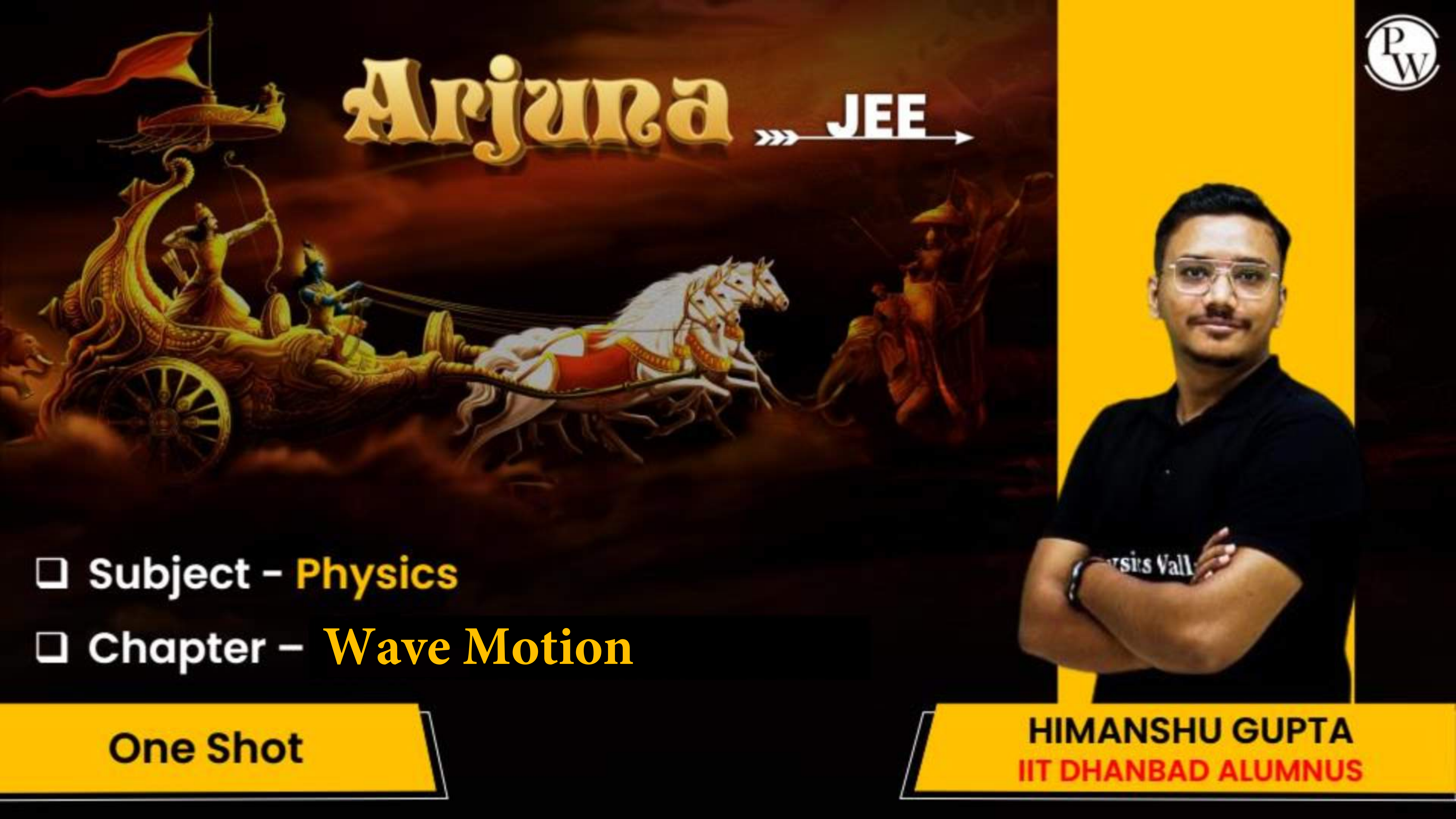

Arjuna
JEE
Subject - Physics
Chapter – Wave Motion
One Shot
HIMANSHU GUPTA
IIT DHANBAD ALUMNUS

Today's Targets

1. Equation of Travelling Wave
2. Wave Speed
3. Wave Interference
   - Wave travelling in same direcn
   - wave travelling in opp. direcn
4. Doppler's Effect

Wave

Disturbance which travel around space.

Energy

Information

Types of Waves

ON BASIS OF MEDIUM

Class-12th [EM waves]

Class-12th [Dual Nature]

Mechanical Waves

These waves require medium to travel.

Ex. Sound Wave, String wave.

Non-Mechanical Waves

These waves do not require any medium to travel.

Ex Light, EM Waves.

Matter Wave

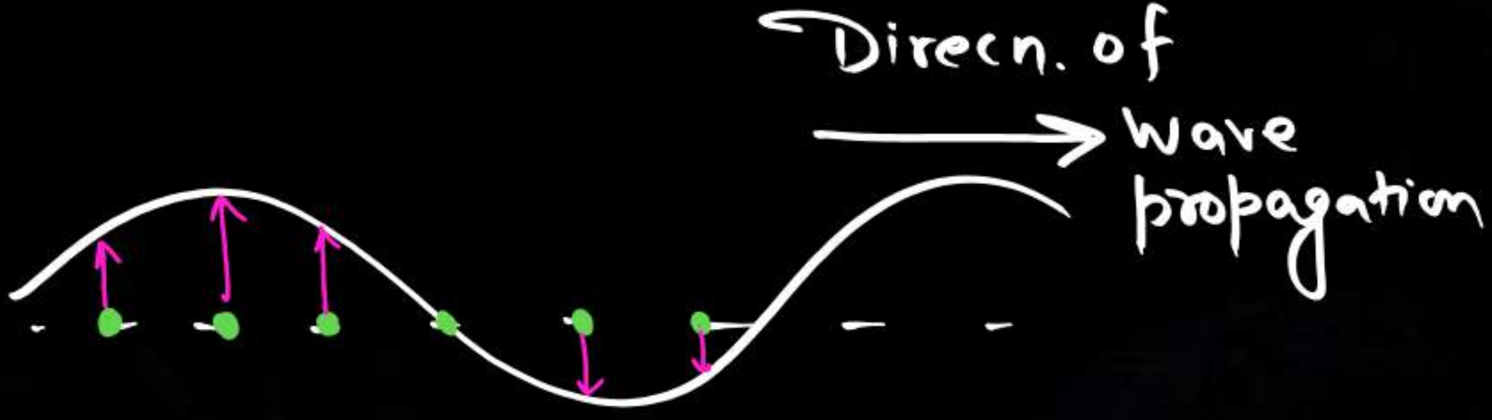
It is the wave associated with wave nature of particles.

Ex Electron wave

Types of Waves

ON BASIS OF OSCILLATION OF PARTICLES

Transverse Wave



Motion of particles is ⊥ to the direcn. of wave propagation.

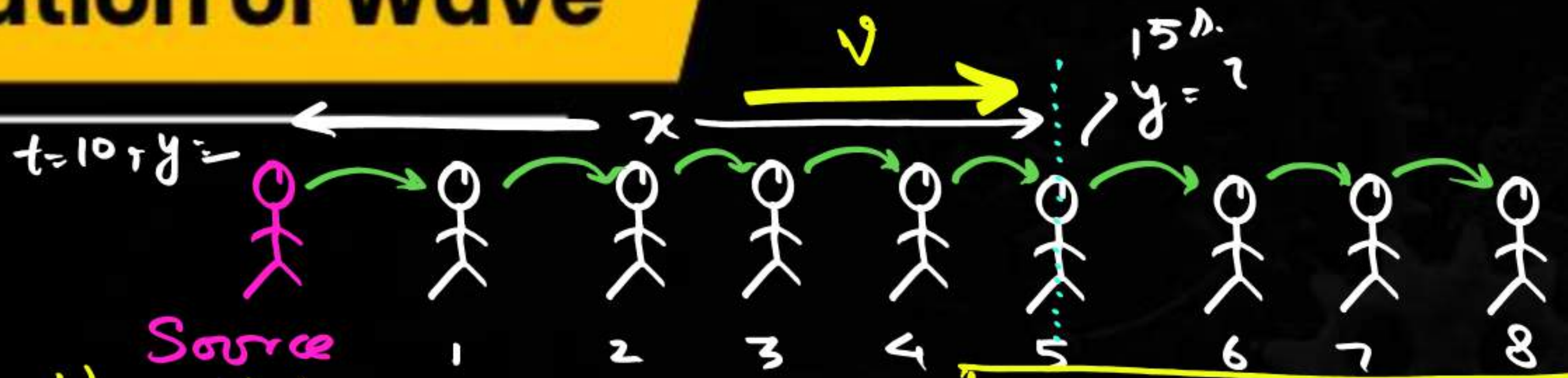
Ex. EM wave, string wave.

Longitudnal Wave

Motion of particles is along the direcn. of wave propagation

Ex Sound wave.

Equation of Wave



known $\rightarrow y(0,t) = f(t)$ $\quad \boxed{y(x,t) = y\left(0, t - \frac{x}{v}\right)}$

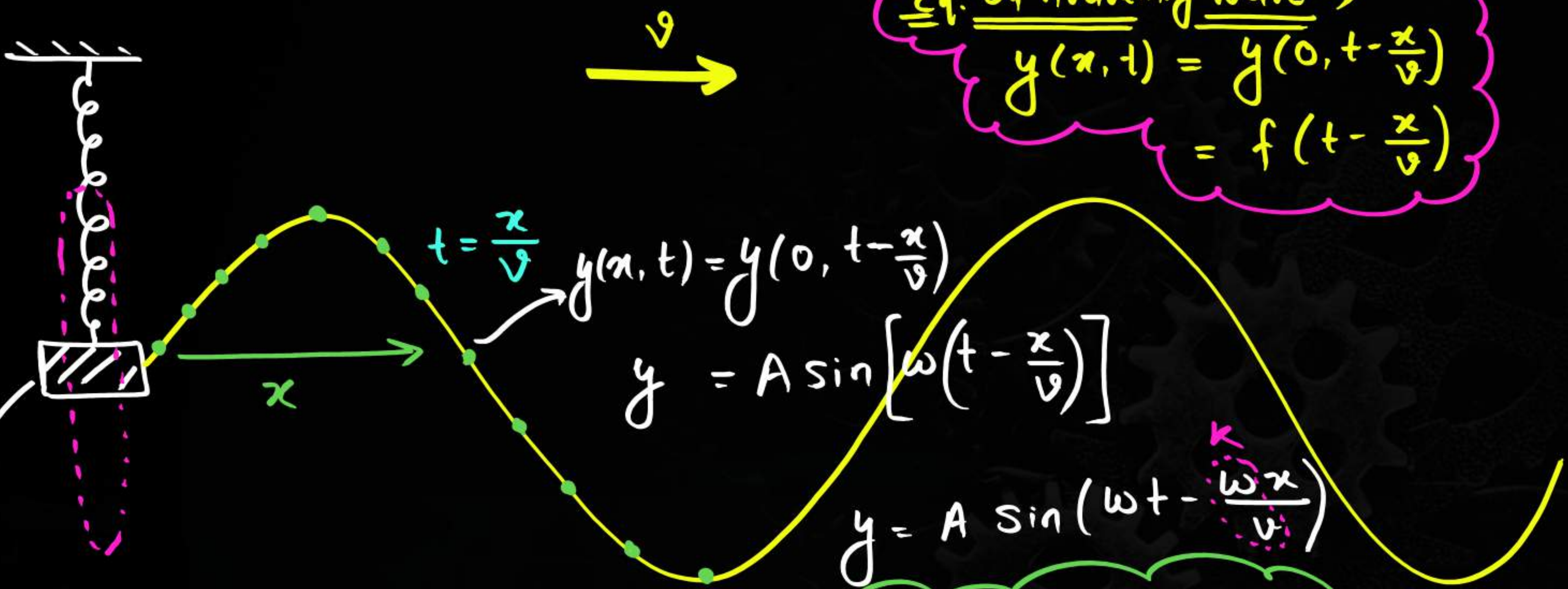
$$t = \frac{x}{v}$$

$$y(x,t) = f\left(t - \frac{x}{v}\right)$$

- Speed of wave propagation is decided by medium
- Shape & frequency of wave is decided by Source.

Eq. of travelling wave →

$$y(x,t) = y\left(0, t - \frac{x}{v}\right)$$
$$= f\left(t - \frac{x}{v}\right)$$



