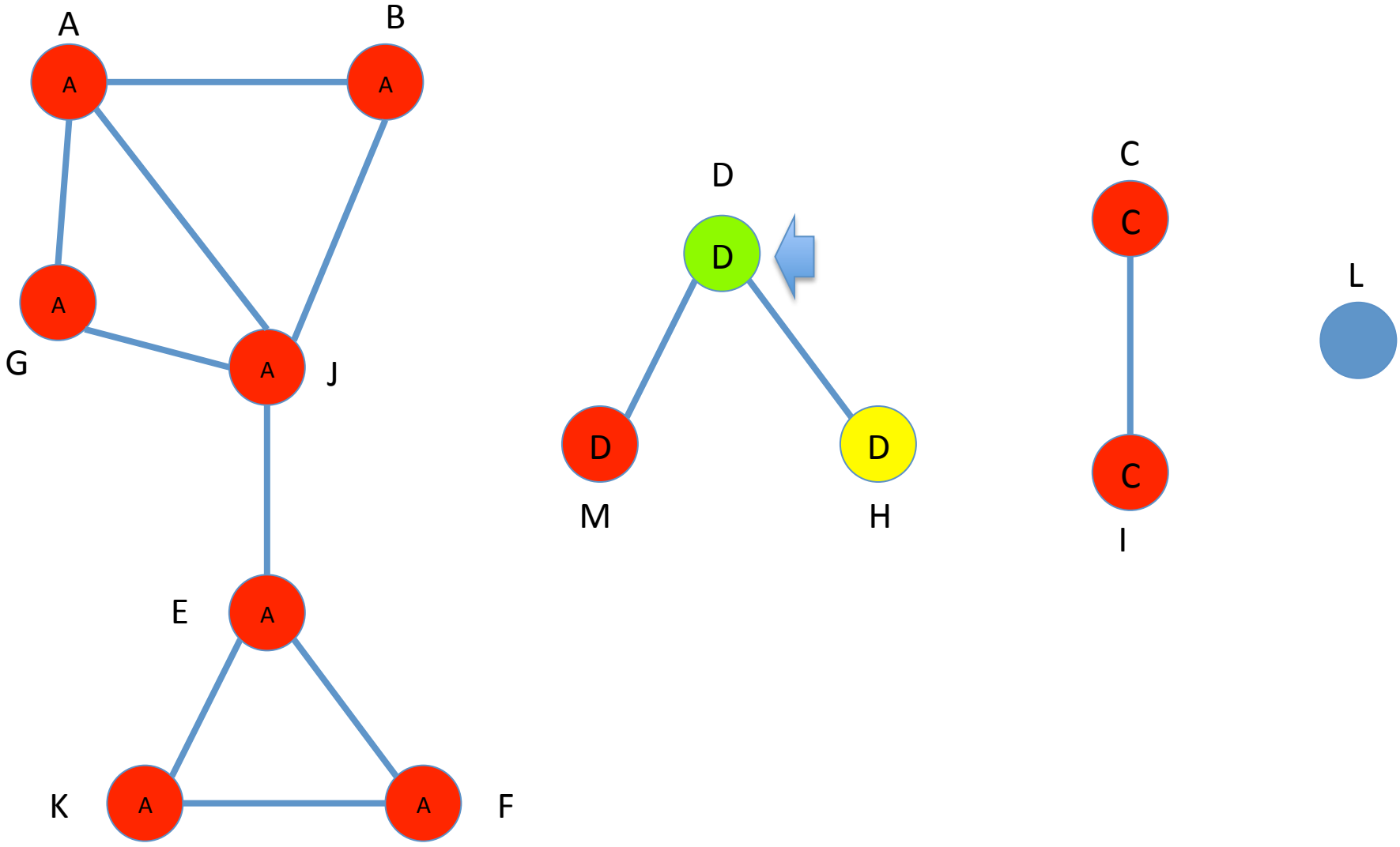

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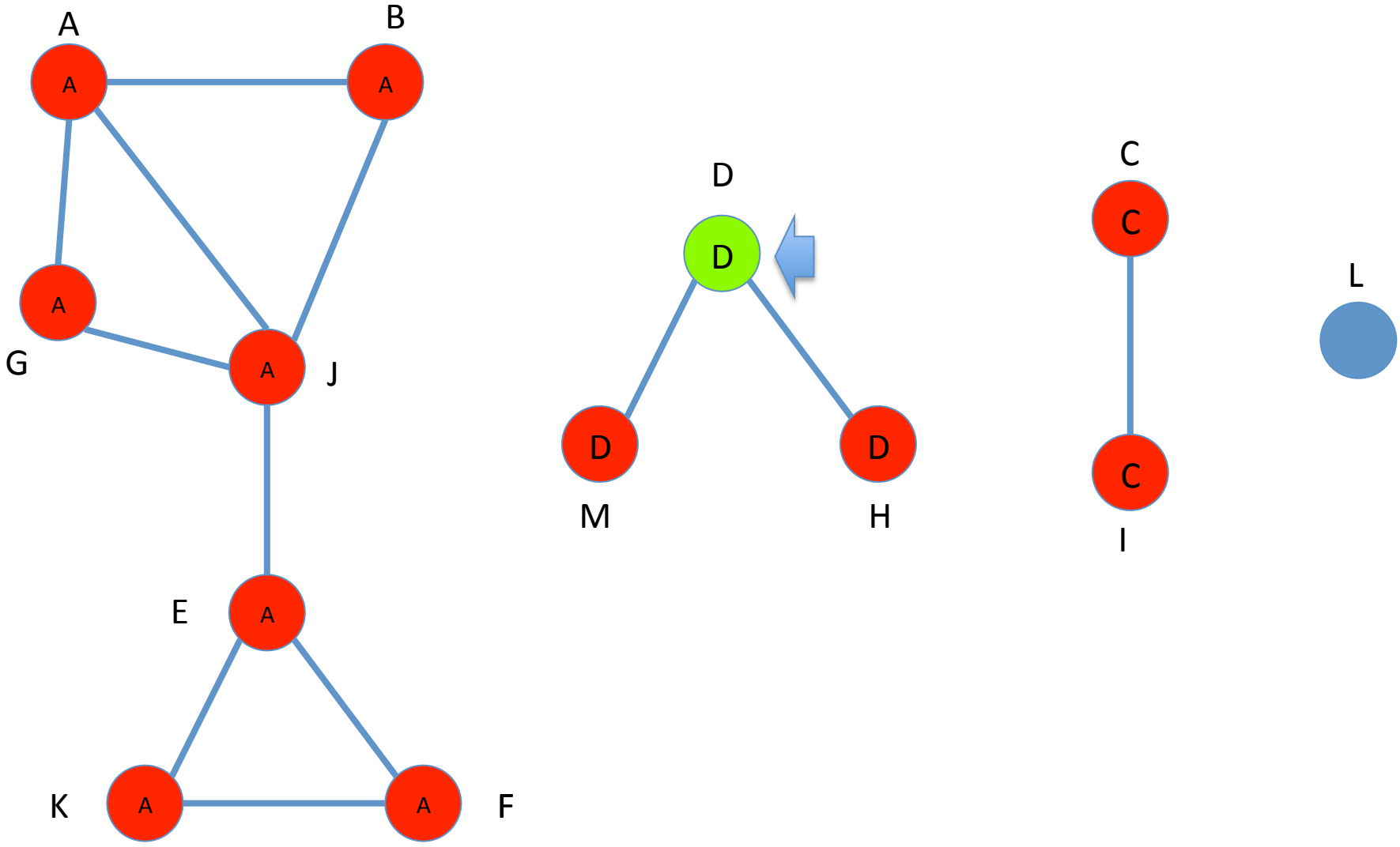

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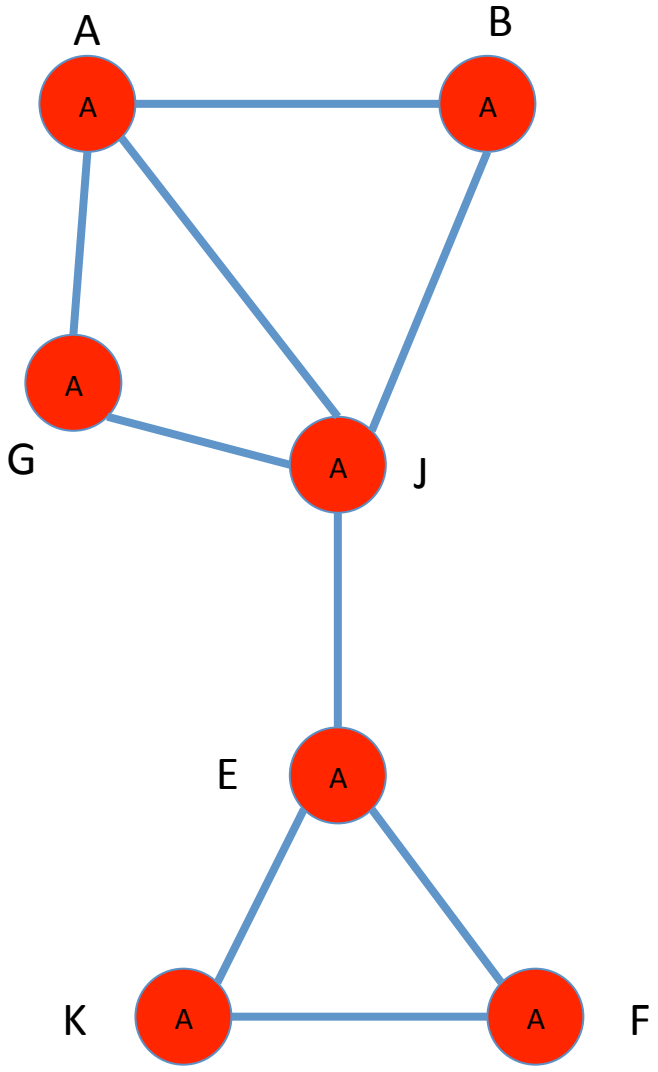

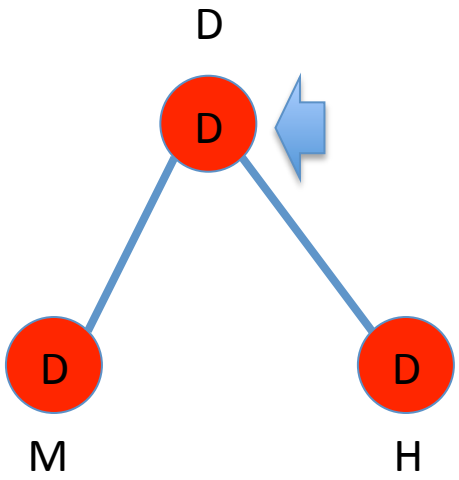

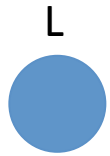

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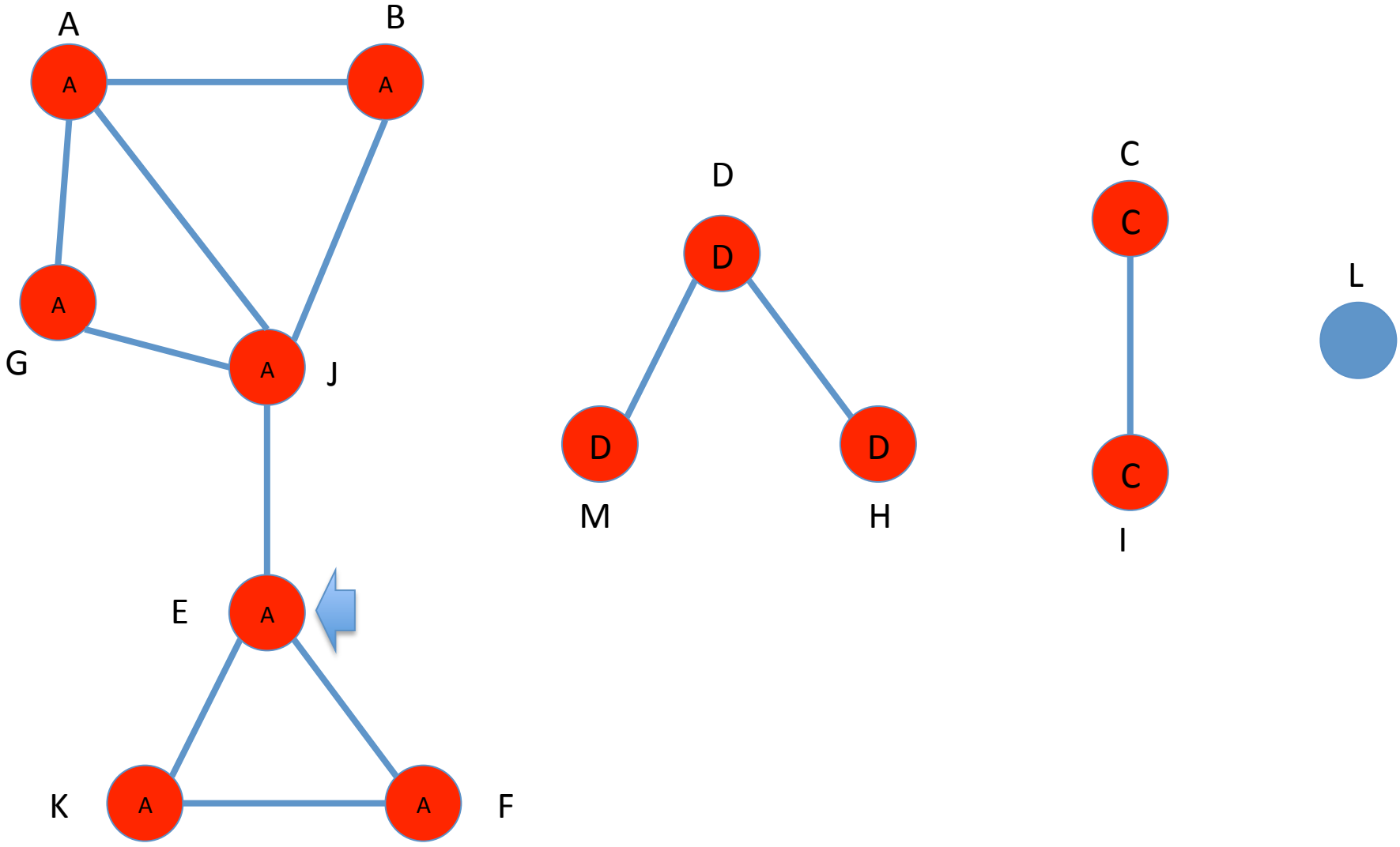

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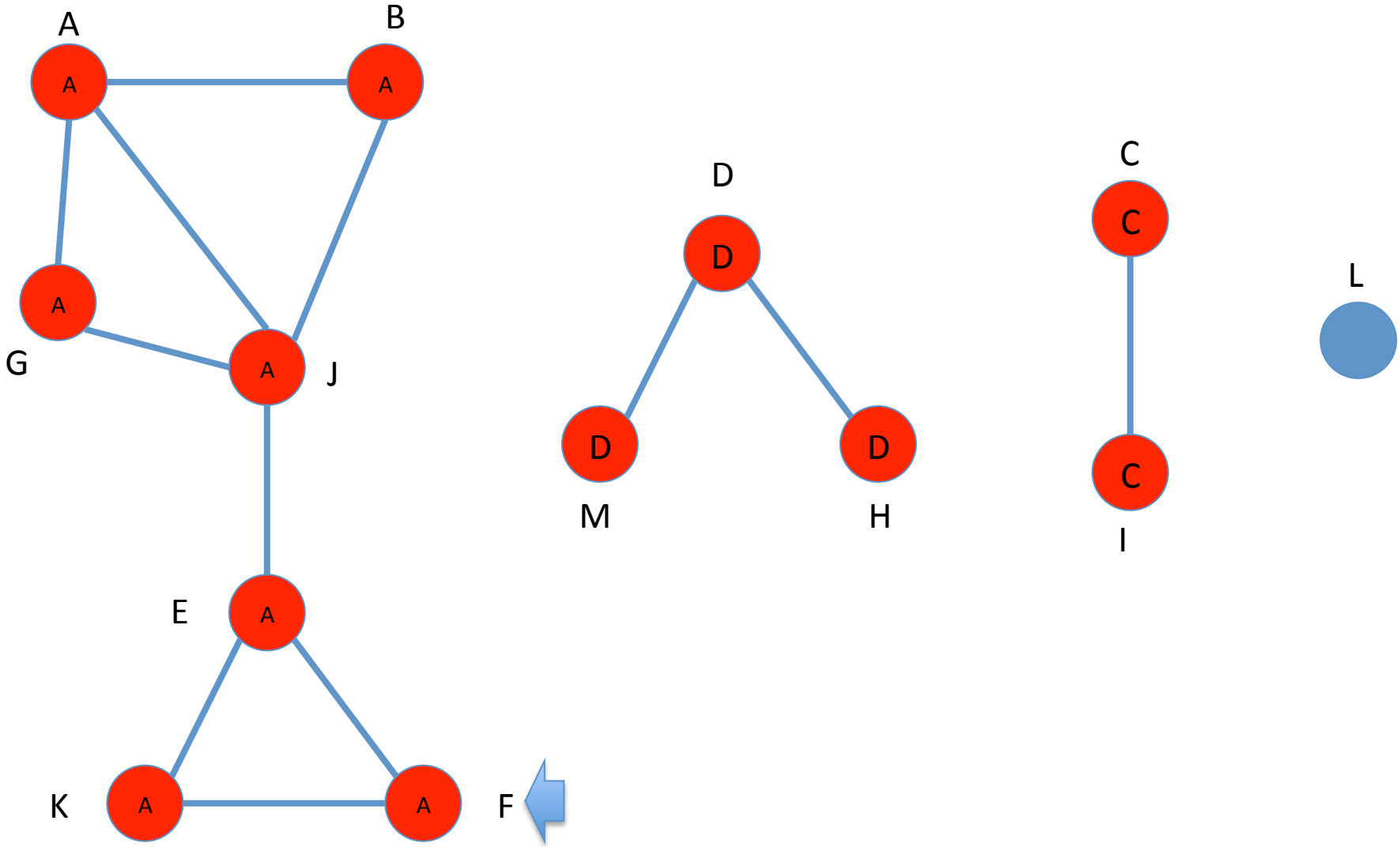

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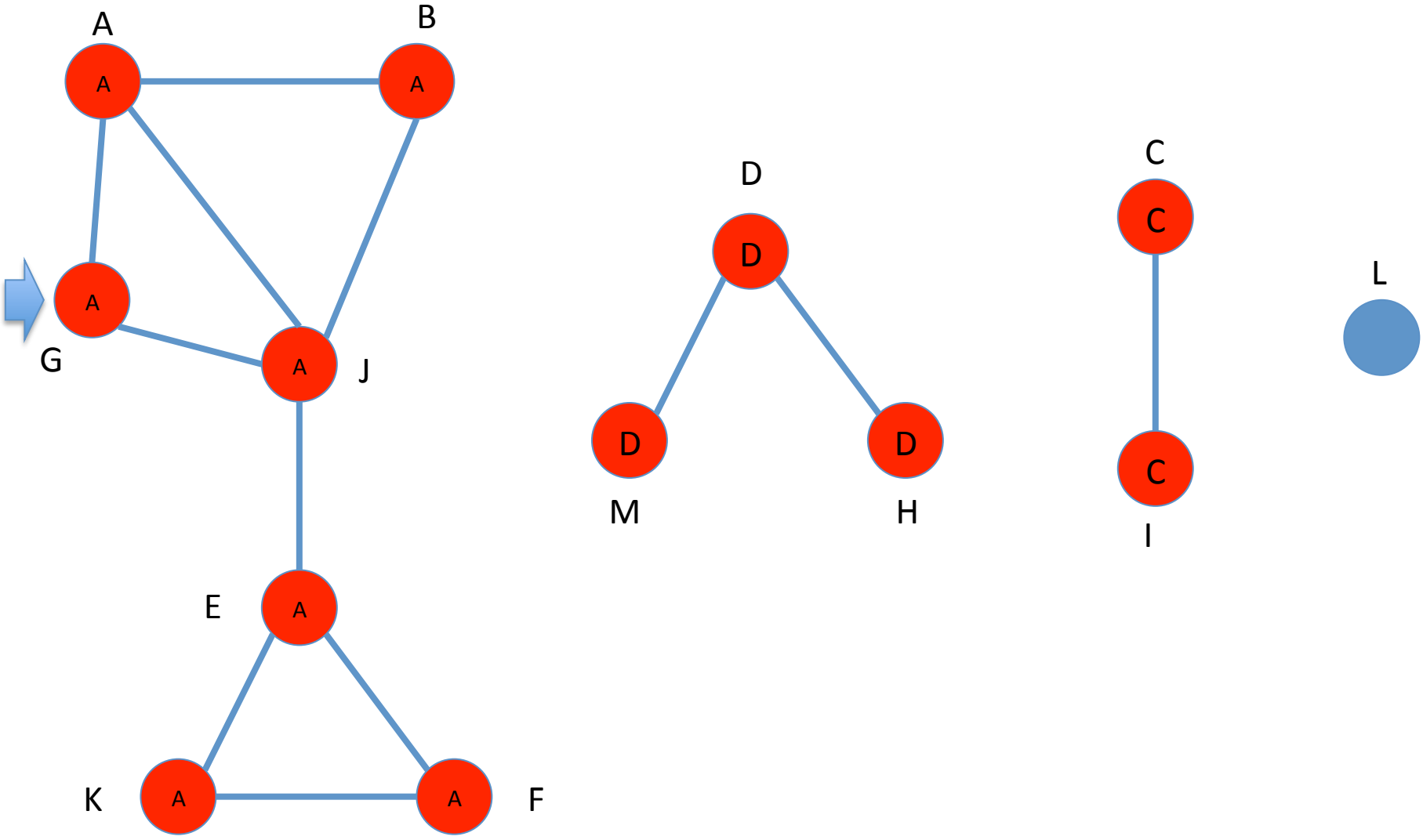

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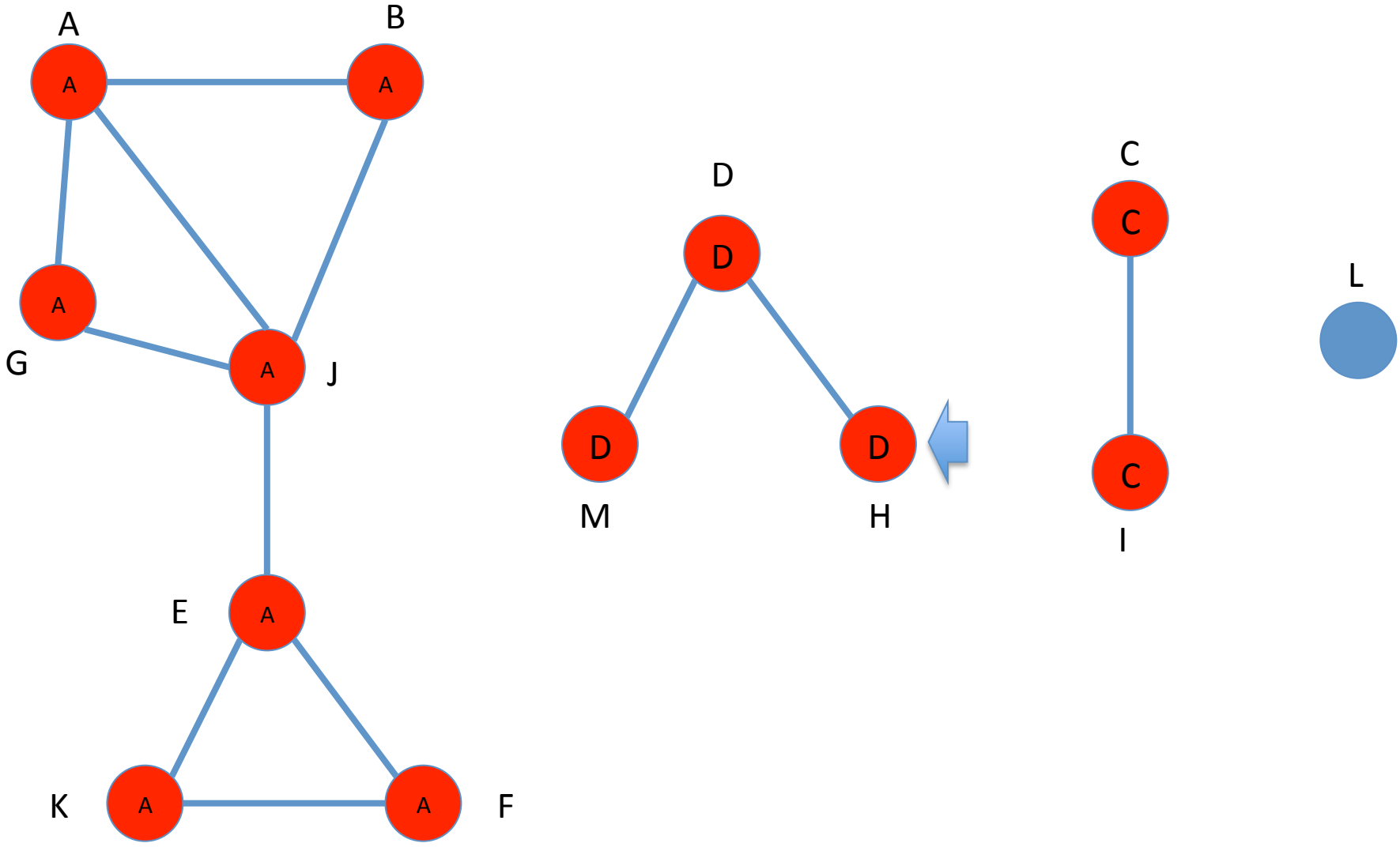

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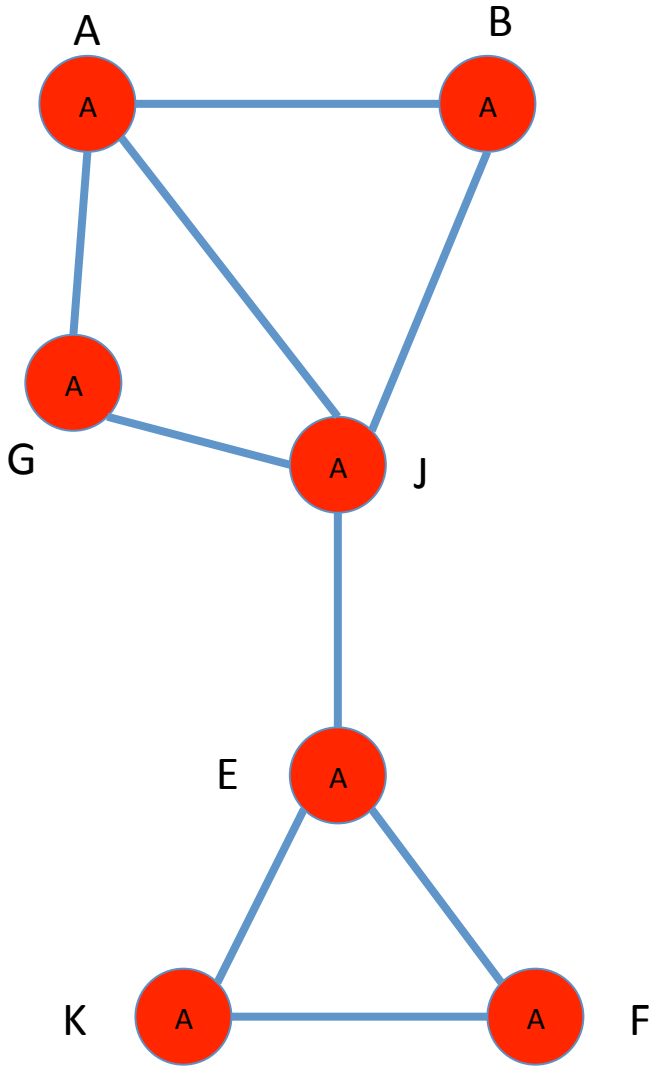

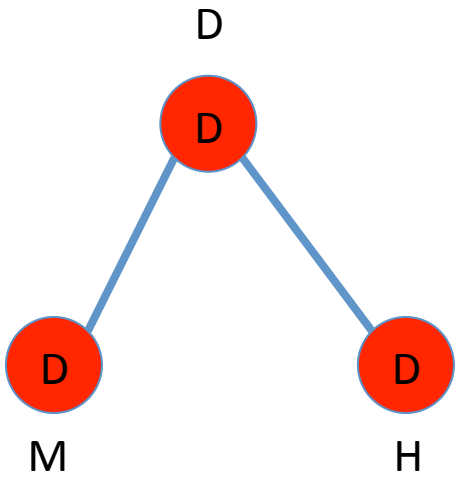

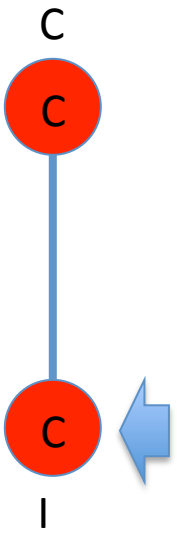

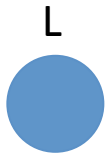

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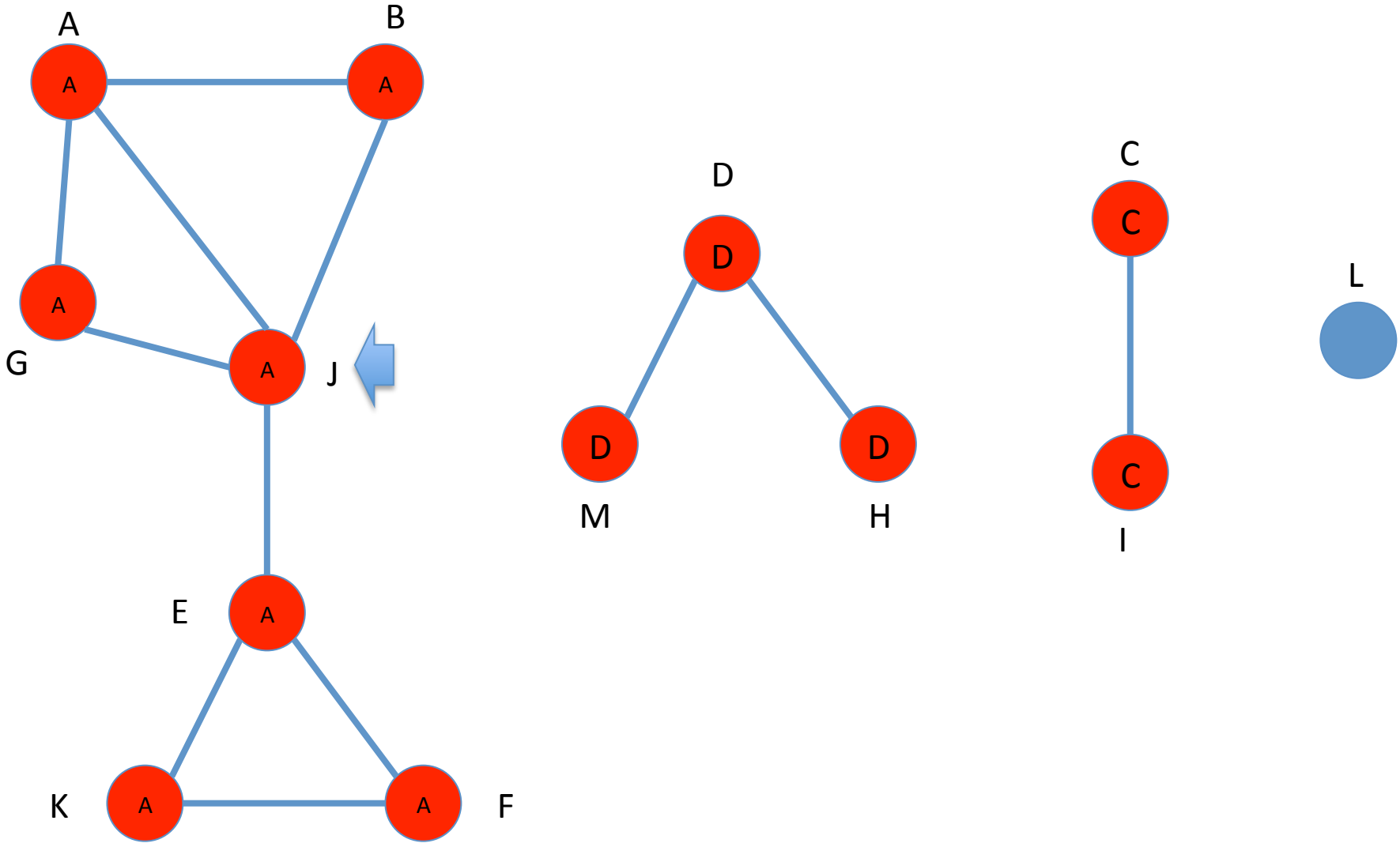

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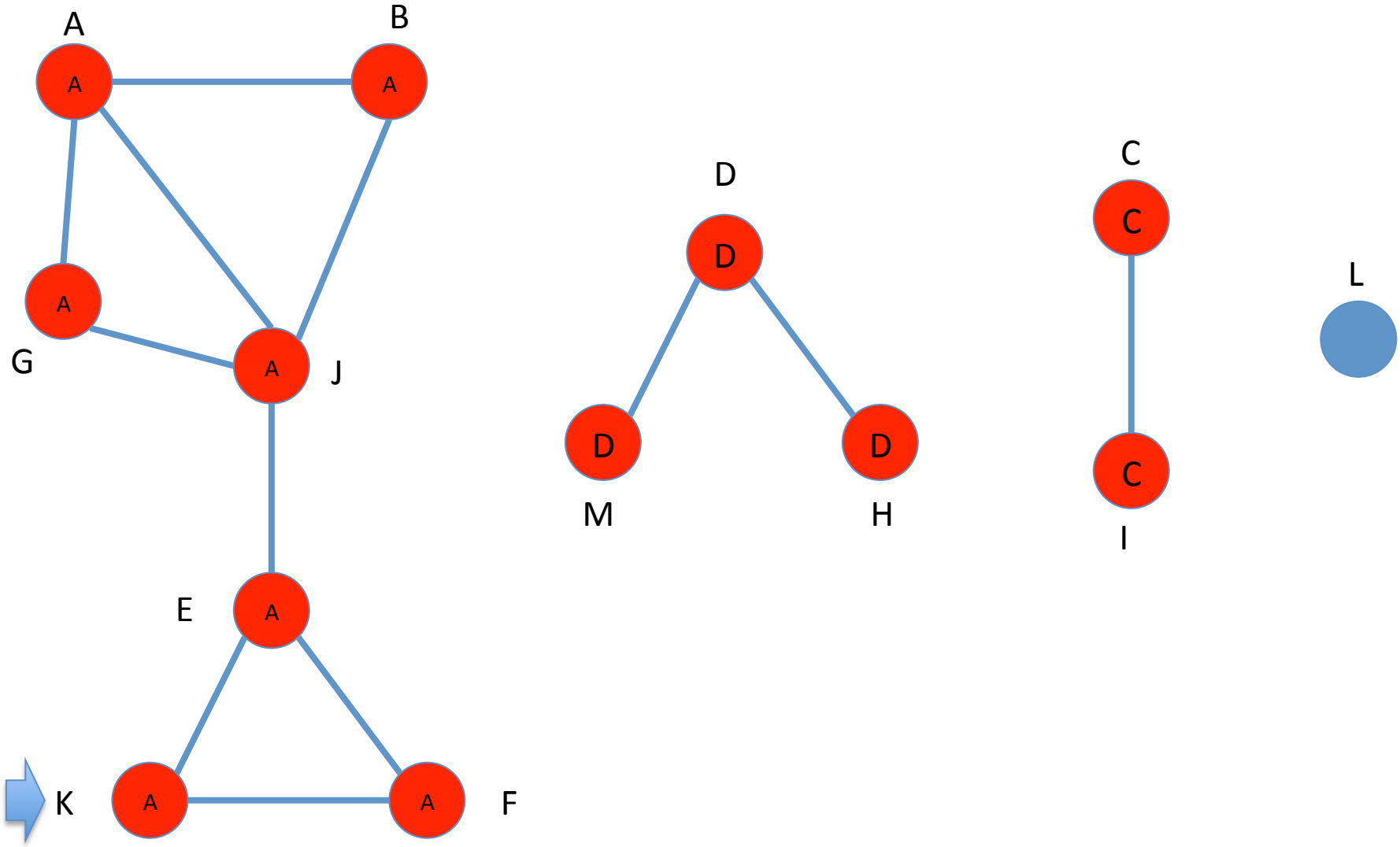

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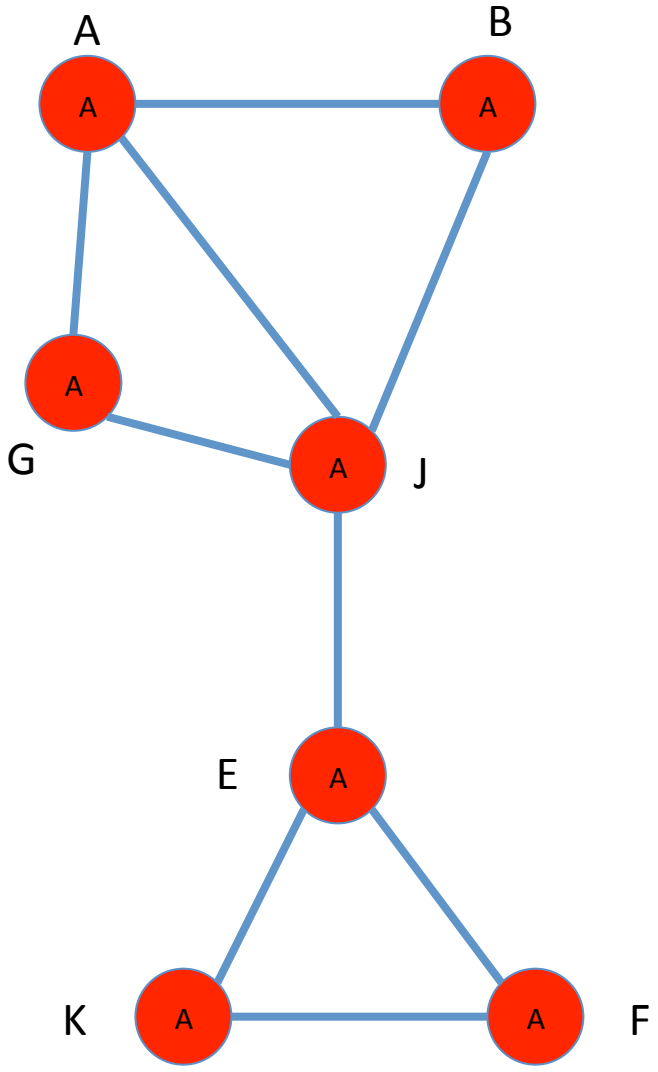

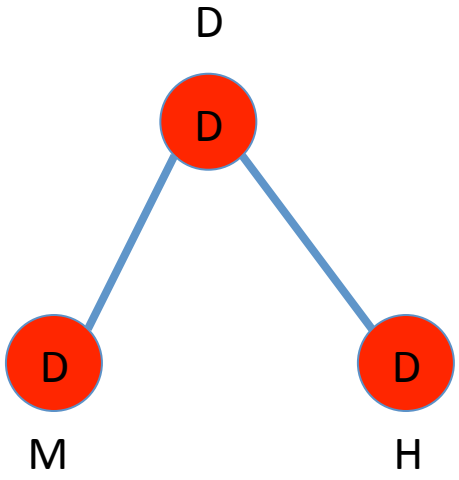

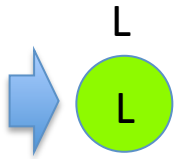

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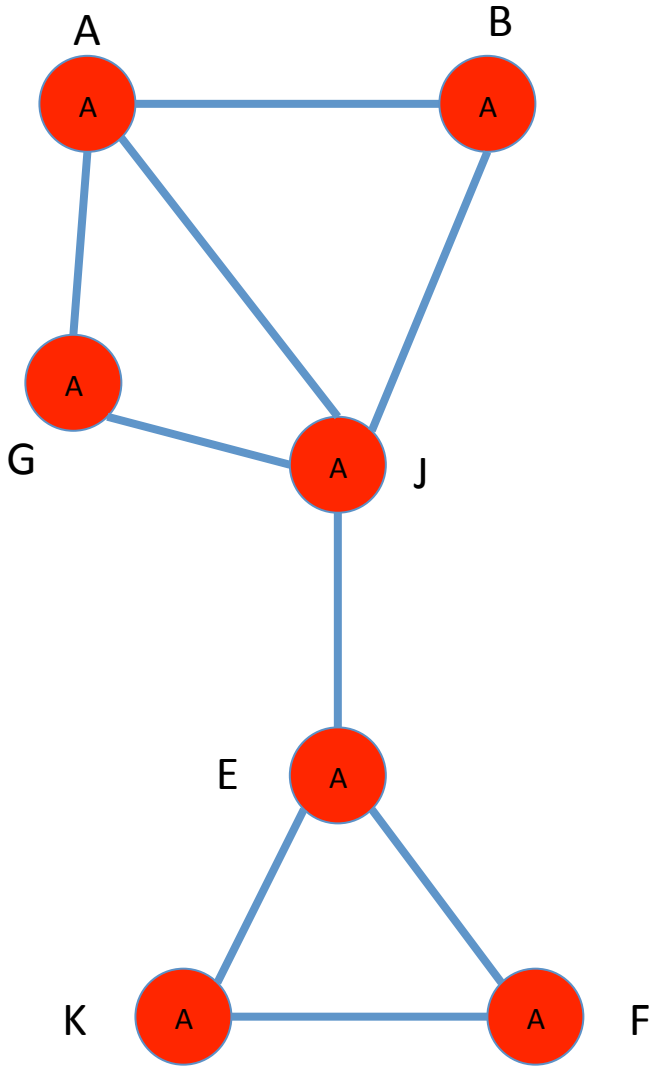
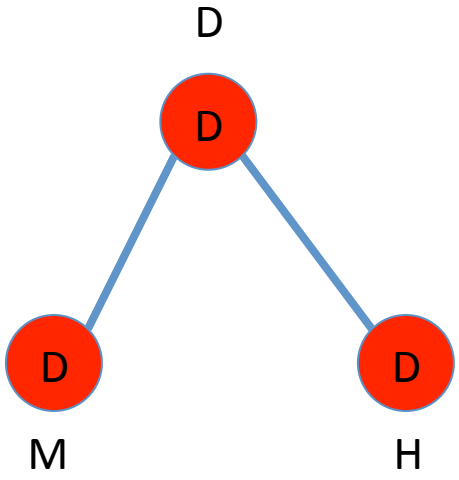
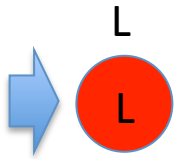
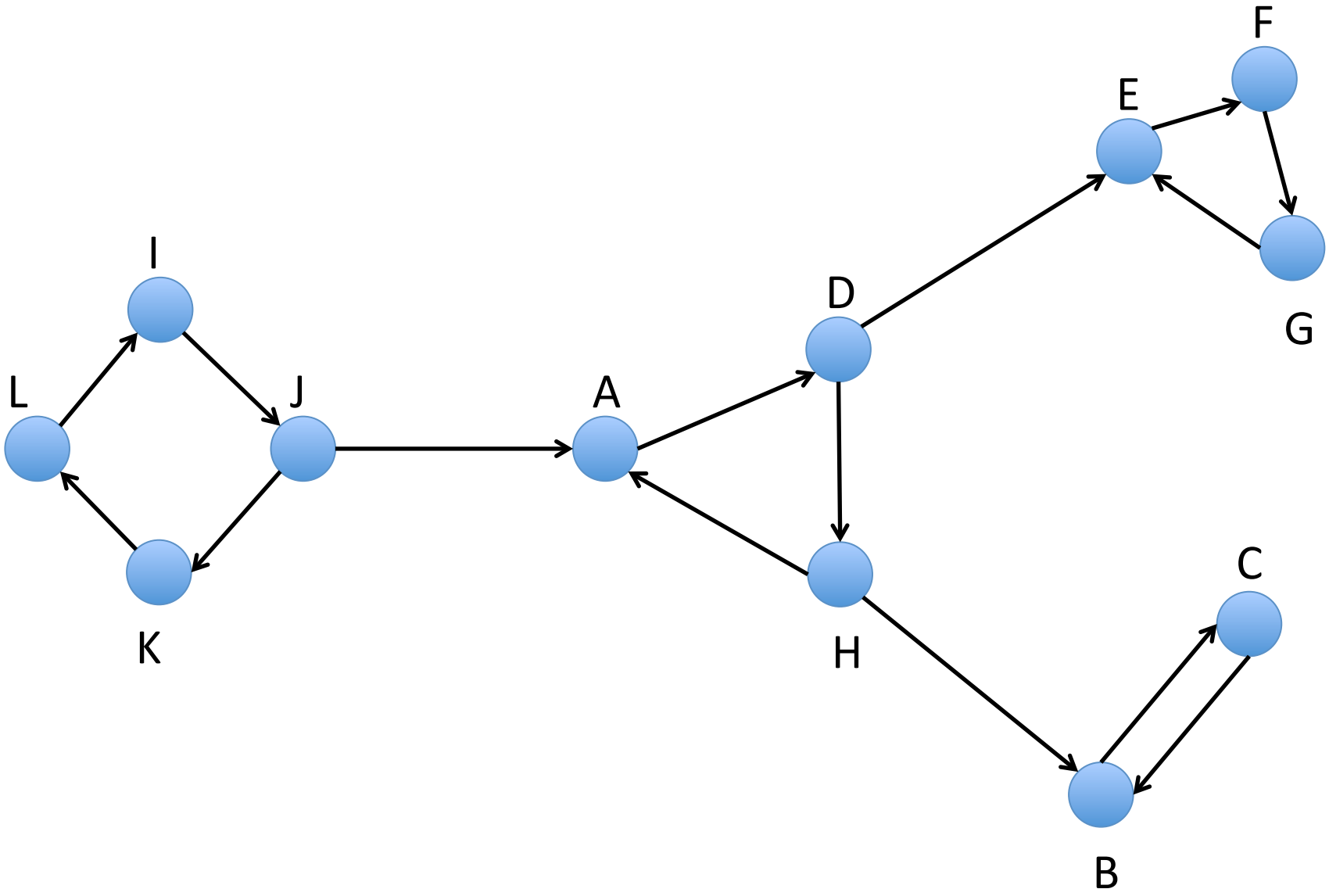






Can DFS/BFS Also Identify SCCs?



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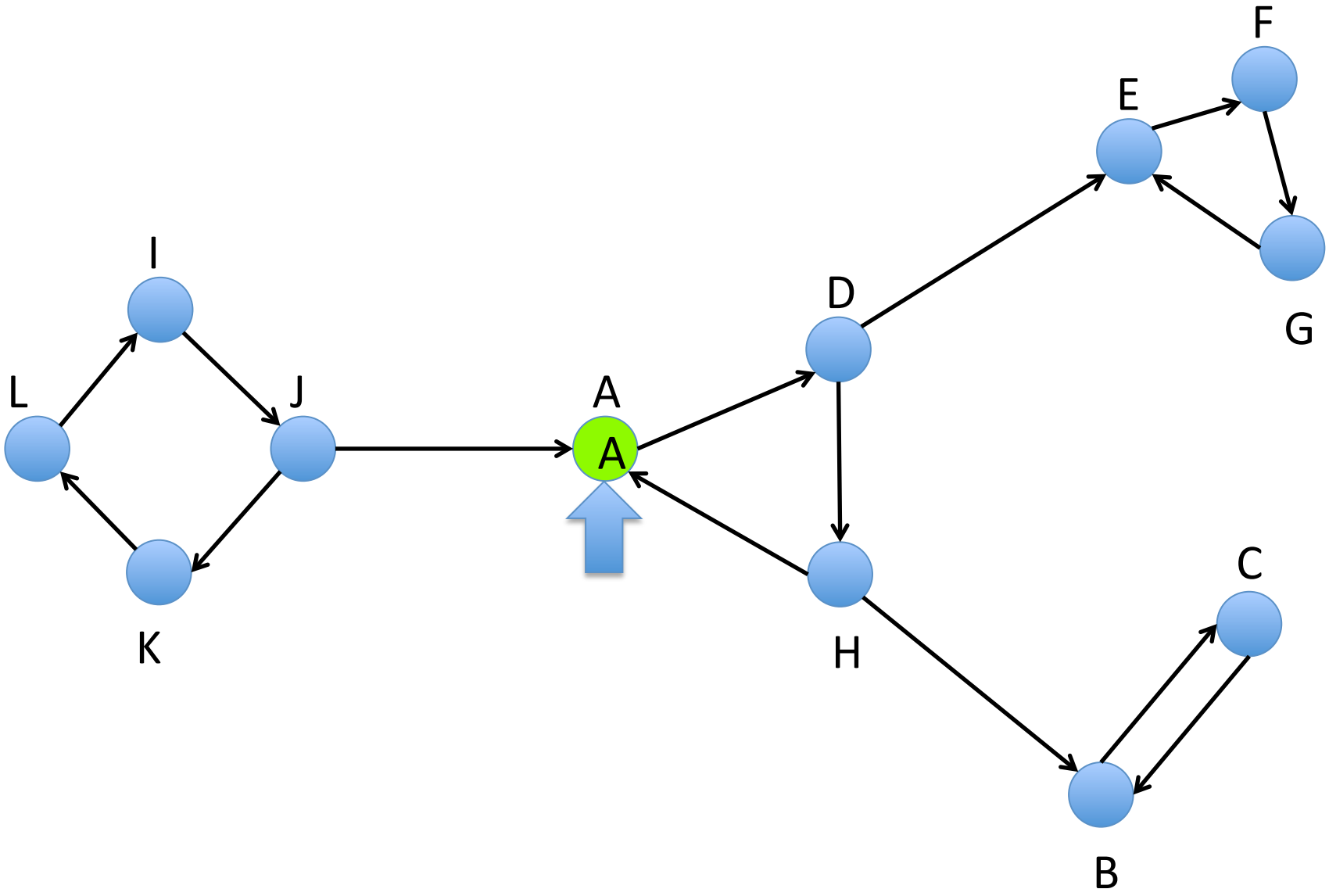

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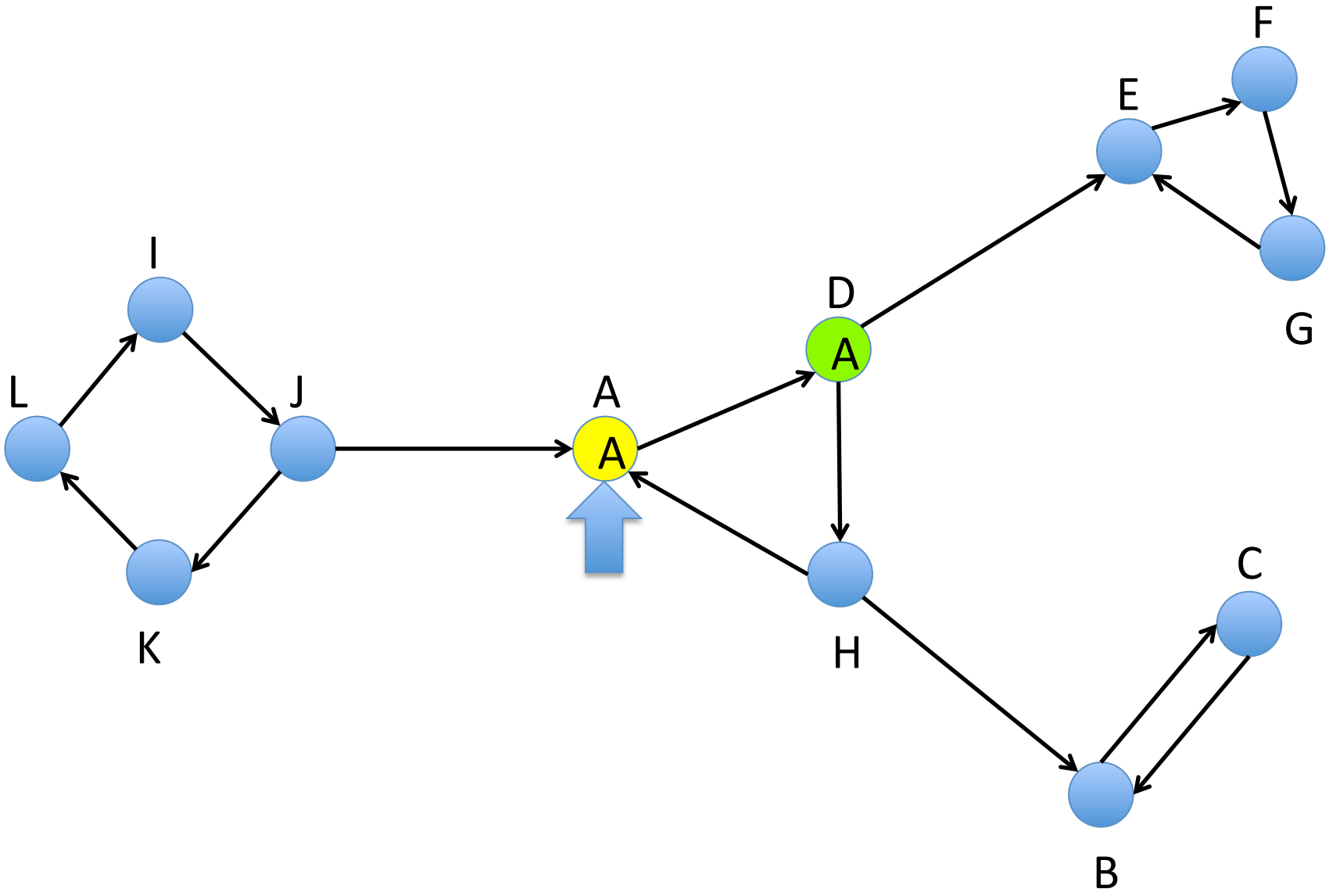

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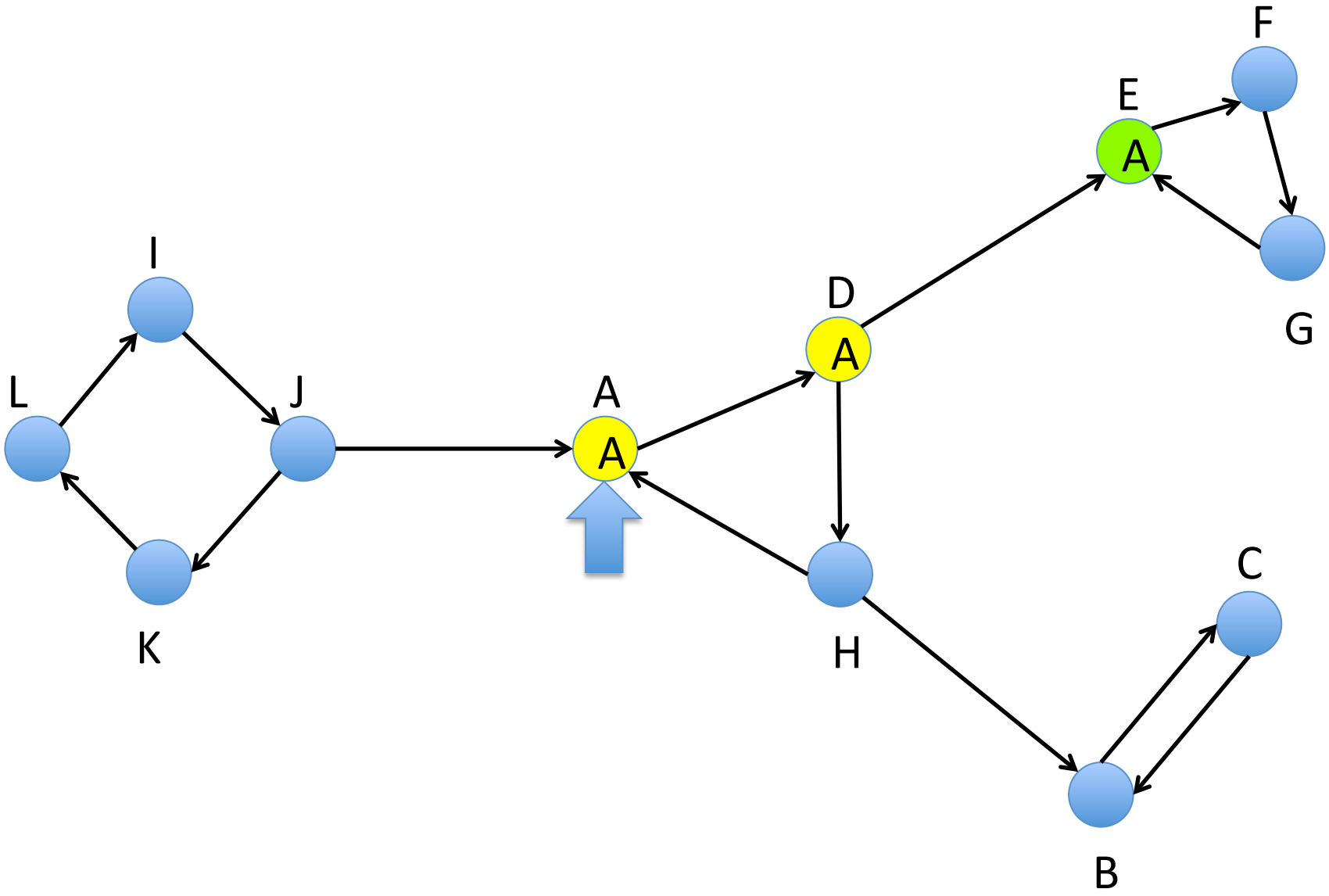

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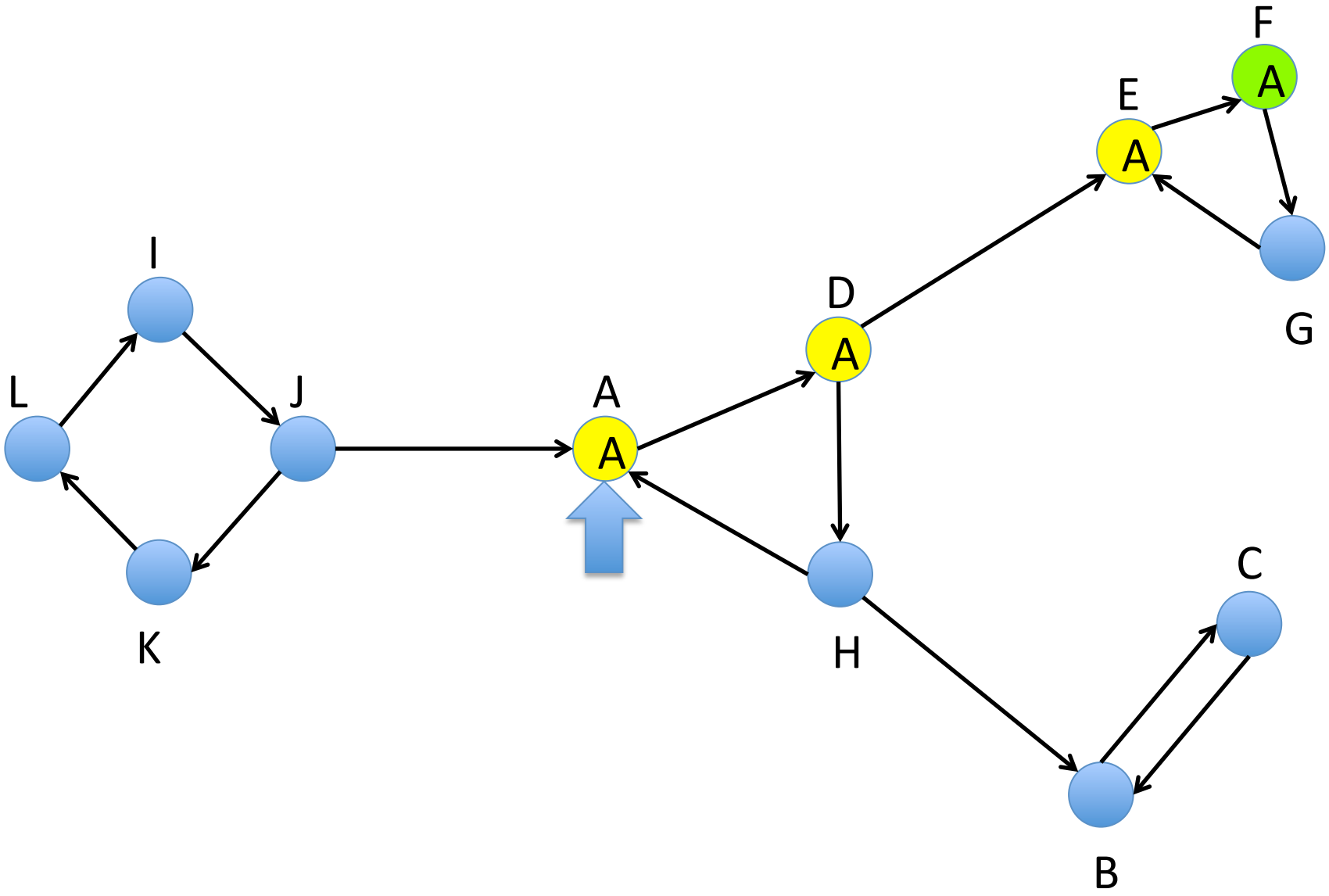

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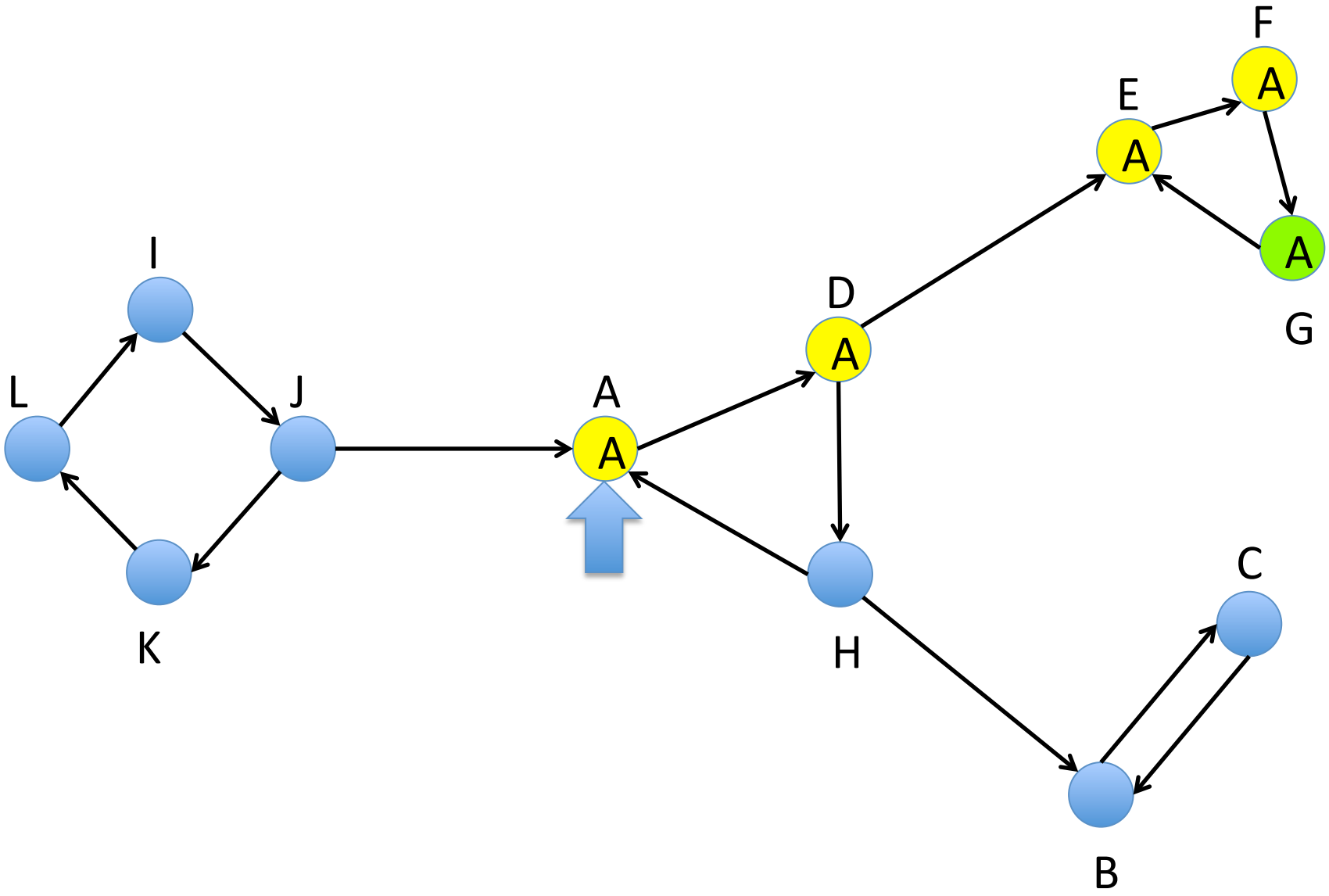

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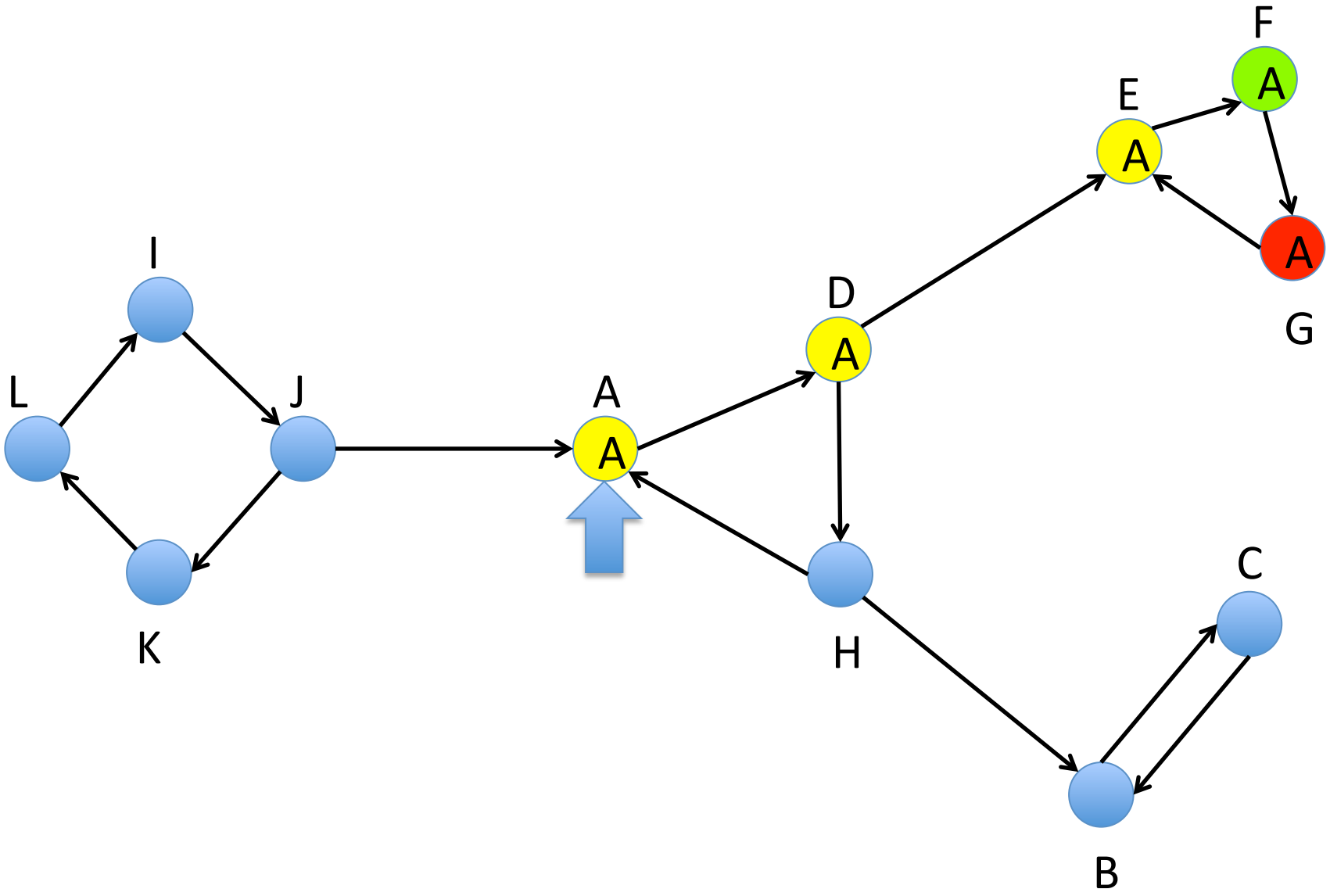

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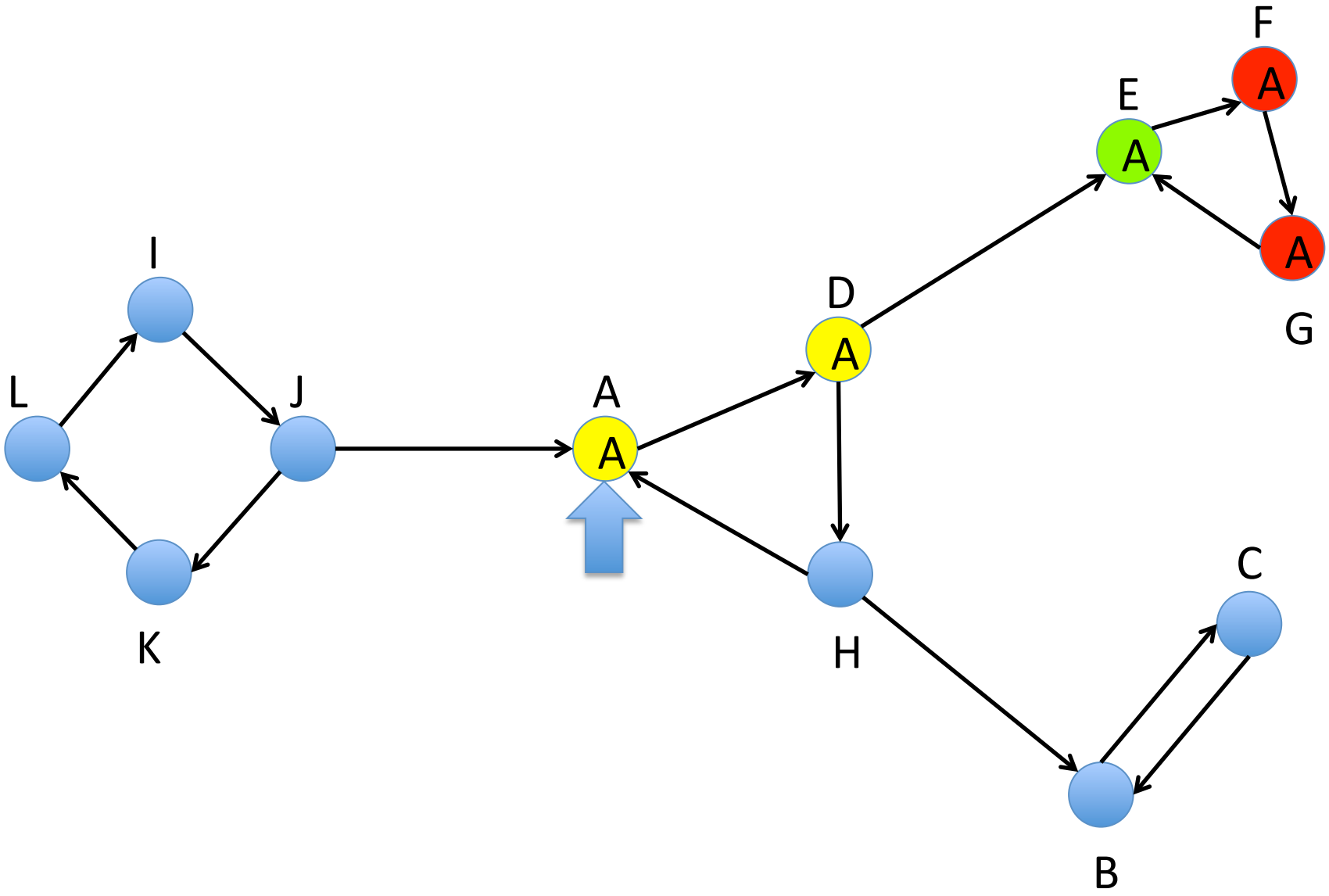

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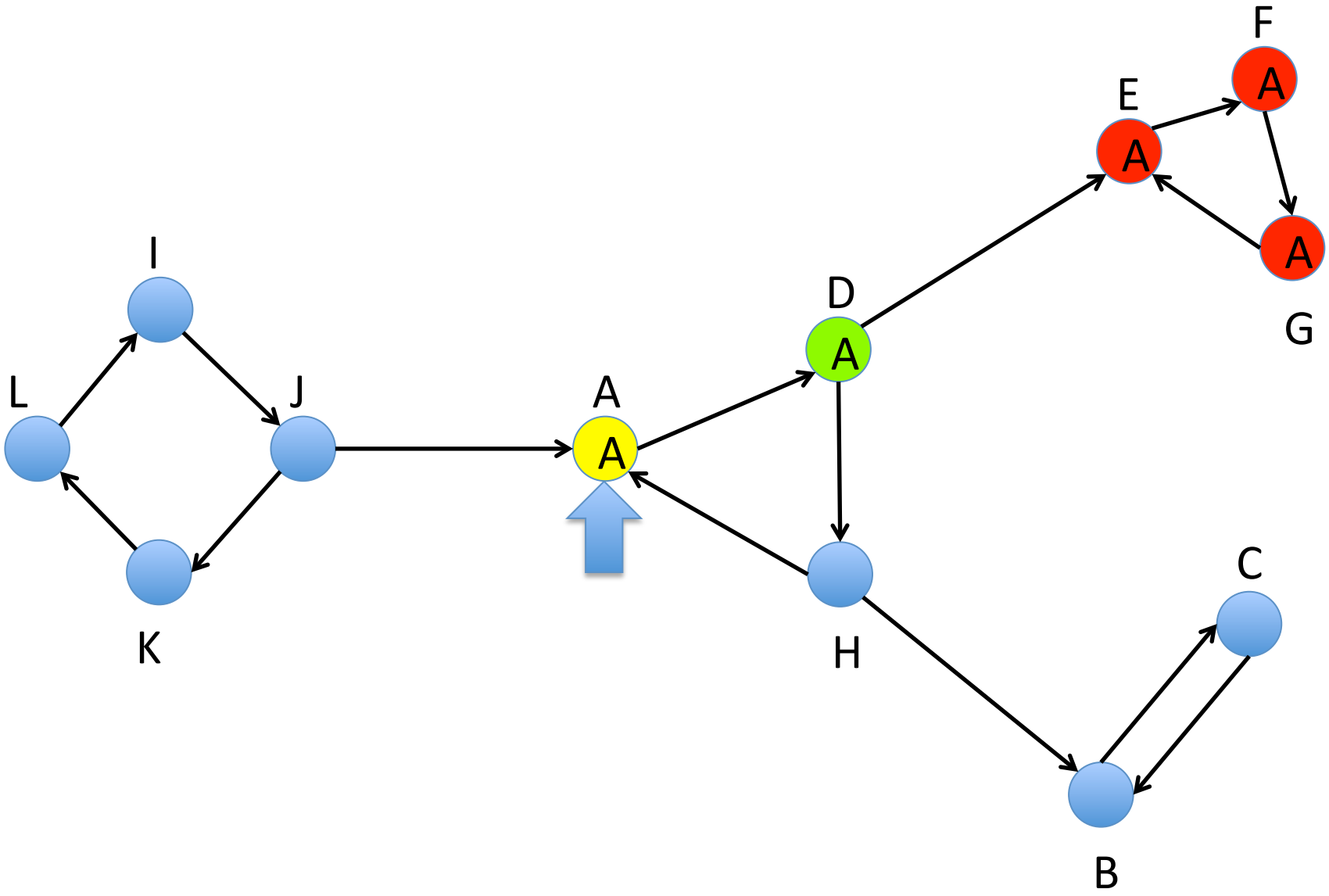

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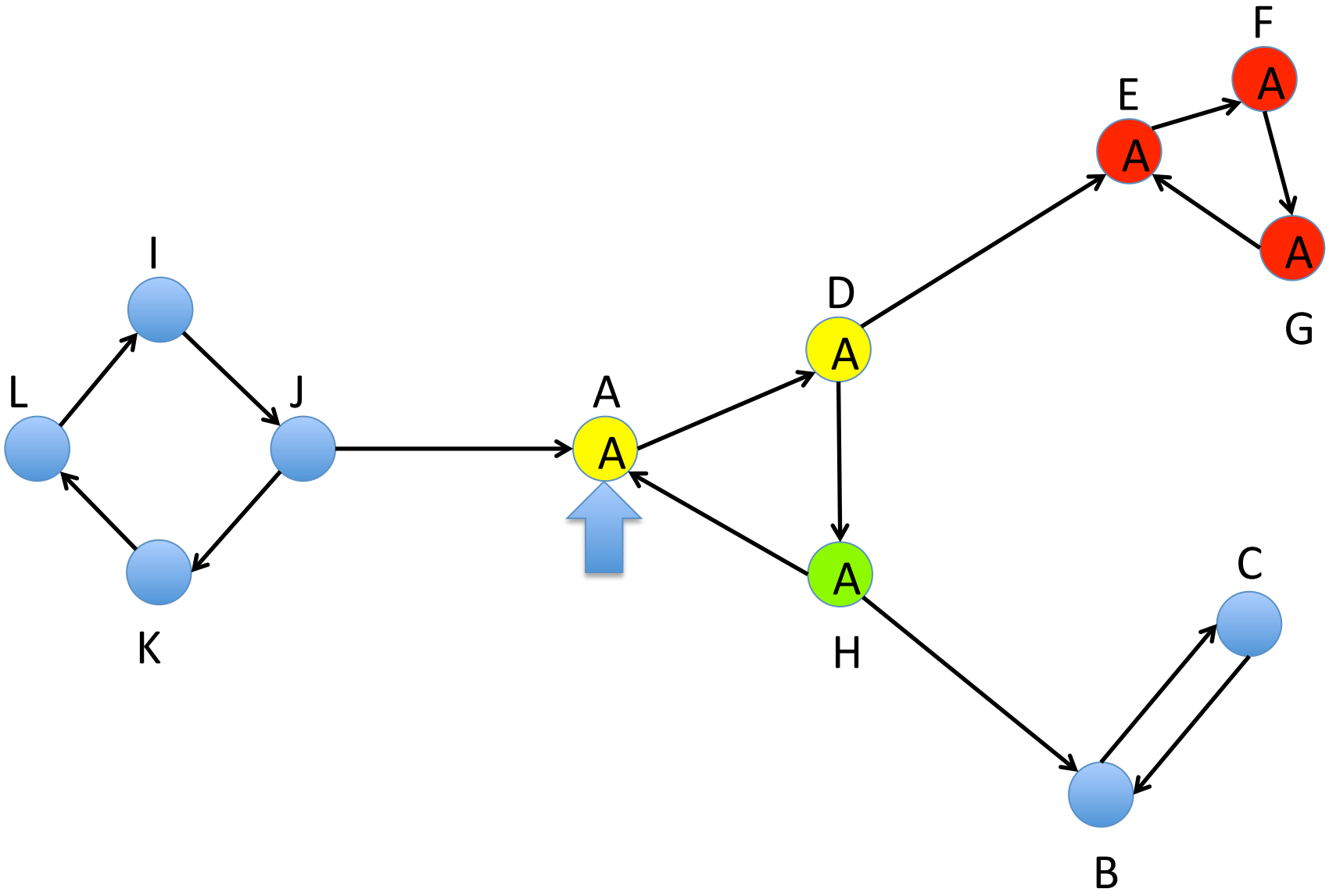

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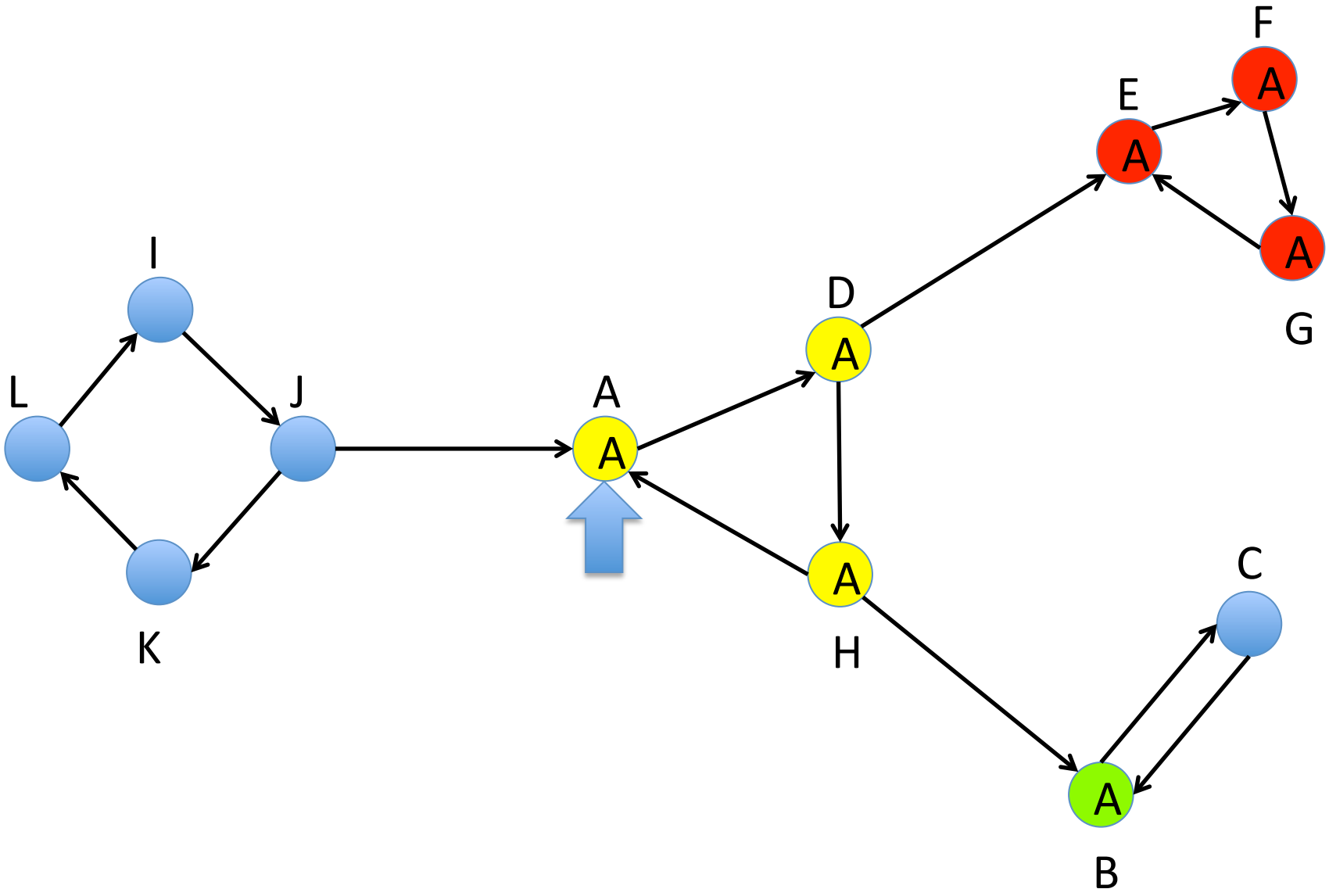

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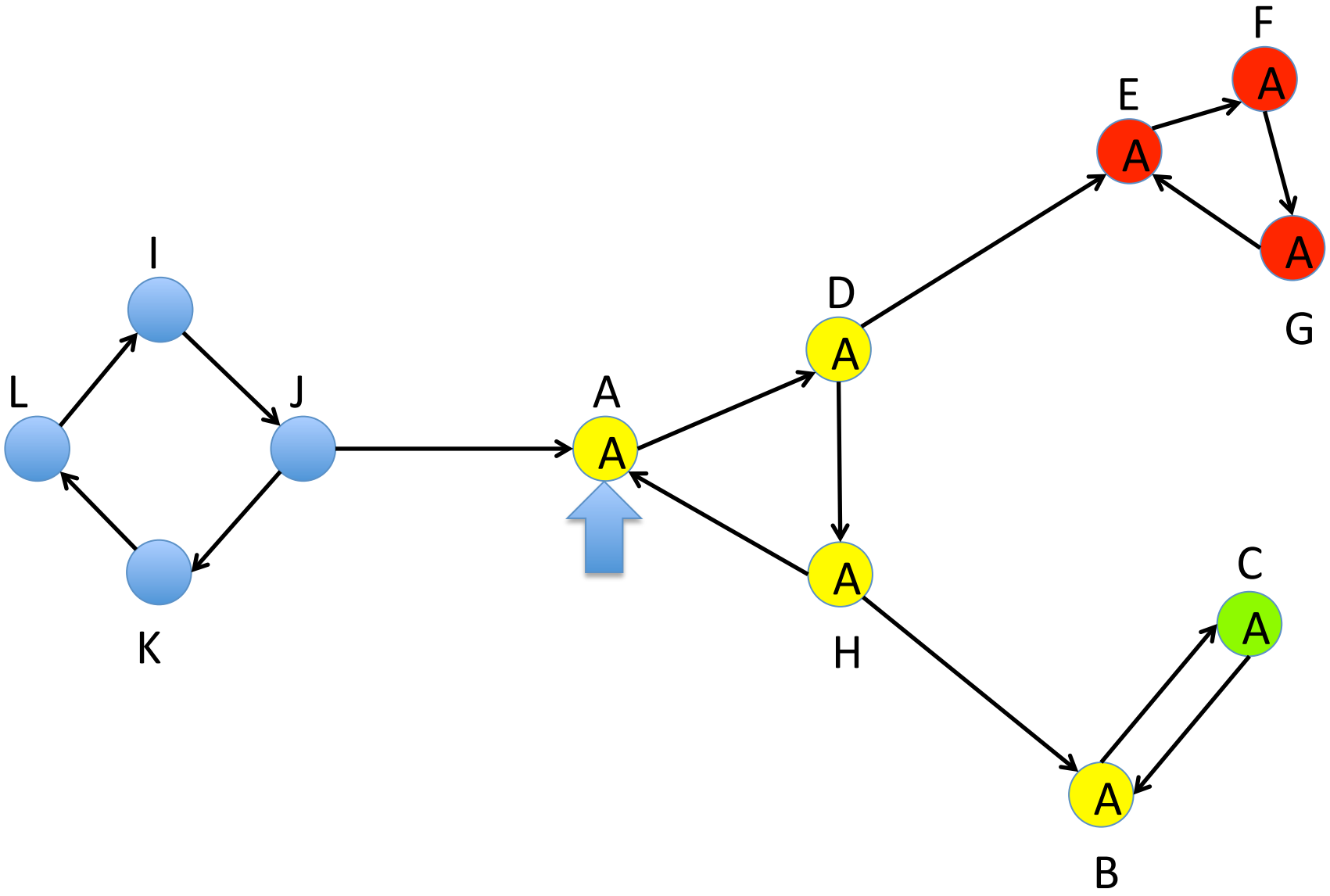

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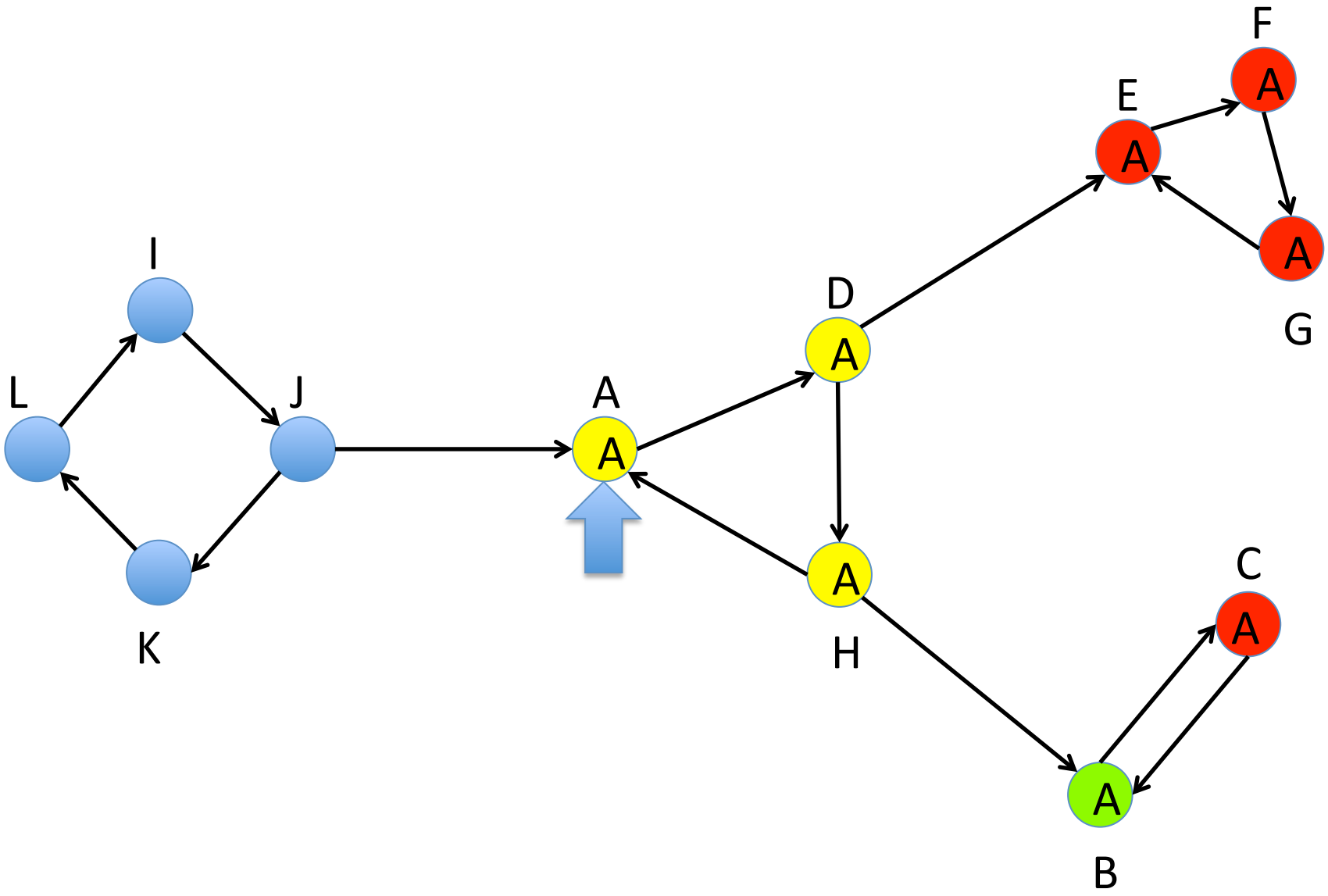

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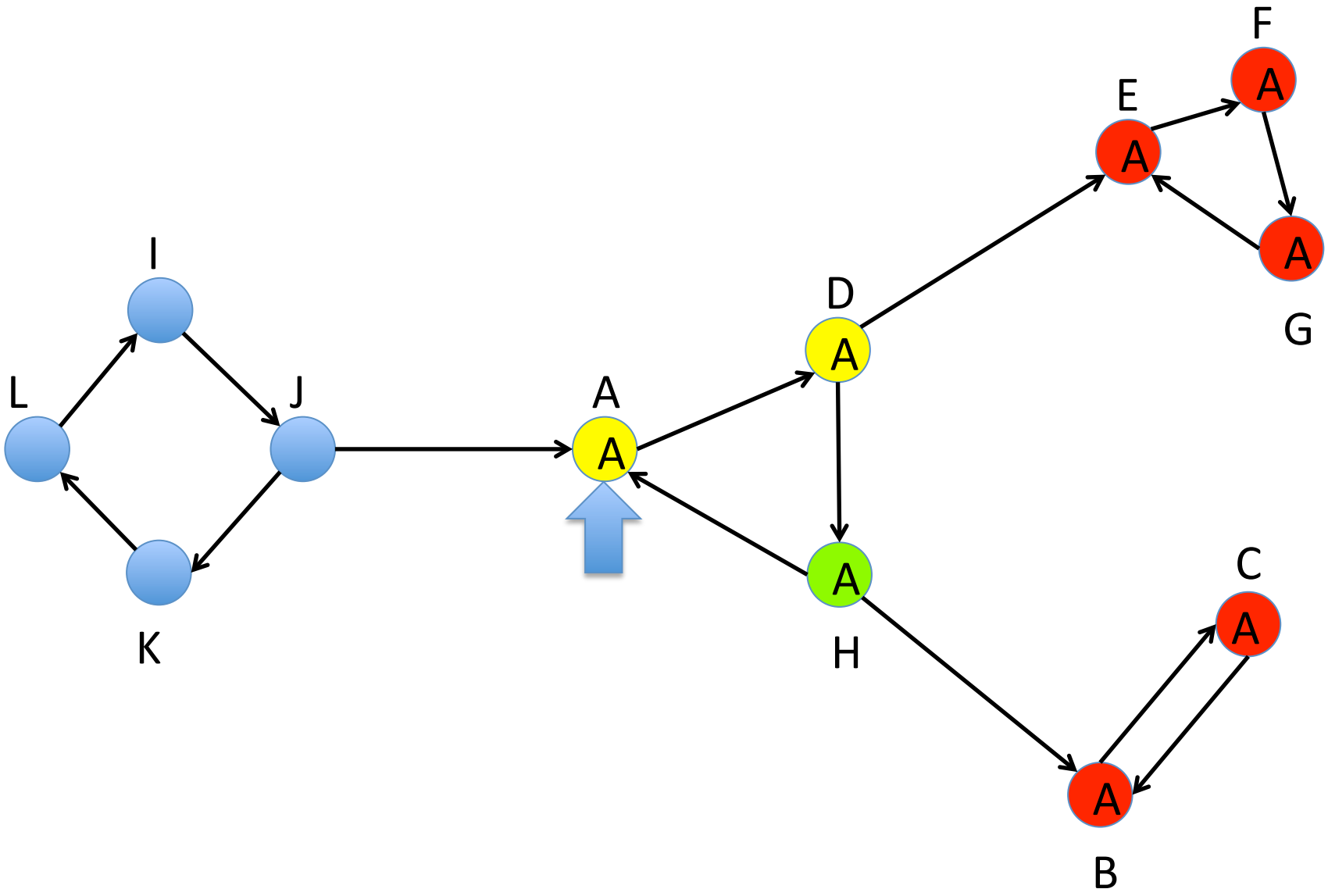

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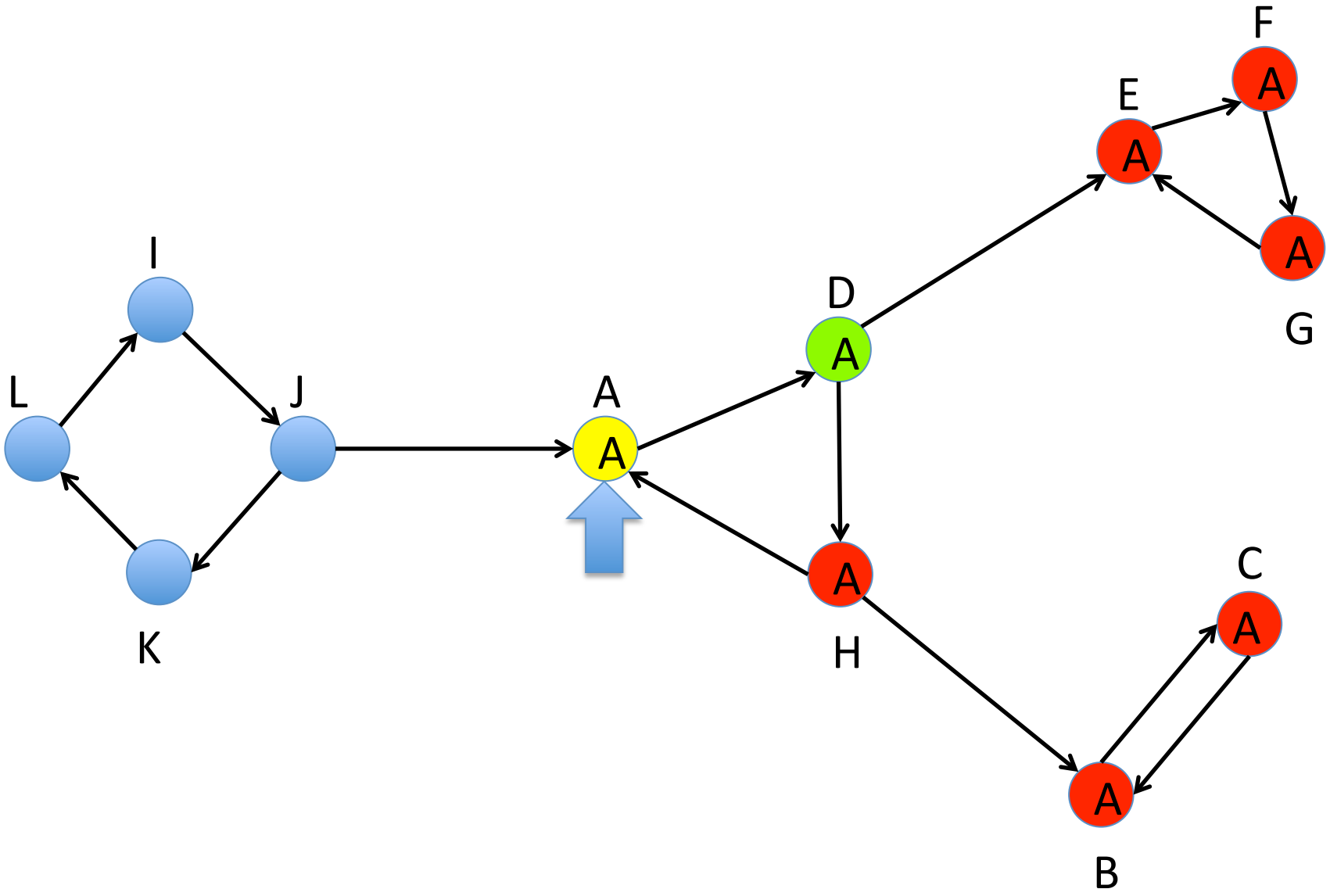

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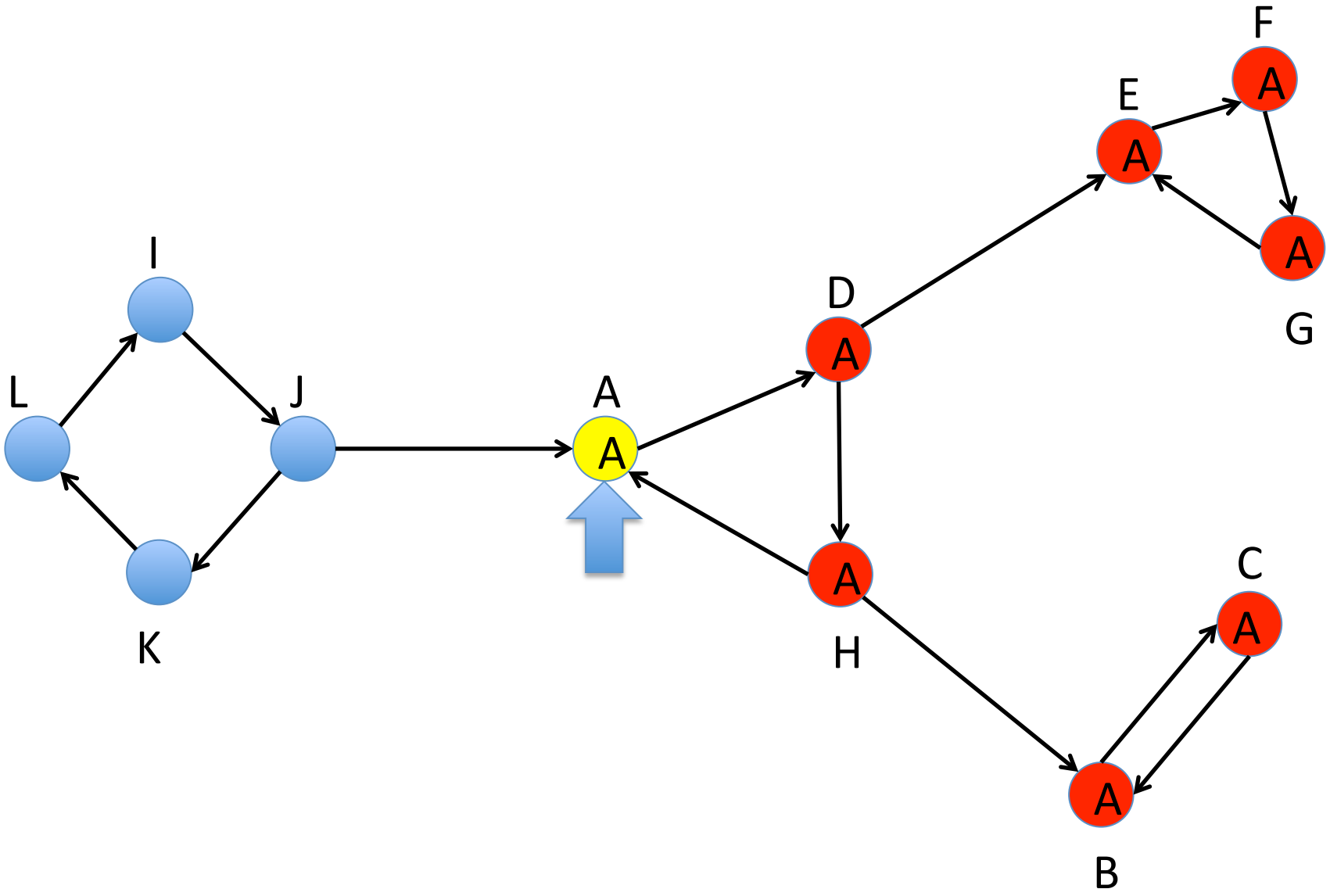

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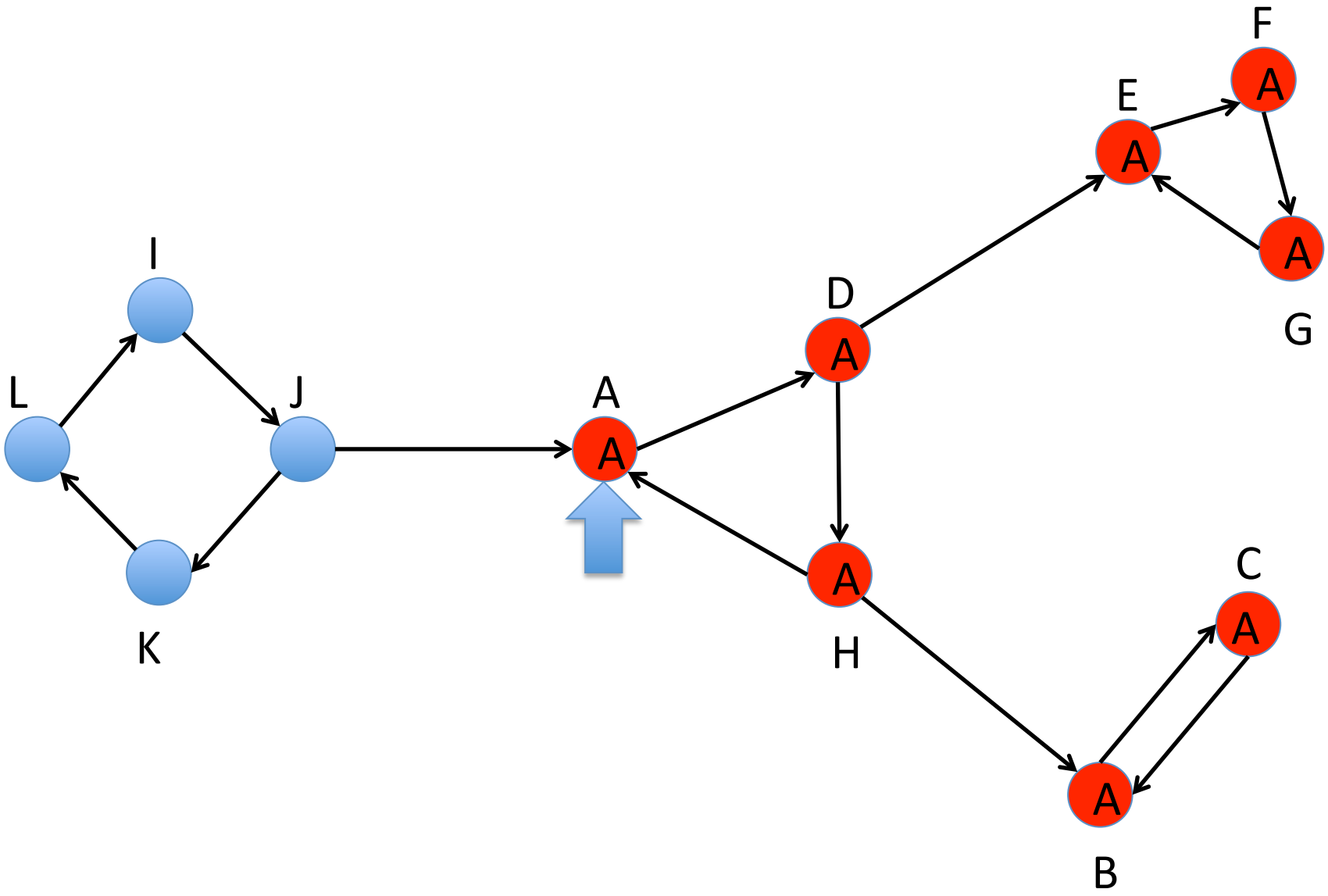

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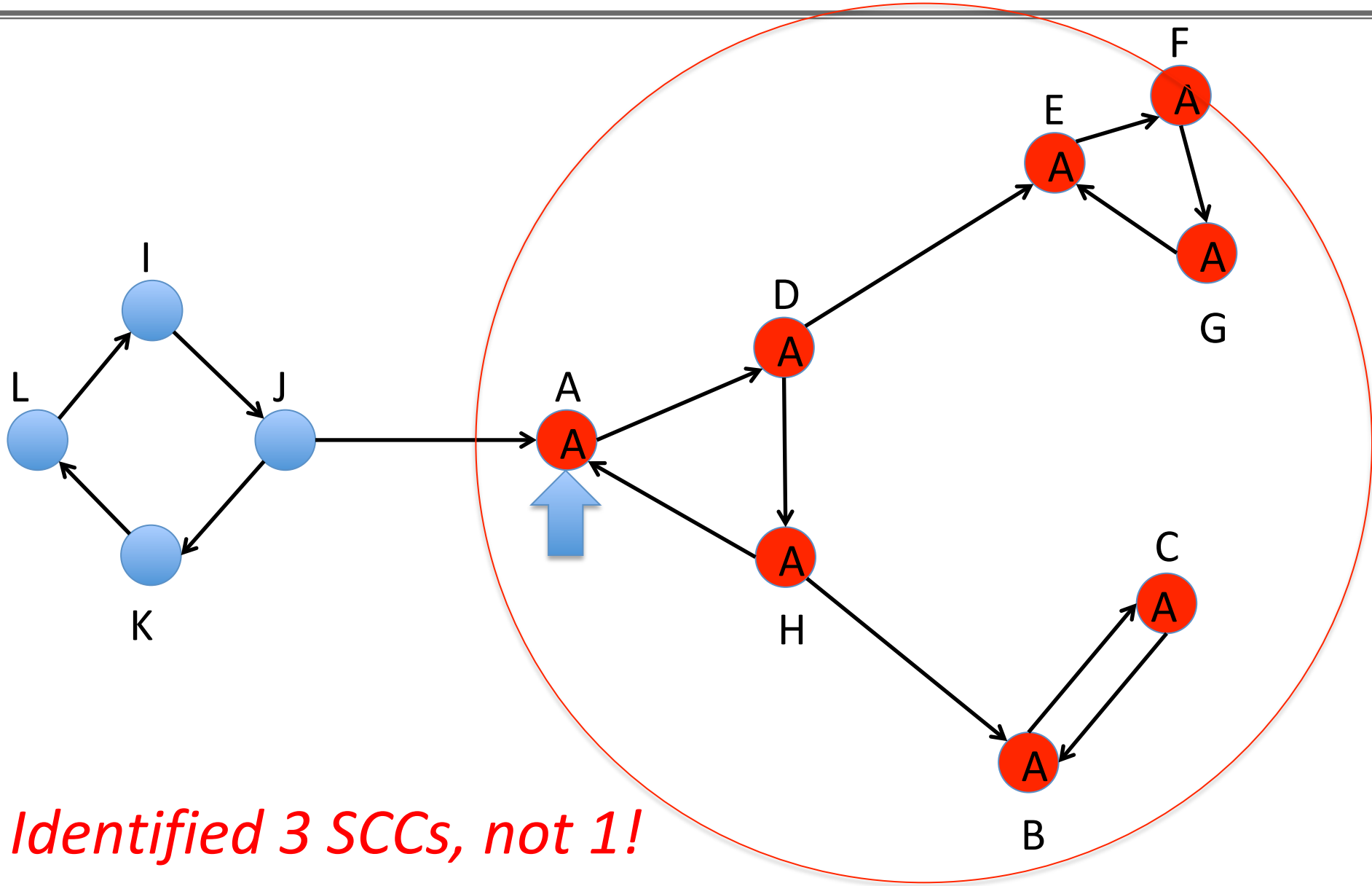
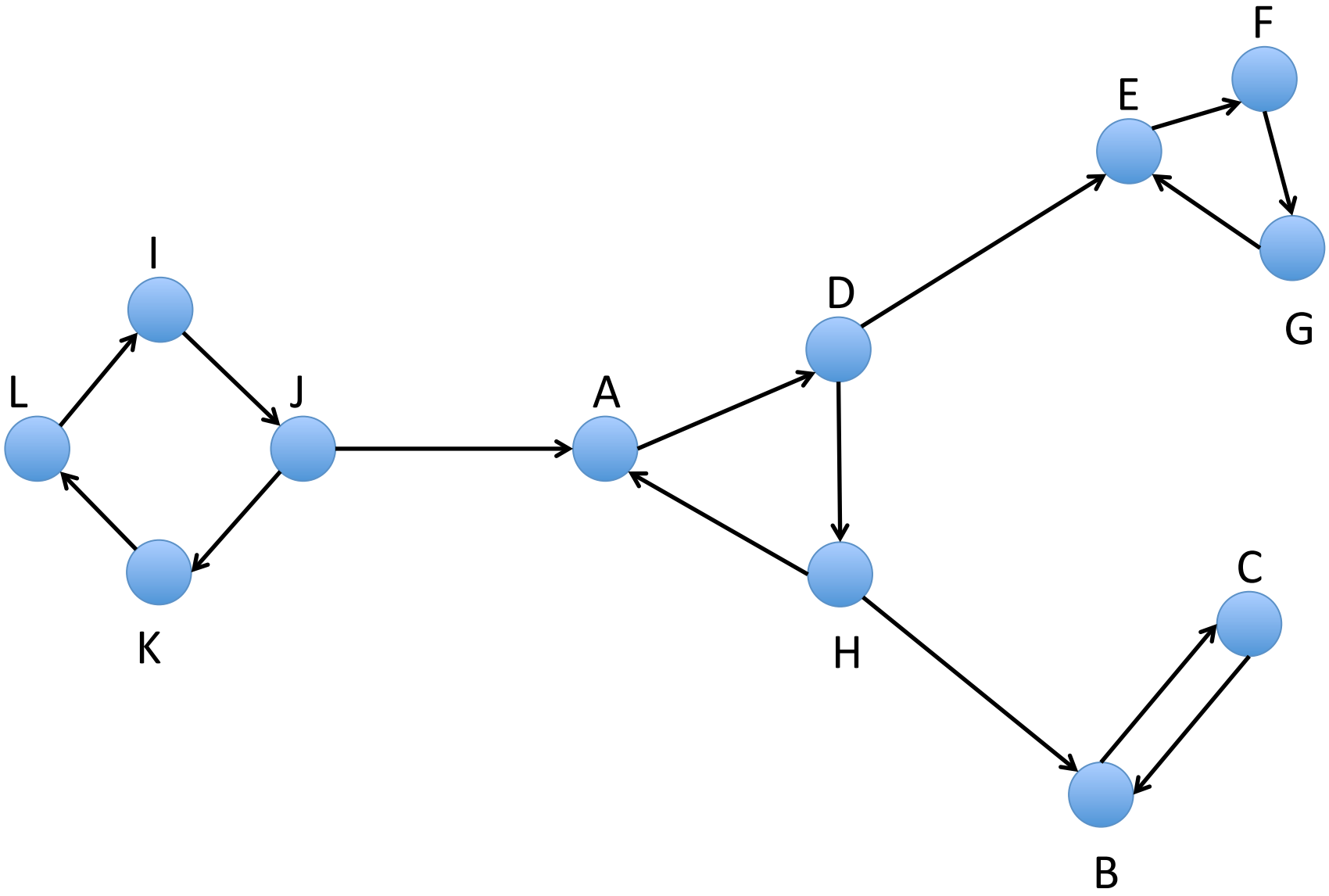


Identified 3 SCCs, not 1!

What If We Got Very Lucky?



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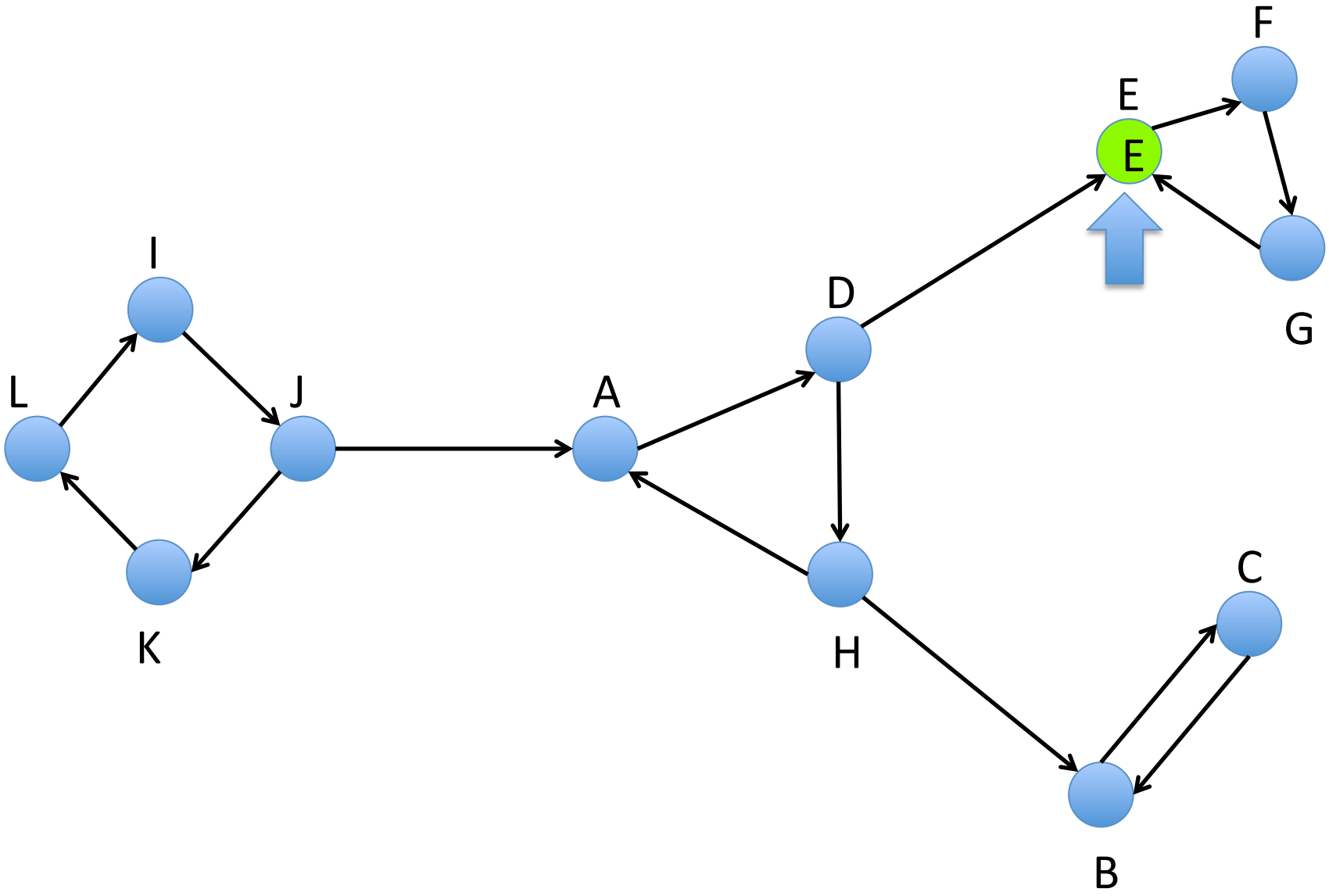

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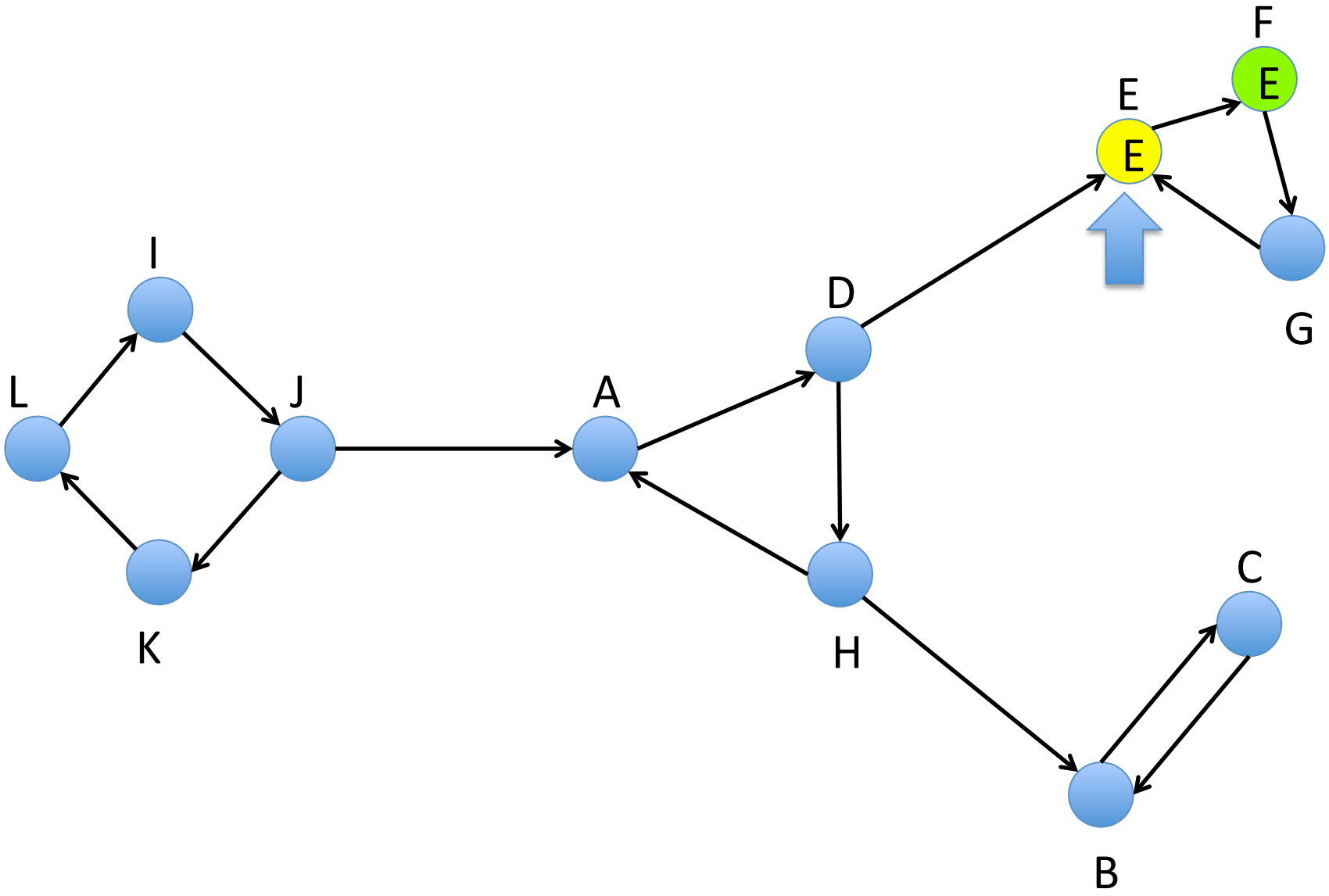

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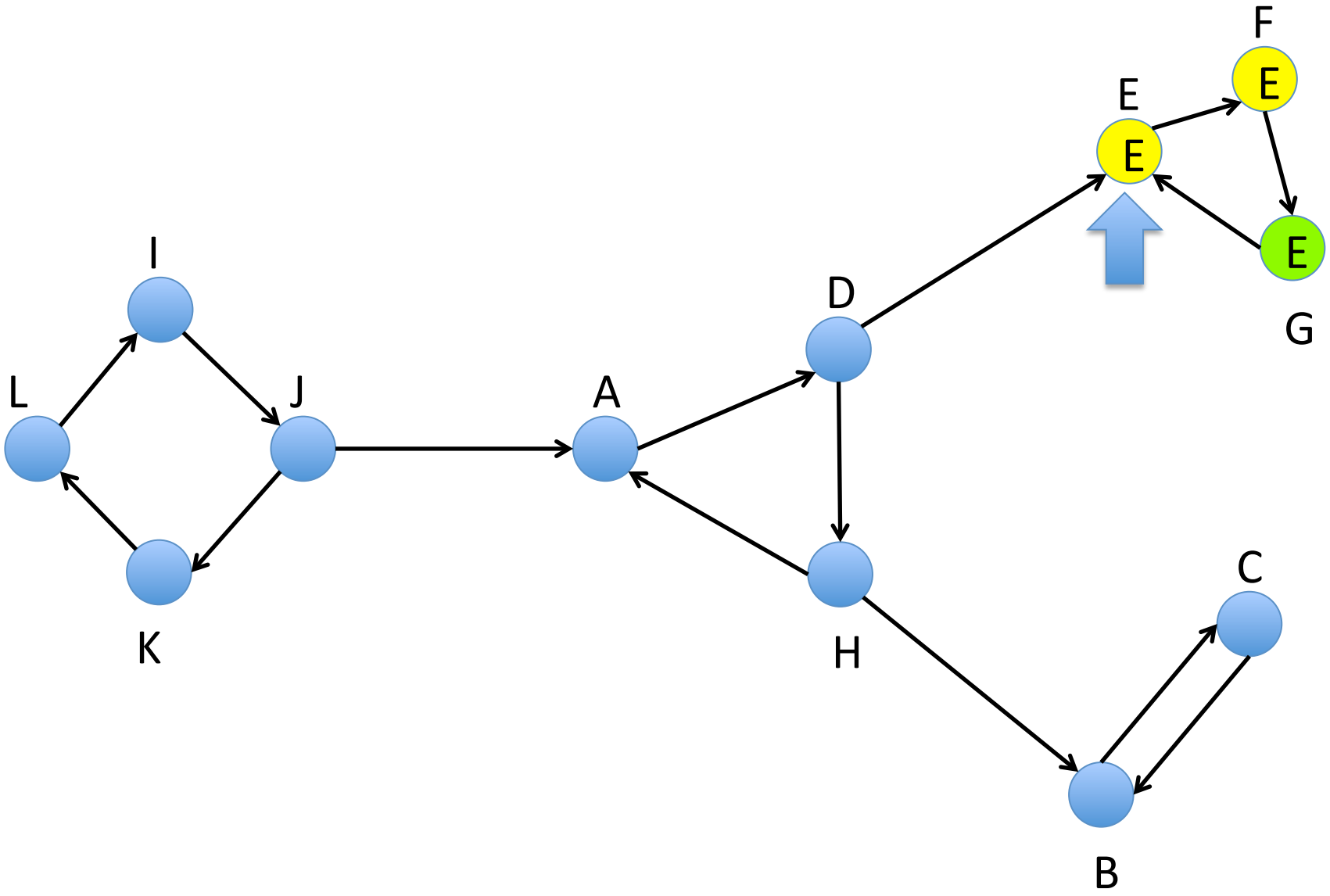

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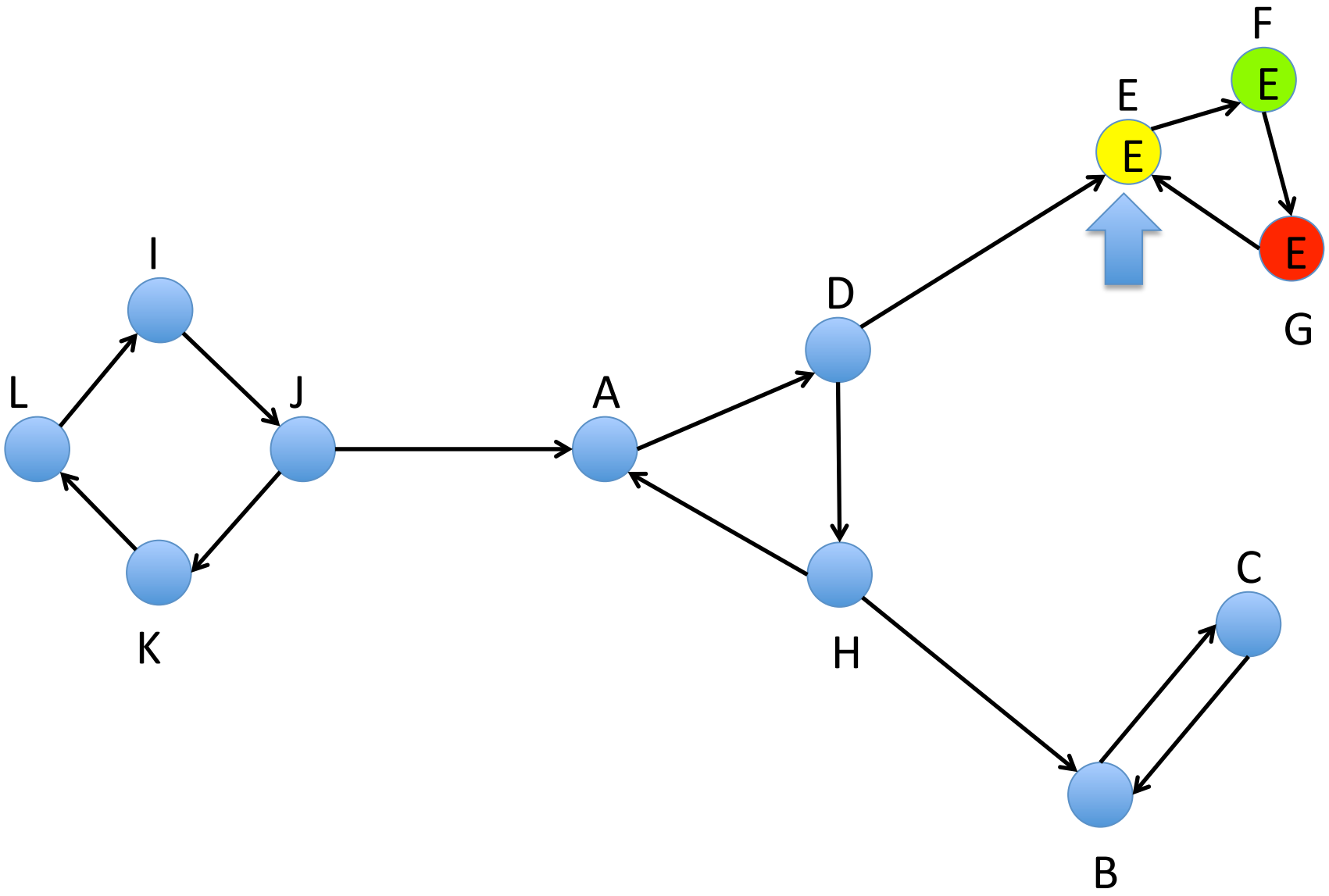

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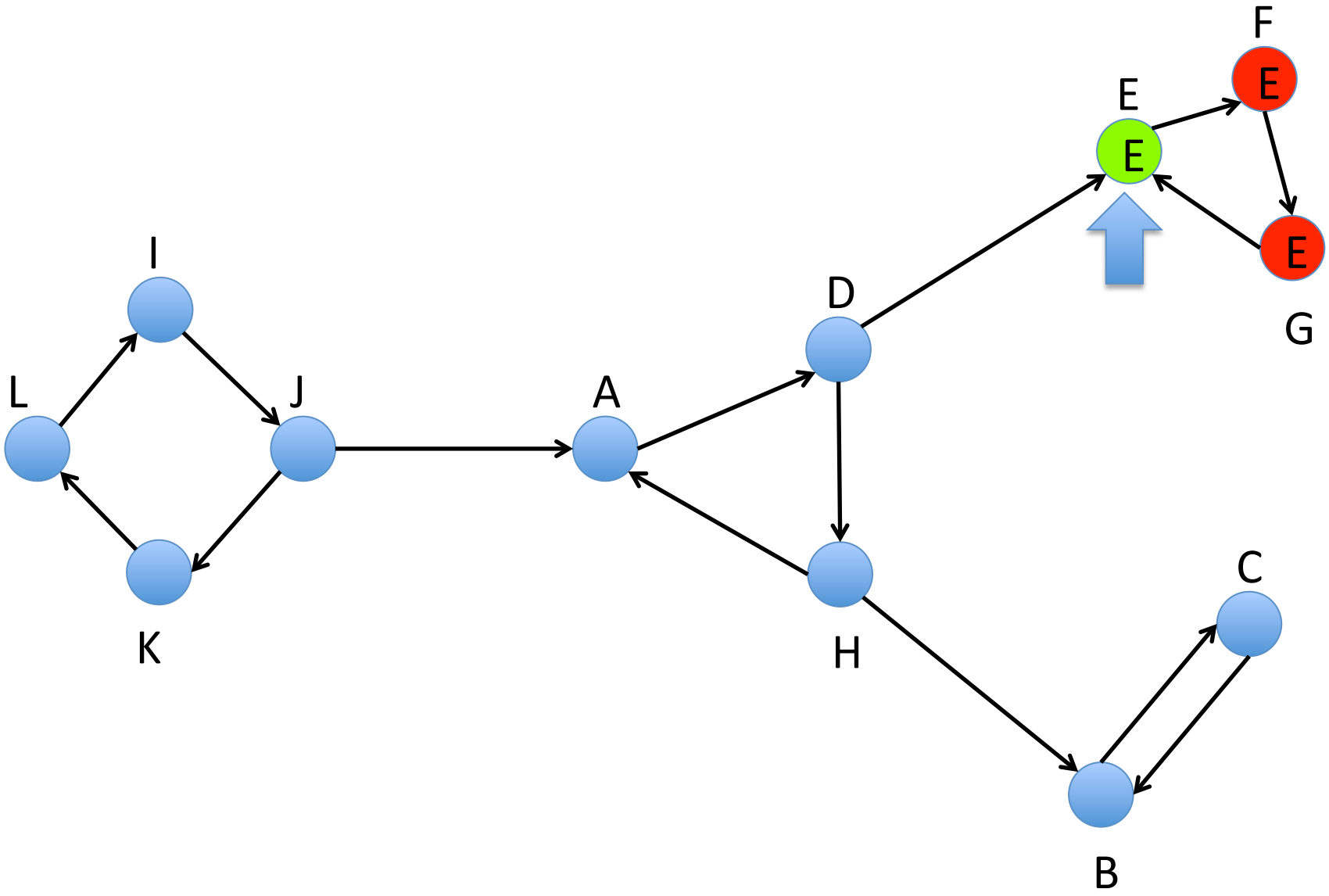

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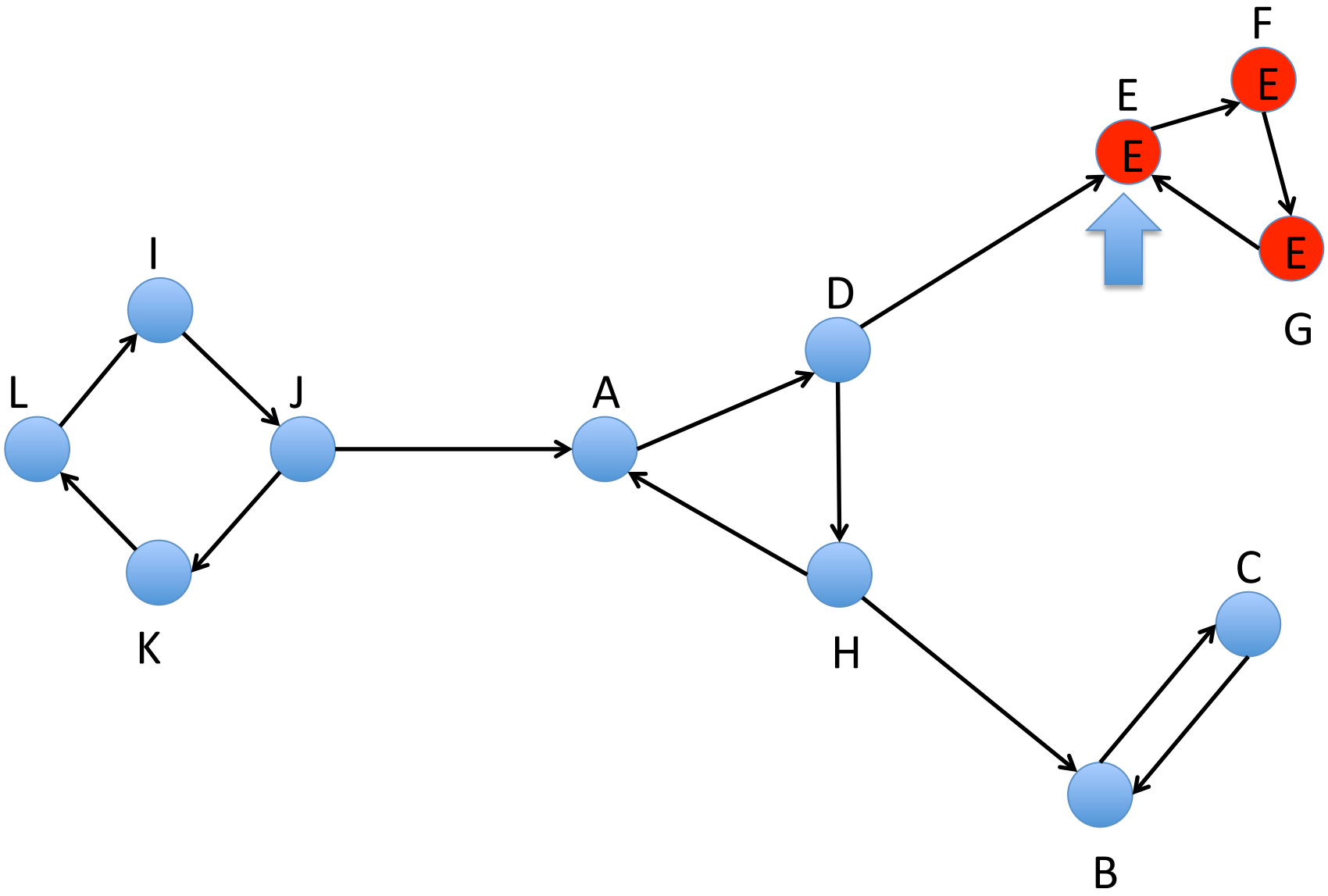

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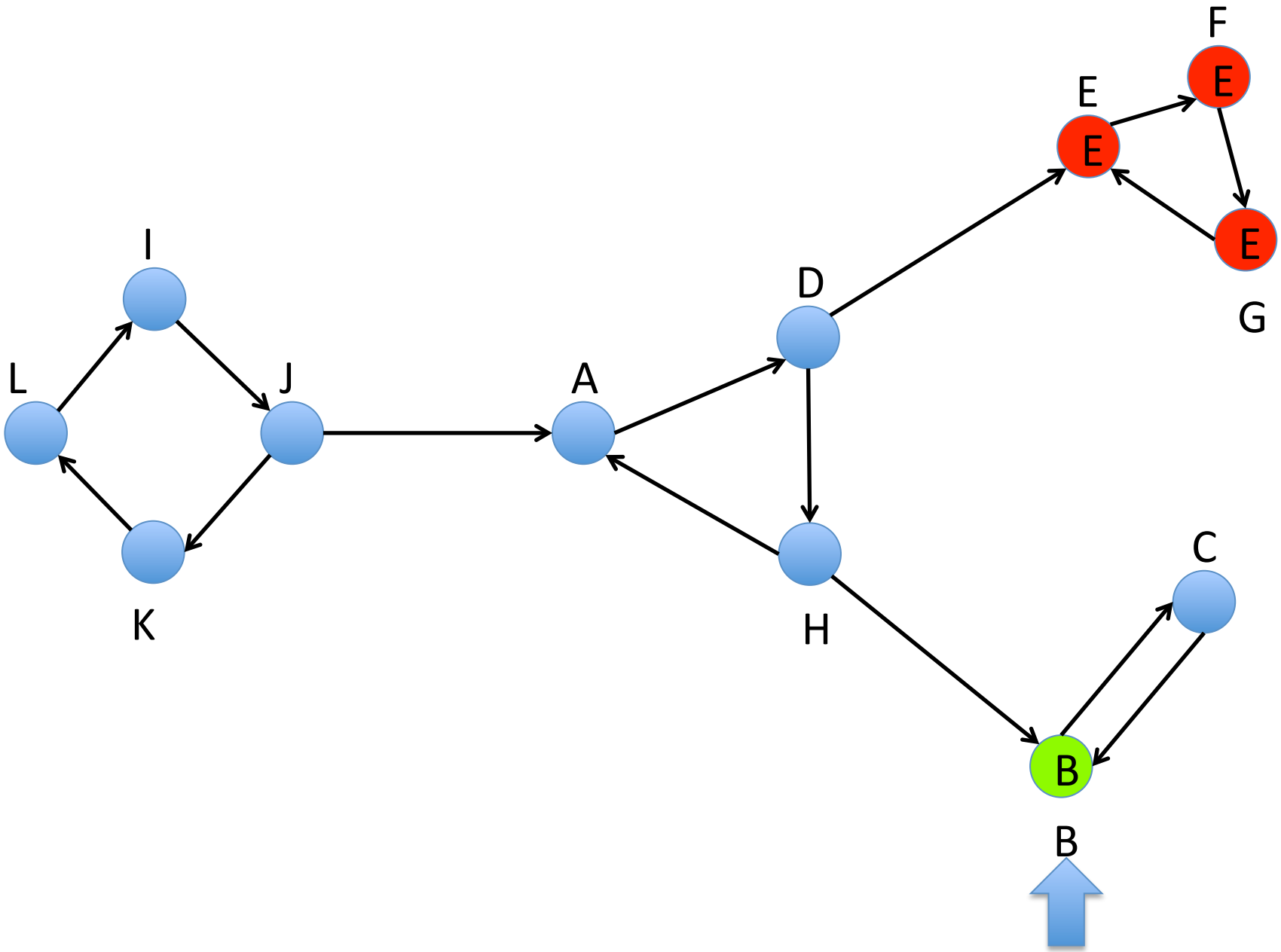

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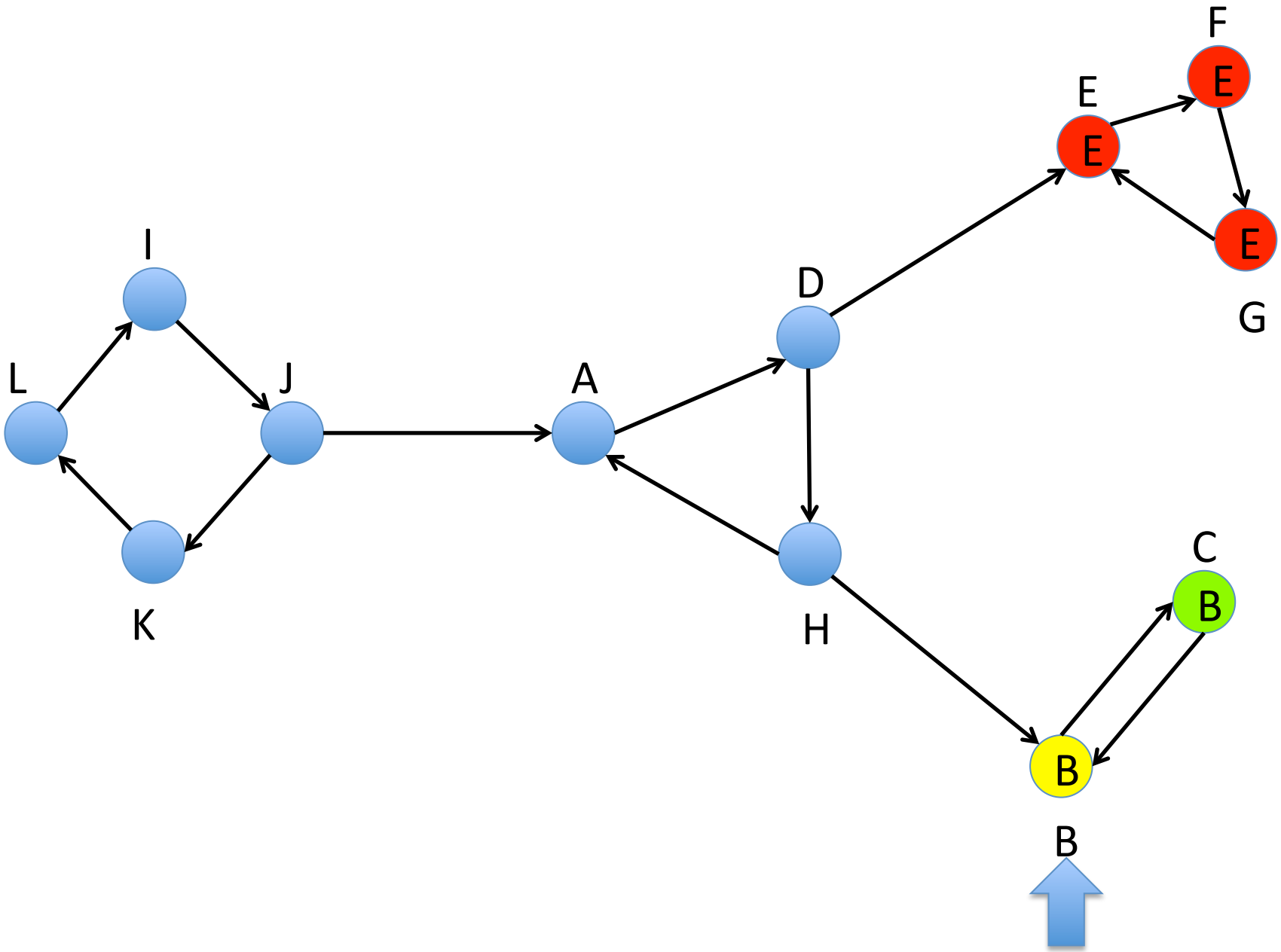

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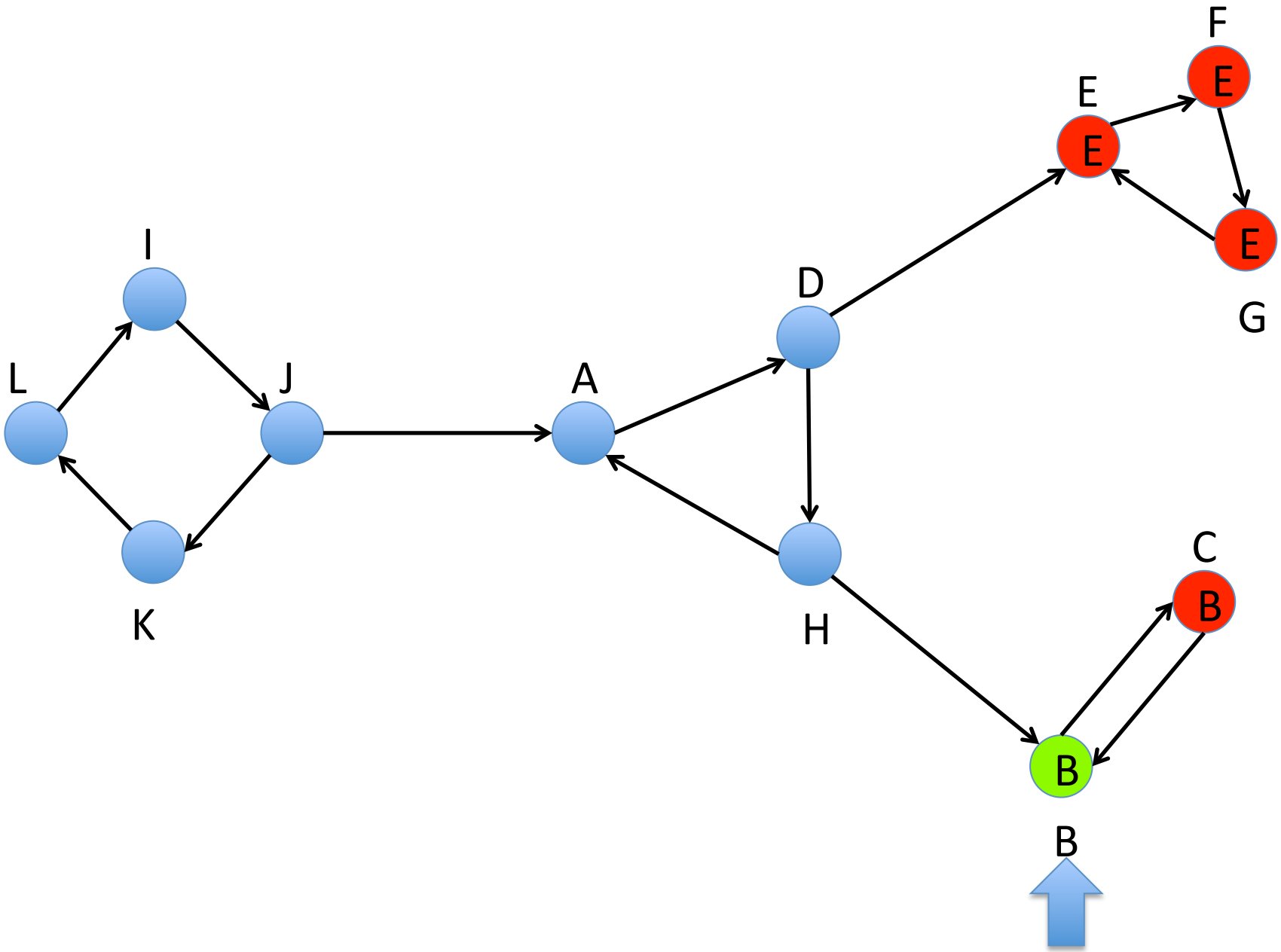

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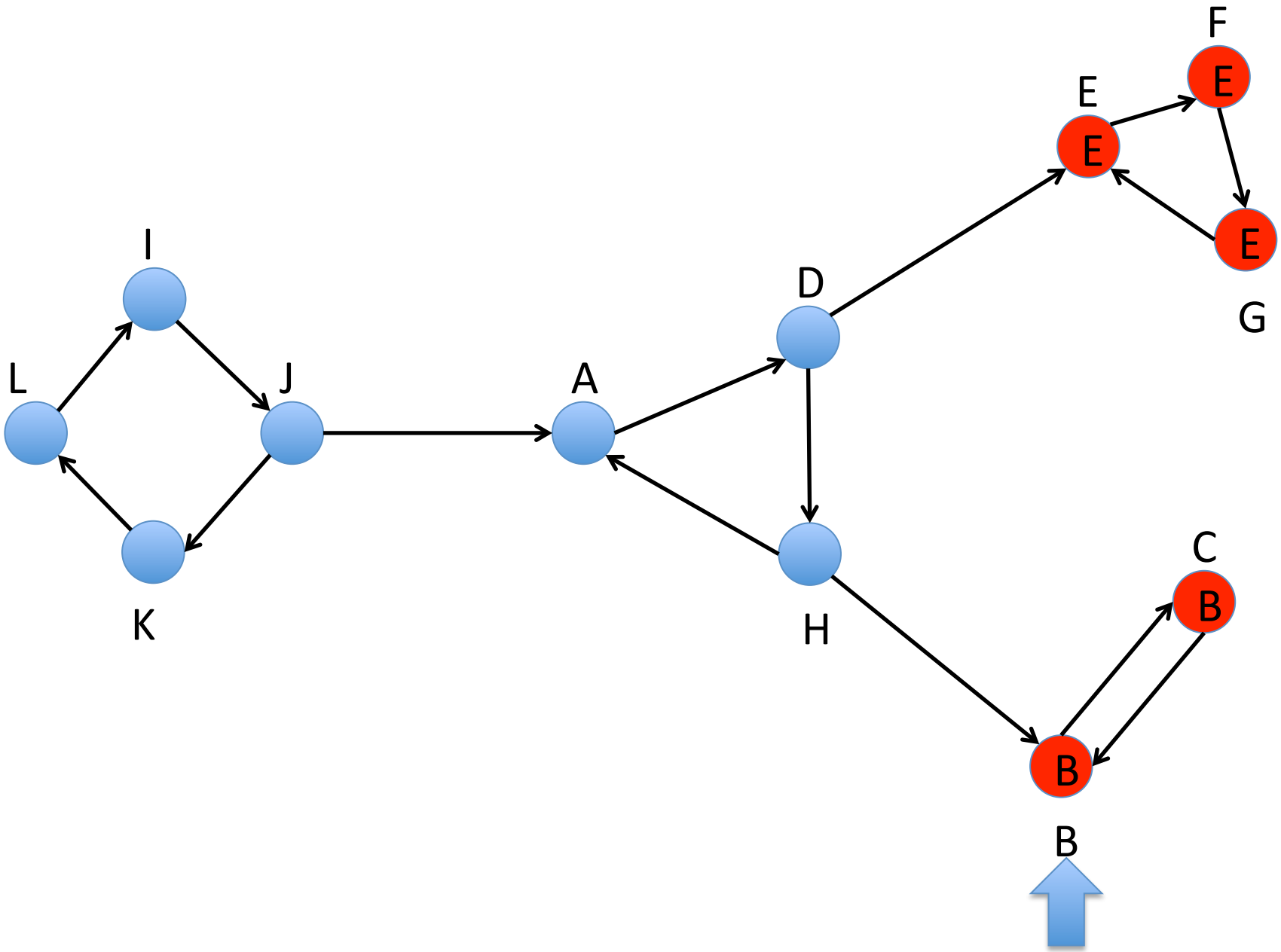

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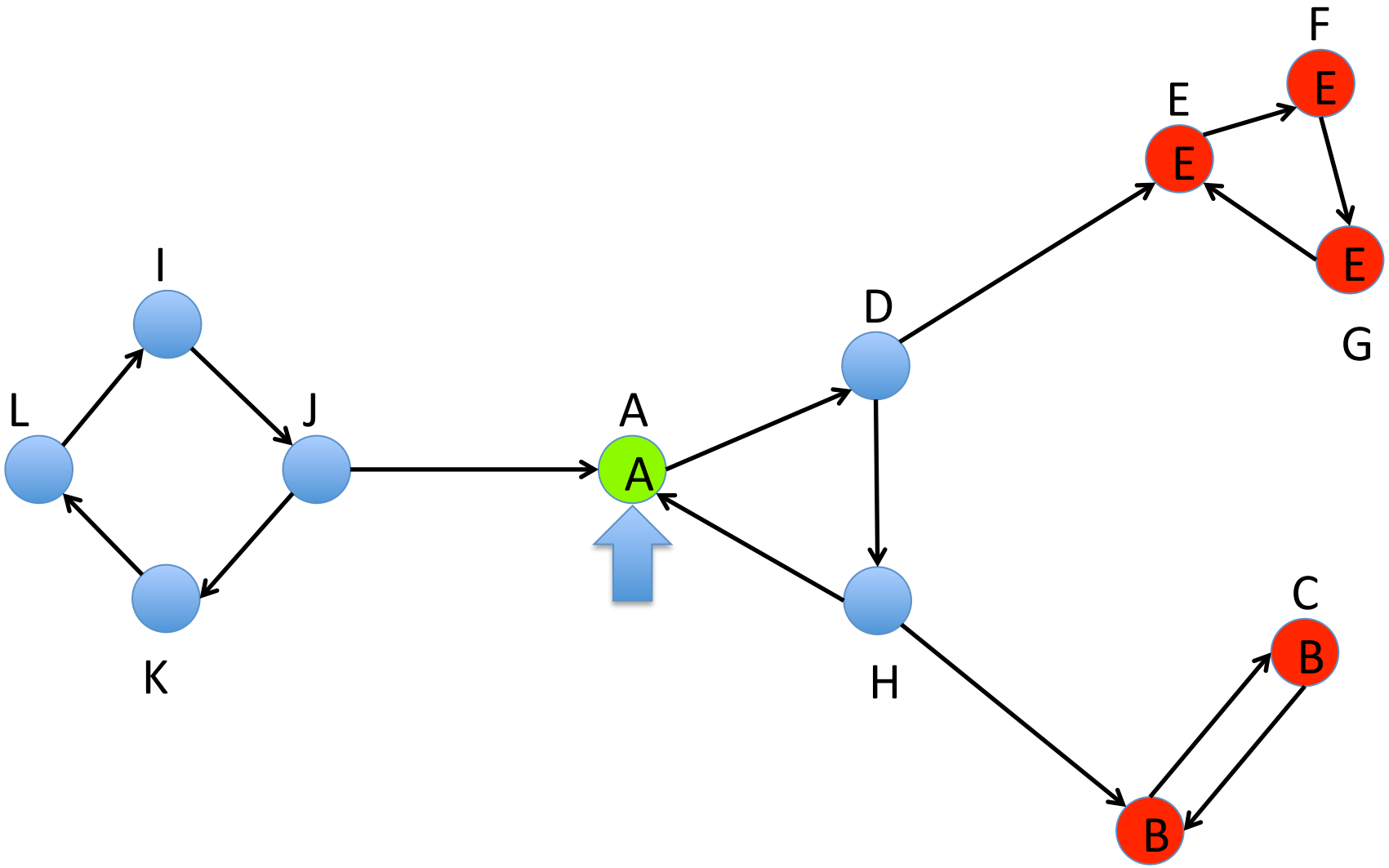

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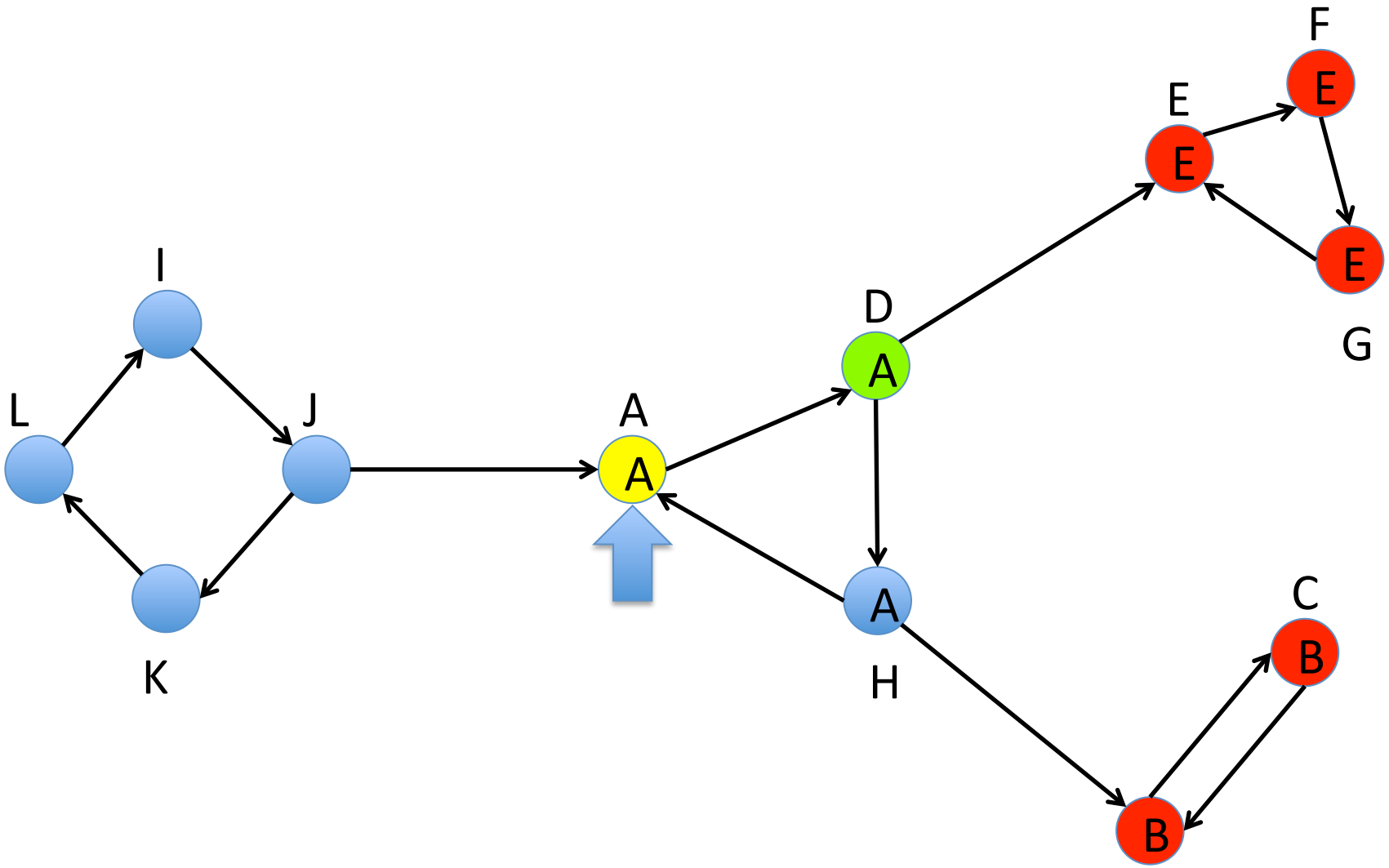

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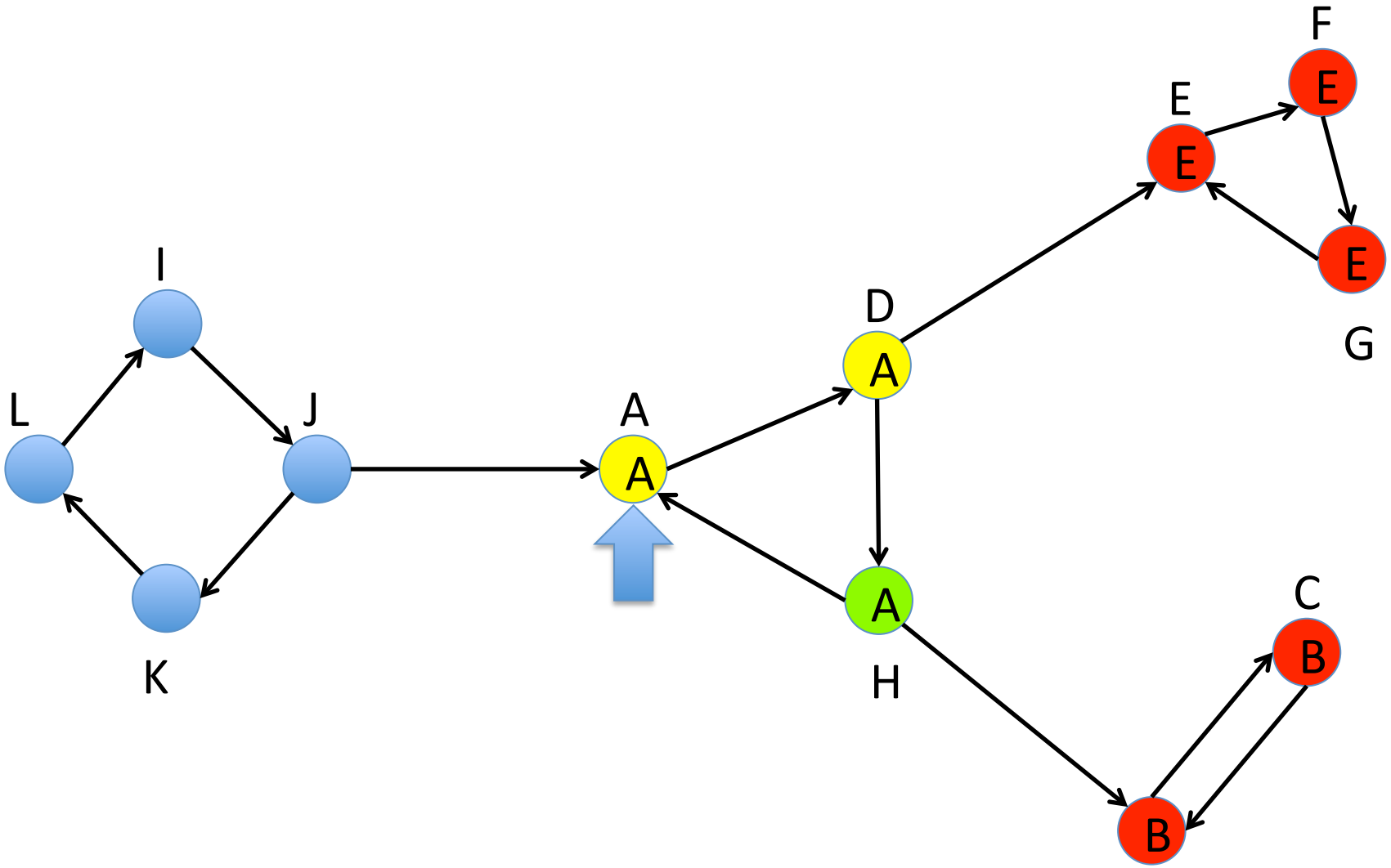

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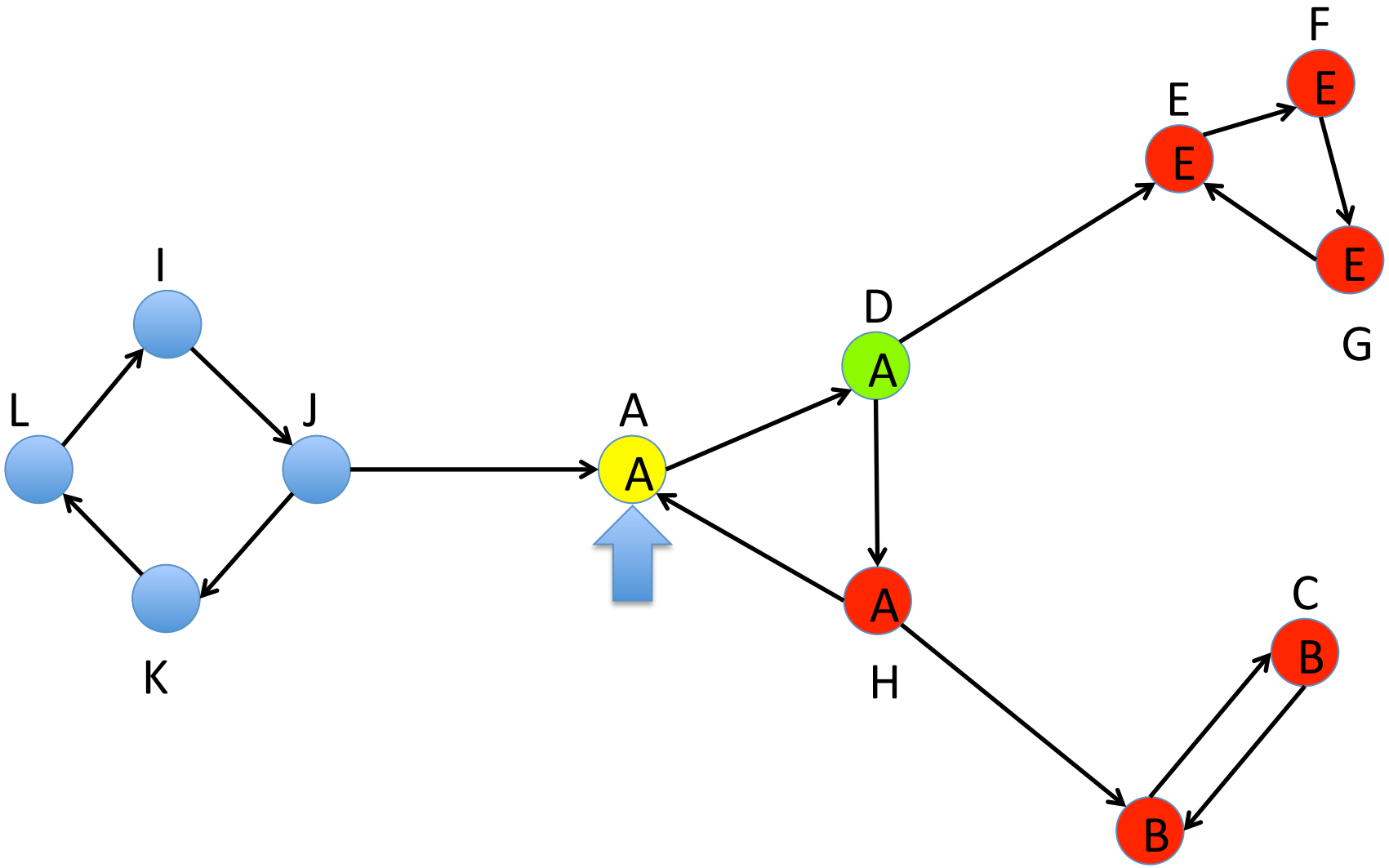

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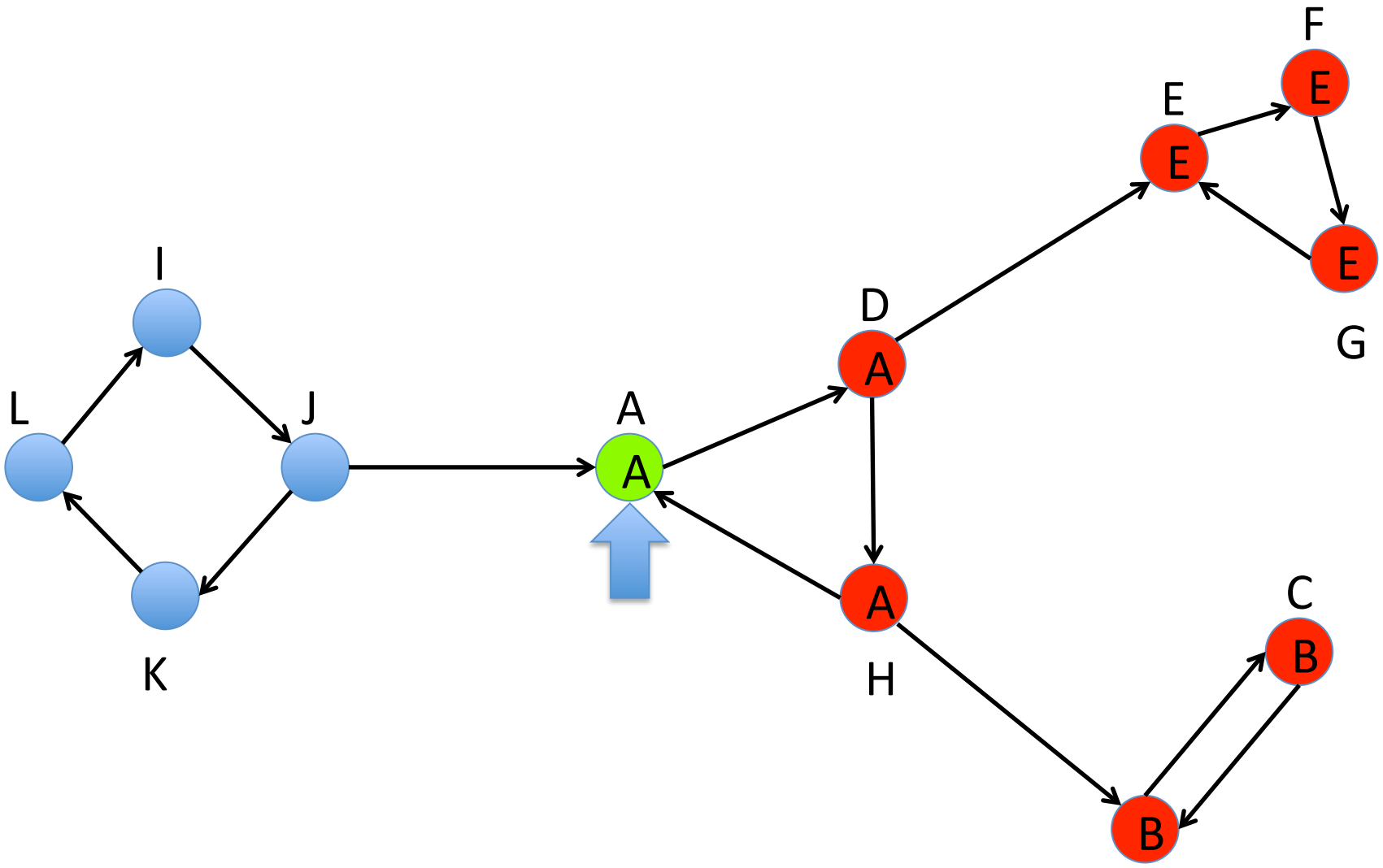

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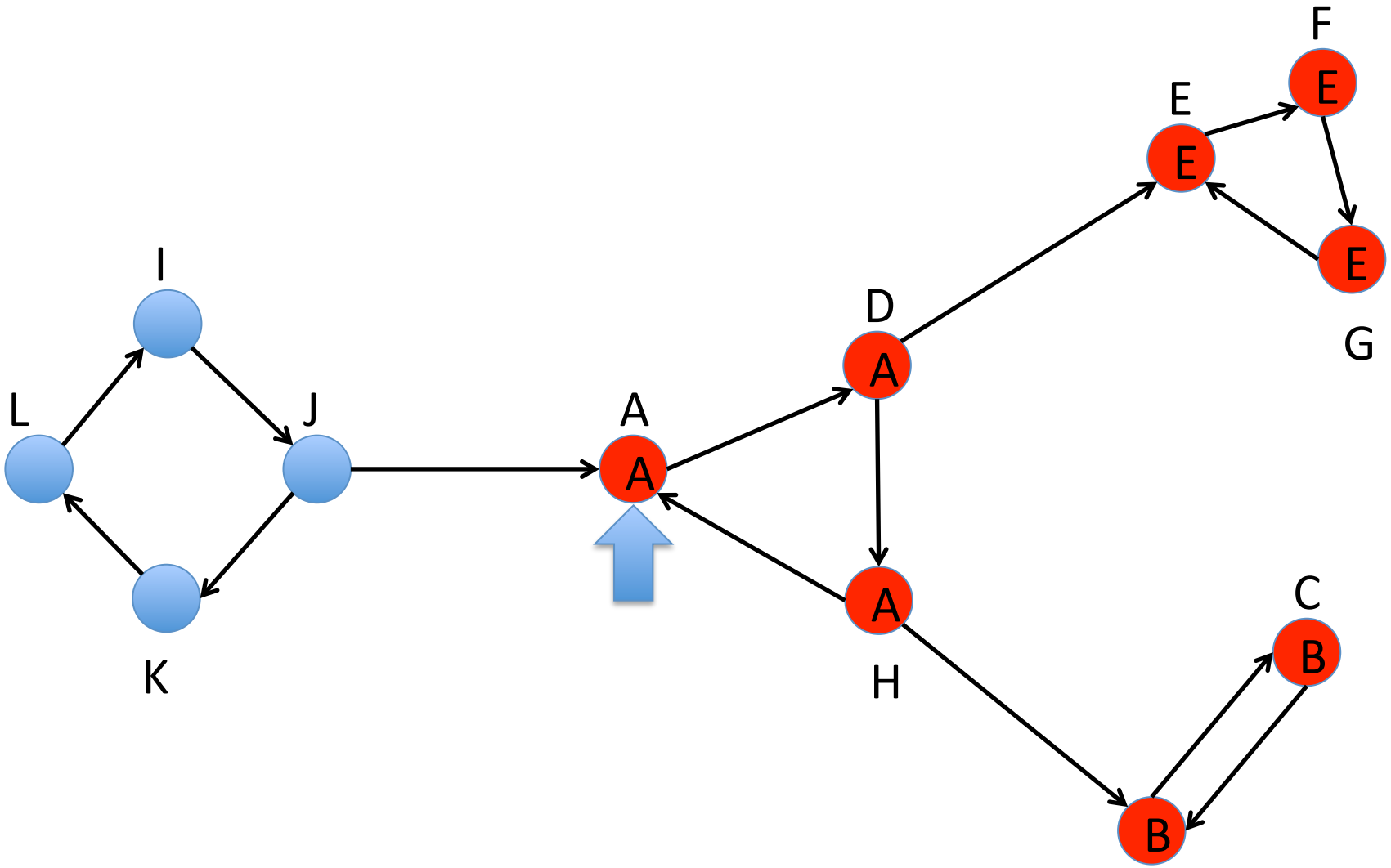

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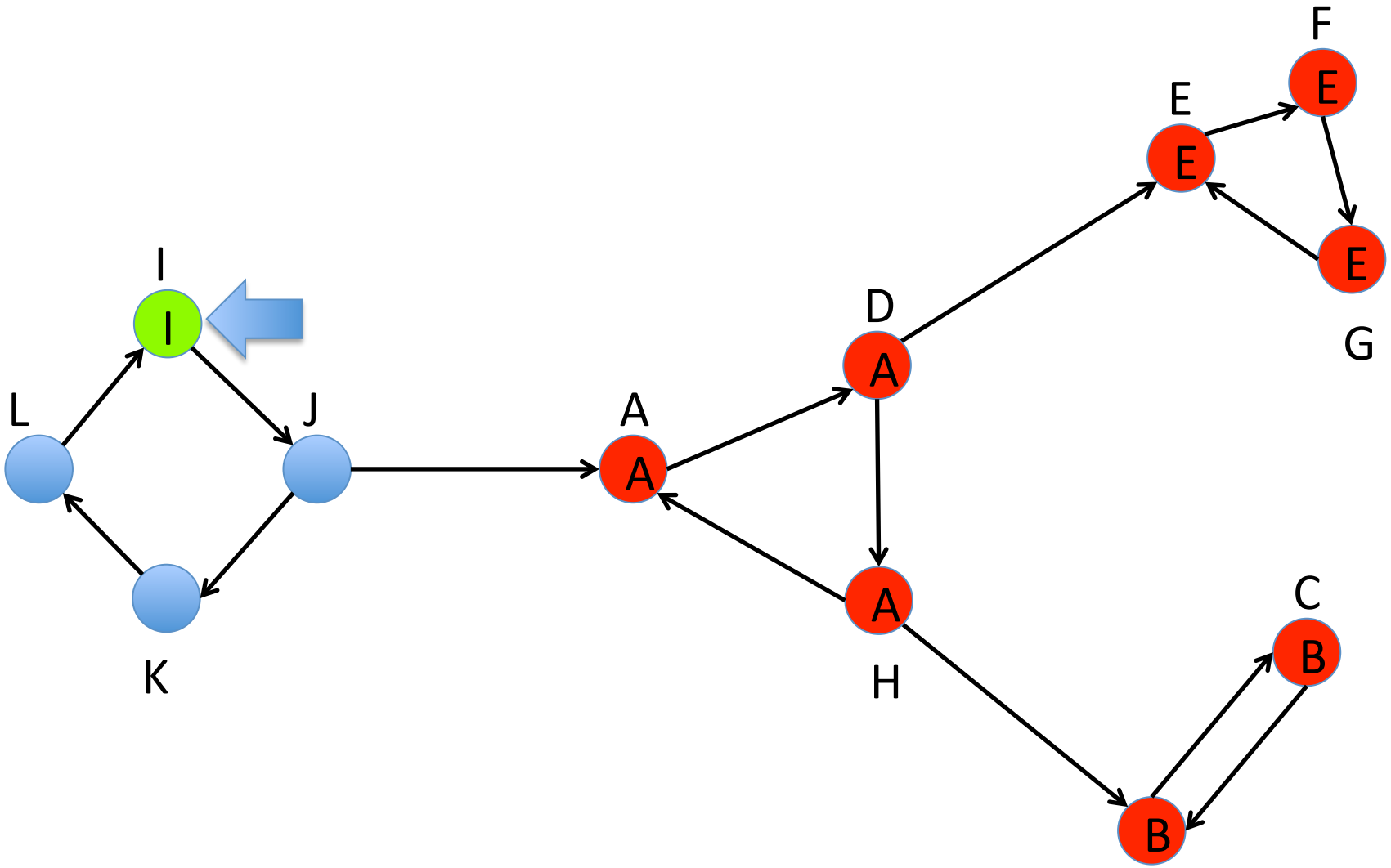

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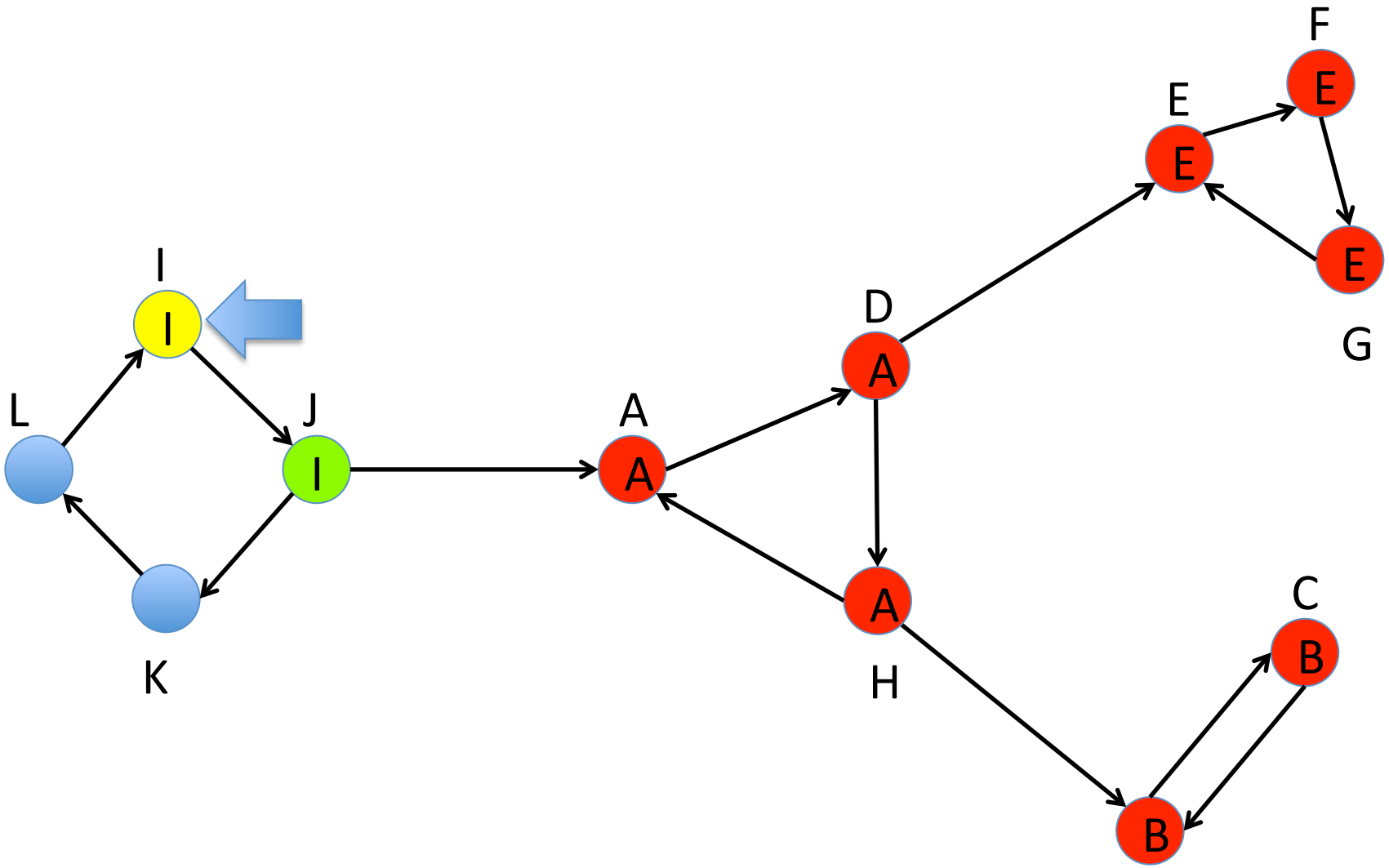

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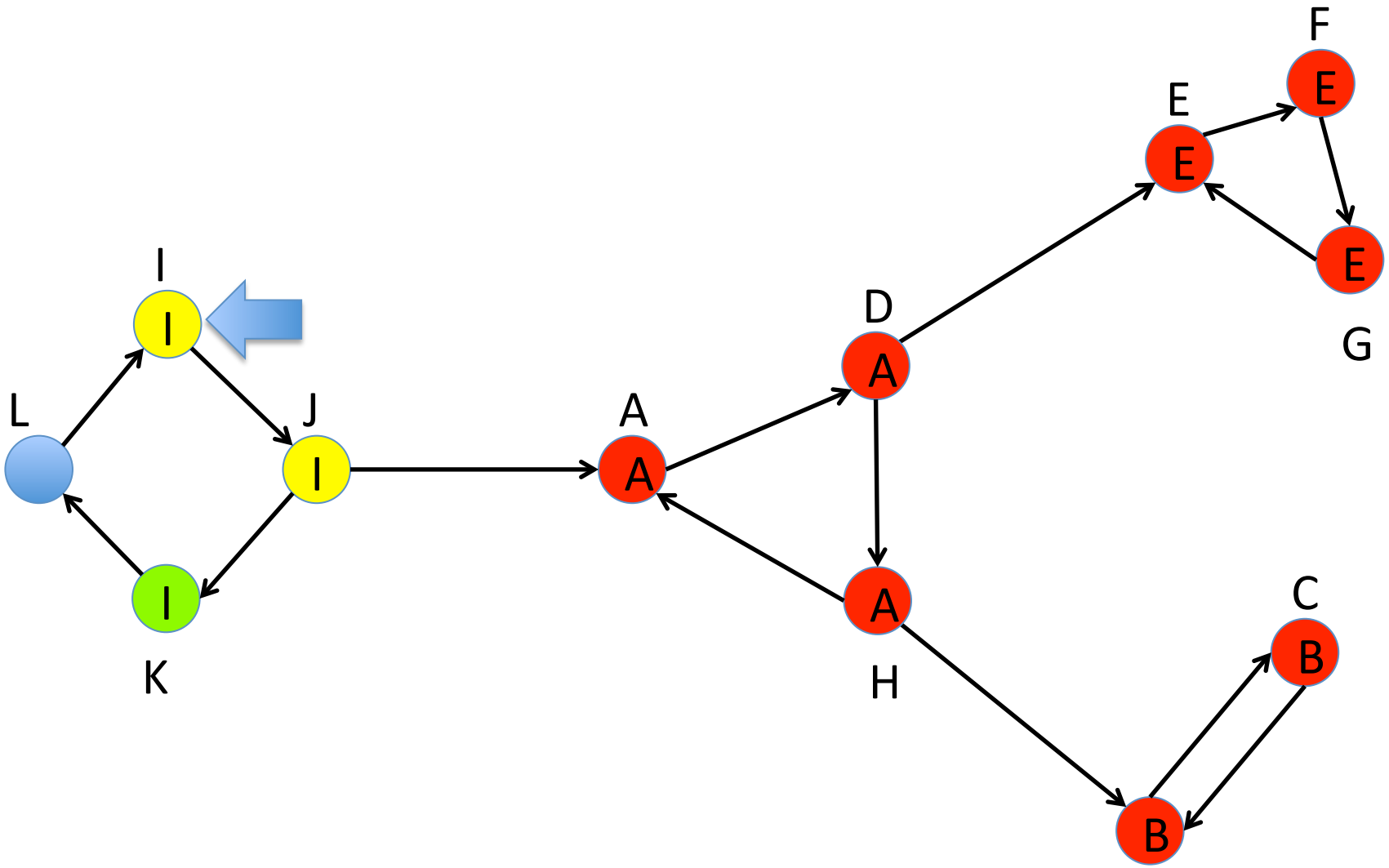

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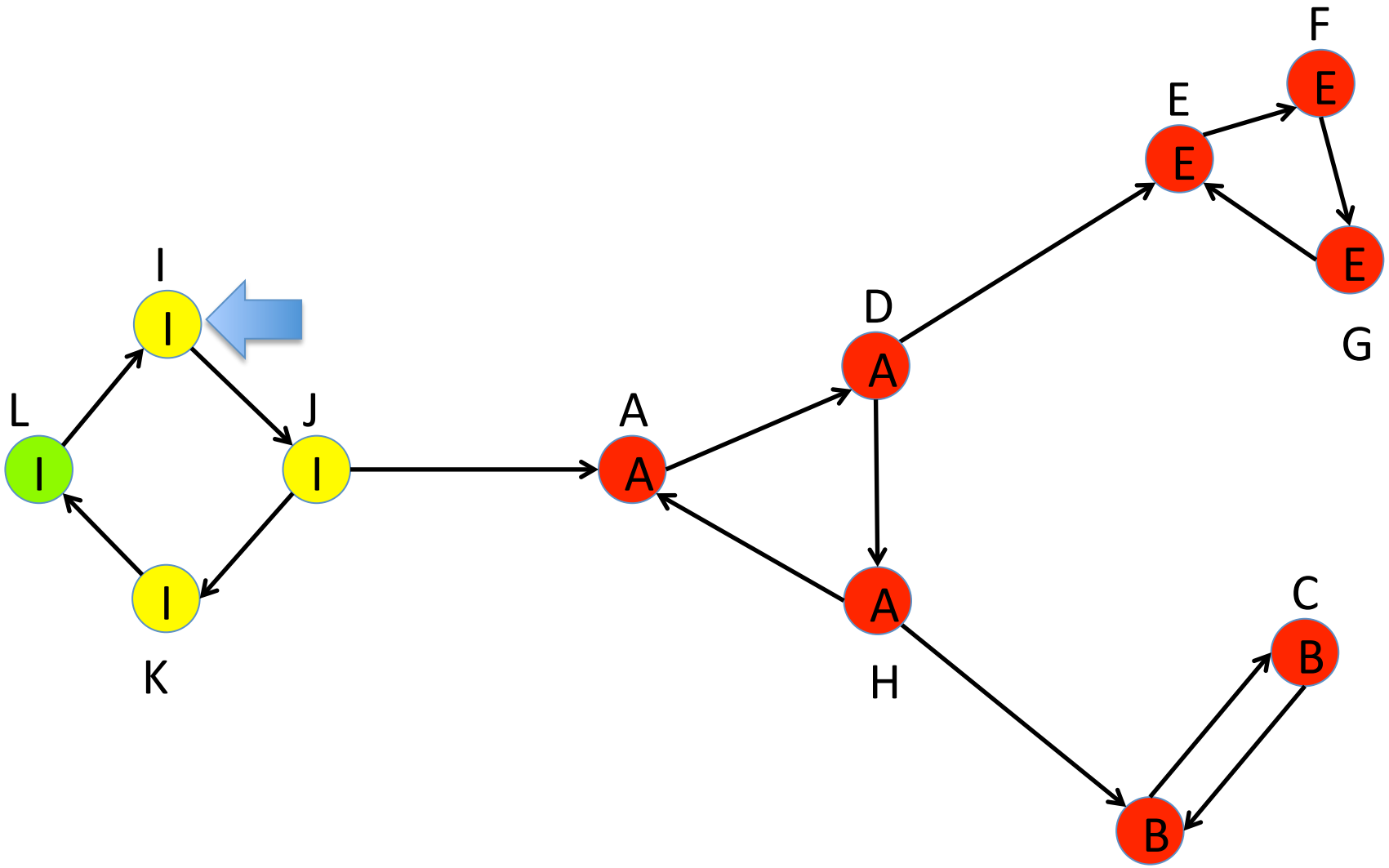

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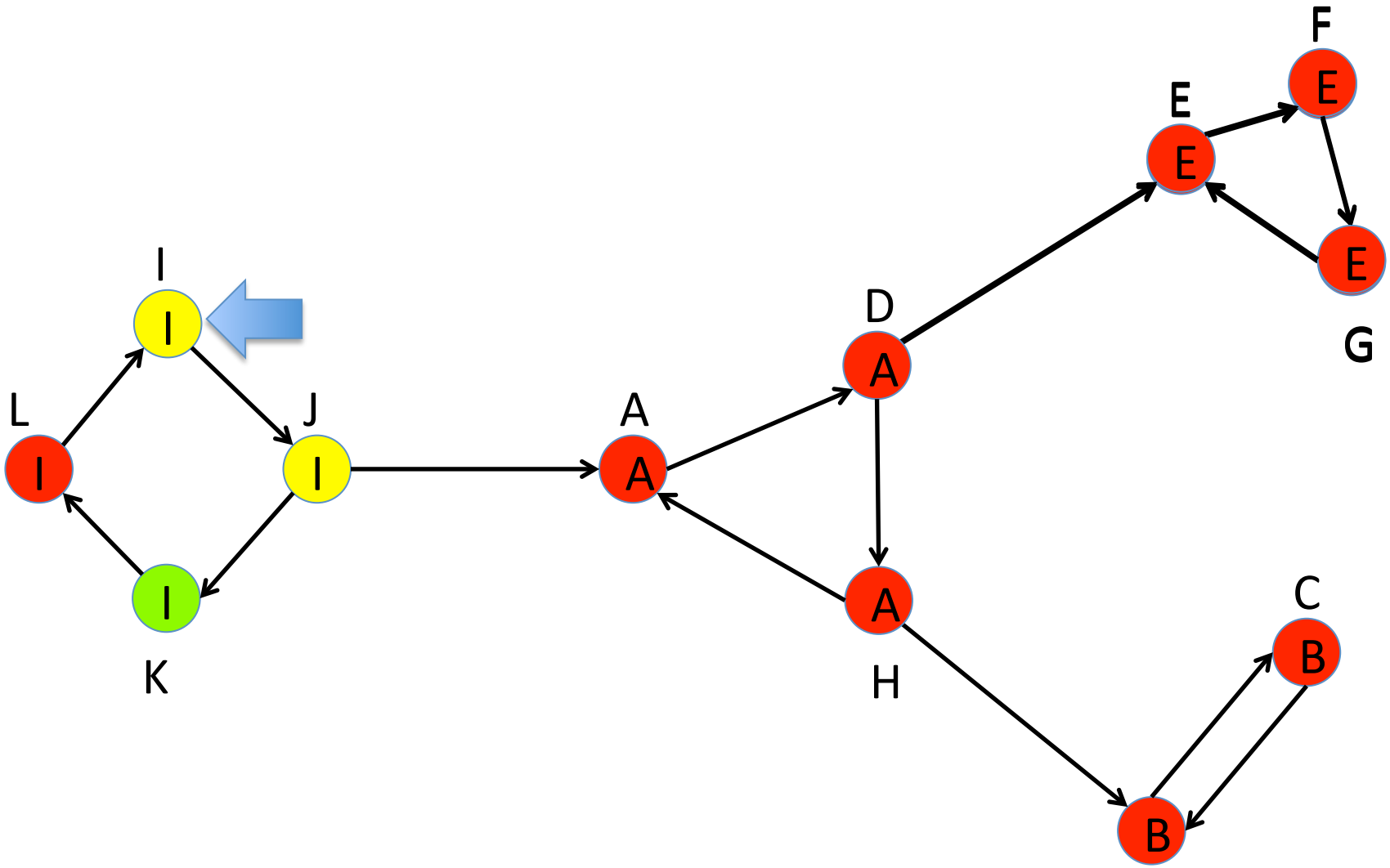

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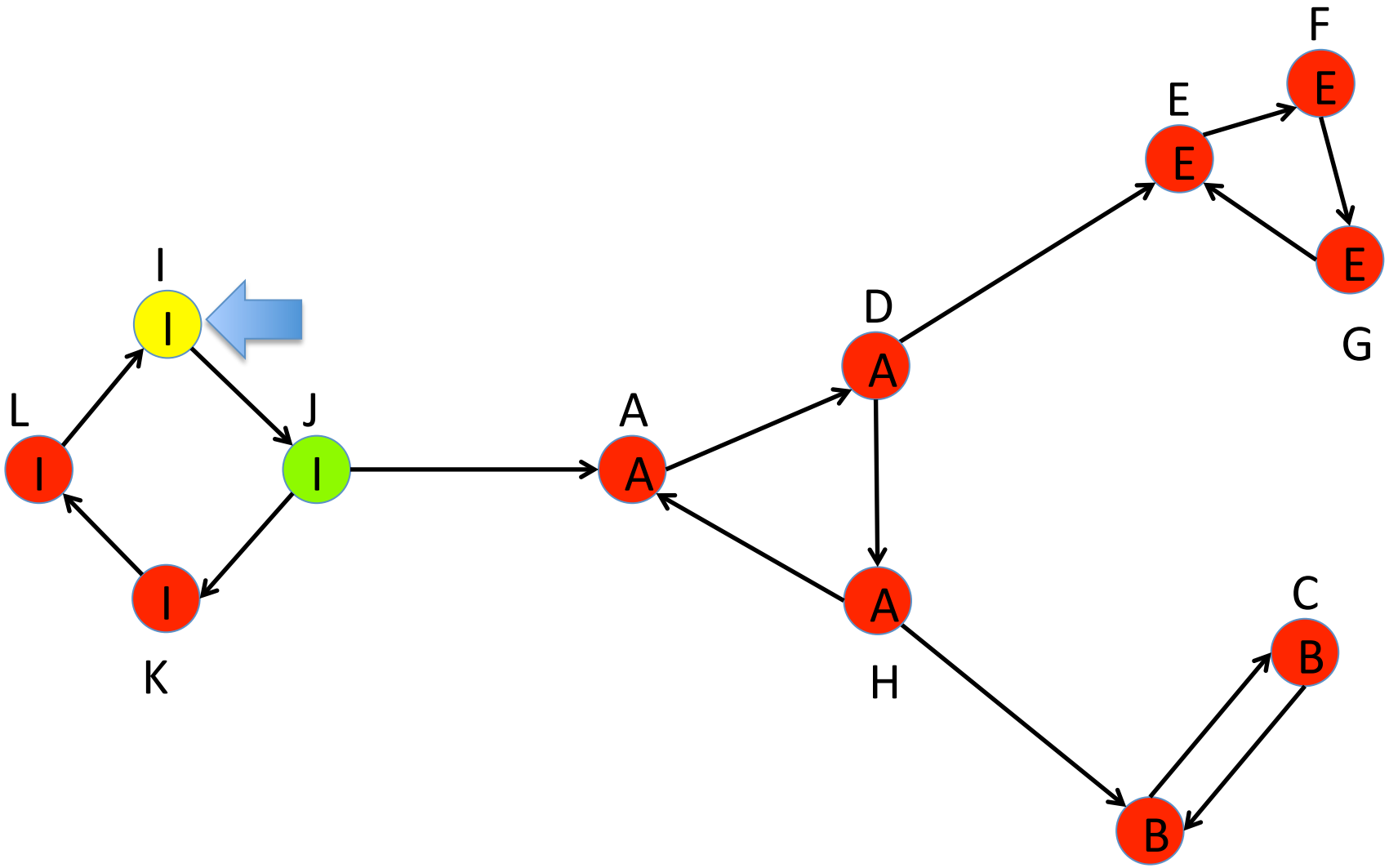

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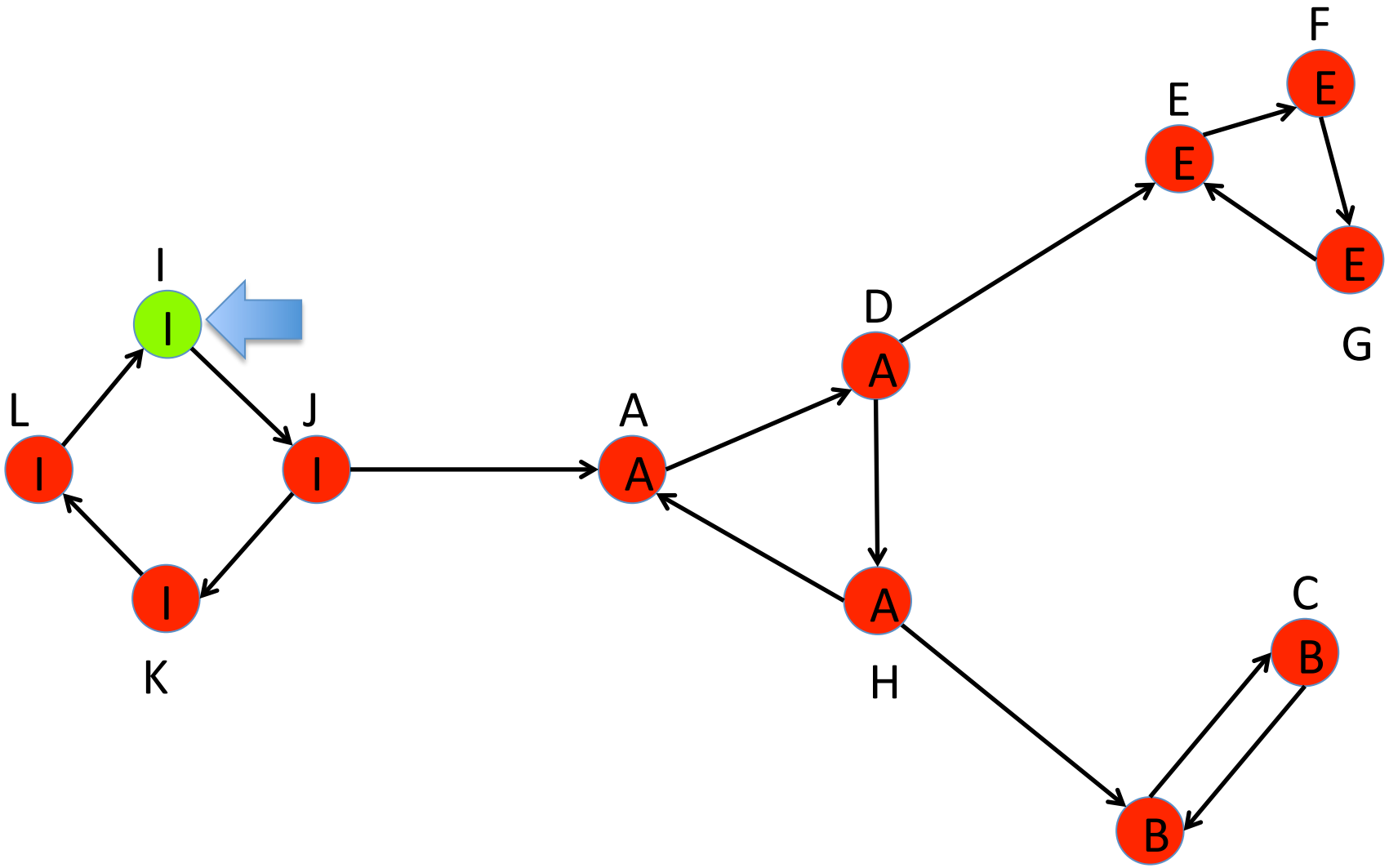

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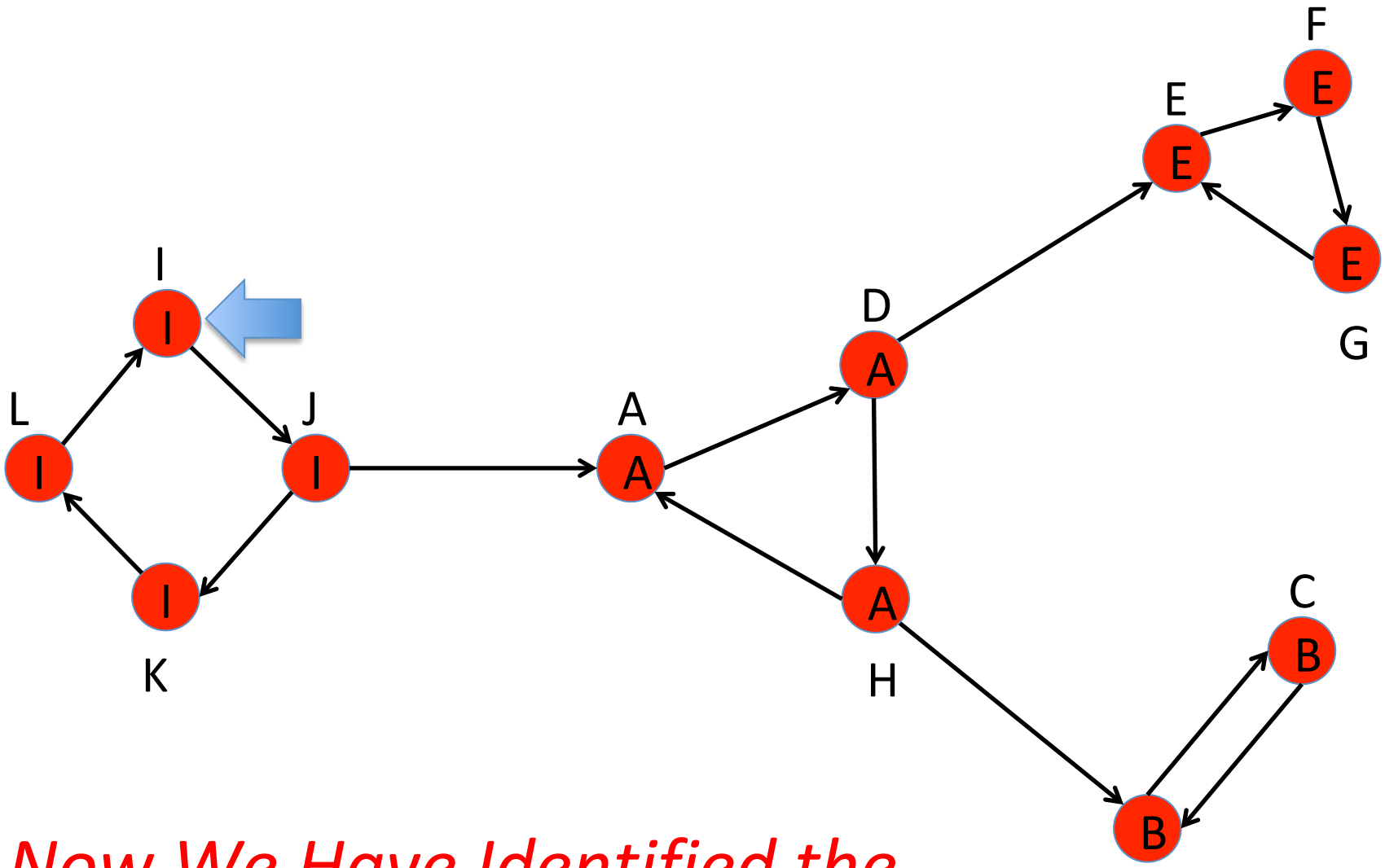
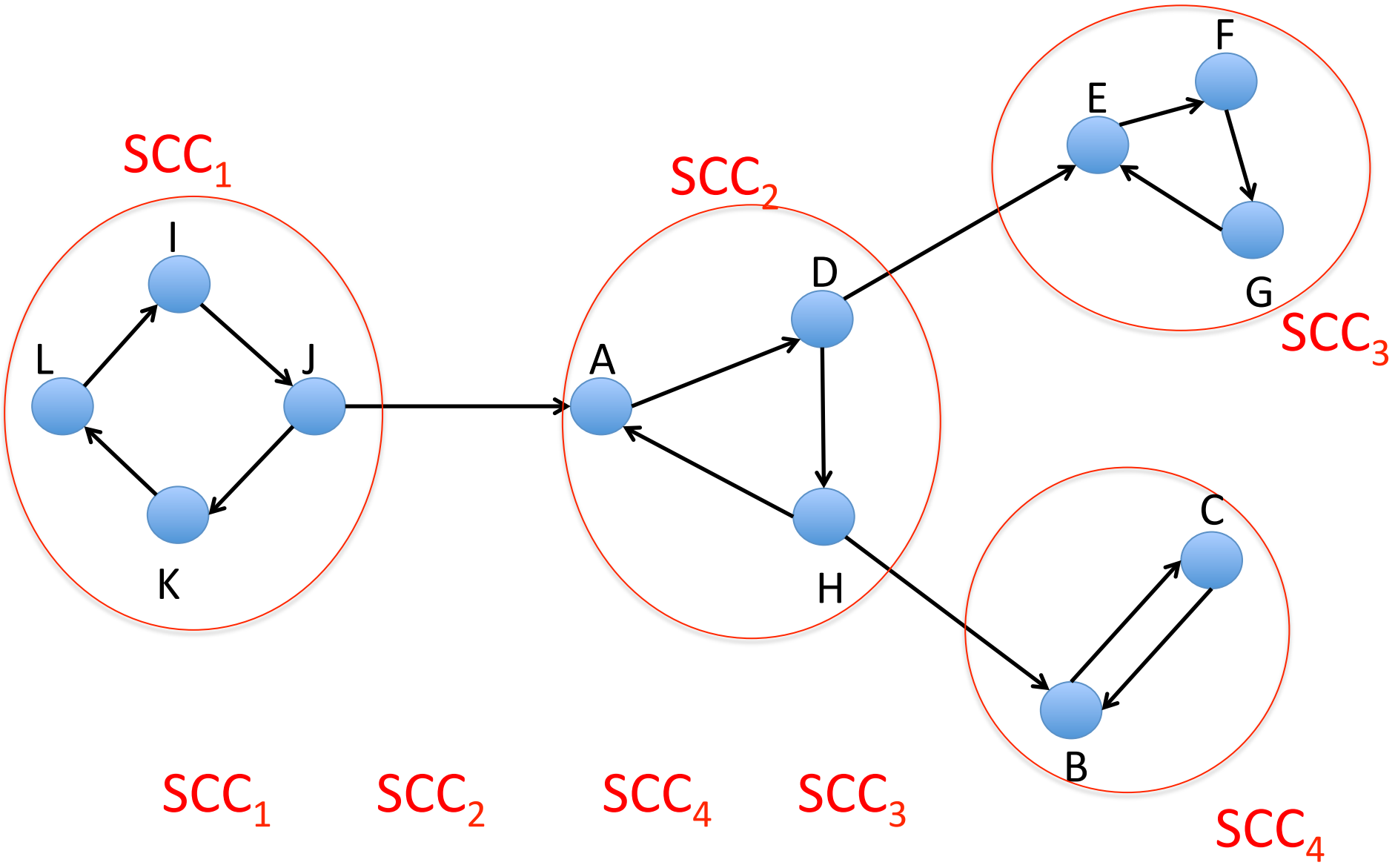


Now We Have Identified the SCCs Correctly!