$$t = \frac{x}{v} \rightarrow y(x,t) = y\left(0, t - \frac{x}{v}\right)$$

$$y = A \sin\left[\omega\left(t - \frac{x}{v}\right)\right]$$

$$y = A \sin\left(\omega t - \frac{\omega x}{v}\right)$$

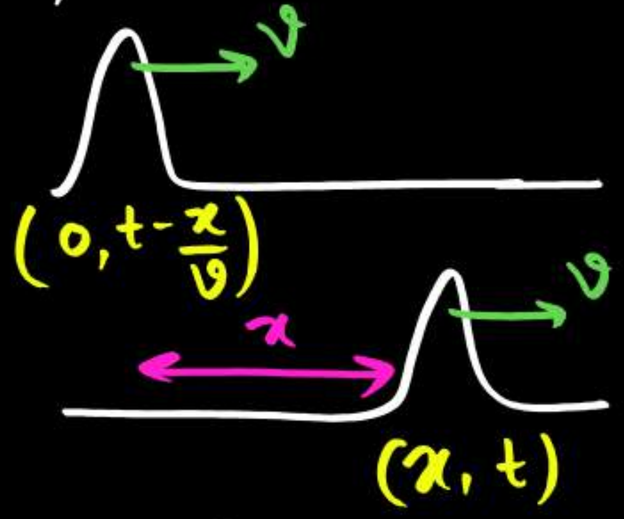
k

$$y(0,t) = A \sin(\omega t)$$

Eq. of a travelling sin wave →

$$\boxed{y = A \sin(\omega t - kx)}$$

known
↑
$f(t)$



Eq. of travelling wave towards +ve x-axis →

$$y(x,t) = y\left(0, t-\frac{x}{v}\right) = f\left(0, t-\frac{x}{v}\right)$$

$$y(x,t) = y(x-vt, 0) = f(x-vt, 0)$$

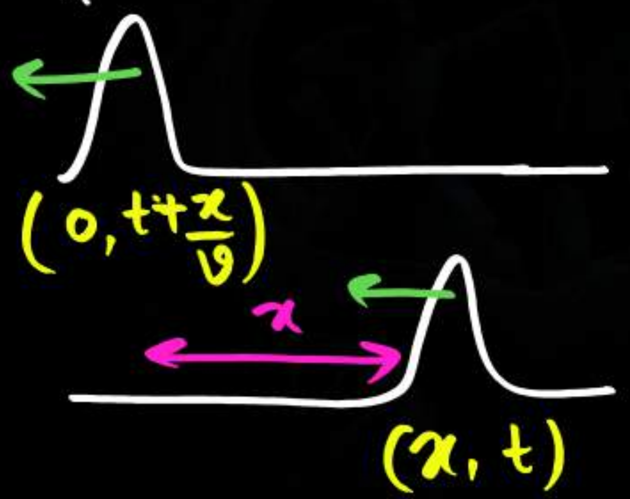
Sine Wave –

$$y(x,t) = A\sin\left[\omega\left(t-\frac{x}{v}\right)\right]$$

$$y(x,t) = A\sin(\omega t - kx)$$

where $k = \frac{\omega}{v}$

known
↑
$f(t)$



Eq. of travelling wave towards −ve x-axis →

$$y(x,t) = y\left(0, t+\frac{x}{v}\right) = f\left(t+\frac{x}{v}\right)$$

$$y(x,t) = y(x+vt, 0) = f(x+vt, 0)$$

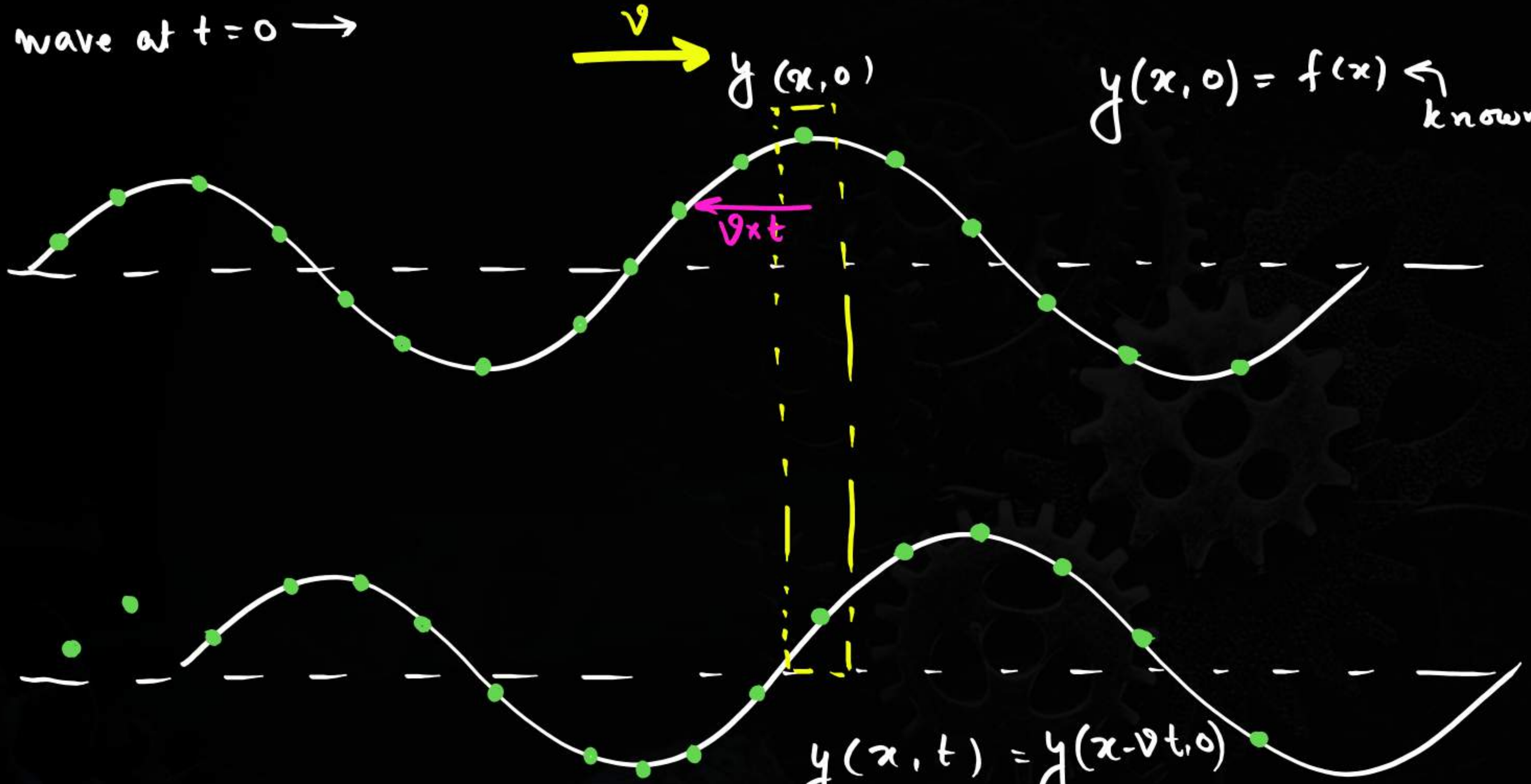
$$y(x,t) = A\sin\left[\omega\left(t+\frac{x}{v}\right)\right]$$

$$y(x,t) = A\sin(\omega t + kx)$$

where $k = \frac{\omega}{v}$

Photo of wave at $t = 0 \rightarrow$

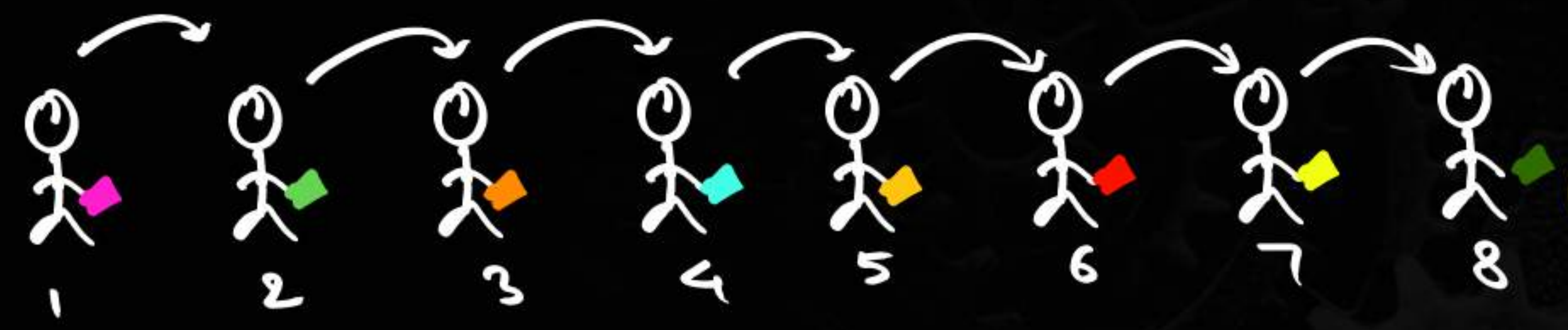
$y(x,0) = f(x)$ ← known



$y(x,t) = y(x - vt, 0)$

Photo at $t = 0 \rightarrow$

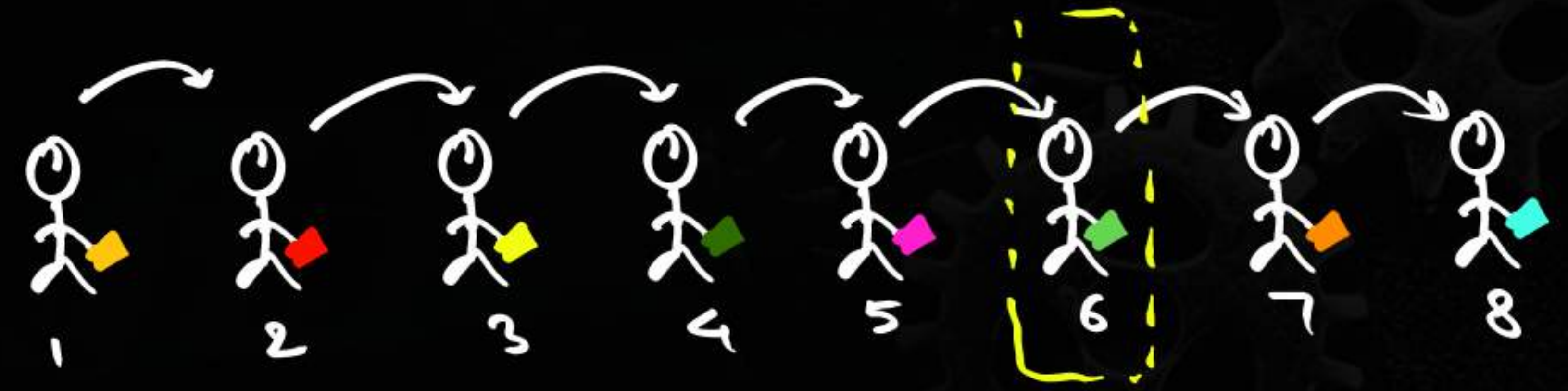
$v = 1$ pillow/sec



$(x - v \times t)$

$t = 4s.$

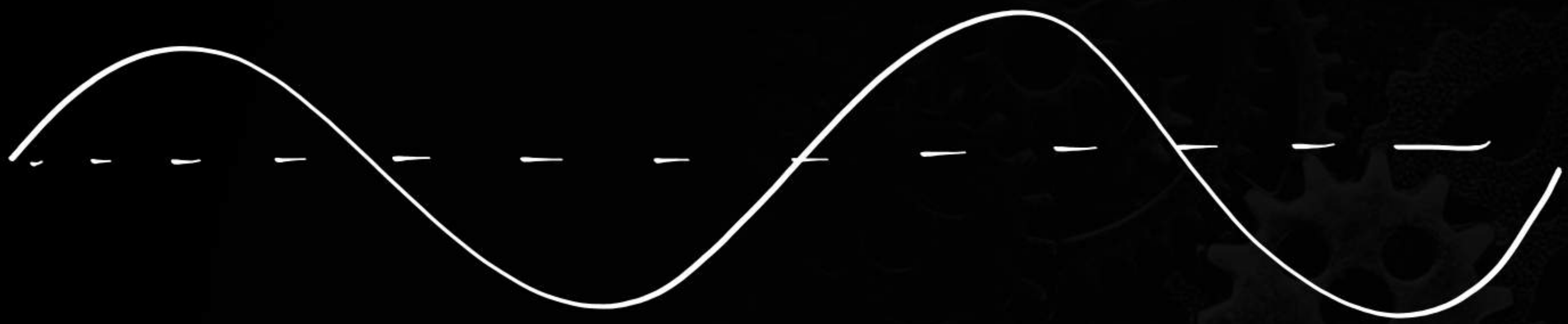
$t = 4s.$



x

$f(x, t) = f(x - vt, 0)$

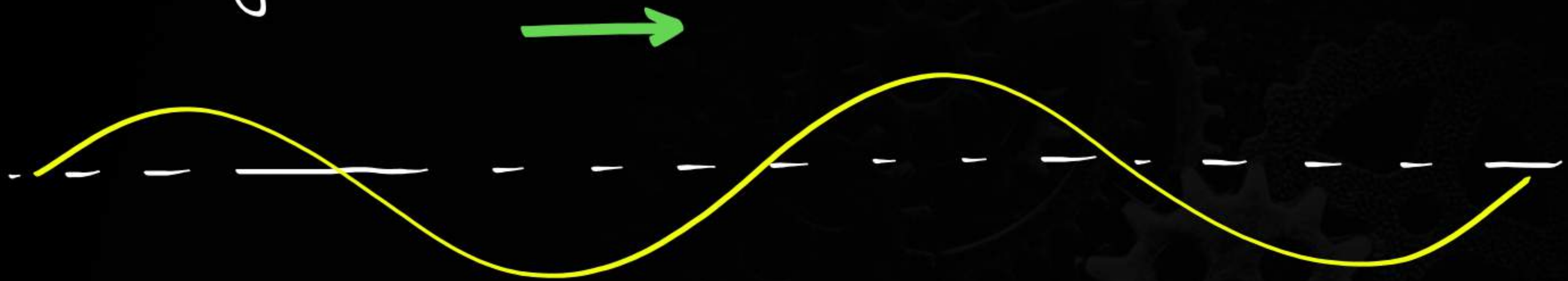
At $t=0 \rightarrow y = A\sin(kx)$



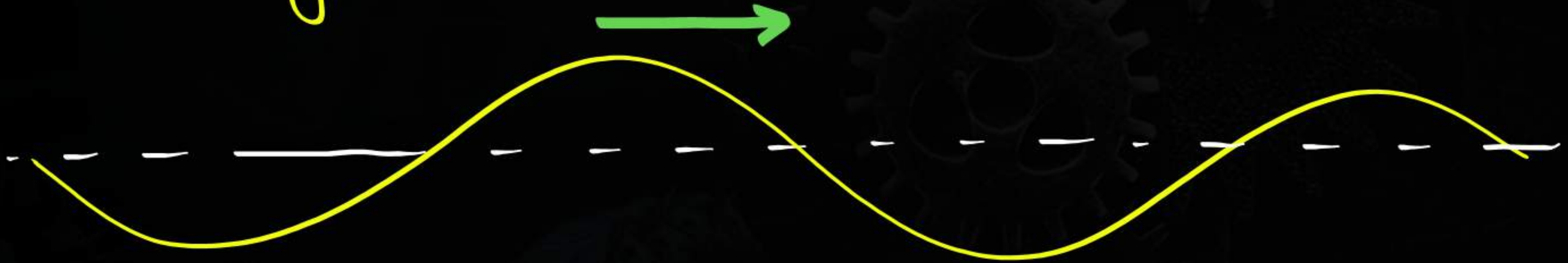
At $t \longrightarrow$

$$y(x,t) = y(x - vt, 0)$$
$$= A\sin[k(x - vt)]$$
$$y = A\sin(kx - \omega t)$$

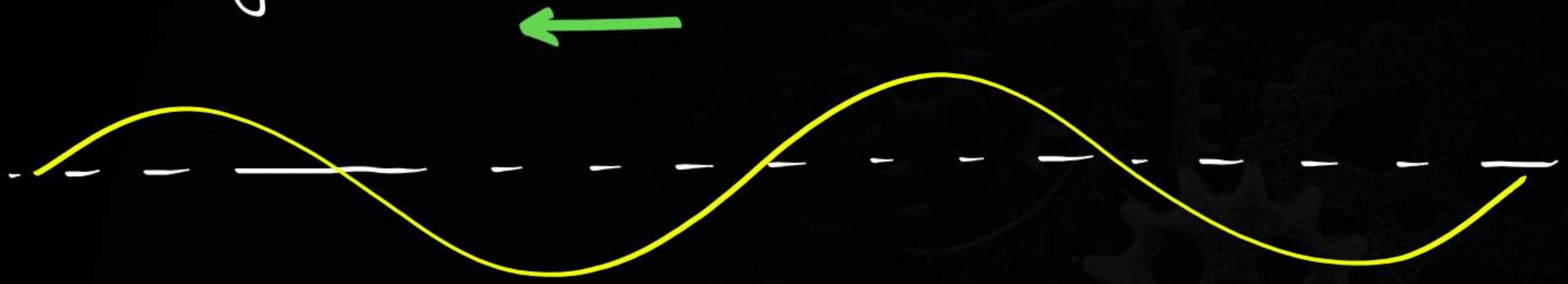
$$y = A\sin(kx - \omega t)$$



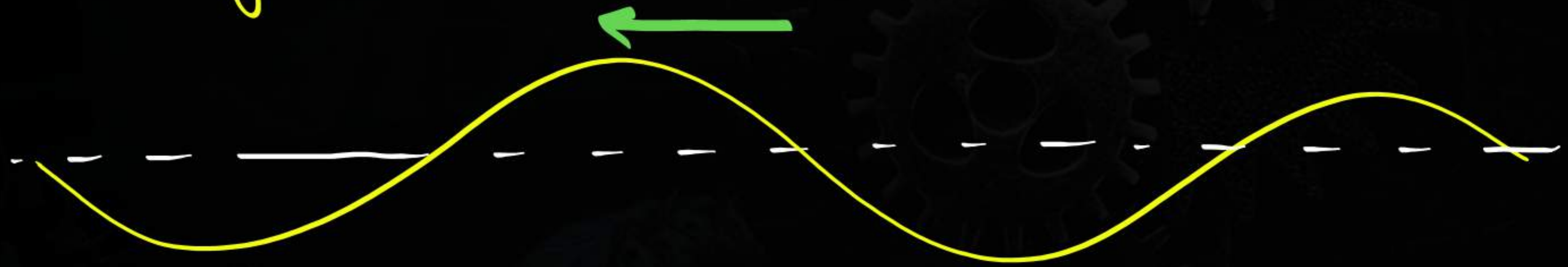
$$y = A\sin(\omega t - kx)$$

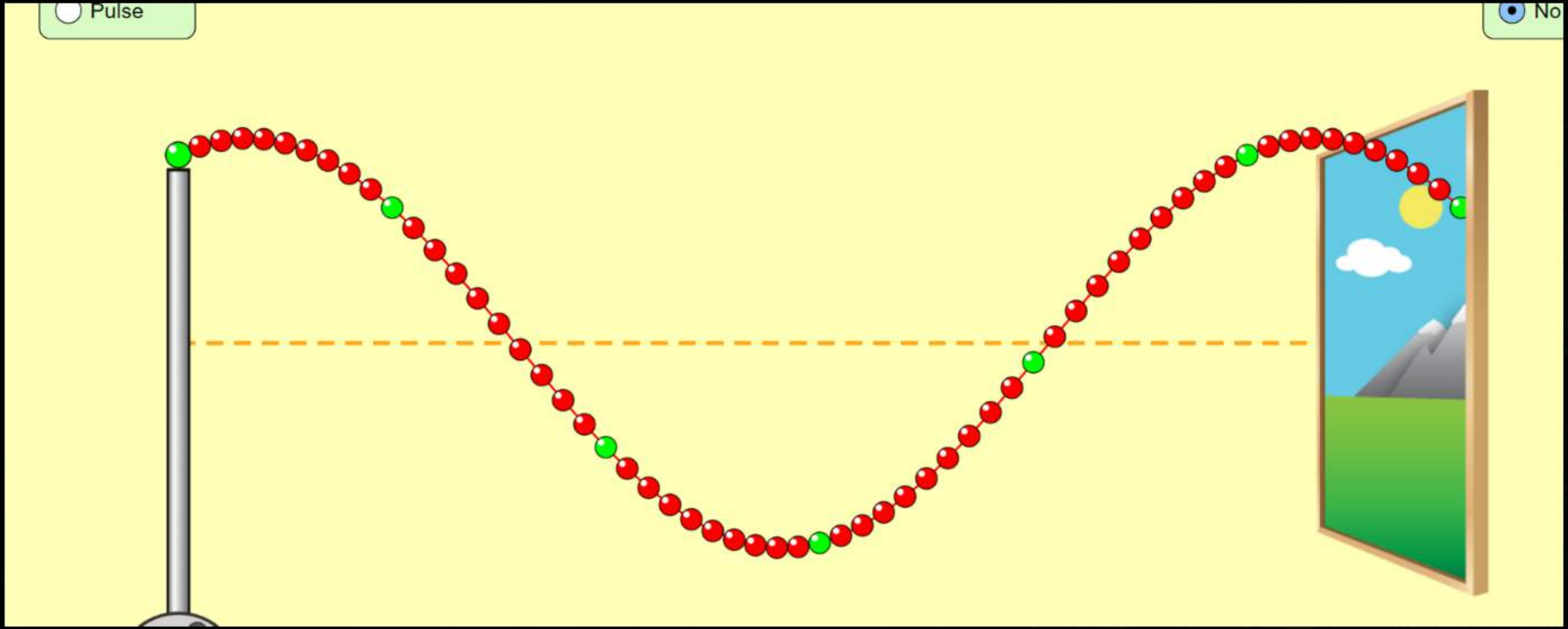


$$y = A\sin(kx + \omega t)$$



$$y = -A\sin(\omega t + kx)$$




Pulse
No

Question

If $y = (6x - 30t)^2$ then Find speed of waves.