Need to start BFS/DFS from a vertex in reverse topological order of G^{SCC}



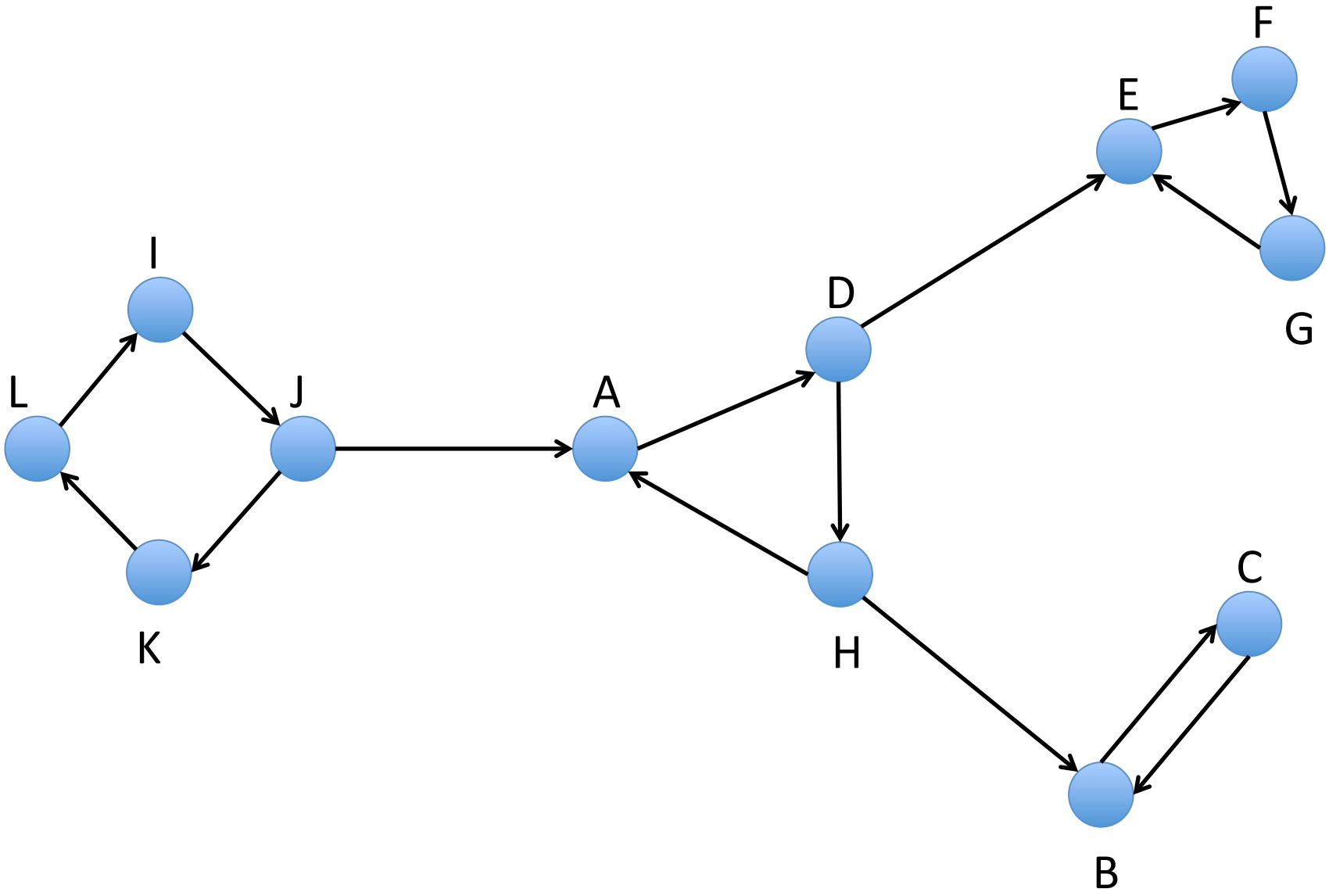
Key Idea in Kosaraju's Algorithm

- If we can start a BFS/DFS from vertices in reverse topological order of G^{SCC}
- In other words, if we can always pick a start vertex from a sink SCC
- We can use BFS/DFS to identify the components correctly!

Goal: Do some preprocessing to guarantee that we always start a traversal from vertex from a sink SCC!

How?

Let's Try DFS & Finishing Times (1)



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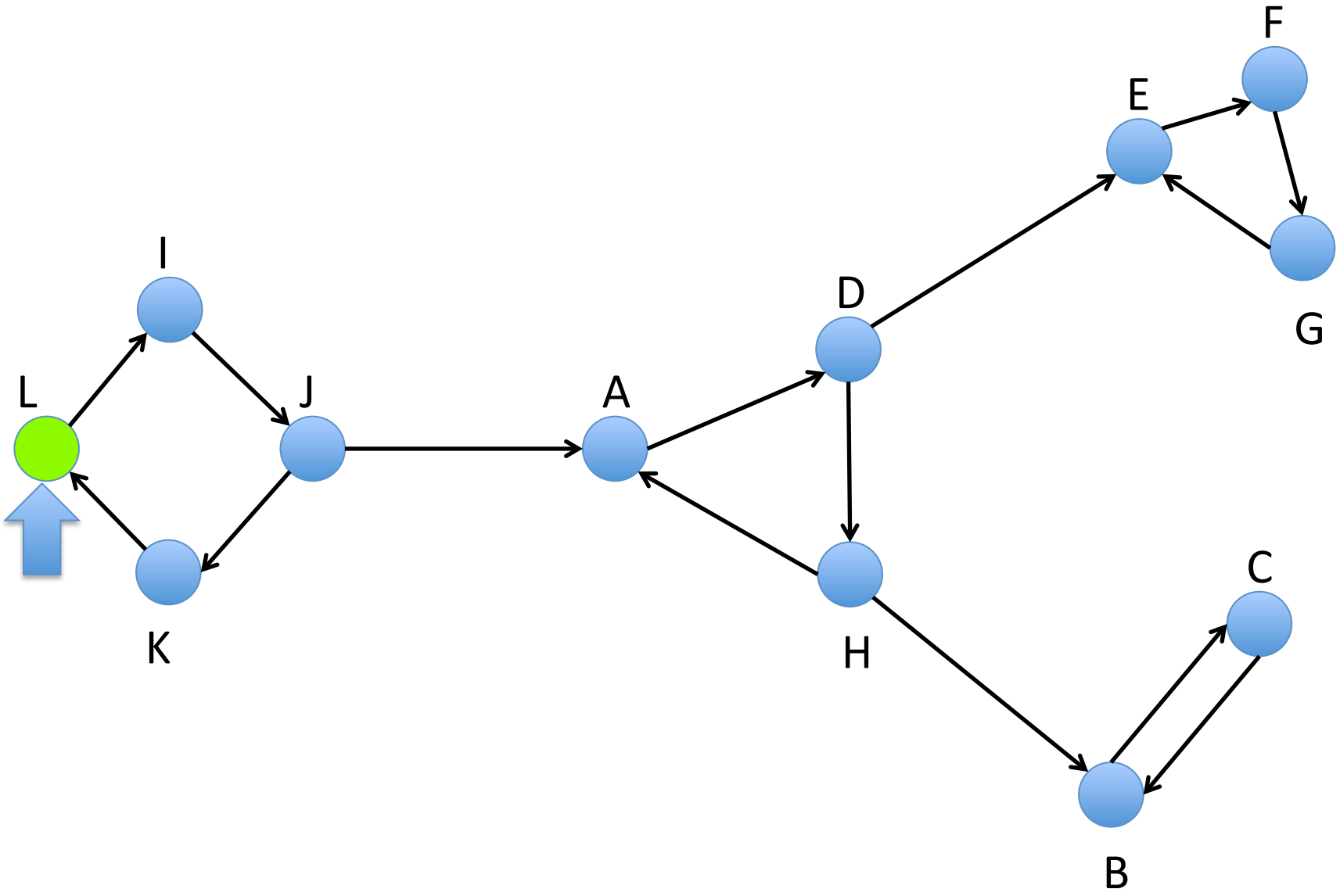

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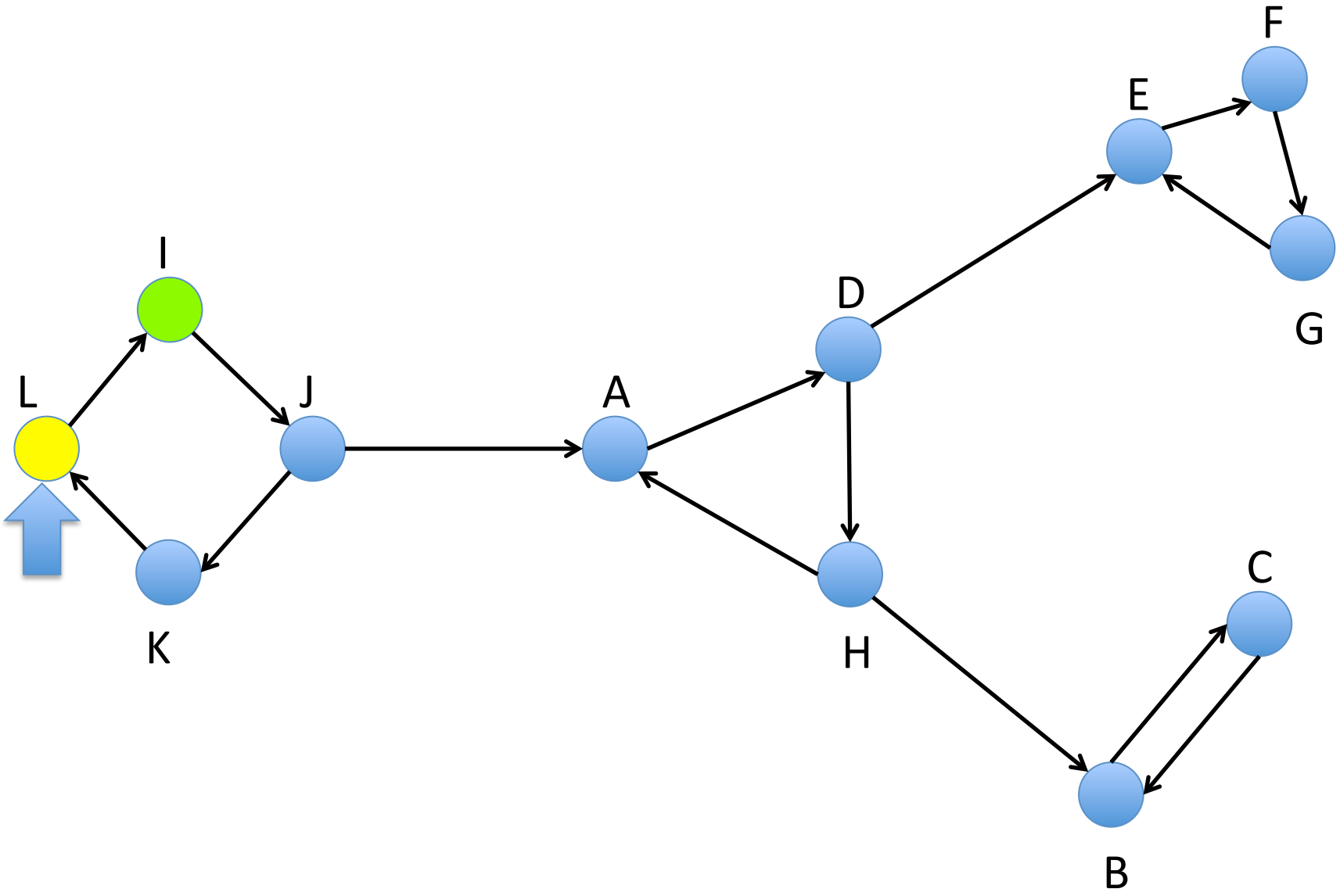

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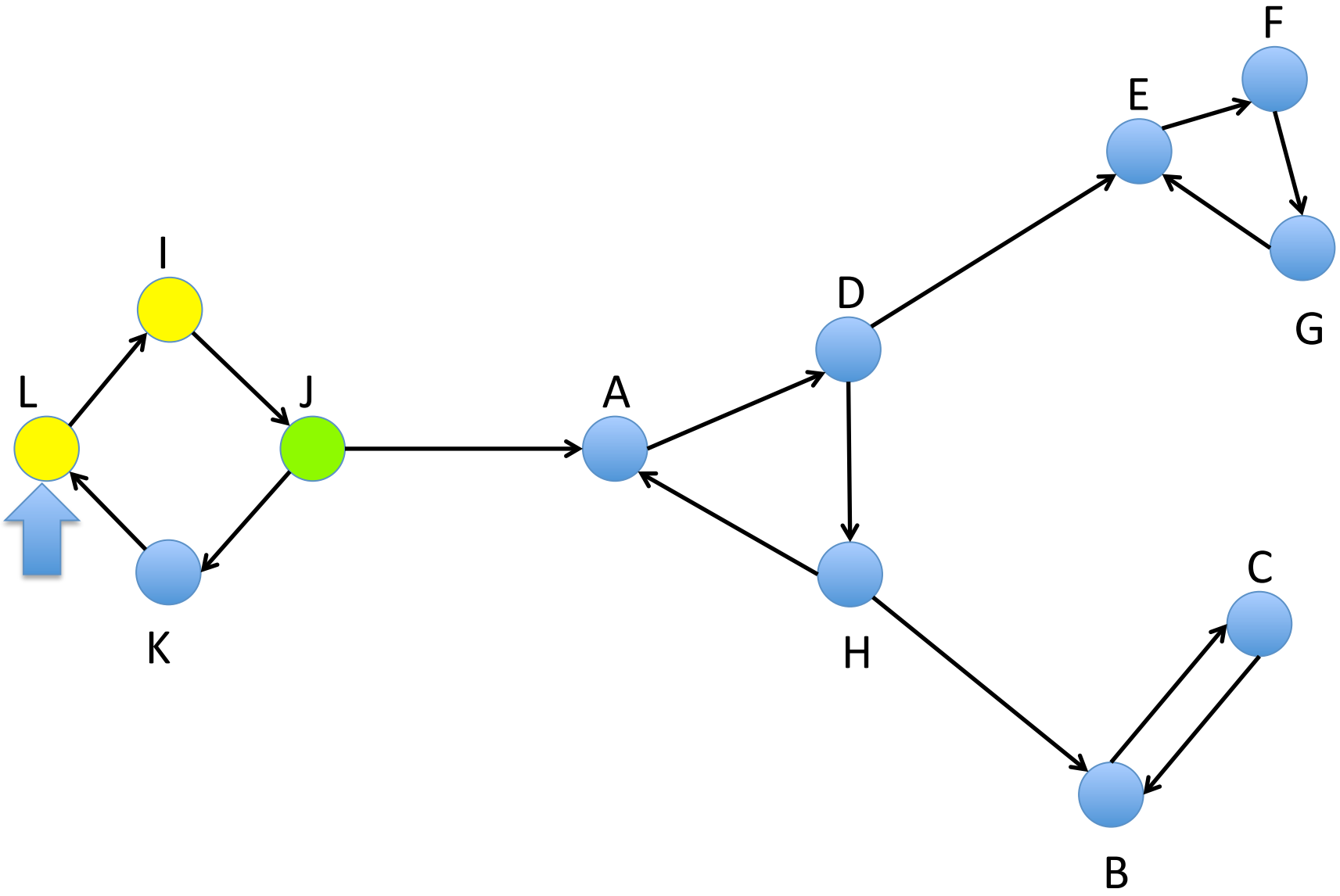

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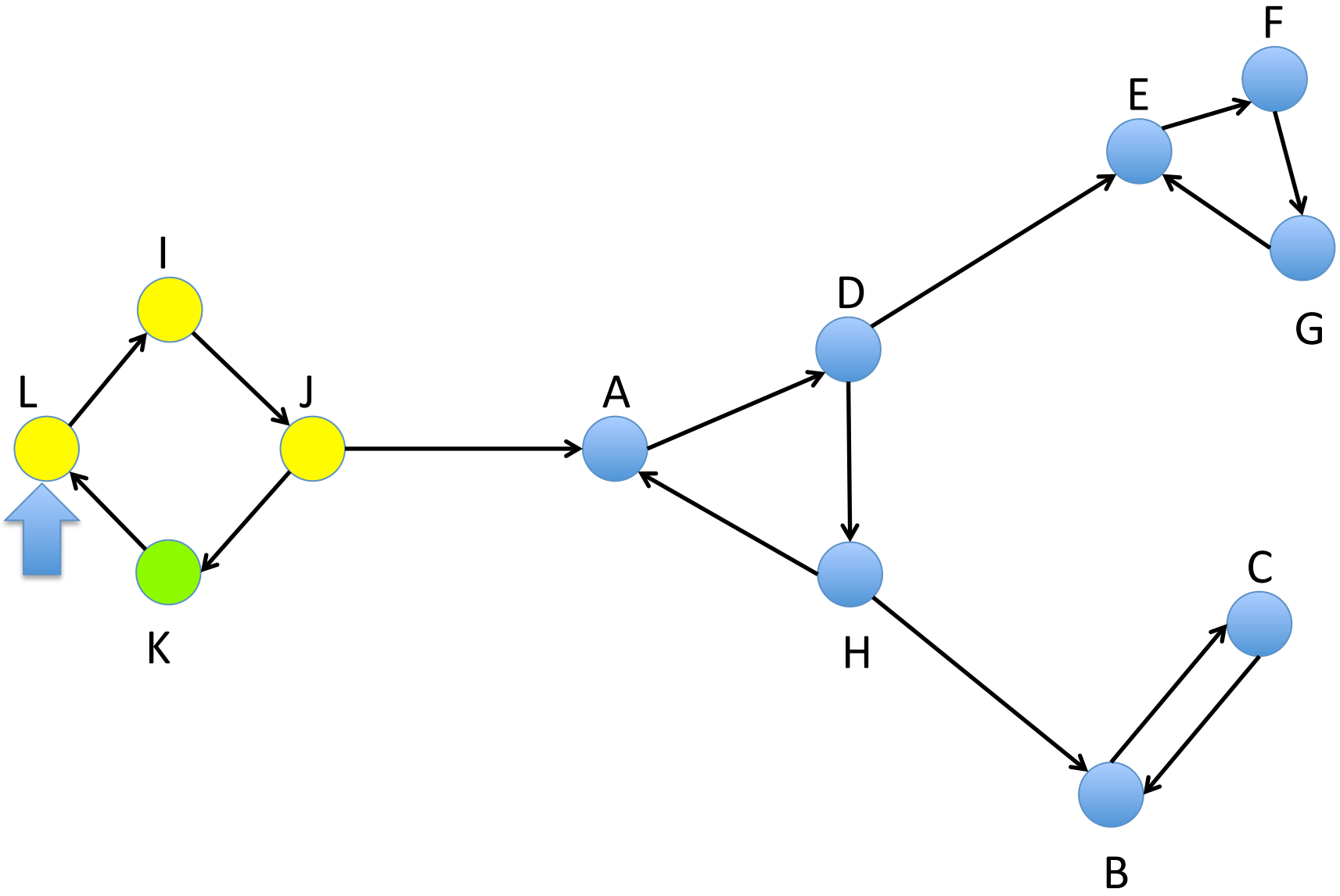

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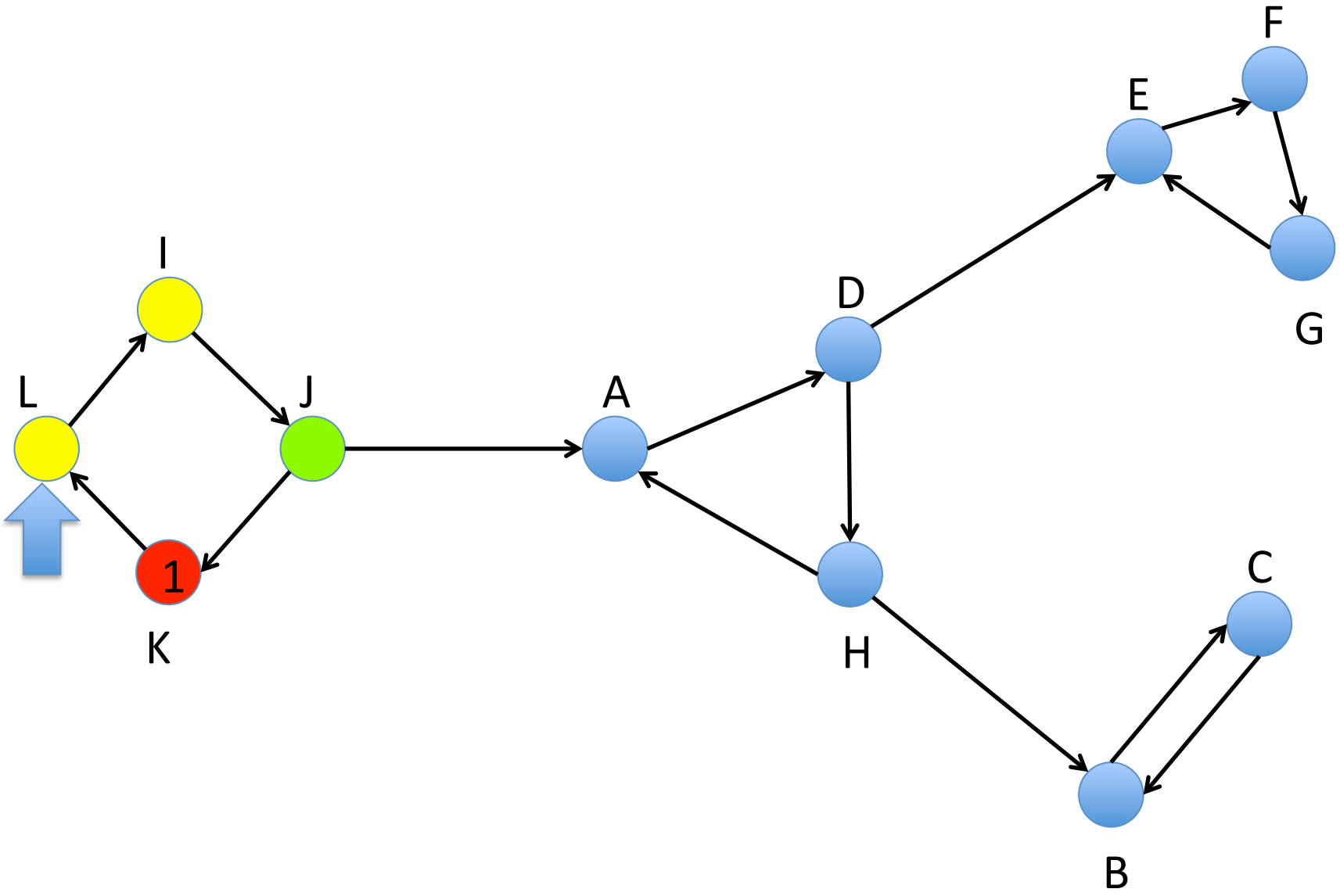

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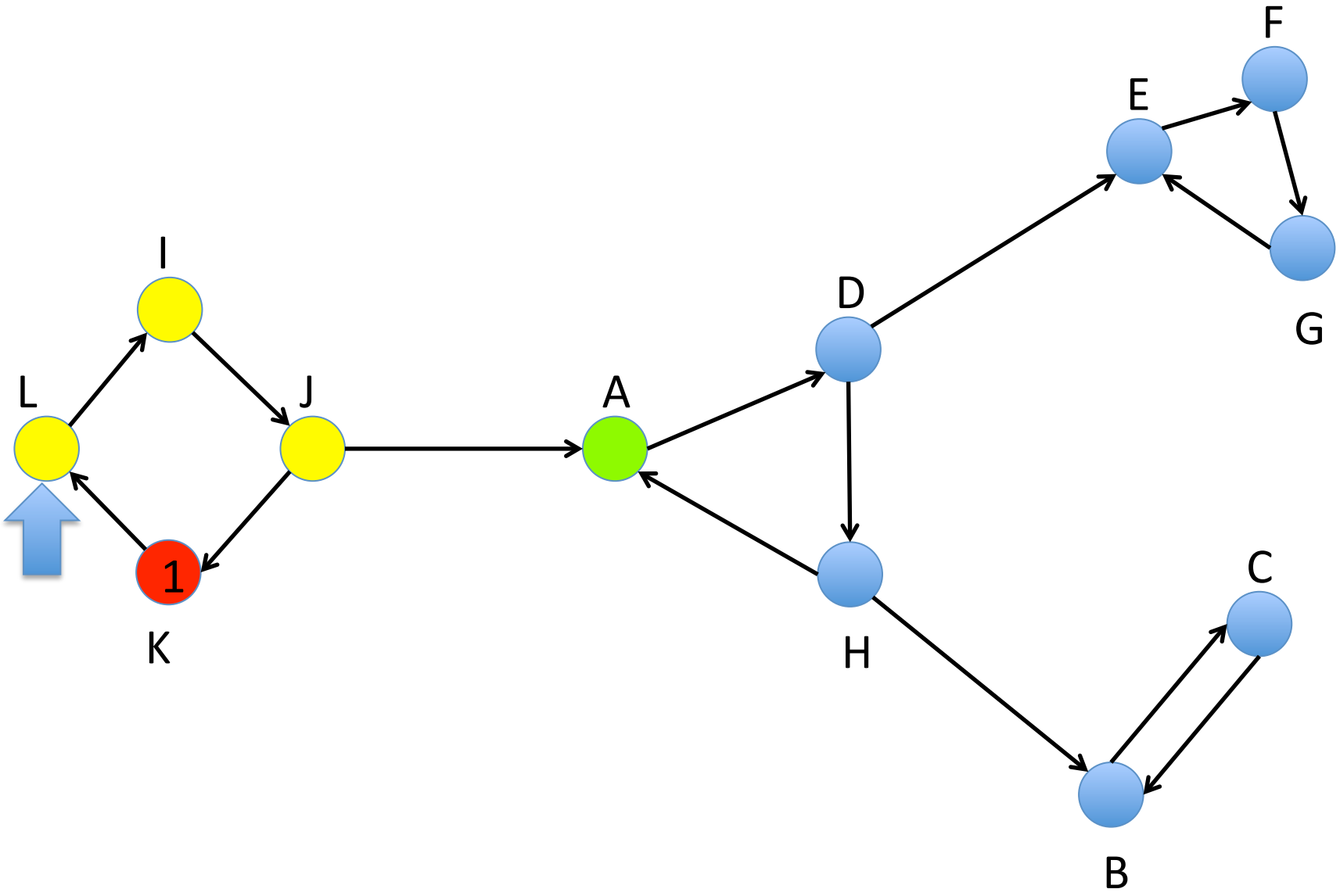

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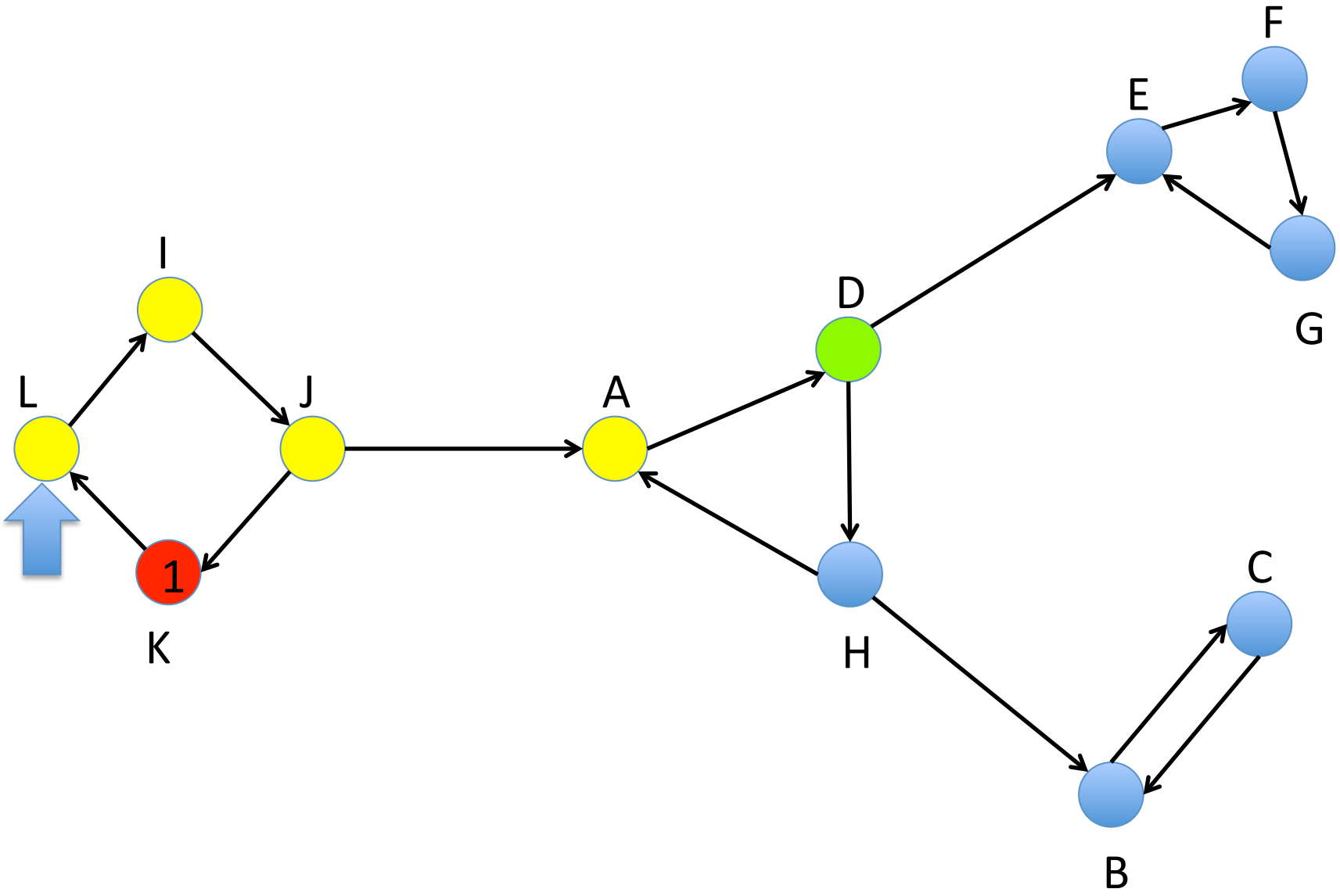

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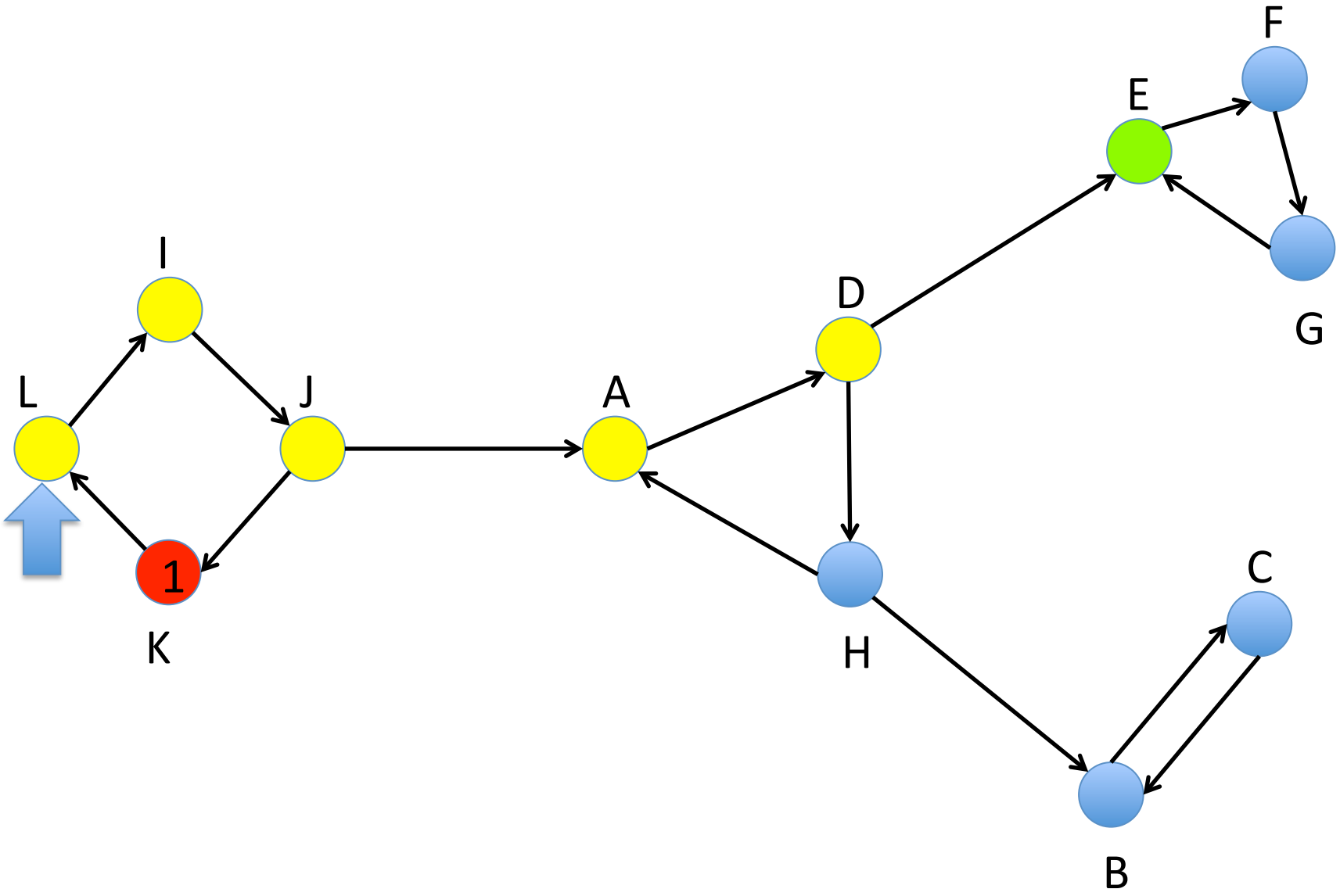

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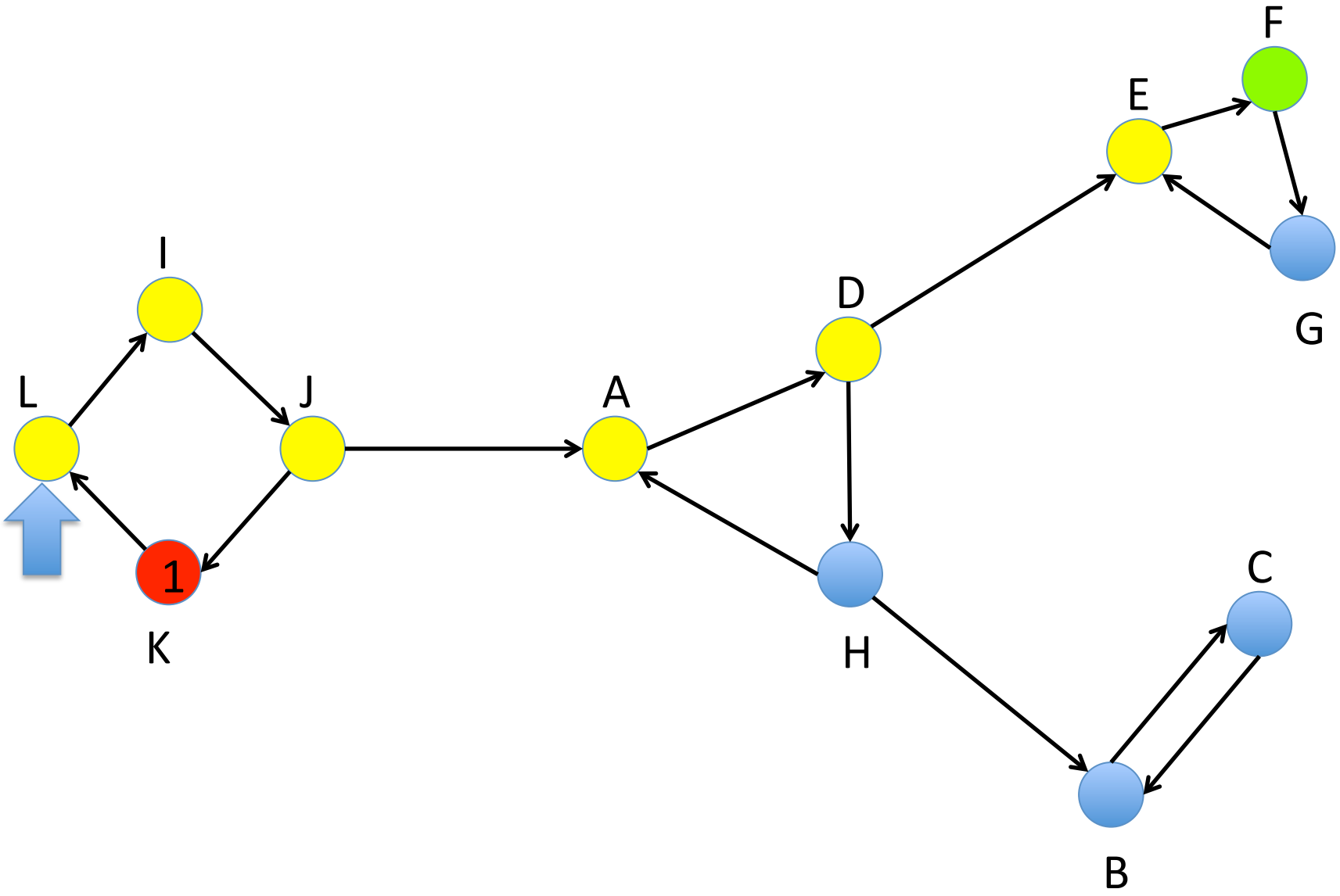

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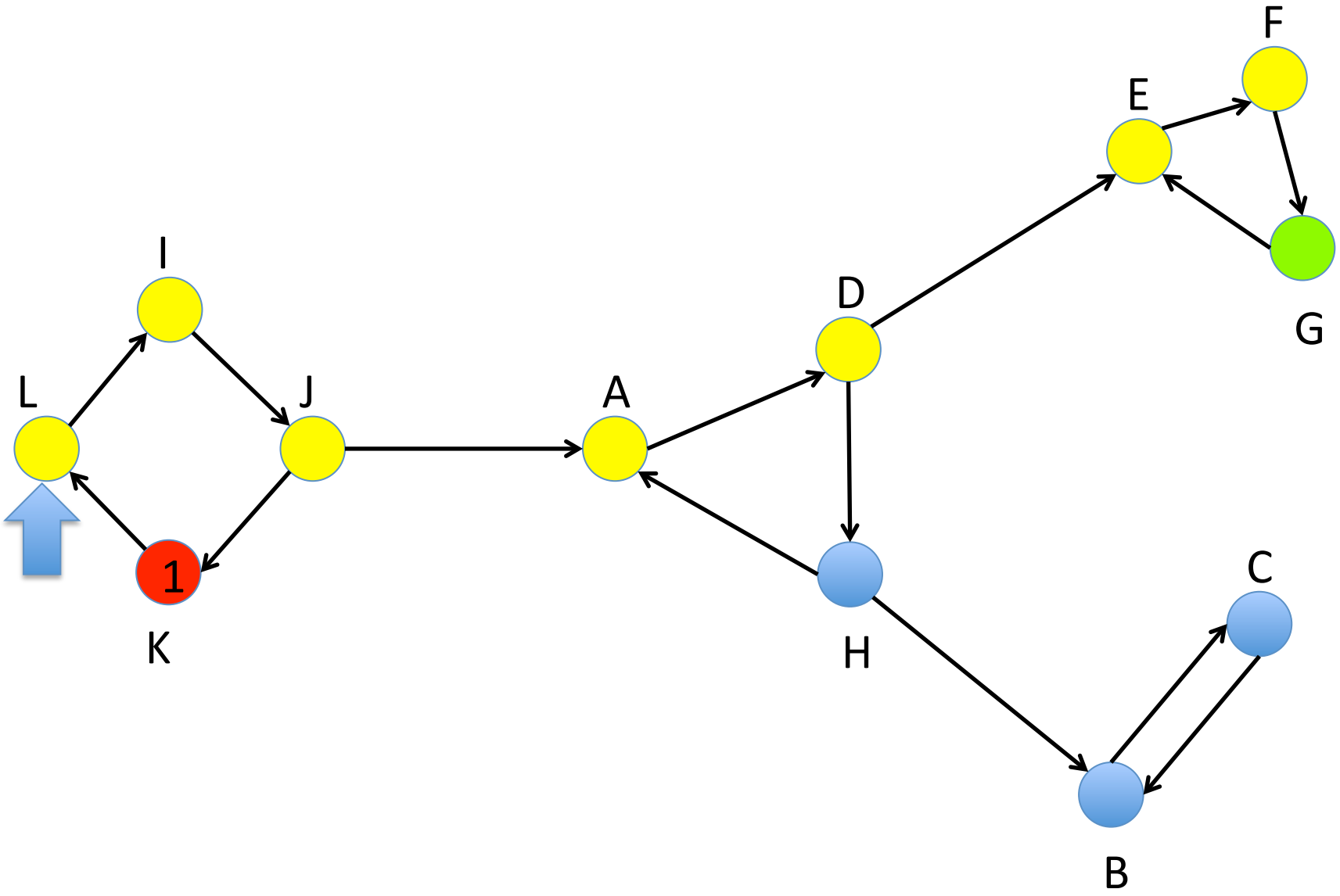

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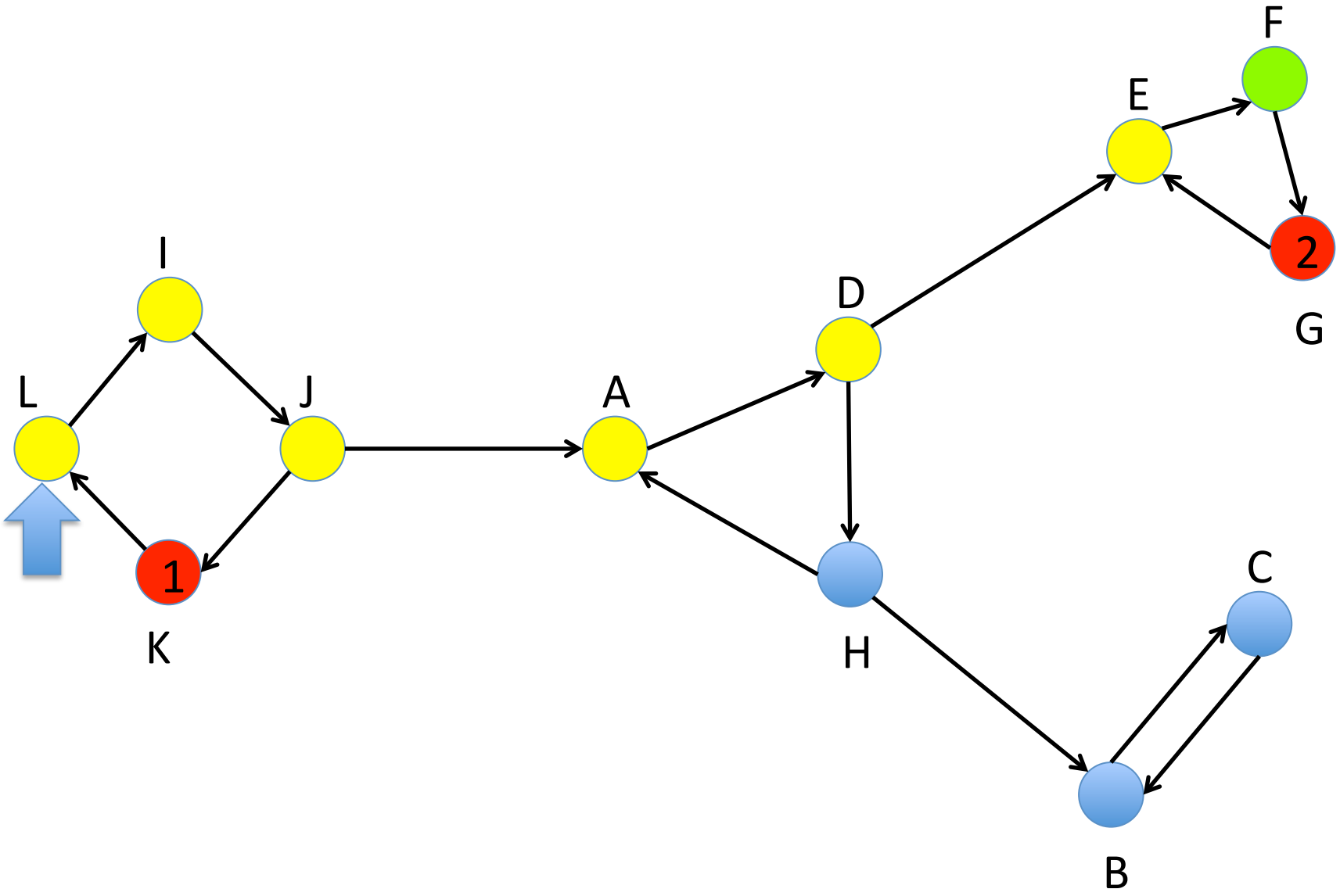

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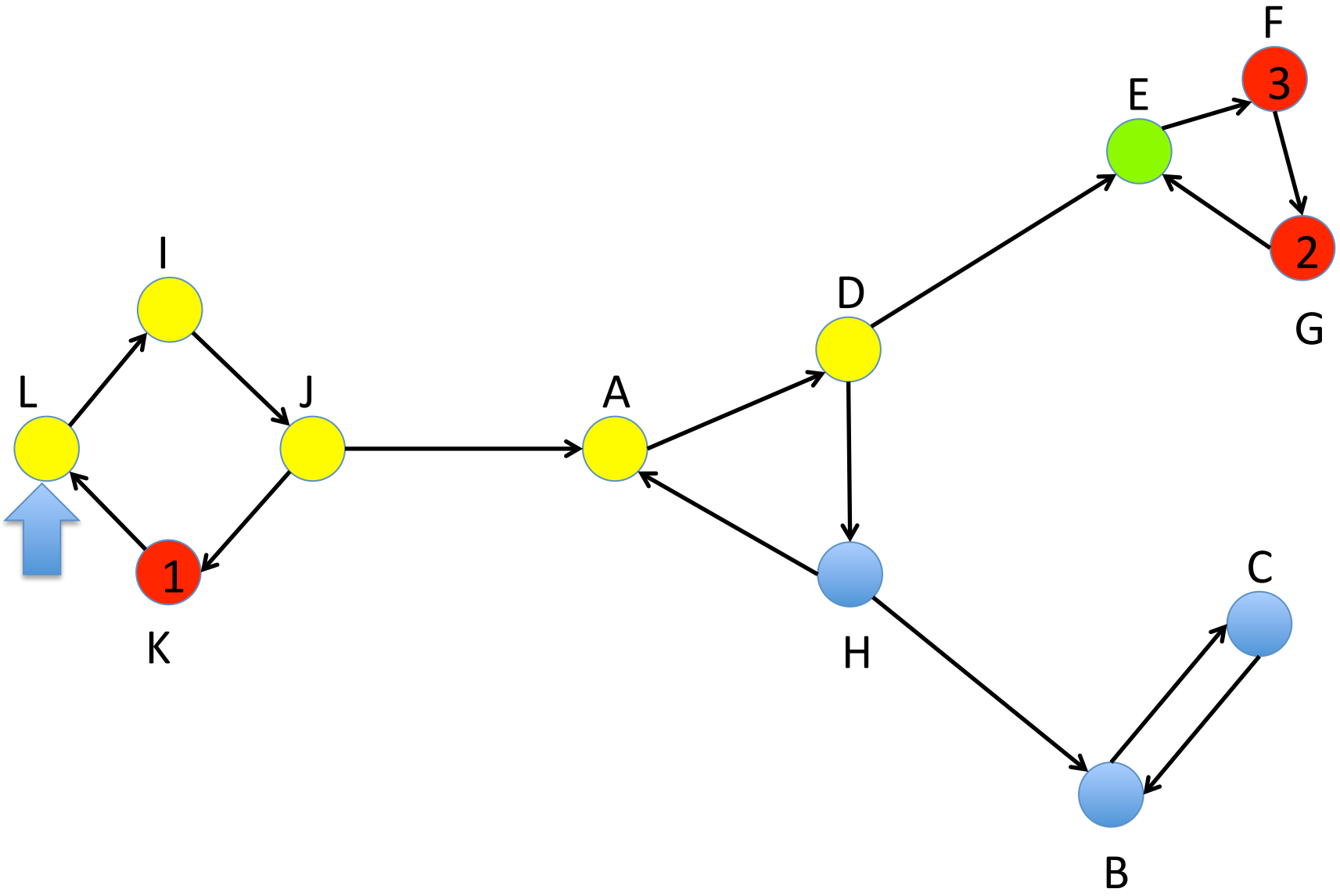

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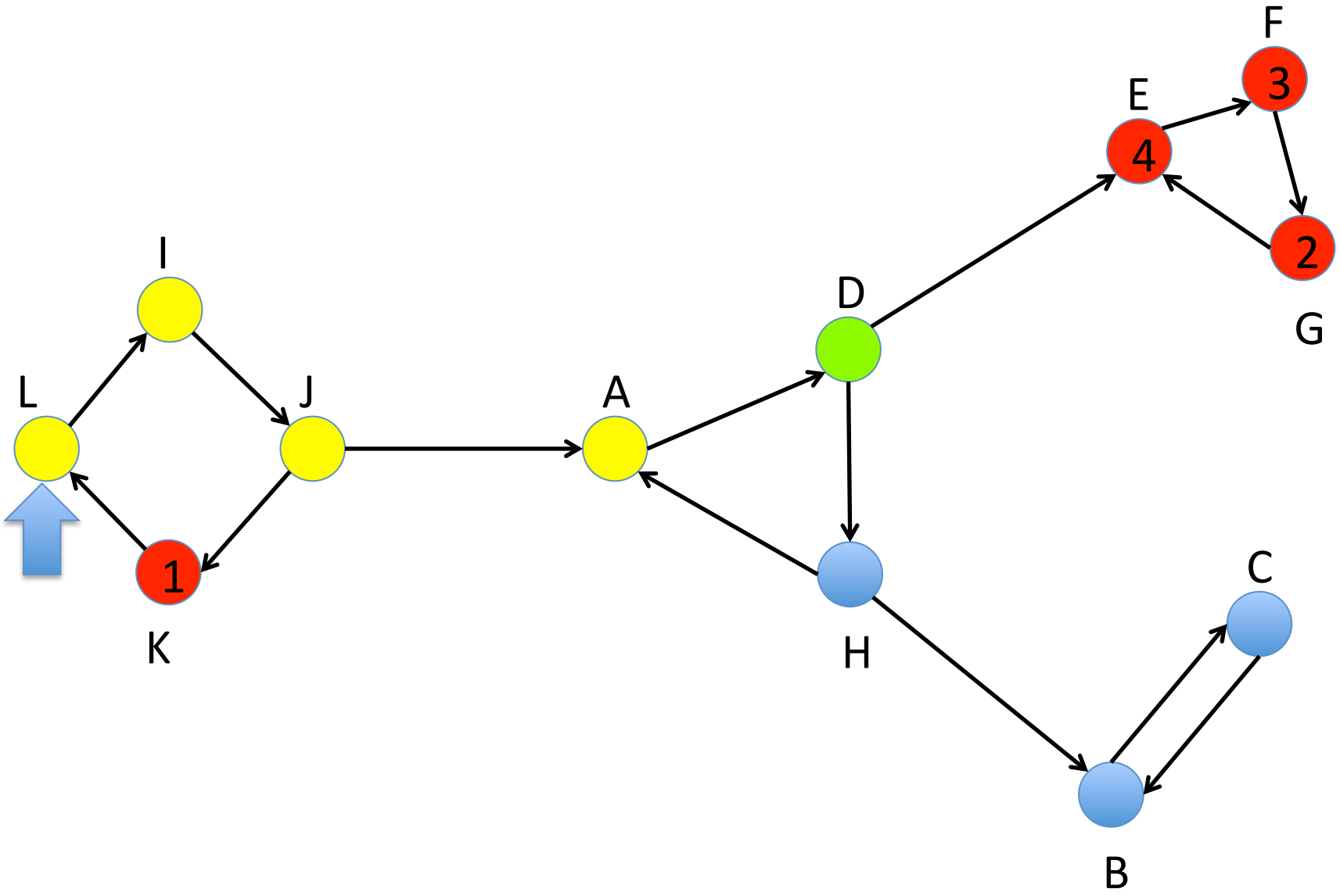

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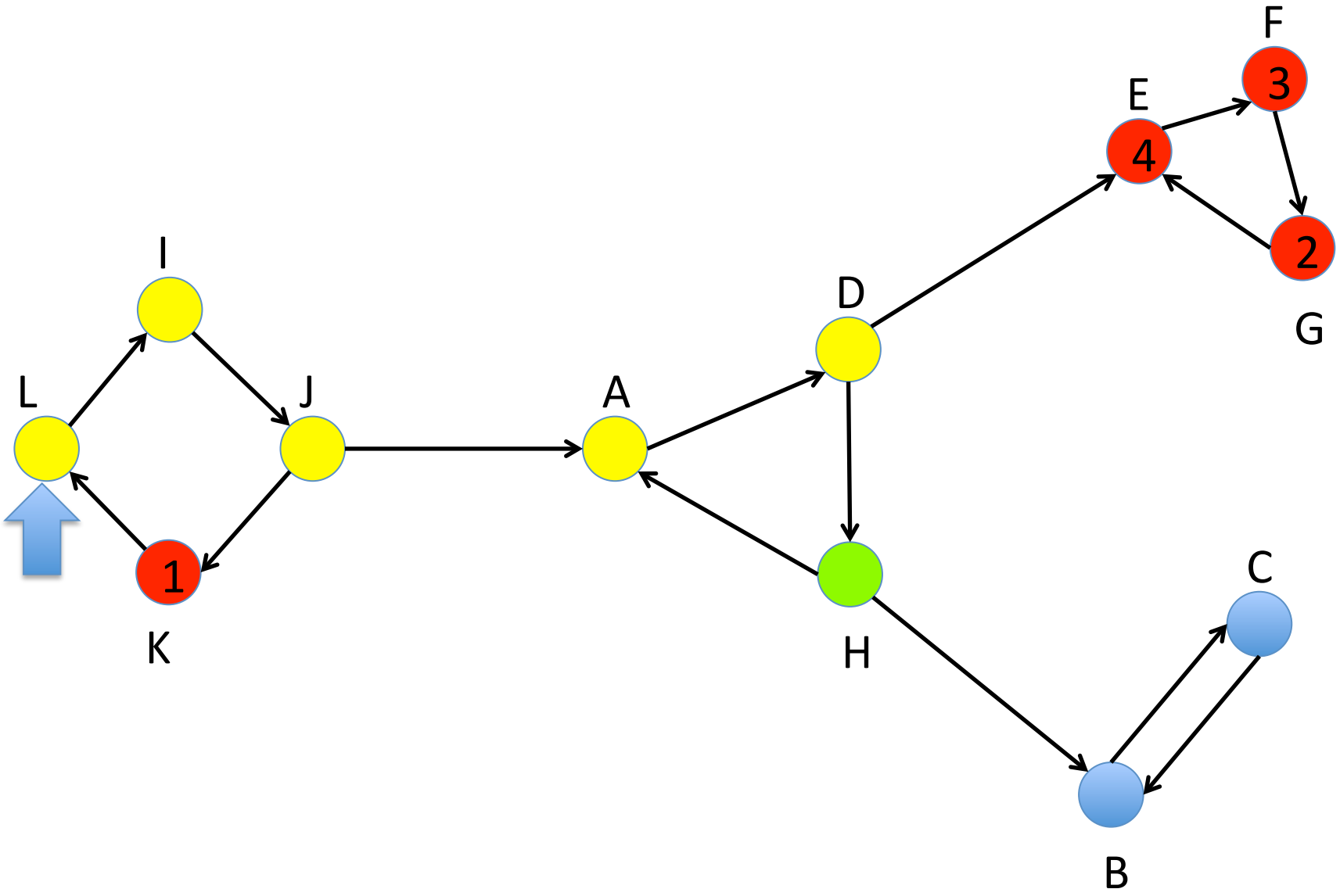

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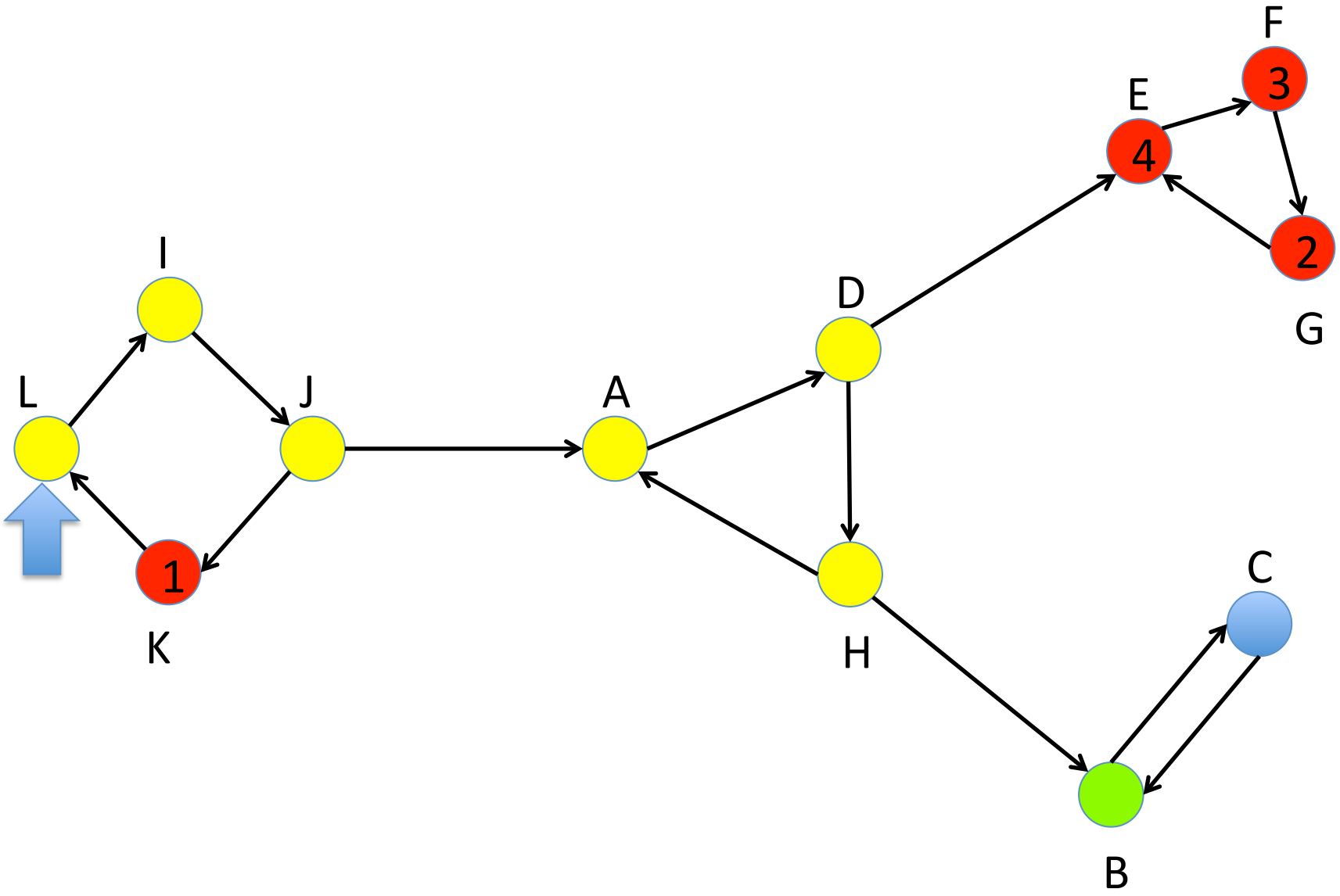

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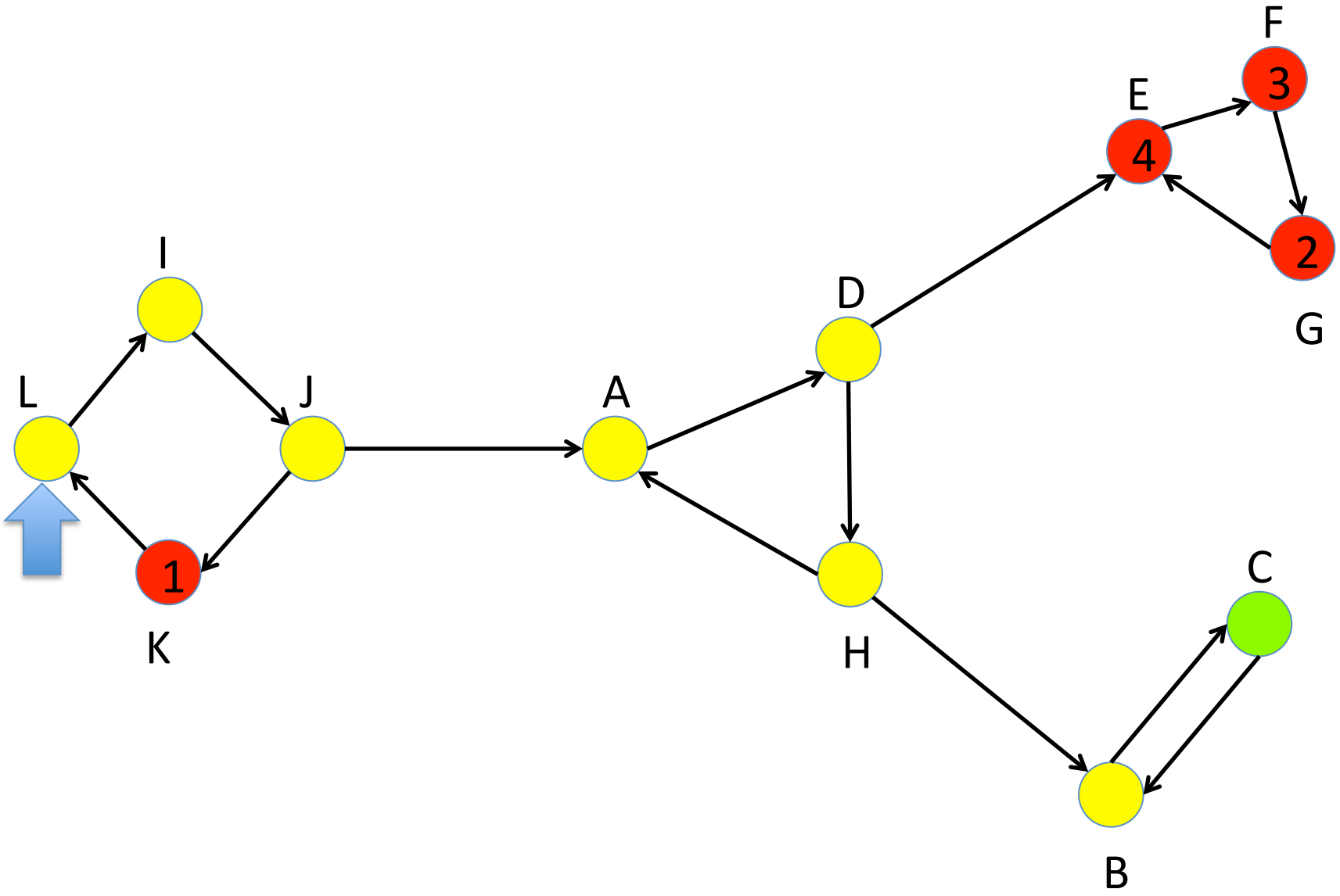

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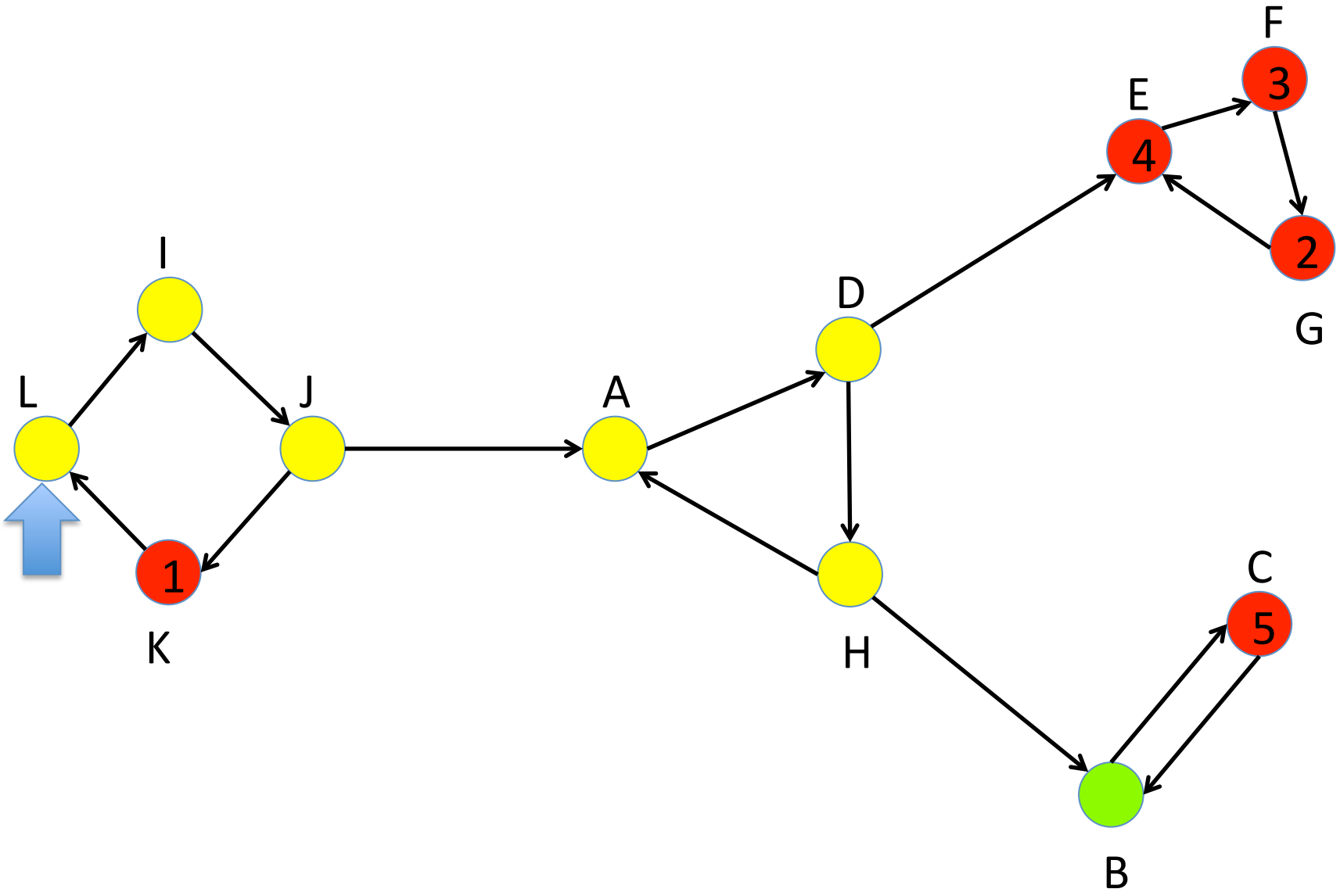

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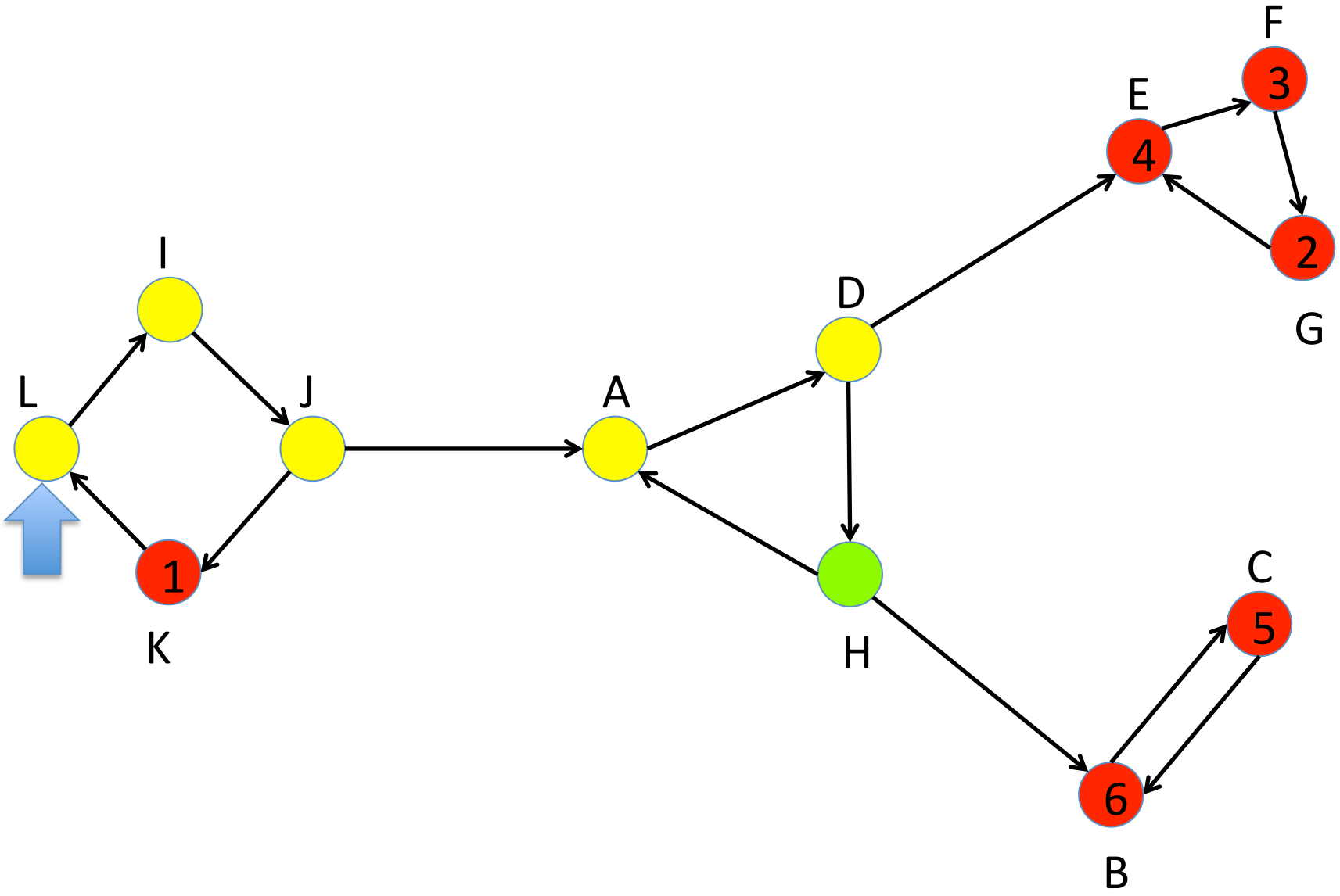

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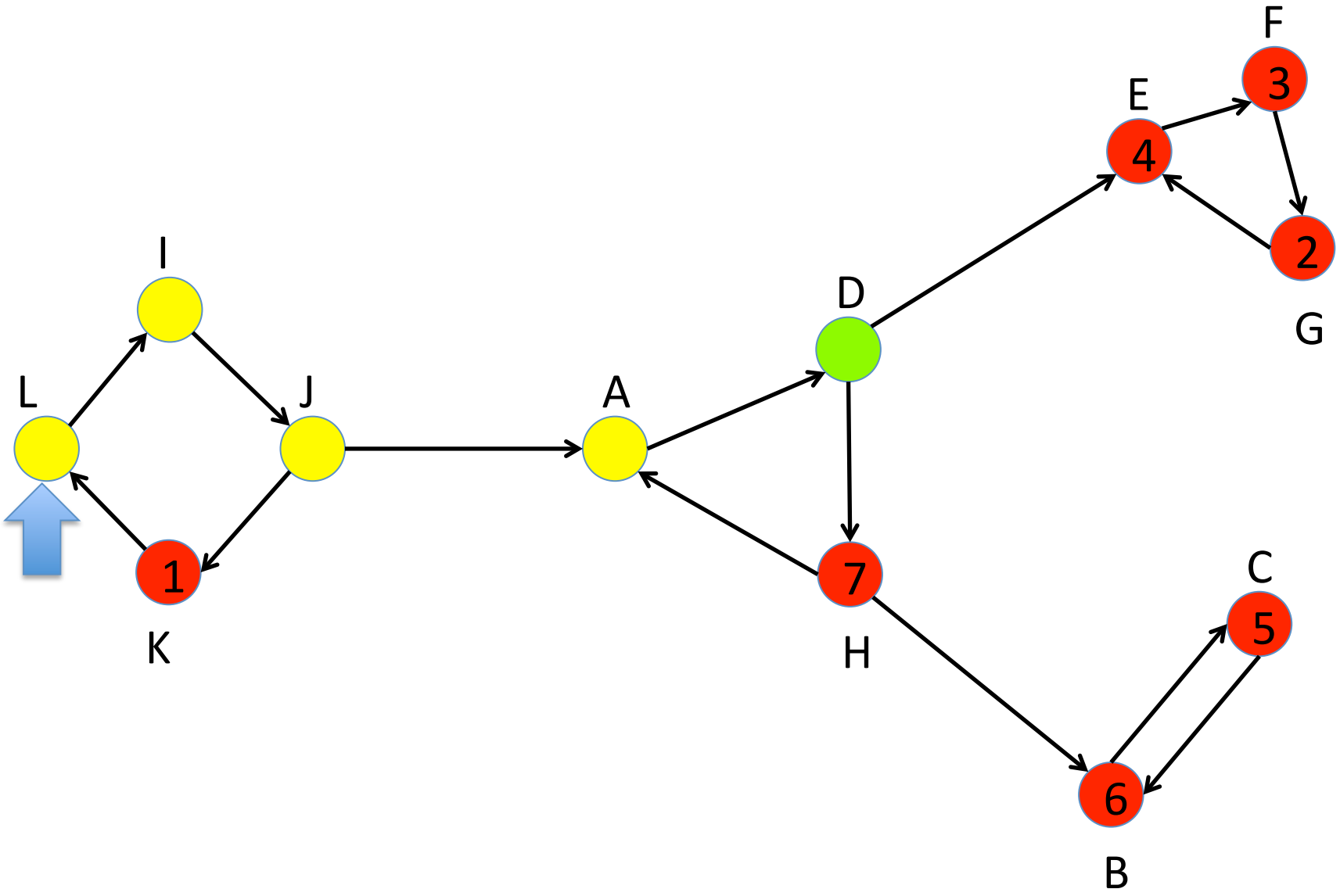

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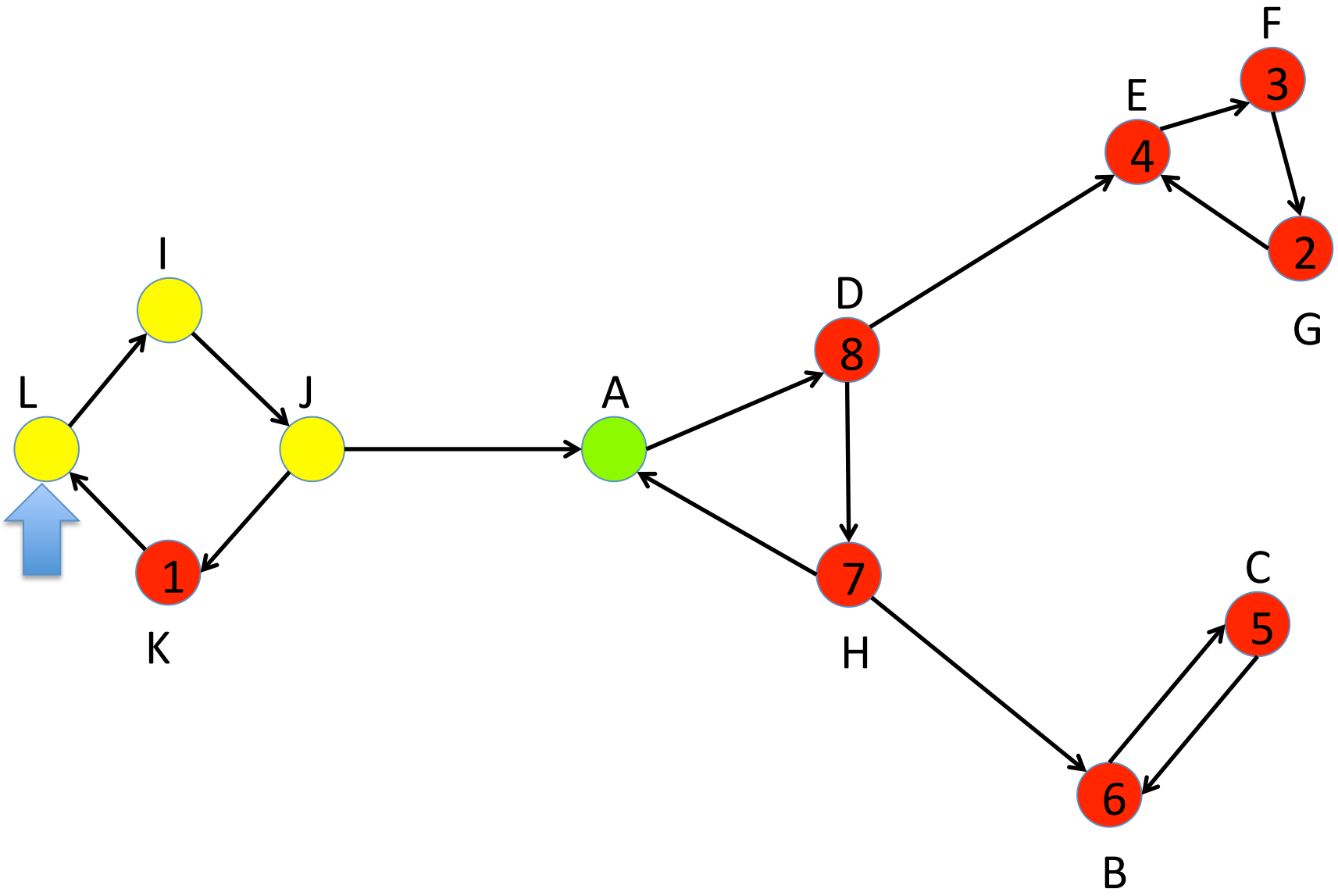

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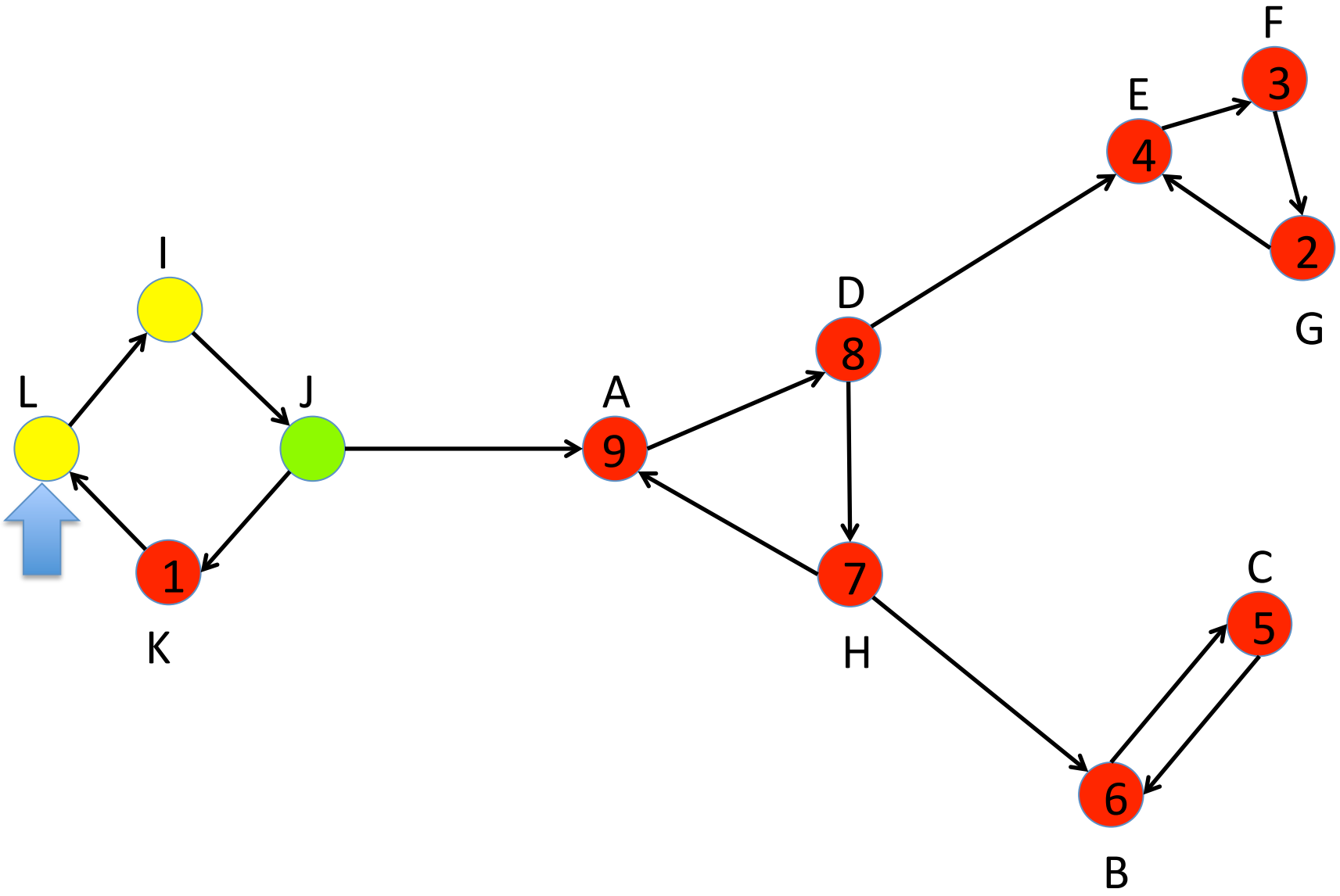

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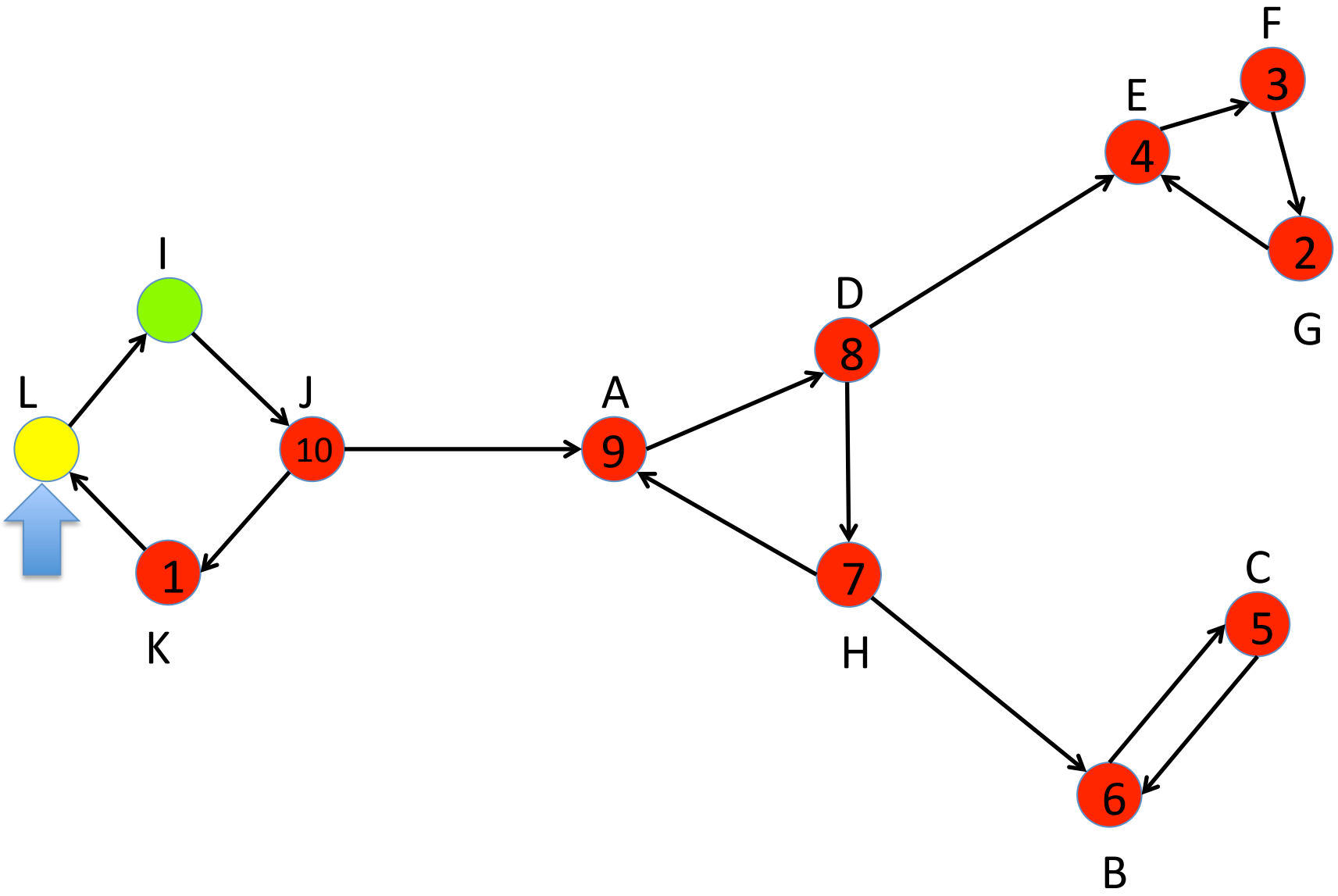

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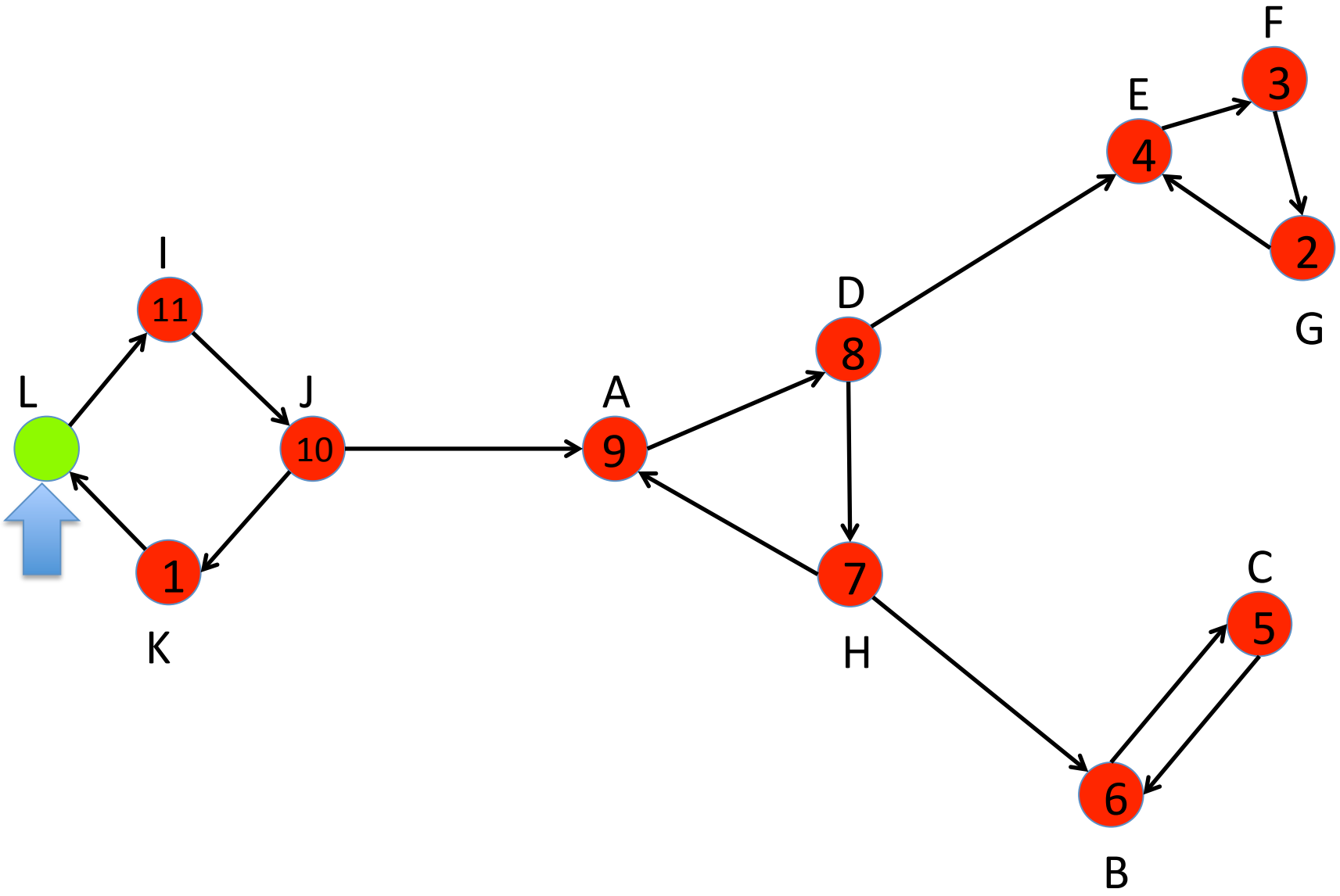

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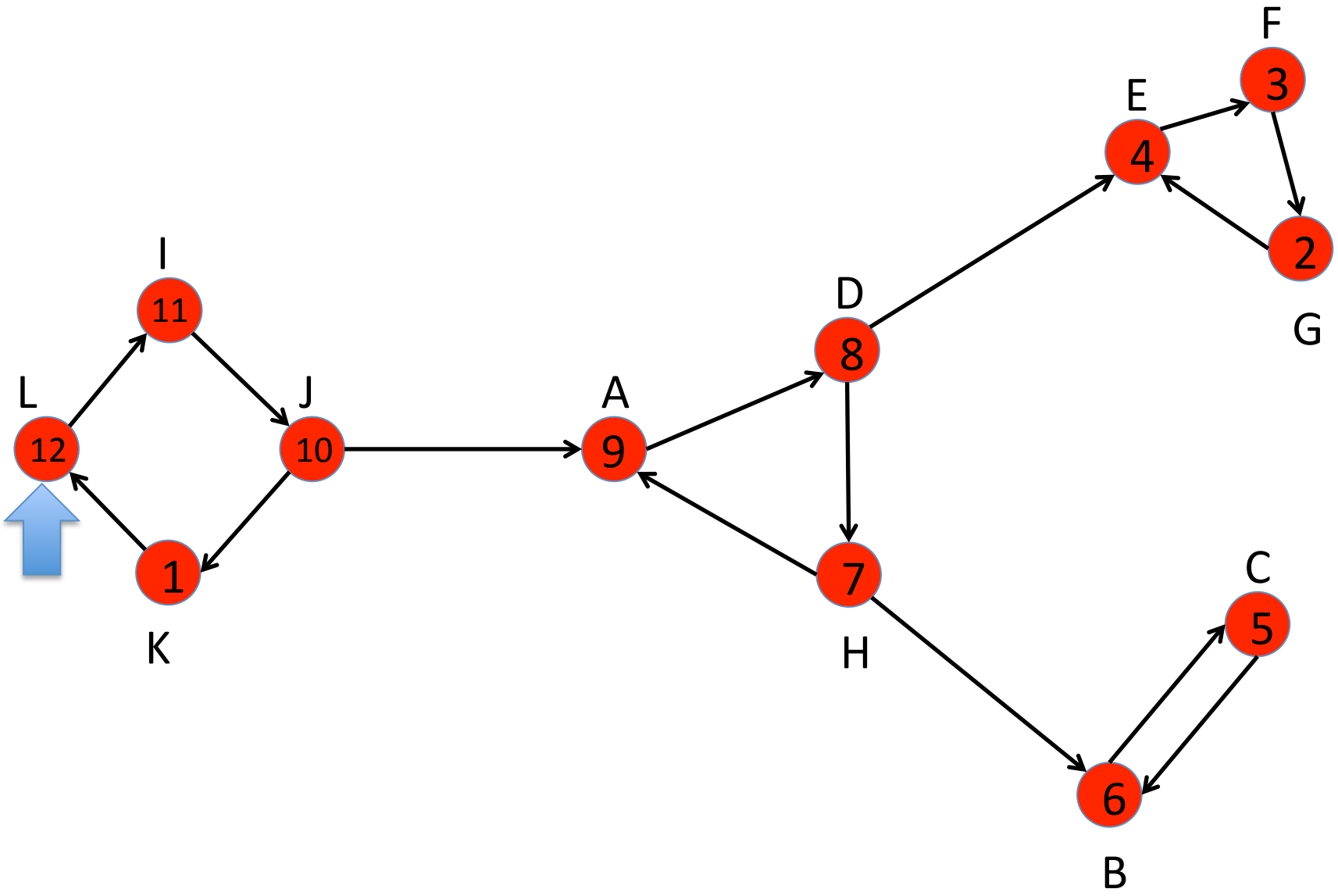

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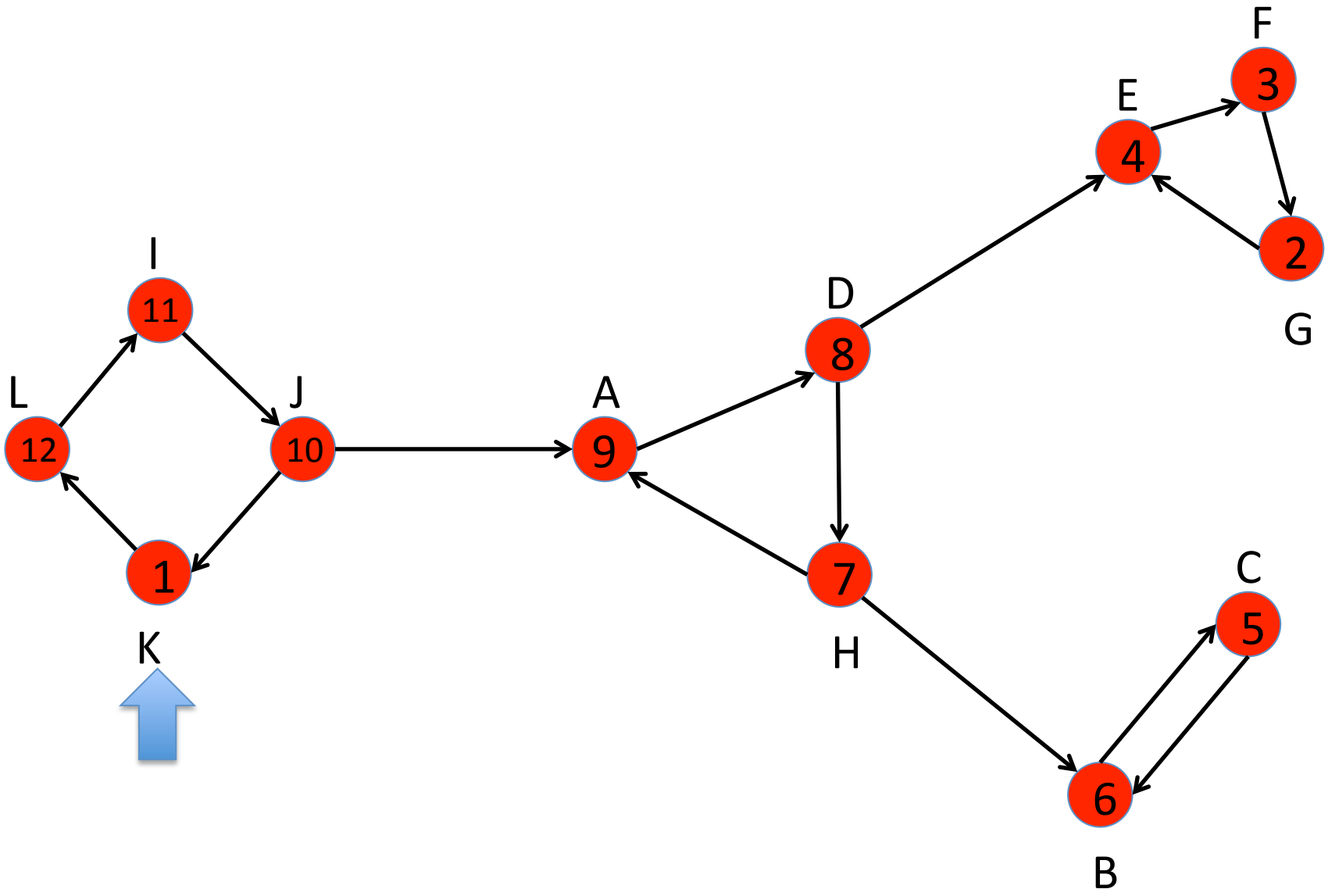

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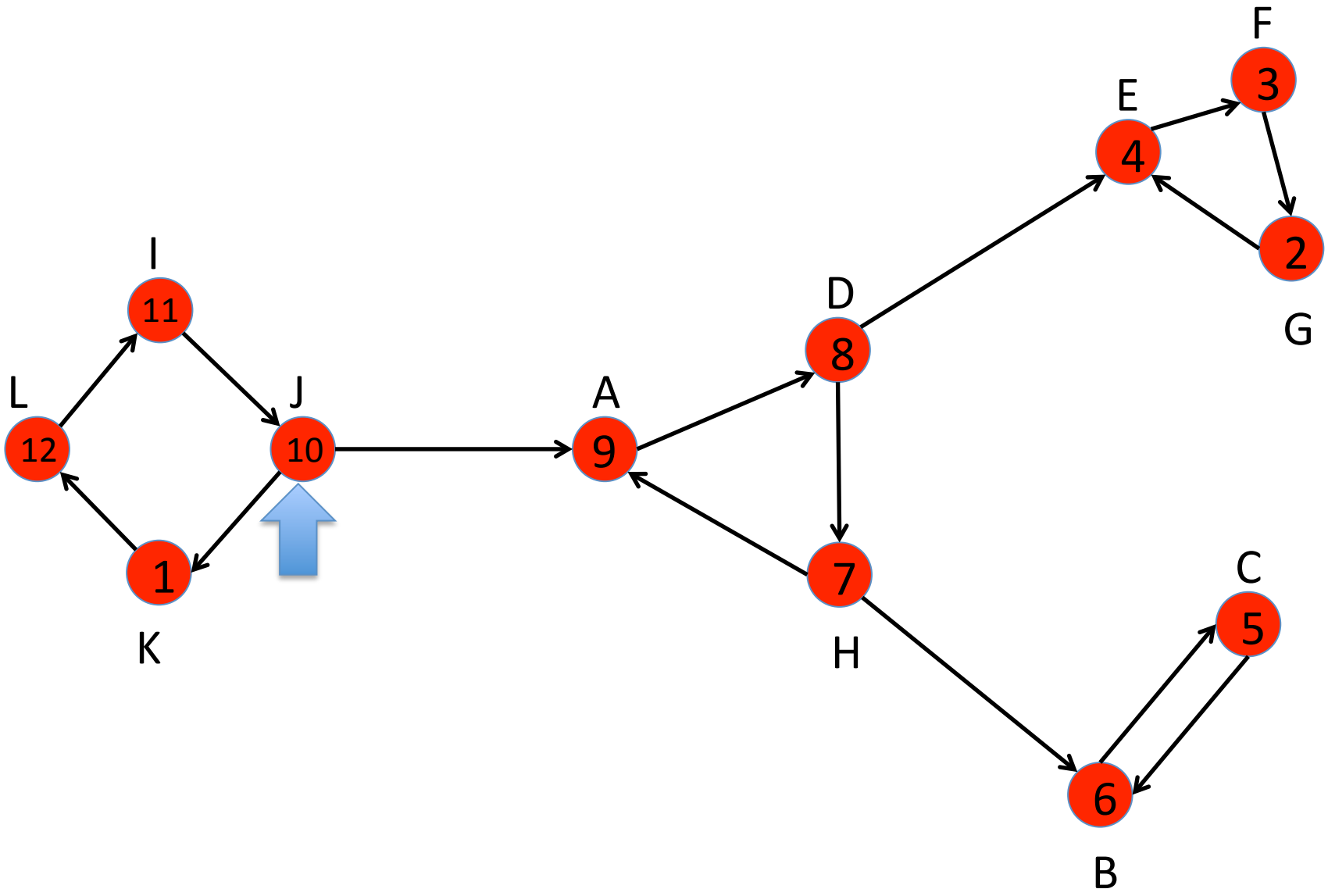

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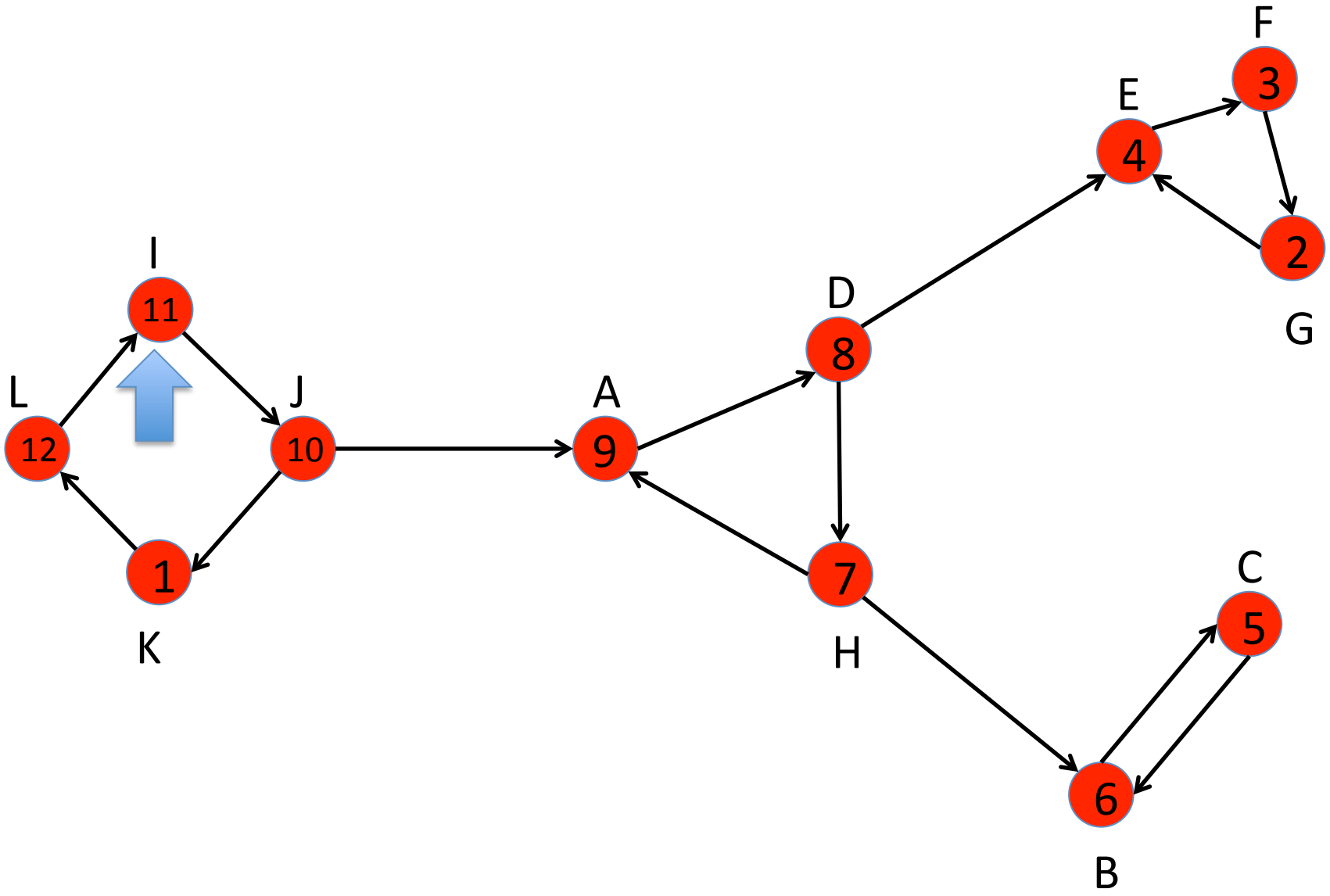

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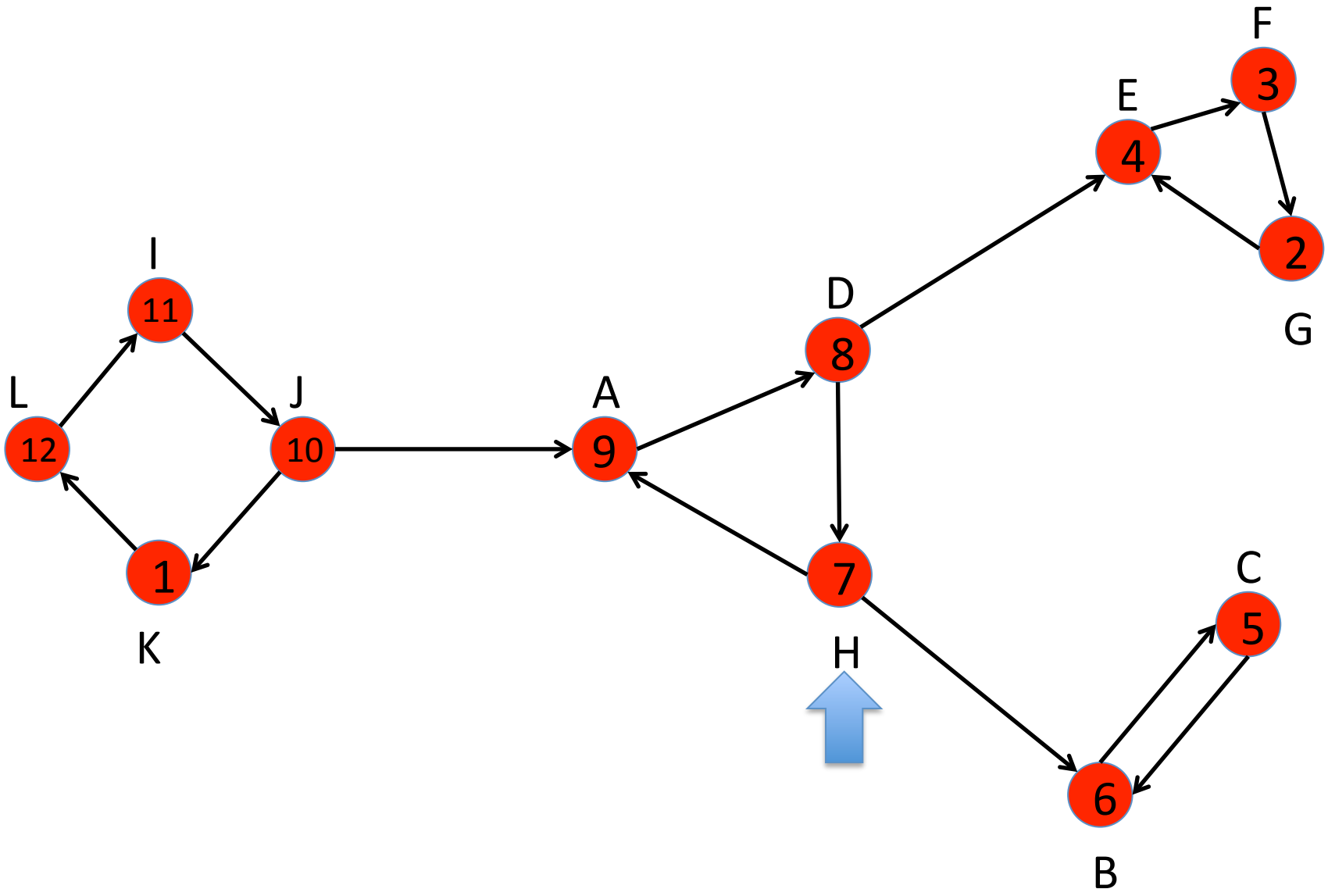

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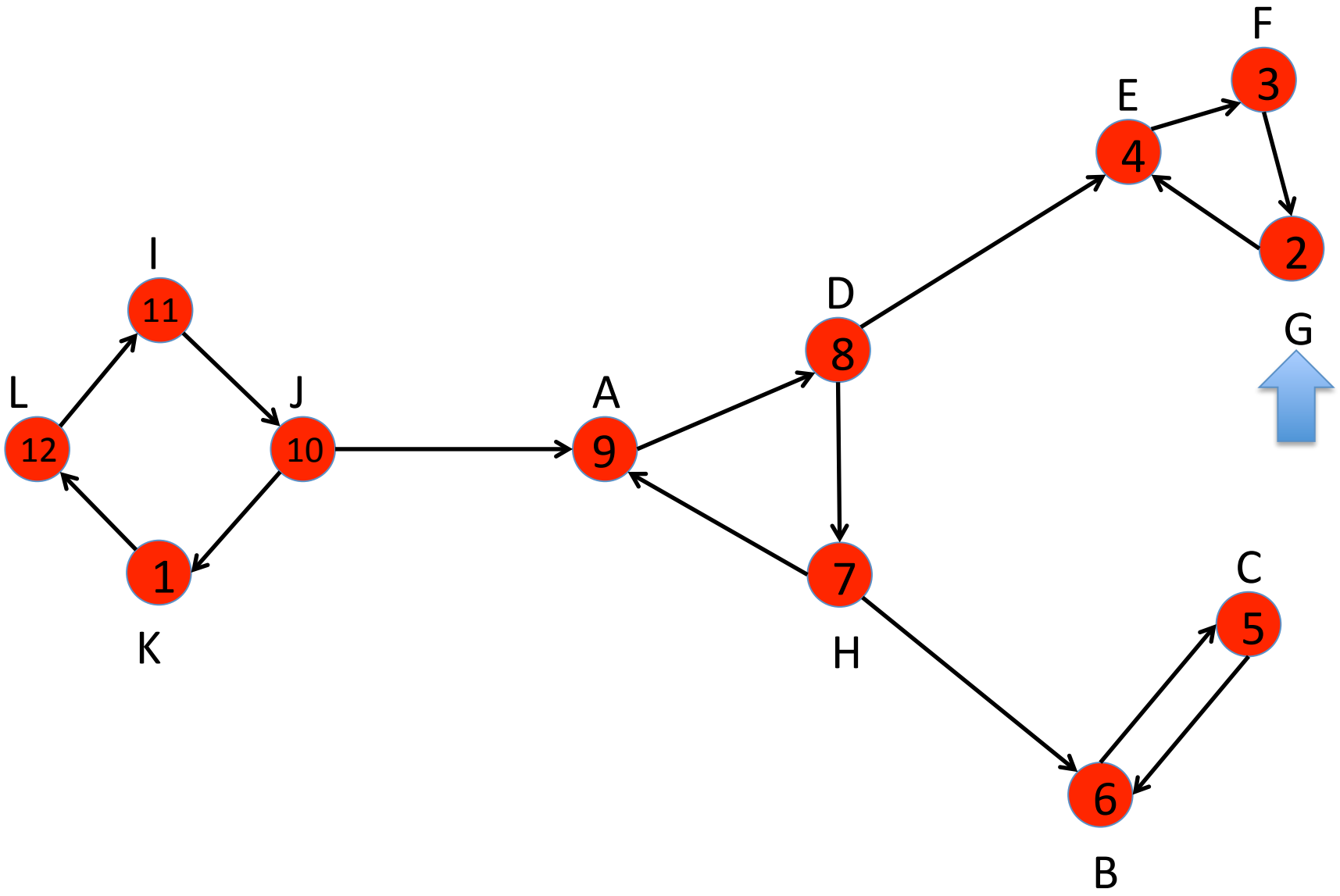

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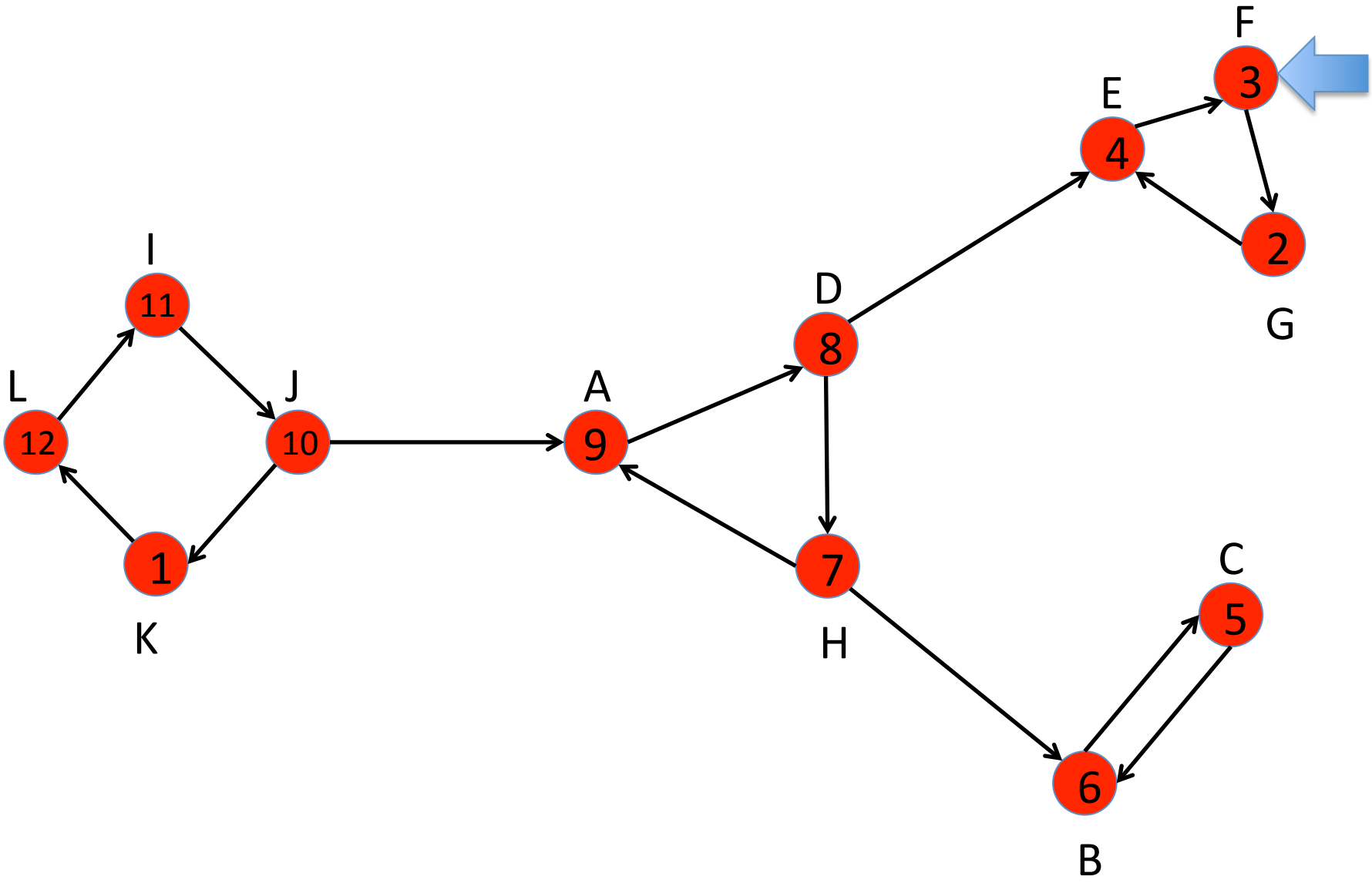

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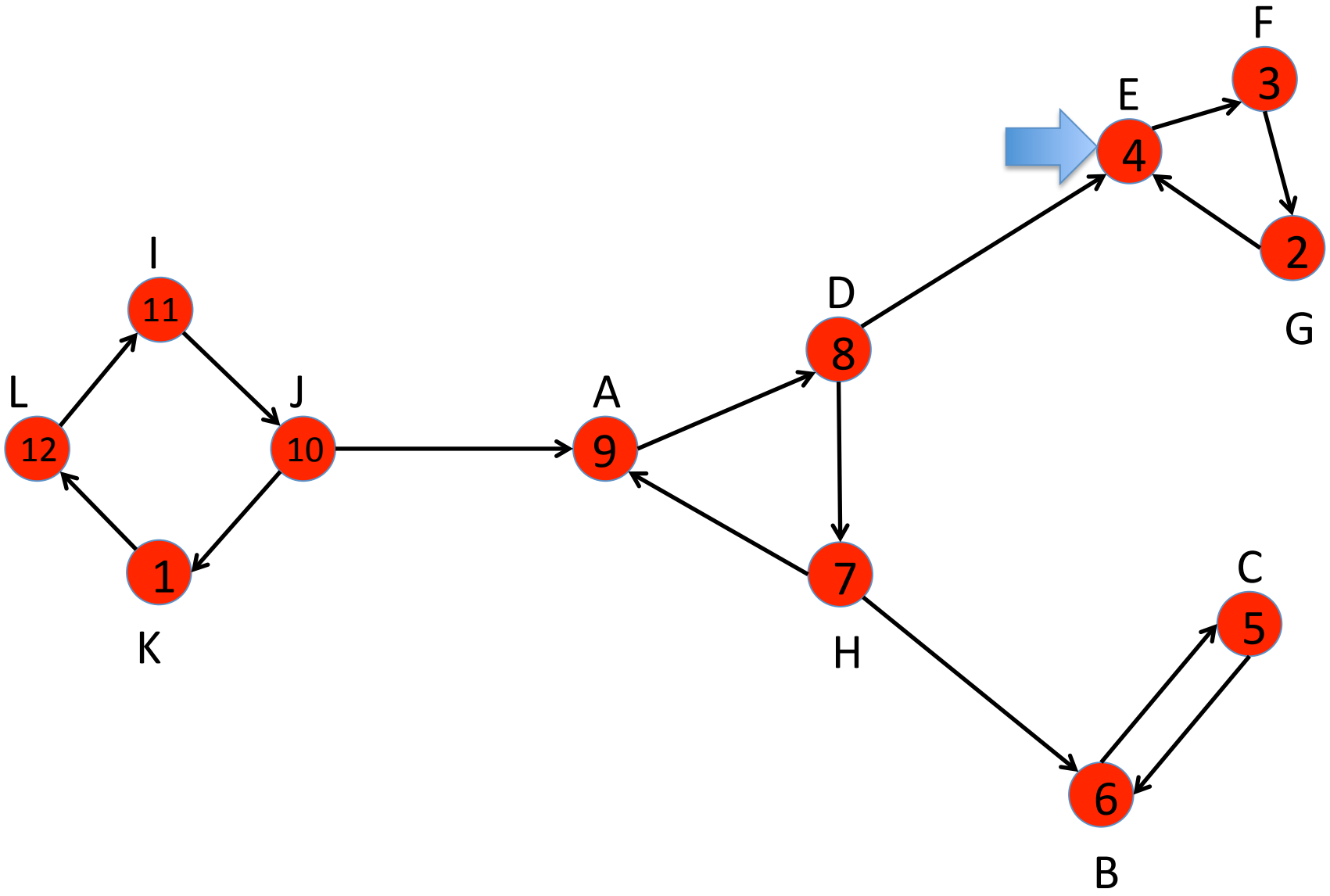

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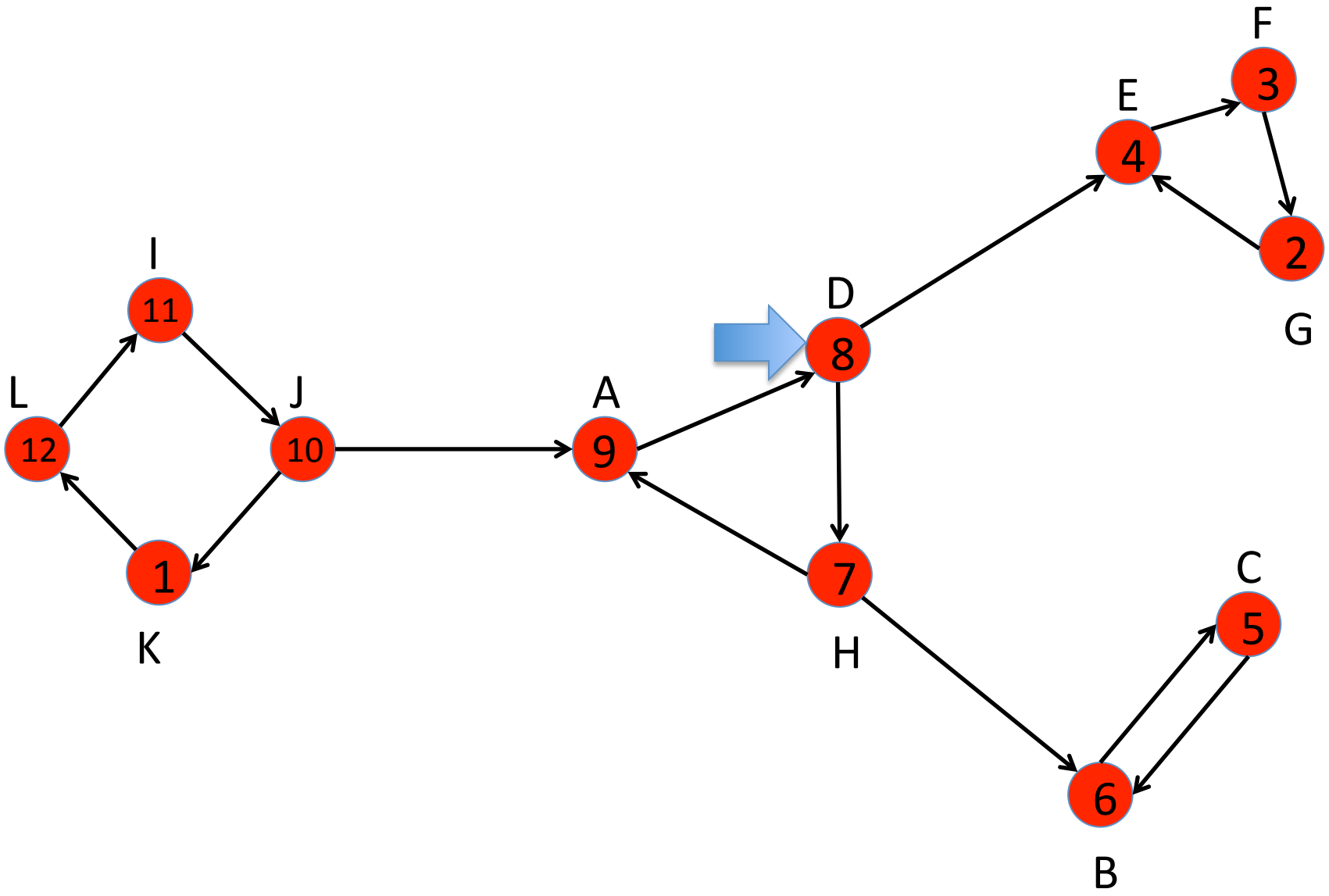

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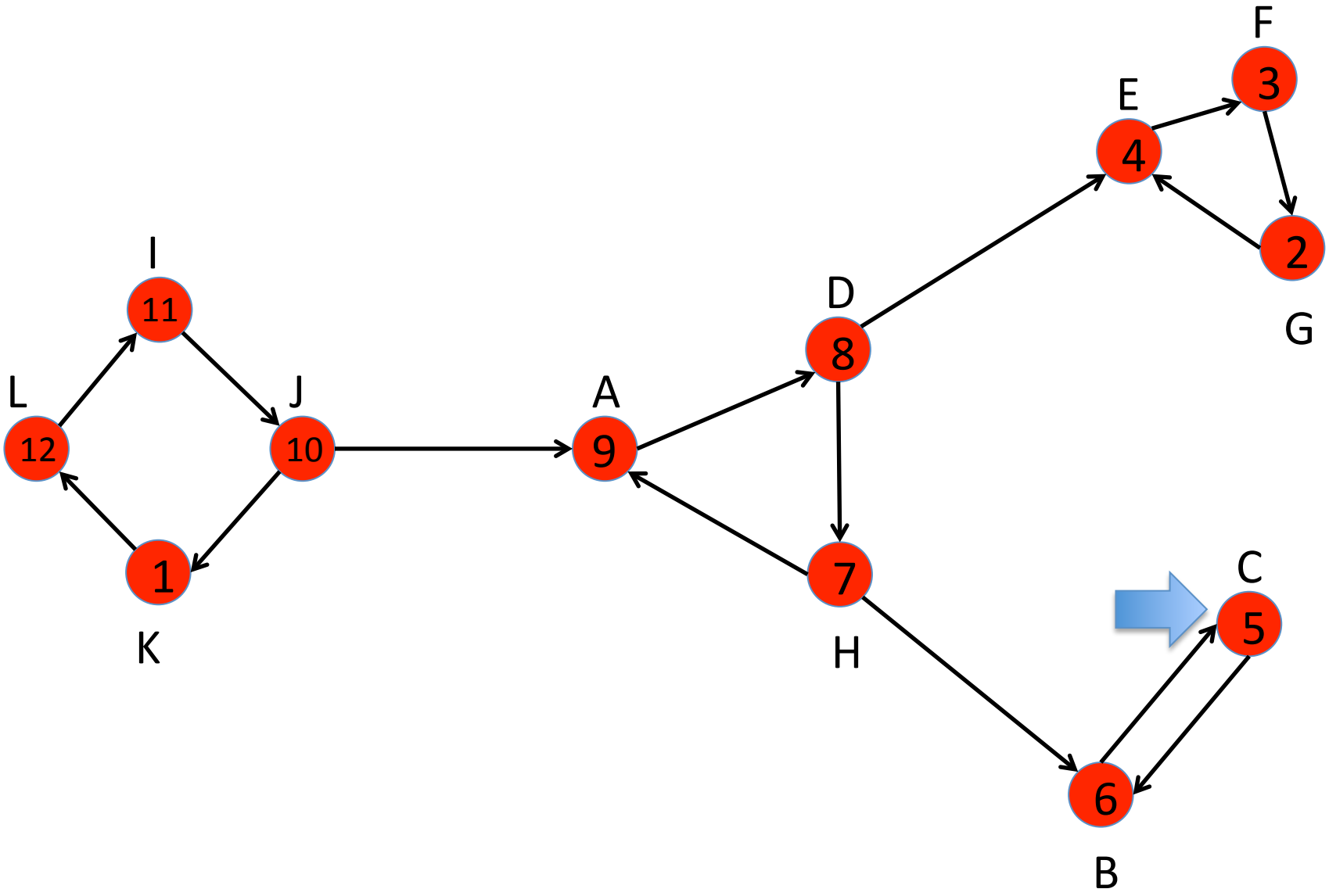

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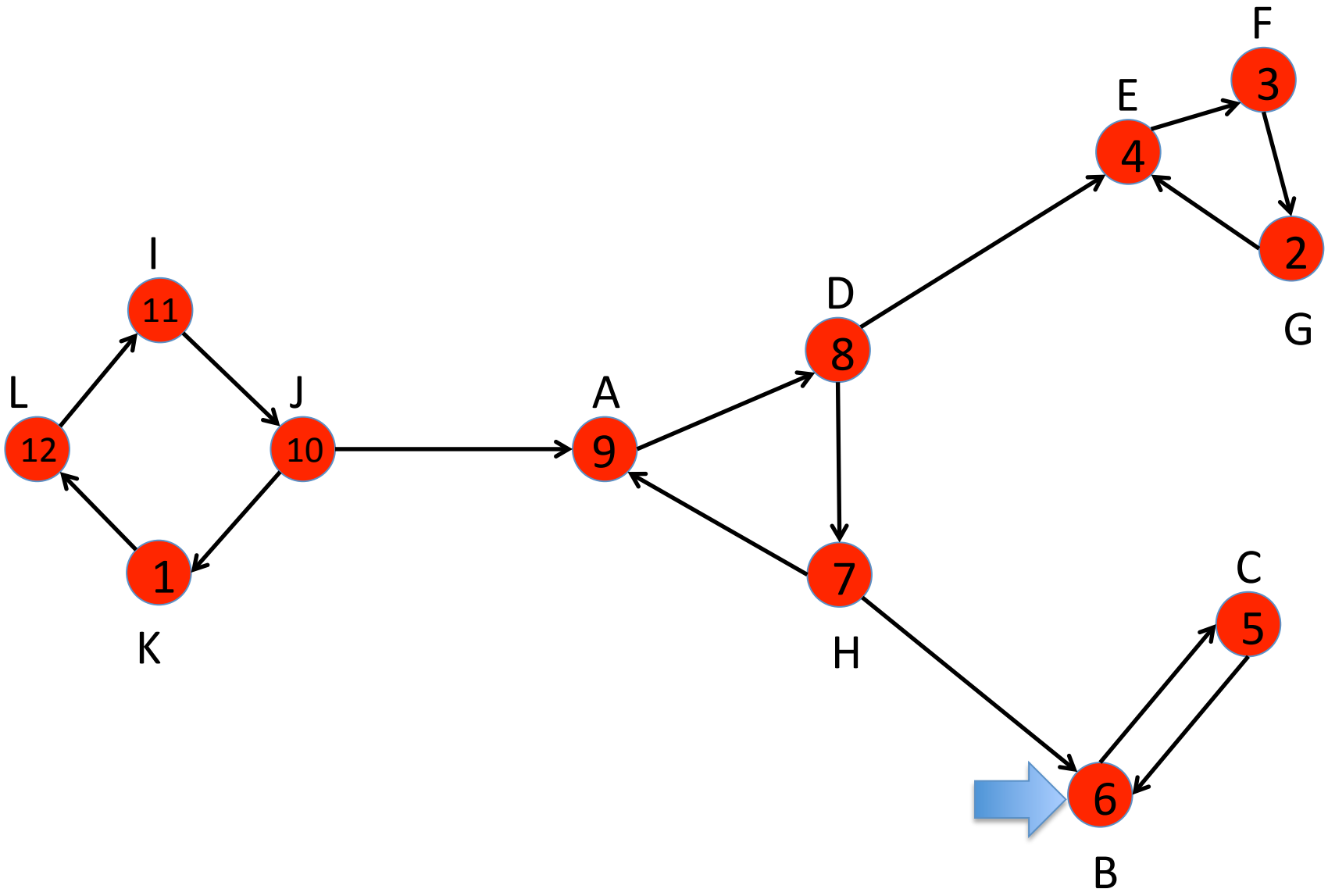

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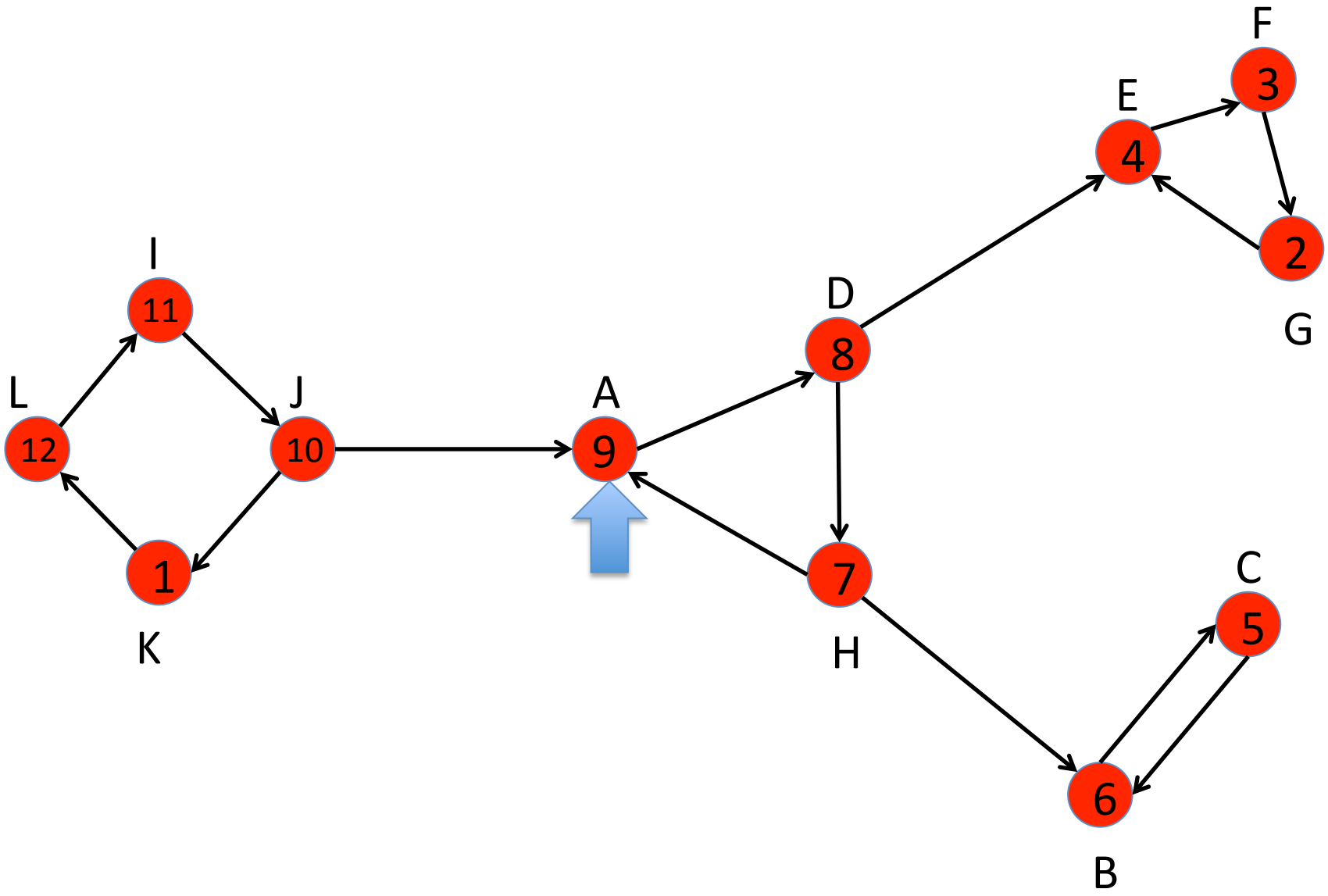

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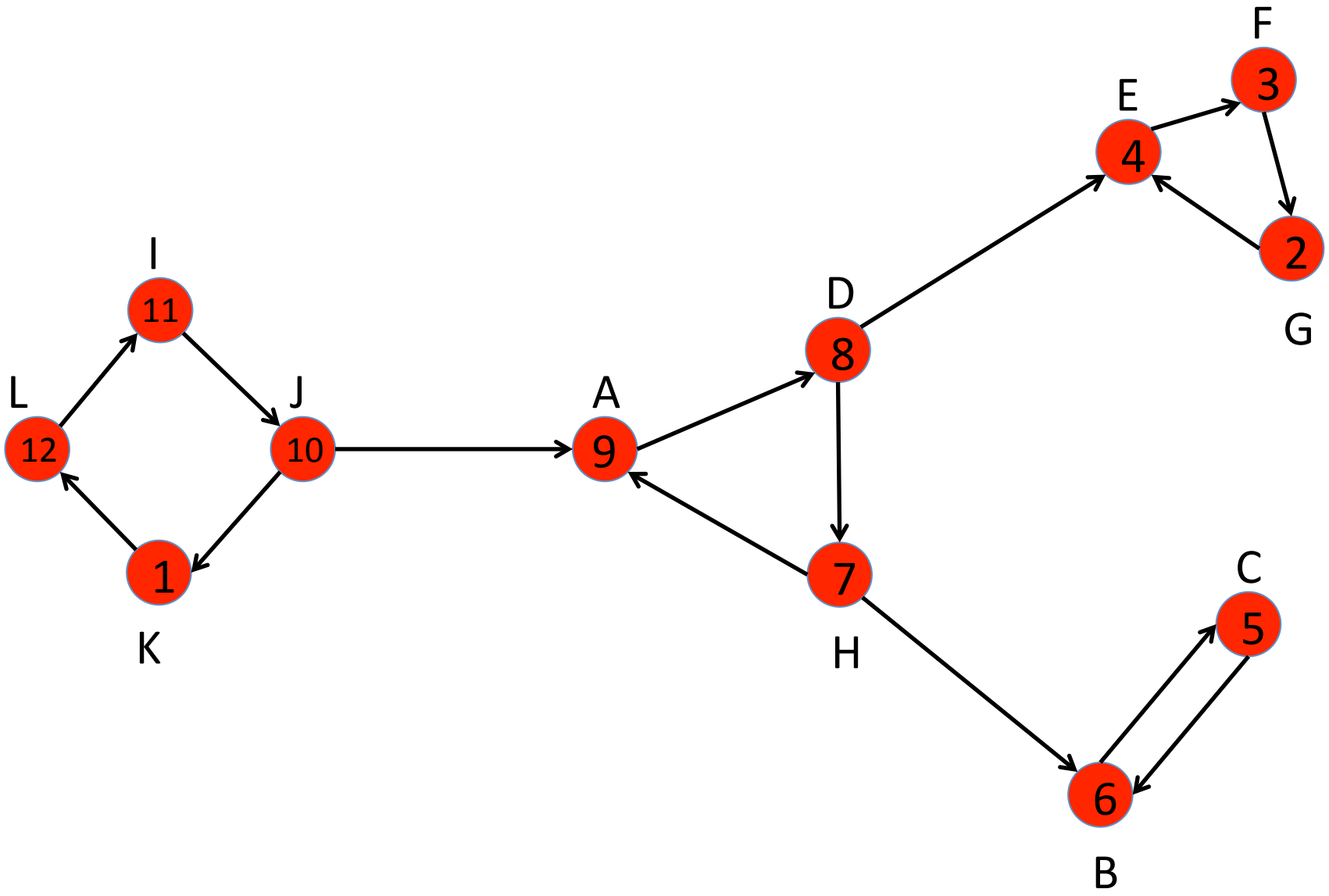

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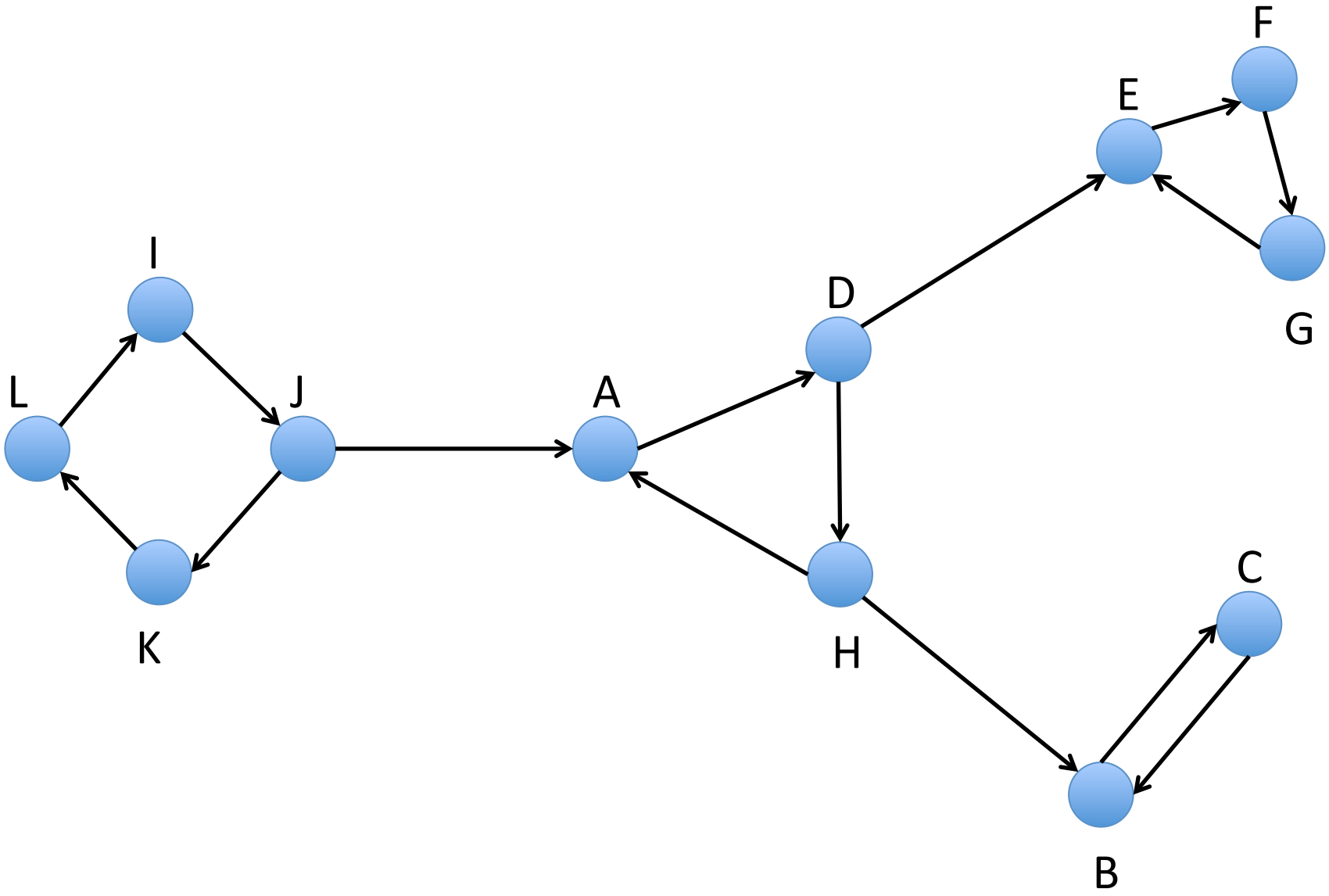

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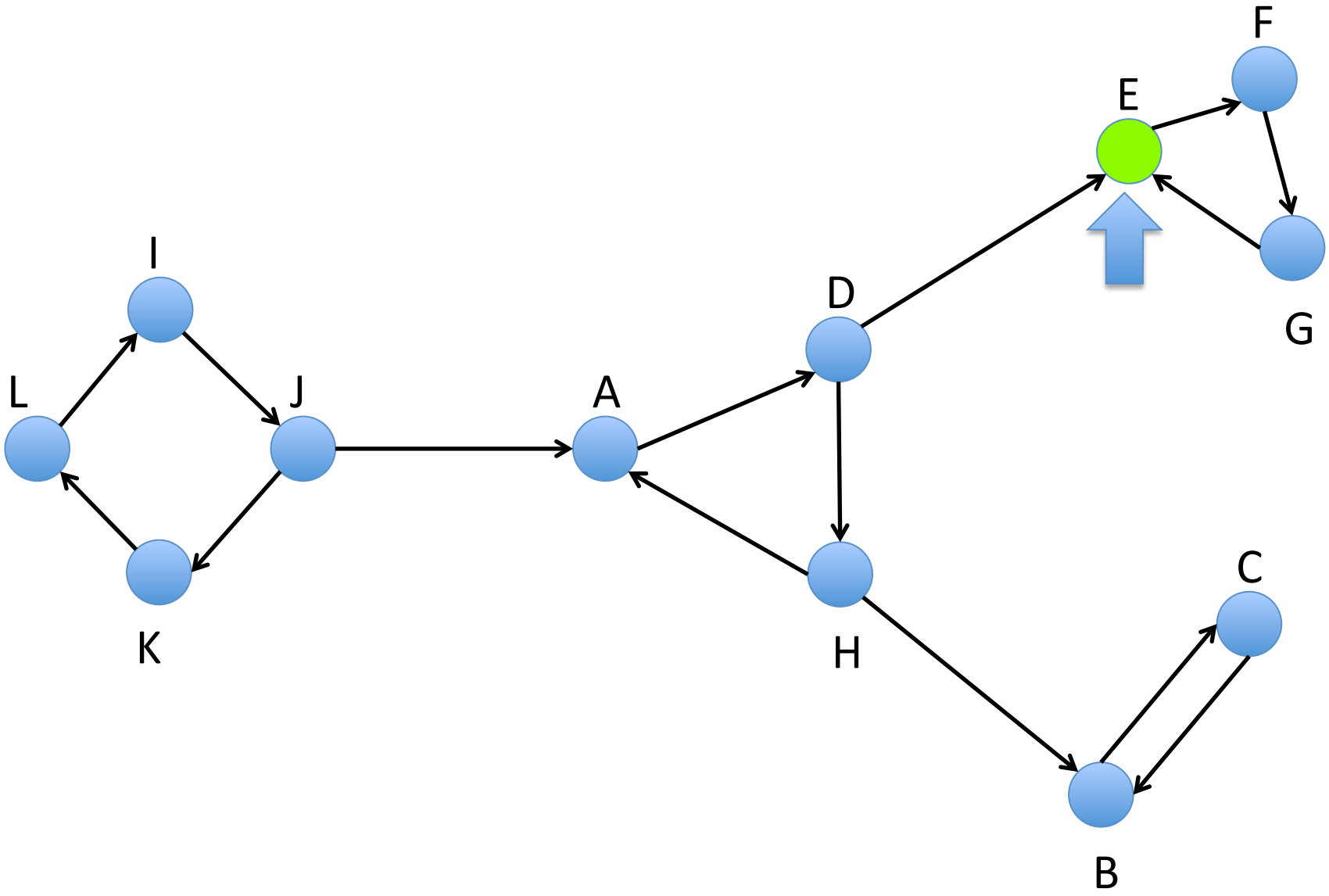

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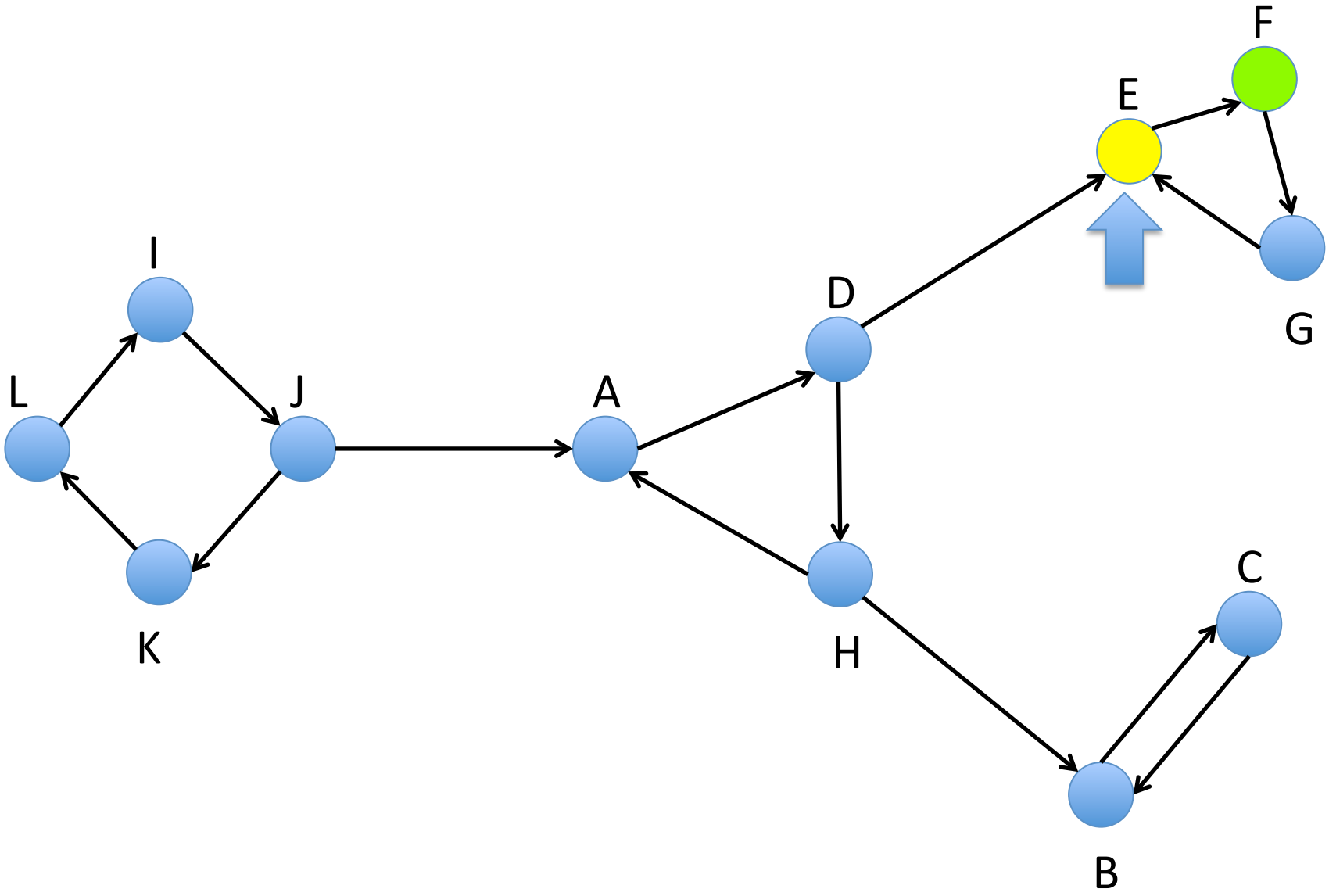

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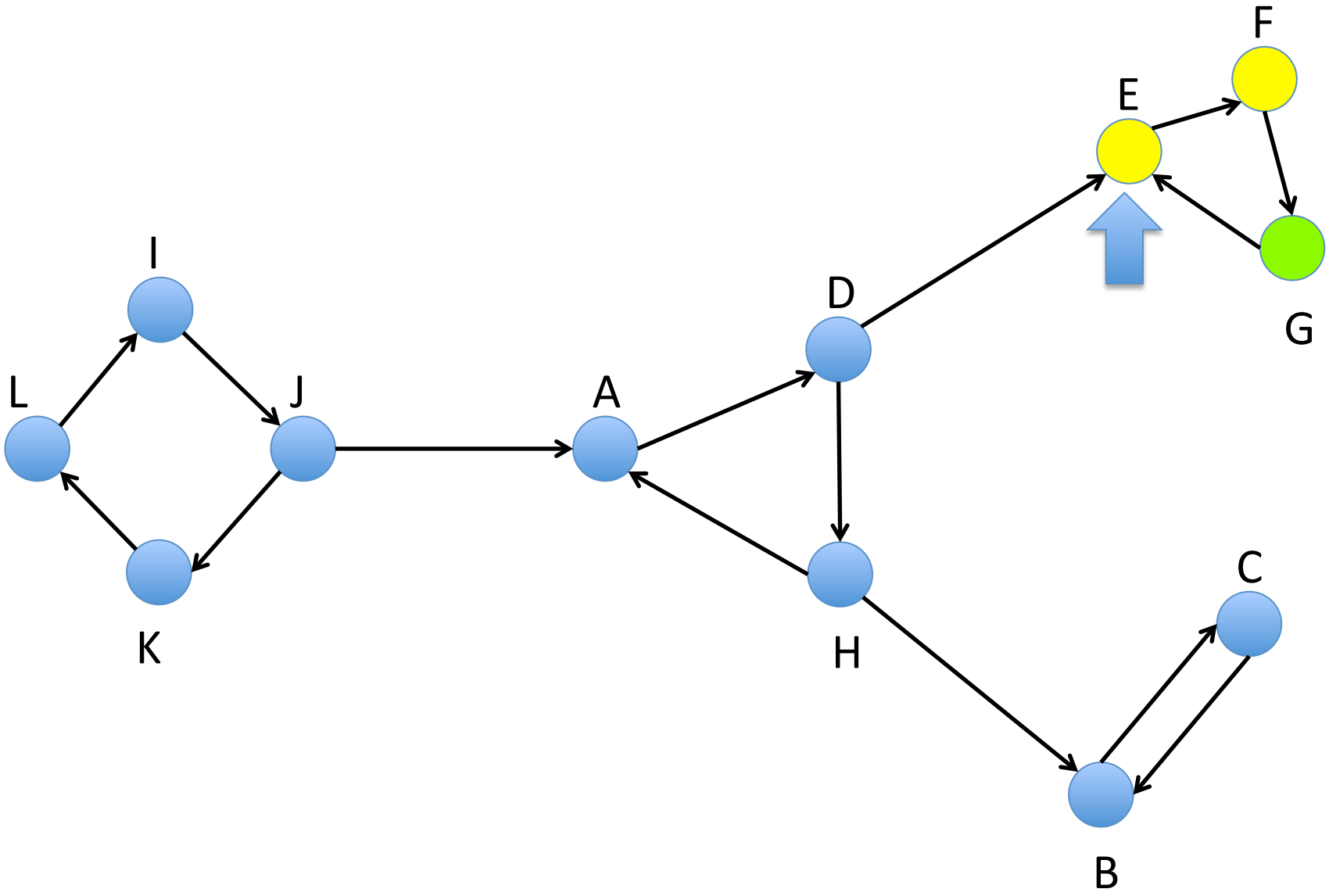

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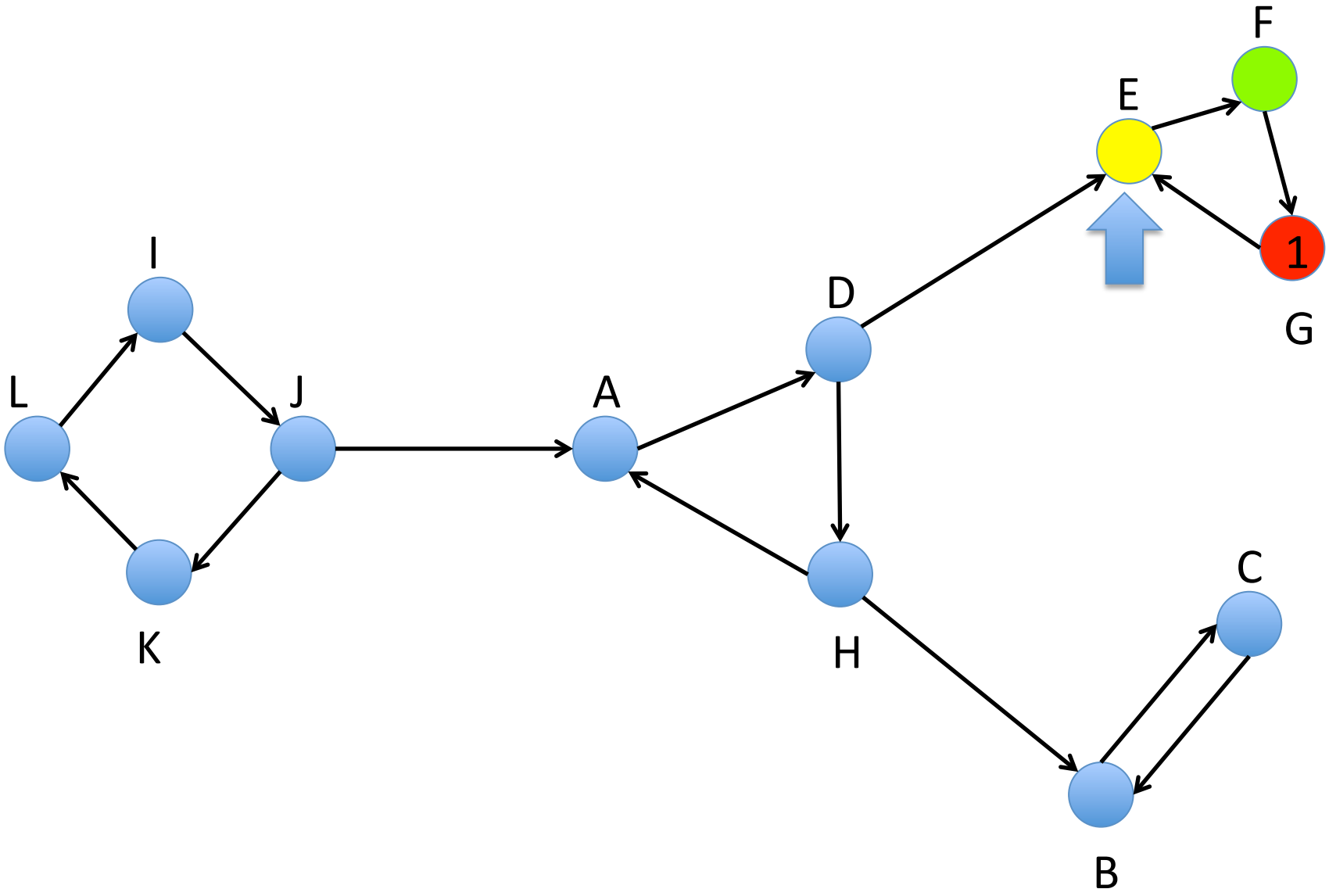

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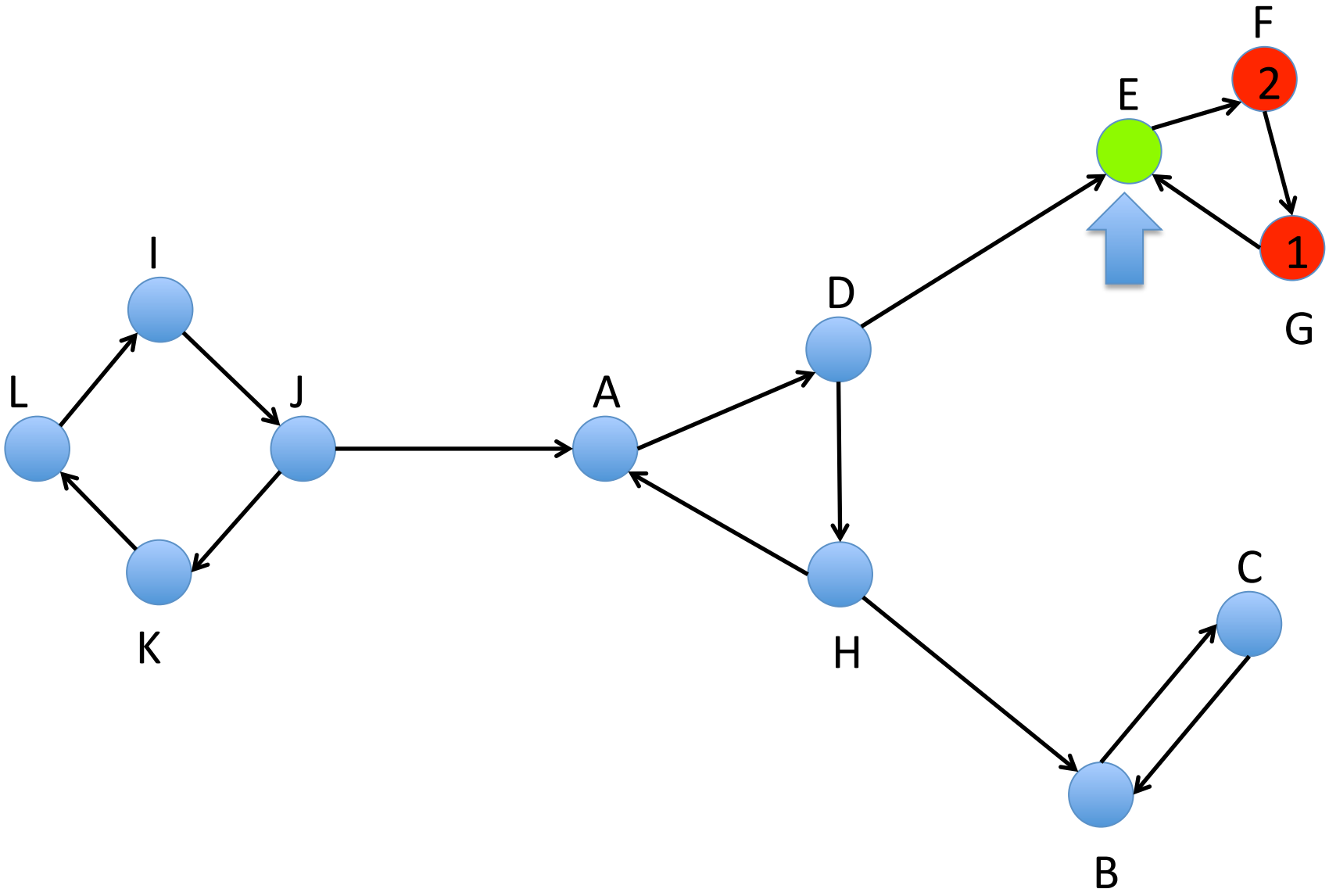

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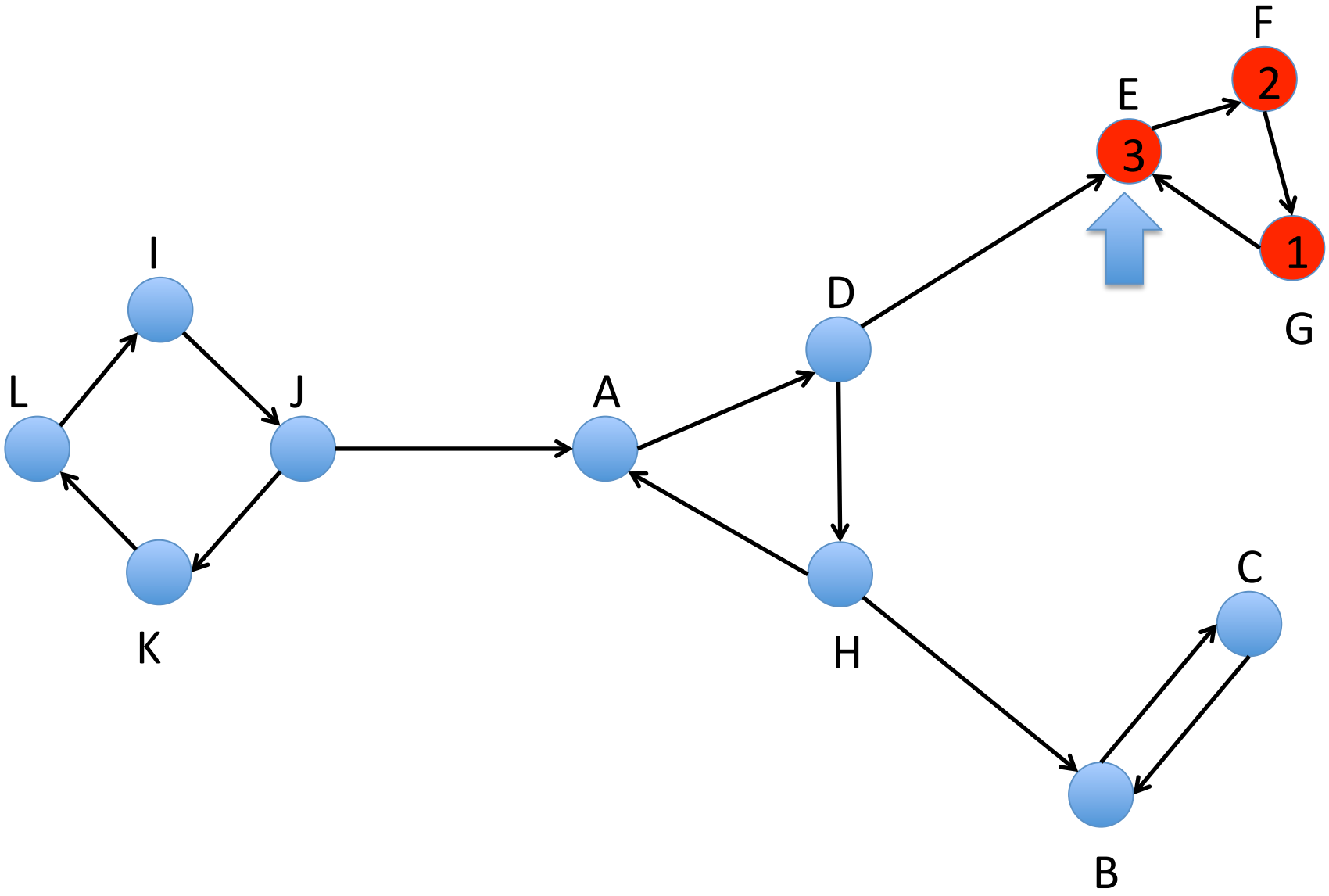

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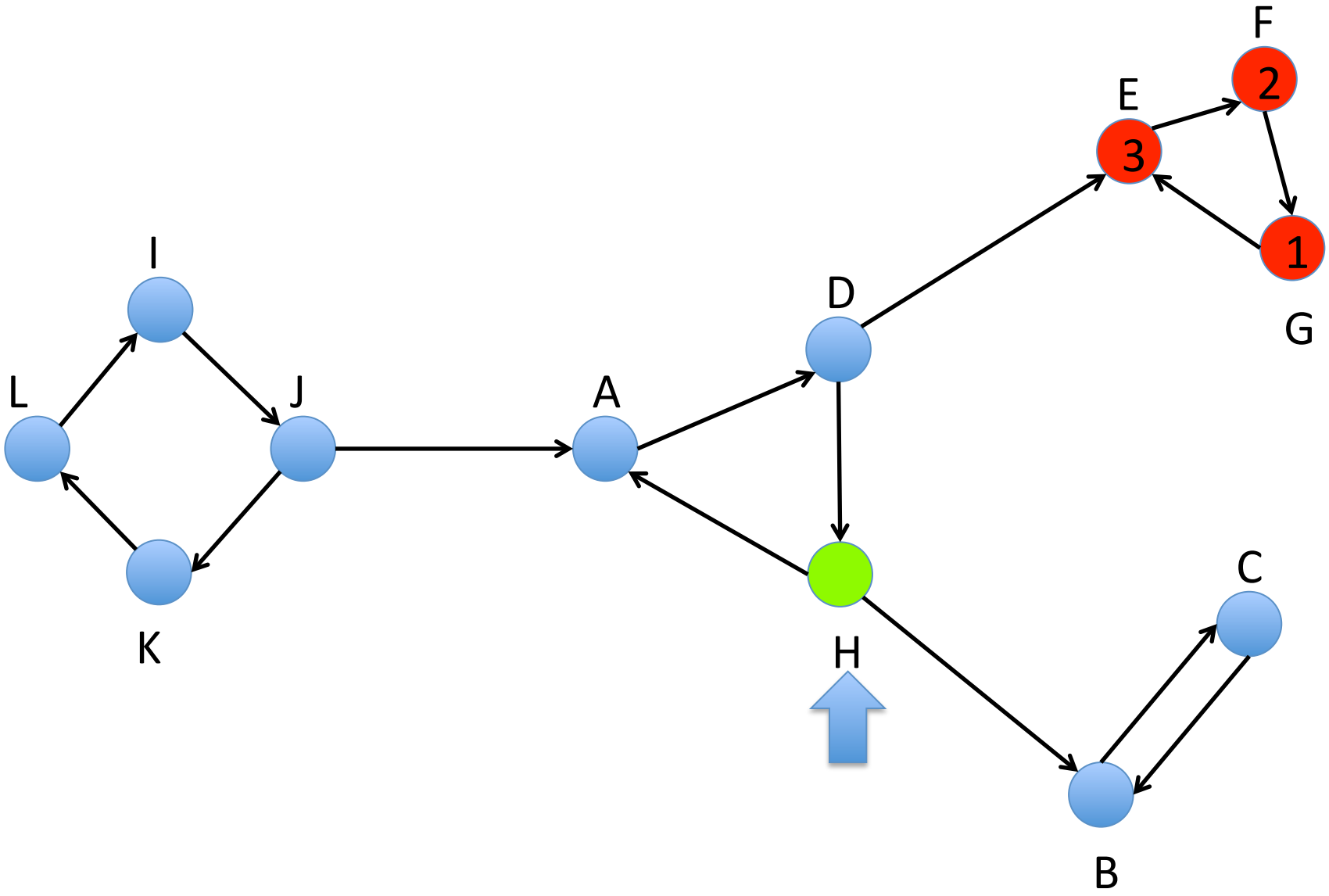

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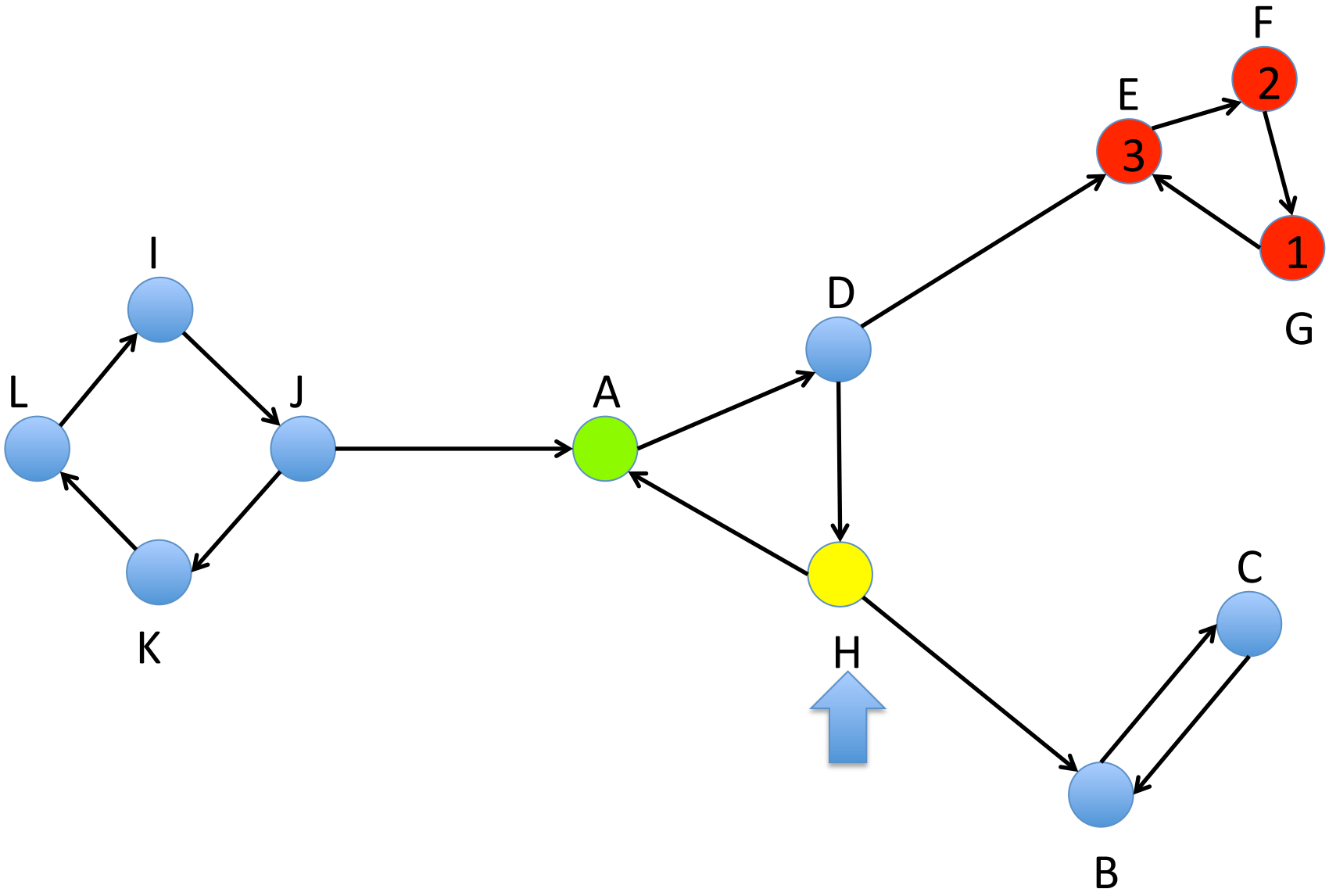

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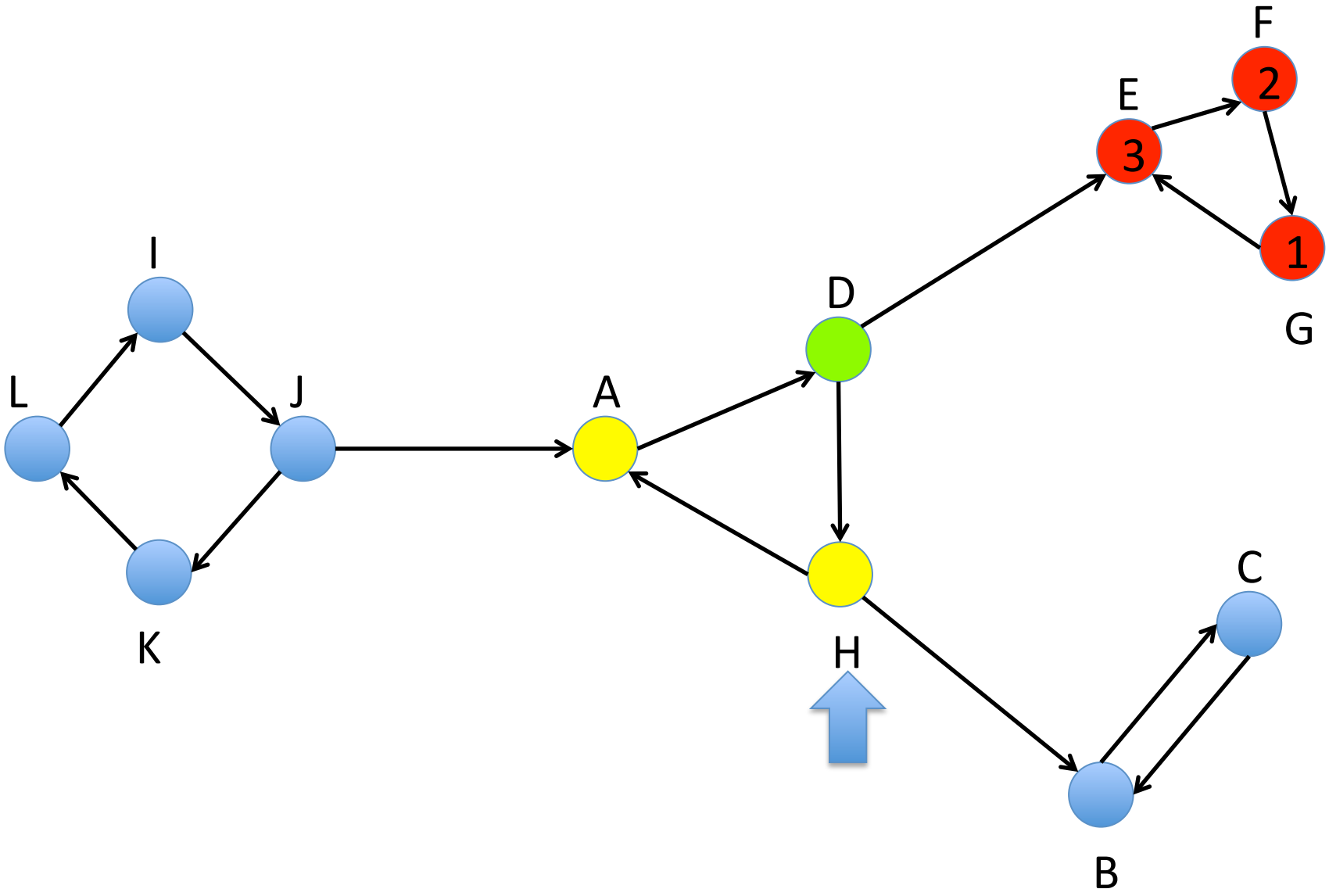

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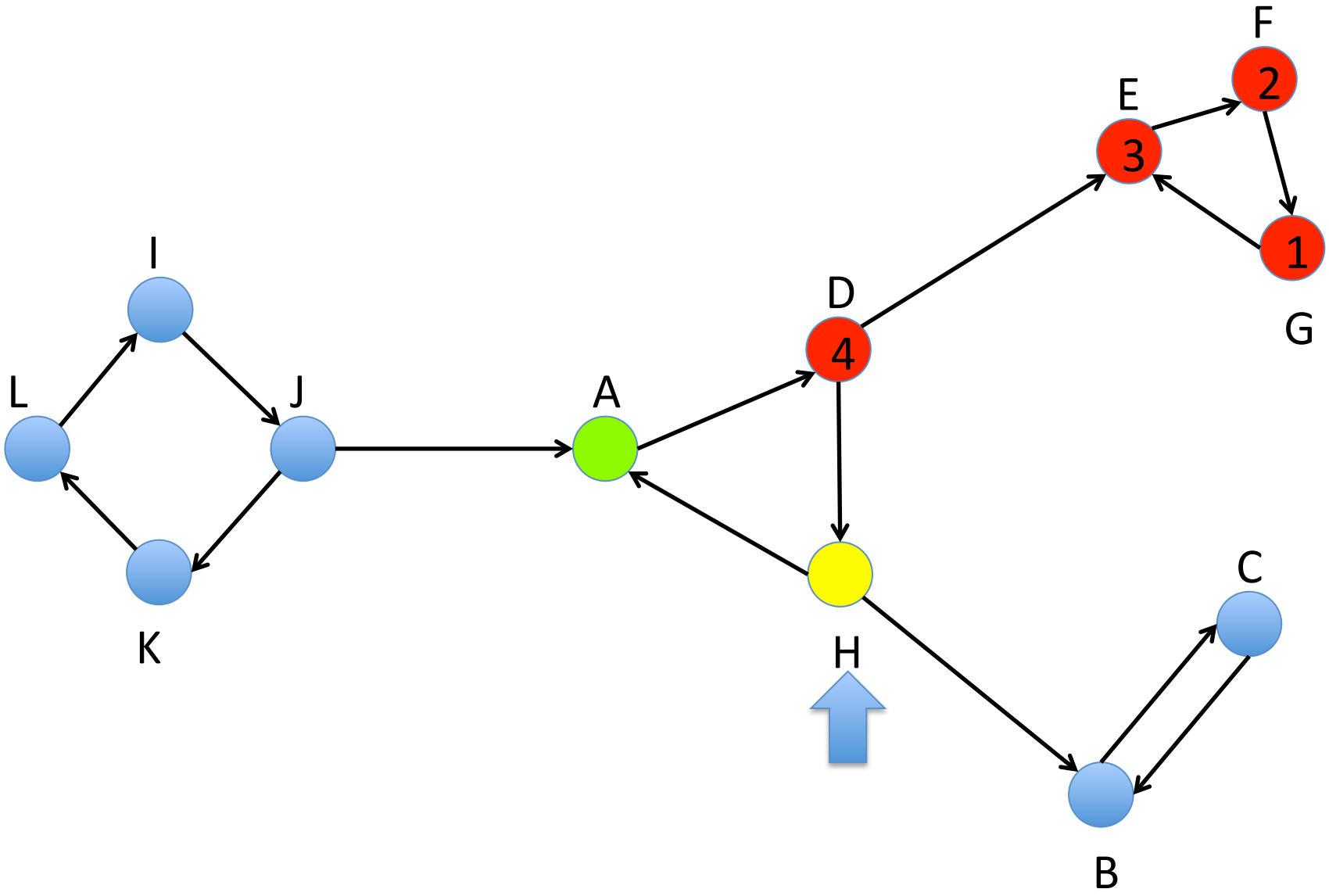

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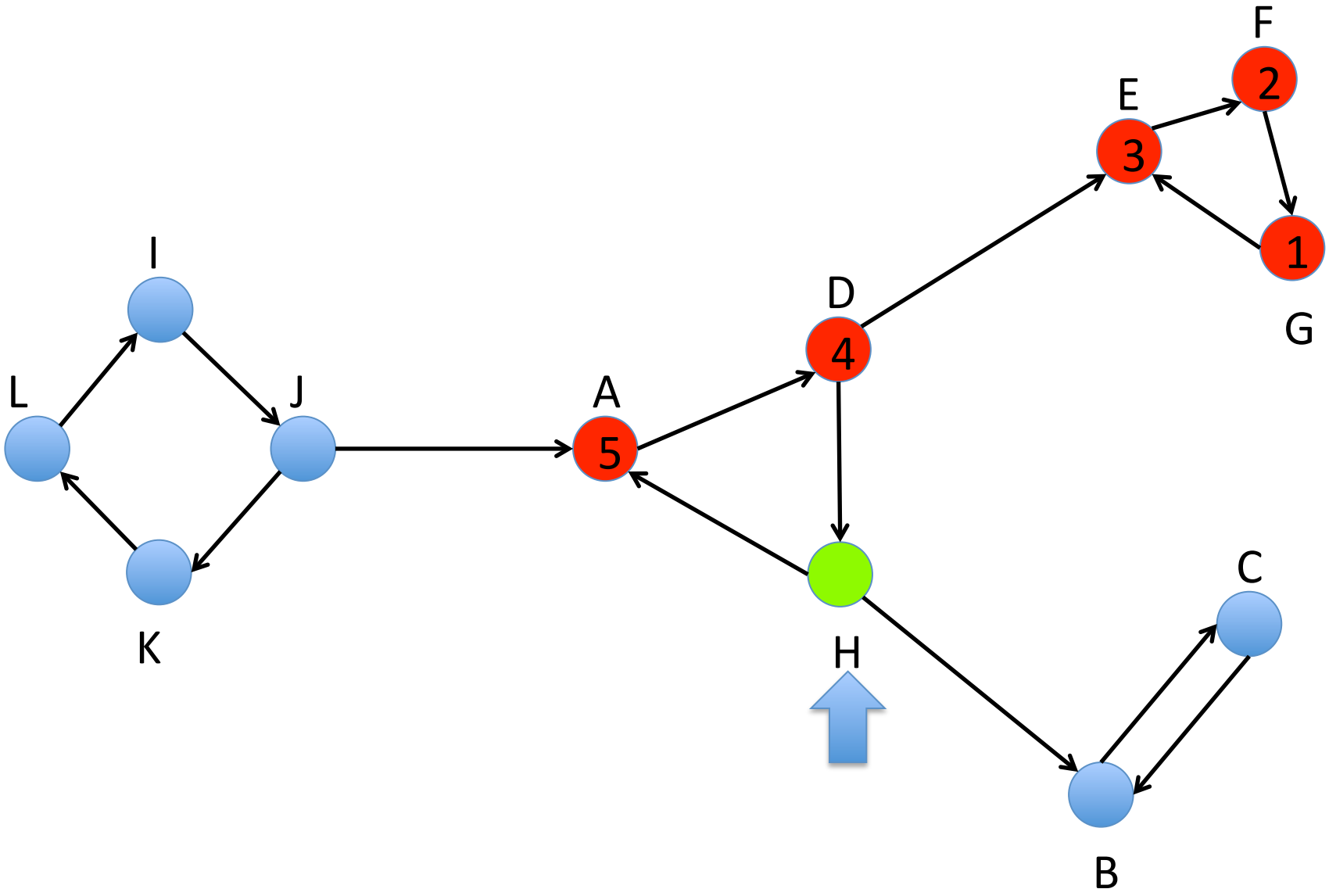

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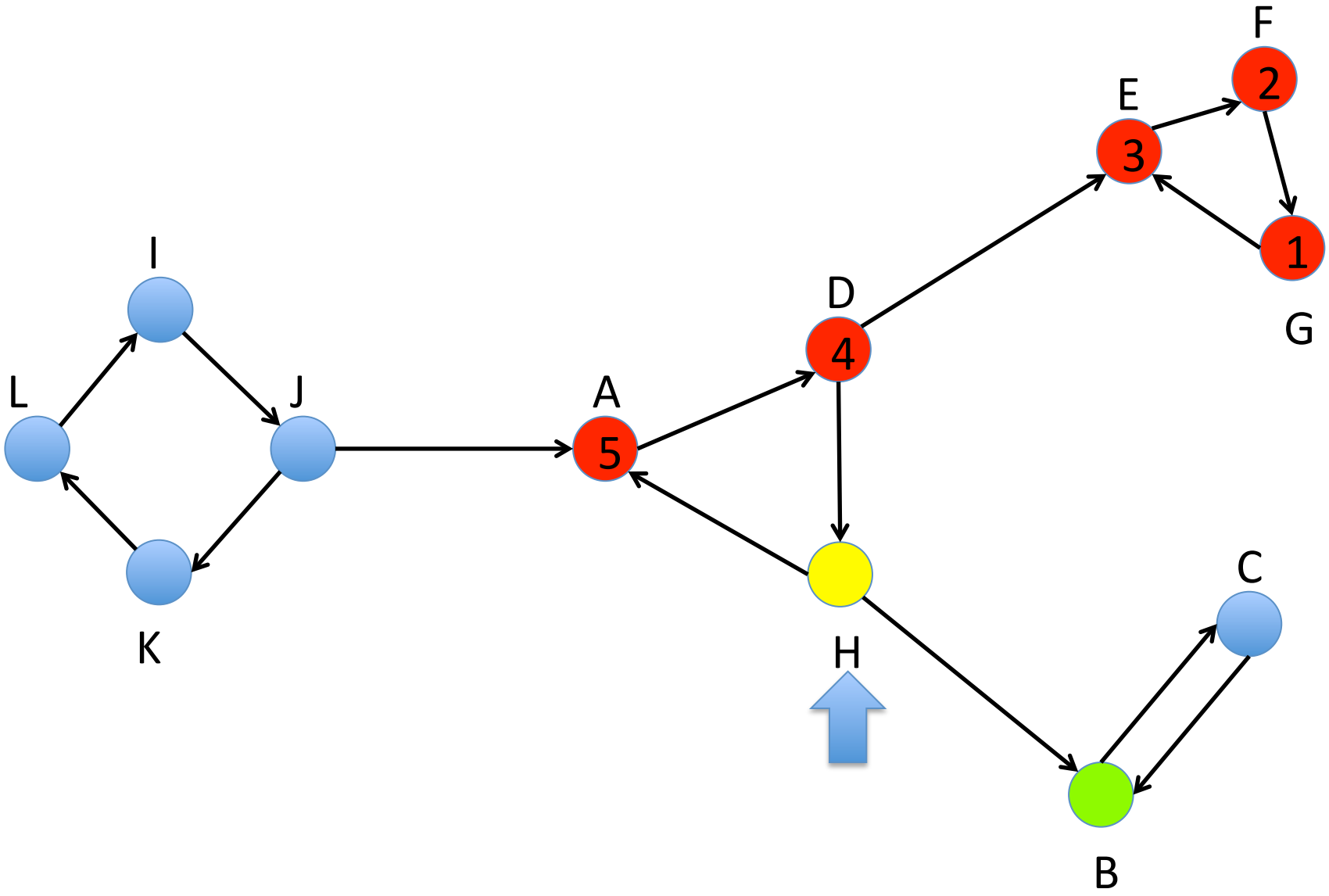