$$y = (6x - 30t)^2 \Rightarrow \left[6\left[x - \underbrace{\frac{30}{6}}_{v} t\right]\right]^2$$

$$f(kx - \omega t)$$

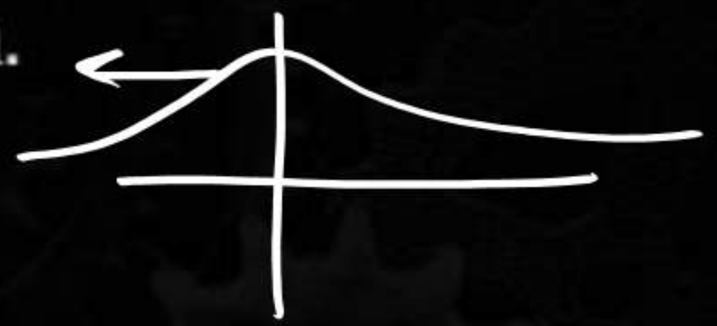
$$v = \frac{\omega}{k} = \frac{30}{6} = 5 \text{ m/s.}$$

$$y = f(t)$$
$$= A\sin(\omega t)$$
$$= A\sin\left(\omega\left(t - \frac{x}{v}\right)\right)$$
$$A\sin\left(\omega t - \underbrace{\left(\frac{\omega}{v}\right)}_{k} x\right)$$

$$k = \frac{\omega}{v}, \quad v = \frac{\omega}{k}$$

Question

If $y = \dfrac{1}{(4x+3t)^2+1}$ then find vel. of wave along with direction.



$$y = \frac{1}{(\boxed{4x+3t})^2+1}$$

$$= \frac{1}{\left[4\left(x+\underbrace{\frac{3}{4}}_{v}t\right)\right]^2+1}$$

$v = \frac{3}{4}$ m/s towards

-ve x axis.

Equation of Travelling Sinusoidal Wave

Eq. of travelling sinosudal wave towards +ve x-axis →

$y = A\sin(kx - \omega t)$ or $y = A\sin(\omega t - kx)$

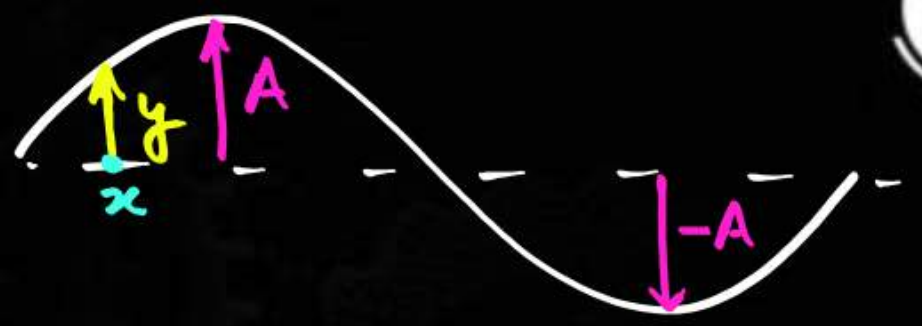
where $k = \frac{\omega}{v}$

Eq. of different waves both travelling along +ve x-axis

Eq. of travelling sinosudal wave towards −ve x-axis →

$y = A\sin(kx + \omega t)$ or $y = -A\sin(kx + \omega t)$

Wave Parameters



$$y = A\sin(kx - \omega t + \phi)$$

y = Displacement of particle at x from mean posn.

A = Amplitude of wave [Max. displacement from mean posn.]

k = Angular wave number $\left[\frac{\omega}{v} = k \ ; \ k = \frac{2\pi}{\lambda}\right]$

ω = Angular wave frequency $\left[\omega = v \cdot k \ ; \ \omega = \frac{2\pi}{T} \ ; \ \omega = 2\pi f\right]$

ϕ = Phase const. (Represents initial phase of wave)

$kx - \omega t + \phi$ = Phase of wave at x at any time t

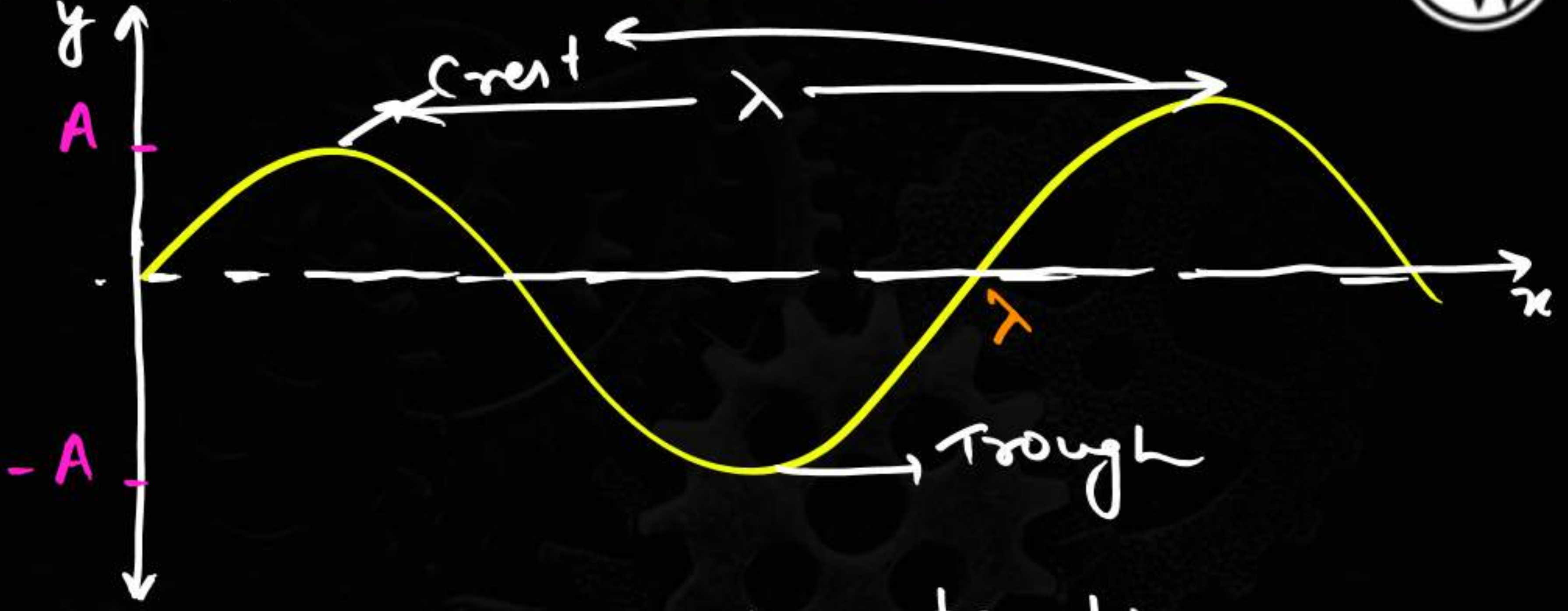
$$y(x,t) = A\sin(kx-\omega t)$$

At $t=0 \longrightarrow$

$$y(x,0) = A\sin(kx)$$

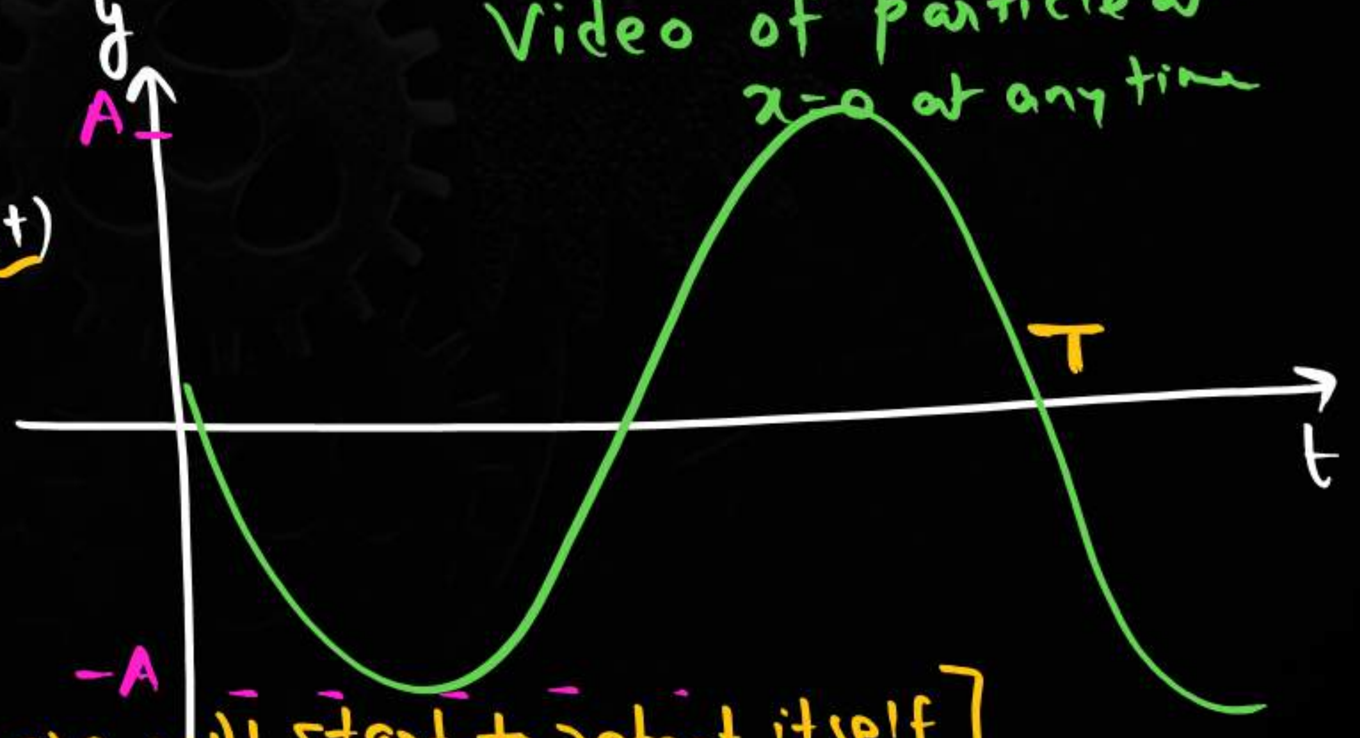
$$k\lambda = 2\pi \Rightarrow \lambda = \frac{2\pi}{k}$$

Photo of wave at $t=0$



λ = Wavelength [Dist. b/w two consecutive crests or troughs

Video of particle at $x=0$ at any time



At $x=0 \longrightarrow$

$$y(0,t) = A\sin(-\omega t) = -A\sin(\omega t)$$

$$= A\sin(\omega t+\pi)$$

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$$

T = Time period (Min. duration after which wave will start to repeat itself] frequency, $f = \frac{1}{T}$ [Total no. of repetitions in 1 sec.].

$f = \frac{\omega}{2\pi}$

Question

Marathon Q.

If $y = 4 \sin\left(\pi x - \frac{\pi}{2} t\right)$ then find

k ω

1. Amplitude of wave ? $= 4\,m$
2. Wavelength = ? $\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2\,m.$
3. Frequency = ? $\Rightarrow f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4}\,Hz.$
4. Wave speed = ? $1/2\,m/s.$
5. Speed of particle at $x = 1$ at $t = 3s.$ $= 0\,m/s$

4) $v = \frac{\omega}{k} = \frac{\lambda}{T} = \frac{\pi/2}{\pi}$

$= \frac{1}{2}\,m/s.$

Dimensional analysis :→

$\sin(\underbrace{kx}_{} - \underbrace{\omega t}_{})$

$[M^0L^0T^0]$

$[k] = [L^{-1}]$ $[\omega] = [T^{-1}]$

5) At $x = 1 \longrightarrow$ $y = 4\sin\left(\pi \times 1 - \frac{\pi}{2} t\right)$

$y = 4\sin\left(\pi - \frac{\pi}{2} t\right)$

$y = 4\sin\left(\frac{\pi}{2} t\right) \Rightarrow v = \frac{dy}{dt} = 4\cos\left(\frac{\pi}{2} t\right) \times \frac{\pi}{2}$

$v = 2\pi\cos\left(\frac{\pi}{2} t\right)$

At $t = 3 \rightarrow$

$v = 2\pi\cos\left(\frac{3\pi}{2}\right) = 0\,m/s$

Question

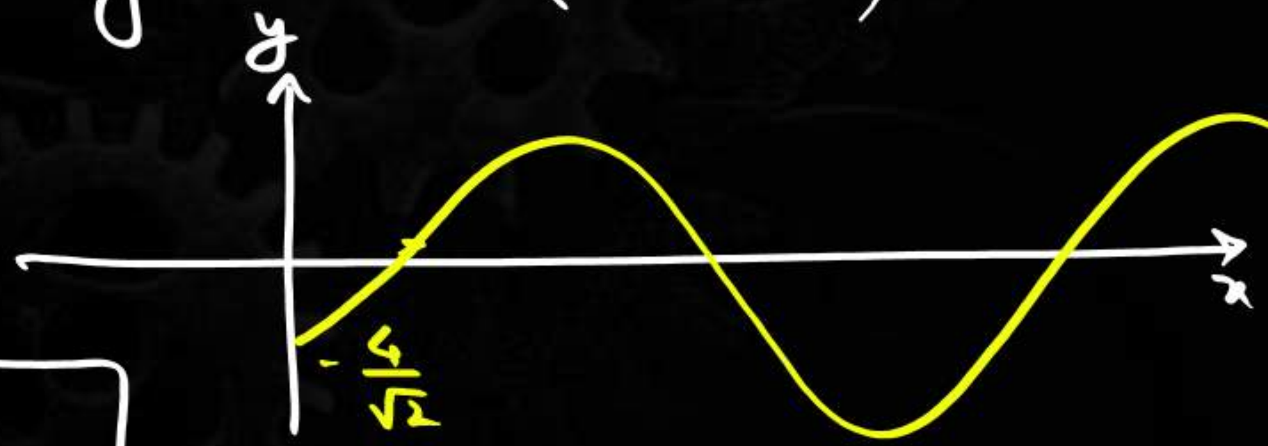
If $y = 4\sin(\pi x - \frac{\pi}{2}t)$ then

find

6. Position of particle at $t = 1$ at $x = 2$. $= -4\,m$
7. Snapshot of wave at $t = 0.5$ s. $4\sin\left(\frac{\pi}{2} - \frac{\pi}{2}t\right) = 4\cos\left(\frac{\pi}{2}t\right)$
8. Position of particle at $x = 0.5$m at any time.
9. Slope of tangent on wave at $x = 1$ at $t = 1$. $= 4\pi$
10. Max. particle velocity. $\rightarrow V_{max} = A\omega = 4 \times \frac{\pi}{2} = 2\pi\ m/s$
11. Phase diff. between two particles separated by 0.5 m. ↳ later

6) $y = 4\sin(\pi \times 2 - \frac{\pi}{2} \times 1)$
$= 4\sin\left(\frac{3\pi}{2}\right)$
$y = -4\,m$

7) $y = 4\sin(\pi x - \frac{\pi}{2} \times 0.5)$
$y = 4\sin\left(\pi x - \frac{\pi}{4}\right)$



9) $y = 4\sin\left(\pi x - \frac{\pi}{2}t\right)$

At $t = 1 \longrightarrow$

$$y = 4\sin\left(\pi x - \frac{\pi}{2}\right)$$

$$y = -4\sin(\pi x)$$

$$\text{Slope} = \tan\theta = \frac{dy}{dx} = -4\cos(\pi x)\times\pi$$

$$= -4\pi\cos\pi x$$

At $x = 1$ —

$$\tan\theta = -4\pi\cos(\pi)$$

$$= -4\pi(-1)$$

$$= 4\pi$$

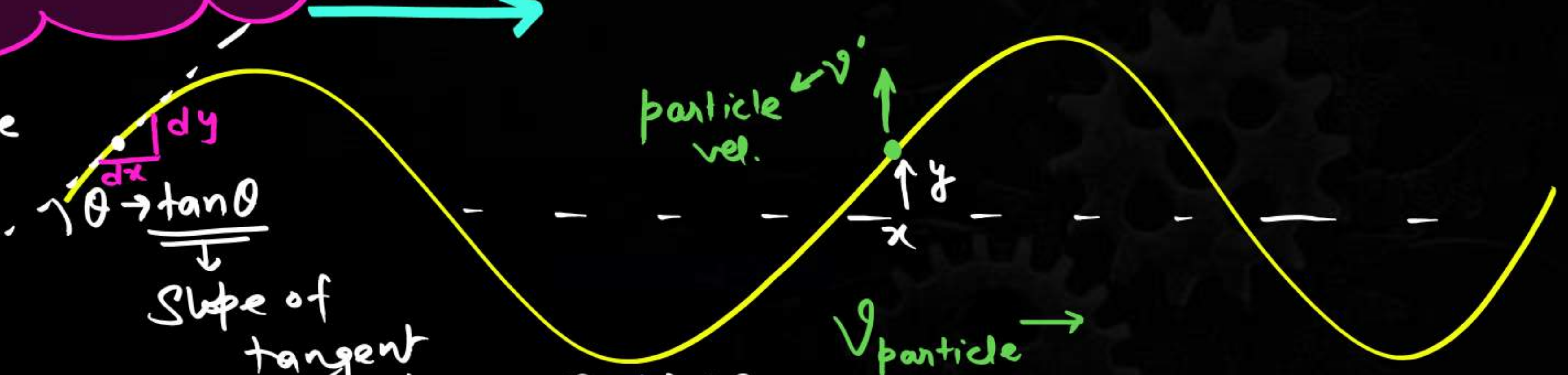
Wave Velocity, Particle Velocity and Slope

$\tan\theta = \frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx)$

↓

Slope of particle at x at t.

$v \rightarrow$ wave vel.



↓

Slope of tangent drawn on wave.

$v_{particle} \rightarrow$

$$v_{wave} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$$y = A\sin(\omega t - kx)$$

$$v_{particle} = \frac{\partial y}{\partial t} = \underbrace{A\omega}_{v_{max}}\cos(\omega t - kx)$$

↓

Partial diff (Treat x as const. while diff. w.r.t t)

Question

A wave travelling in the positive x-direction having displacement along y-direction as 1 m, wavelength 2π m and frequency of $\frac{1}{\pi}$ Hz is represented by

A

λ

f

(A) $y = \sin(x - 2t)$

(B) $y = \sin(2\pi x - 2\pi t)$

(C) $y = \sin(10\pi x - 20\pi t)$

(D) $y = \sin(2\pi x + 2\pi t)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi} = 1$

$f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f \Rightarrow \omega = 2\pi \times \frac{1}{\pi} = 2$

$y = A\sin(kx - \omega t)$

$= 1\sin(x - 2t)$

Question

$f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\ Hz.$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \frac{1}{5} = 0.2\ m$

The wave described by

$$y = 0.25 \sin (\underbrace{10\pi}_{k} x - \overbrace{2\pi}^{\omega} t),$$

where, x and y are in metre and t in second, is a wave travelling along the

(A) negative x-direction with frequency 1Hz

(B) positive x-direction with frequency π Hz and wavelength $l = 0.2$m

(C) positive x-direction with frequency 1Hz and wavelength $l = 0.2$m

(D) negative x-direction with amplitude 0.25m and wavelength $l = 0.2$m

Question

A transverse wave is represented by $\boxed{y = A\sin(\omega t - kx)}$. For what value of the wavelength is the wave velocity equal to $\underbrace{\text{the maximum particle velocity?}}$

$$v_{wave} = \frac{\omega}{k}$$

$$v_{max} = A\omega$$

(A) $\pi A/2$

(B) πA

(C) $2\pi A$

(D) A

$$v_{wave} = v_{max}.$$

$$\frac{\cancel{\omega}}{k} = A\cancel{\omega} \Rightarrow k = \frac{1}{A}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{(1/A)} = 2\pi A$$ Ans

Question

A transverse wave propagating along x-axis is represented by

$$y(x,t) = 8\sin\left(\overset{k}{0.5\pi}x - \overset{\omega}{4\pi}t - \frac{\pi}{4}\right)$$

where, x is in metre and t is in second. The speed of the wave is

$$v_{wave} = \frac{\omega}{k} = \frac{4\pi}{0.5\pi} = 8\ m/s.$$

A. 4π m/s

B. 0.5π m/s

C. $\pi/4$ m/s

D. 8 m/s

Question

A wave in a string has an amplitude of 2 cm. The wave travels in the positive direction of x-axis with a speed of 128 ms^{-1} and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is

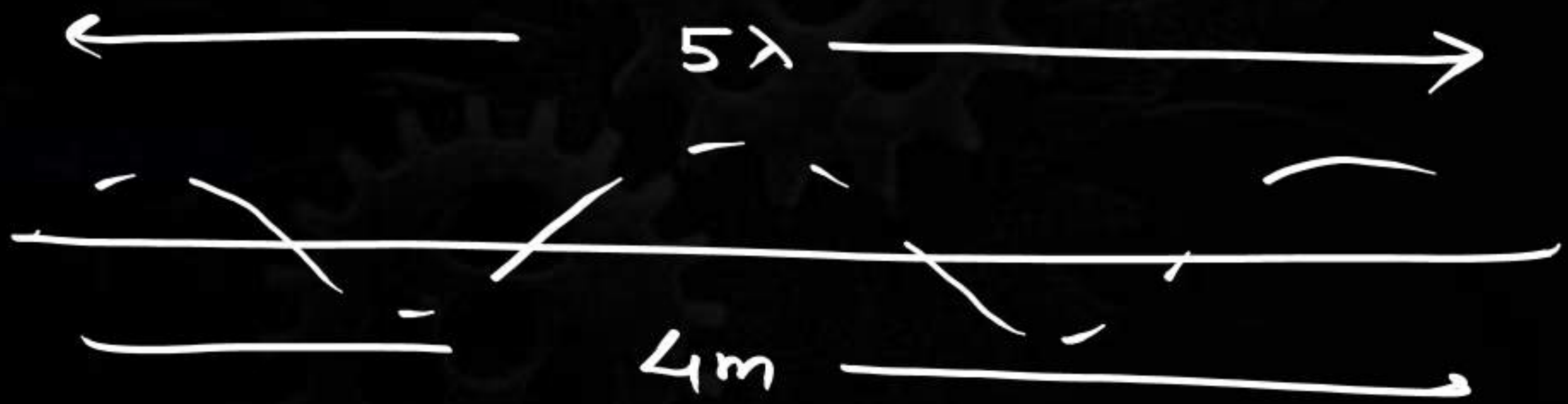
(A) $y = (0.02)\text{m} \sin(7.85x + 1005\,t)$

(B) $y = (0.02)\text{m} \sin(15.7x - 1010\,t)$

(C) $y = (0.02)\text{m} \sin(15.7x + 1010\,t)$

(D) $y = (0.02)\text{m} \sin(7.85x - 1005\,t)$

$v = 128$ m/s.



$5\lambda = 4 \Rightarrow \lambda = \frac{4}{5}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4/5} = \frac{5\pi}{2}$

$= 2.5\pi$

Question

H.W

A wave of amplitude $a = 0.2\text{m}$, velocity $v = 360\text{m m/s}$ and wavelength 60m is travelling along positive x-axis, then the correct expression for the wave is

(A) $y = 0.2 \sin 2\pi\left(6t + \frac{x}{60}\right)$

(B) $y = 0.2 \sin \pi\left(6t + \frac{x}{60}\right)$

(C) $y = 0.2 \sin 2\pi\left(6t - \frac{x}{60}\right)$

(D) $y = 0.2 \sin \pi\left(6t - \frac{x}{60}\right)$

Question

H.W

The equation of wave is given by $y = a\sin\left(100t - \frac{x}{10}\right)$, where x and y are in metre and t in second, then velocity of wave is

A. 0.1 m/s

B. 10 m/s

C. 100 m/s

D. 1000 m/s

Question

A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin (50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave?

[JEE Mains 2019]

$v = \frac{\omega}{k} = \frac{50}{2} = 25 m/s$

A. The wave is propagating along the negative x-axis with speed 25 ms^{-1}.

B. The wave is propagating along the positive x-axis with speed 100 ms^{-1}.

C. The wave is propagating along the positive x-axis with speed 25 ms^{-1}.

D. The wave is propagating along the negative x-axis with speed 100 ms^{-1}.

Wave Speed

$$y = A\sin(kx - \omega t)$$

$$v_{wave} = \frac{\omega}{k} = \frac{\lambda}{T}$$

1) Speed of wave in string →

$$v \propto \sqrt{\text{Elasticity}}$$

$$\propto \sqrt{\text{Inertia}}$$

for string →

$$v = \sqrt{\frac{T}{\mu}}$$

T = Tension in string

μ = Linear mass density.

Velocity of Sound

1) In any medium –

$$v = \sqrt{\frac{\text{Elasticity}}{\text{Inertia}}}$$

In rod →

$$v = \sqrt{\frac{Y}{\rho}}$$

Y = Young's modulus

ρ = vol. density.

In any medium –

$$v = \sqrt{\frac{B}{\rho}}$$

B = Bulk modulus.

Newton's formula & Laplace's correction

Speed of sound in air →

$$\vartheta = \sqrt{\frac{B}{\rho}}$$

According to Newton gases compress & expand isothermally.

$$\Delta P = -B\frac{\Delta V}{V}$$

for isothermal process —

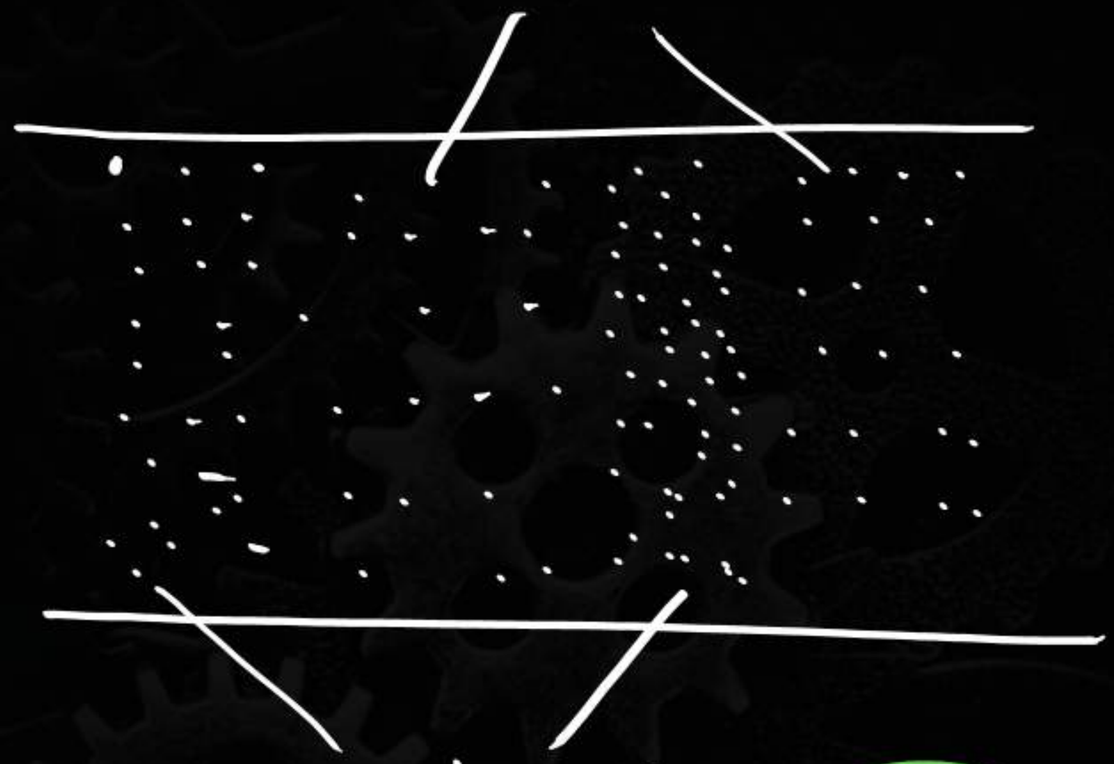
$$B = P$$

$PV = \text{const.}$

$$\Delta P . V + P . \Delta V = 0$$

$$\Delta P V = -P\Delta V$$

$$\Delta P = -\underbrace{P}_{B}\left(\frac{\Delta V}{V}\right)$$



$$\boxed{\vartheta = \sqrt{\frac{P}{\rho}}}$$

Laplace proposed that the compression & expansion of gas are an adiabatic process & not isothermal because of its quick nature heat does not gets exchanged.

$$PV^{\gamma} = \text{const.}$$

$$\Rightarrow \Delta P \cdot V^{\gamma} + P \cdot \gamma V^{\gamma-1} \Delta V = 0$$

$$\Rightarrow \Delta P \cdot V^{\gamma} = -\gamma P \cdot V^{\gamma-1} \Delta V$$

$$\Delta P = -\gamma P \cdot \frac{V^{\cancel{\gamma-1}}}{\cancel{V^{\gamma}}} \Delta V$$

$$\Delta P = -\underbrace{\gamma P}_{B} \frac{\Delta V}{V}$$

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

for air / diatomic gas — $(f = 5)$

$$\frac{C_P}{C_V} = \gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = 1.4$$

Question

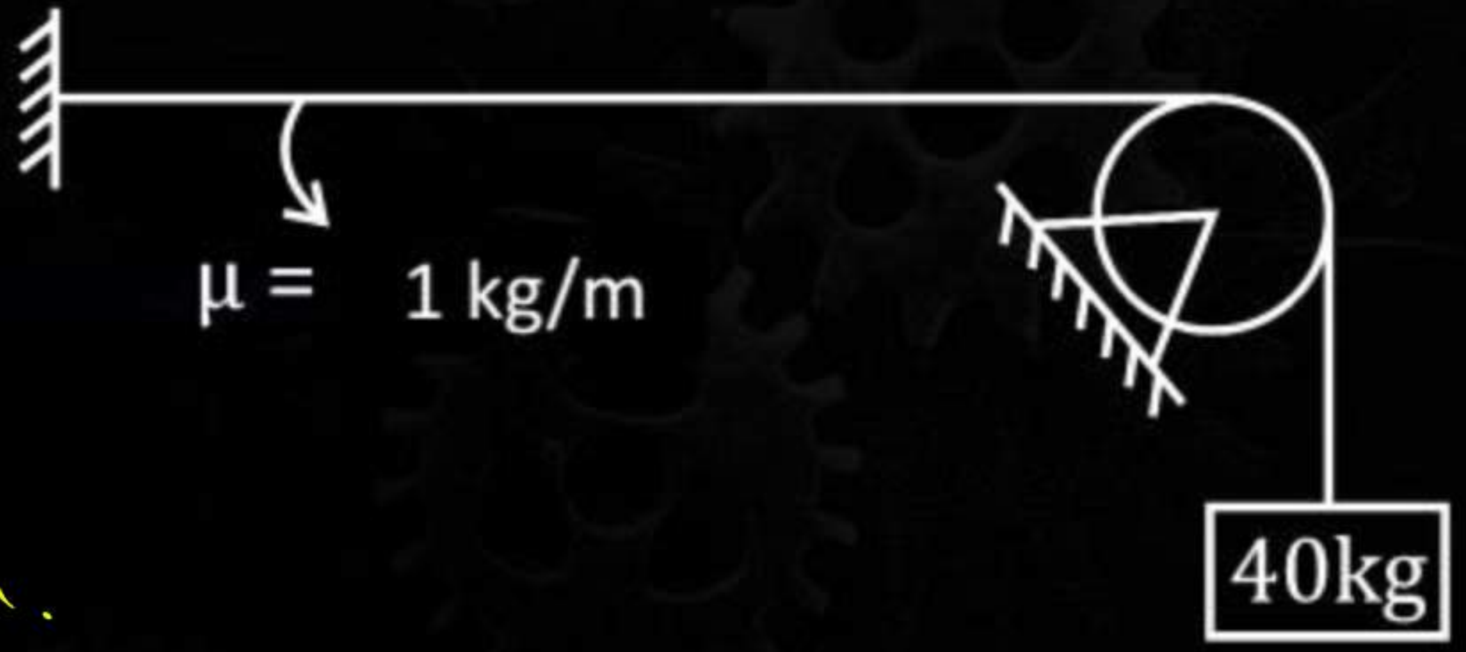
Find velocity of wave in string. If a source of frequency 100 Hz is used, what will be the wavelength of travelling wave.