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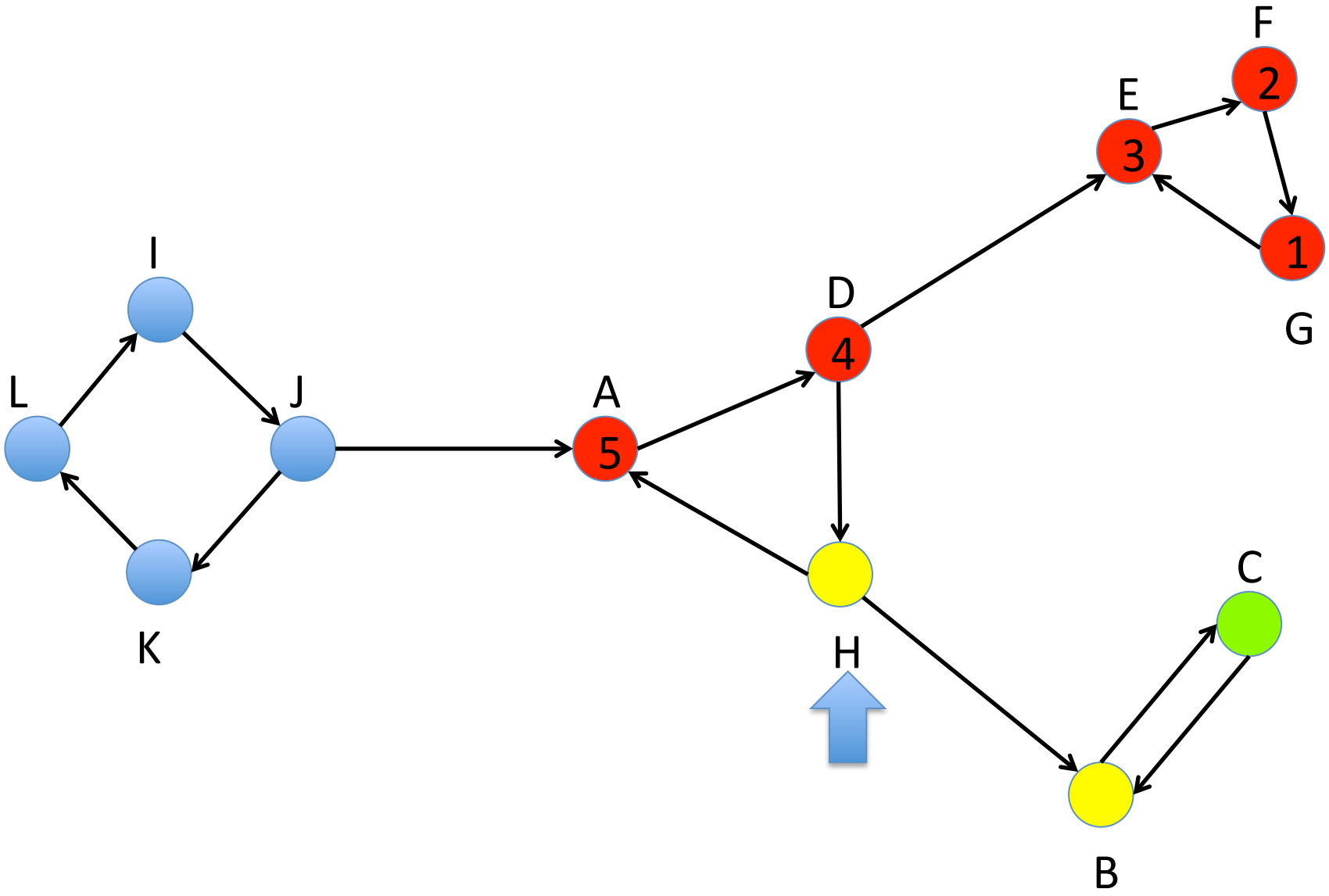

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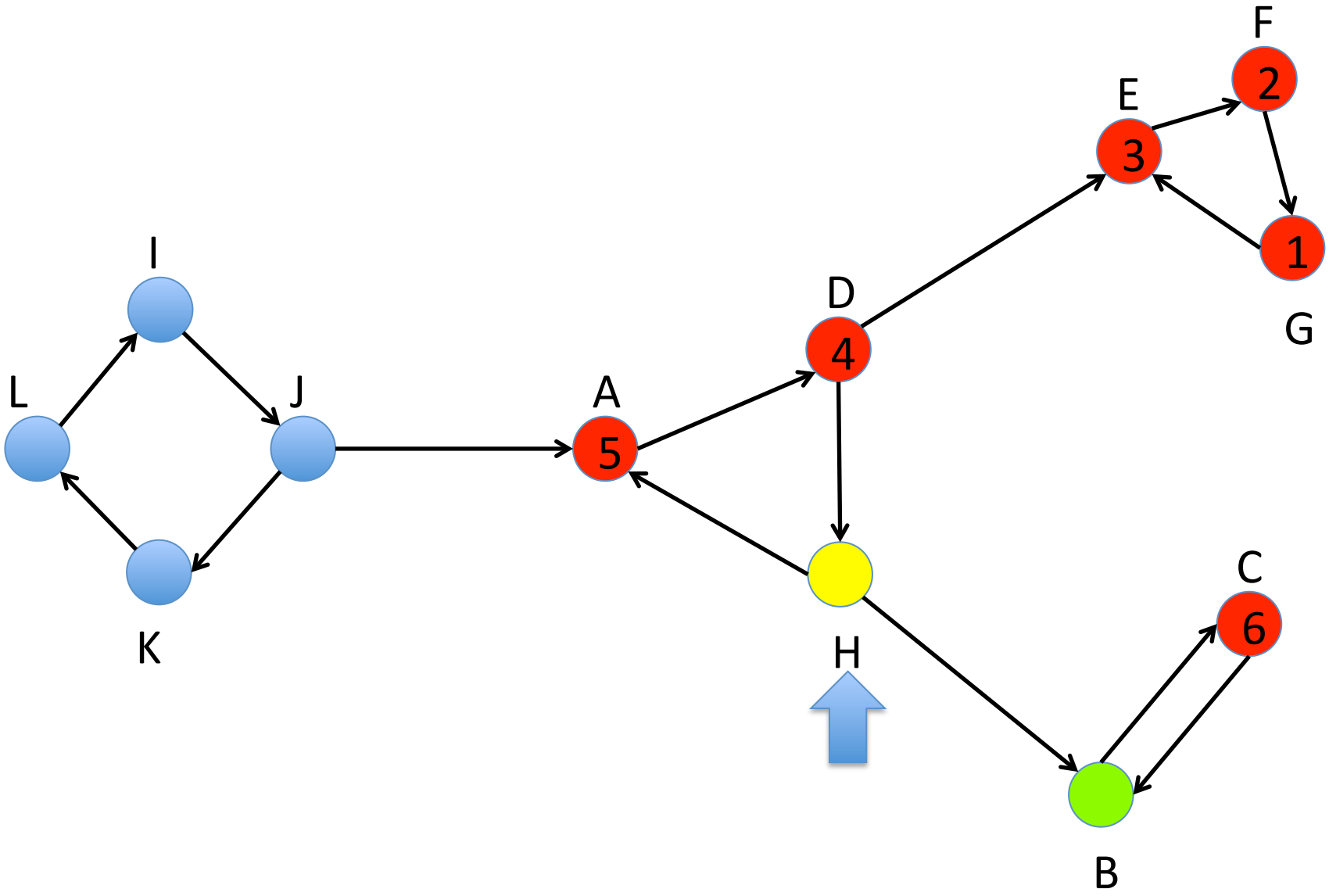

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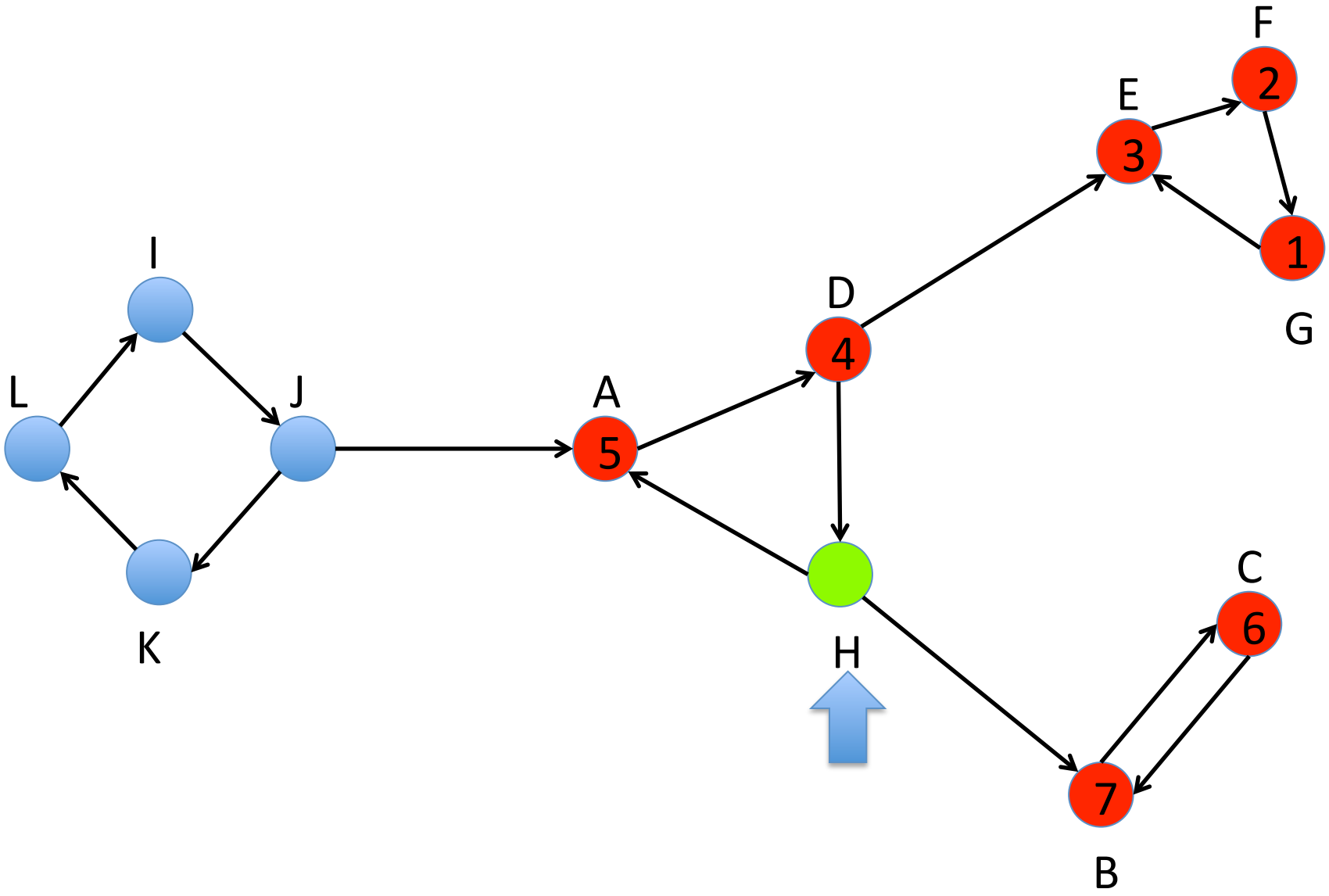

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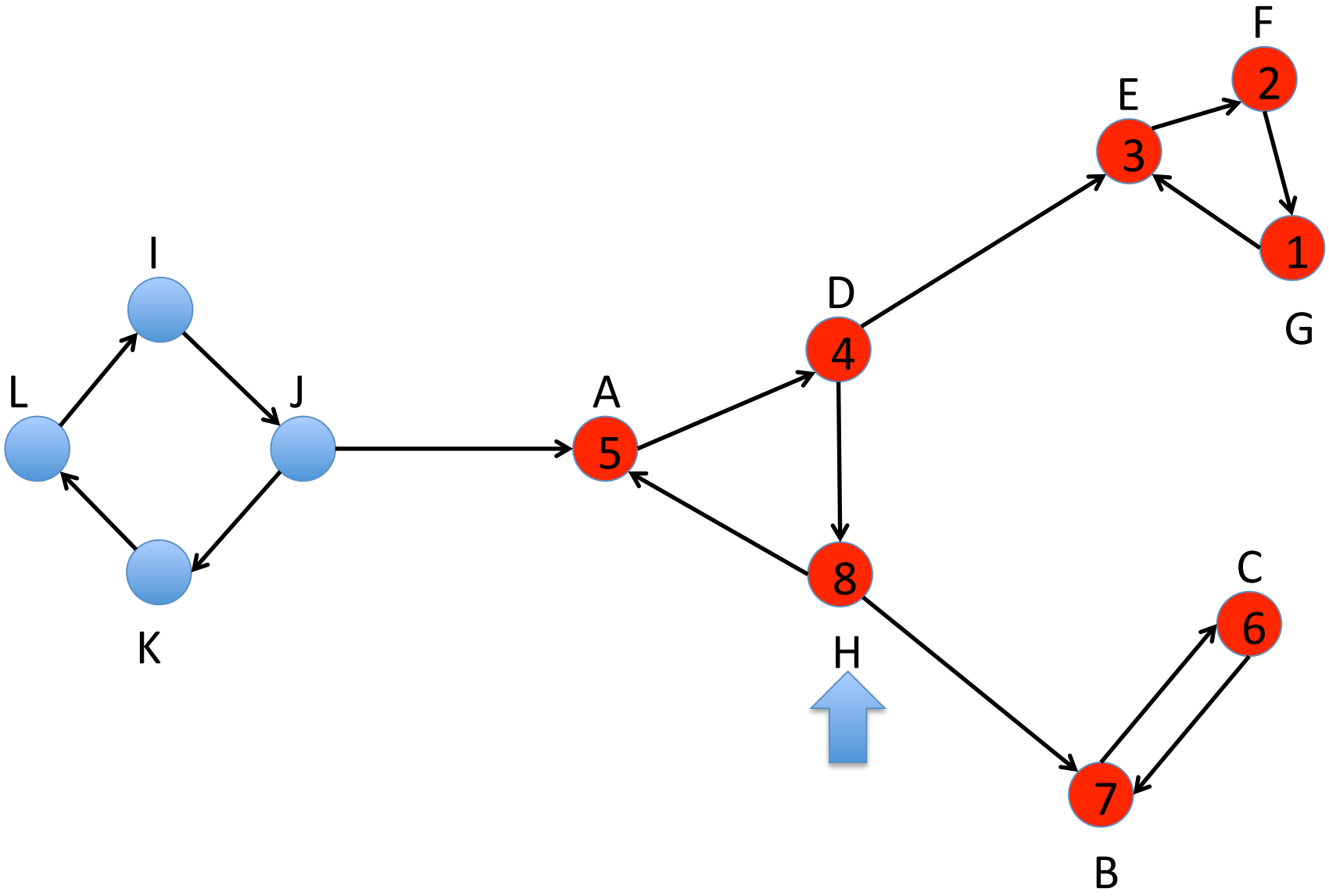

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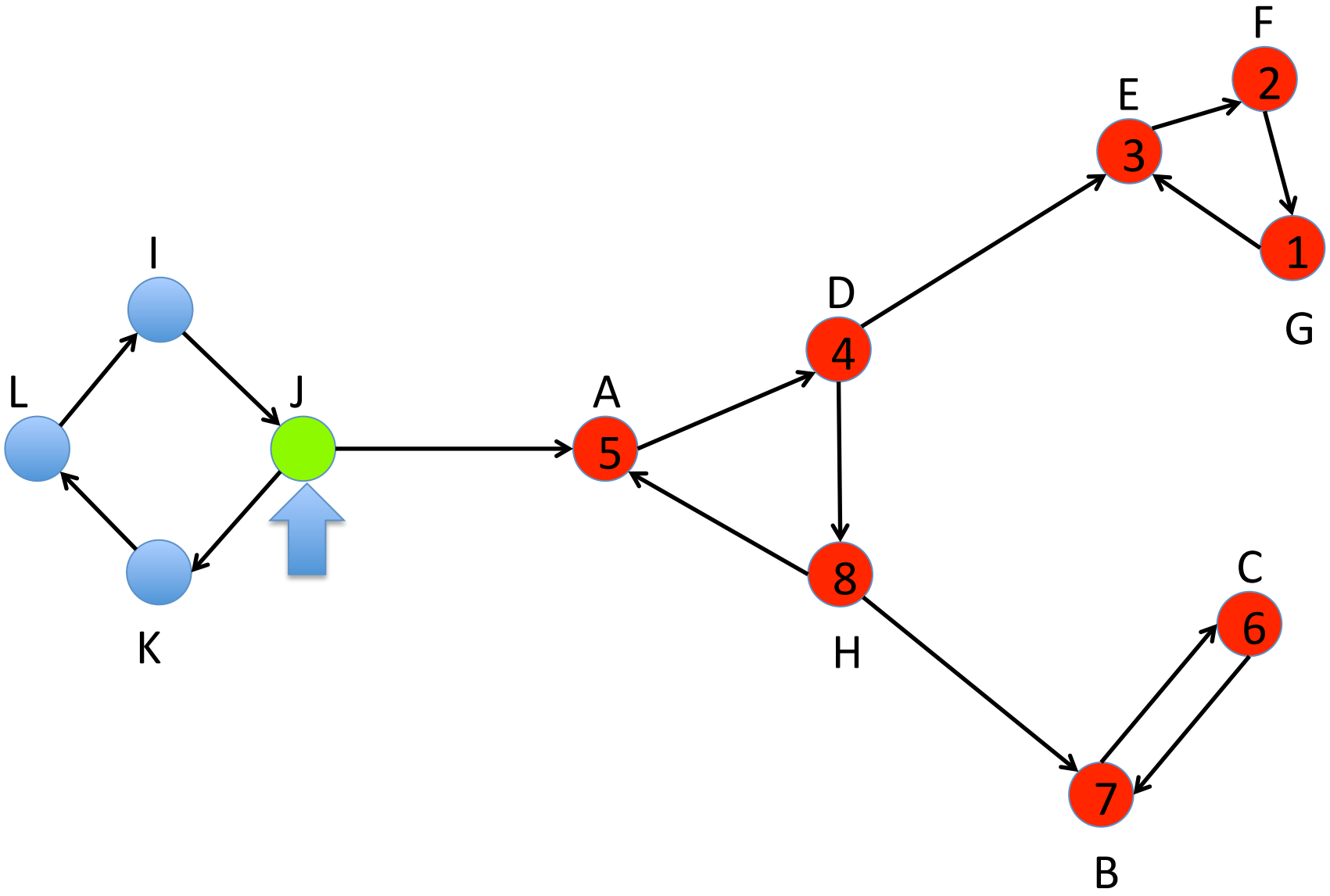

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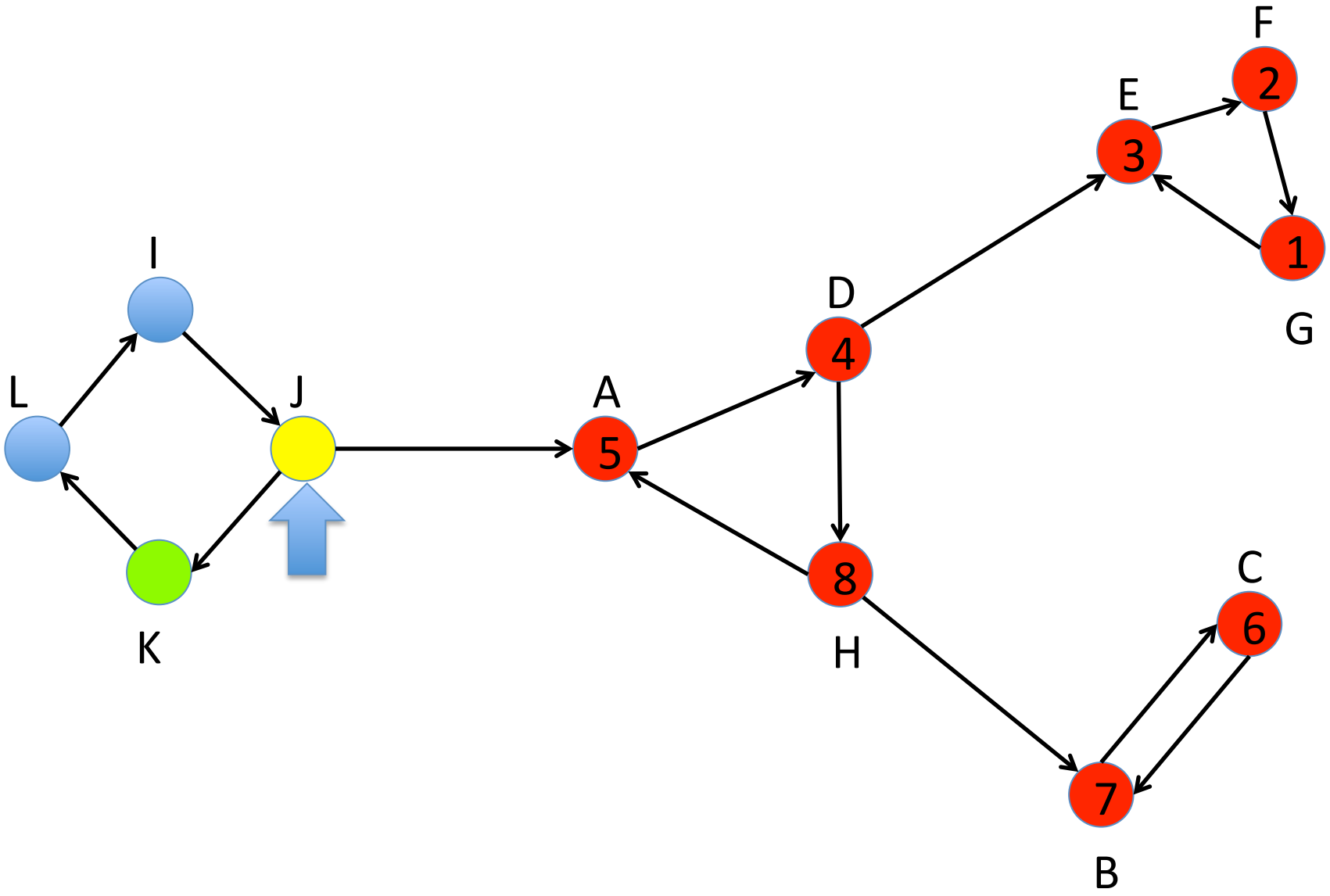

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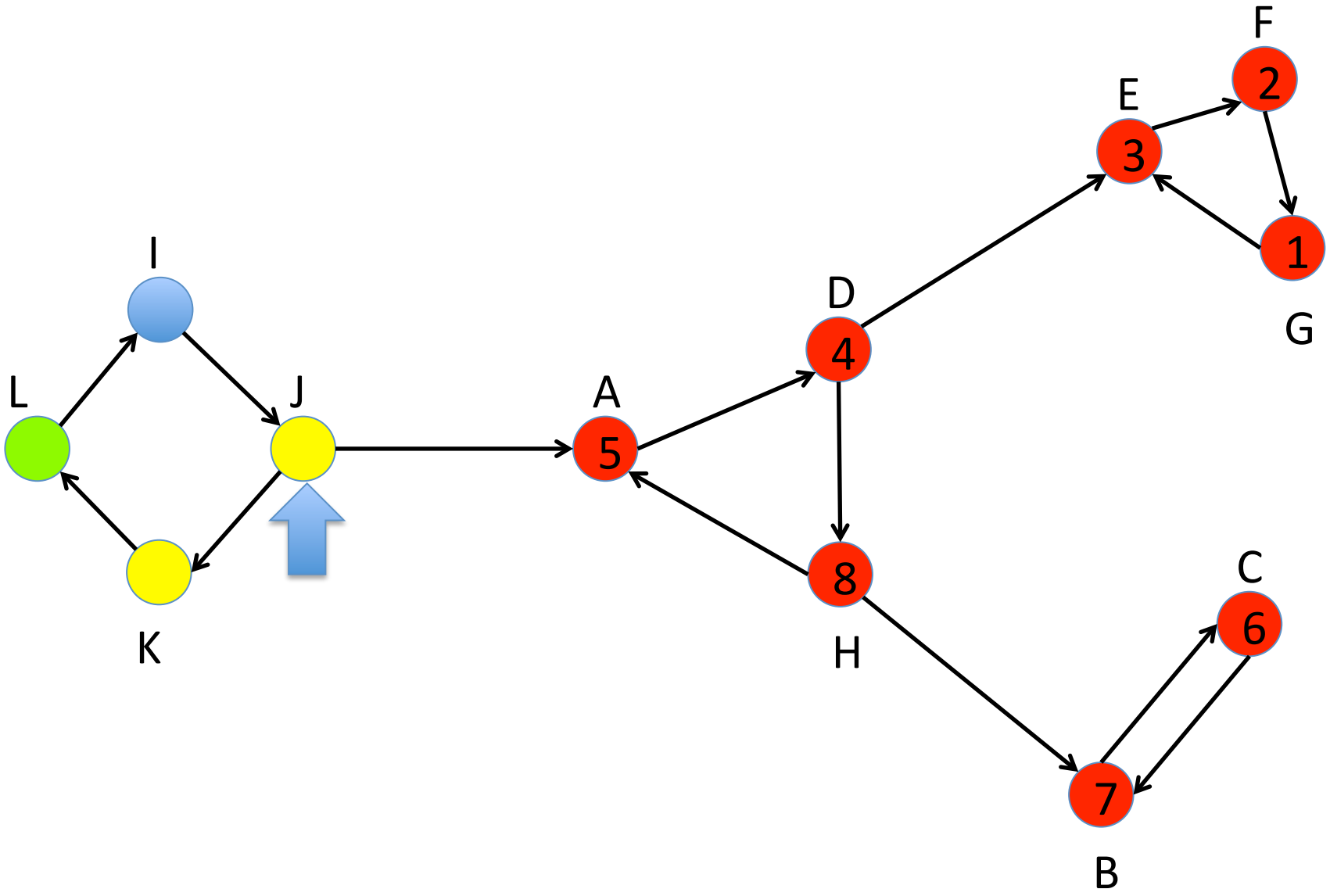

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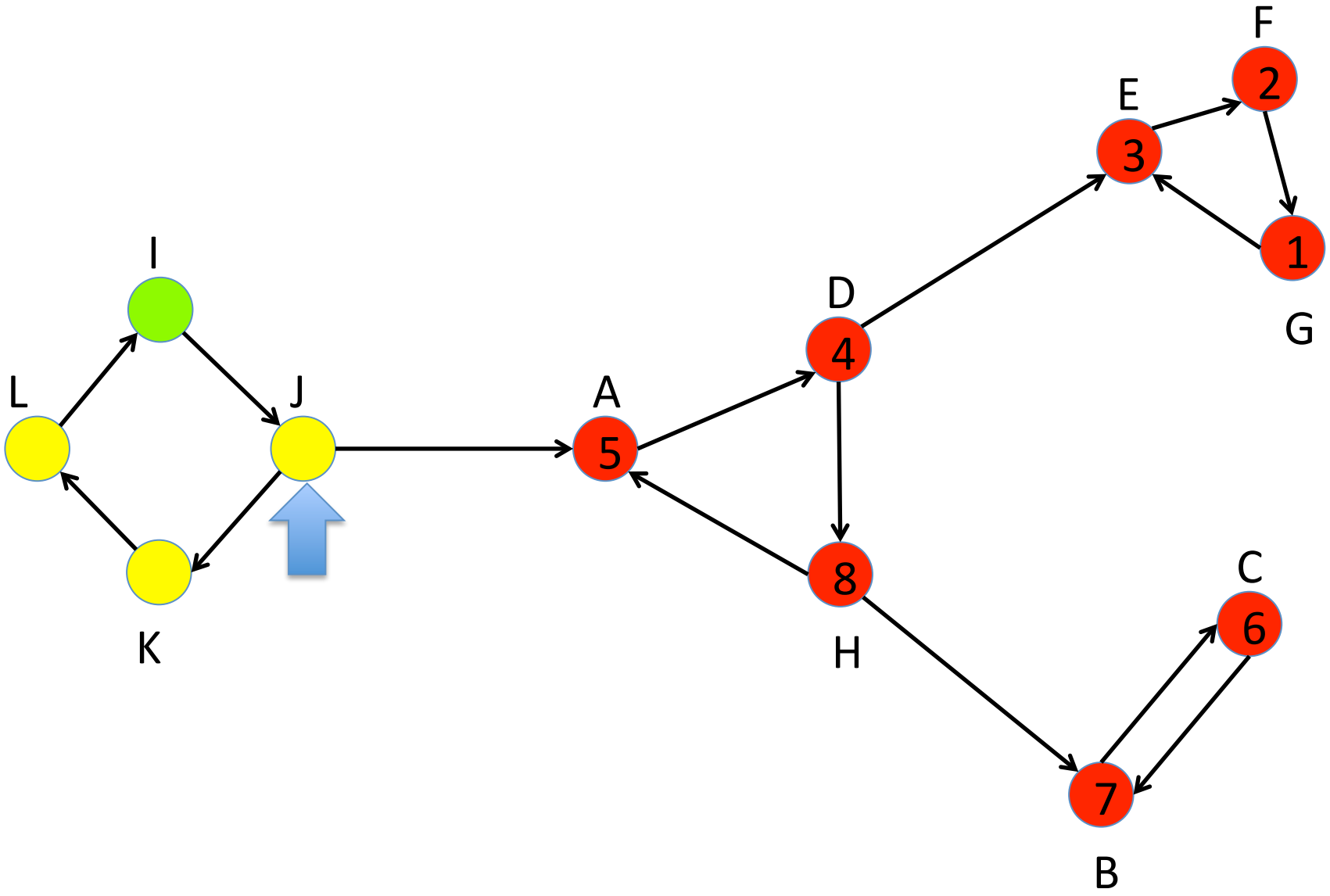

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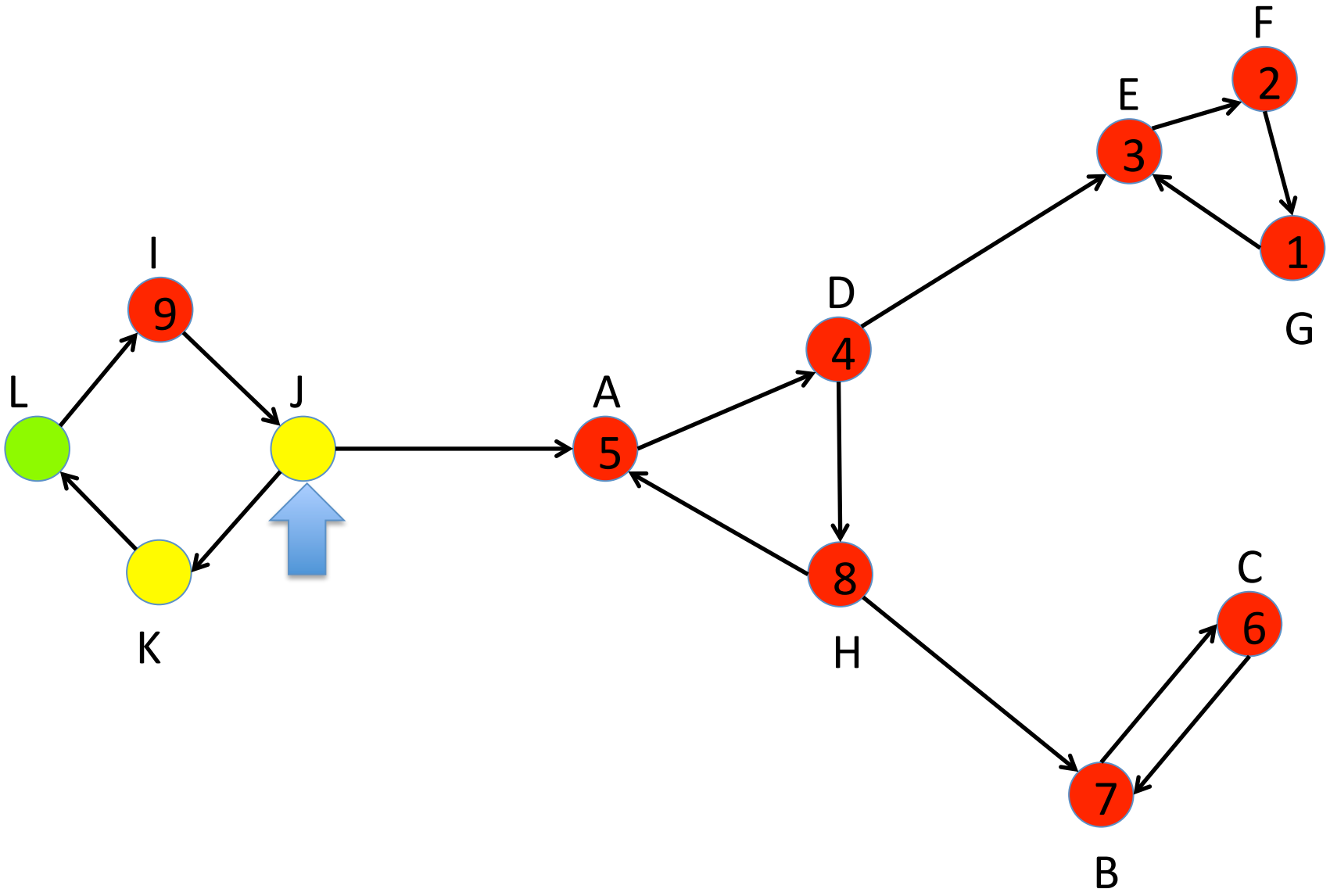

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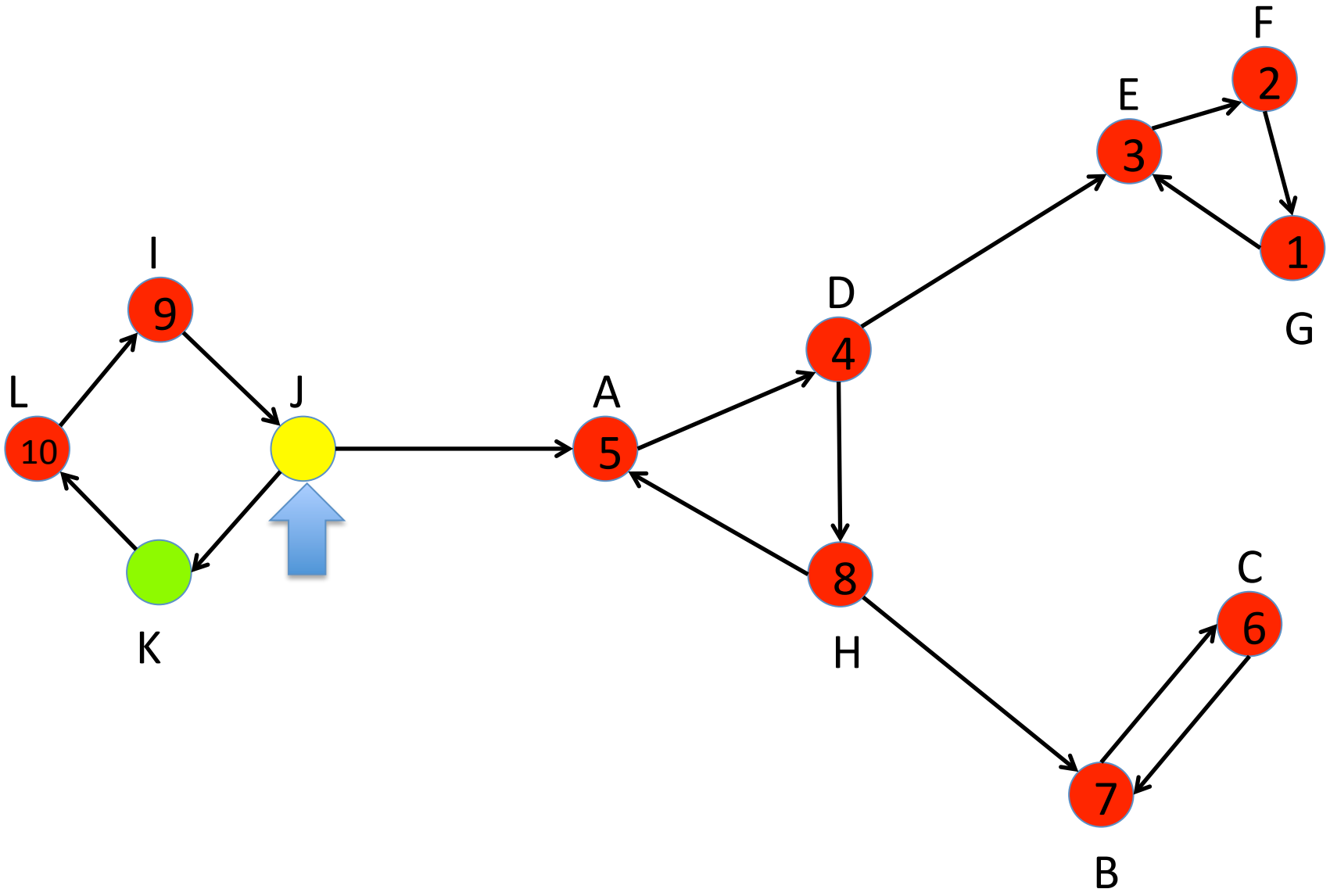

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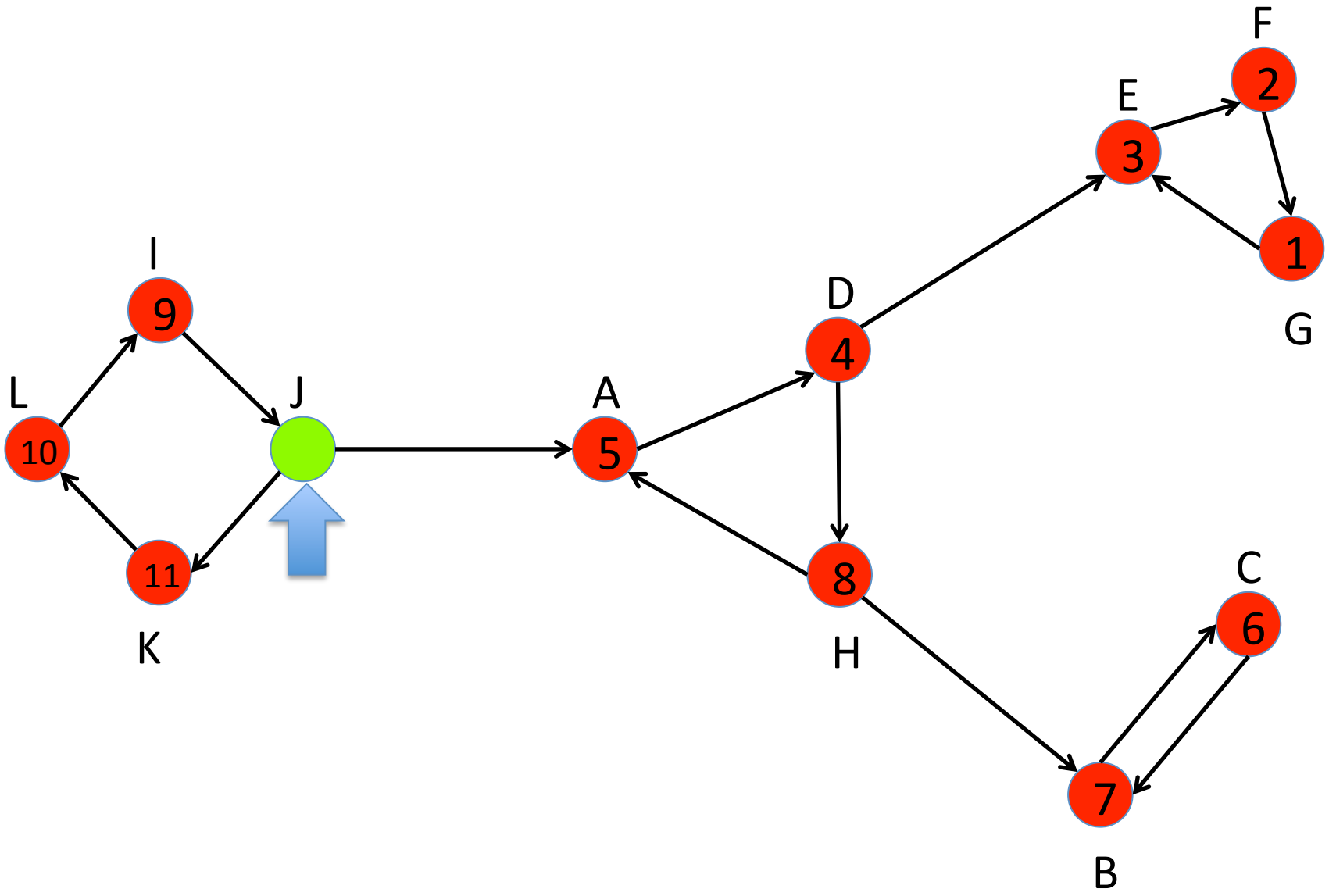

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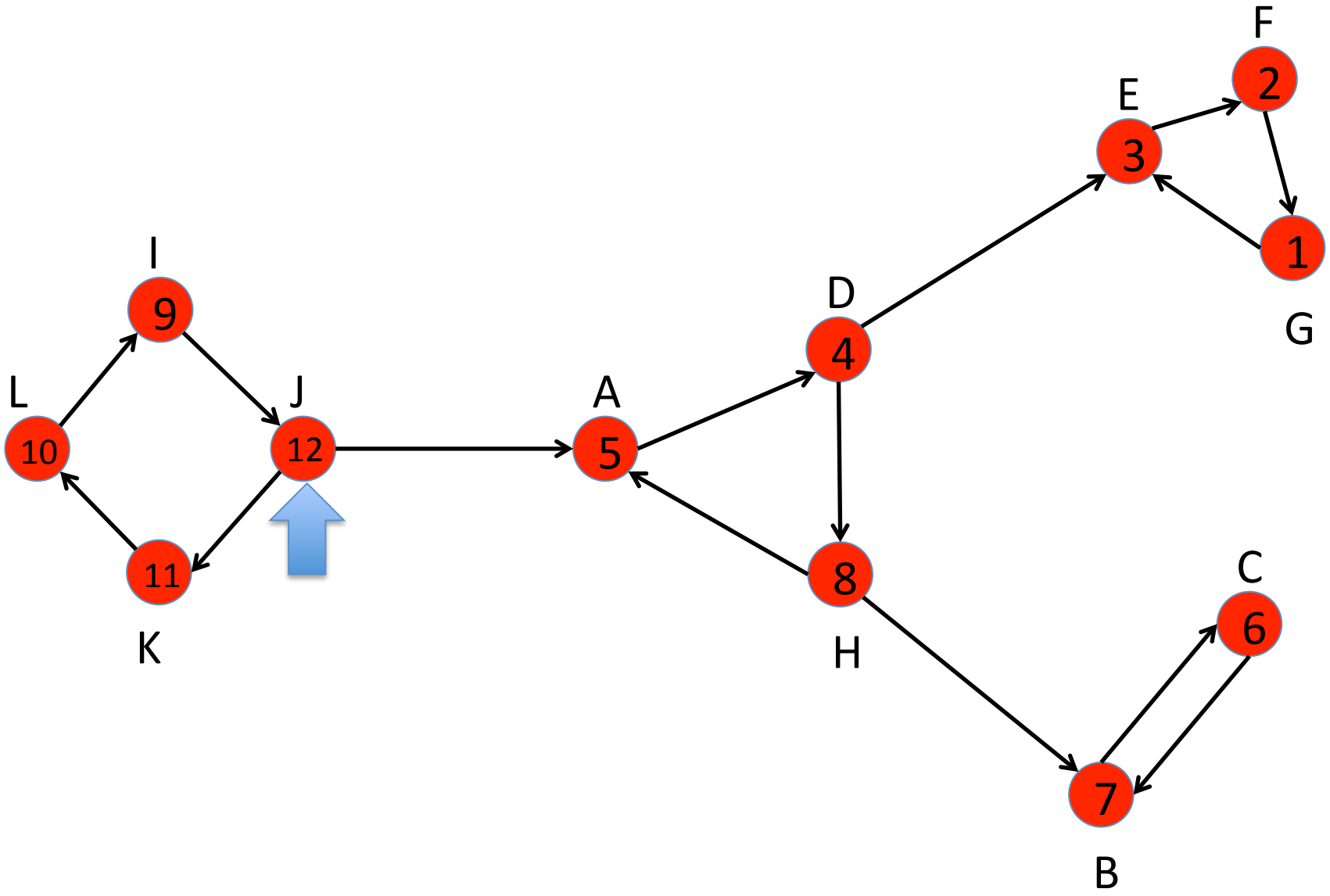

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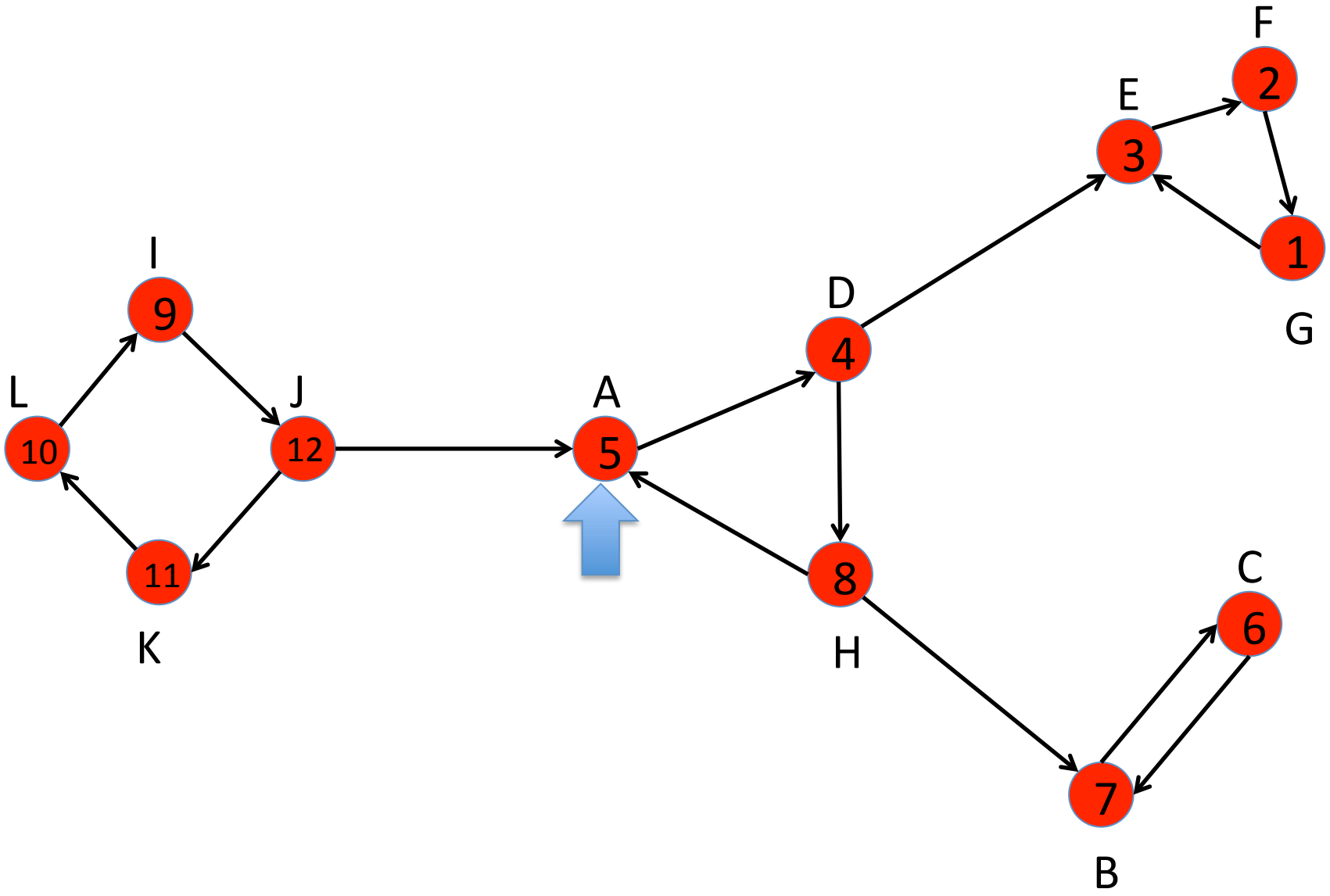

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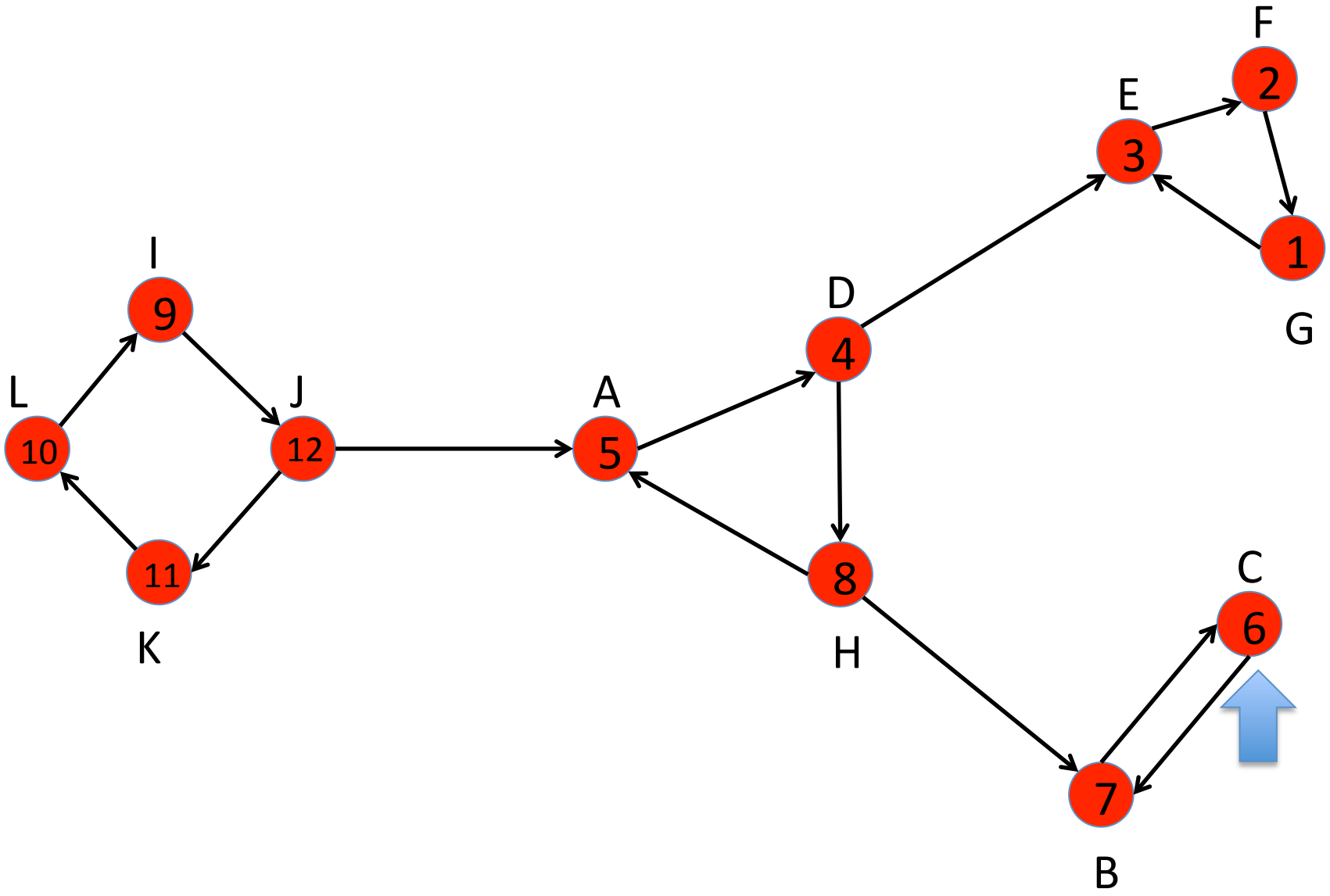

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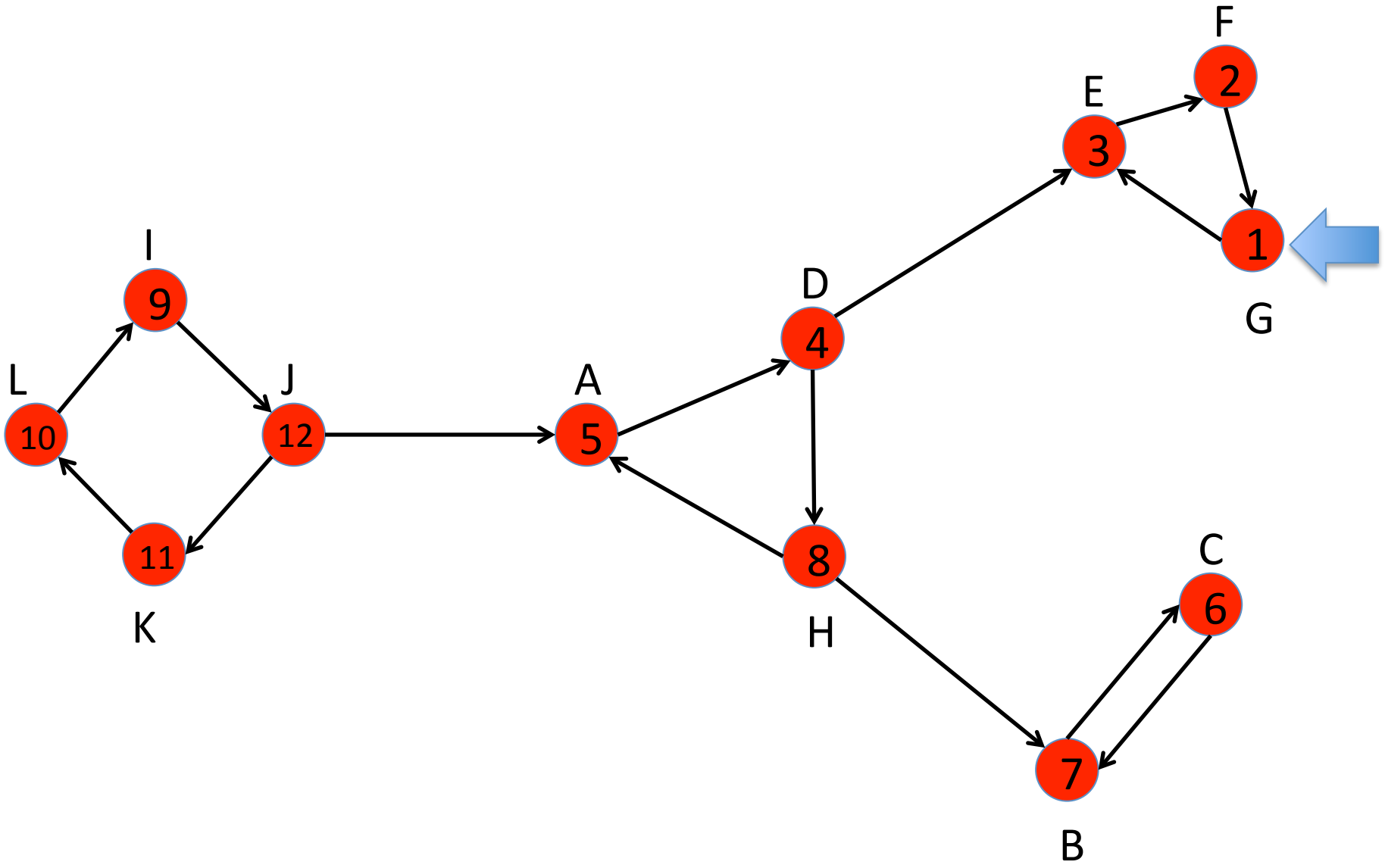

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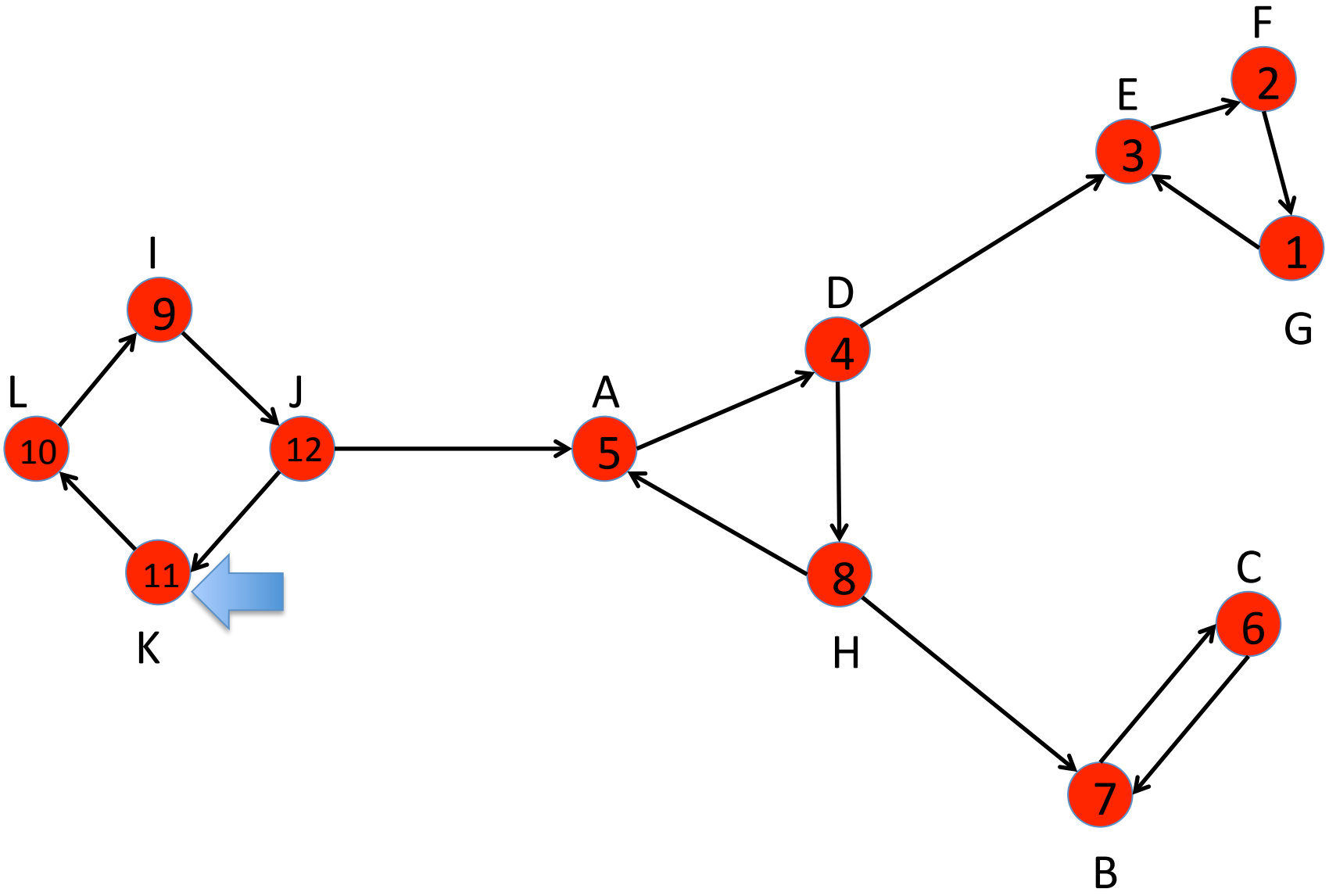

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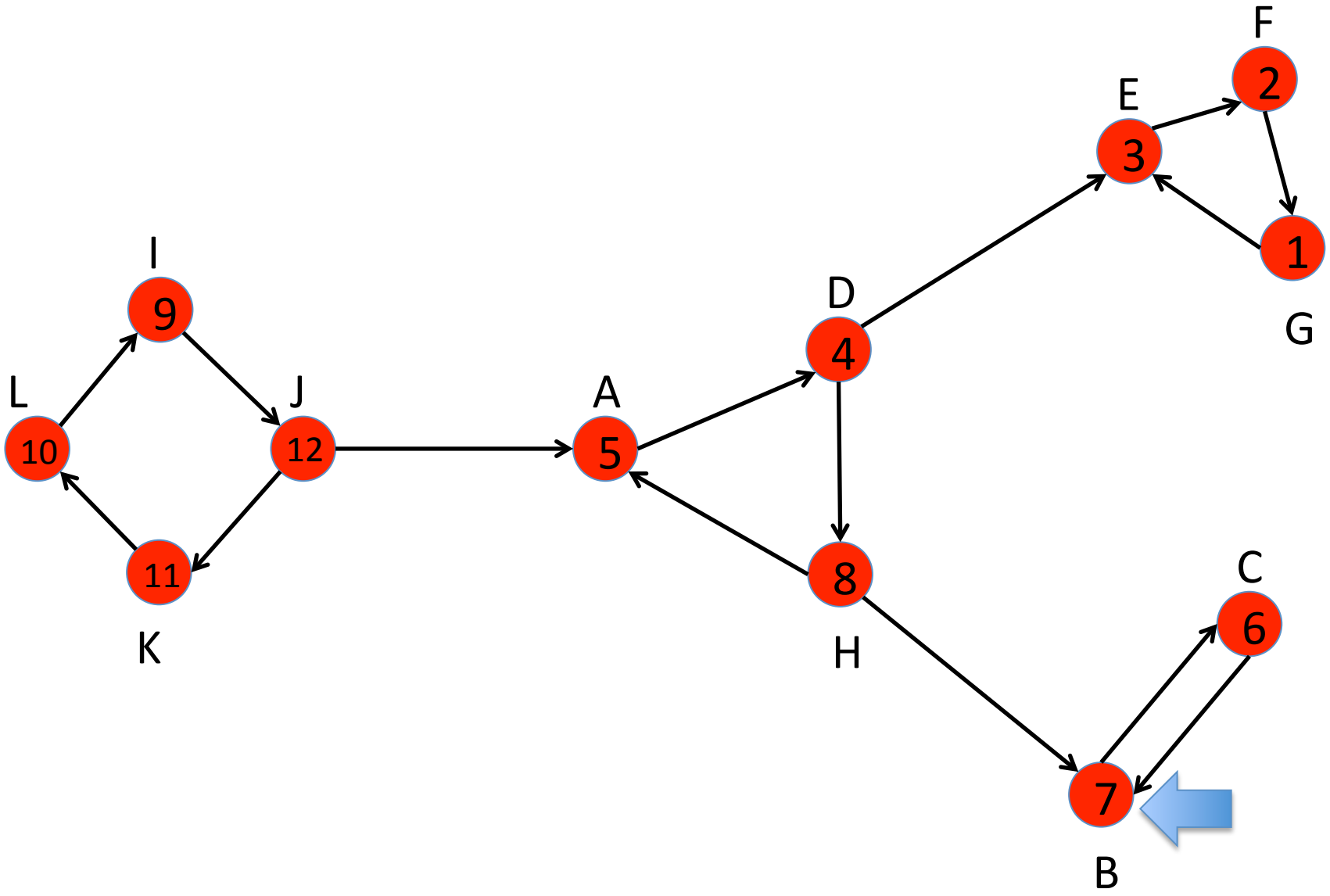

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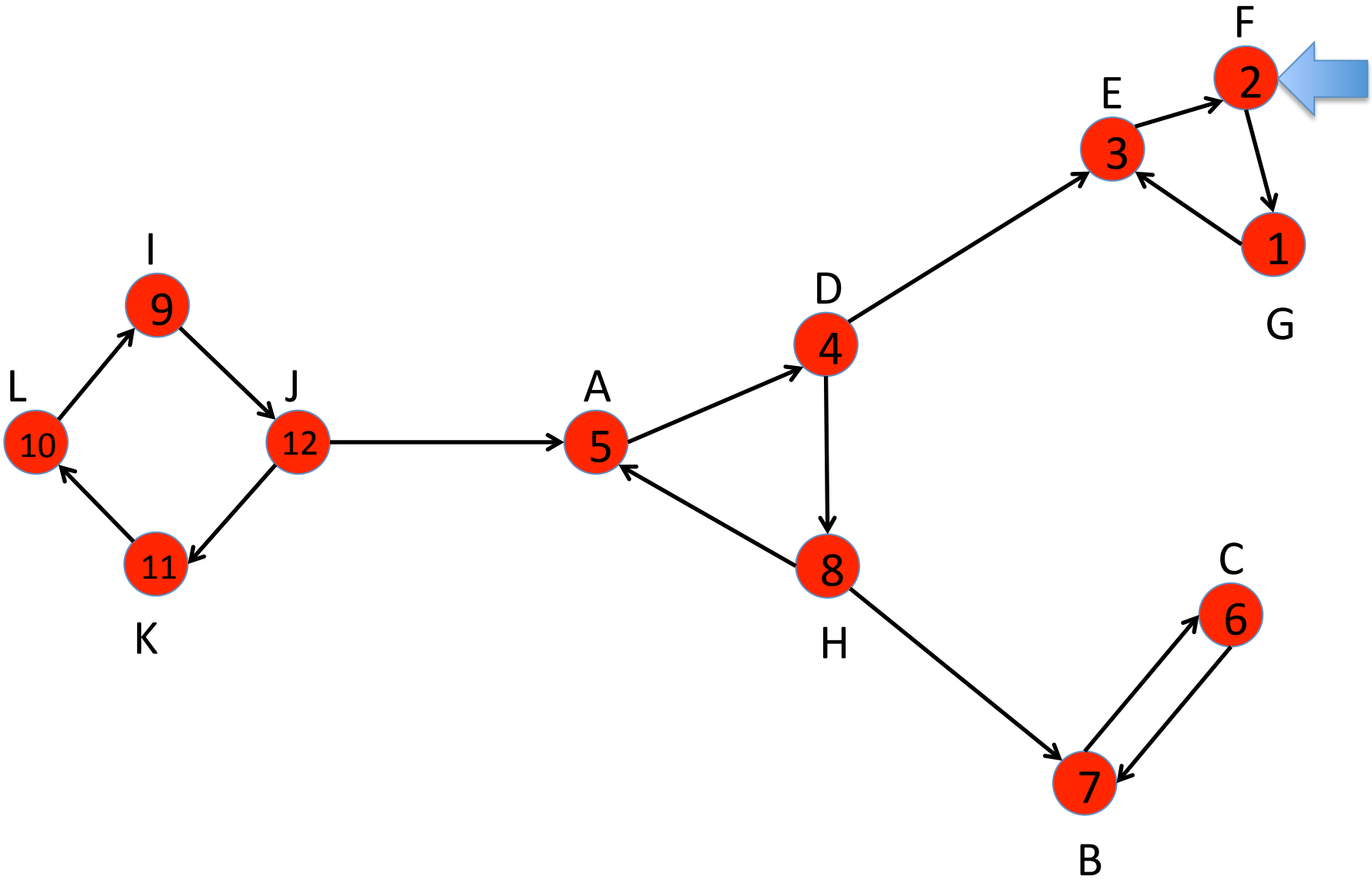

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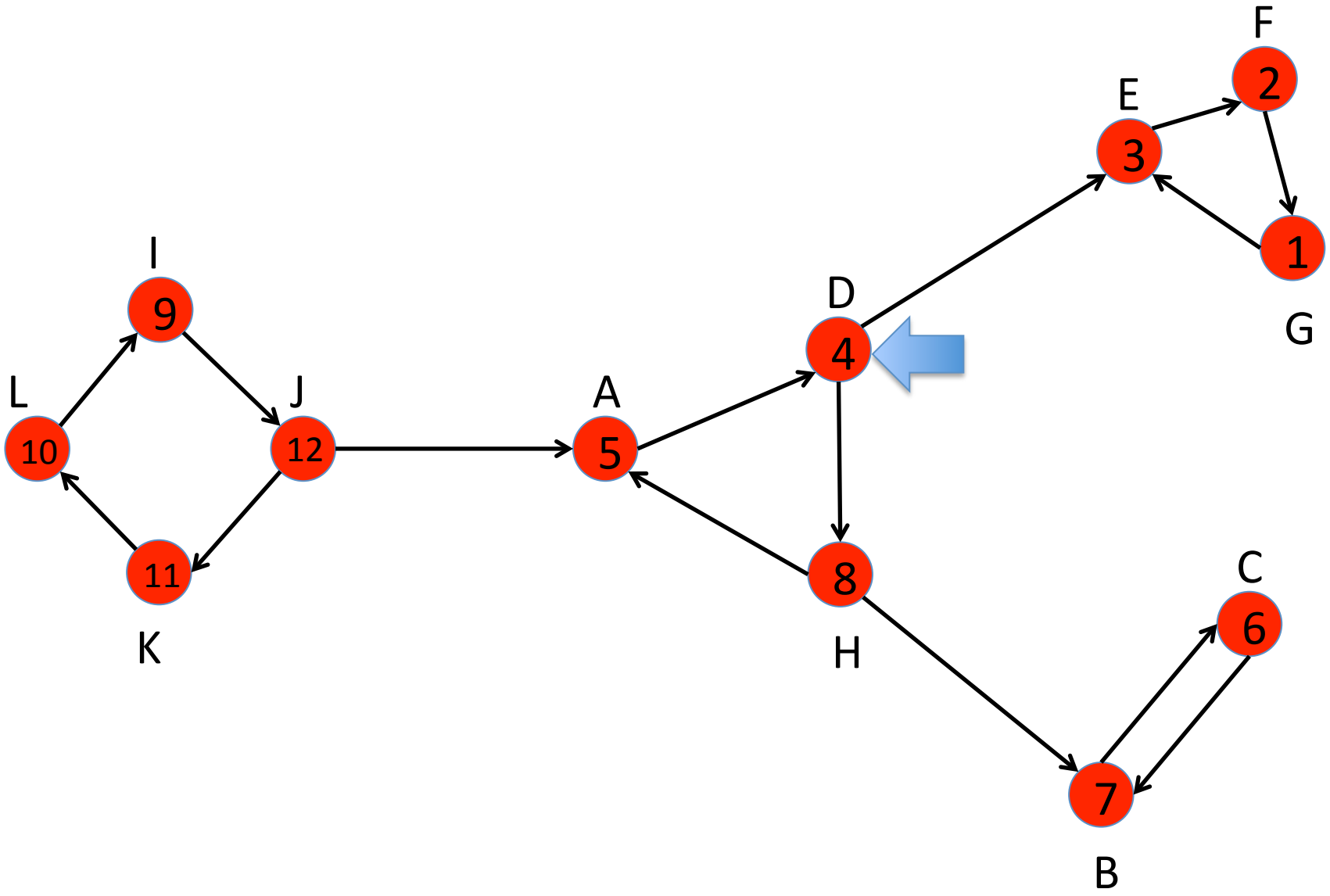

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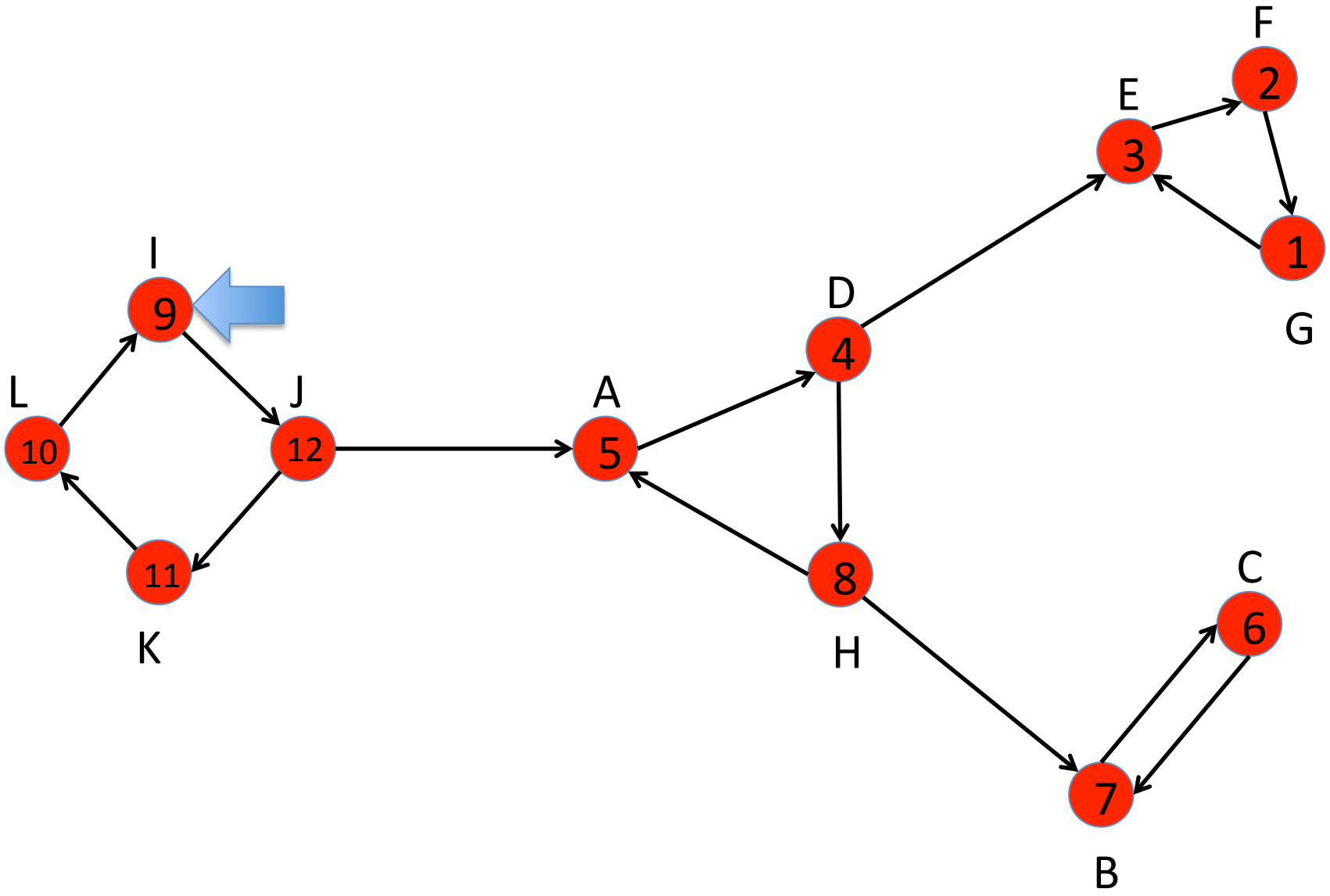

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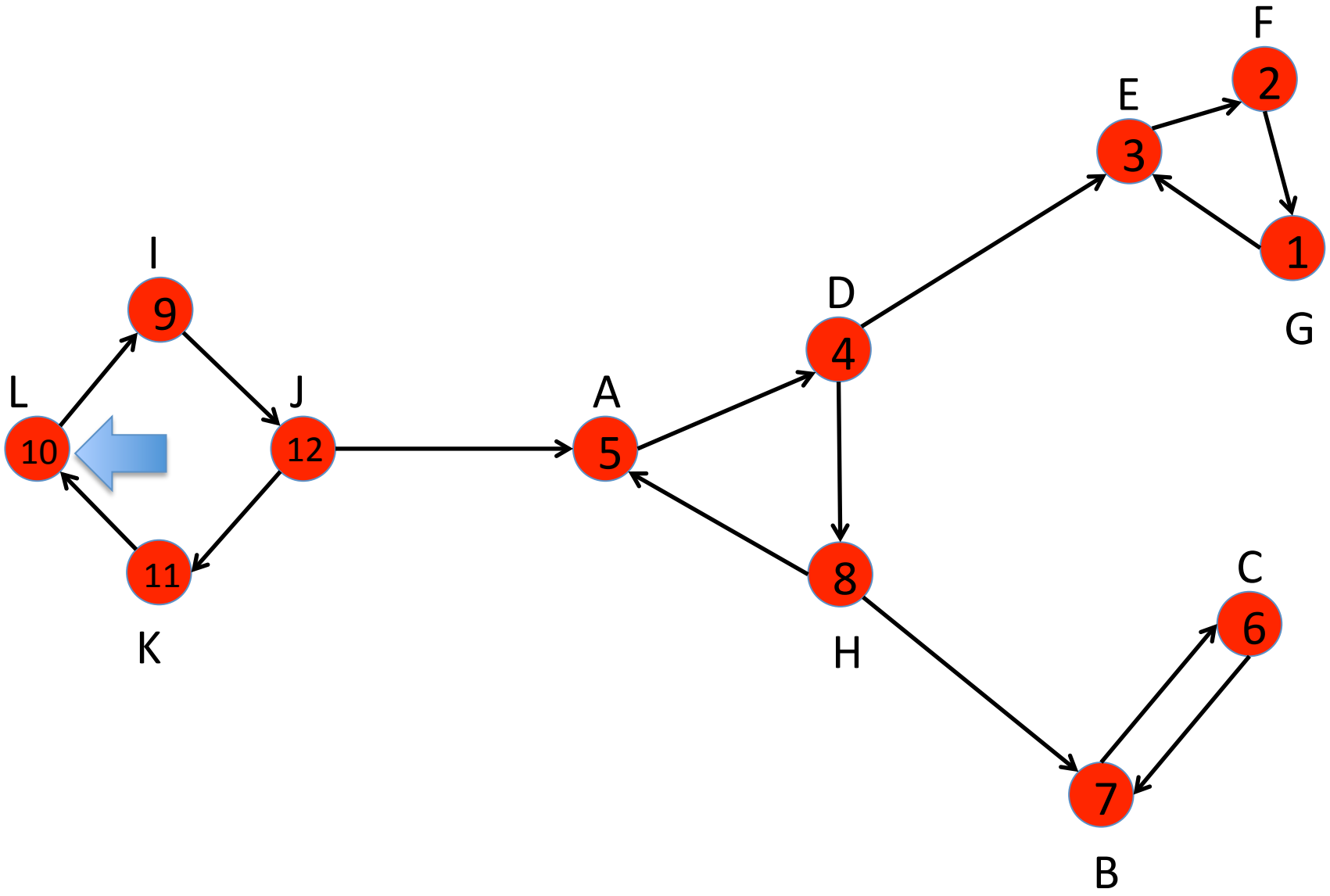
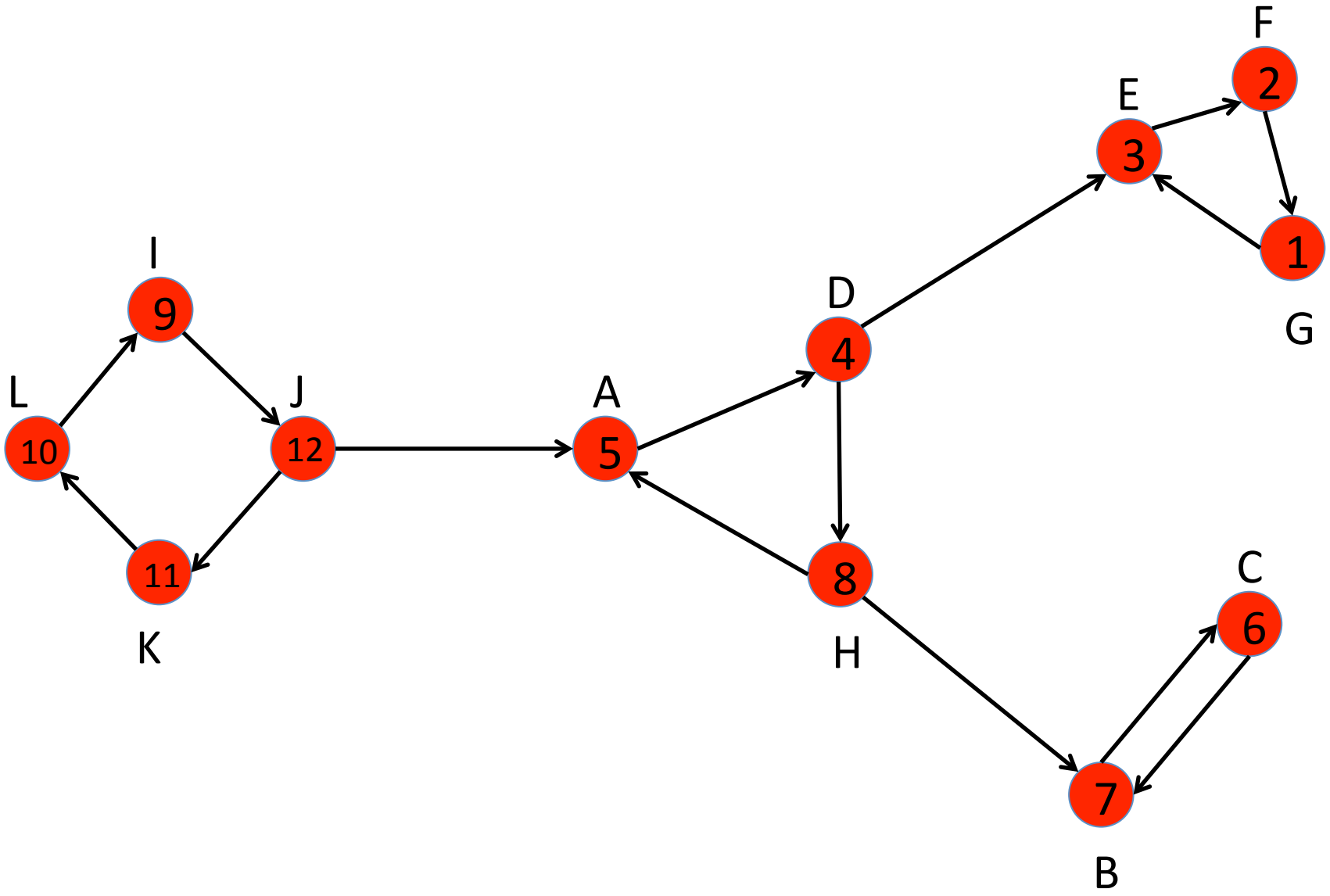


DFS & Finishing Times (2)



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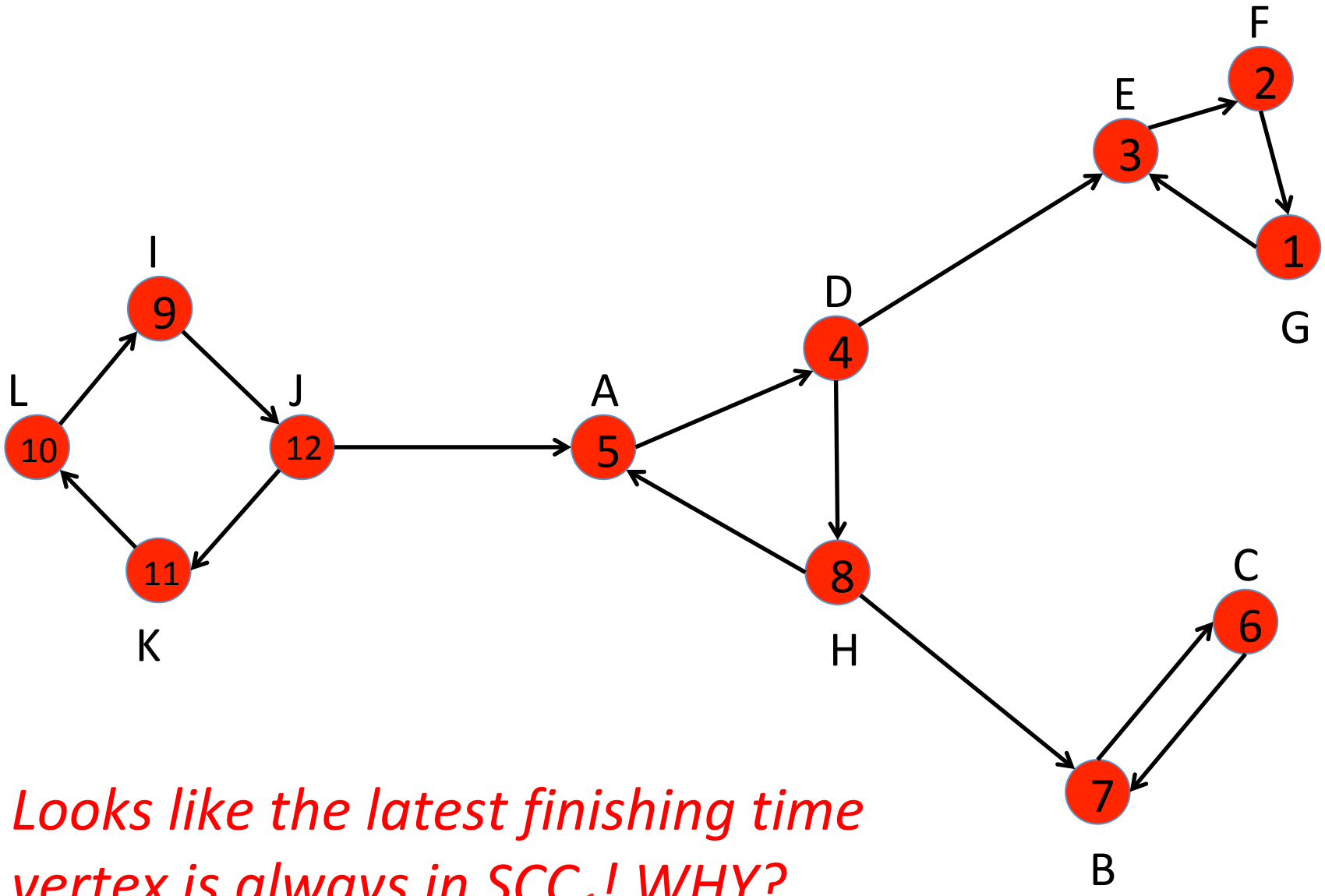


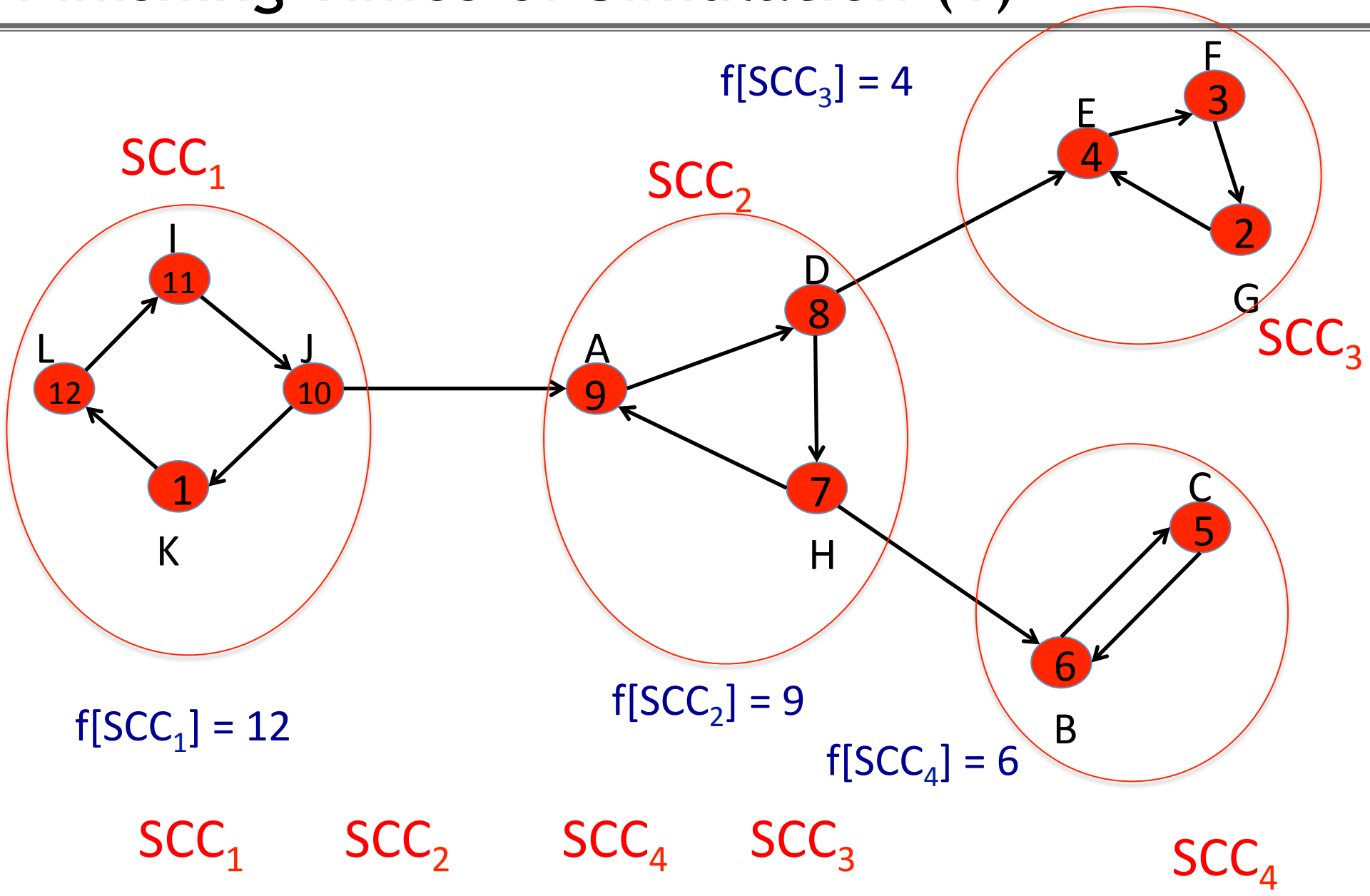
Looks like the latest finishing time vertex is always in SCC_1! WHY?

Definition: Finishing Time of an SCC

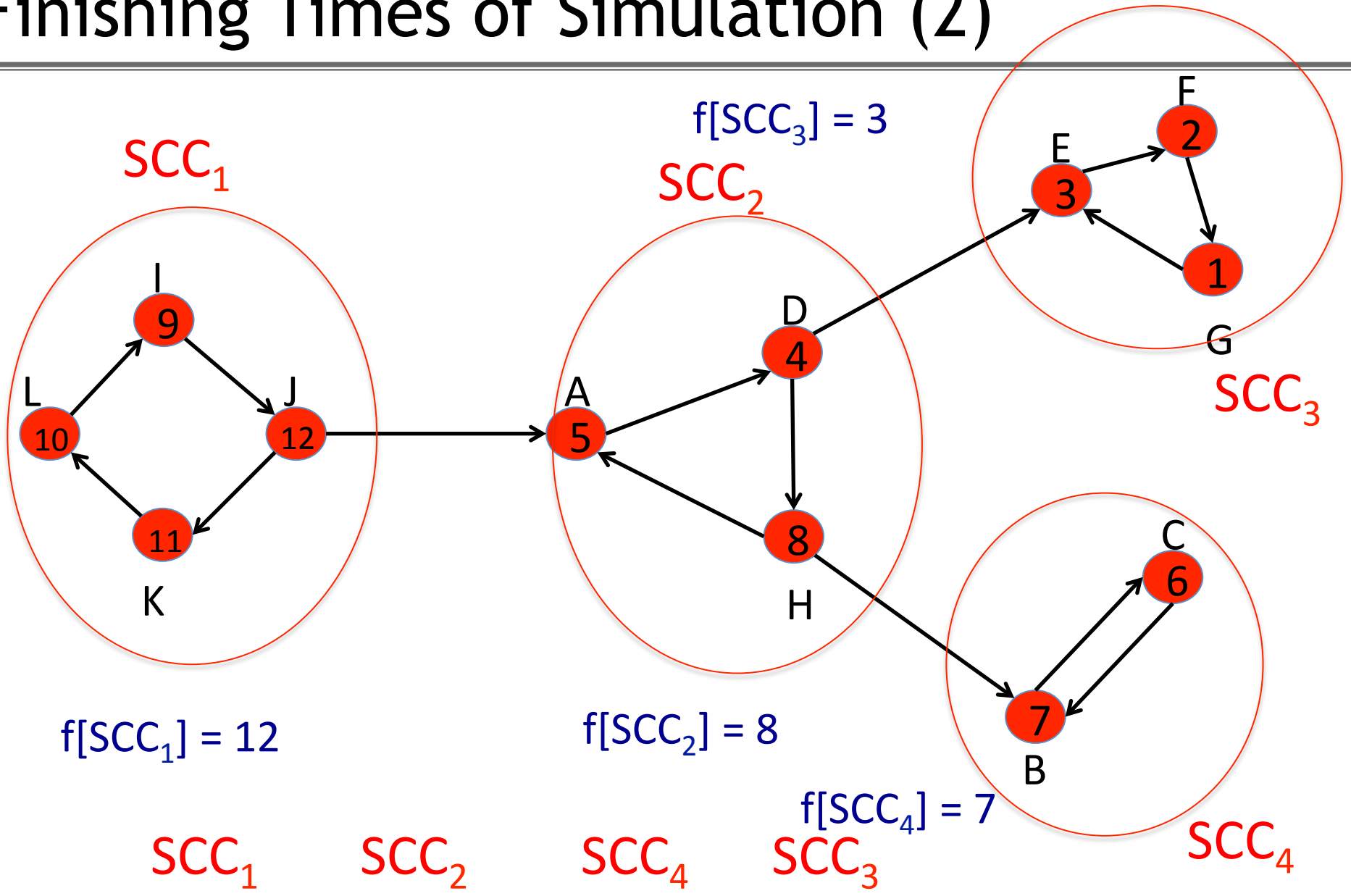
- Let "finishing time of an SCC_i" in DFS traversal be the maximum finishing time of the vertices in SCC_i

$$f[SCC_i] = \max_{v \in SCC_i} f[v]$$

Finishing Times of Simulation (1)



Finishing Times of Simulation (2)



Key Lemma about finishing times of SCCs

In G^{SCC} if $SCC_i \rightarrow SCC_j$, then
$***f[SCC_i] > f[SCC_j]***$

Will prove this later

Exstended Key Lemma

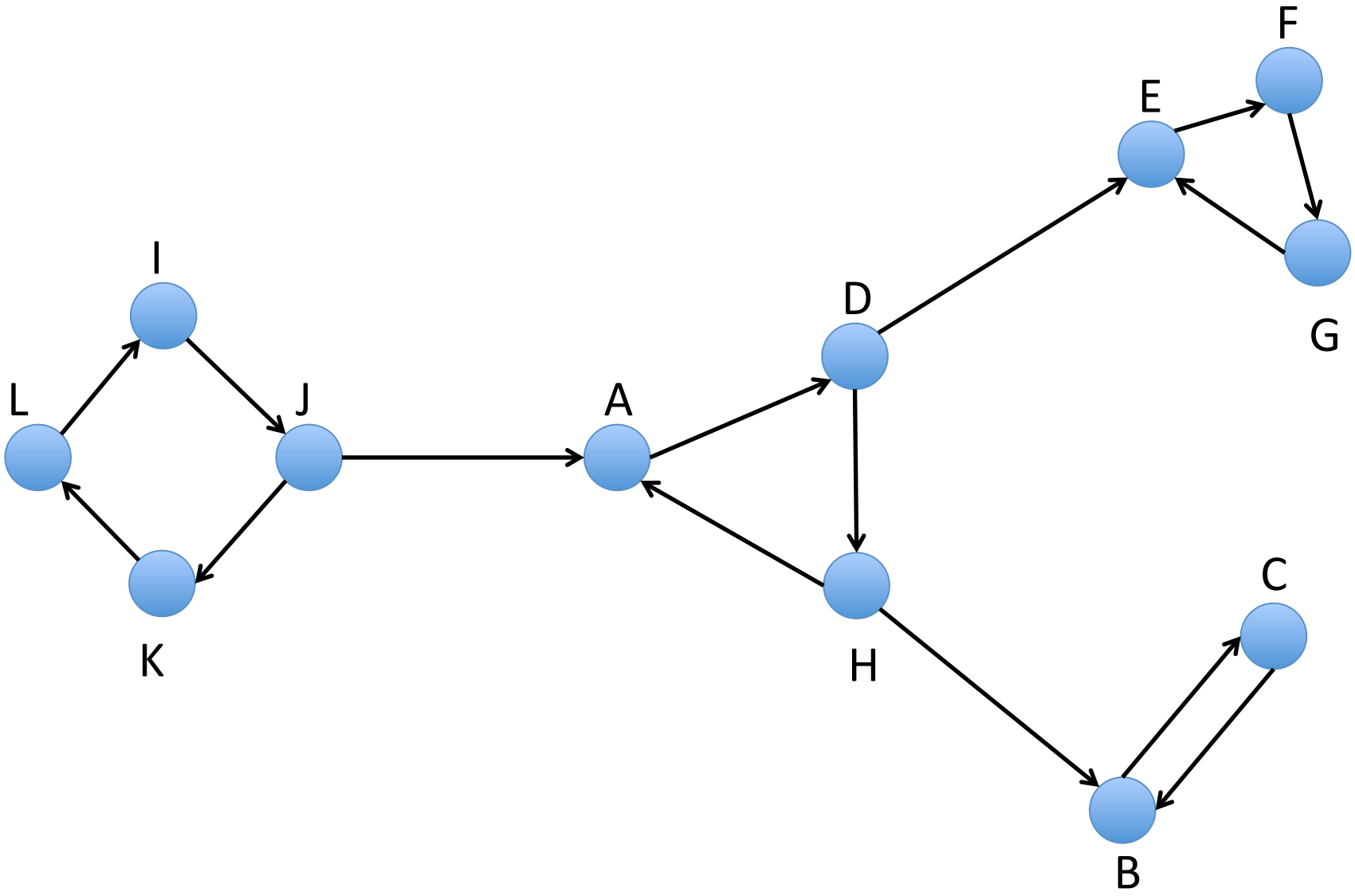
In G^{SCC} if $SCC_i \rightsquigarrow SCC_j$, then
****$f[SCC_i] > f[SCC_j]$****

Can prove by induction on the length of the paths using Key Lemma.