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{400}{1}}$$

$$v = 20\, m/s$$

$$\lambda = v \times T$$

$$\lambda = \frac{v}{f} = \frac{20}{100} = 0.2\, m.$$



Question

$\gamma = 1 + \frac{2}{f}$

Find ratio of speed of sound in O_2 & H_2 at same temperature.

$$\frac{v_{O_2} = \sqrt{\frac{\cancel{\gamma}\cancel{R}\cancel{T}}{\cancel{32}\ 16}}}{v_{H_2} = \sqrt{\frac{\cancel{\gamma}\cancel{R}\cancel{T}}{\cancel{2}}}}$$

$$= \frac{1}{\sqrt{16}} = \frac{1}{4}$$

$$v\uparrow = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma R T\uparrow}{M\downarrow}} = \sqrt{\frac{\gamma k_B T}{m}}$$

T = Absolute temp.

M = molar mass

R = Universal gas const

$k_B = \frac{R}{N_A}$ = Boltzmann const

m = molecular mass

$PV = nRT$

$$P = \frac{nRT}{V}\frac{M}{M}$$

$$= \frac{(nM)RT}{V \times M}$$

$$= \left(\frac{\text{mass}}{\text{Vol}}\right)\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

Effect of Humidity on speed of sound

Atmosphere

O_2 & N_2

32g 28g

Atmosphere with Humidity

O_2 & N_2 & $H_2O\uparrow$

32g 28g 18g

$$v\uparrow = \sqrt{\frac{\gamma RT}{M\downarrow}}$$

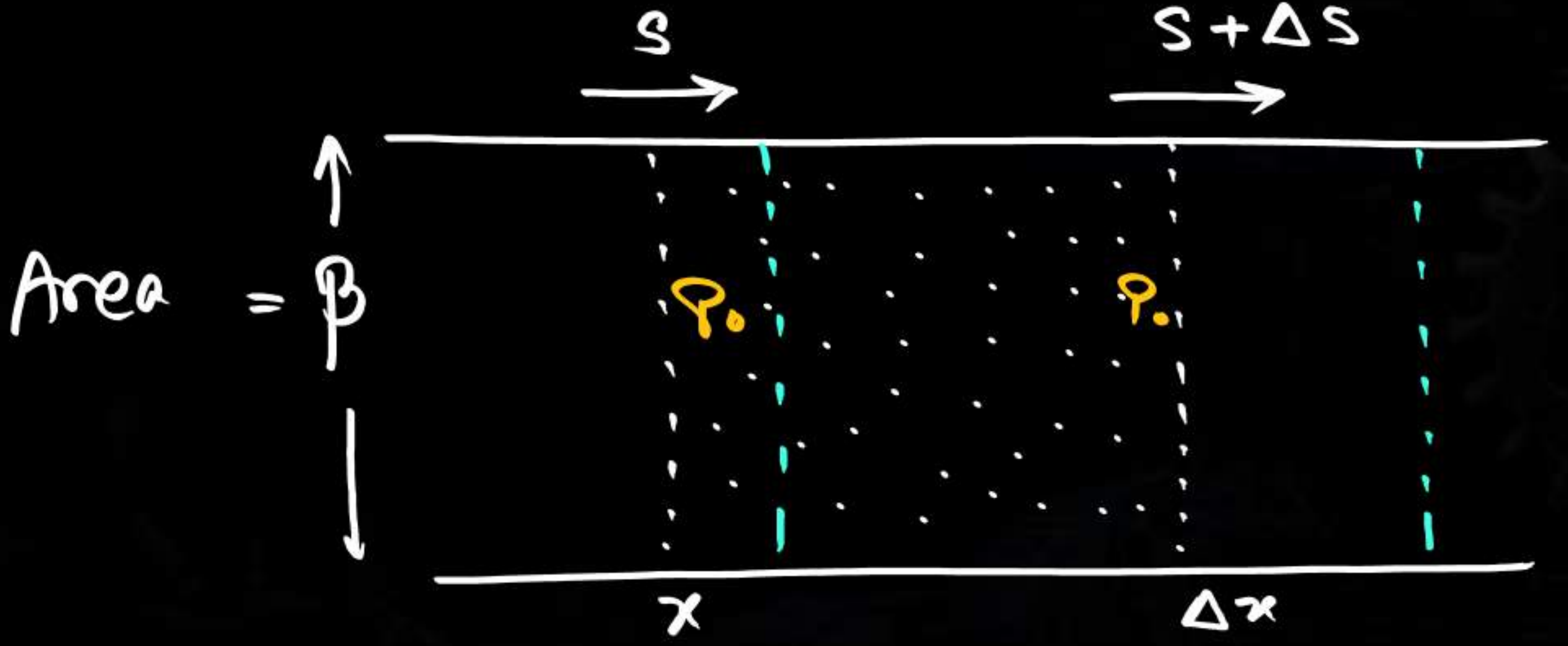
⇒ Speed of sound in Humid environment is more.

Pressure Wave

→ for sound we prefer to study pressure.

$$S = A \sin(\omega t - kx)$$

$$\Delta S = A \cos(\omega t - kx) \times (-k\Delta x)$$

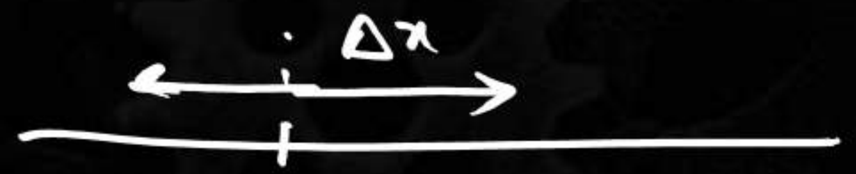


Transwerse wave -

$$\Delta y = A \sin(kx - \omega t)$$

Longitudnal wave -

$$\Delta x = A \sin(kx - \omega t)$$



$$V = \beta \Delta x$$

$$\Delta V = \beta \Delta S$$

$$\Delta V = \beta(-kA\Delta x \cos(\omega t - kx))$$

$$\Delta P = -B\,\frac{\Delta V}{V}$$

$$\Delta P = -B \cdot \frac{\cancel{S}(-kA\cancel{\Delta x}\cos(\omega t - kx))}{\cancel{S}\,\cancel{\Delta x}}$$

$$\Delta P = \underbrace{BAk}\cos(\omega t - kx)$$

Phase diff. b/w pressure & disp. wave is $\frac{\pi}{2}$.

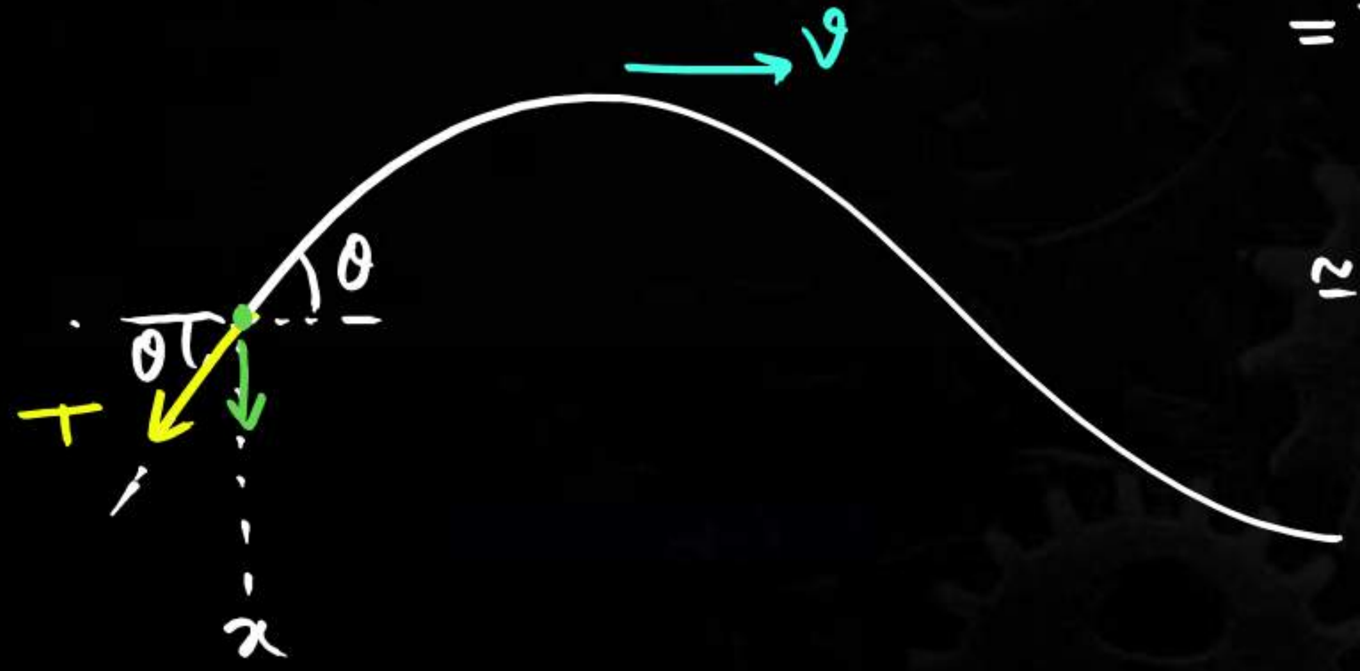
$$\Delta P = P_0 \cos(\omega t - kx)$$

(याद करो)

where $P_0 = BAk$

Energy Trasfer

for string $\longrightarrow$



$$\sin\theta \simeq \tan\theta \simeq \frac{\partial y}{\partial x}$$

$$y = A\sin(kx - \omega t)$$

$$\frac{\partial y}{\partial x} = Ak \cdot \cos(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

$$\text{Power} = T \cdot v_{particle}$$

$$= T\sin\theta \cdot \frac{\partial y}{\partial t}$$

$$\simeq T \cdot \tan\theta \, \frac{\partial y}{\partial t}$$

$$\simeq T \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial y}{\partial t}$$

$$= T \cdot \left[Ak\cos(kx - \omega t)\right]\left[-A\omega\cos(kx - \omega t)\right]$$

$$|P| = T \cdot A^2 \omega k \cos^2(kx - \omega t)$$

$$\langle P \rangle = T \cdot A^2 \omega k \cdot \left[\frac{1}{2}\right]$$

$$P_{avg} = \frac{1}{2} T A^2 \omega k$$

$$P_{avg} = \frac{1}{2} T \frac{A^2 \omega^2}{v}$$

$$P \propto A^2$$

$$\propto \omega^2$$

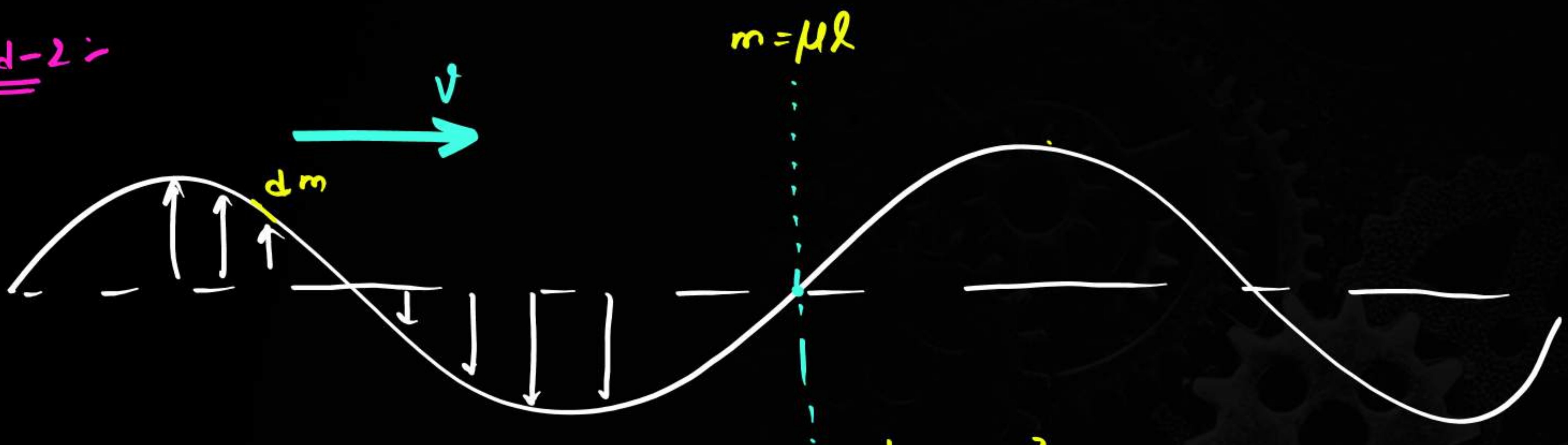
$$k = \frac{\omega}{v}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$E = P \cdot t$$

$$T = \mu v^2$$

Method-2 :-



Total Energy of dm — $dE = \frac{1}{2} k A^2$

$= \frac{1}{2}(dm\,\omega^2)A^2$

$E = \frac{1}{2}\omega^2 A^2 \int dm$

$E = \frac{1}{2}\omega^2 A^2 \mu \cdot v t \Rightarrow \frac{E}{t} = \frac{1}{2}\omega^2 A^2 \mu v$

Question

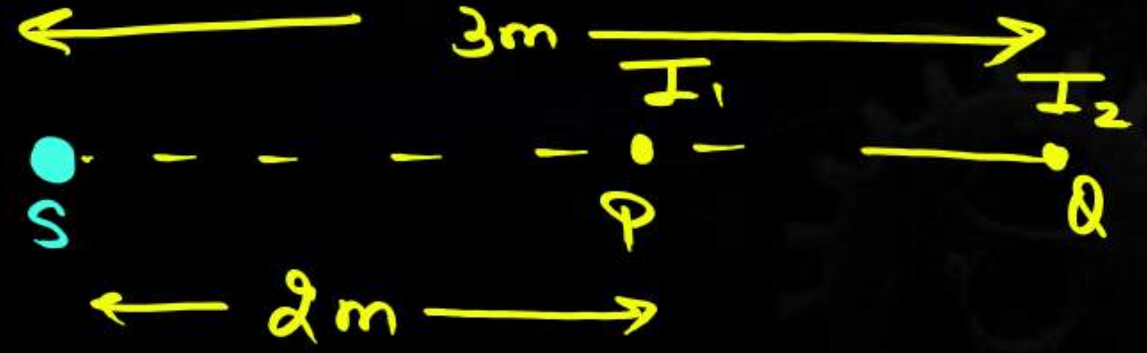
A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distance of 2m and 3m respectively from the source. The ratio of the intensities of the waves at P and Q is

(A) 9 : 4

(B) 2 : 3

(C) 3 : 2

(D) 4 : 9



$$I \propto \frac{1}{r^2}$$

$$\frac{I_1}{I_2} = \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}.$$

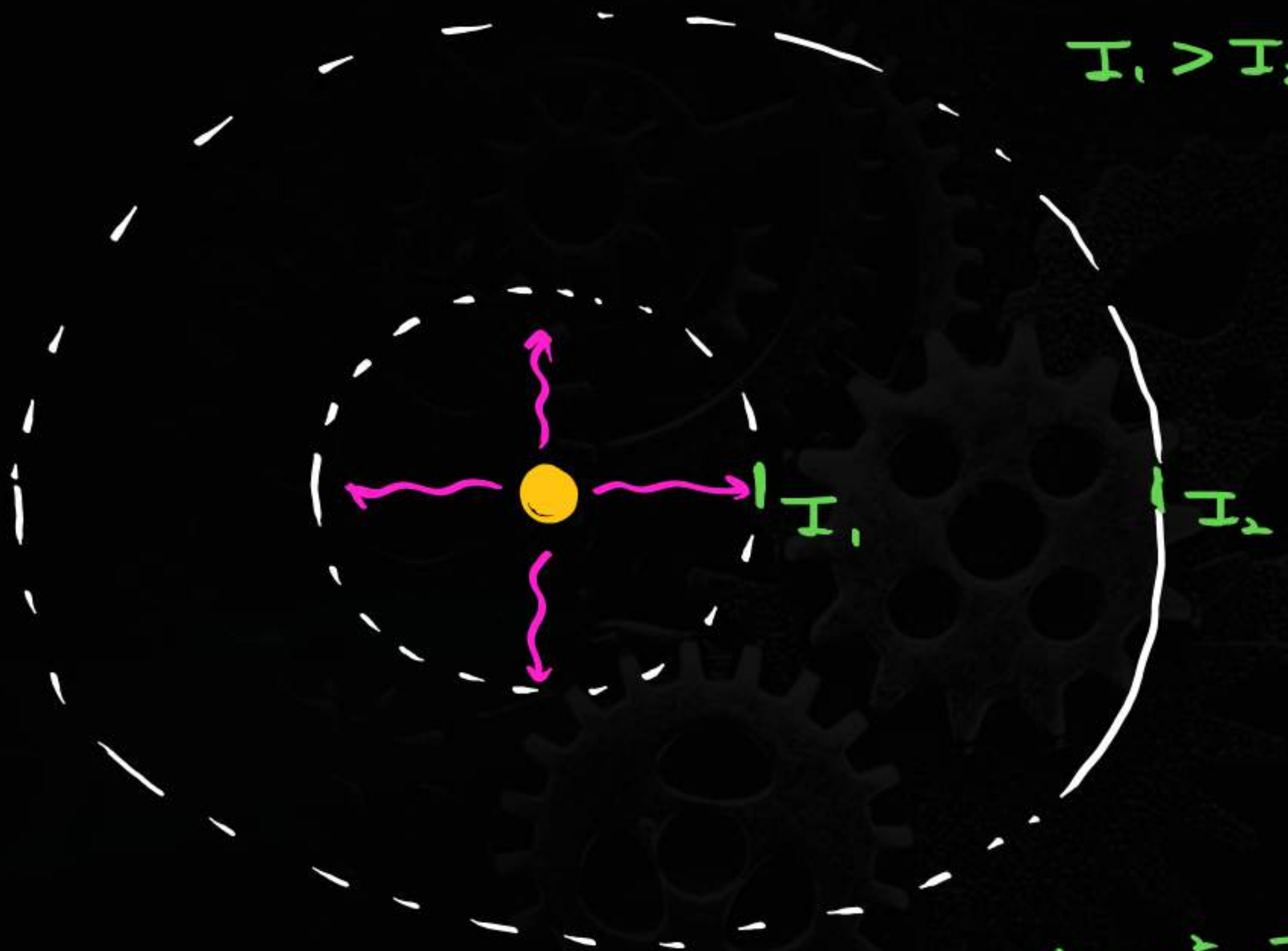
$\omega = 2\pi f$

$E \propto A^2$

$E \propto \omega^2$

$\propto f^2$

Intensity → $\frac{\text{Energy}}{\text{Area} \times \text{time}}$



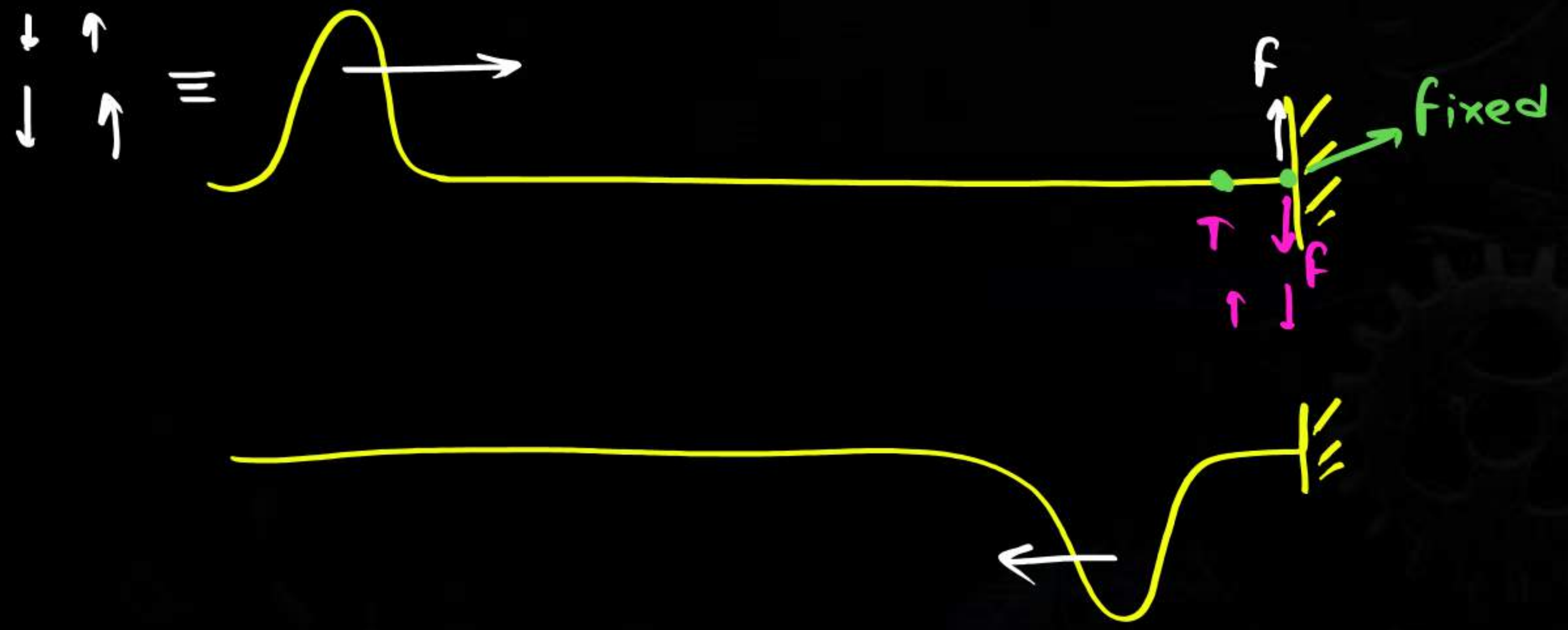
$I_1 > I_2$

$E = I_1 \times 4\pi r_1^2 = I_2 \times 4\pi r_2^2$

$I = \frac{E}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

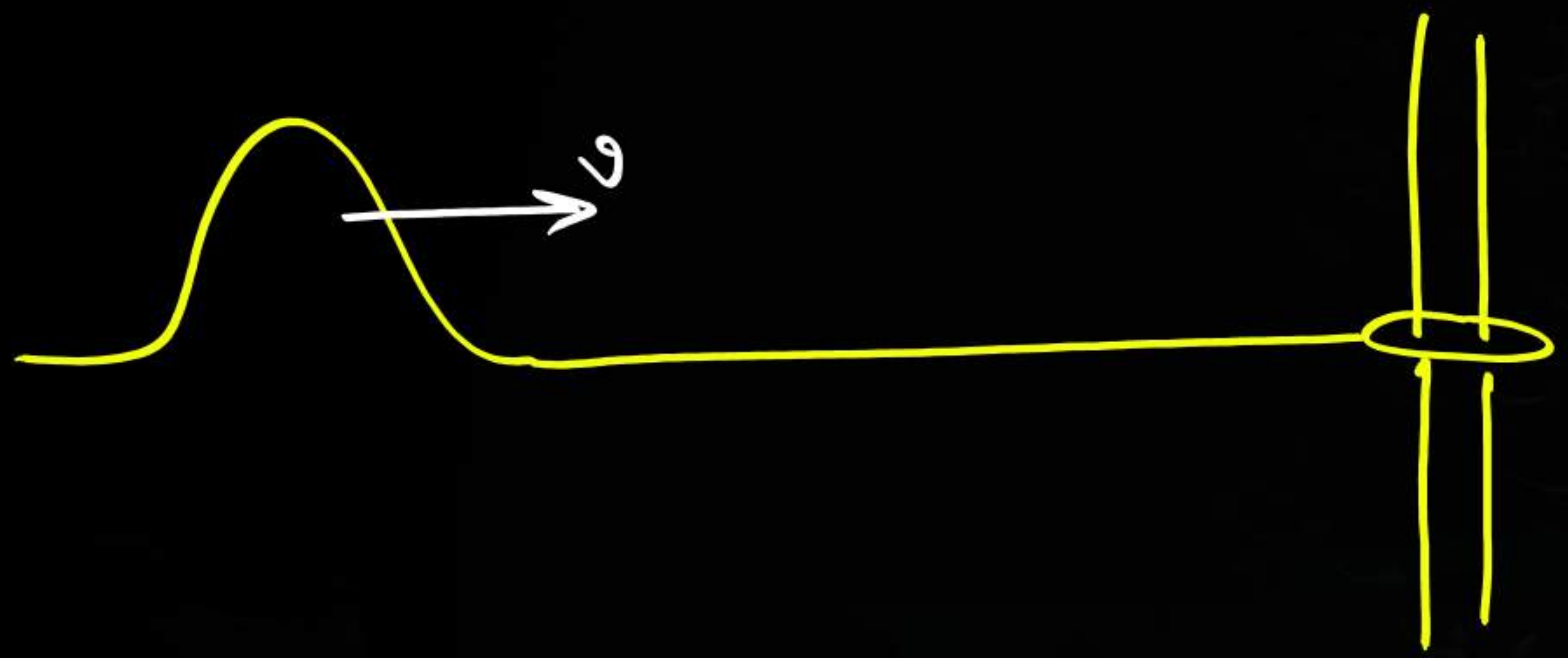
Wave Reflection and Refraction

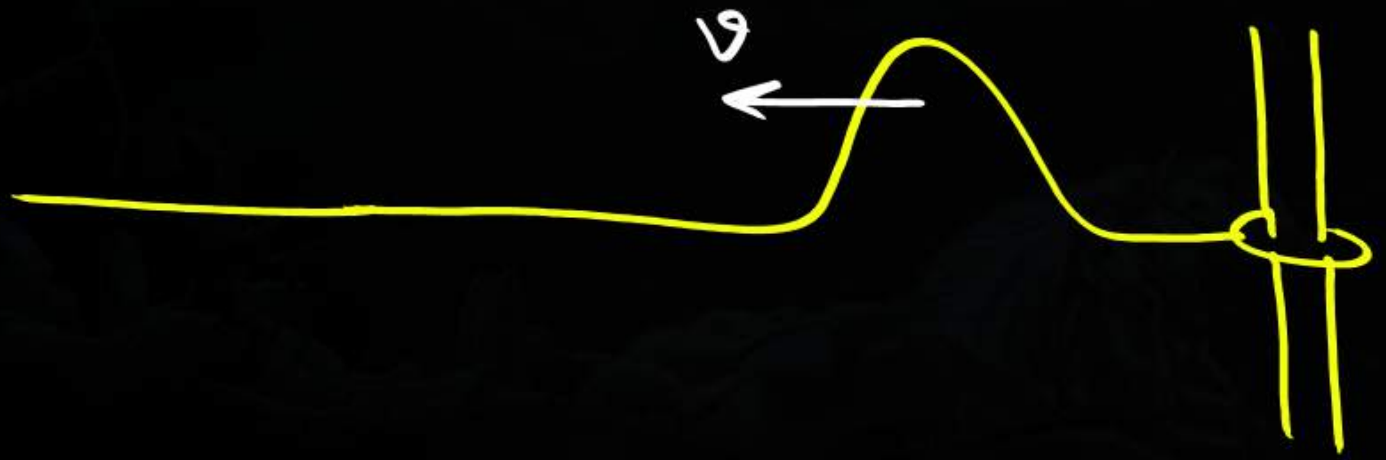
1) Reflected from fixed End



After reflection from fixed end, the wave gets inverted.

2) Reflection from free end →



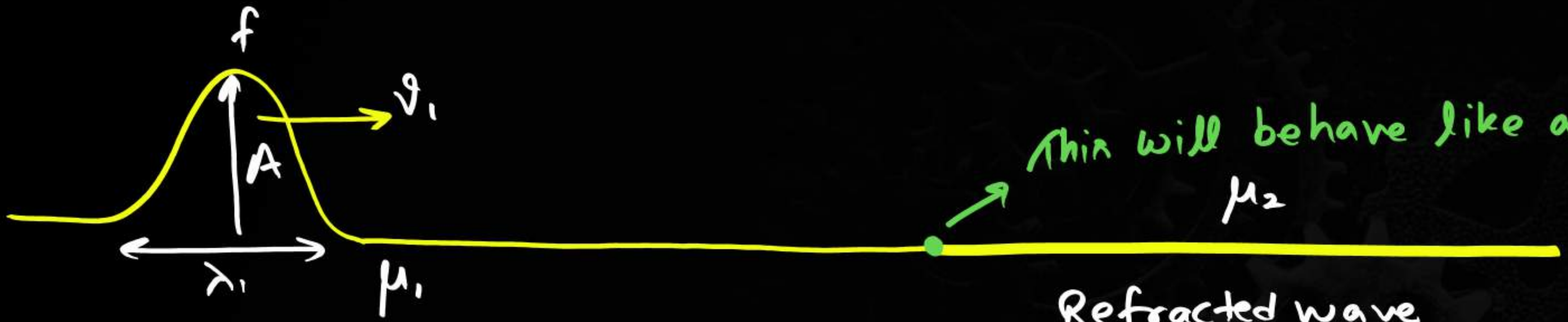


⇒ upon reflection from free end, no inversion of wave takes place.

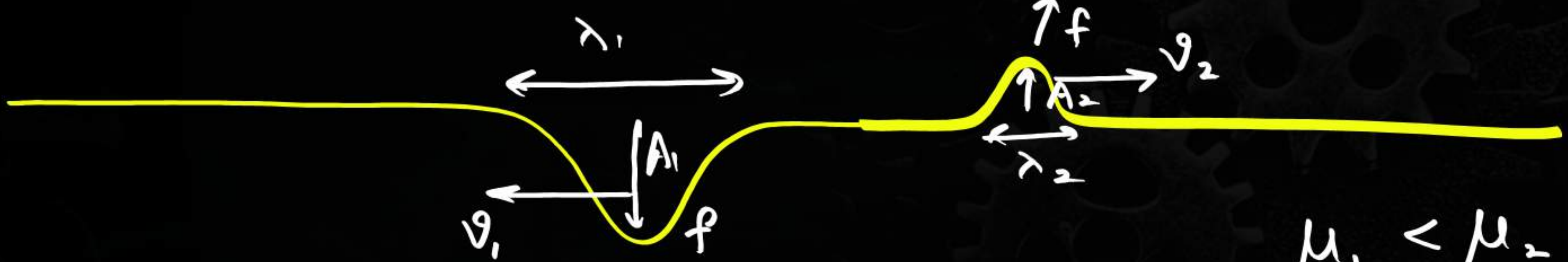
Rarer medium ⟶ Denser medium

$\lambda = v \times T$

This will behave like a fixed end.



Refracted wave



Reflected wave.