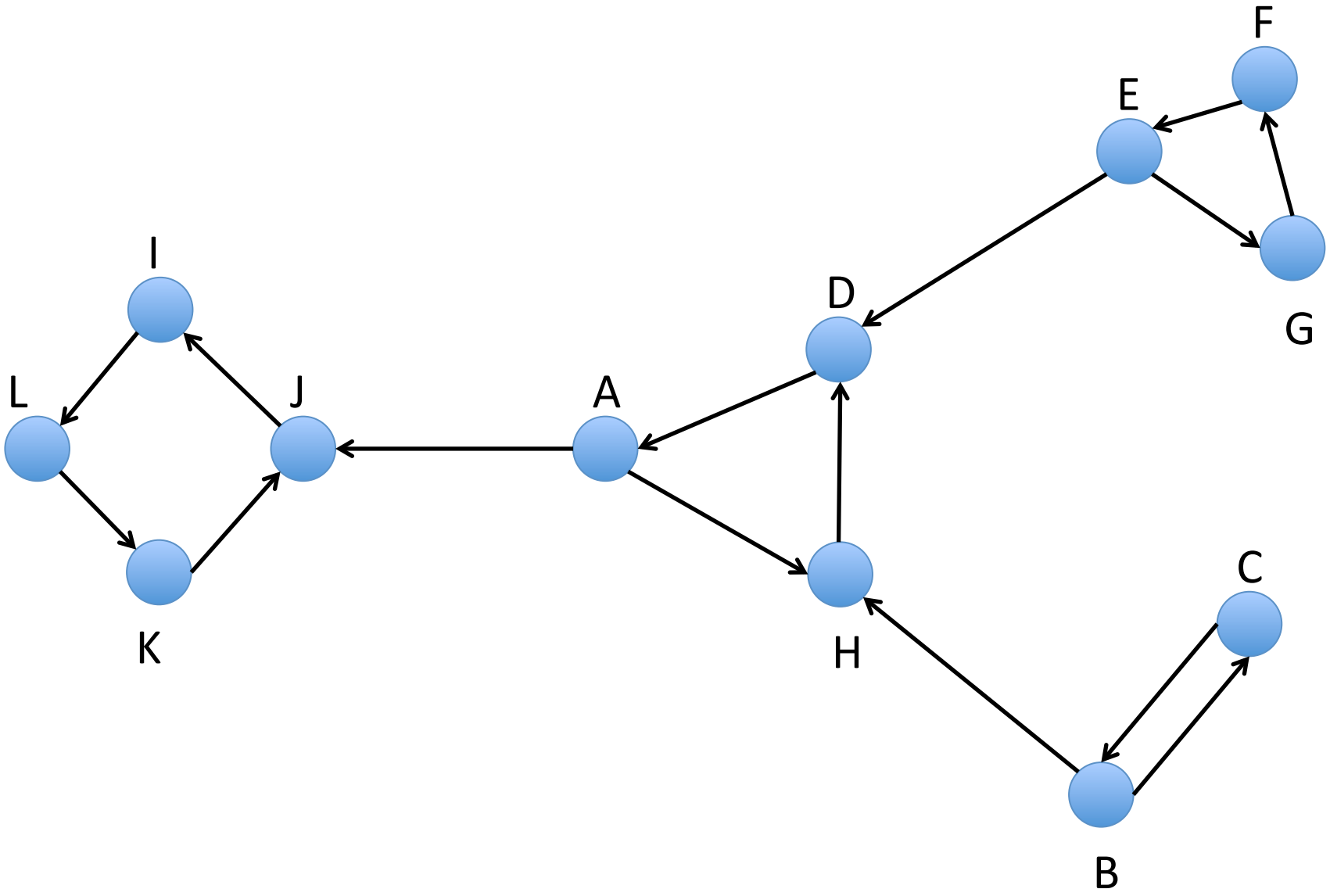
Final Tricks

- Recall we want to identify sinks NOT sources!
- So far it looks like we can identify sources.
- What if we took the reverse of G => G_{rev}
- And ran DFS on G_{rev} and studied the finishing times

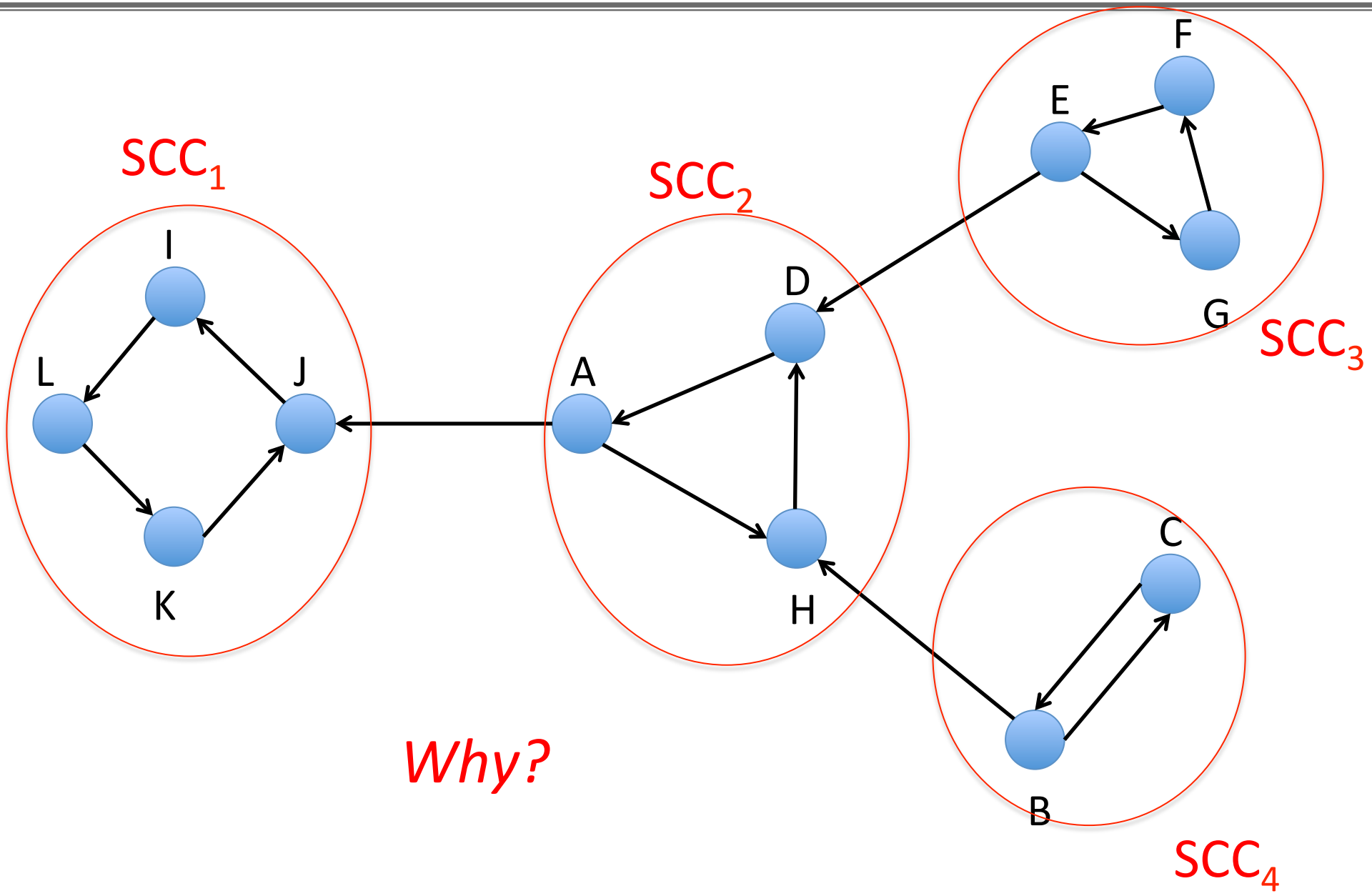
Original Graph G



G_{rev}



G_{rev} has the exact same SCCs as G!

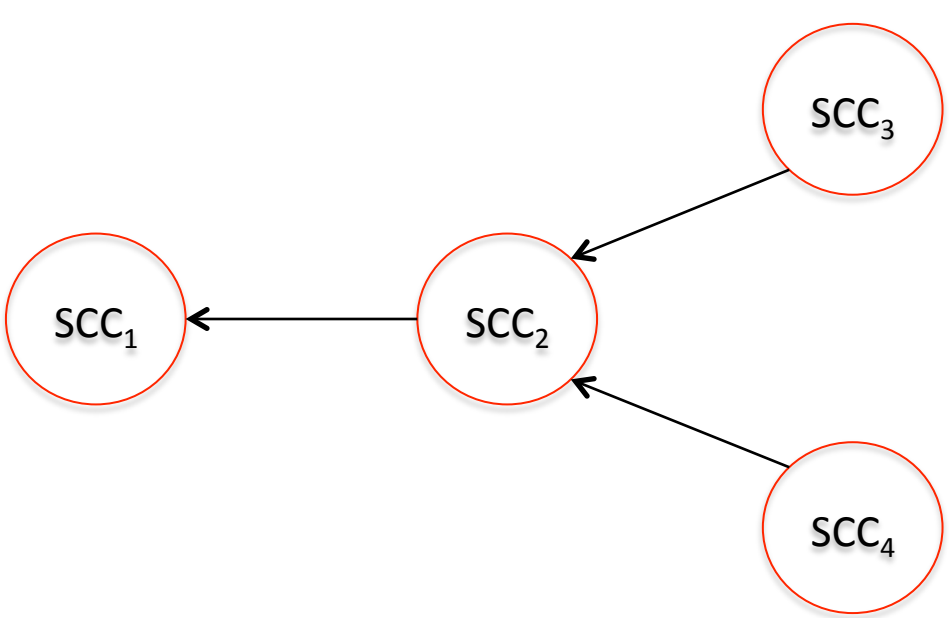


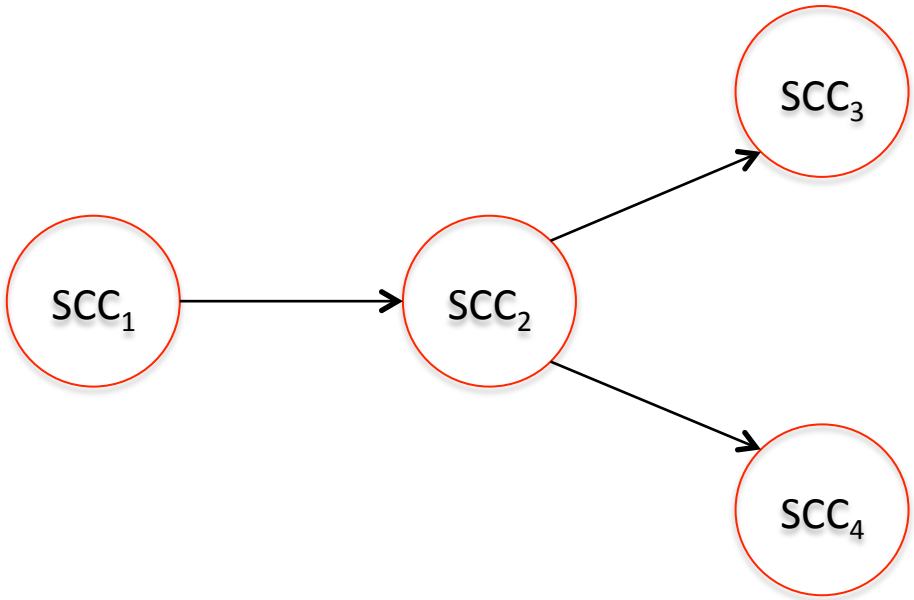
Properties of G_{rev}

1. G_{rev} and G have the same SCCs
2. The $(G_{rev})^{SCC}$ is the reverse of G^{SCC}

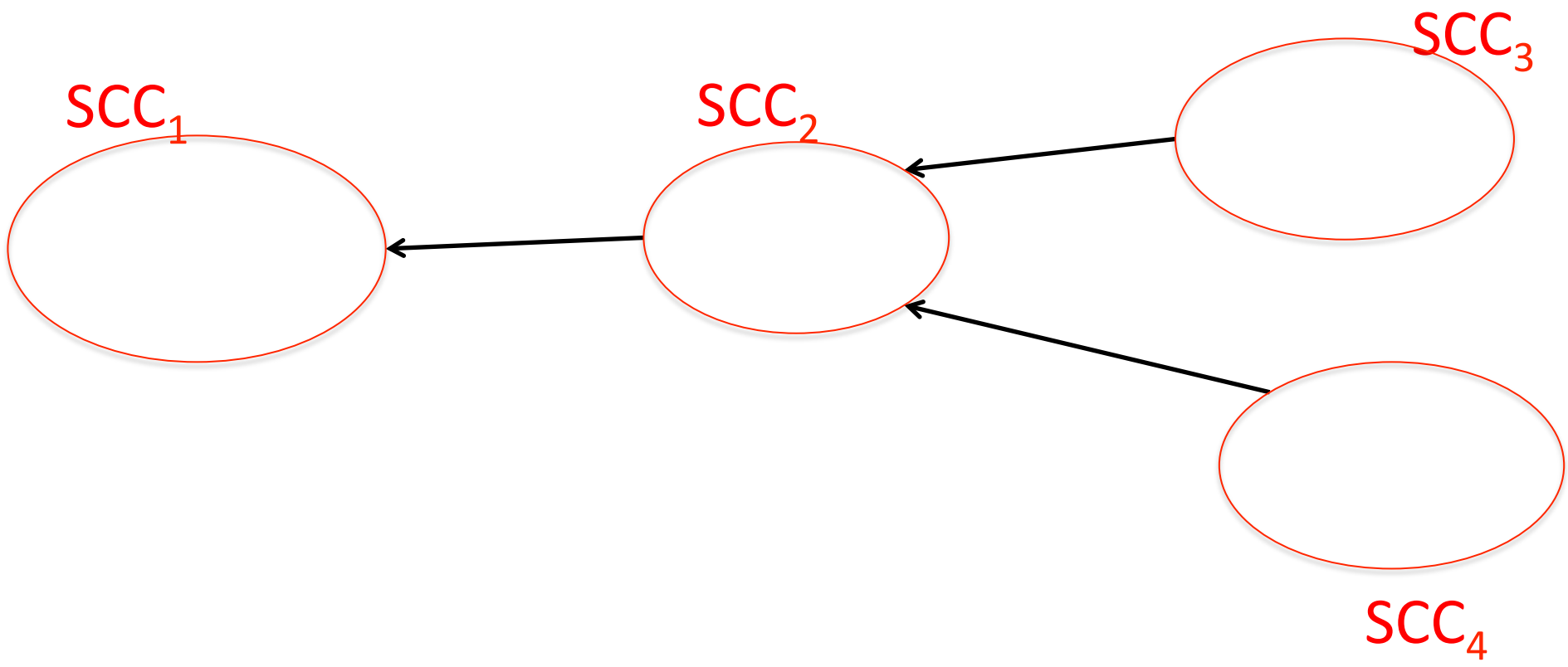
SCC meta-graph of G_{rev}

SCC meta-graph of G





Running DFS on G_{rev}

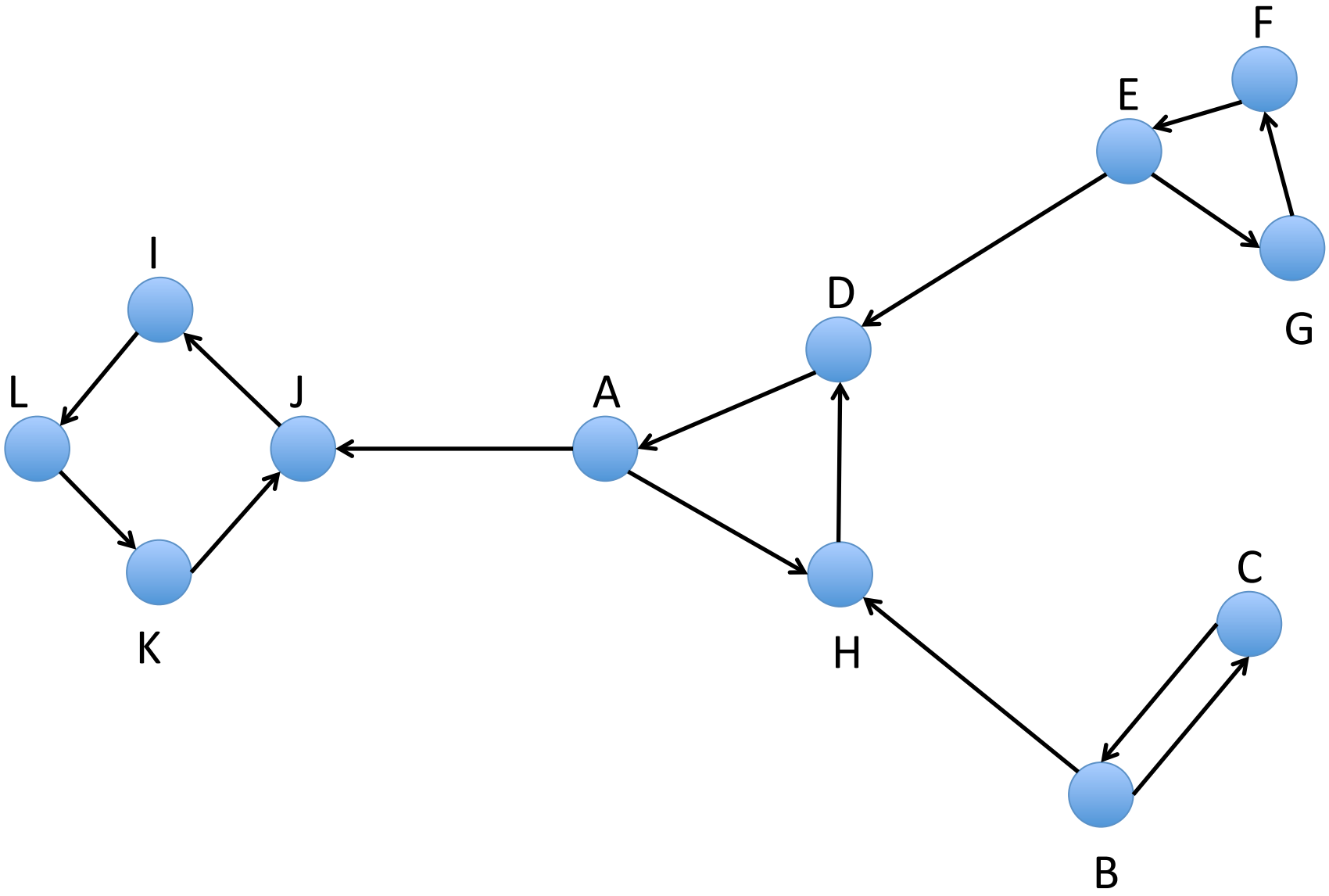


By Key Lemma we should get:

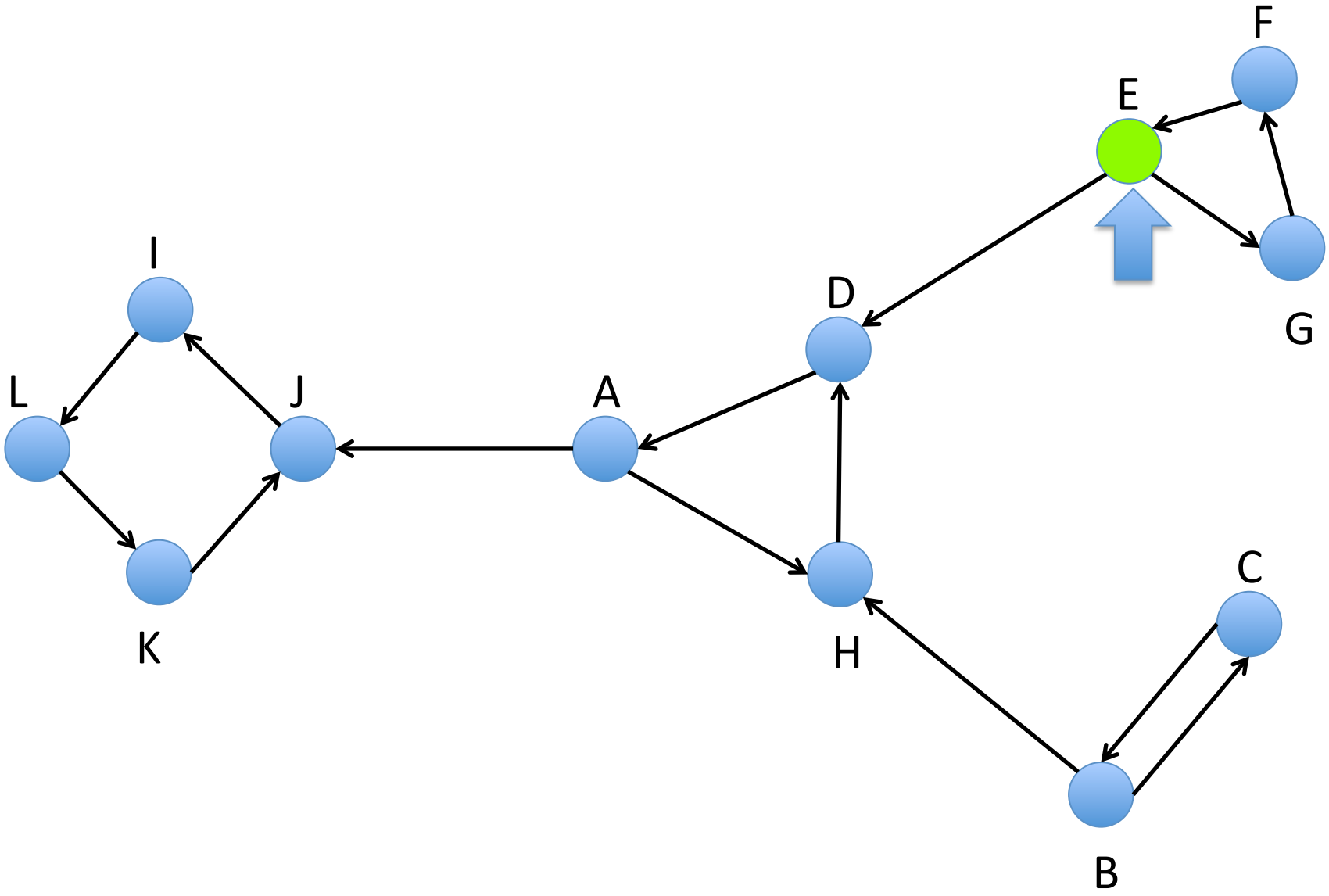
$f[SCC_3] > f[SCC_4] > f[SCC_2] > f[SCC_1]$ OR

$f[SCC_4] > f[SCC_3] > f[SCC_2] > f[SCC_1]$

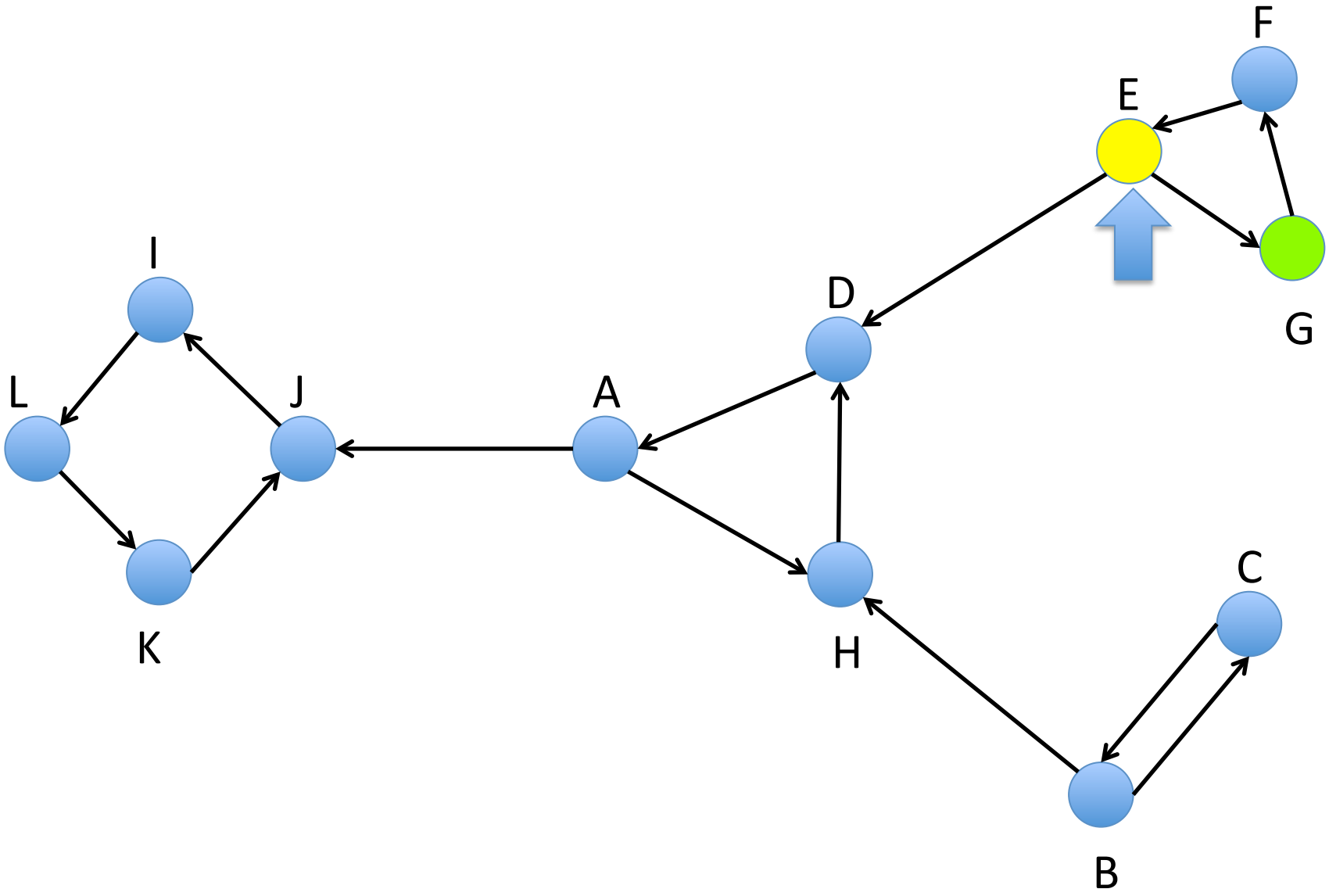
DFS & Finishing Times on Grev



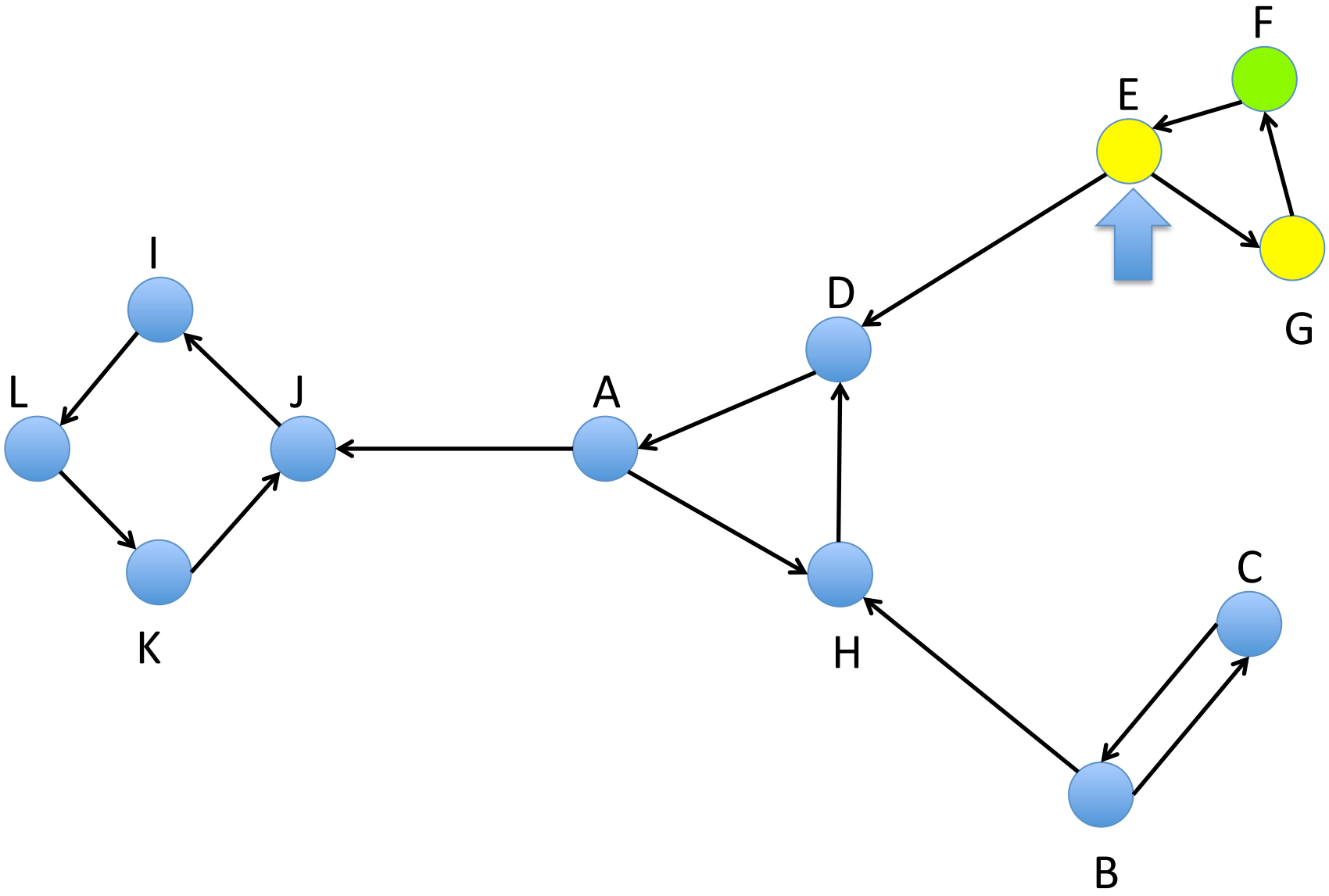
DFS & Finishing Times on Grev



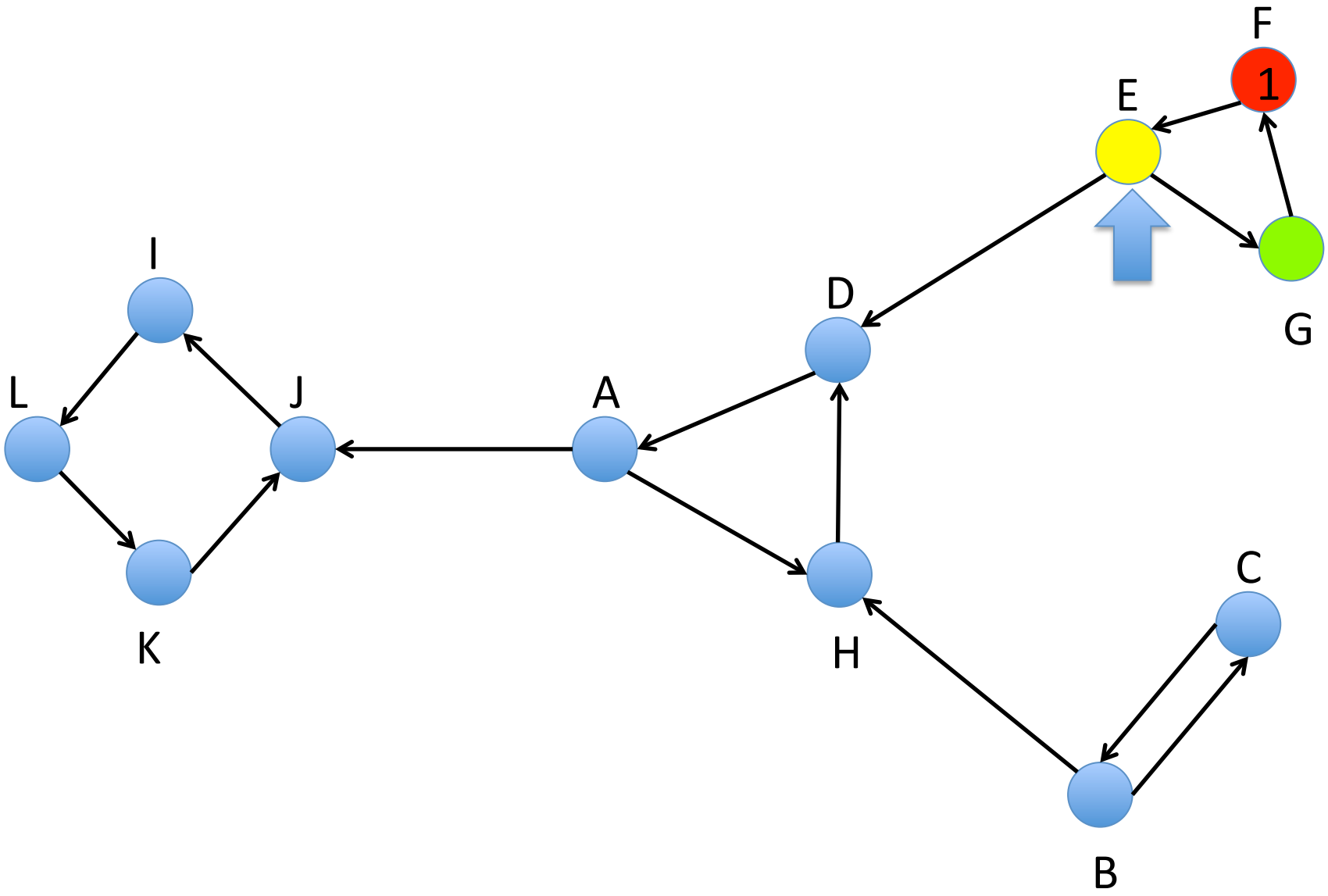
DFS & Finishing Times on Grev



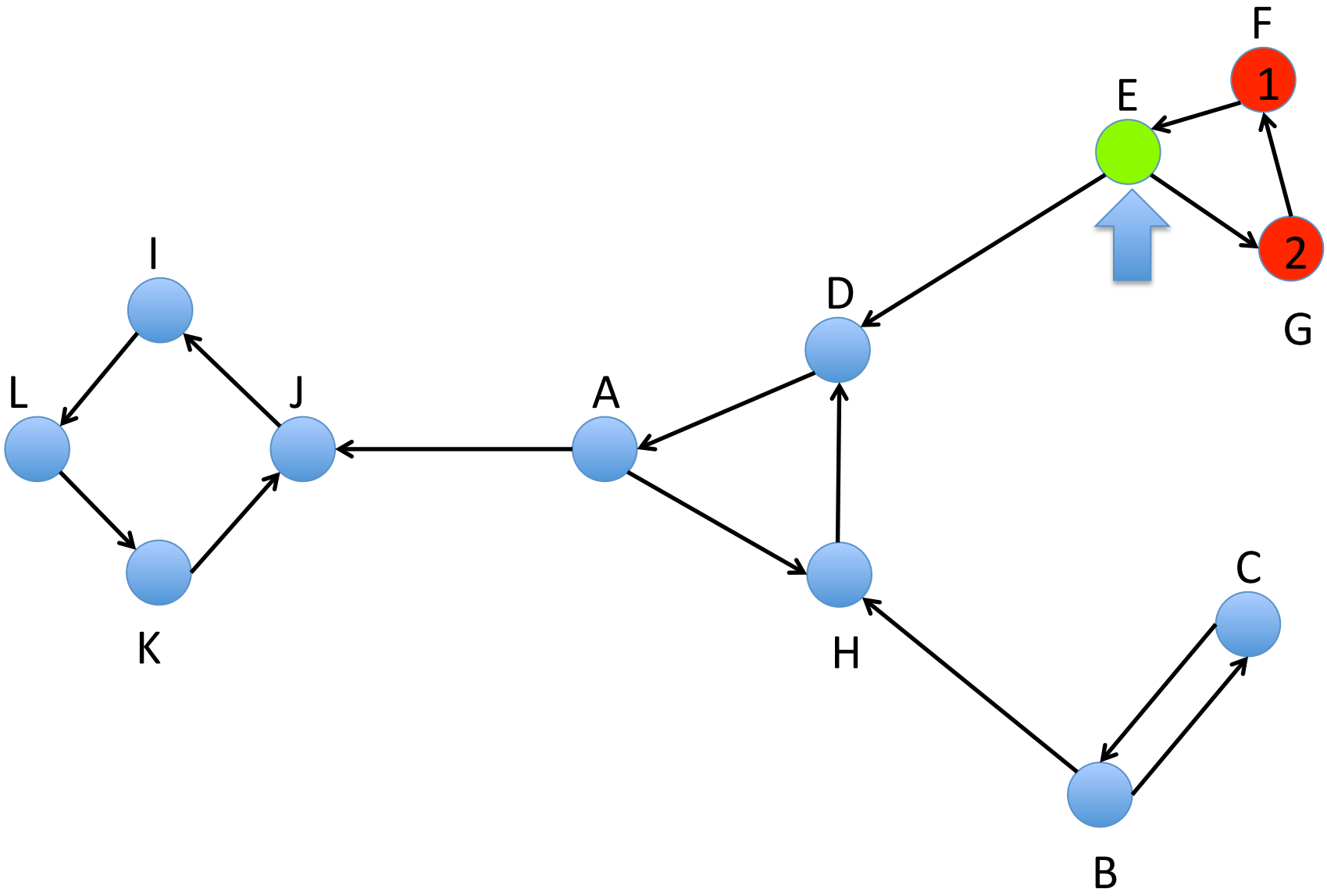
DFS & Finishing Times on Grev



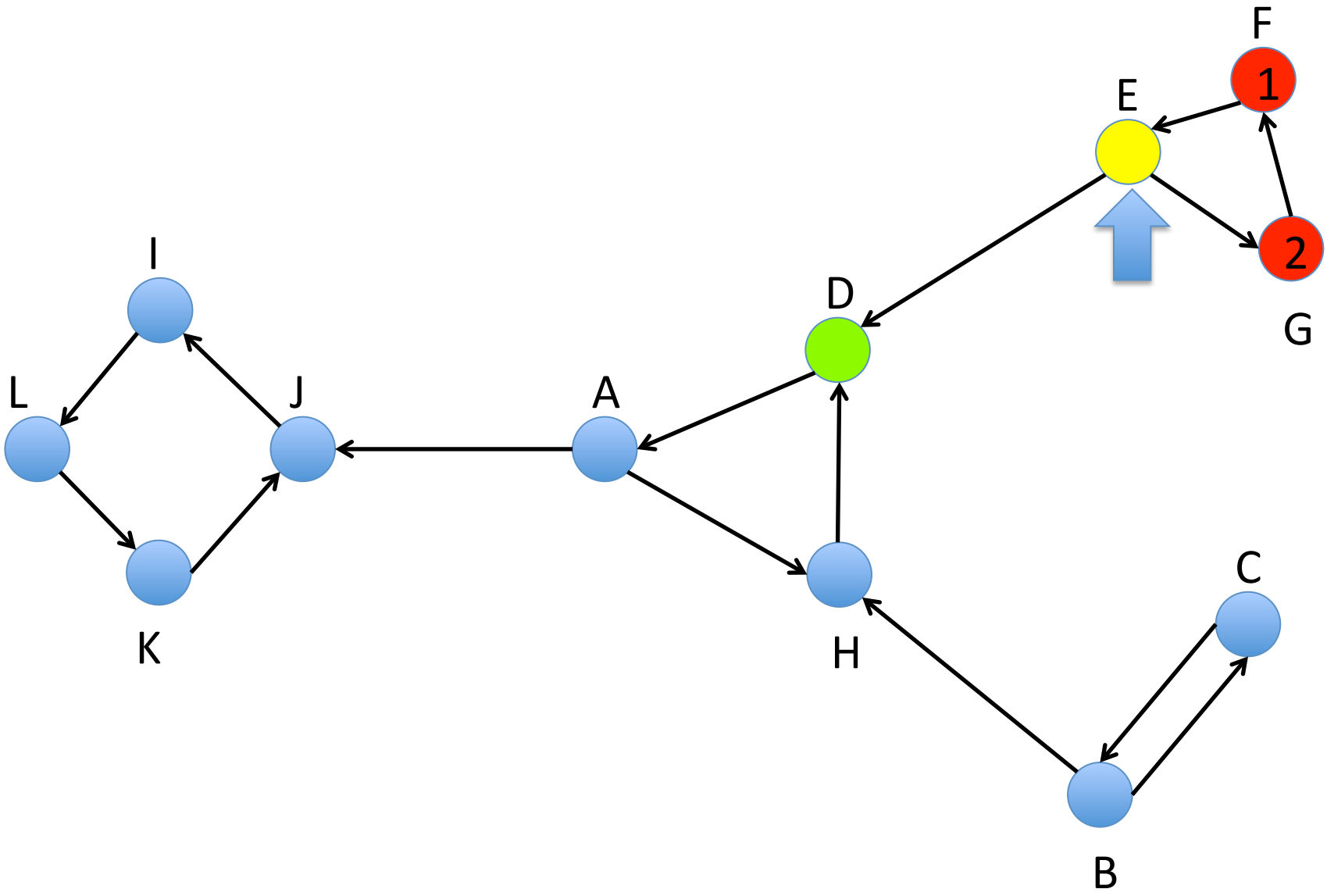
DFS & Finishing Times on Grev



DFS & Finishing Times on Grev



DFS & Finishing Times on Grev



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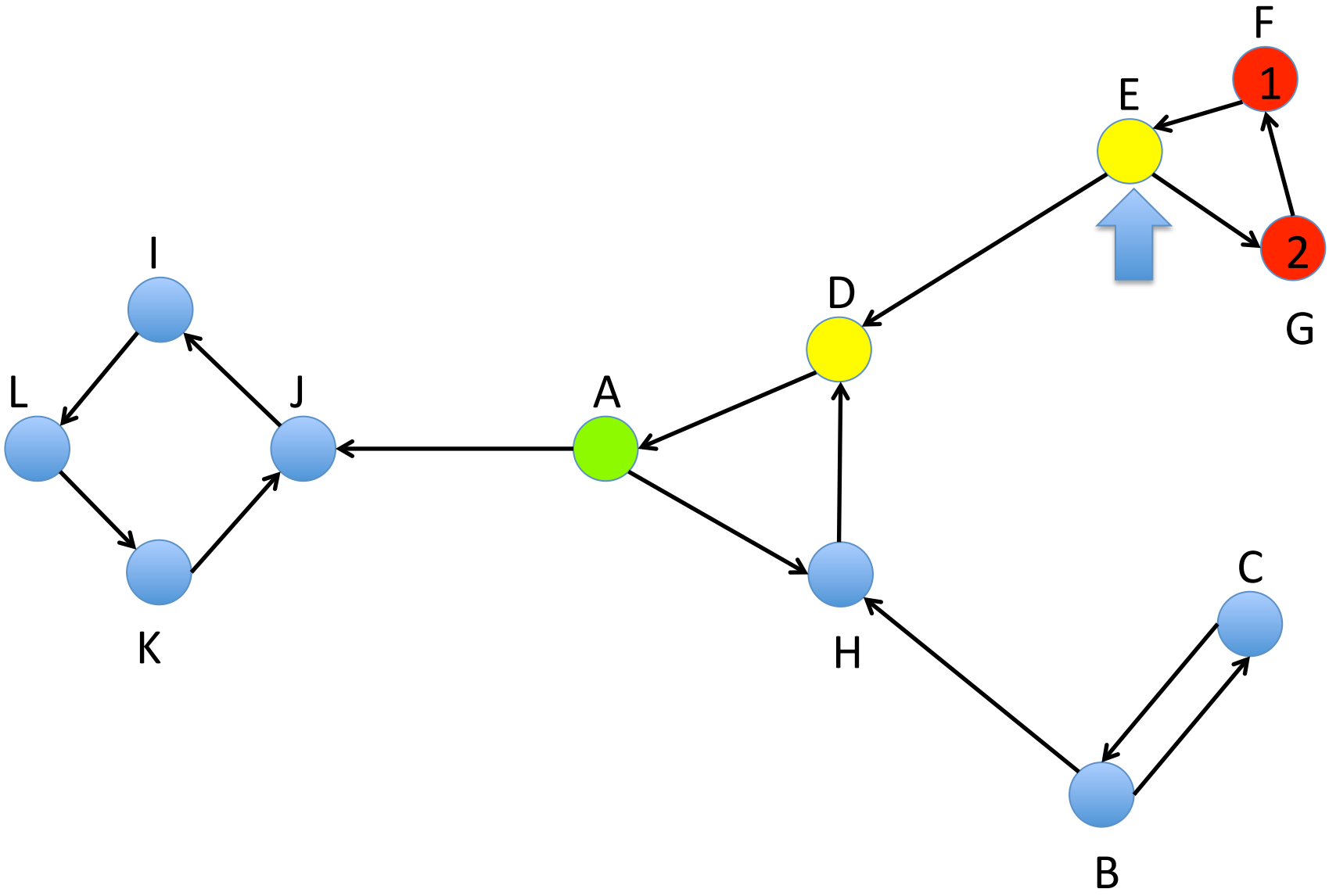
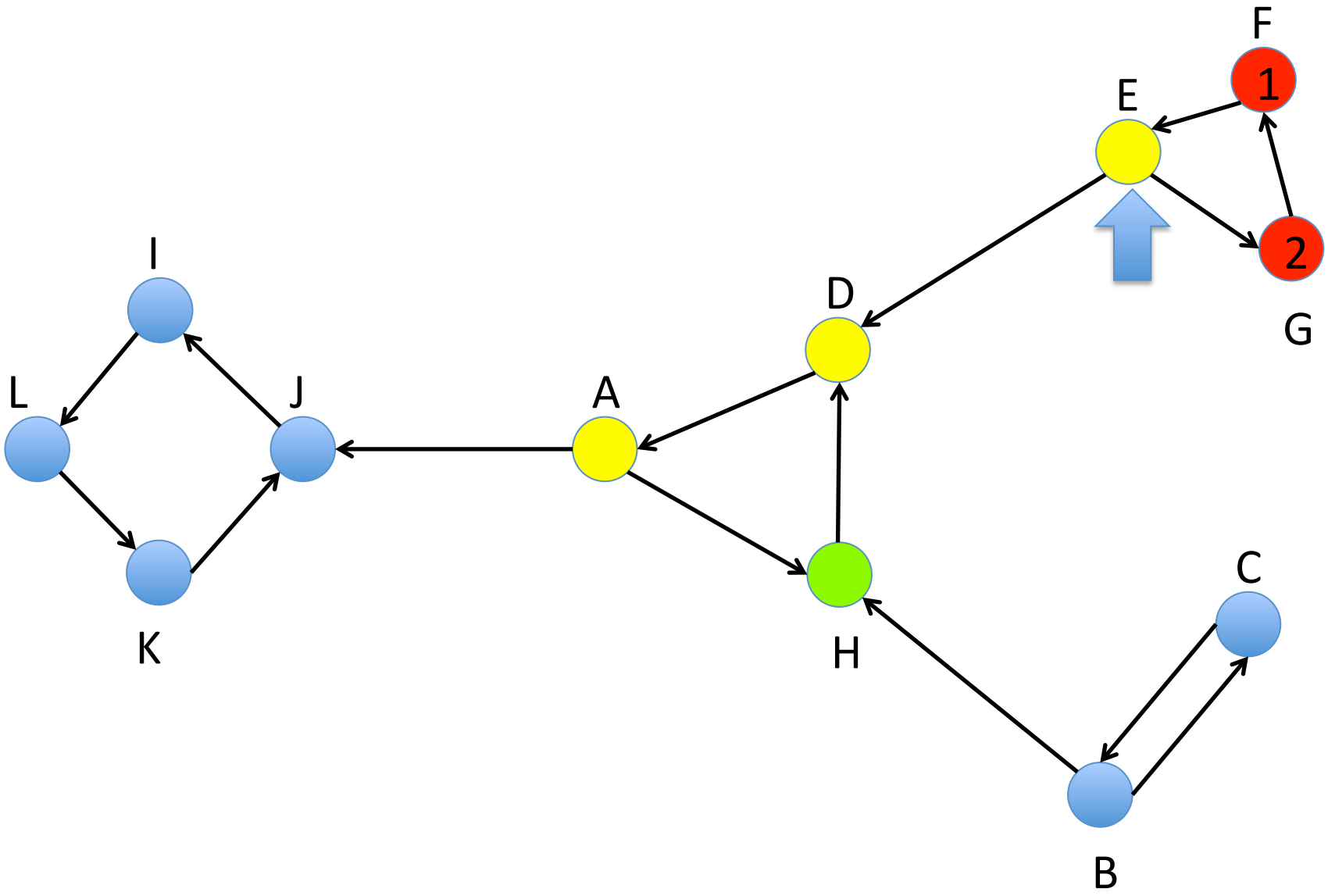


DFS & Finishing Times on Grev



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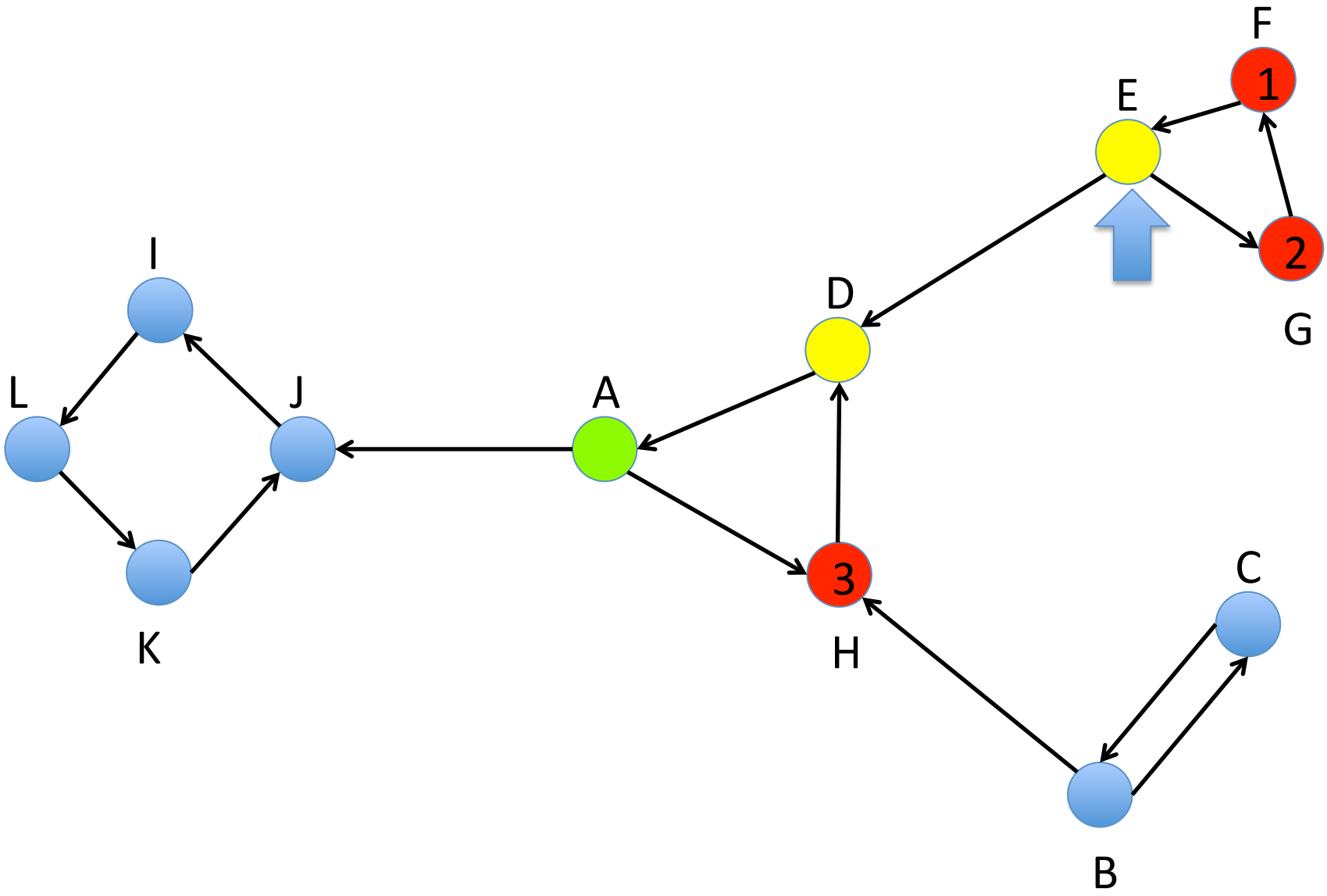

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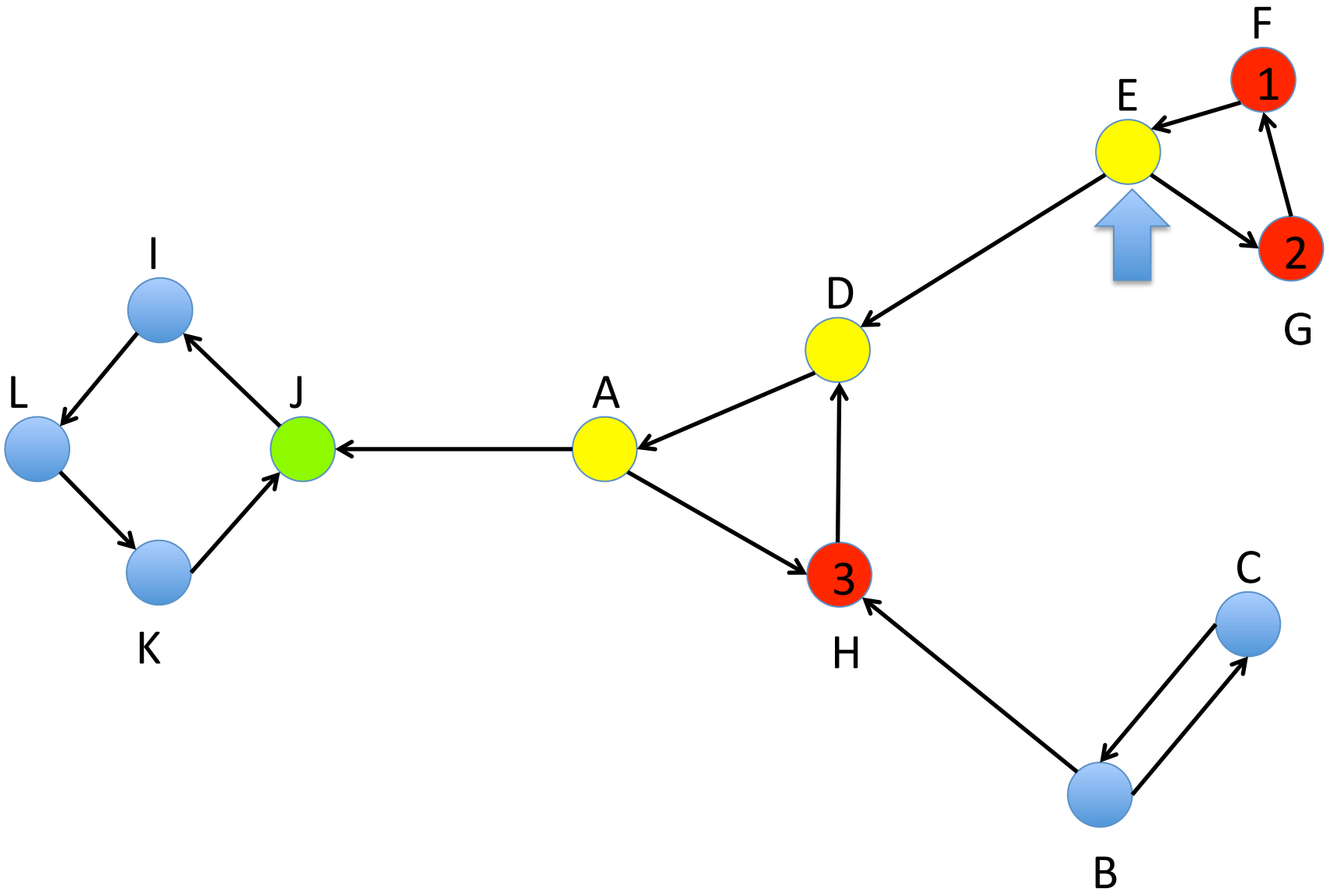

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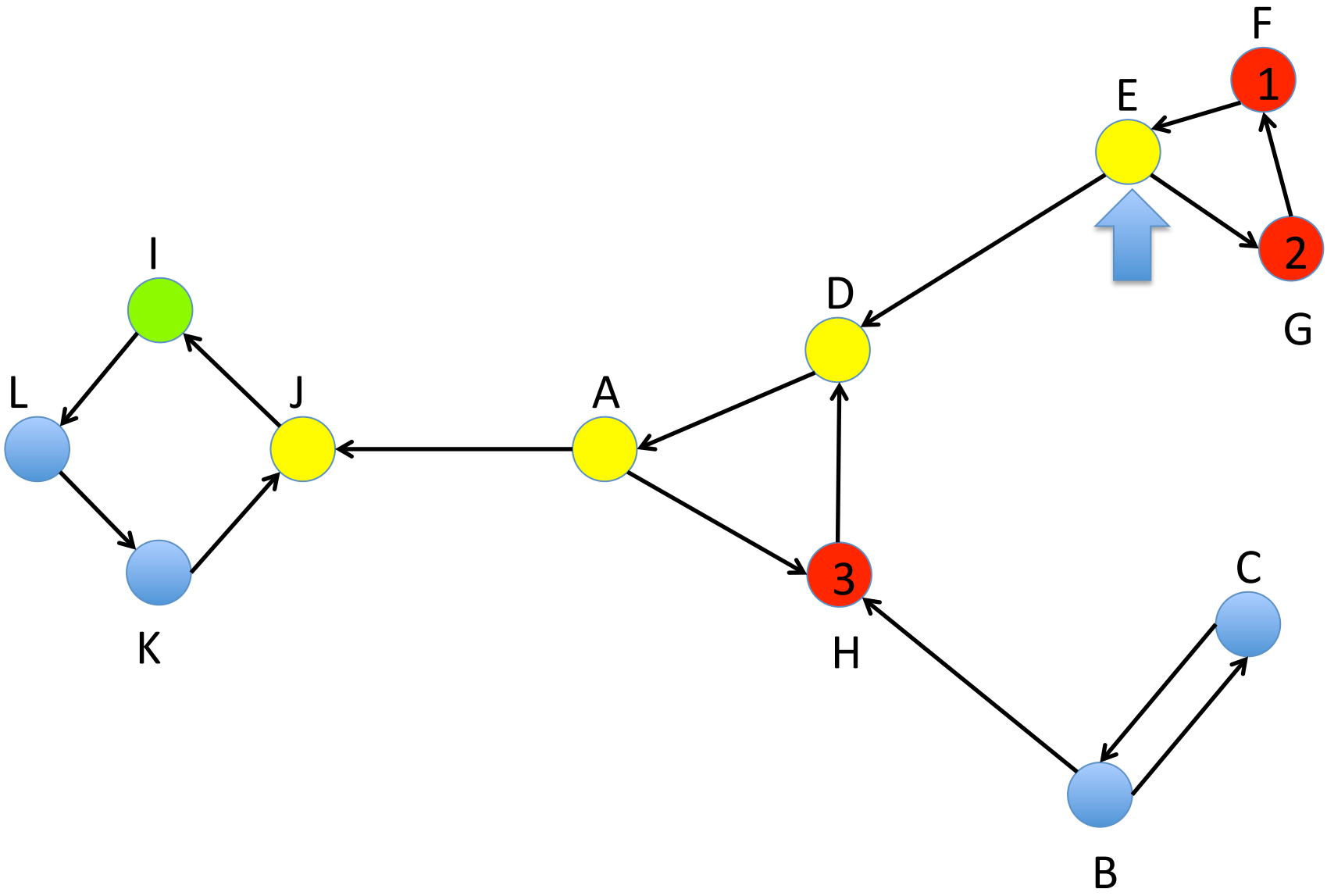

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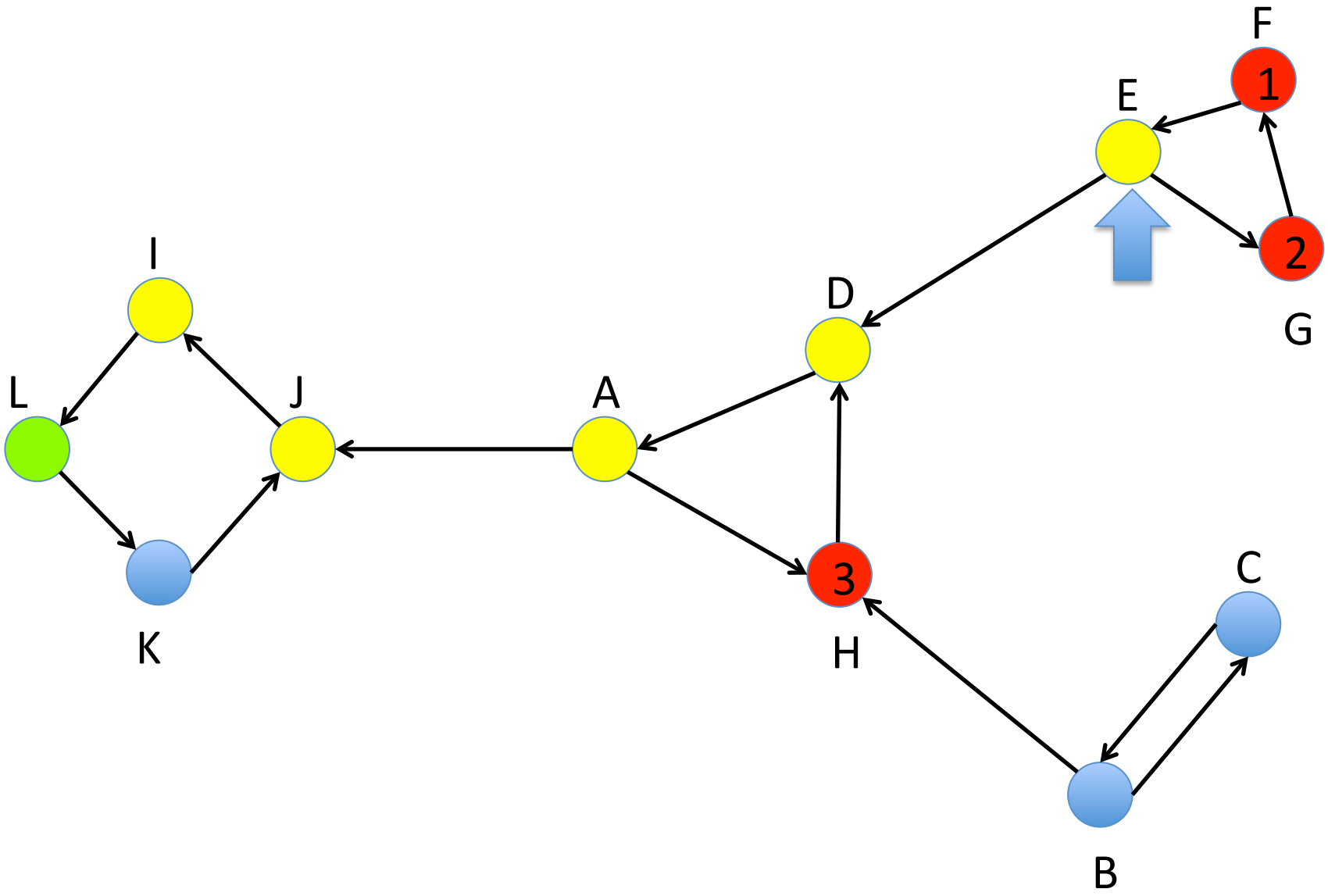

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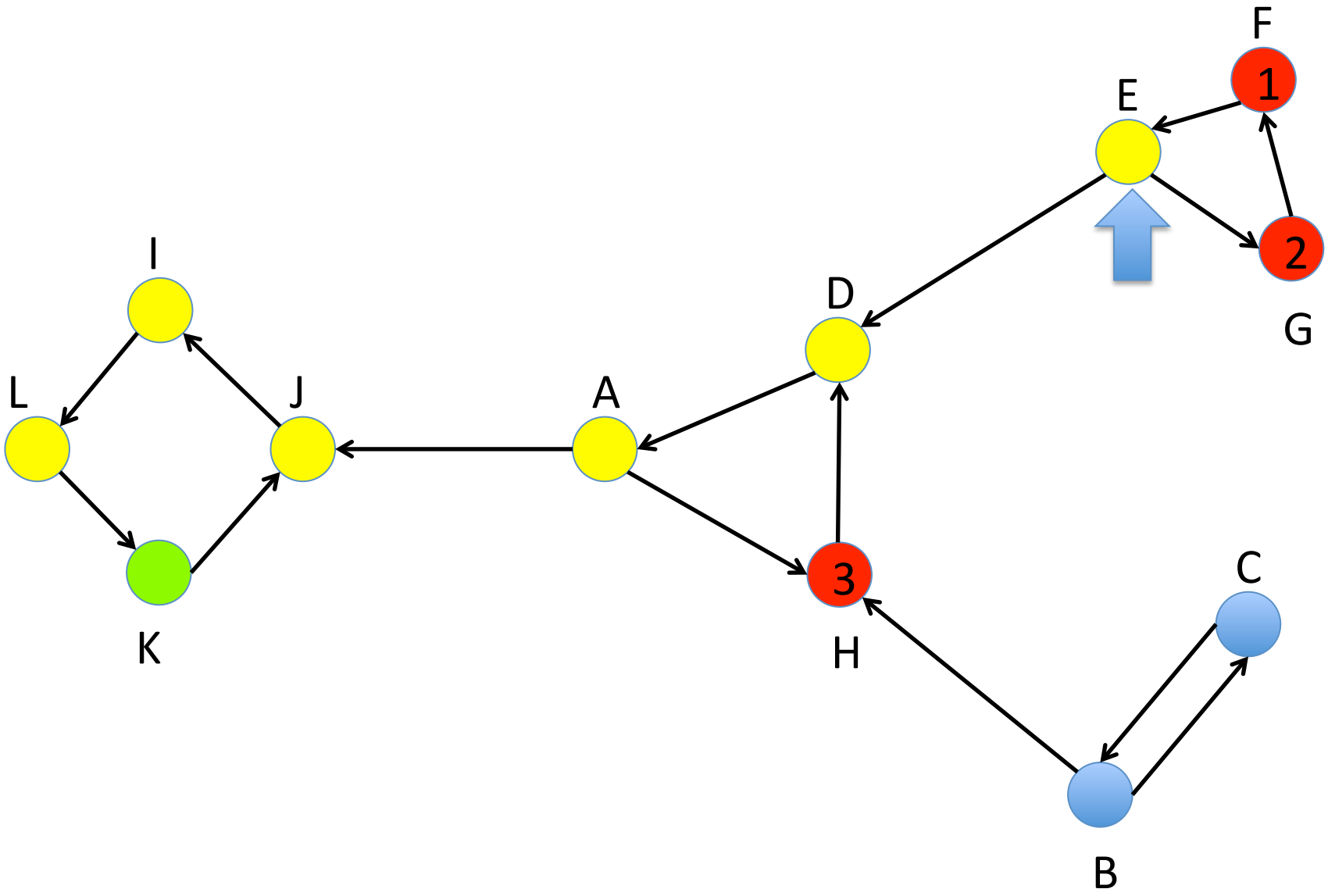

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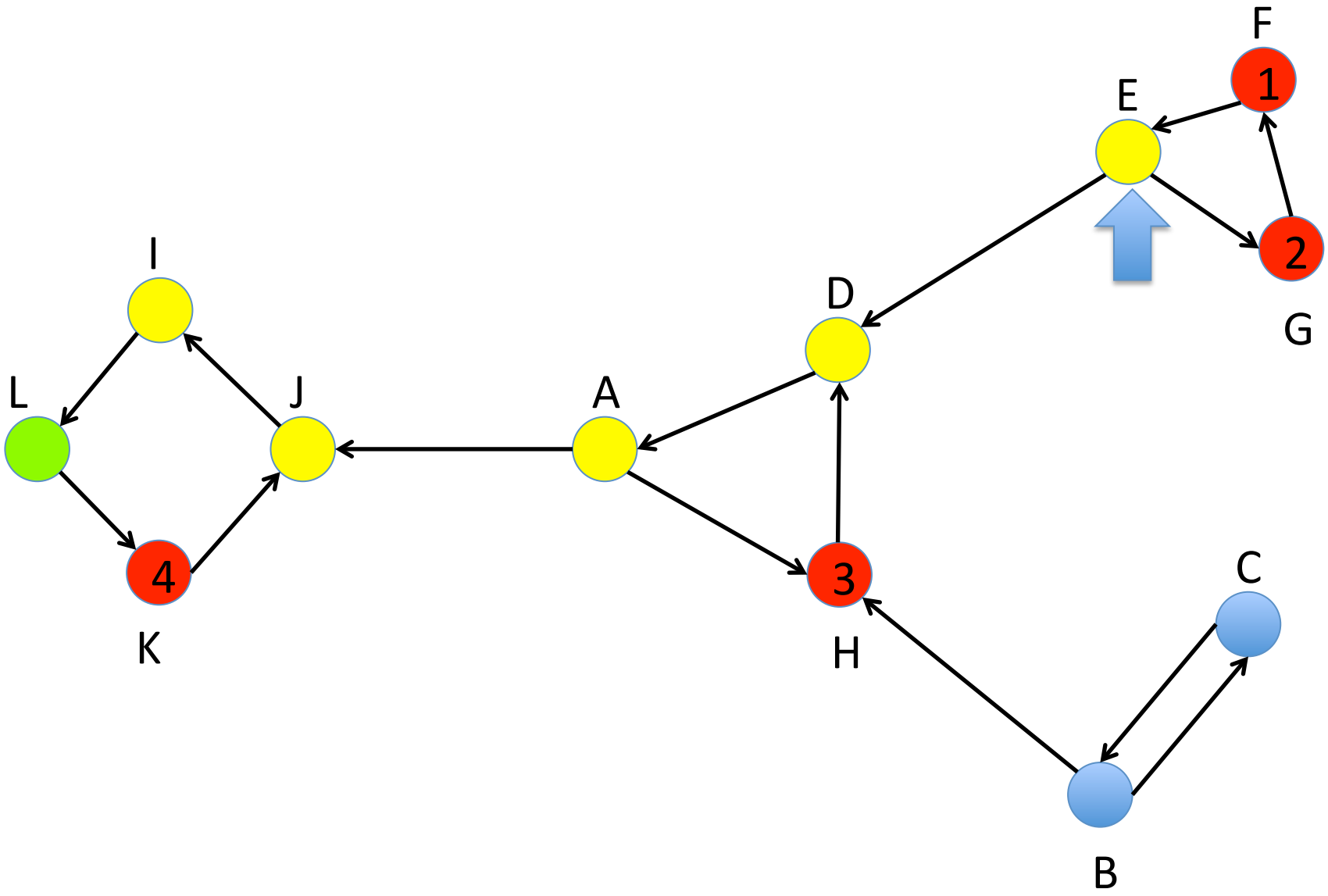

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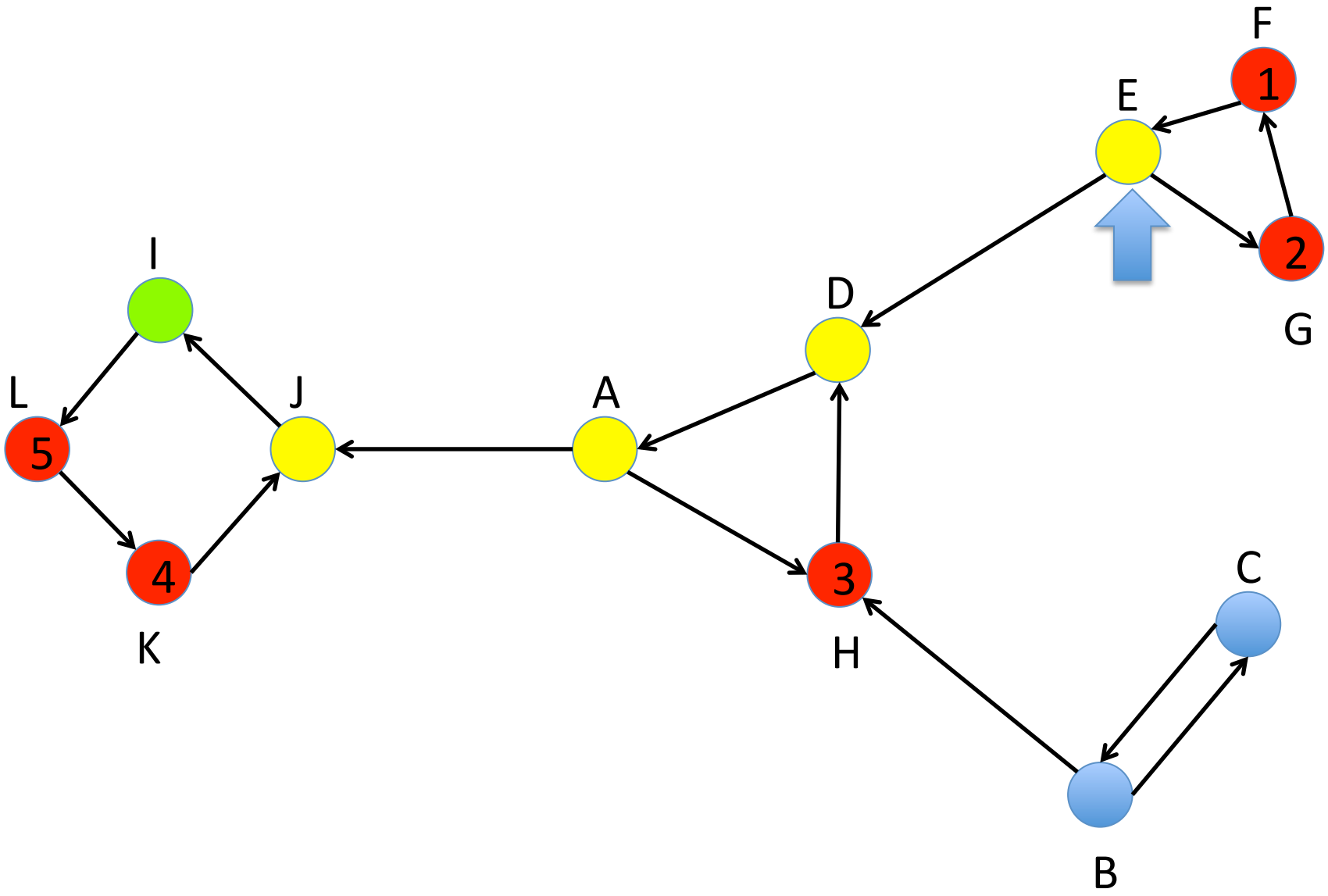

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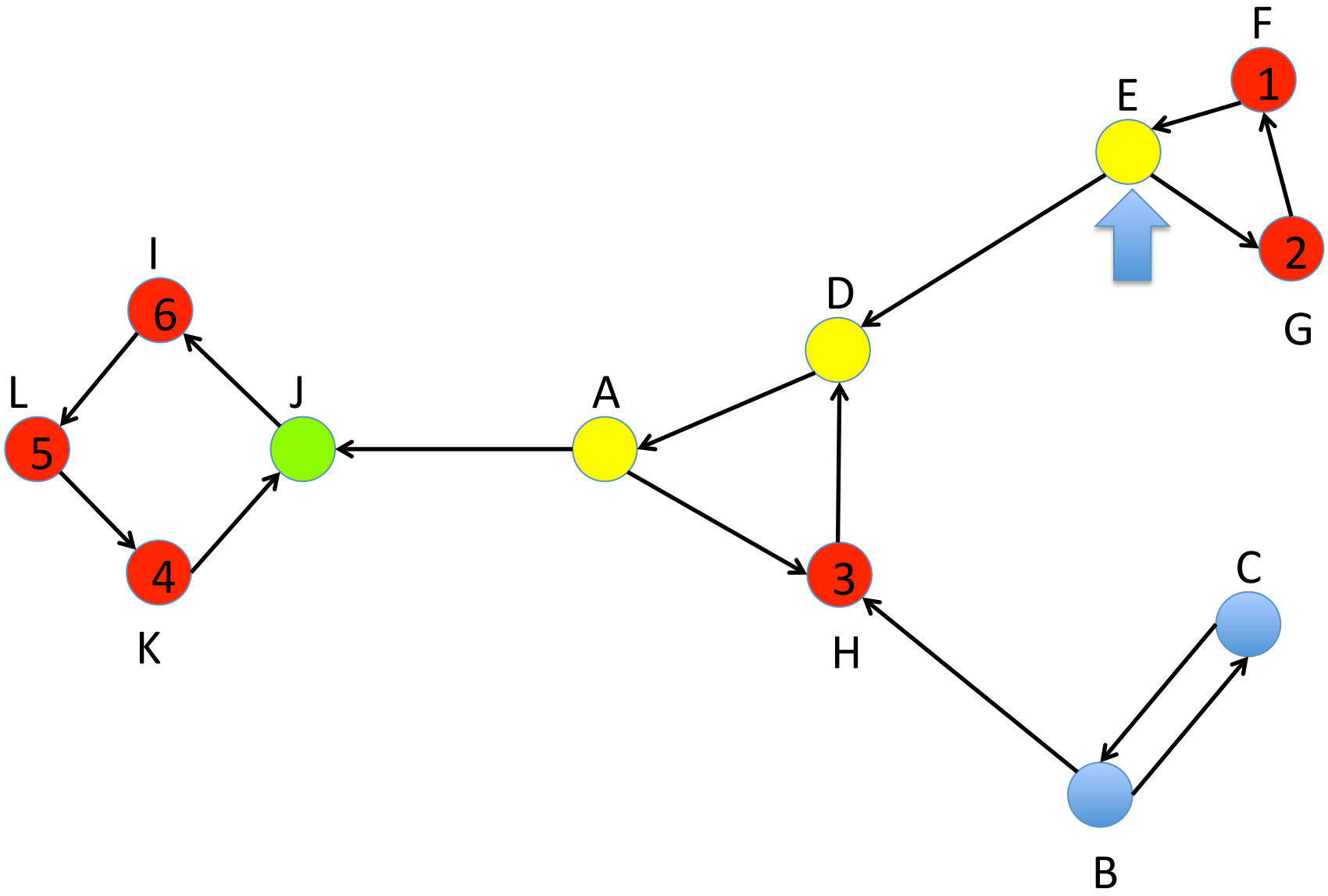

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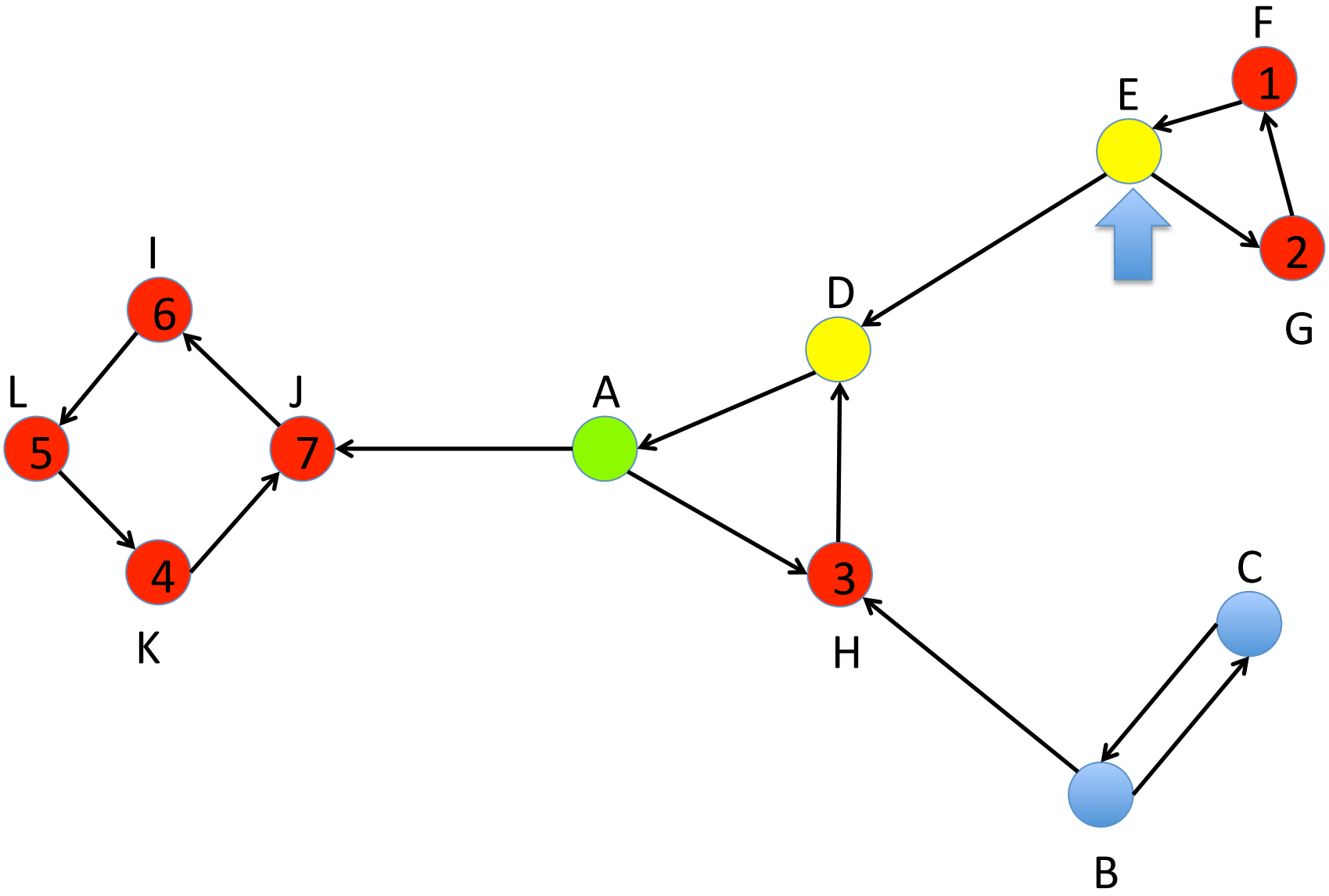

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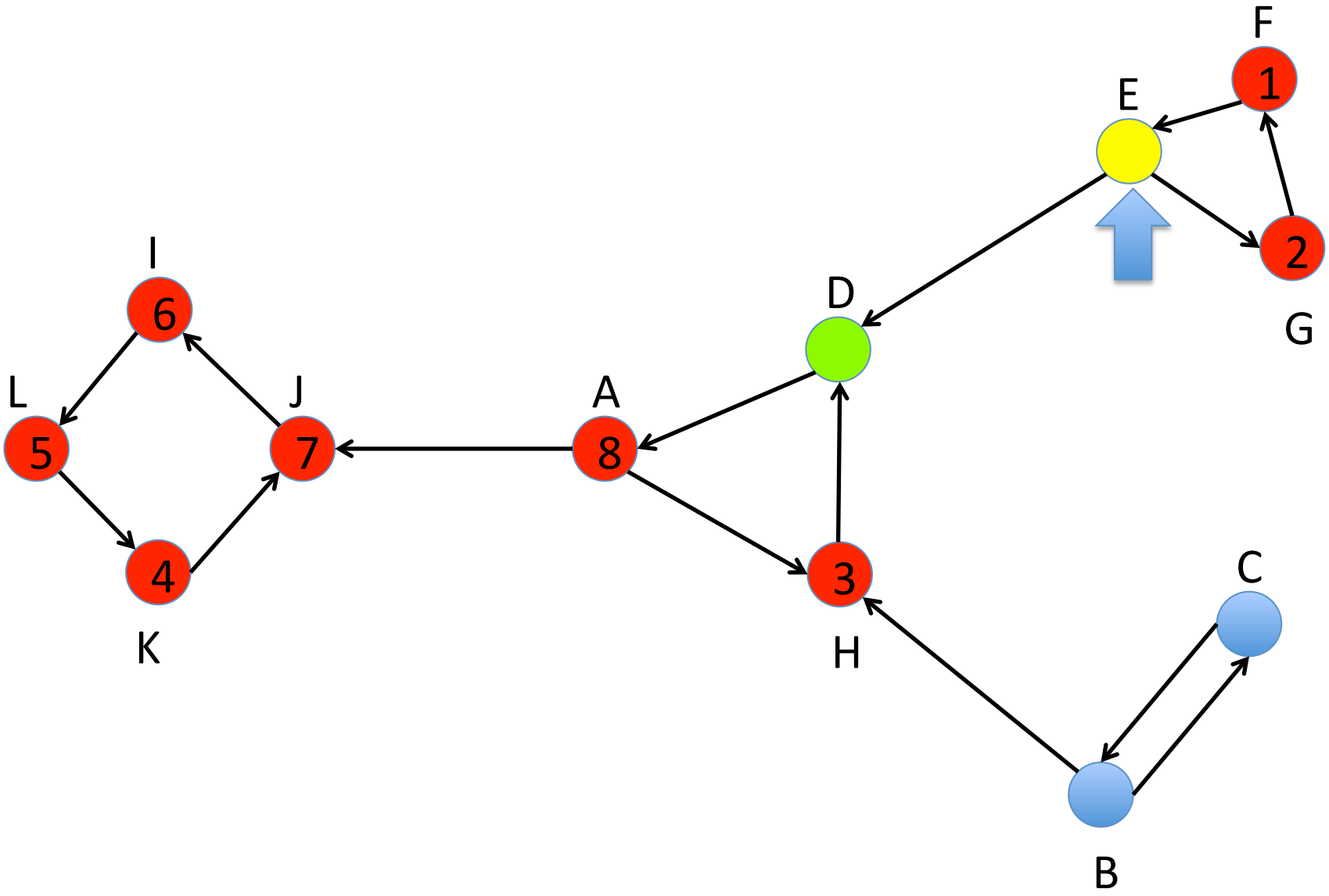

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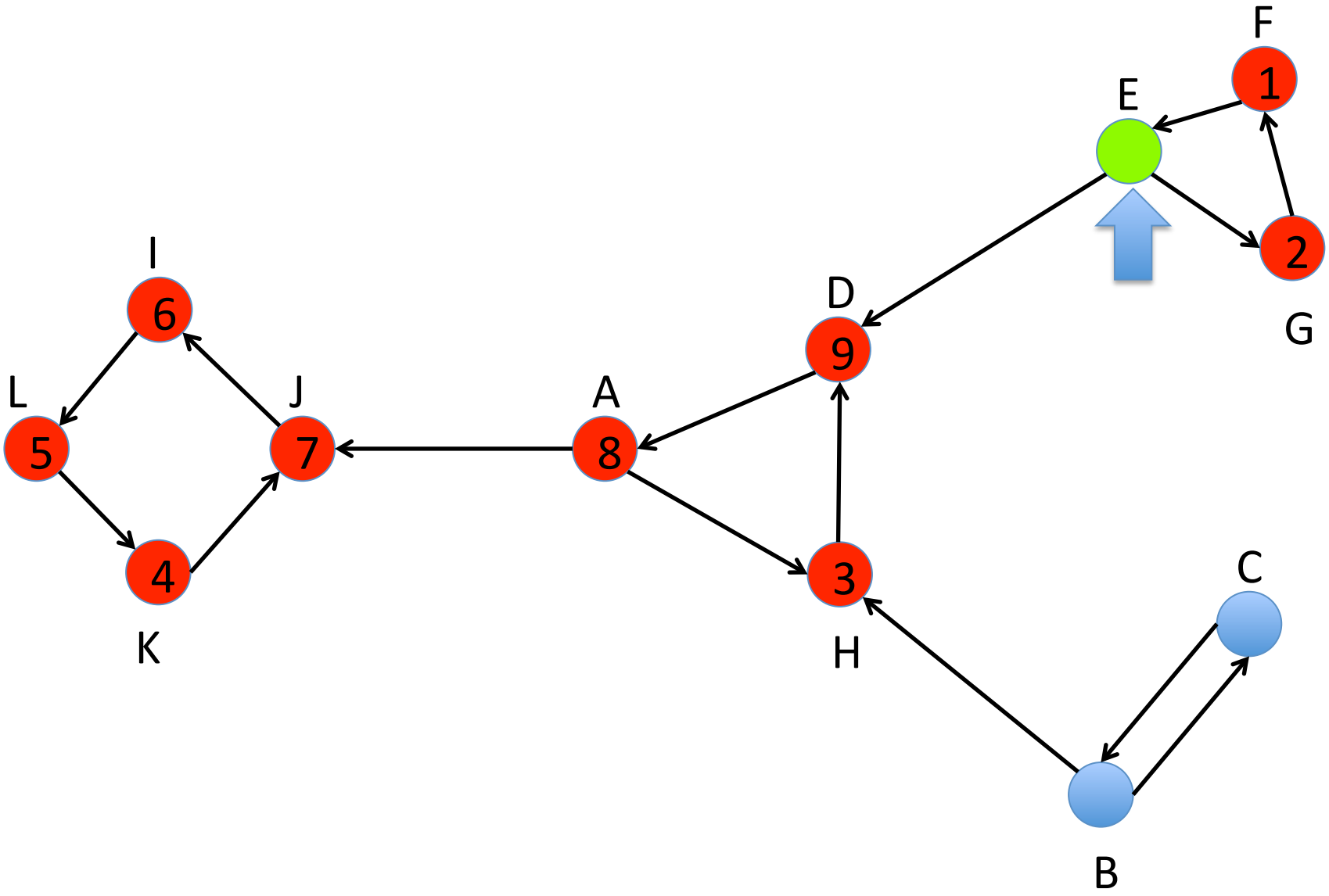

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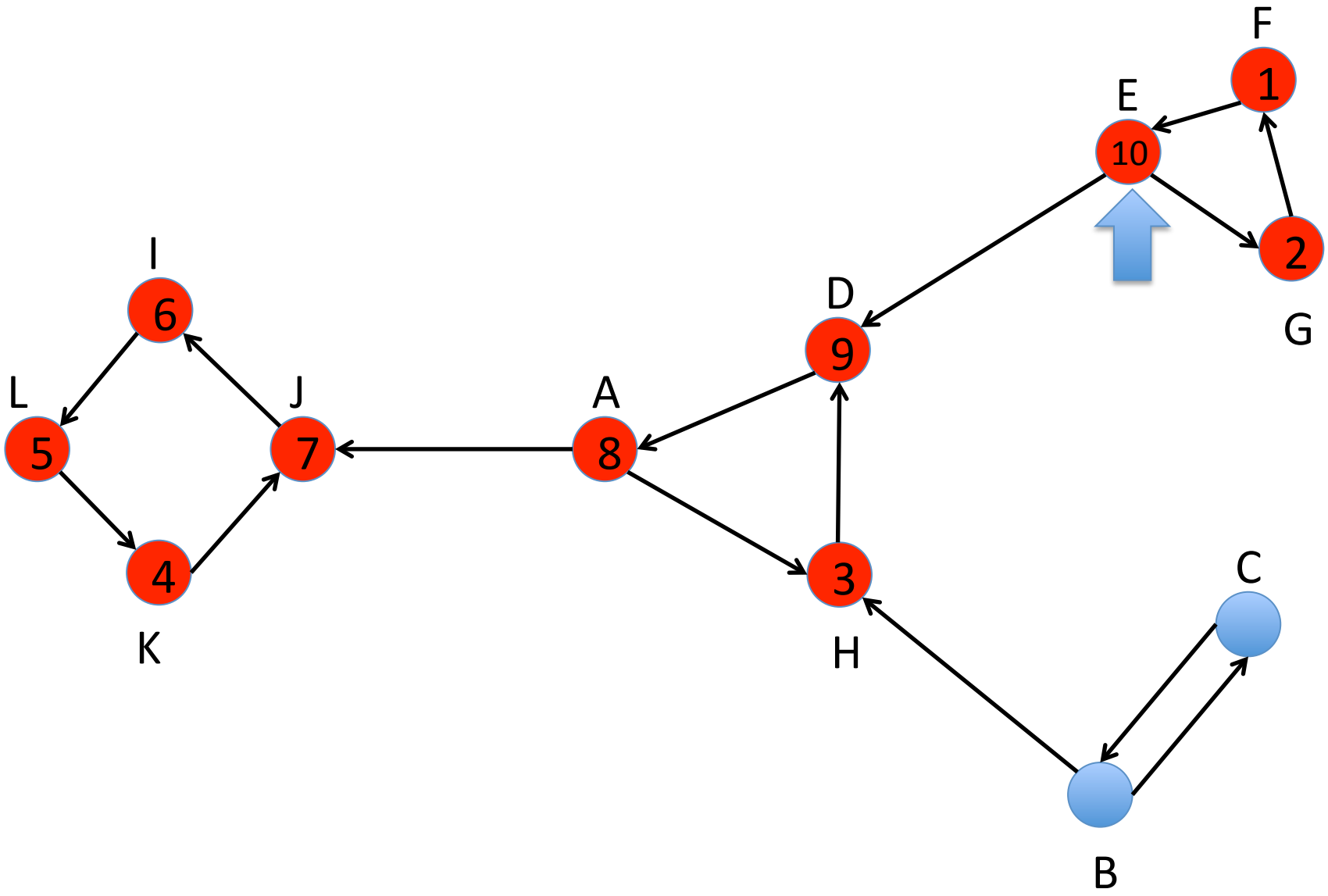

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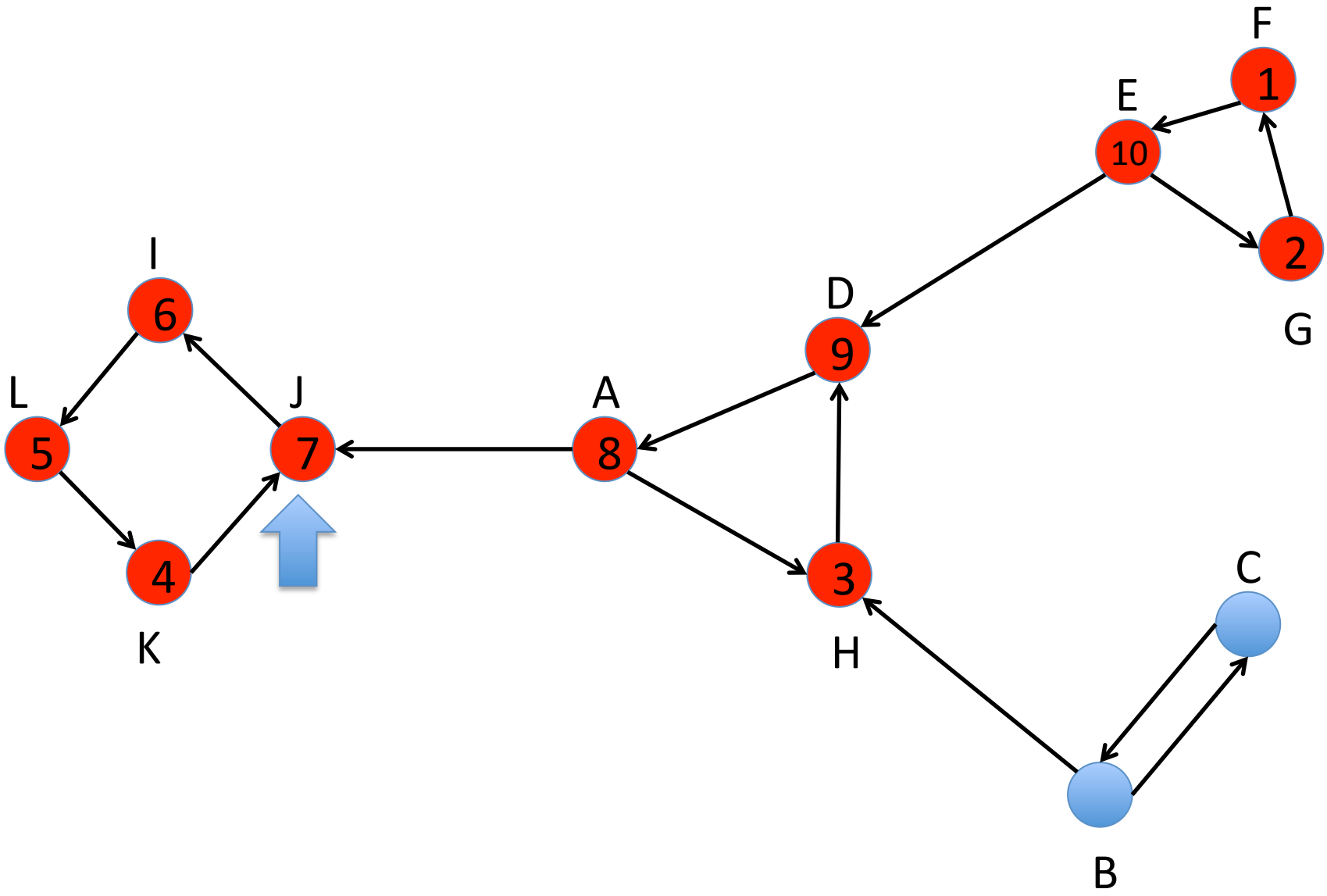