$\mu_1 < \mu_2$

$v_1 > v_2$

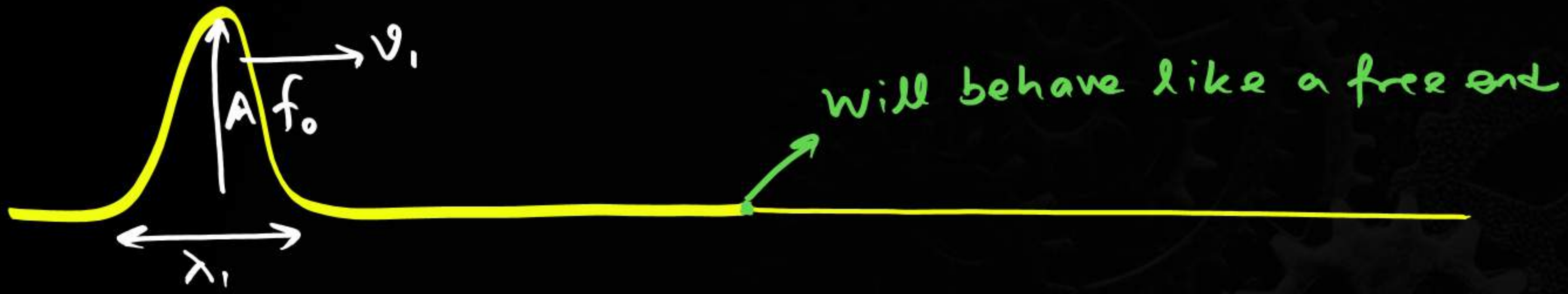
$\lambda_1 > \lambda_2$

$f_1 = f_2$

$A_1 < A$

$A_2 < A$

Denser medium ⟶ Rarer medium →



$\mu_1 > \mu_2$

$v_1 < v_2$

$\lambda_1 < \lambda_2$

$f_0 = f_0$

Question

$v' = 10v$

Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a $\underbrace{\text{700 Hz}}_{f}$ acoustic wave as it enters brass from warm air.

$\lambda^{\times 10} = \frac{v^{\times 10}}{f}$

(A) increases by a factor 20

(B) increases by a factor 10

(C) decreases by a factor 20

(D) decreases by a factor 10

Interference of Waves

⇒ Addition of two waves.

$$y = y_1 + y_2$$

Interference of Waves

$f_1 = f_2$

$\Rightarrow \omega_1 = \omega_2$

$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$

1. Both waves of same frequency and moving in same direction:

$$y_1 = A_1 \sin(kx - \omega t)$$

$$y_2 = A_2 \sin(kx - \omega t + \phi)$$

$$y = y_1 + y_2 = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$$

$$y = A_1 \sin(kx-\omega t) + A_2 \sin(\underbrace{kx-\omega t}_{A} + \underbrace{\phi}_{B})$$

$$= A_1 \sin(kx-\omega t) + A_2 [\sin(kx-\omega t)\cdot\cos\phi + \cos(kx-\omega t)\sin\phi]$$

$$\{\sin(A+B) = \sin A\cos B + \cos A\cdot\sin B\}$$

$$= (A_1 + A_2\cos\phi)\sin(kx-\omega t) + A_2\sin\phi\cos(kx-\omega t)$$

$$= \sqrt{(A_1+A_2\cos\phi)^2 + (A_2\sin\phi)^2}\ \sin\left(kx-\omega t + \tan^{-1}\left(\frac{A_2\sin\phi}{A_1+A_2\cos\phi}\right)\right)$$

$$\left\{A_1\sin\theta + A_2\cos\theta = \sqrt{A_1^2+A_2^2}\ \sin\left(\theta + \tan^{-1}\frac{B}{A}\right)\right\}$$

$$= \sqrt{A_1^2 + \underline{A_2^2\cos^2\phi} + 2A_1A_2\cos\phi + \underline{A_2^2\sin^2\phi}}\ \sin(kx-\omega t+\delta)$$

where $\delta = \tan^{-1}\left(\frac{A_2\sin\phi}{A_1+A_2\cos\phi}\right)$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}\ \sin(kx-\omega t+\delta)$$

$$= A\sin(kx-\omega t+\delta)$$ where $A = \sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$

Brahmastra

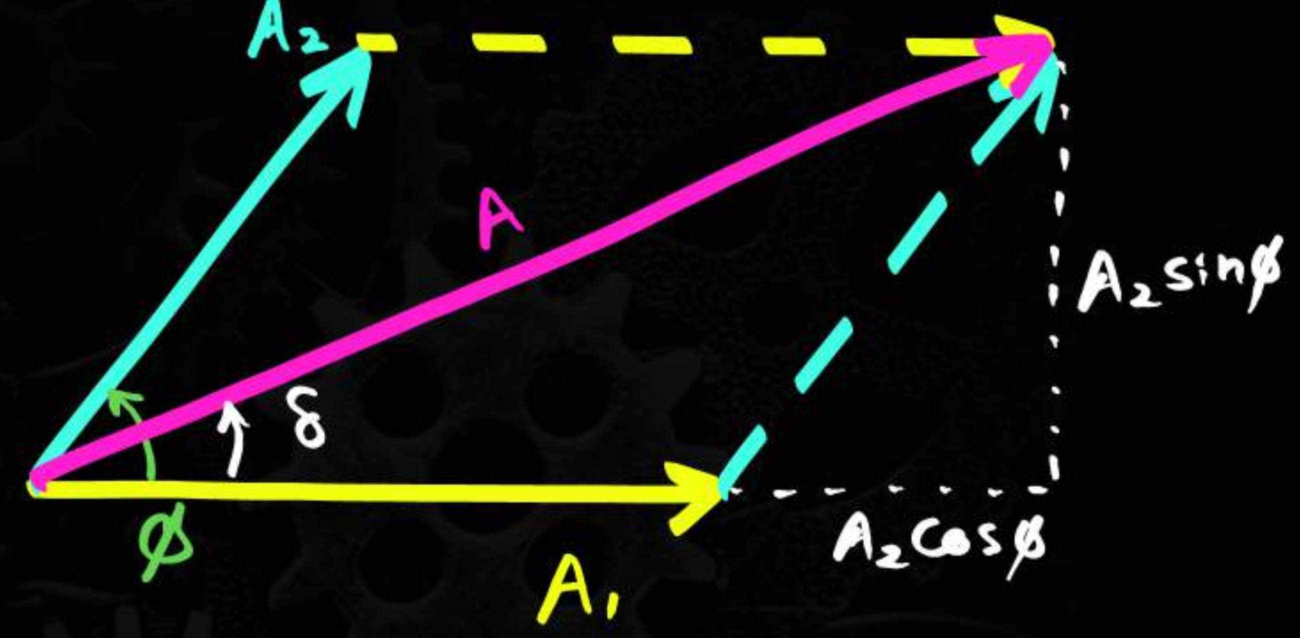
$$y_1 = A_1 \sin(kx - \omega t)$$

$$y_2 = A_2 \sin(kx - \omega t + \phi)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\tan\delta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

$$y = y_1 + y_2 = A \sin(kx - \omega t + \delta)$$



Phasor.

Special cases

$I \propto A^2$

(1) $\phi = 0 \rightarrow$

$$y_1 = A_1 \sin(kx - \omega t) \qquad I_1 \propto A_1^2$$

$$y_2 = A_2 \sin(kx - \omega t) \qquad I_2 \propto A_2^2$$

$$y = y_1 + y_2 = (A_1 + A_2) \sin(kx - \omega t)$$

(Constructive Interference)

$$I \propto (A_1 + A_2)^2$$

$$I \propto A_1^2 + A_2^2 + 2A_1A_2$$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}}$$

(2) $\phi = \pi$ (Destructive Interference)

$y_1 = A_1 \sin(kx - \omega t)$ ———— $I_1 \propto A_1^2$

$y_2 = A_2 \sin(kx - \omega t + \pi)$ ———— $I_2 \propto A_2^2$

$= -A_2 \sin(kx - \omega t)$

$y = y_1 + y_2 = (A_1 - A_2) \sin(kx - \omega t)$ ———— $I \propto (A_1 - A_2)^2$

$I \propto A_1^2 + A_2^2 - 2A_1A_2$

$I = I_1 + I_2 - 2\sqrt{I_1 I_2}$

Question

If $A_1 : A_2 = 3 : 2$ then find the ratio of I_{max}/I_{min}. upon interference of two waves.

$3A \longleftarrow A_1$
$2A \longleftarrow A_2$

$$\frac{A_1}{A_2} = \frac{3}{2}$$

Constructive Interference :-

$$A_{con.} = (3A + 2A)$$
$$= 5A$$
$$I_{max} \propto (5A)^2$$

Destructive Interference :-

$$A_{Dest.} = 3A - 2A$$
$$= A$$
$$I_{min.} \propto A^2$$

$$\frac{I_{max}}{I_{min.}} = \frac{(5\cancel{A})^2}{\cancel{A^2}}$$

$$= 25 : 1 \text{ Ans}$$

Question

Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to: [9 Jan. 2020 I]

(A) $5.8\, I_0$

(B) $0.2\, I_0$

(C) $3\, I_0$

(D) I_0

$A_{net} = A_0 + 2A_0 \cos 45°$

$= A_0 + \sqrt{2} A_0$

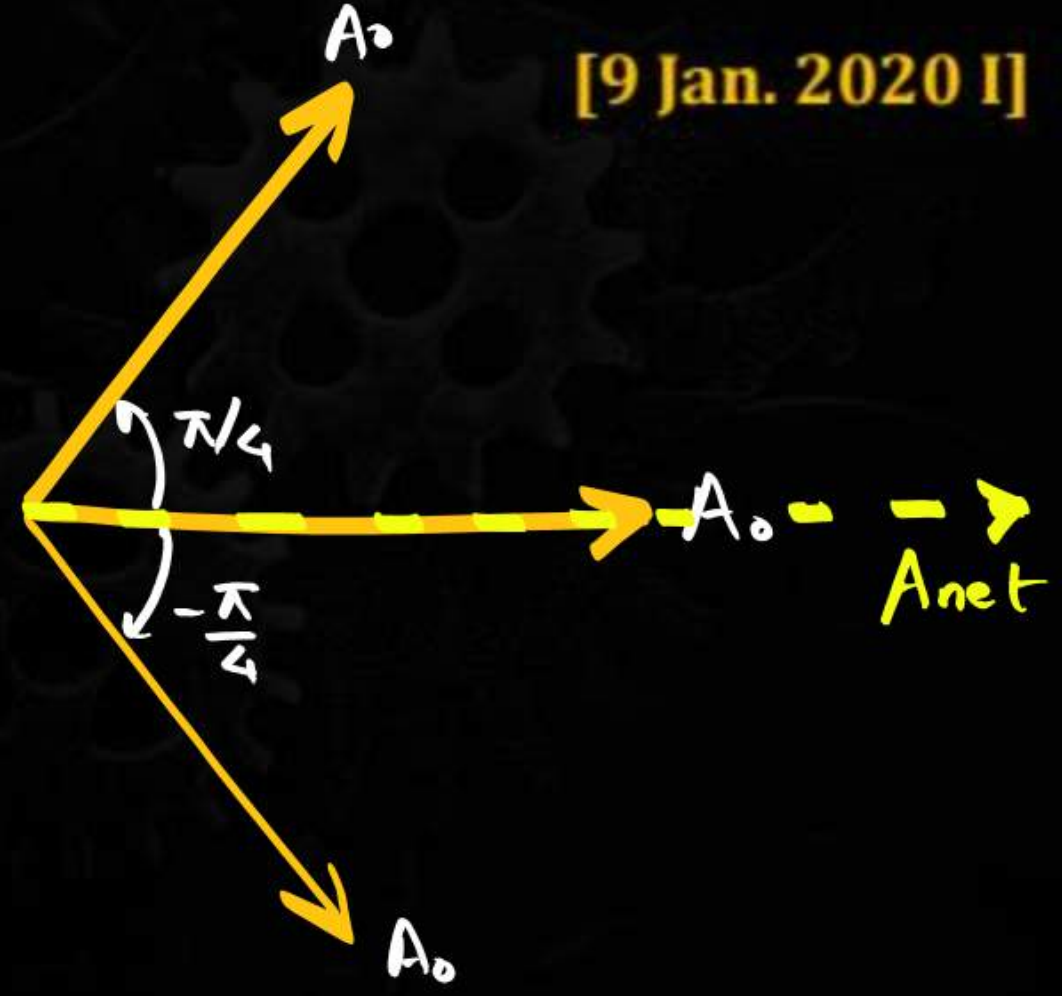
$A_{net} = A_0(1+\sqrt{2})$

$I \propto A_{net}^2$

$\propto (A_0(1+\sqrt{2}))^2$

$I \propto A_0^2 (1+\sqrt{2})^2$

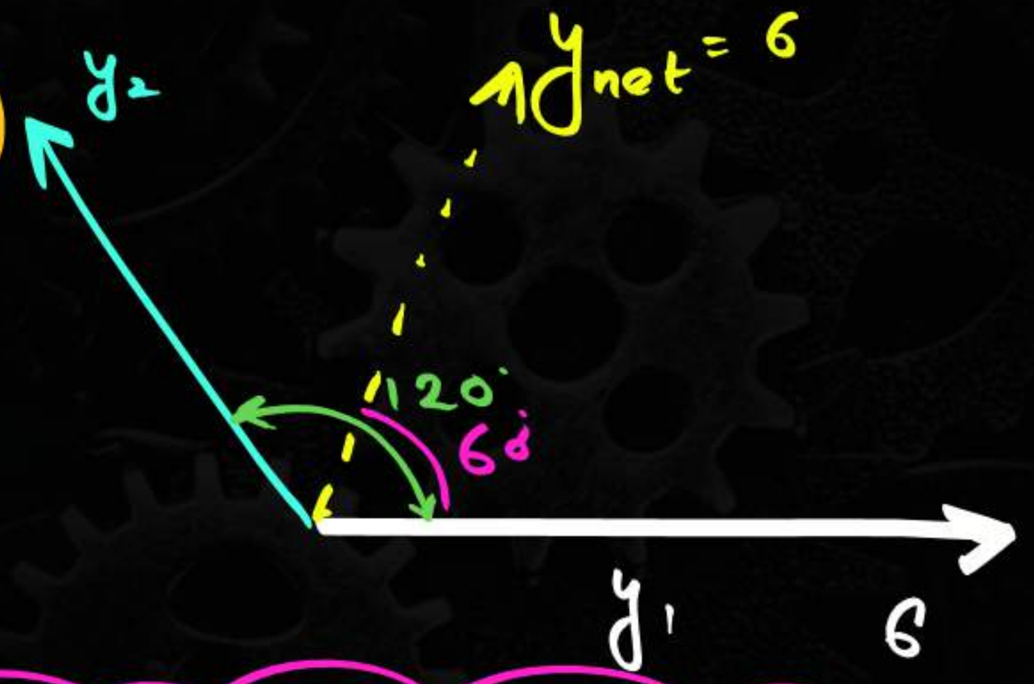
$I = I_0(1+2+2\sqrt{2})$



Q. Find the resultant wave upon interference of following two waves →

$$y_1 = 6\sin\left(\pi x - \frac{\pi}{2}t\right)$$

$$y_2 = 6\sin\left(\pi x - \frac{\pi}{2}t + \underset{120^\circ}{\frac{2\pi}{3}}\right)$$



$$A = \sqrt{6^2 + 6^2 + 2\times 6\times 6\times \cos(120^\circ)}$$

$$= \sqrt{36 + \cancel{36} + \cancel{72\times\left(-\frac{1}{2}\right)}}$$

$$= 6$$

$$\boxed{y_{net} = 6\sin\left(\pi x - \frac{\pi}{2}t + \frac{\pi}{3}\right)}$$ Ans.

Interference of Waves

2. Both waves of same frequency and amplitude and moving in opposite direction:

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t + \delta)$$

$$y = y_1 + y_2 = A\left[\sin(kx - \omega t) + \sin(kx + \omega t + \delta)\right]$$

$$y = y_1 + y_2 = A\left[\sin(kx-\omega t) + \sin(kx+\omega t+\delta)\right]$$

$$= A\left\{2\sin\left[\frac{kx-\cancel{\omega t}+kx+\cancel{\omega t}+\delta}{2}\right]\cdot\cos\left[\frac{\cancel{kx}-\omega t-(\cancel{kx}+\omega t+\delta)}{2}\right]\right\}$$

$$\left\{\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right\}$$

$$= 2A\sin\left(\frac{2kx+\delta}{2}\right)\cdot\cos\left(\frac{-2\omega t-\delta}{2}\right)$$

Standing wave →

$$y = \left[2A\sin\underbrace{\left(kx+\frac{\delta}{2}\right)}_{0,\ \pi,\ 2\pi}\right]\cdot\cos\left(\omega t+\frac{\delta}{2}\right)$$

$$\{\cos(-\theta) = \cos\theta\}$$

Pts. where, $kx+\frac{\delta}{2} = n\pi$, $y = 0$ for all time → Nodes.