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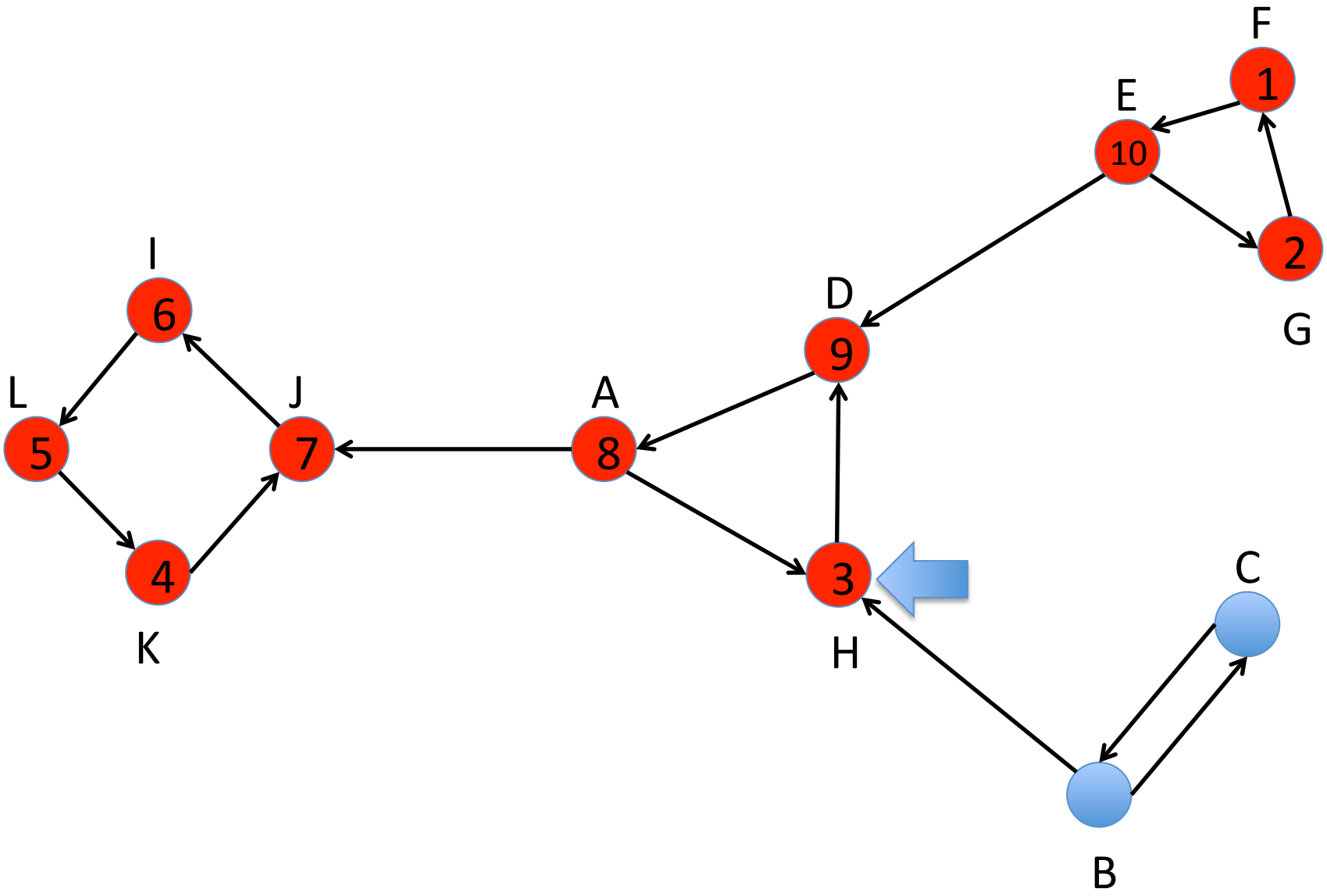

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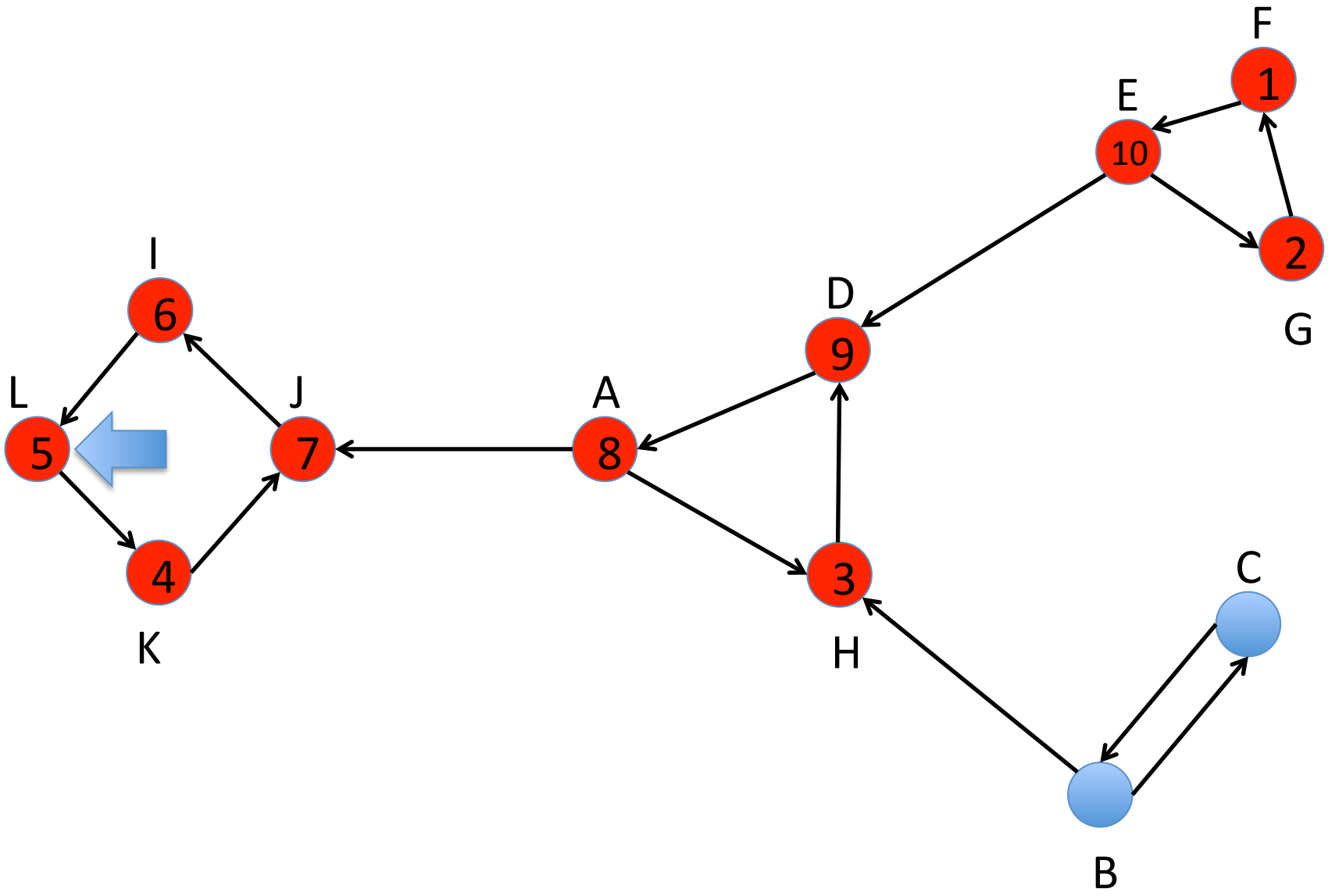

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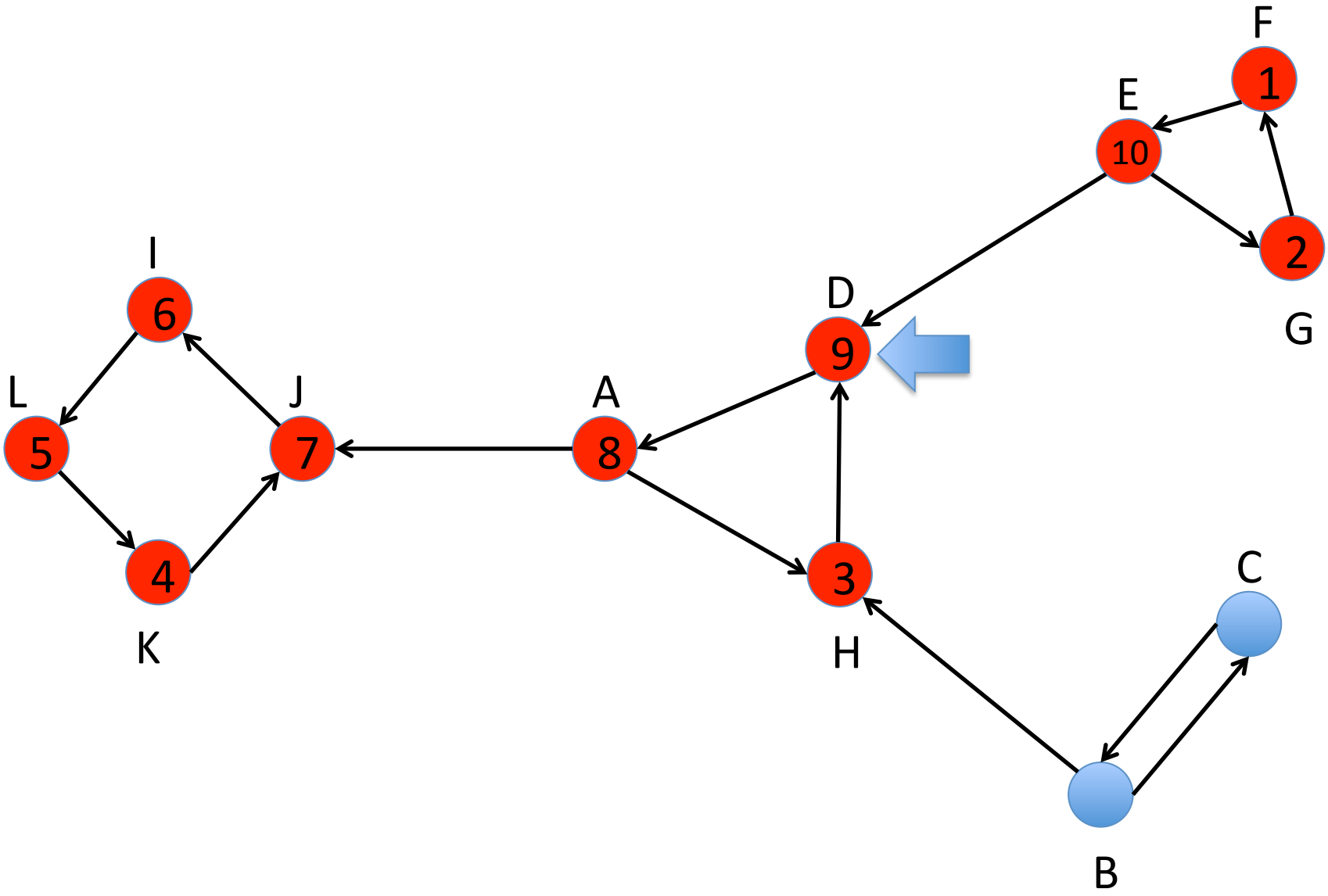

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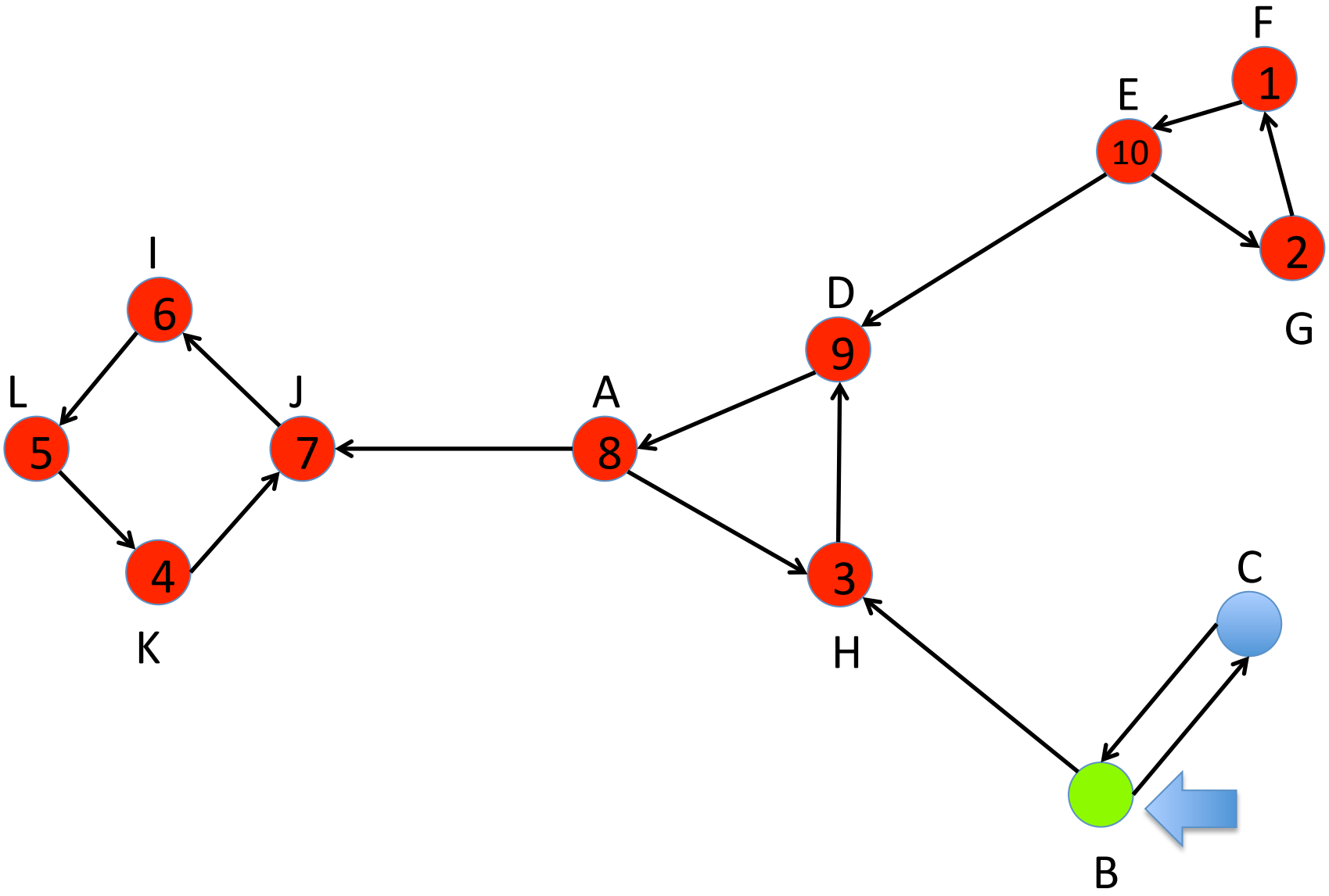

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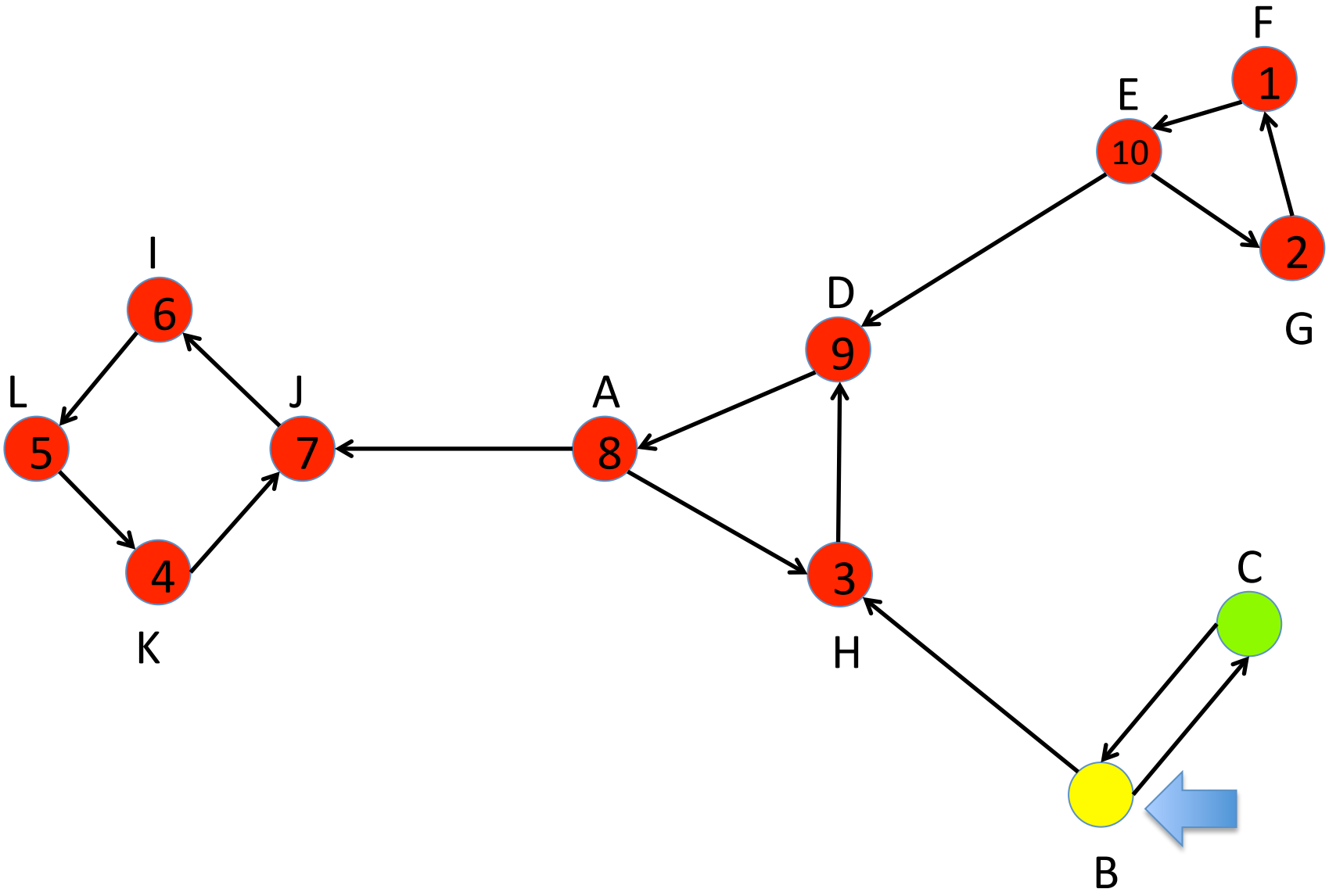

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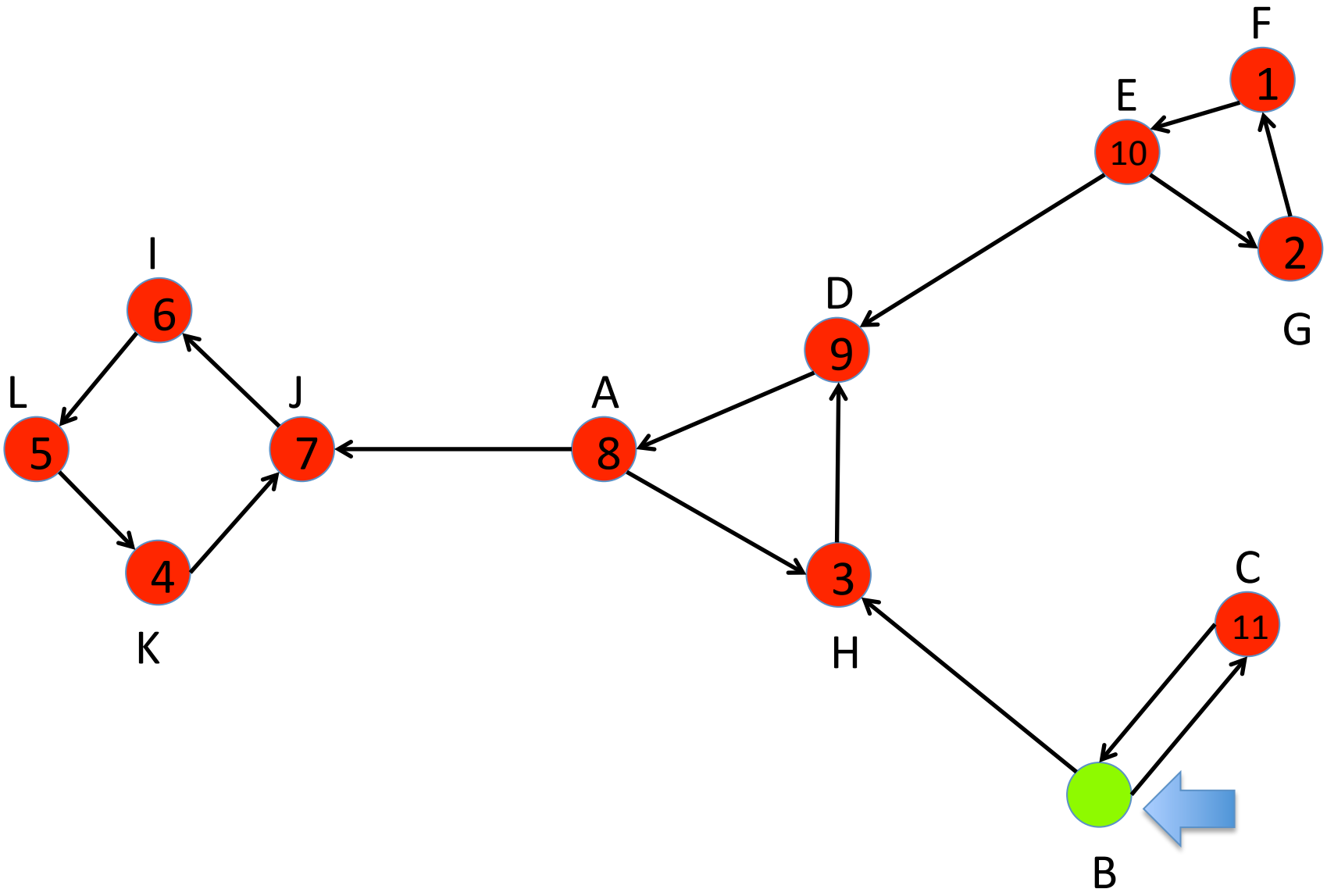

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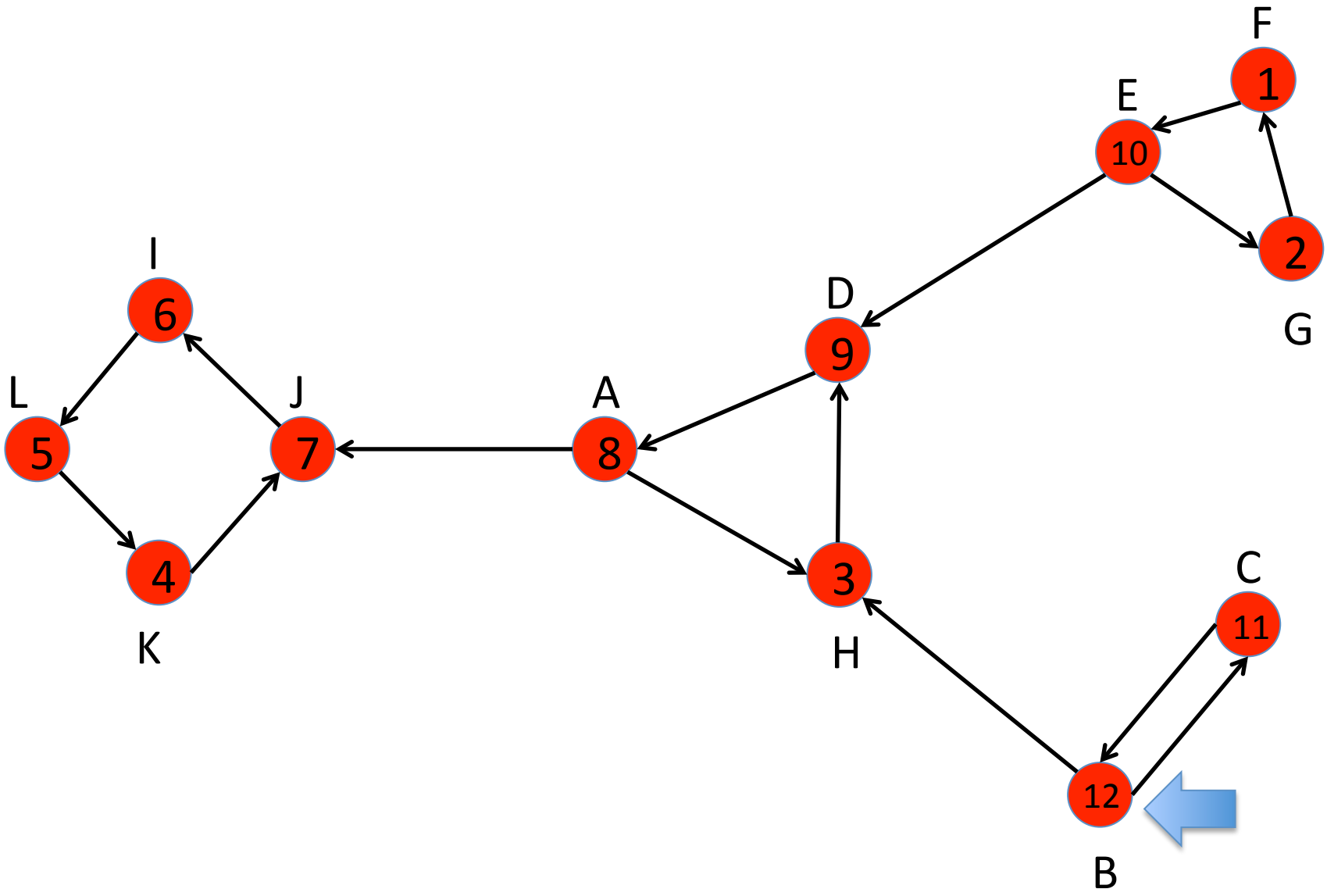

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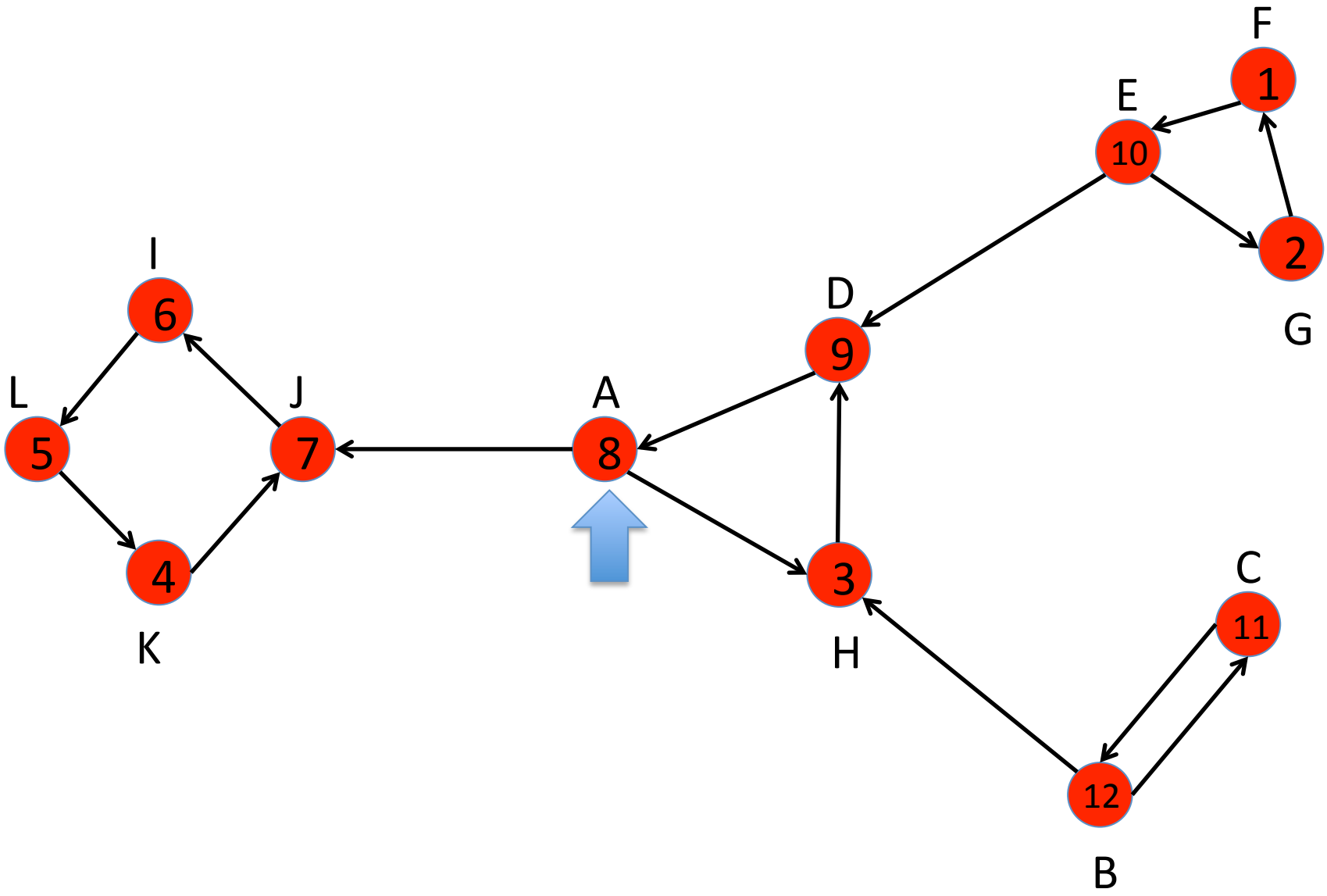

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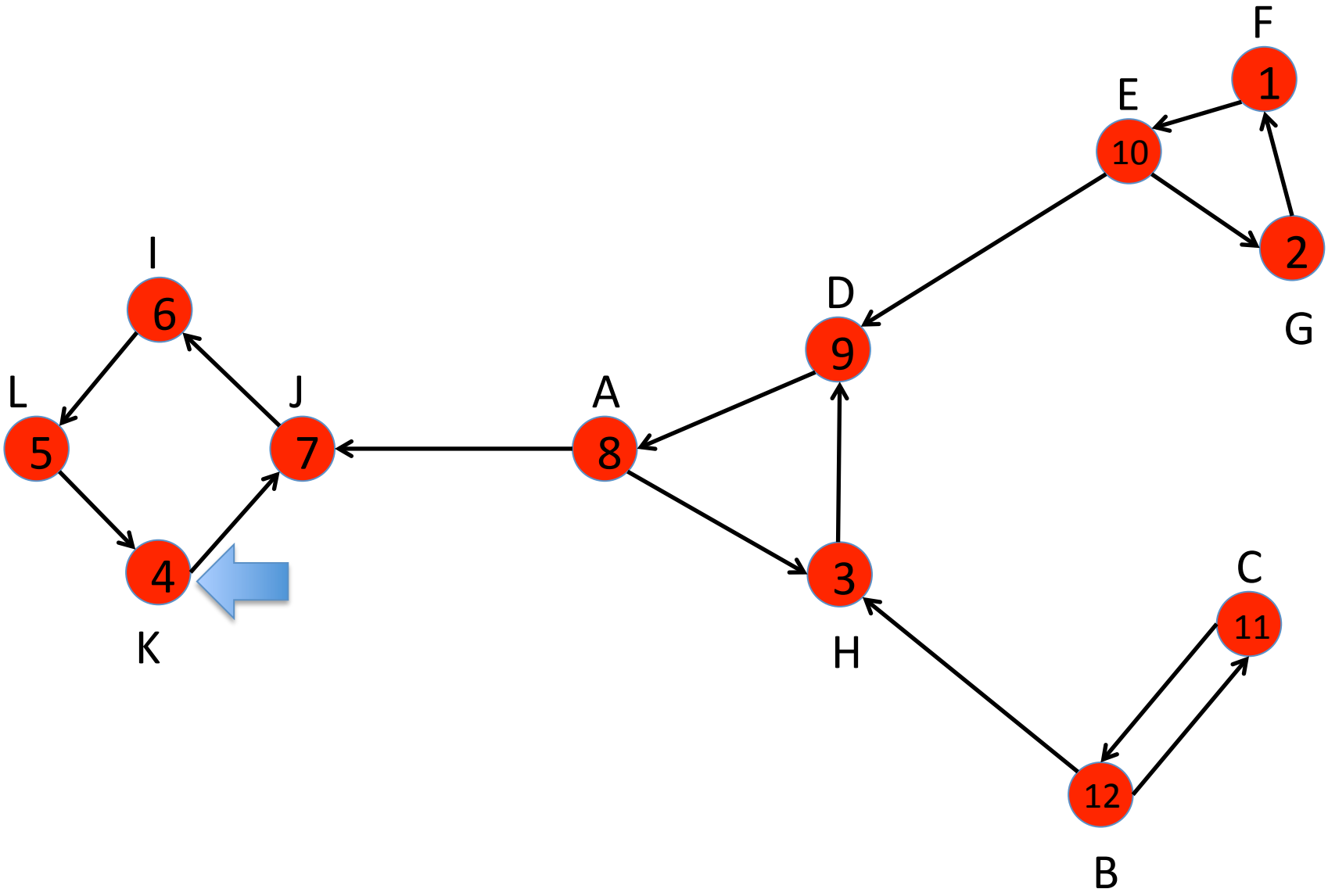

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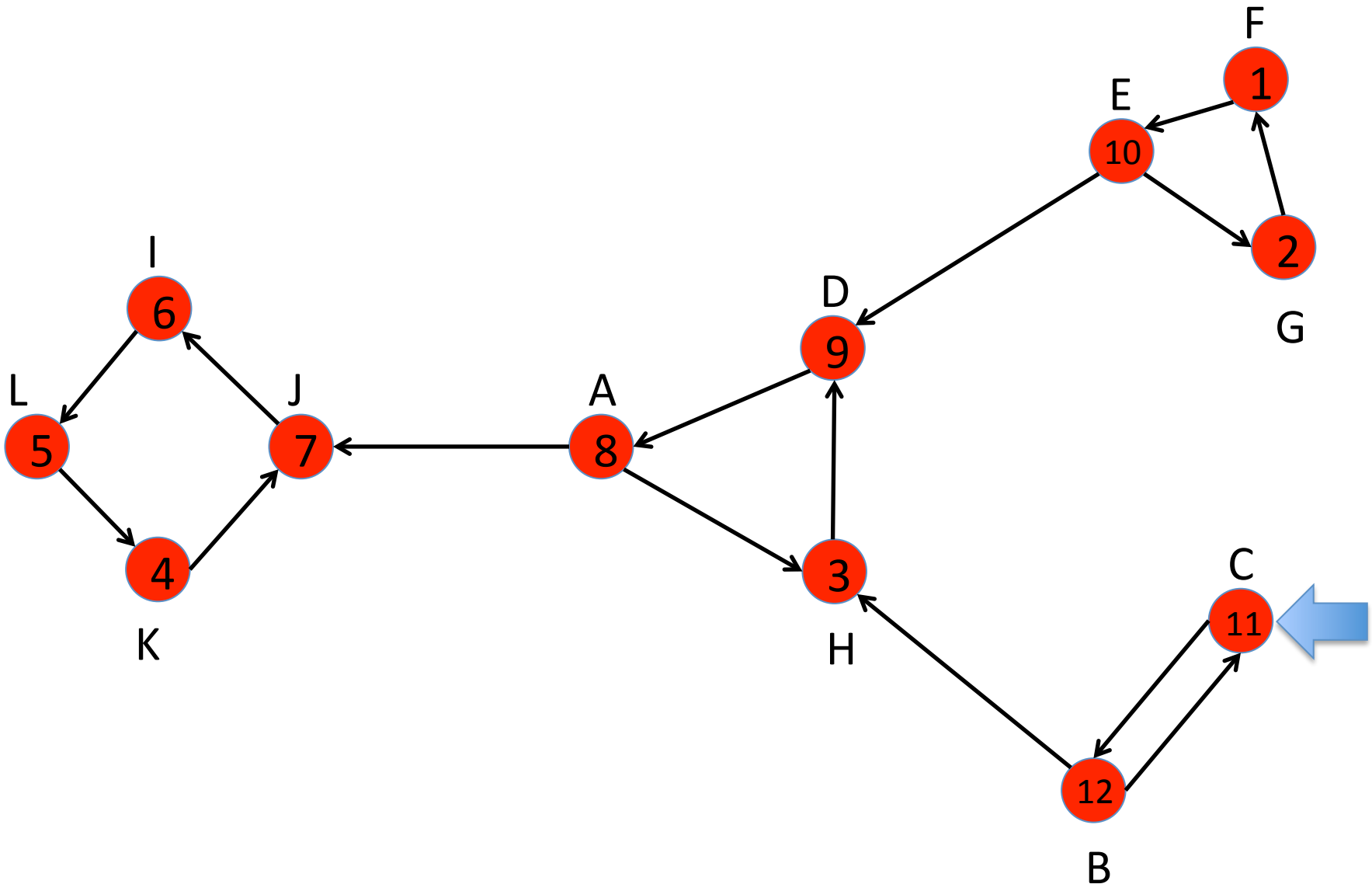

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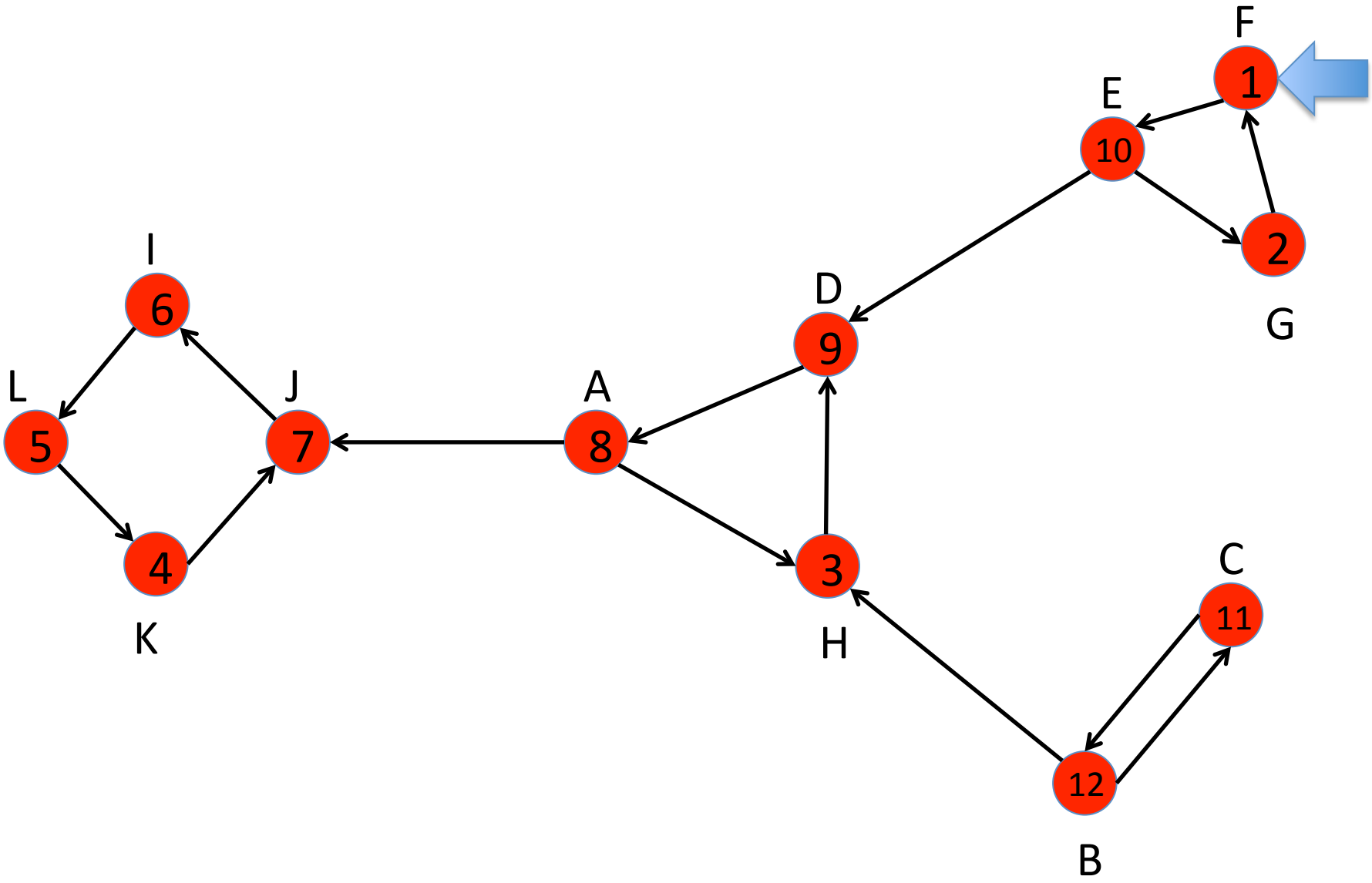

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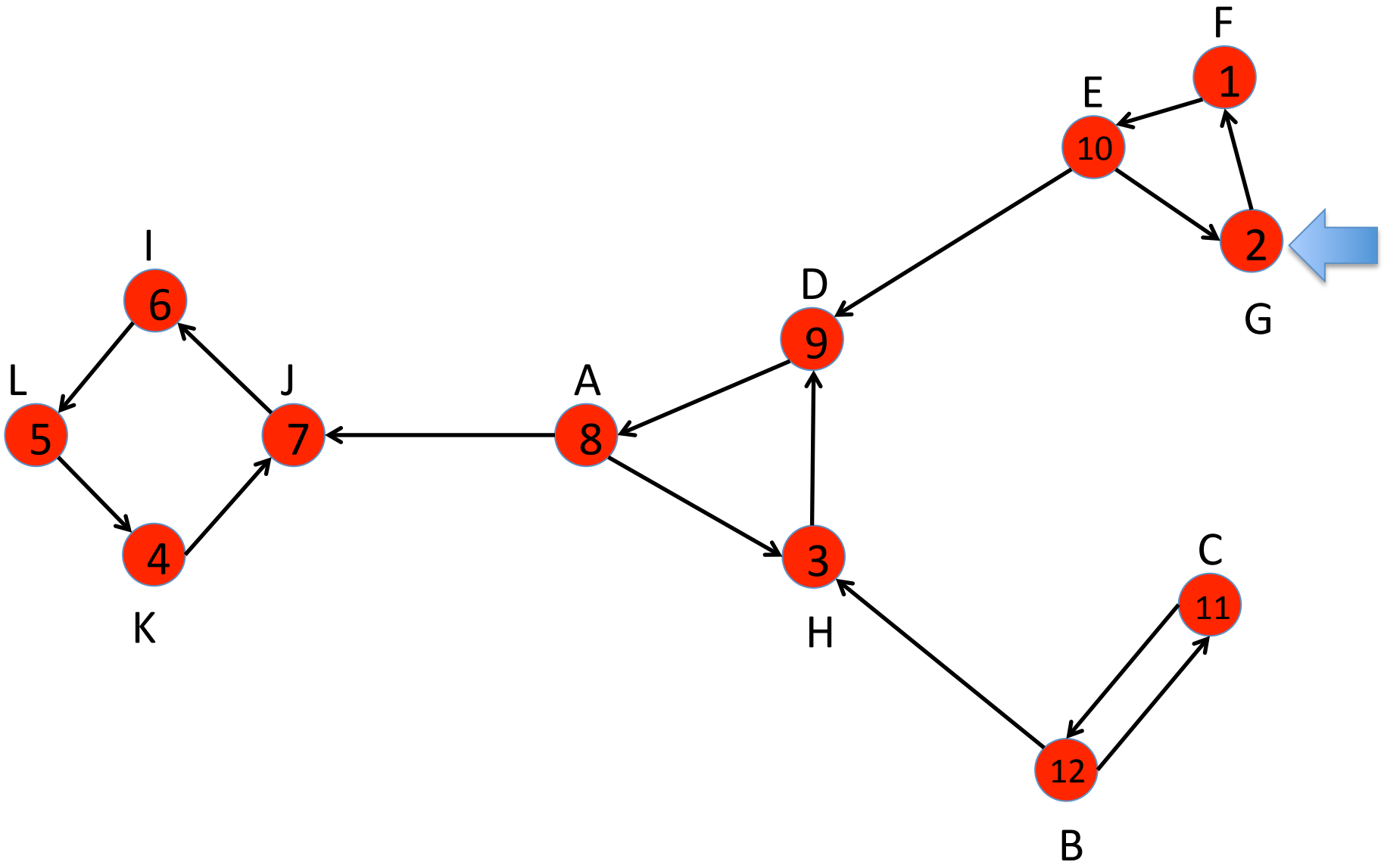

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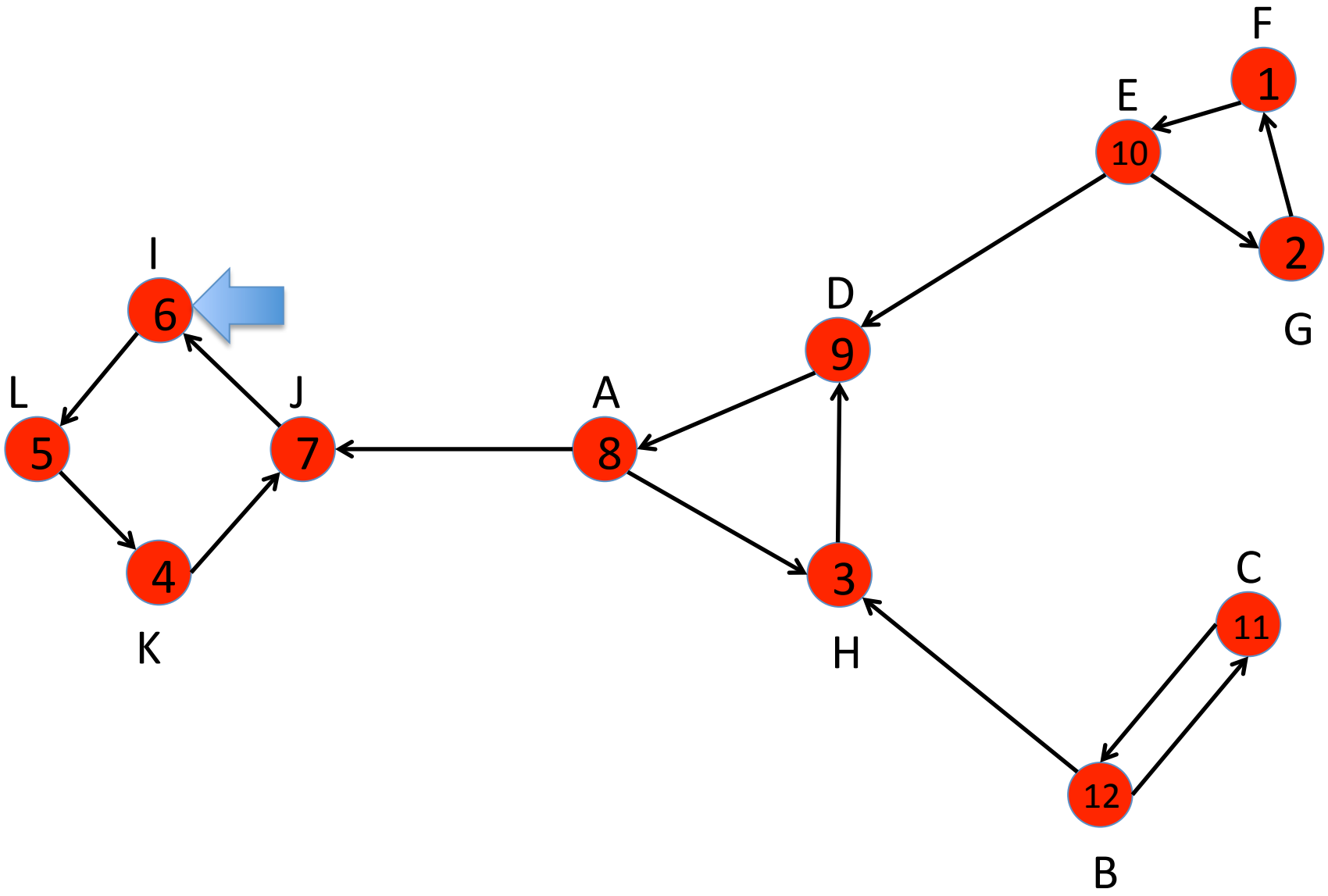
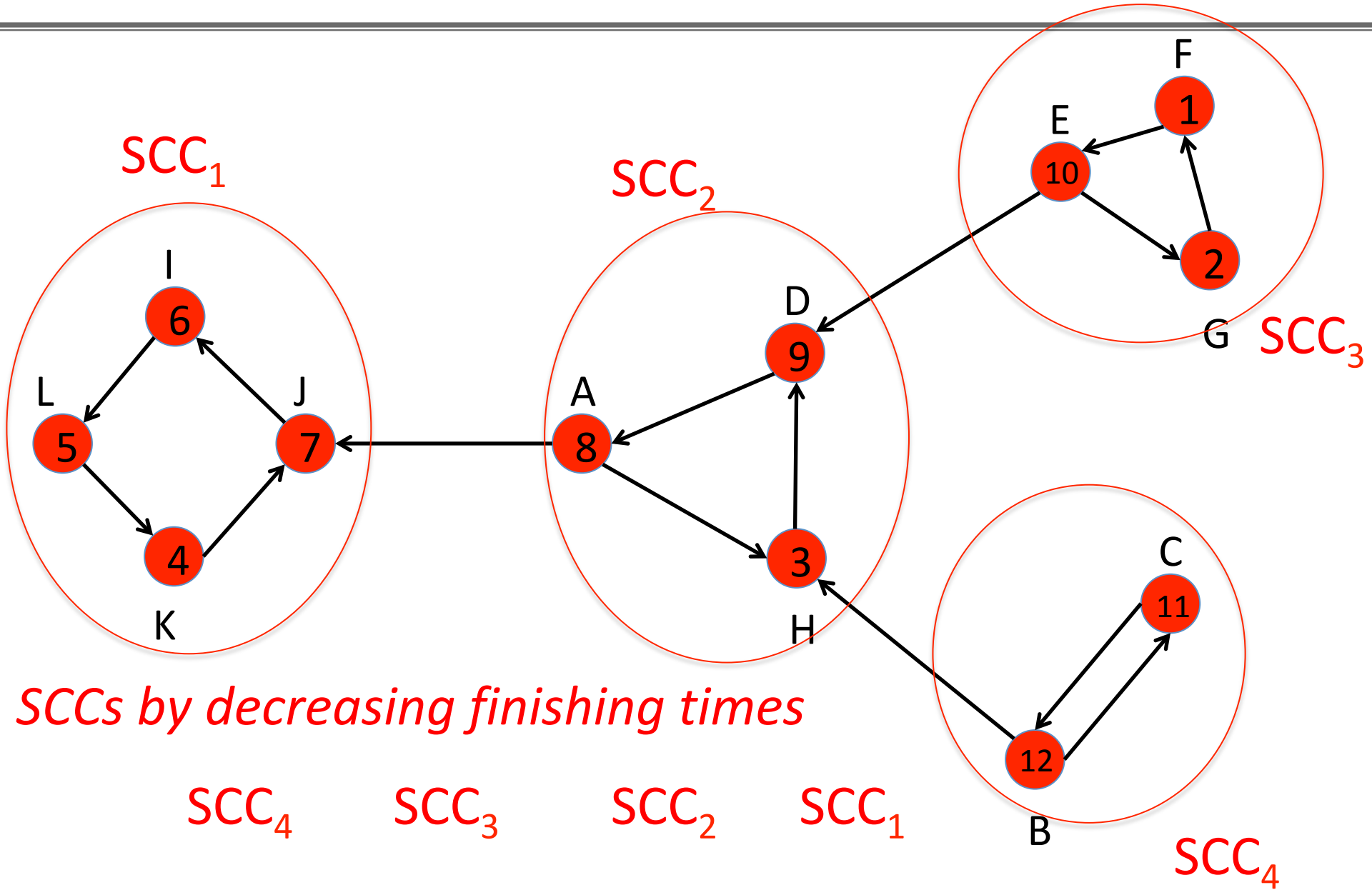


DFS & Finishing Times on Grev



SCCs by decreasing finishing times

SCC_4 SCC_3 SCC_2 SCC_1

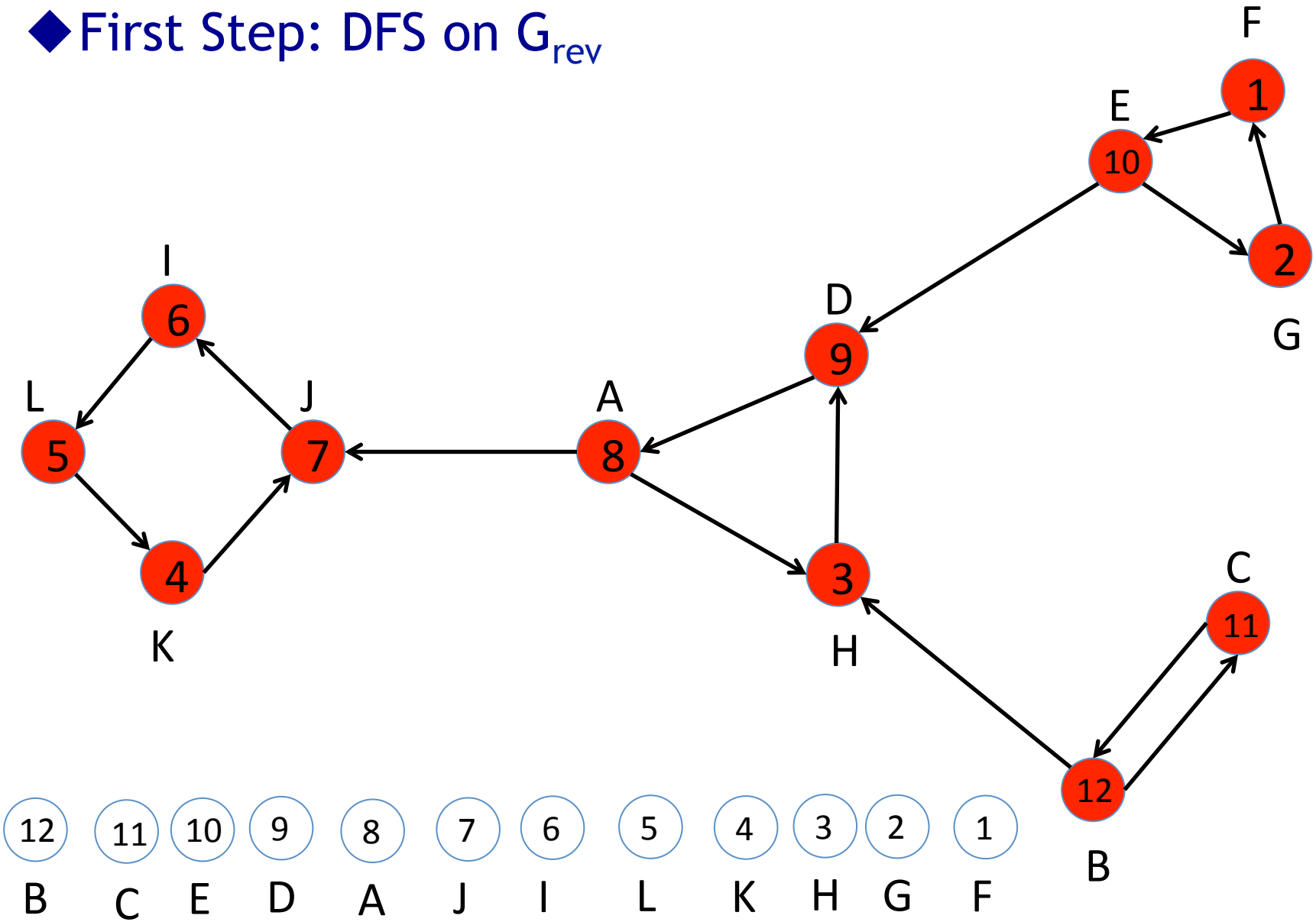
Finally: Kosaraju's Algorithm

```
procedure kosarajusSCC(DAG G):
    1. run DFS on G_rev and keep finishing times f
    2. run DFS/BFS in G:
        pick start vertices by decreasing f times
        propagate the ID of each new start vertex
        during traversal
```

Again, the labels will implicitly identify the SCCs!

Kosaraju's Algorithm Simulation

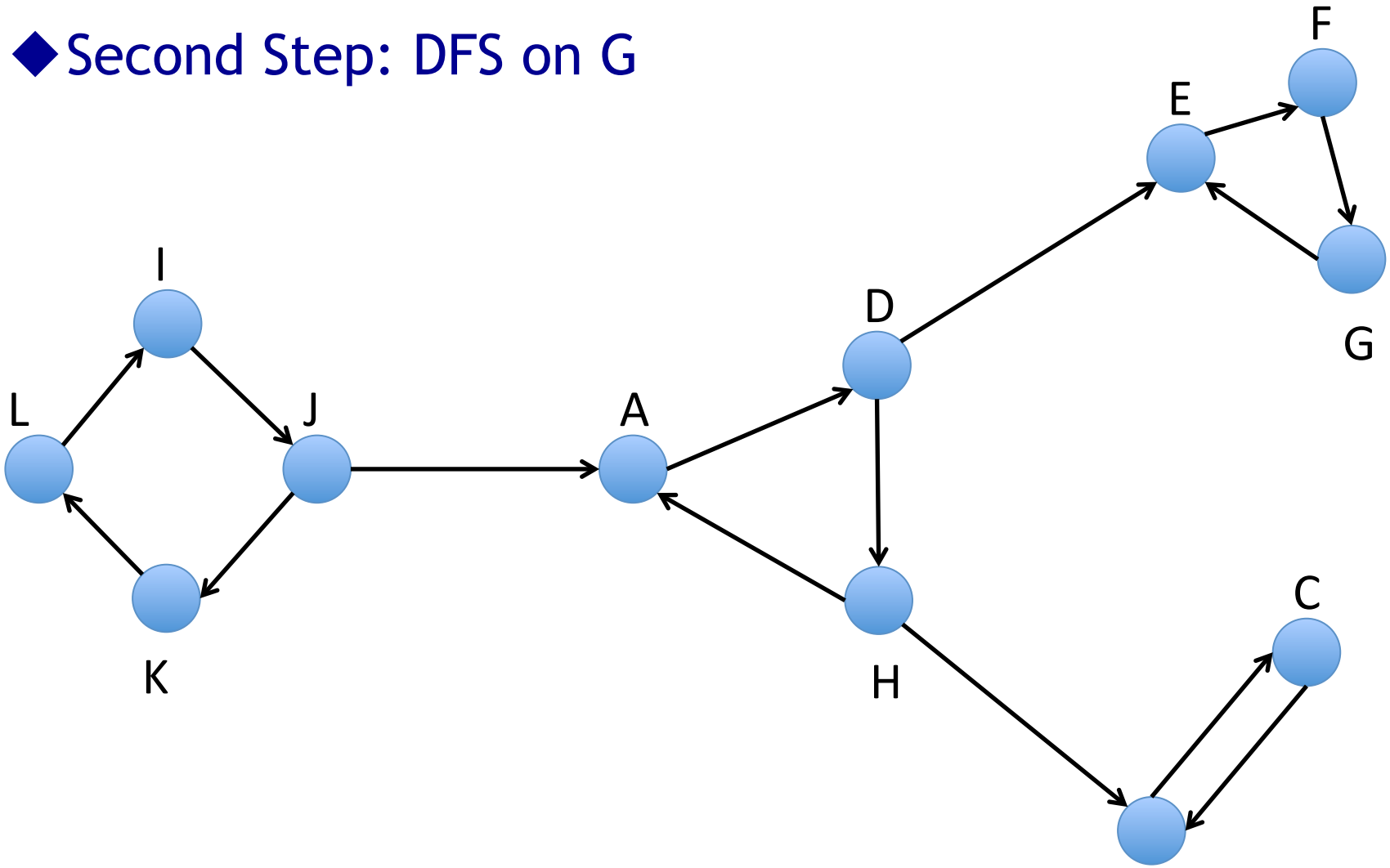
- First Step: DFS on G_{rev}



12	11	10	9	8	7	6	5	4	3	2	1
B	C	E	D	A	J	I	L	K	H	G	F

Kosaraju's Algorithm Simulation

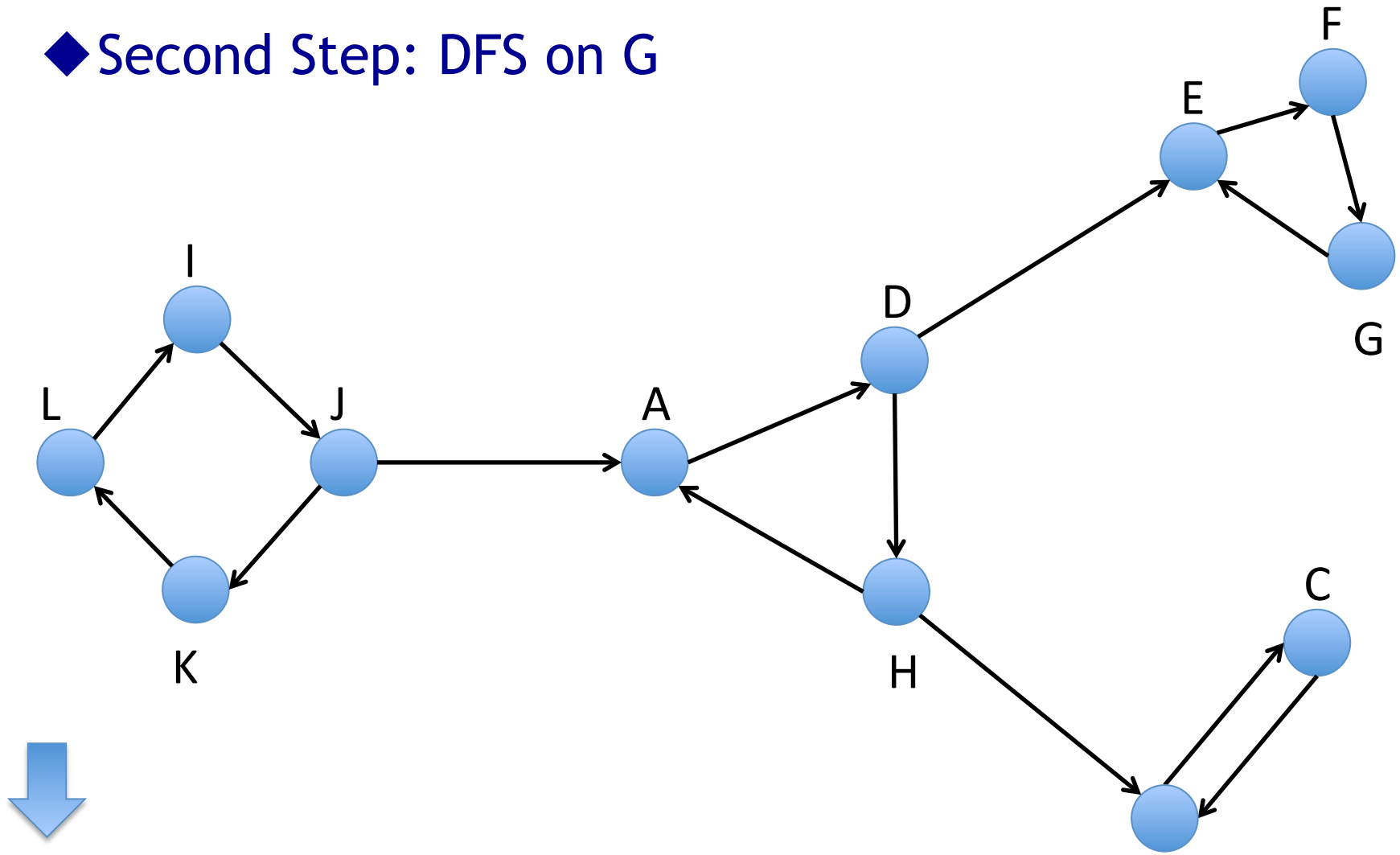
- Second Step: DFS on G



12	11	10	9	8	7	6	5	4	3	2	1
B	C	E	D	A	J	I	L	K	H	G	F

Kosaraju's Algorithm Simulation

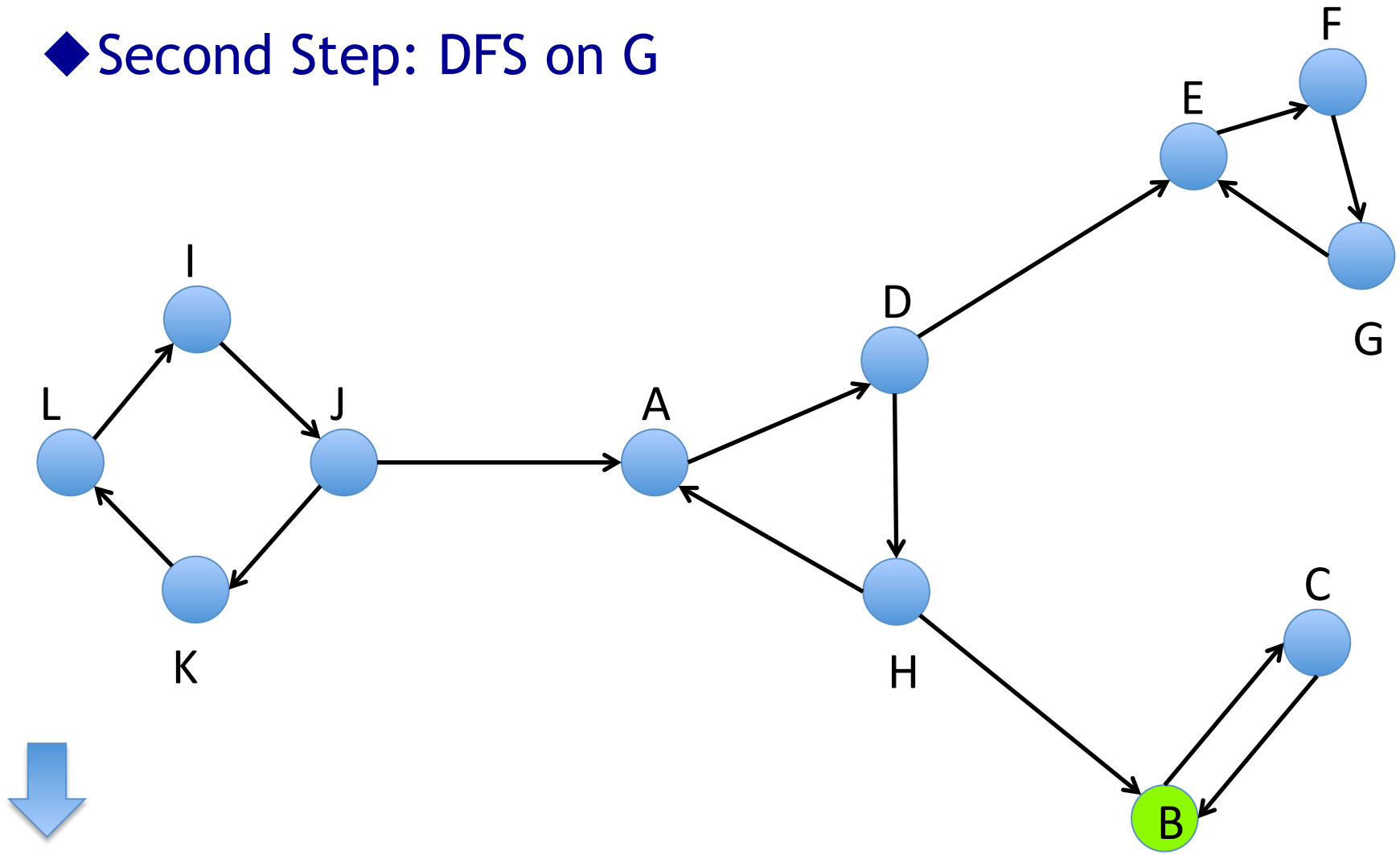
- Second Step: DFS on G



12	11	10	9	8	7	6	5	4	3	2	1
B	C	E	D	A	J	I	L	K	H	G	F

Kosaraju's Algorithm Simulation

- Second Step: DFS on G

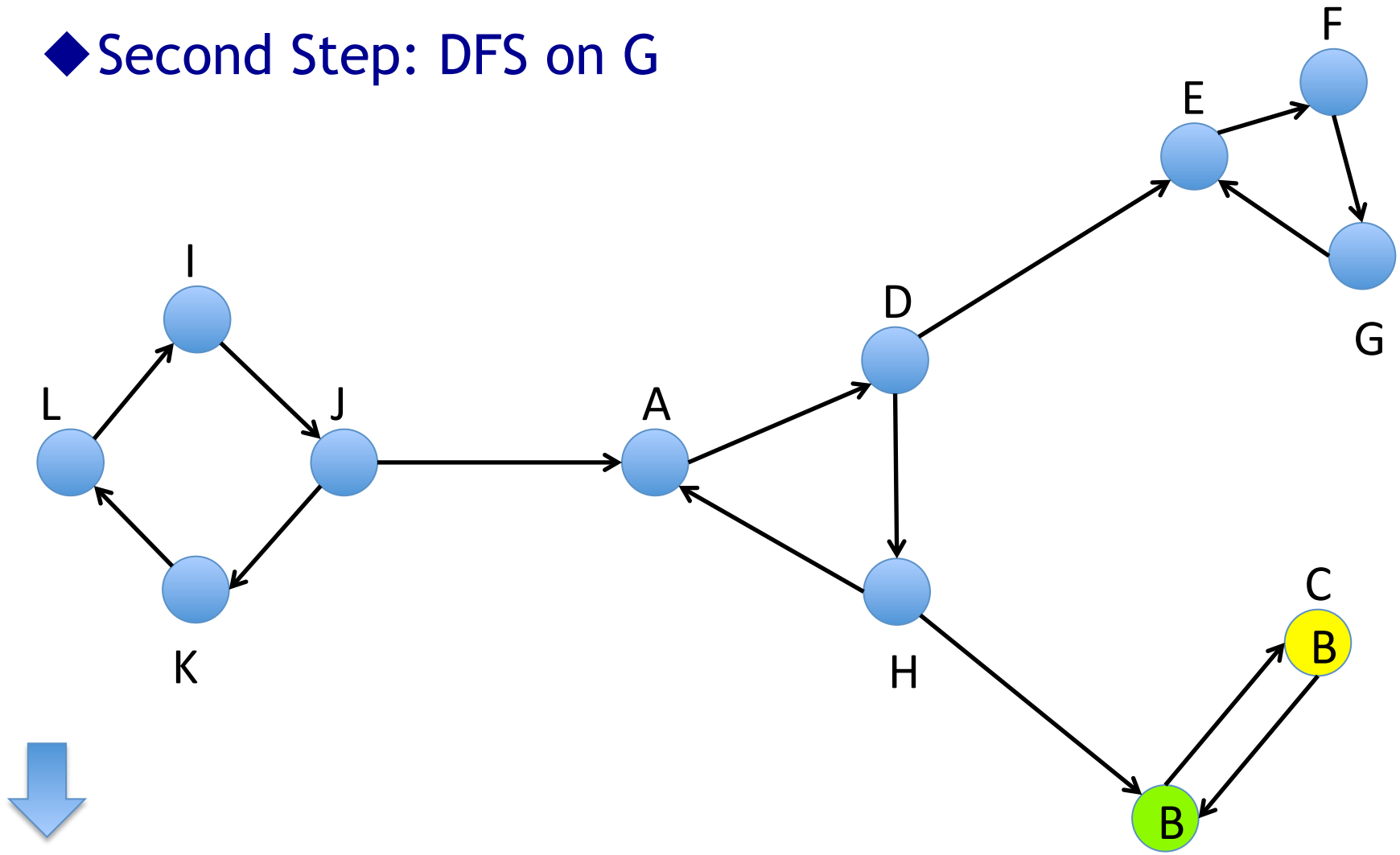


12	11	10	9	8	7	6	5	4	3	2	1
B	C	E	D	A	J	I	L	K	H	G	F

B

Kosaraju's Algorithm Simulation

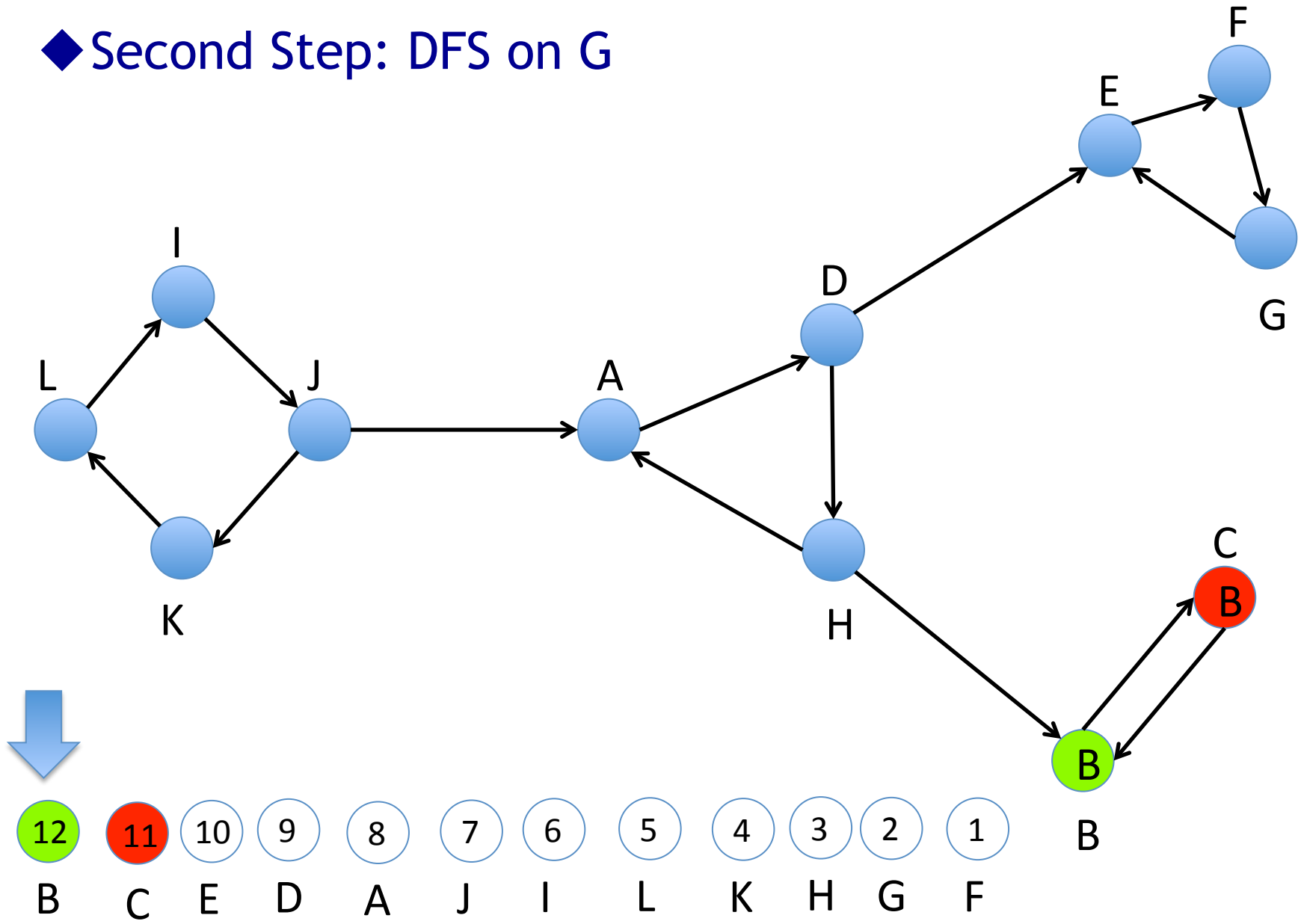
- Second Step: DFS on G



12	11	10	9	8	7	6	5	4	3	2	1
B	C	E	D	A	J	I	L	K	H	G	F

Kosaraju's Algorithm Simulation

- Second Step: DFS on G



Kosaraju's Algorithm Simulation

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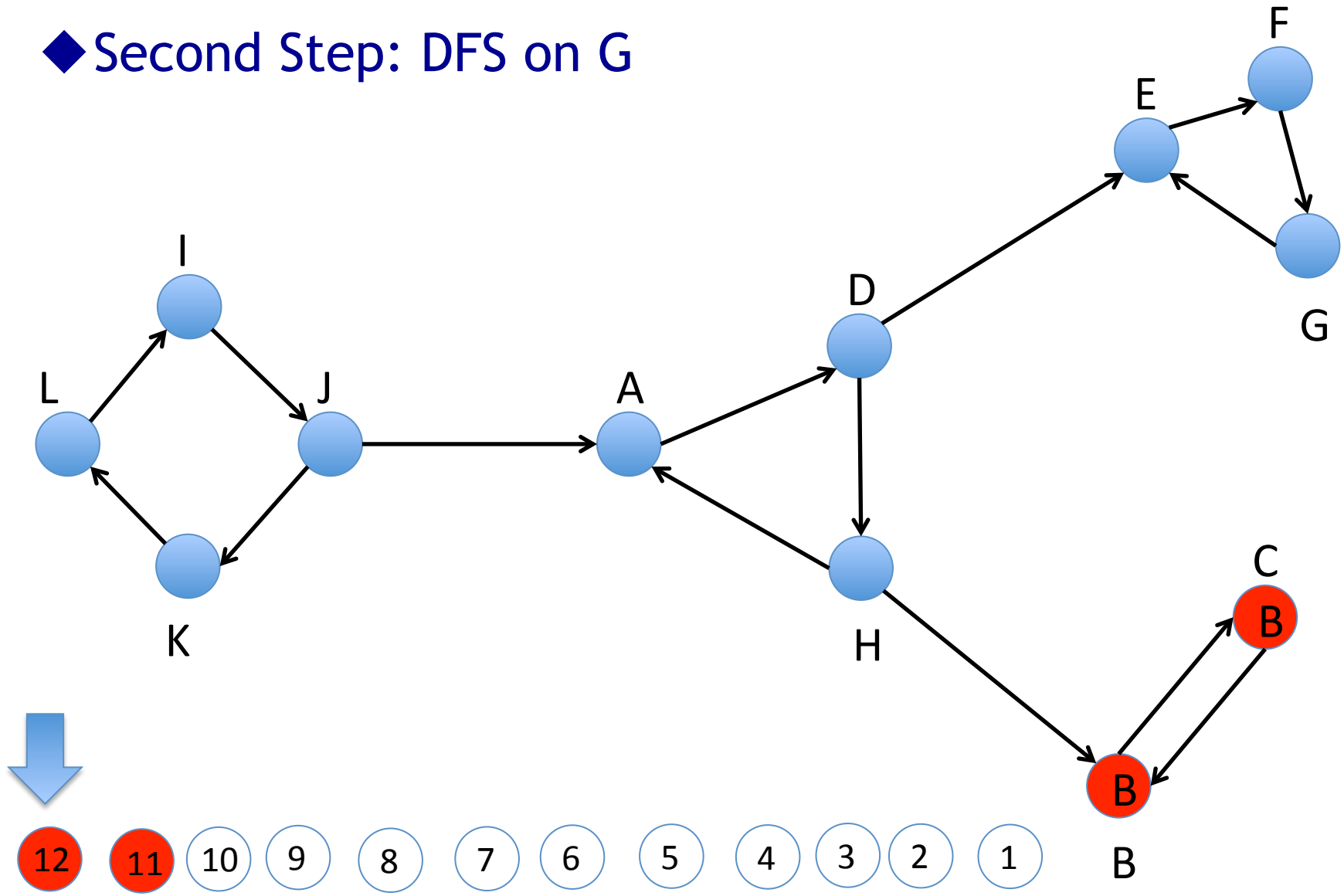

12	11	10	9	8	7	6	5	4	3	2	1
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Kosaraju's Algorithm Simulation

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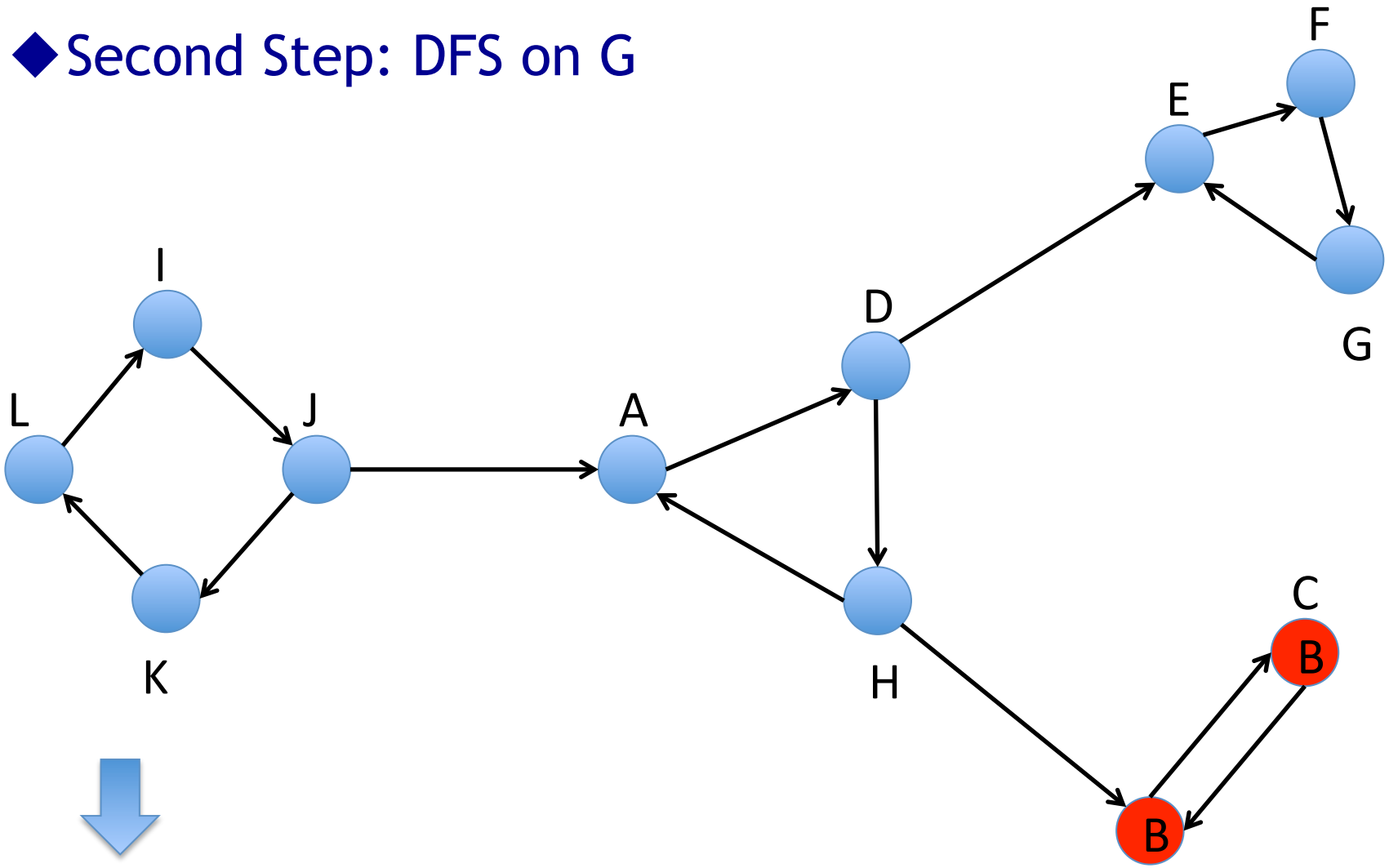

12	11	10	9	8	7	6	5	4	3	2	1
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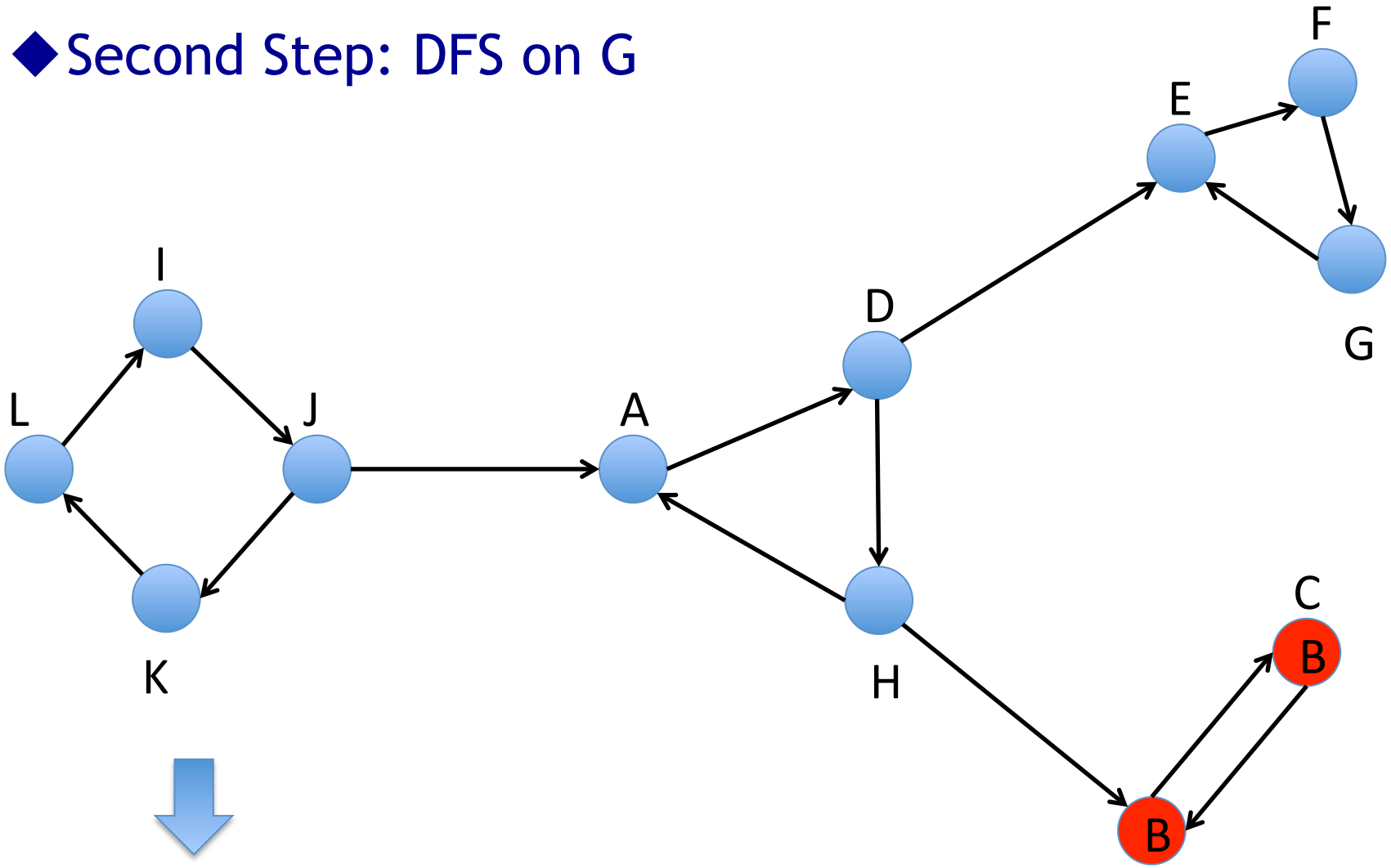
Kosaraju's Algorithm Simulation

- Second Step: DFS on G



12	11	10	9	8	7	6	5	4	3	2	1
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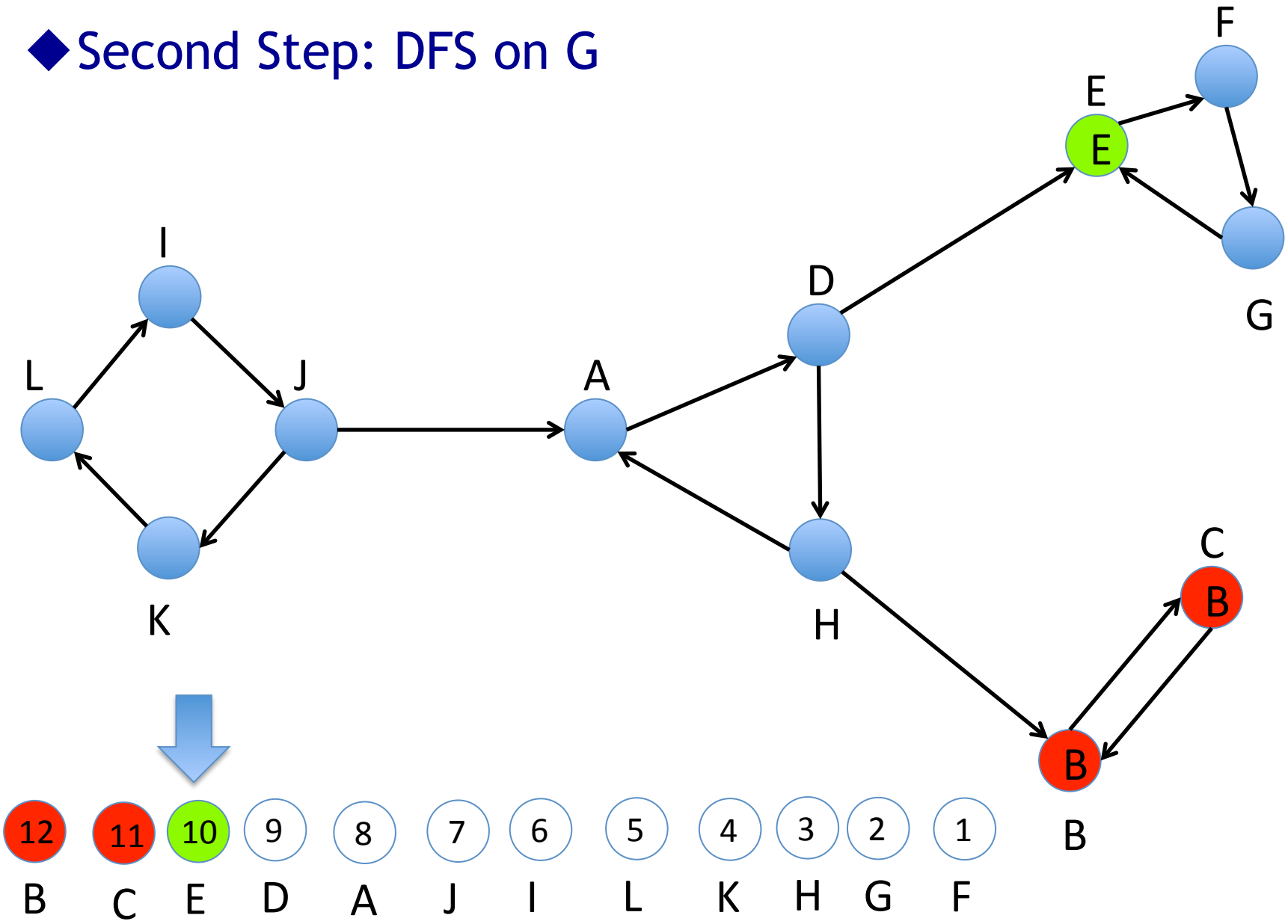
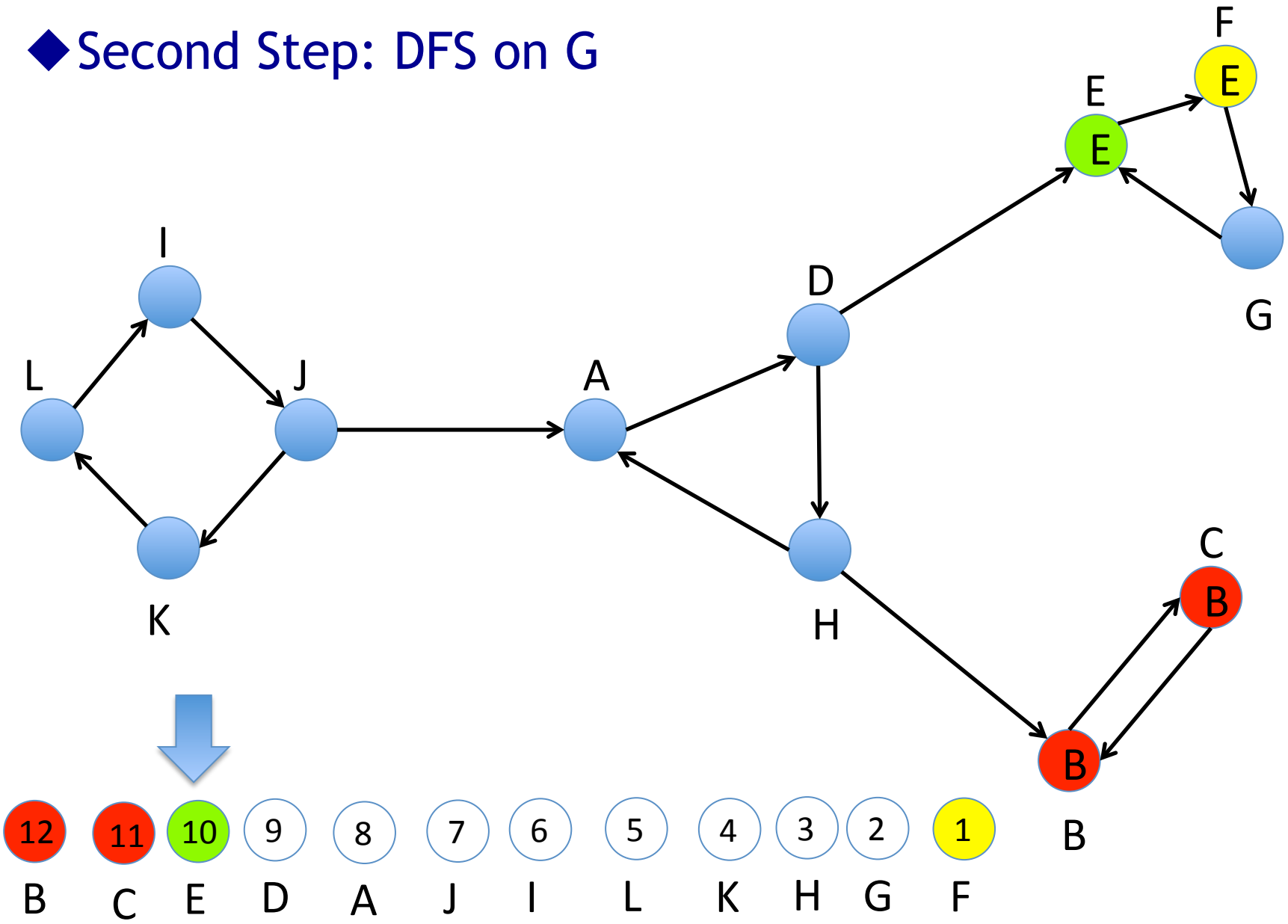
Kosaraju's Algorithm Simulation

◆Second Step: DFS on G



Kosaraju's Algorithm Simulation

- Second Step: DFS on G



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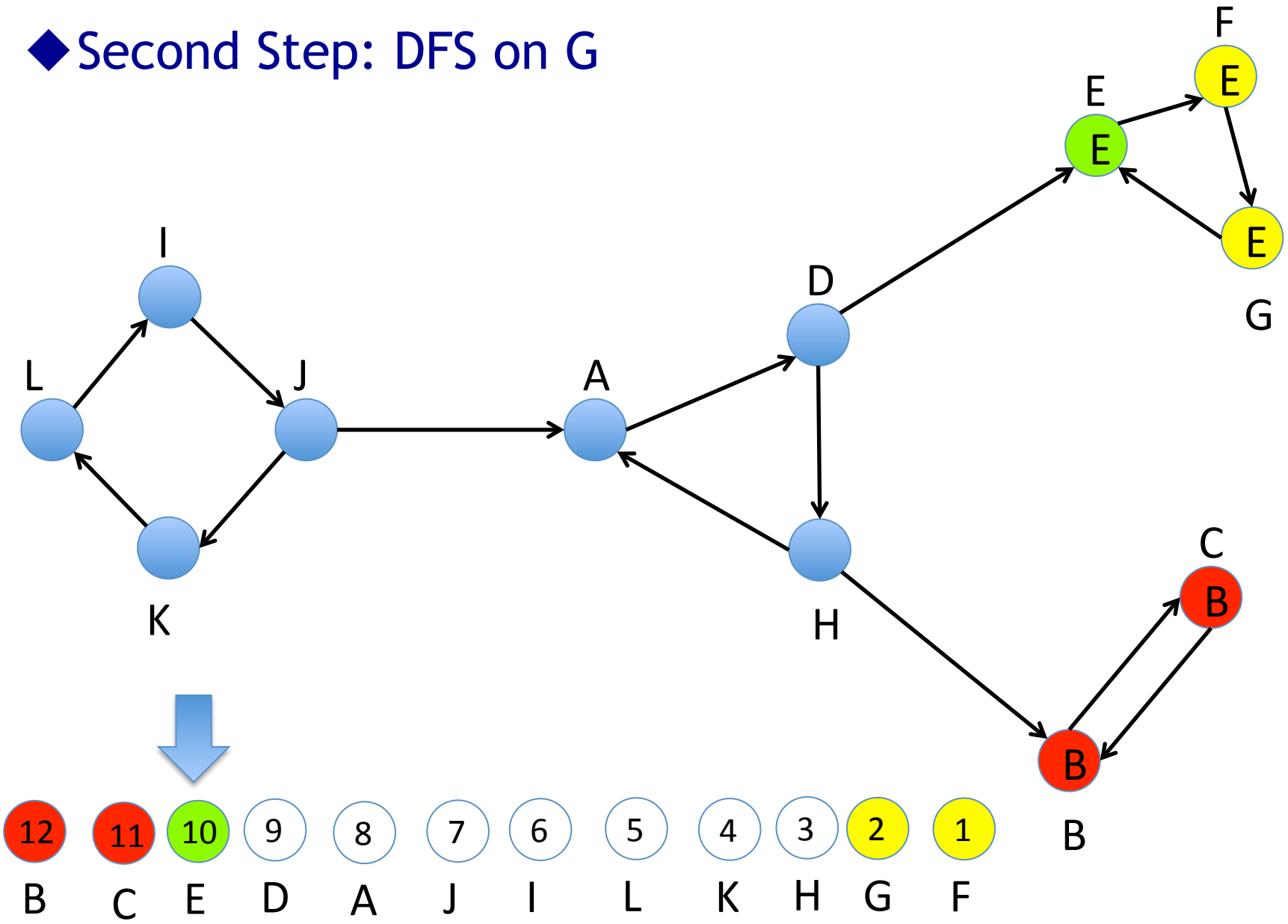

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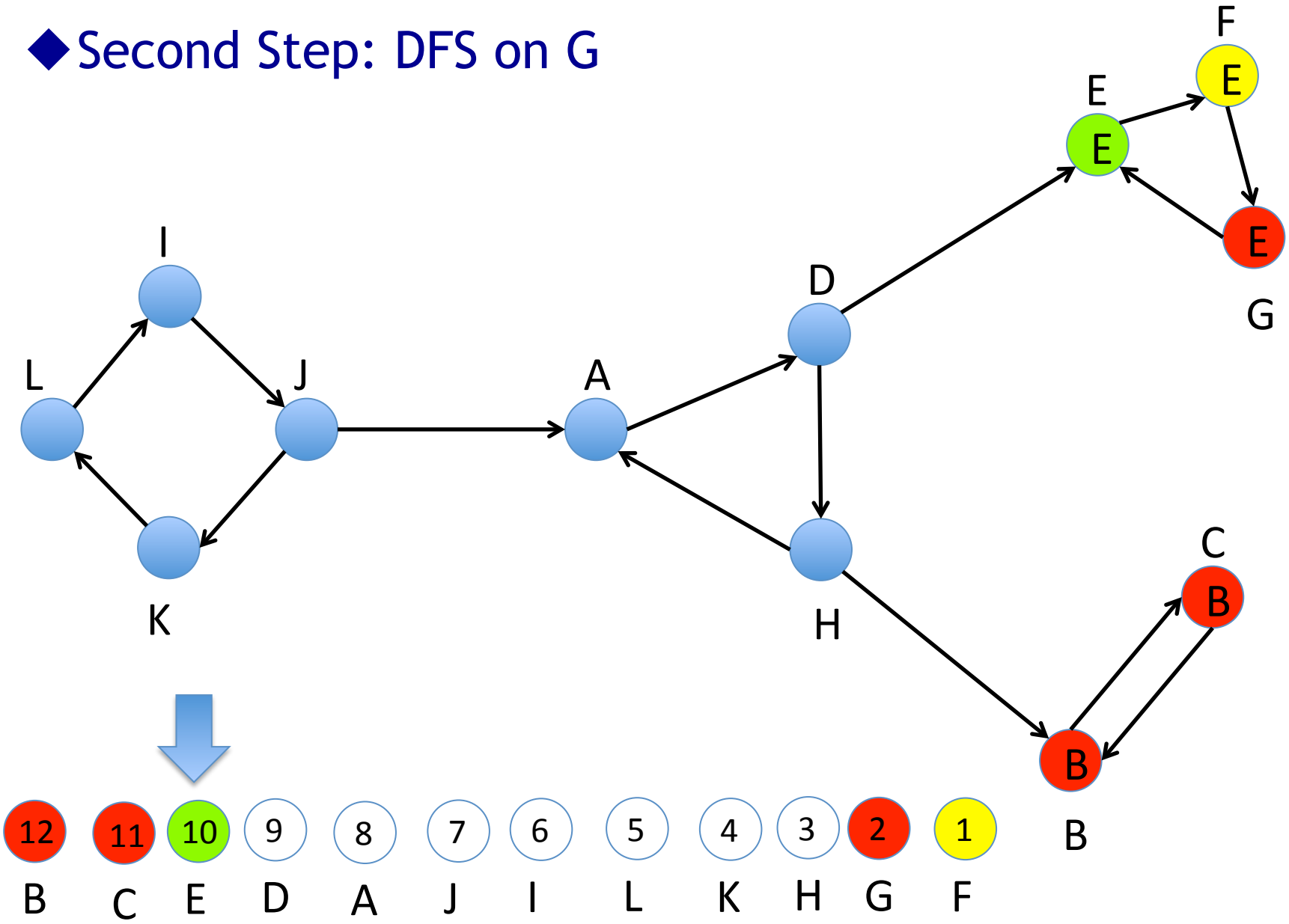

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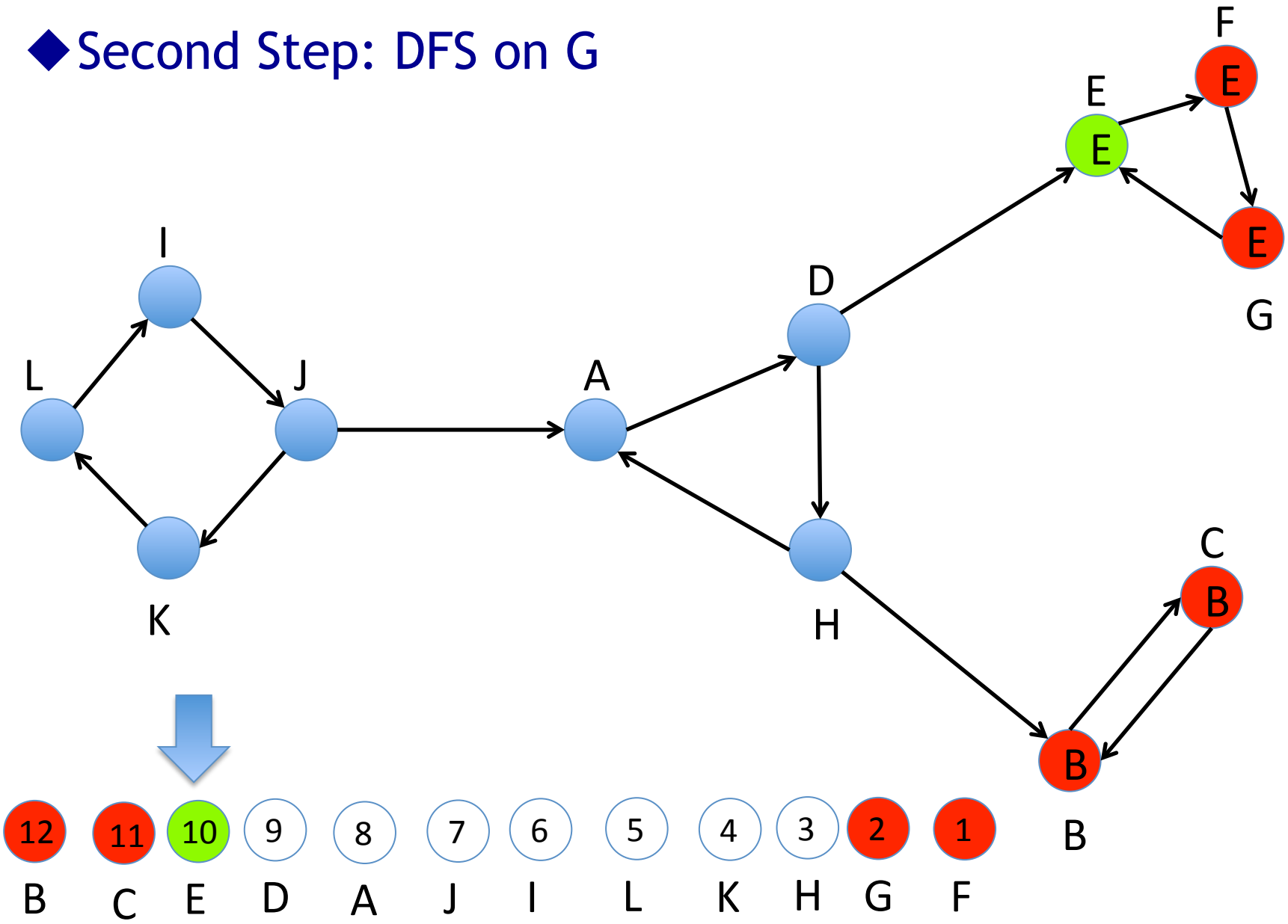

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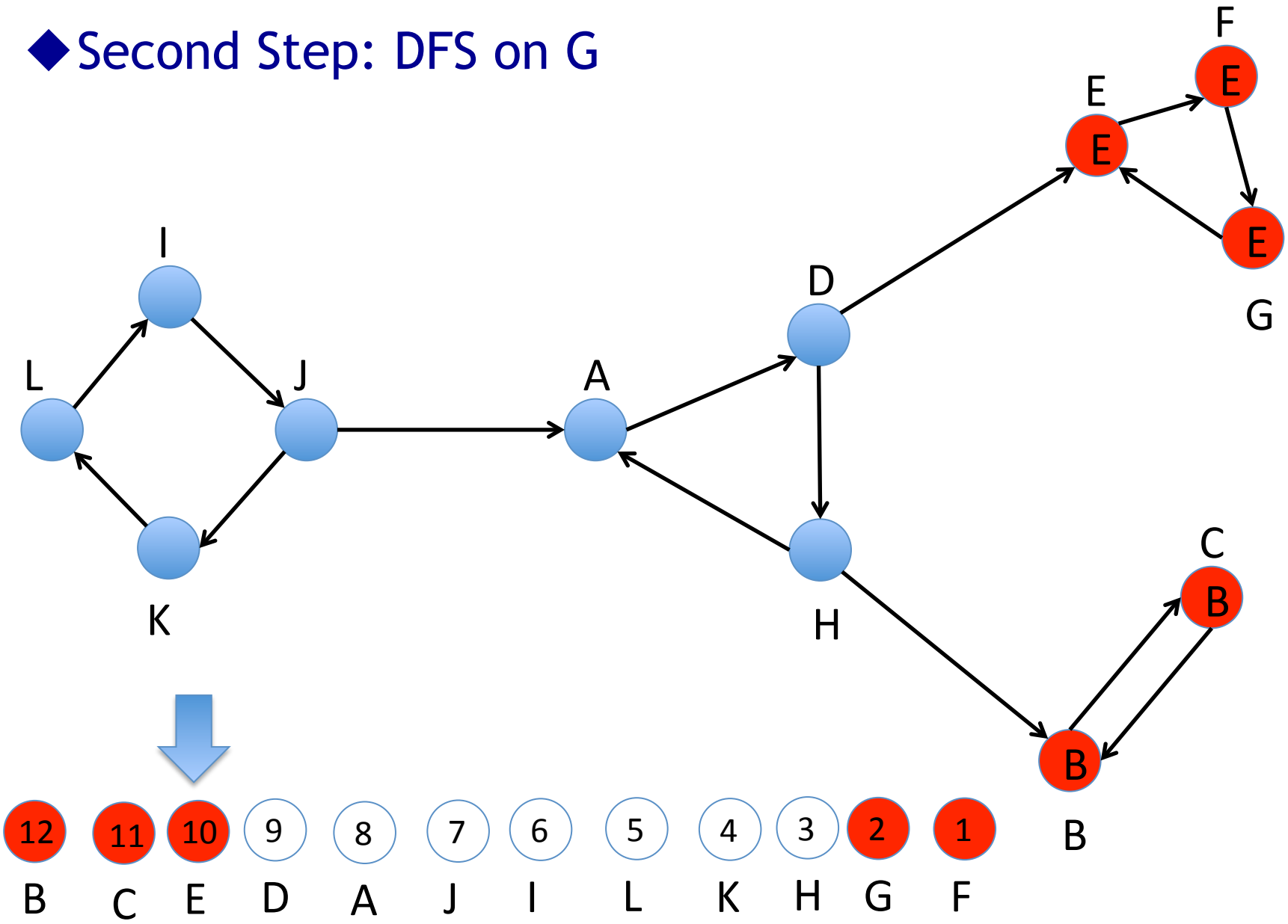

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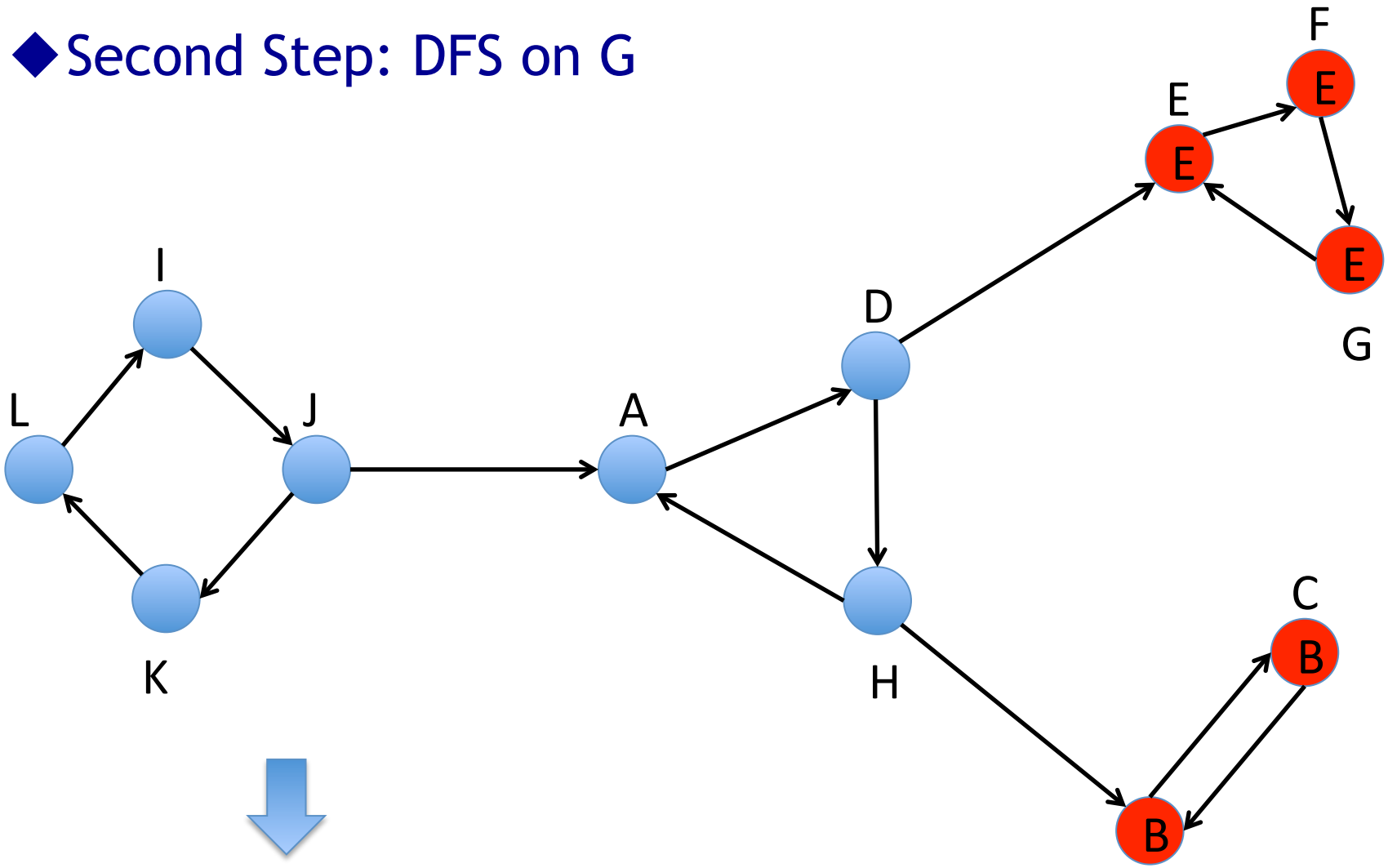

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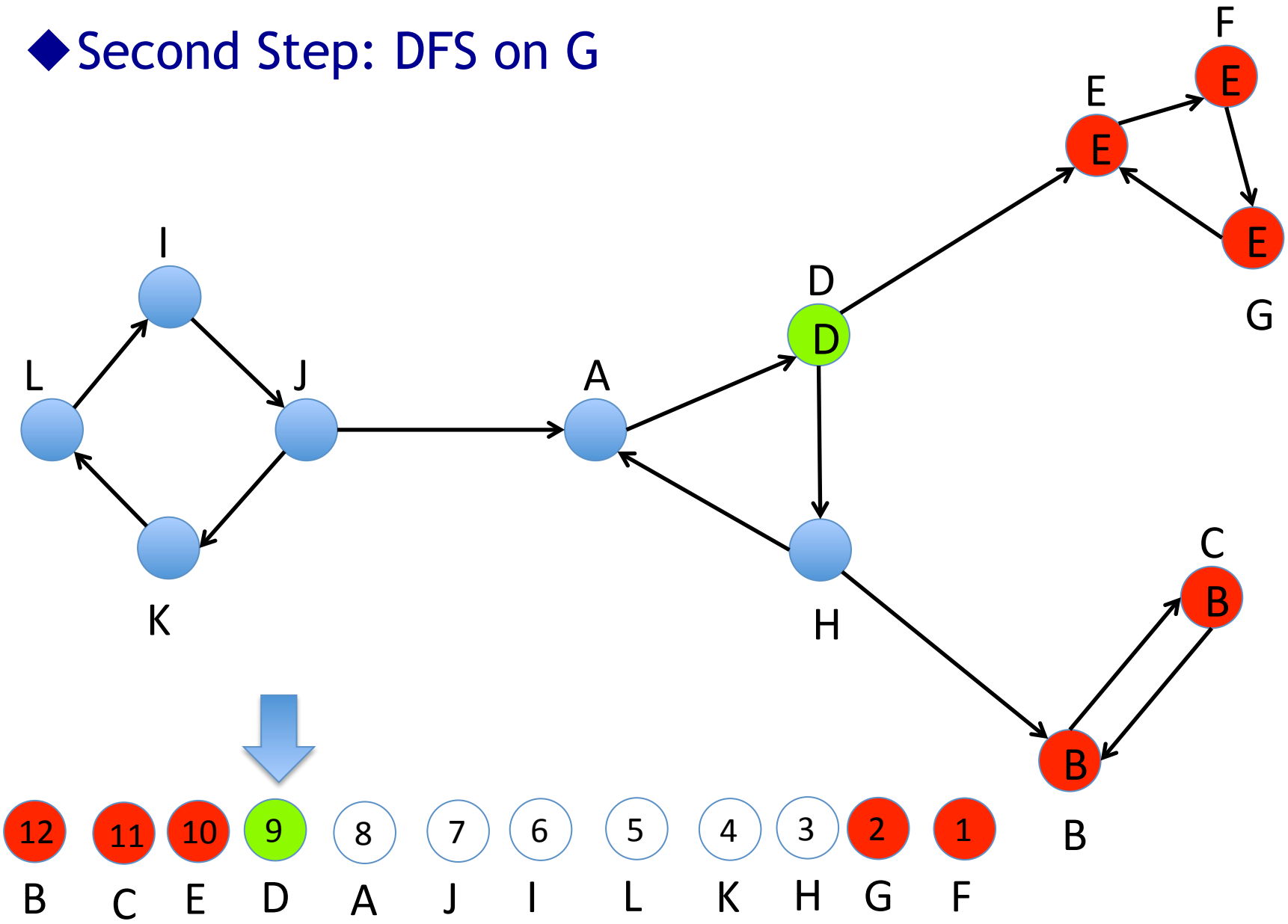

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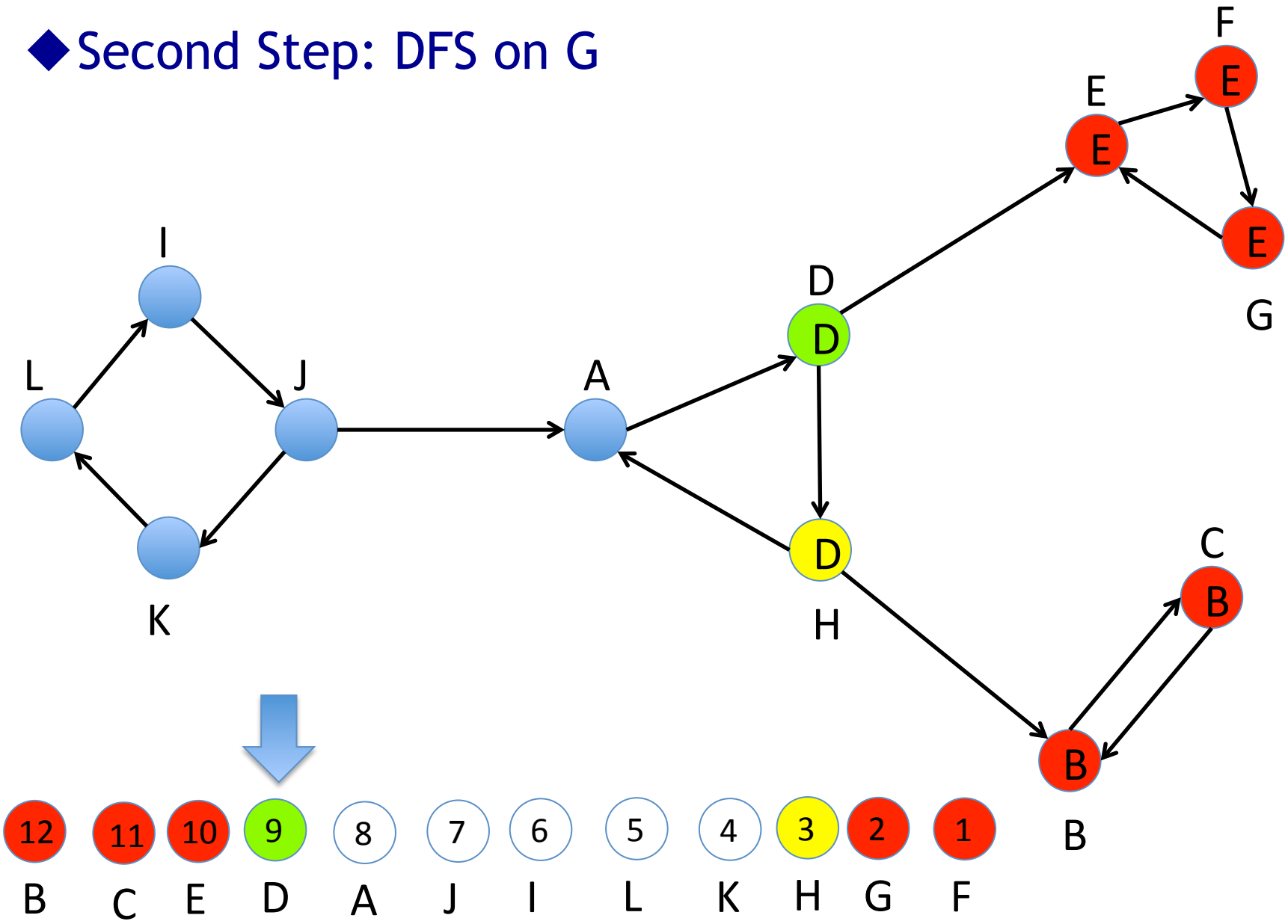

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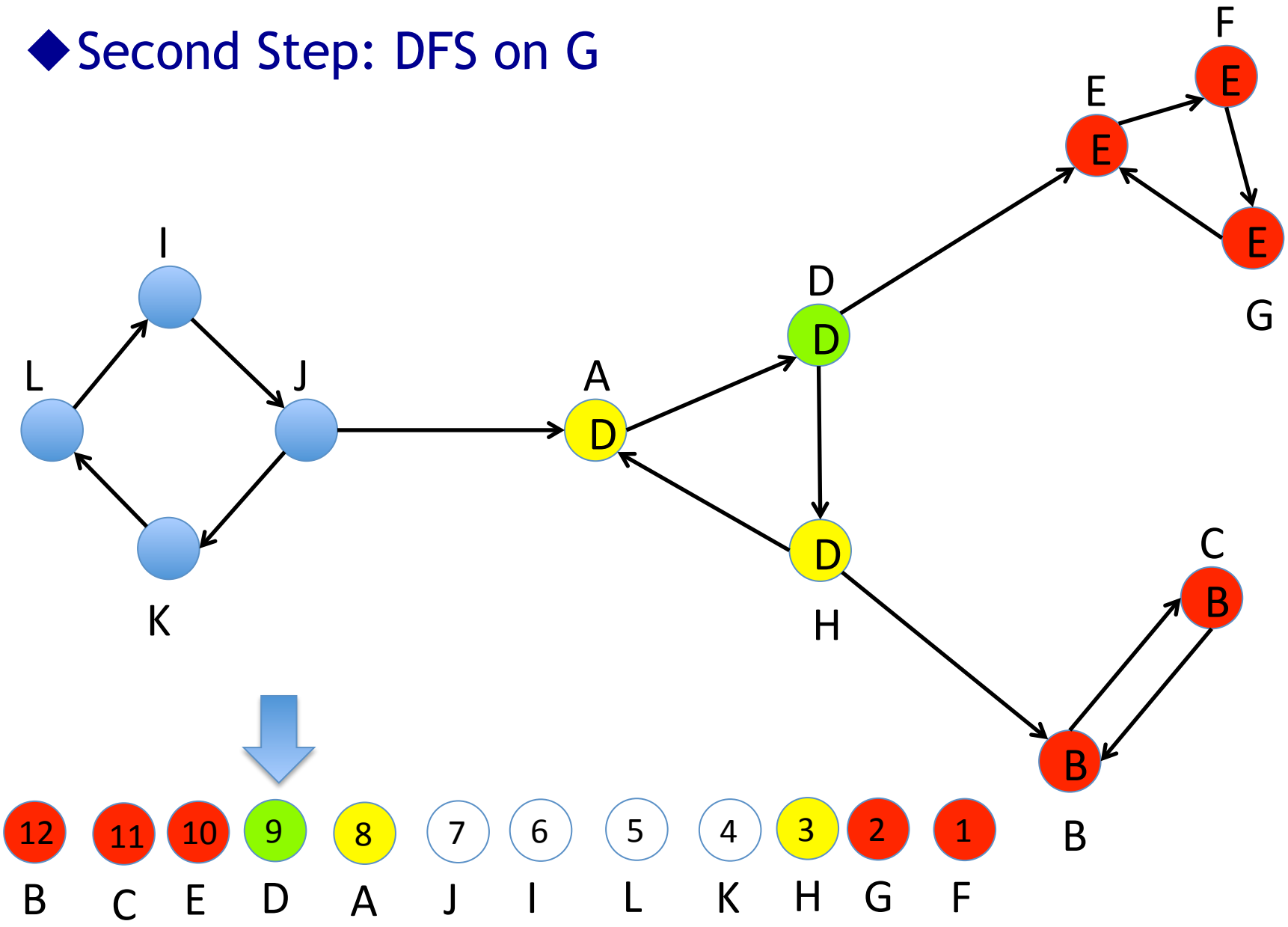

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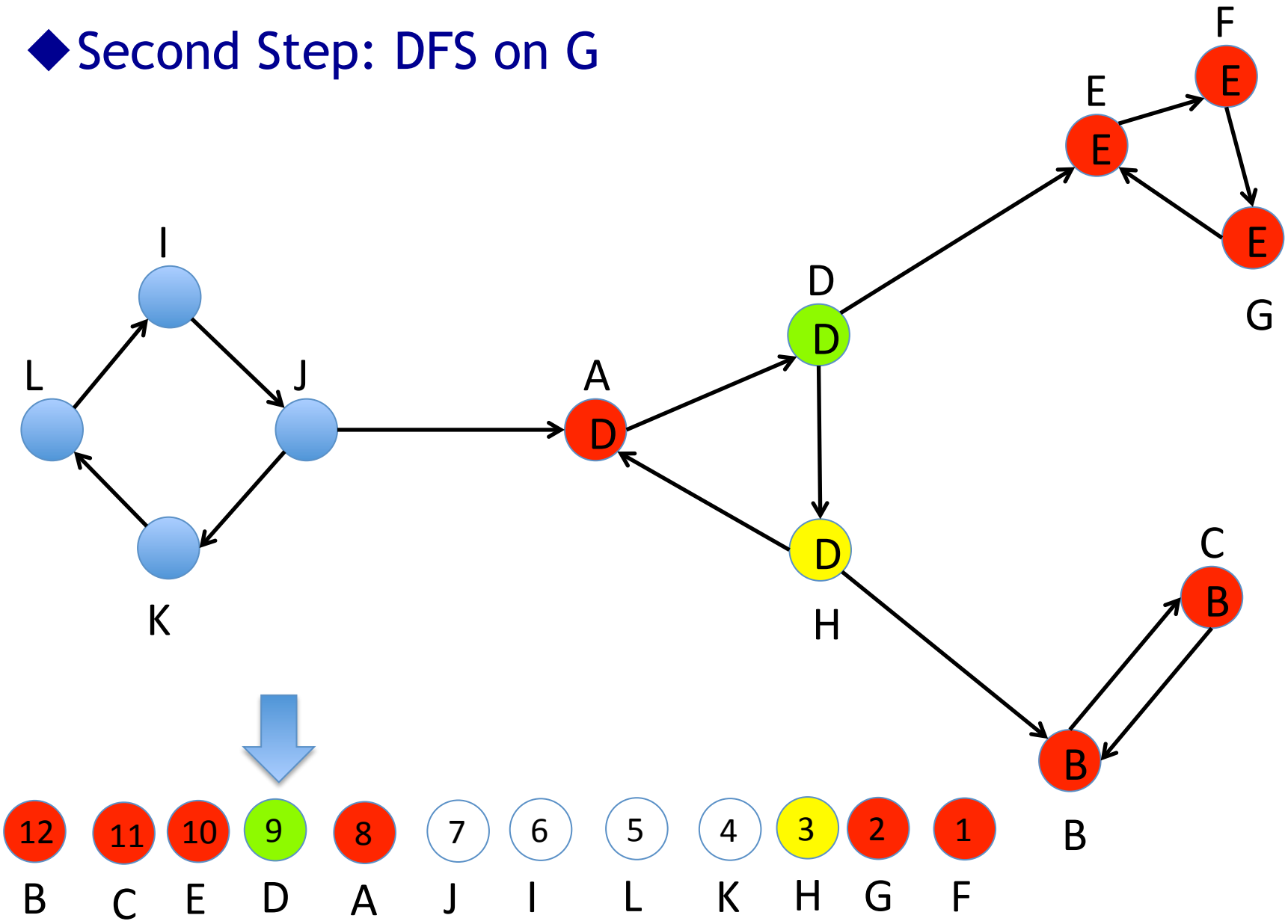

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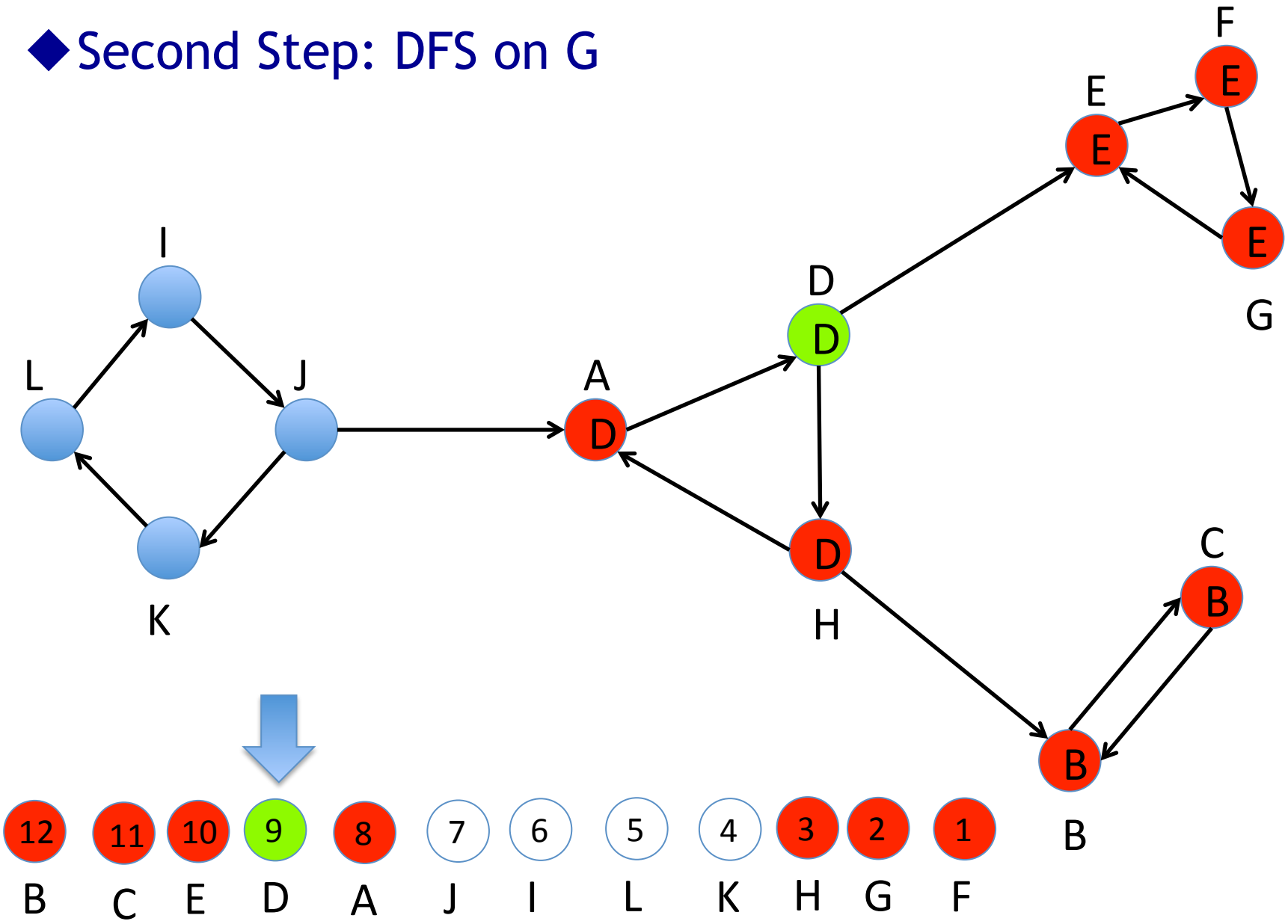

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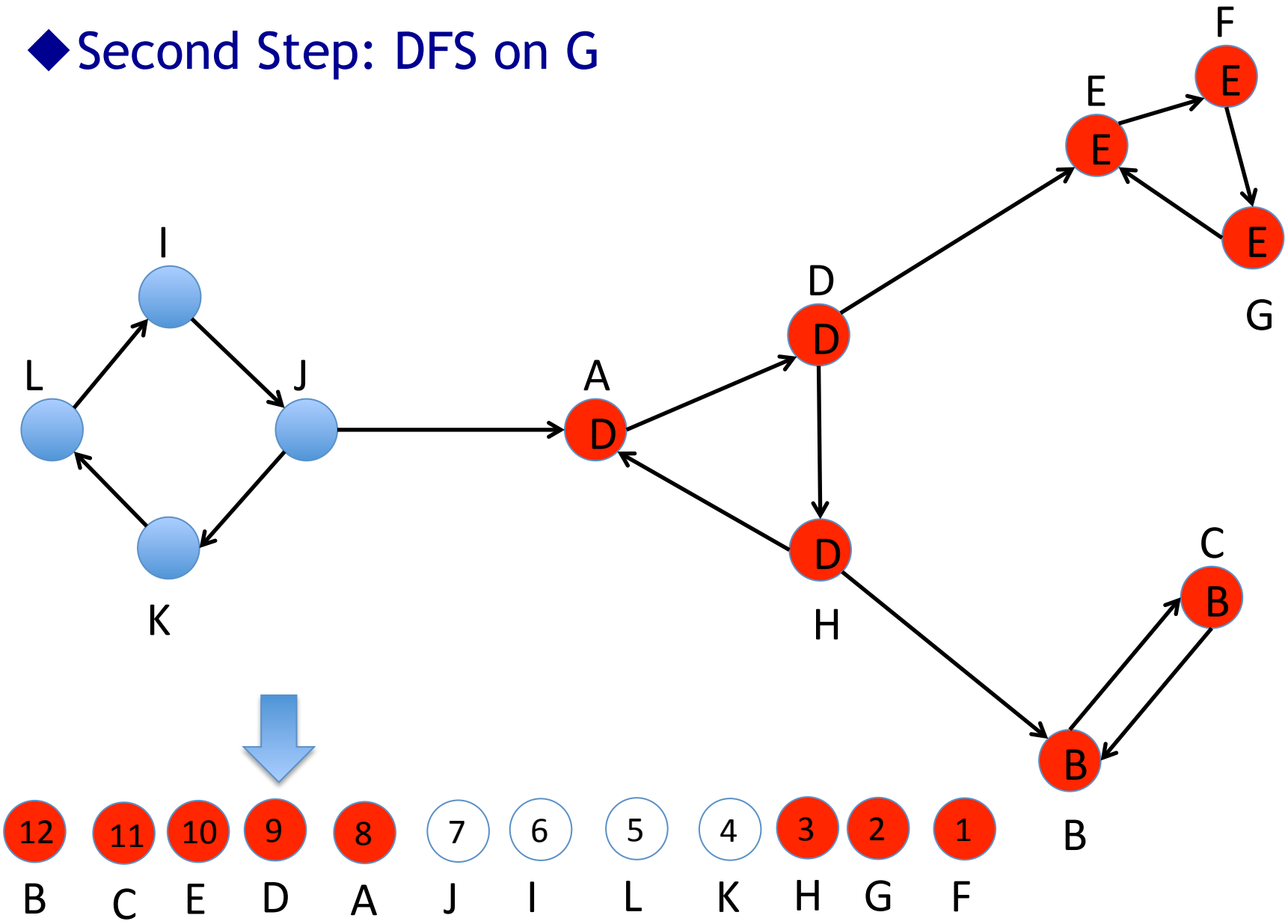

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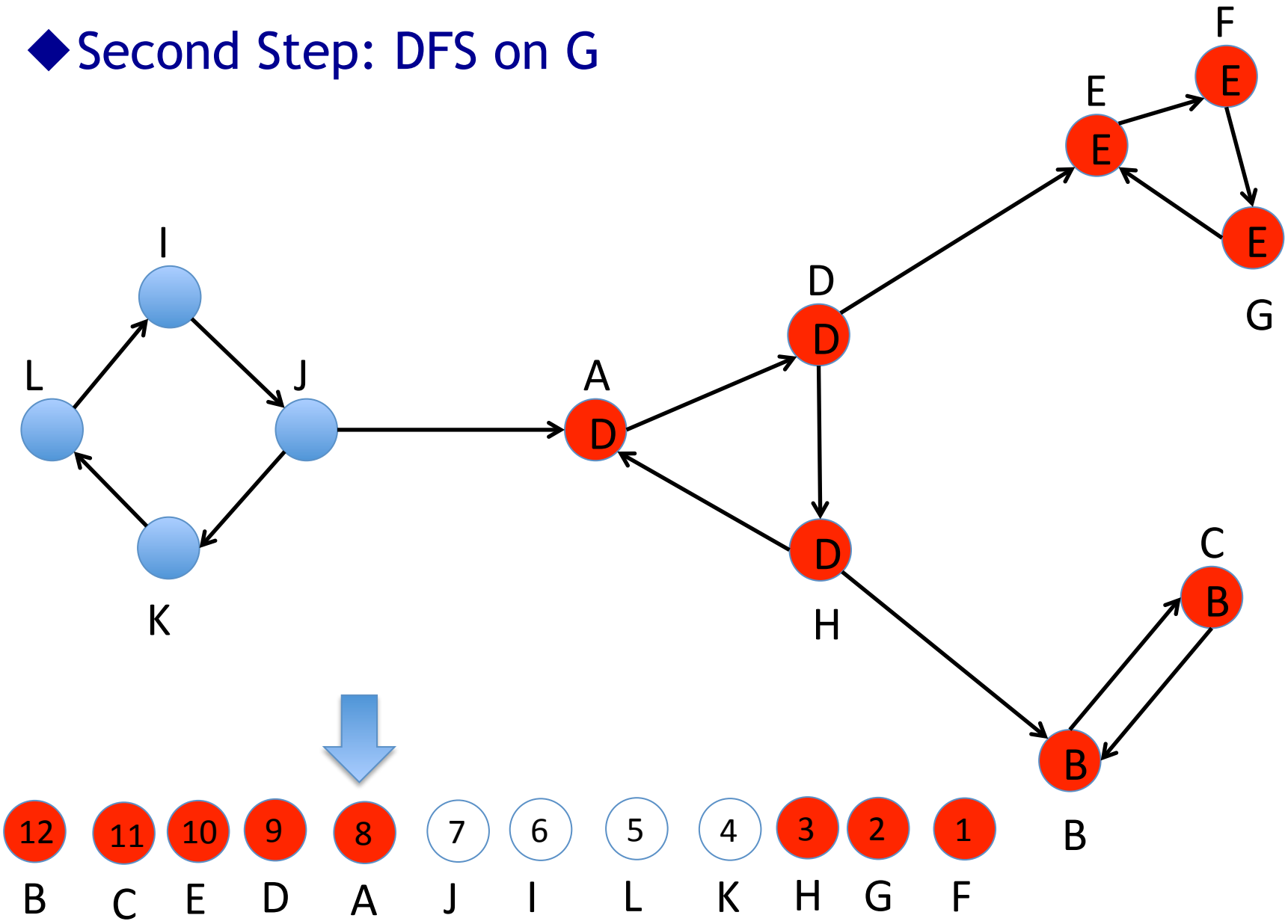

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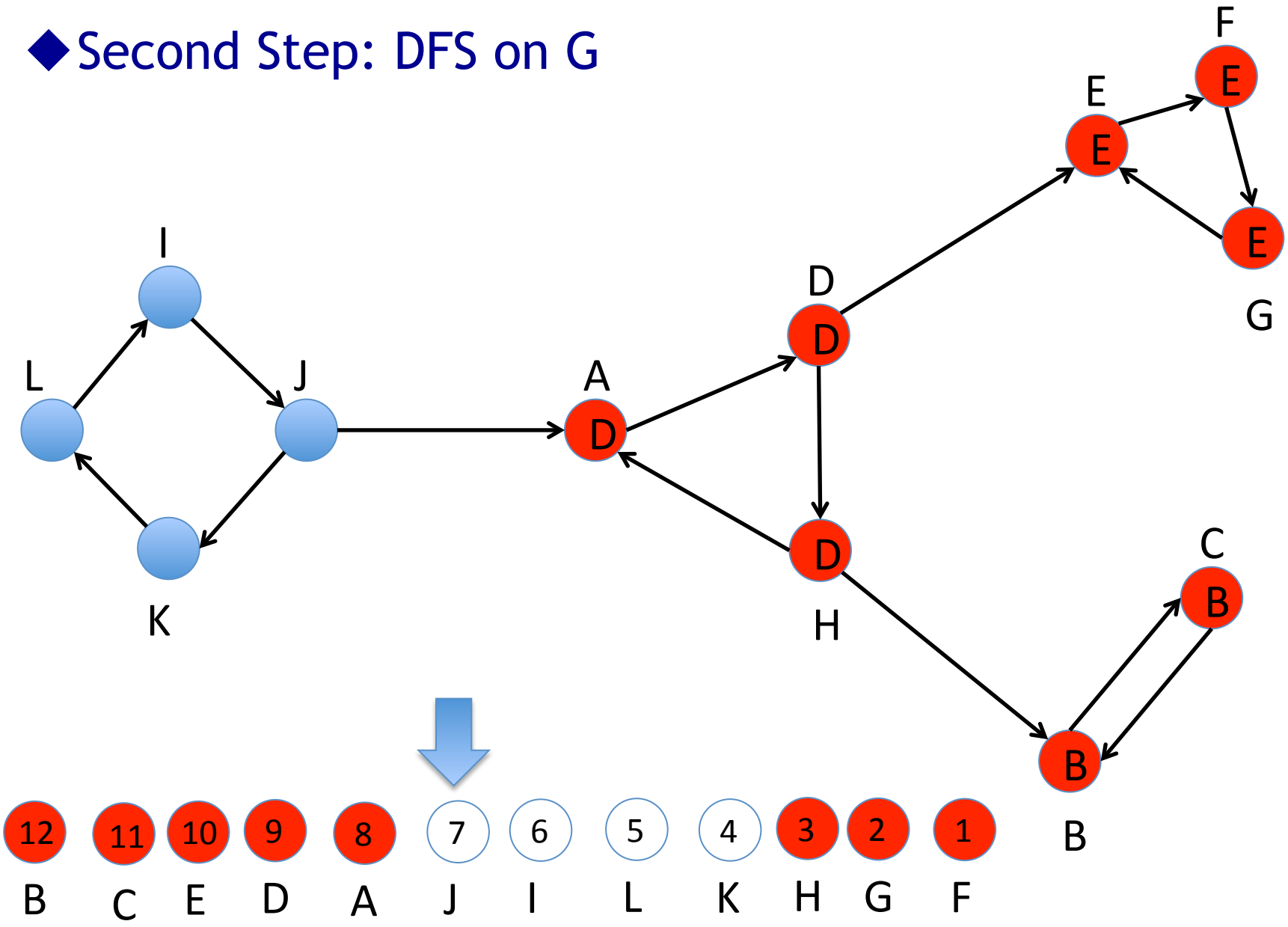

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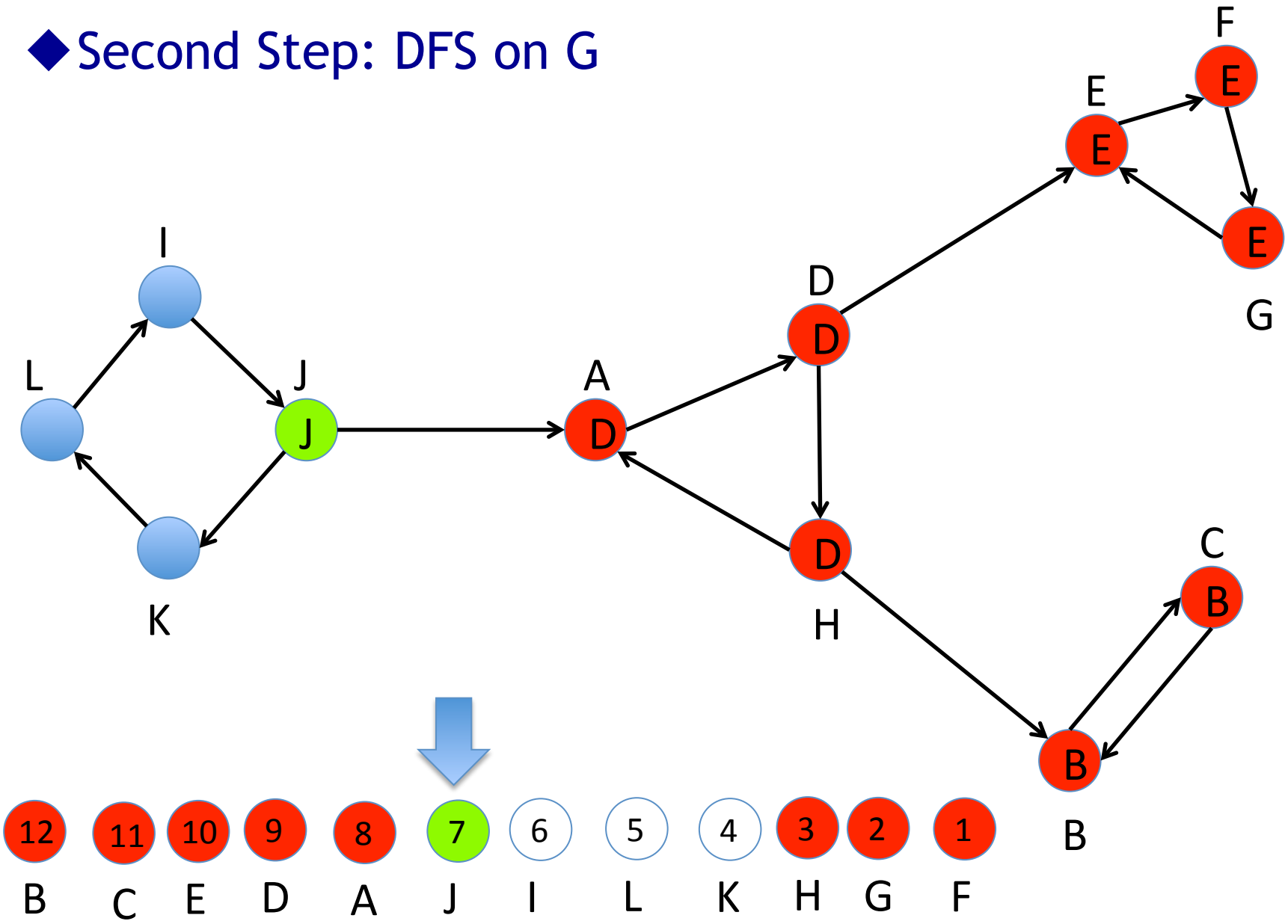

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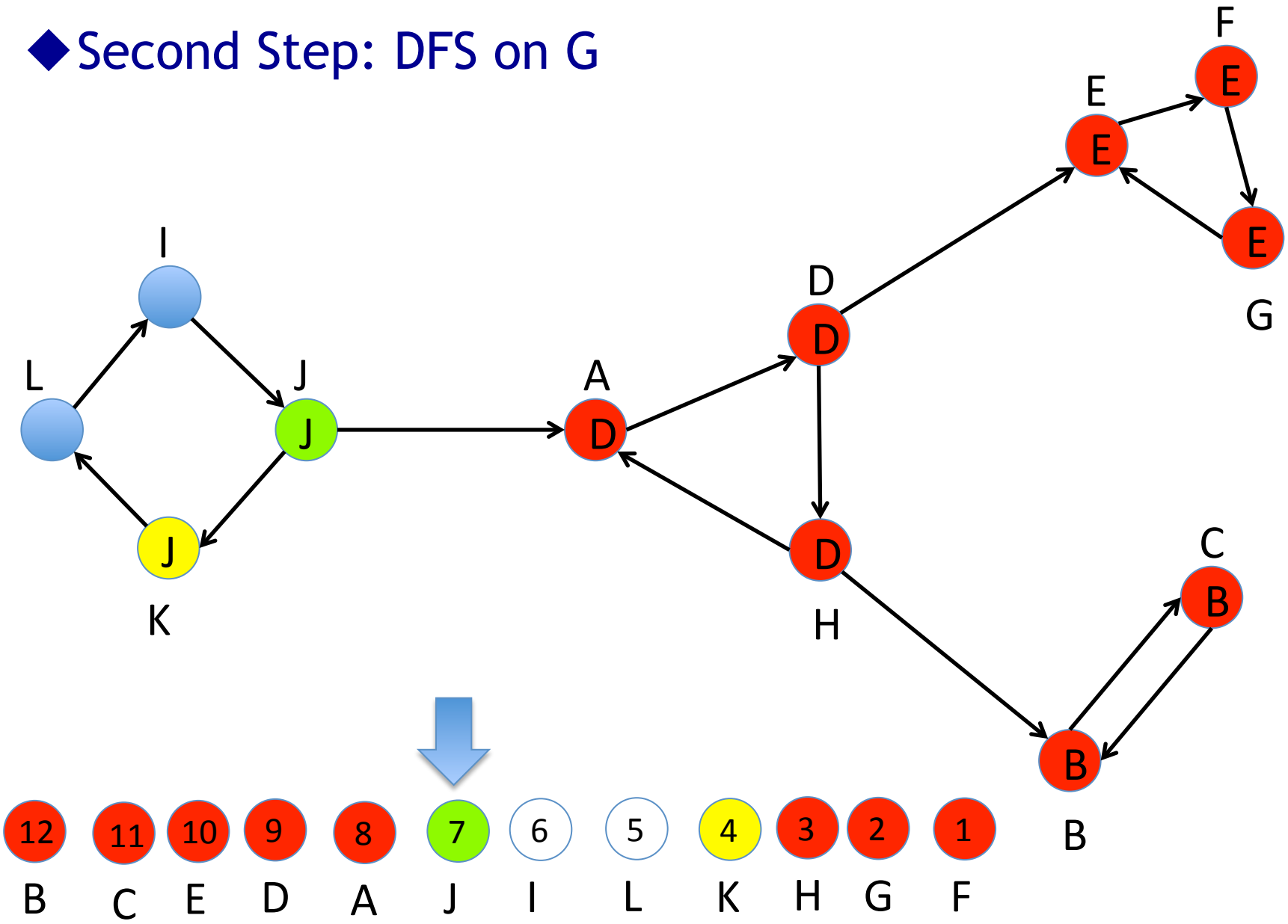

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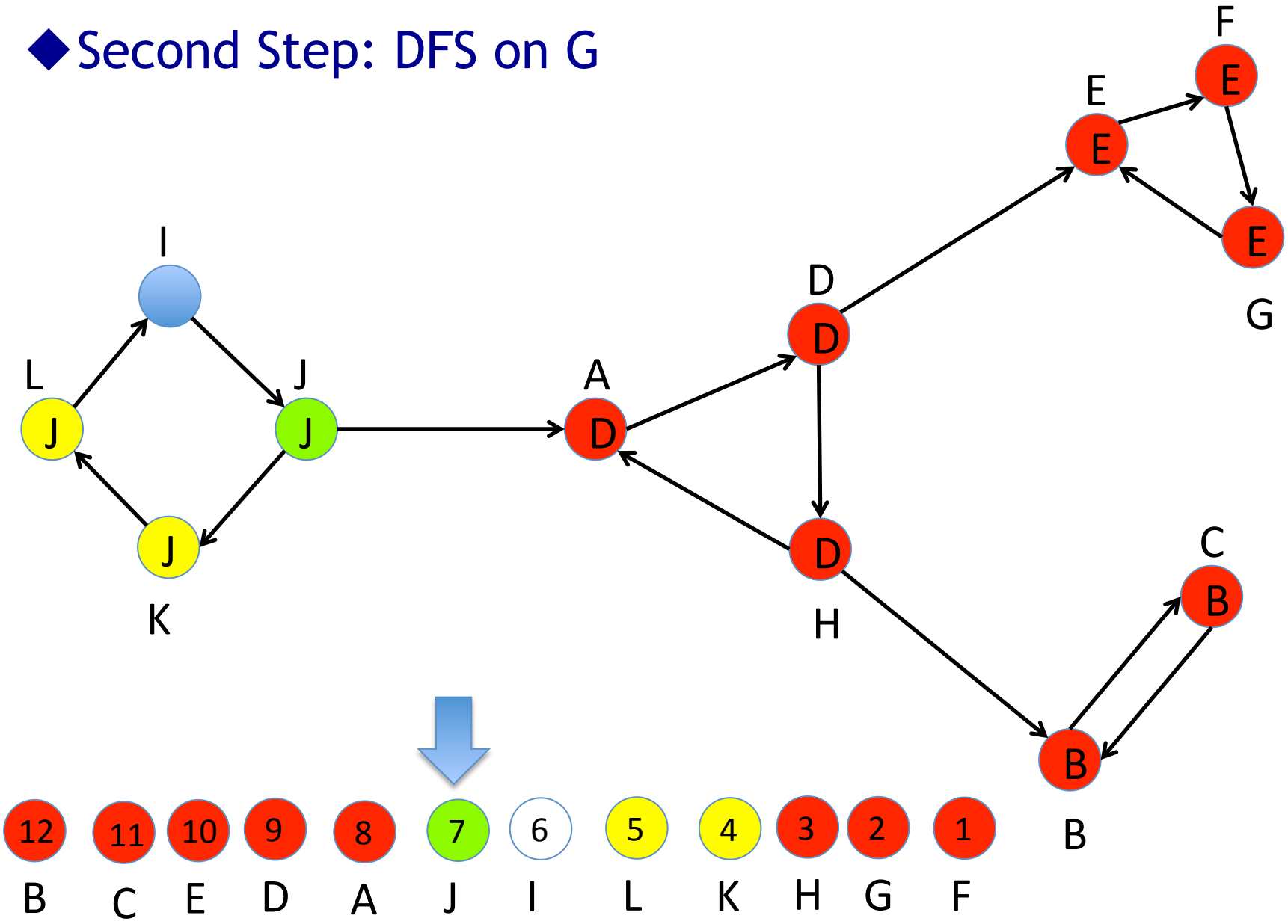

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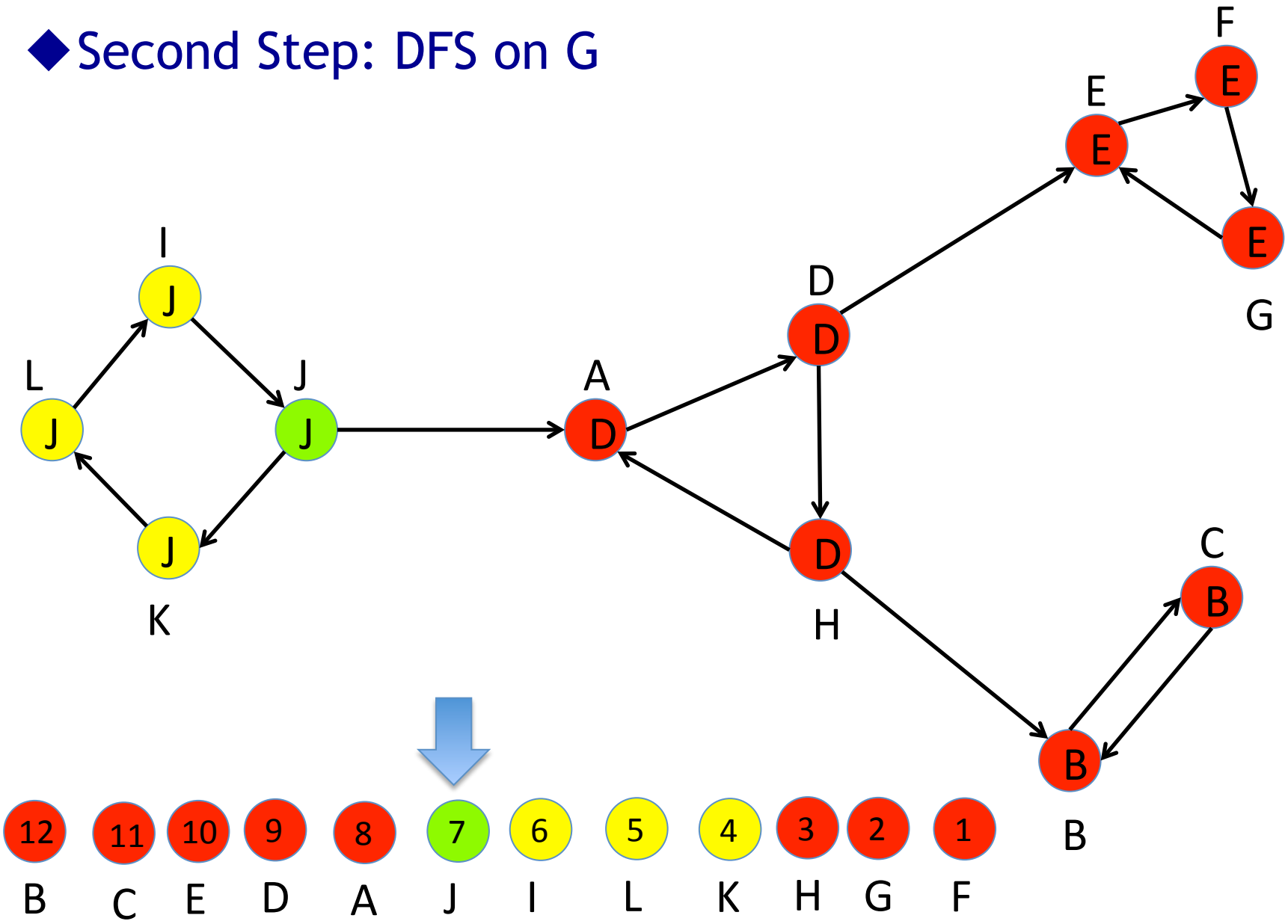

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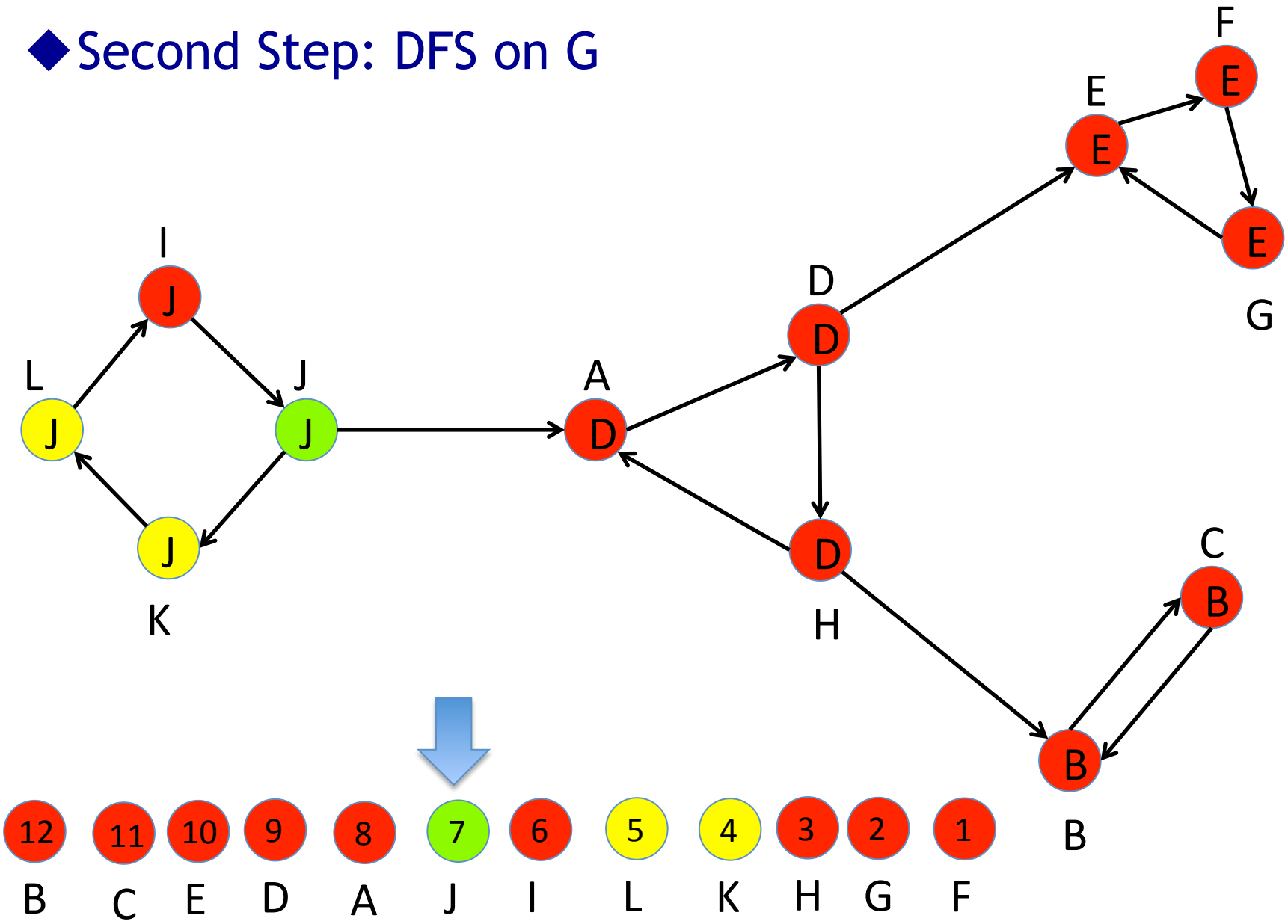

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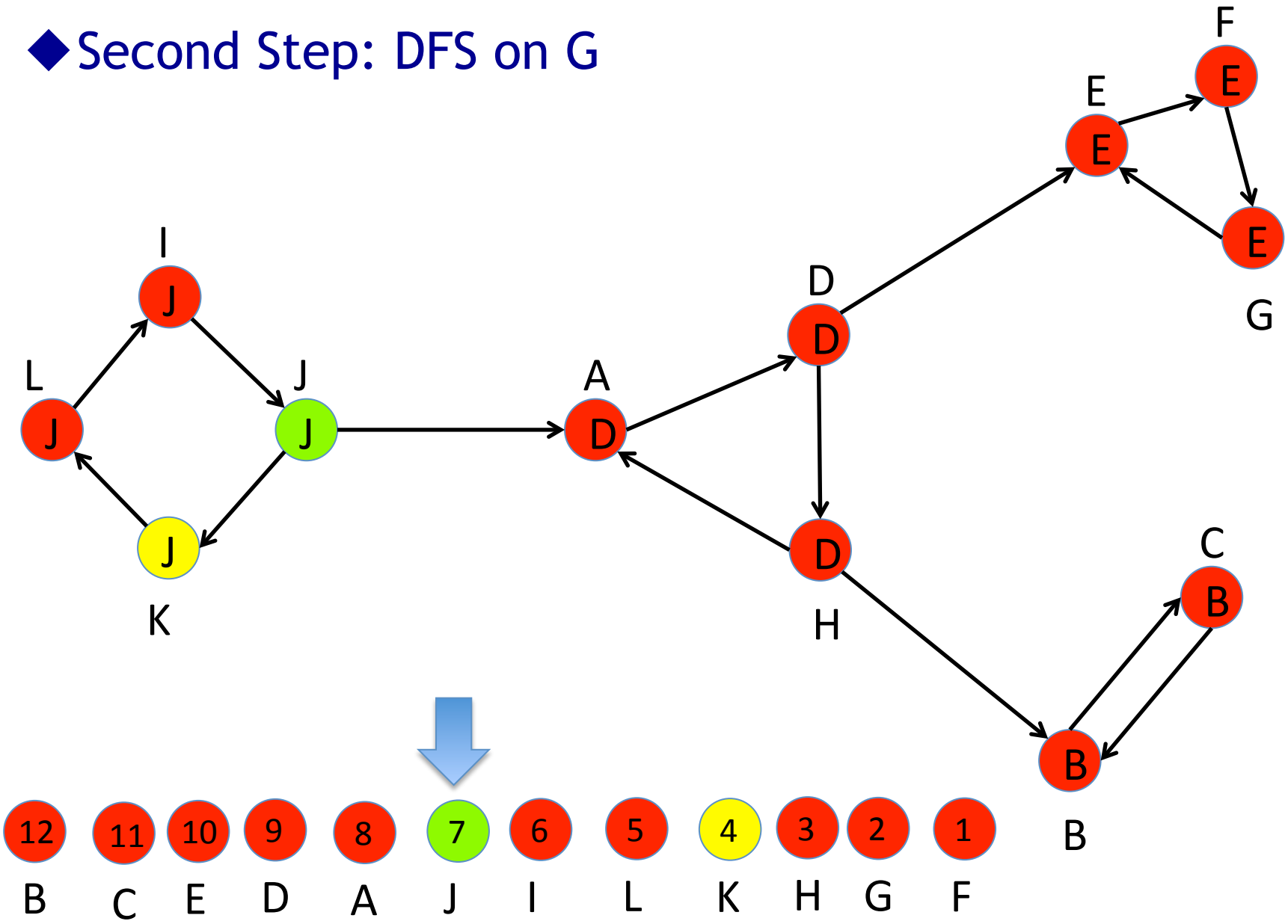

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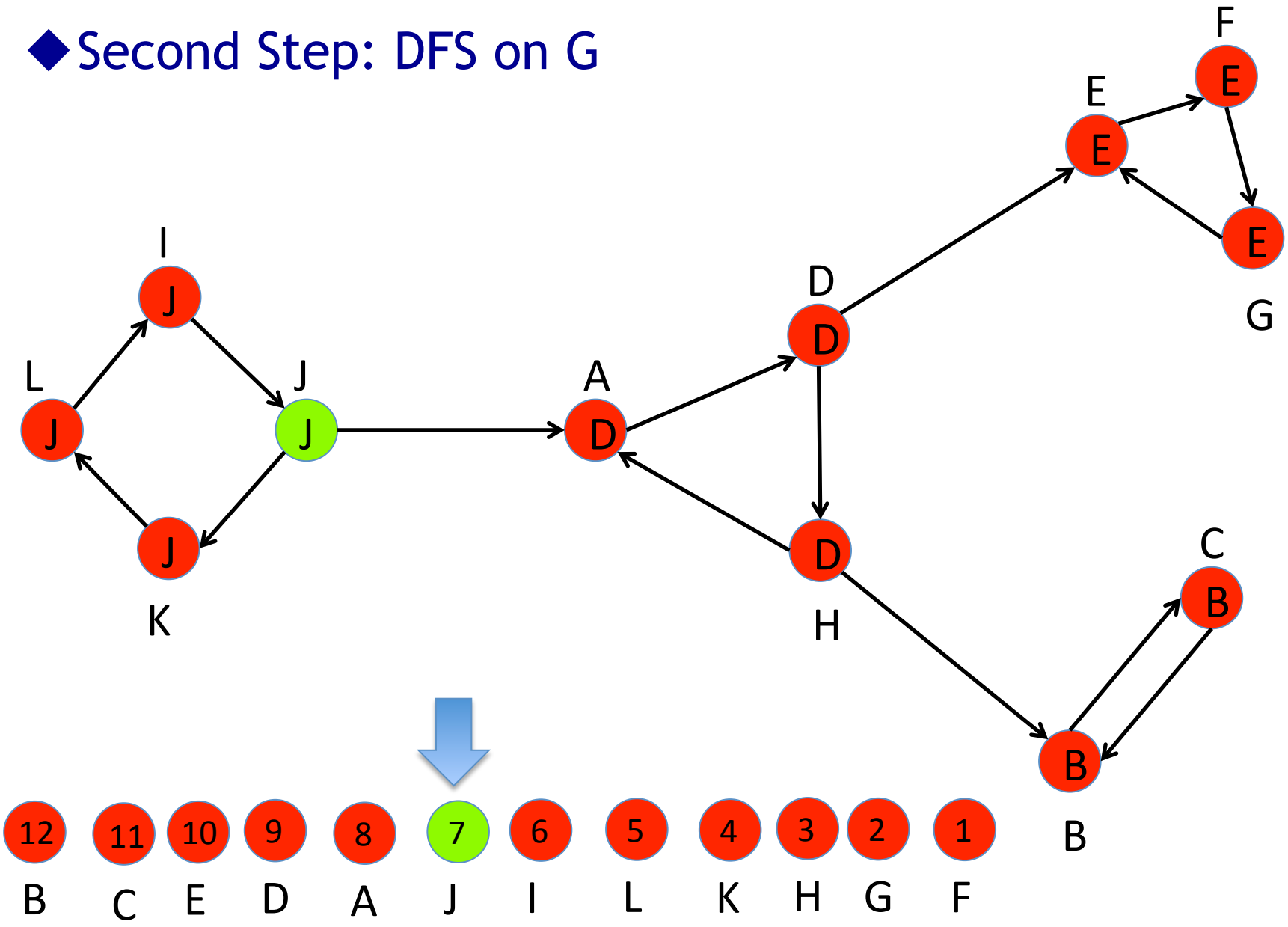

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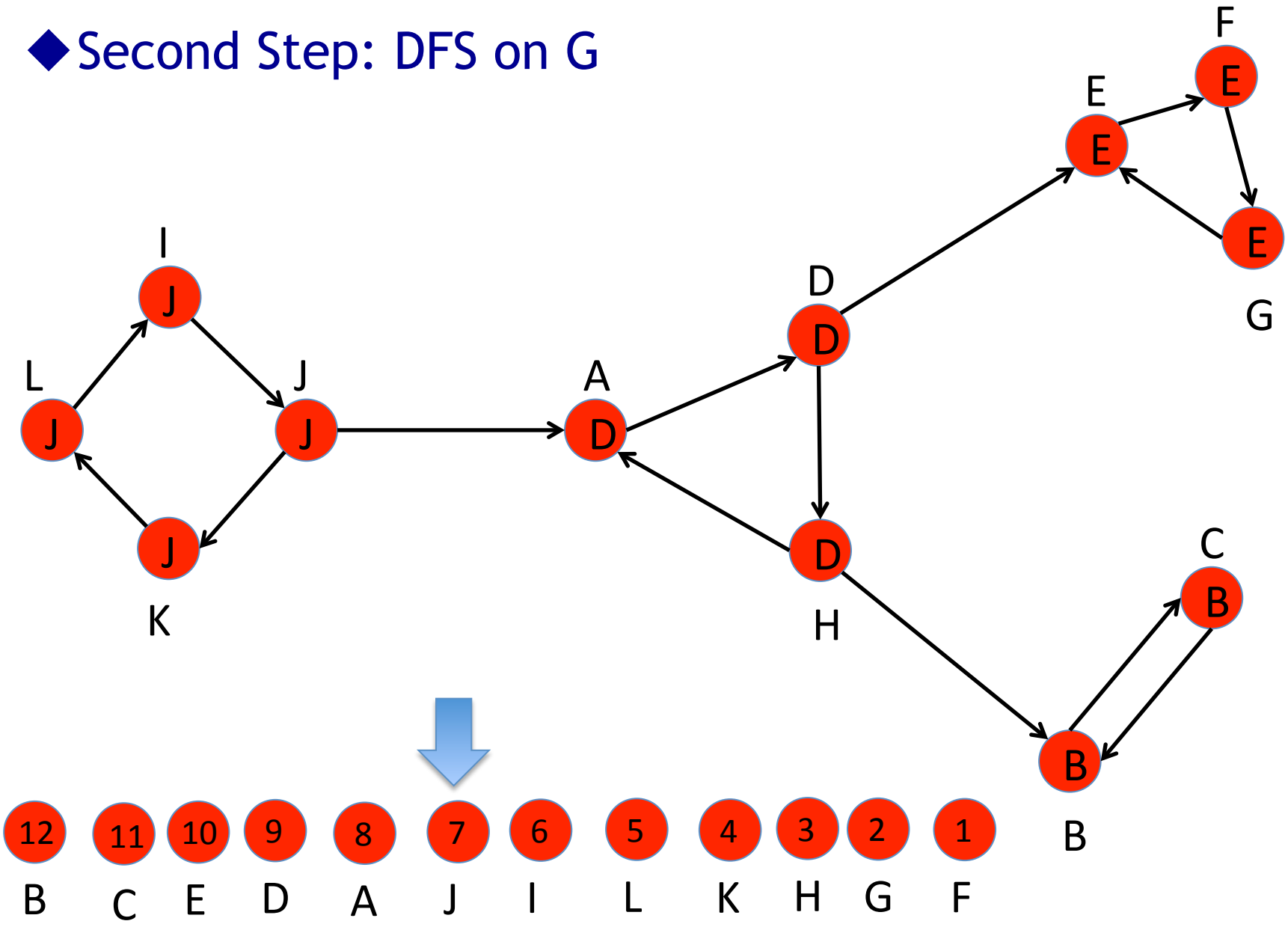

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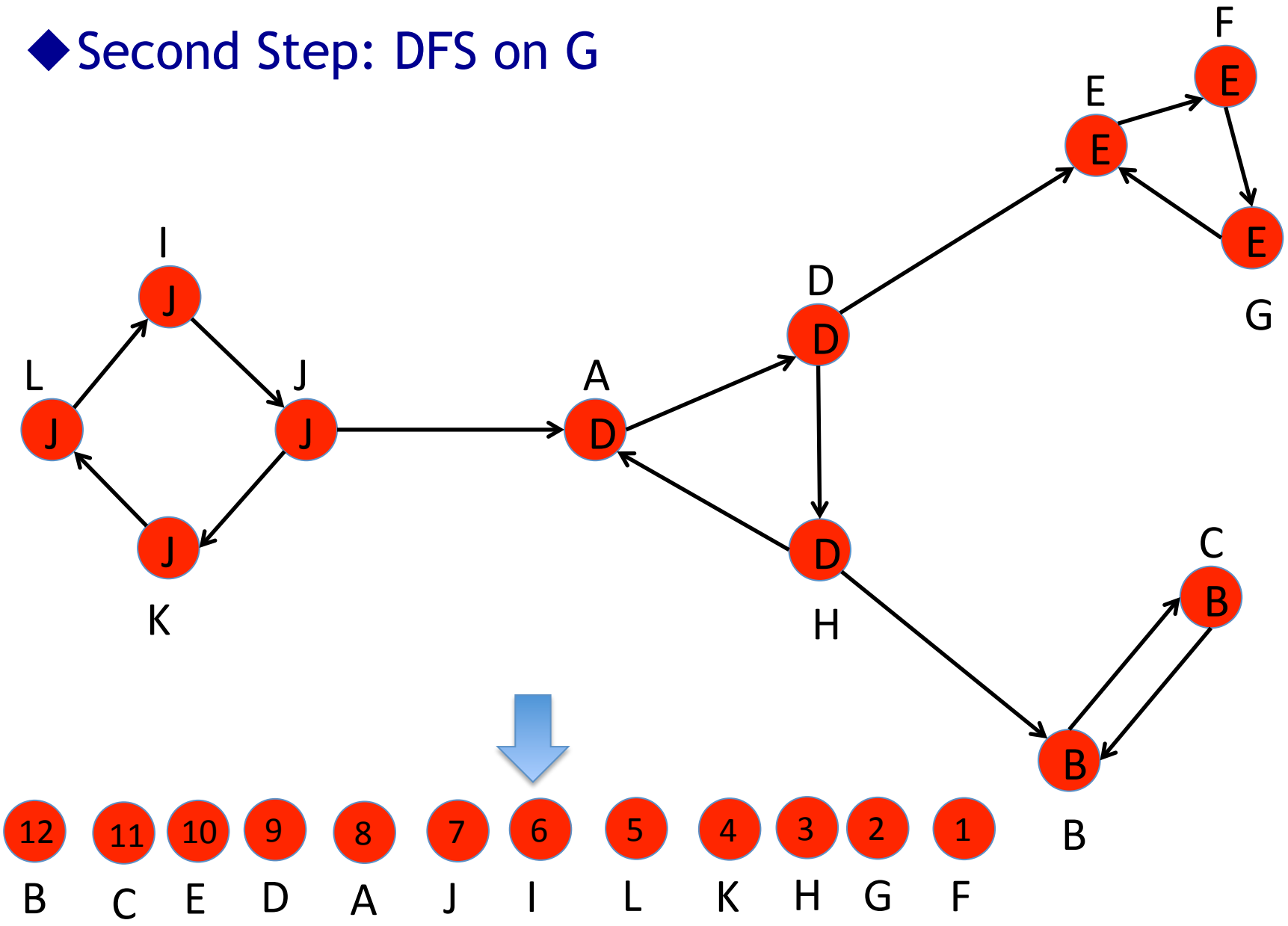

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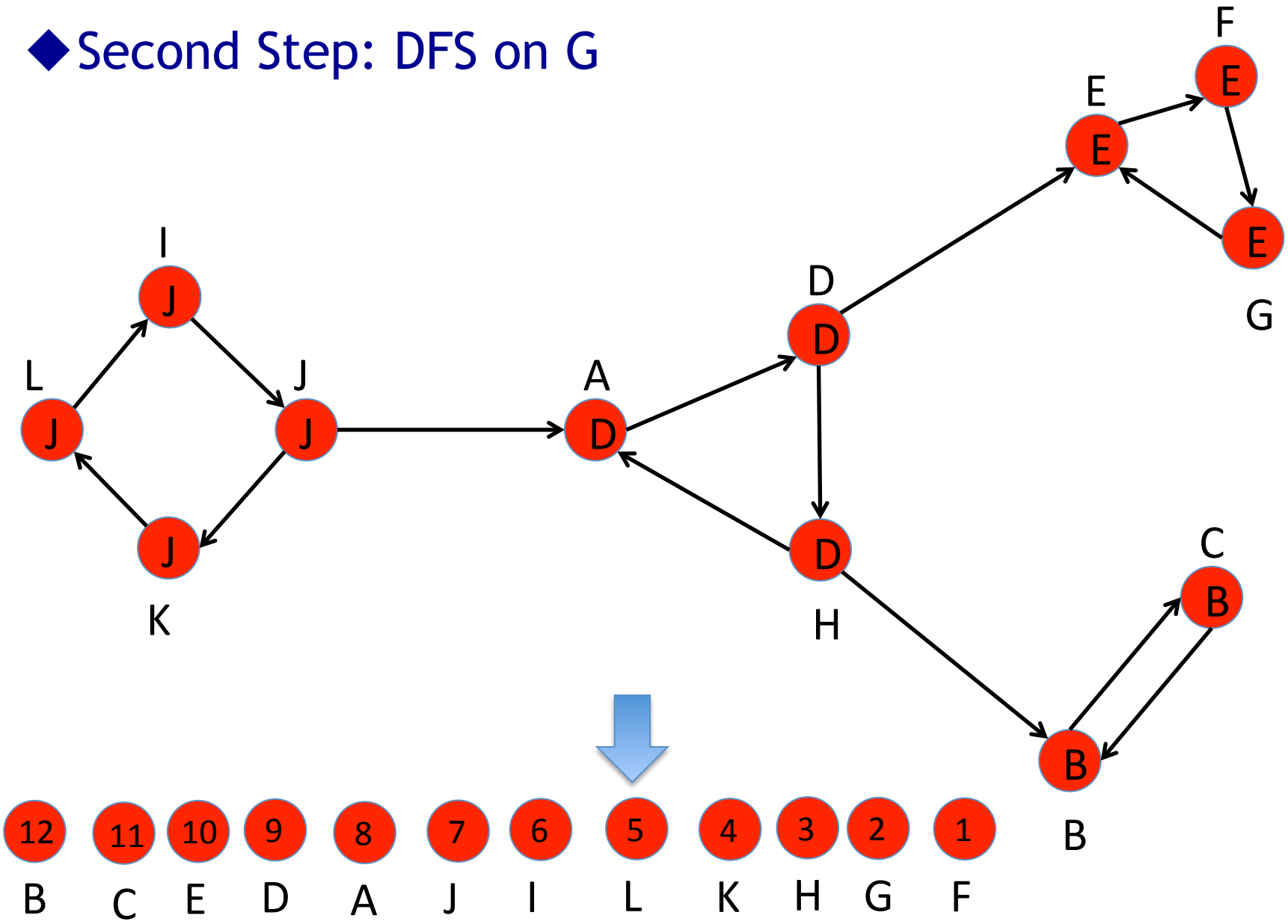

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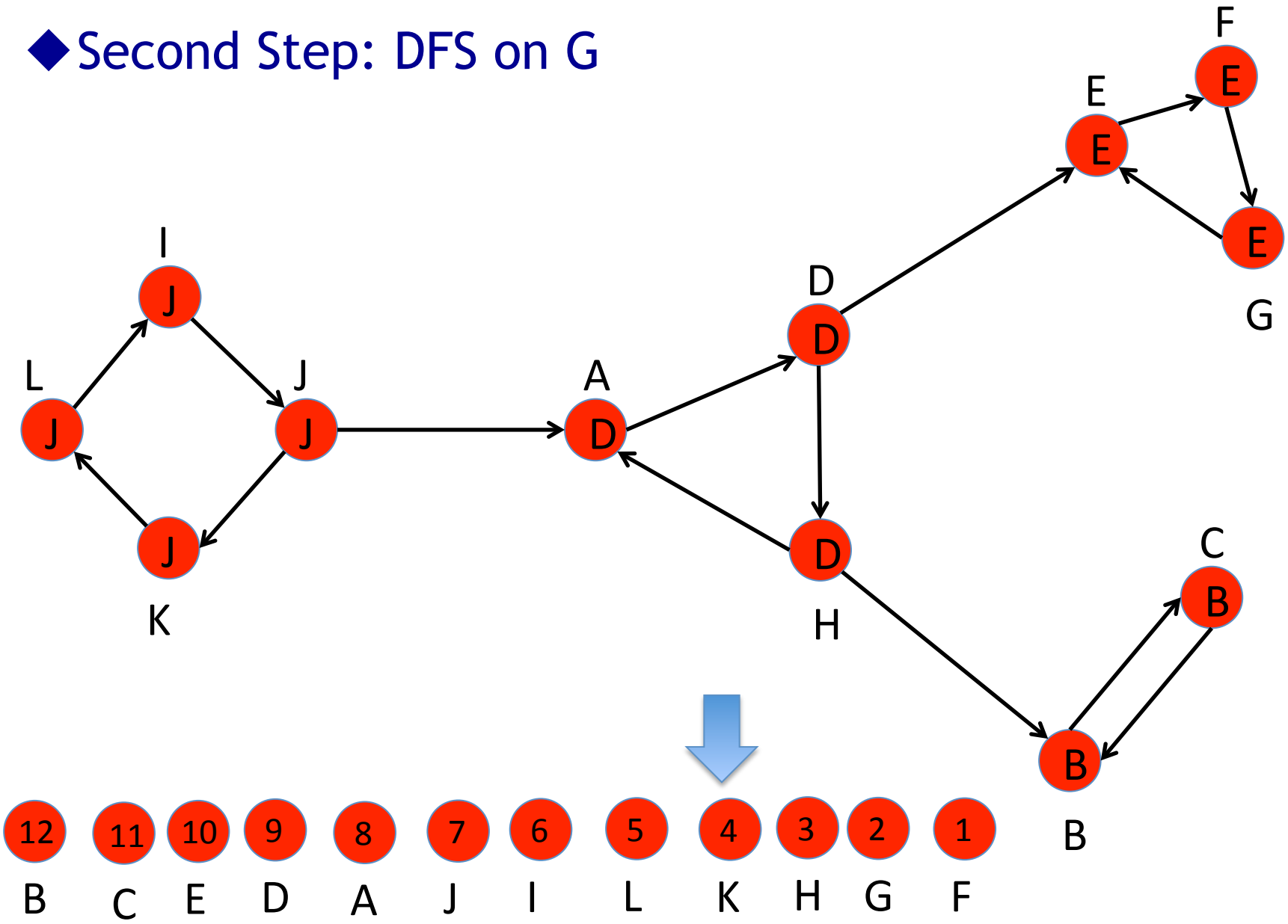

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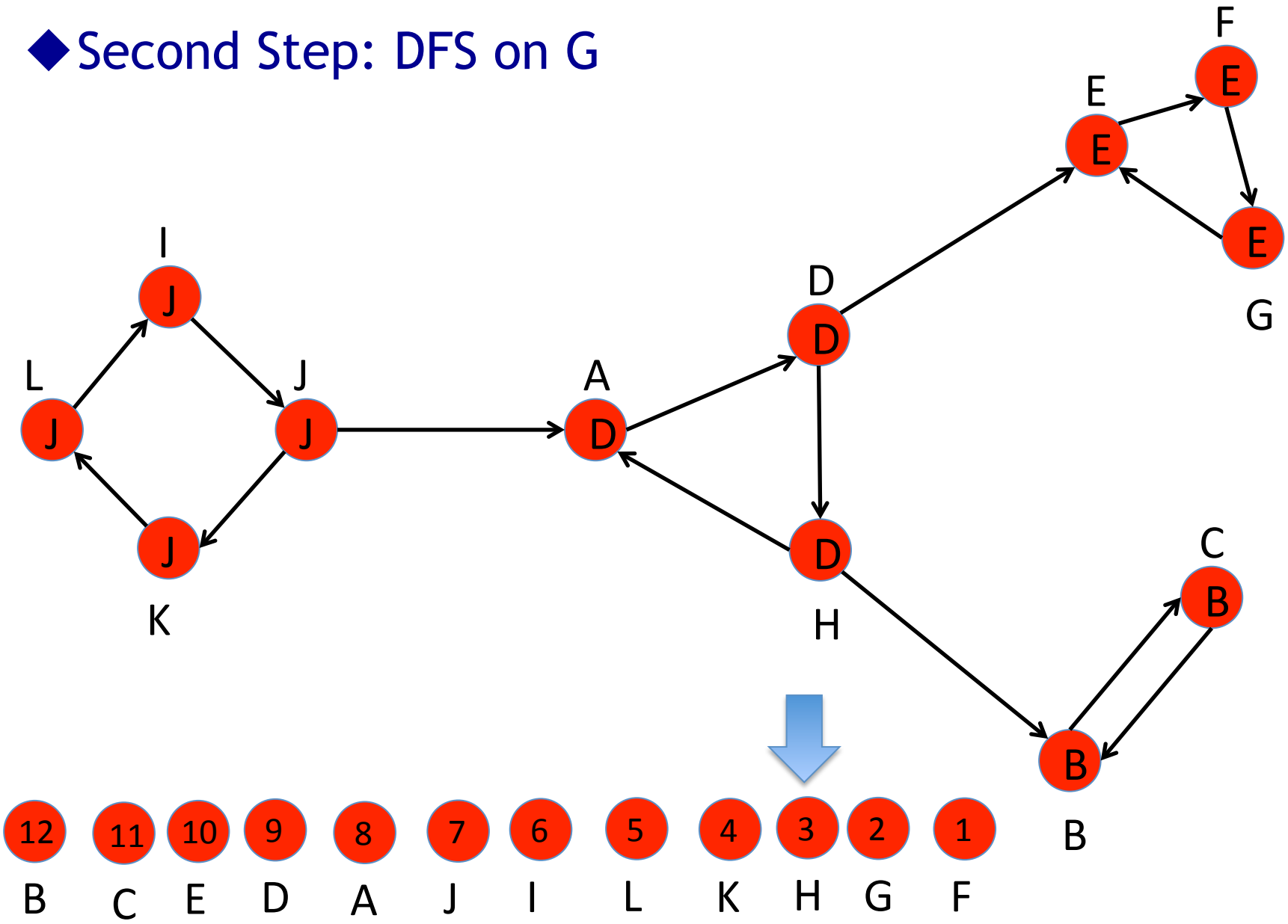

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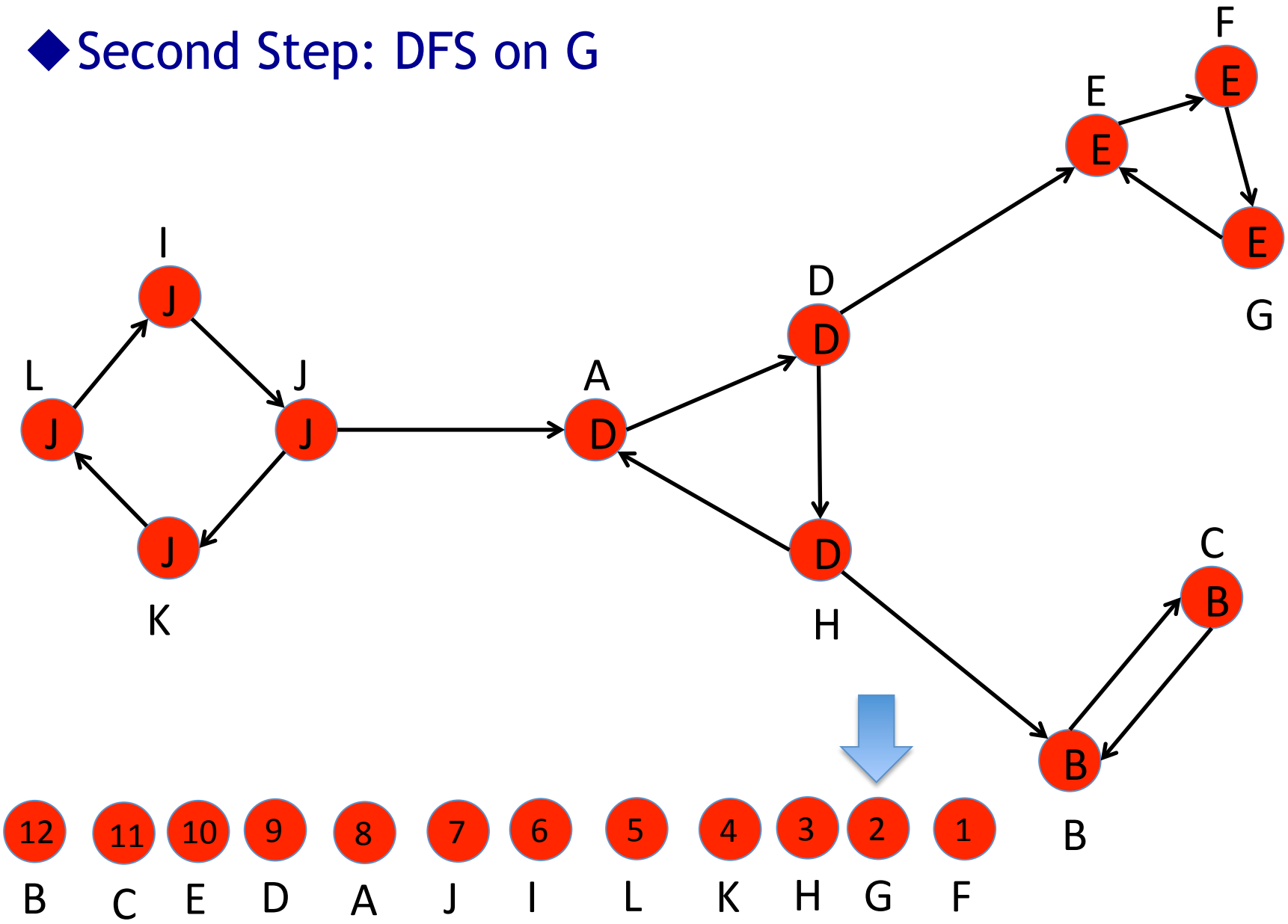

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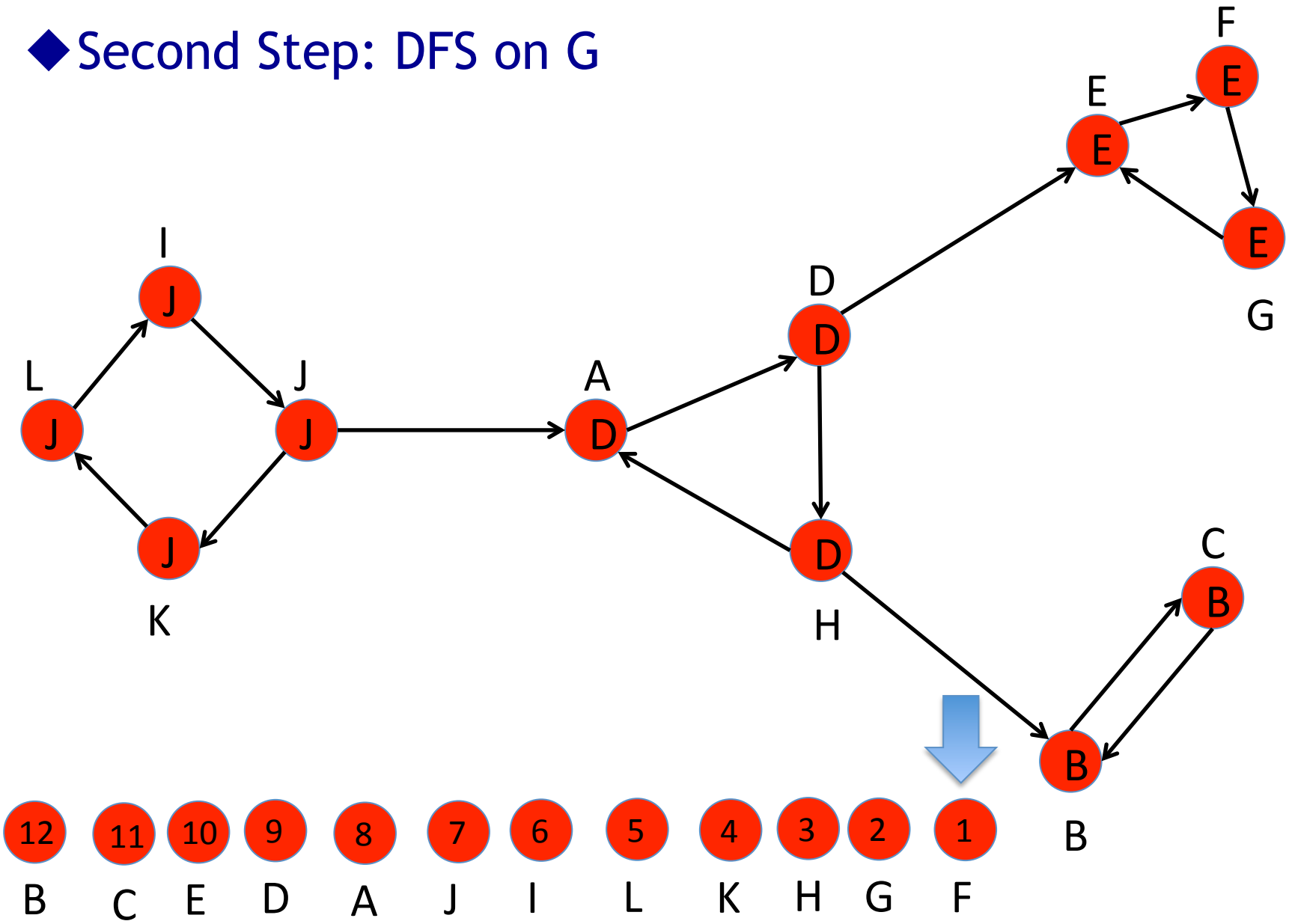


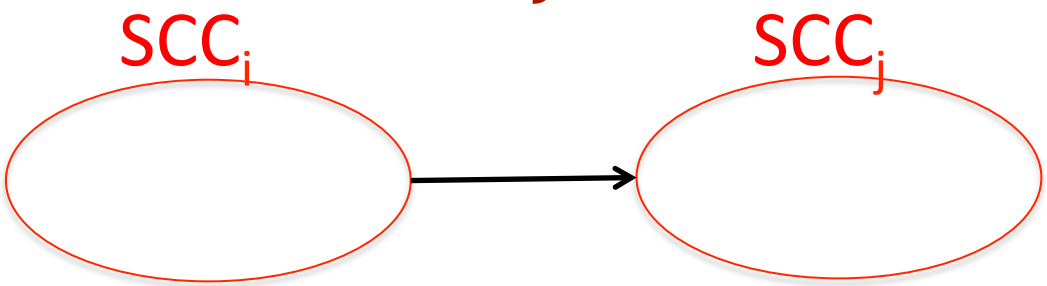
Kosaraju's Algorithm Analysis

- *Run-time: O(n + m)!*
- *Correctness Intuition:*

Recall Key Lemma: $SCC_i \rightarrow SCC_j$, then
$***f[SCC_i] > f[SCC_j]***$
Corallary: In G_{rev}, the highest f[v] will be in a source SCC => In G v will be in a sink SCC
⇒ A BFS/DFS from v will only identify v's SCC
⇒ Once v's SCC is "peeled-off" next highest f[v`] also has to be a sink by induction
⇒ A BFS/DFS from v` will only identify v`s SCC
⇒ Inductively finding all SCCs

Proof of Key Lemma:

In G^{SCC} if $SCC_i \rightarrow SCC_j$, then $f[SCC_i] > f[SCC_j]$



Let u be the first vertex visited in SCC_i or SCC_j

Again, break into 2 cases by whether u is in SCC_i or SCC_j

Case 1: u is in SCC_i. Then:

by extended key lemma 1 (EKL1) of Topological Sort

f[u] will be greater than any vertex in SCC_j

Case 2: u is in SCC_j. Then by (EKL1):

$f[u] > f[y]$ for any vertex in SCC_j

$f[u] < f[x]$ for any vertex in SCC_i

$\Rightarrow f[SCC_i] > f[SCC_j]$