Pts. where, $kx+\frac{\delta}{2} = (2n+1)\frac{\pi}{2}$, $y = \pm 2A\cos\left(\omega t+\frac{\delta}{2}\right)$ ⇒ Amplitude is max. → Antinode.

Standing Wave

- Generated due to interference of waves travelling in opposite direcn.
- Few points in the wave never move. ⟶ Nodes.
- Few points in the wave have max. amplitude ⟶ Antinodes
- All points in a standing cross their mean posn. at same time. & they also reach their extreme positions at same time.
- Different pts. have different amplitude.
- Phase diff. b/w pts. on wave is either 0 or π.

(Same dabba → $\Delta\phi = 0$
Consecutive dabba → $\Delta\phi = \pi$)

Standing Wave	Travelling Wave
• Wave appear to be standing at a point & does not transfer energy.	• Wave appear to be travelling & carrying some energy with it.
• Few points on standing wave never move. (node)	• There is no point on travelling wave which is always at rest.
• Different pts. have different amplitude.	• All pts. have same amplitude
• Phase diff. is either 0 or π.	• Phase diff. varies b/w 0 & 2π

Standing Waves

$\rightarrow y_1 = A\sin(kx-\omega t)$

$\rightarrow y_2 = A\sin(kx+\omega t)$

$$y = \left\{2A\sin\left(kx+\frac{\delta}{2}\right)\right\}\cos\left(\omega t+\frac{\delta}{2}\right)$$

1. String fixed at both ends:

Node at $x = 0 \Rightarrow$ $2A\sin\left(k\times 0+\frac{\delta}{2}\right) = 0$

$$\sin\frac{\delta}{2} = 0$$

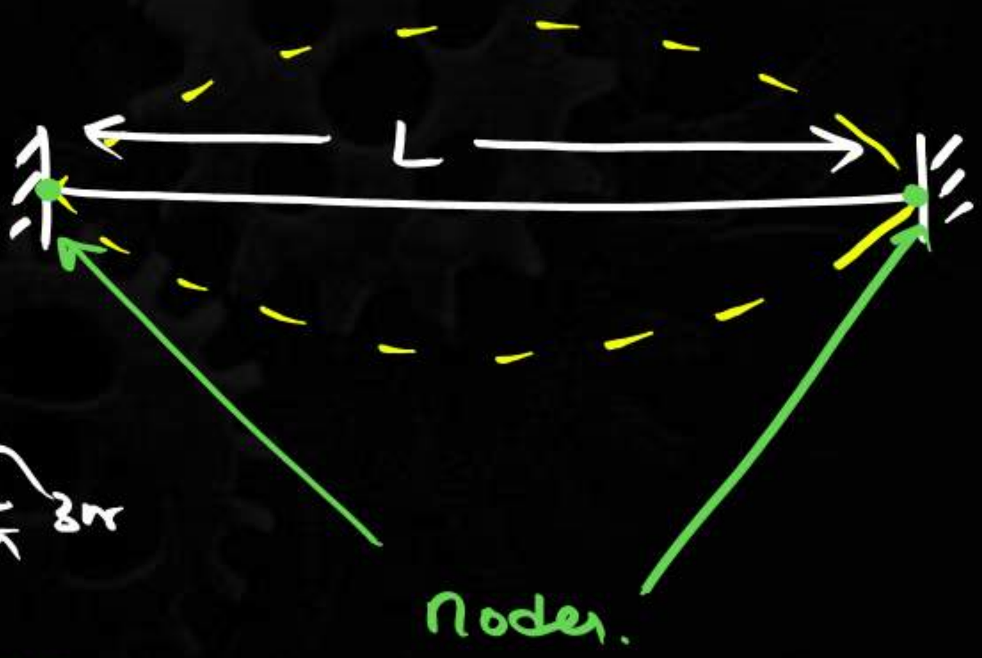
$$\Rightarrow \delta = 0$$

Node at $x = L \Rightarrow$

$$2A\sin(kL) = 0$$

$$kL = n\pi$$

$$L = \frac{n\pi}{k} = \frac{n\pi}{(2\pi/\lambda)} \Rightarrow \boxed{L = \frac{n\lambda}{2}}$$



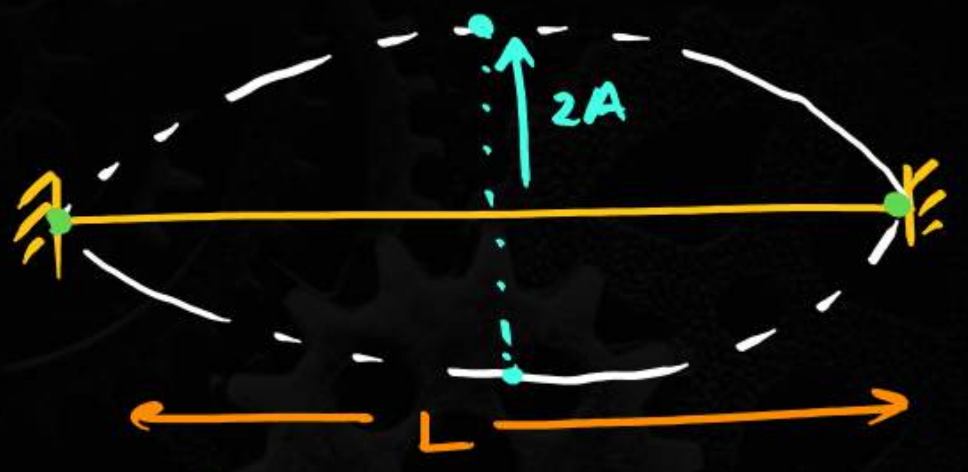
Harmonics of a Standing wave in a string fixed at both ends $\longrightarrow$ $v = \sqrt{\frac{T}{\mu}}$

① First Harmonic / Fundament Harmonic $\longrightarrow$

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$$

fundamental freq.

or Natural frequency $\rightarrow$ $f = \frac{v}{\lambda} = \frac{v}{2L}$



Posn. of nodes : 0, L

no. of nodes = 2

Posn. of anti-nodes = $\frac{L}{2}$

No. of anti-nodes = 1

② 2^{nd} Harmonic (I^{st} overtone) $\longrightarrow$

$$L = 2 \times \frac{\lambda}{2} \Rightarrow \lambda = L$$

$$f = \frac{v}{\lambda} = \frac{2v}{2L}$$

Posn. of nodes : 0, $\frac{L}{2}$, L

no. of nodes = 3

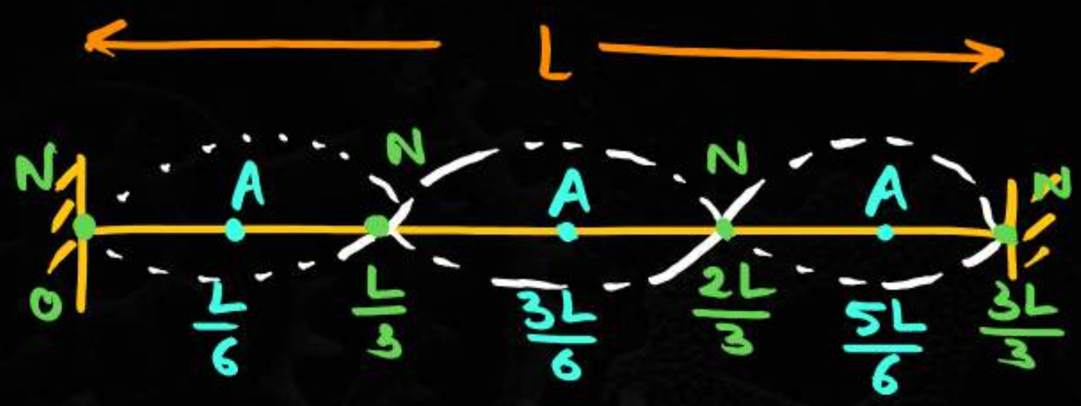
Posn. of anti-nodes = $\frac{L}{4}$, $\frac{3L}{4}$

No. of anti-nodes = 2

(3) 3^{rd} Harmonic (2^{nd} overtone) :-

$$L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3}$$

$$f = \frac{v}{\lambda} = \frac{3v}{2L}$$



Total no. of nodes = 4

Posn. of nodes = $0, \frac{L}{3}, \frac{2L}{3}, L$

No. of antinodes = 3

Posn. of anti-nodes = $\frac{L}{6}, \frac{3L}{6}, \frac{5L}{6}$

⋮

(n) n^{th} Harmonic ($(n-1)^{th}$ overtone) →

$$L = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L}$$

Total no of nodes = $n+1$

Posn. of nodes = $0, \frac{L}{2}, \frac{2L}{n}, \frac{3L}{n} \ldots, \frac{nL}{n}$

Total no. of anti-nodes = n

Posn. of anti-nodes = $\frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \ldots, \frac{(2n-1)L}{2n}$

Standing wave for sound in a tube open at both ends :—

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

(1) Fundamental / first Harmonic →

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$$

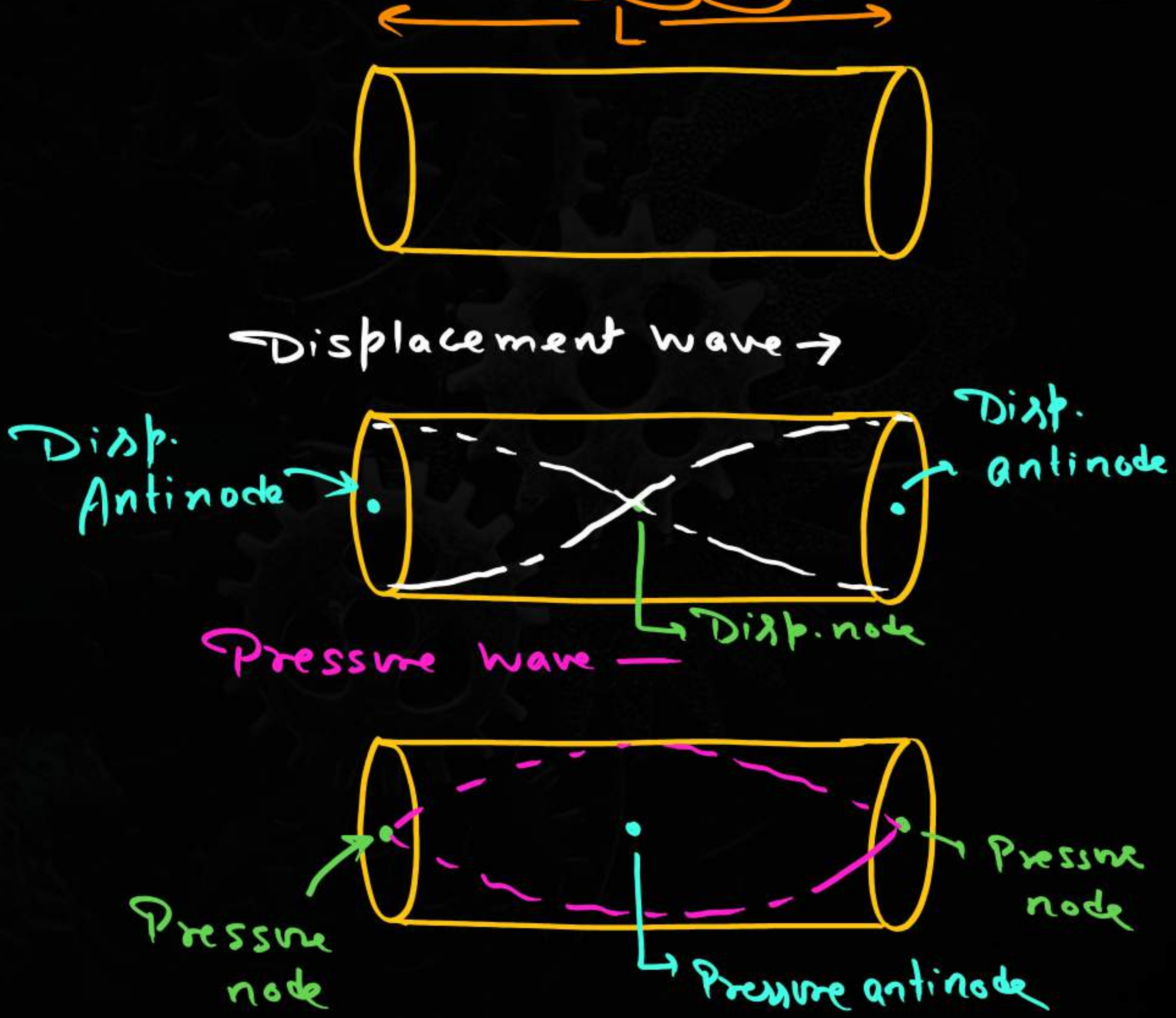
$$\boxed{f = \frac{v}{\lambda} = \frac{v}{2L}}$$

⋮

(n) n^{th} Harmonic ($(n-1)^{th}$ overtone) —

$$L = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

$$\boxed{f = \frac{v}{\lambda} = \frac{nv}{2L}}$$



Standing Waves

$$y = \underbrace{\{2A\sin(kx)\}}_{A_{eff.}}\cos(\omega t)$$

2. String fixed at one end only:

$$y = 2A\sin\left(kx + \frac{\delta}{2}\right)\cos\left(\omega t + \frac{\delta}{2}\right)$$

At $x = 0$, node — $2A\sin\left(k \times 0 + \frac{\delta}{2}\right) = 0$

$$\sin\frac{\delta}{2} = 0$$

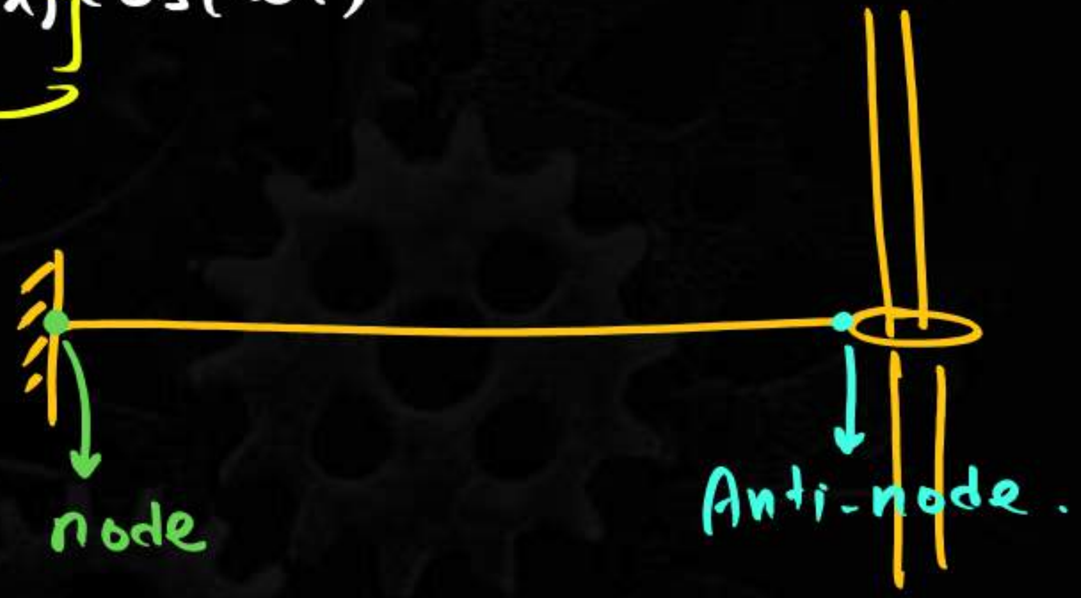
$$\delta = 0$$

At $x = L$, anti node —

$$2A\sin(kL) = \pm 2A$$

$$\sin(kL) = \pm 1$$

$$kL = \pm(2n-1)\frac{\pi}{2} \quad \{n = 1, 2, 3 \ldots\}$$



$$L = \frac{(2n-1)\pi}{2k} \qquad k = \frac{2\pi}{\lambda}$$

$$L = \frac{(2n-1)\cancel{\pi}}{2 \times \frac{2\cancel{\pi}}{\lambda}} = \frac{(2n-1)\lambda}{4}$$

Harmonics of standing wave in a string fixed at one end only →

① fundamental / first Harmonic →

only odd multiples (Harmonics) are obtained in this case.

$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$

fundamental/ Natural freq. → $\boxed{f = \frac{v}{\lambda} = \frac{v}{4L}}$

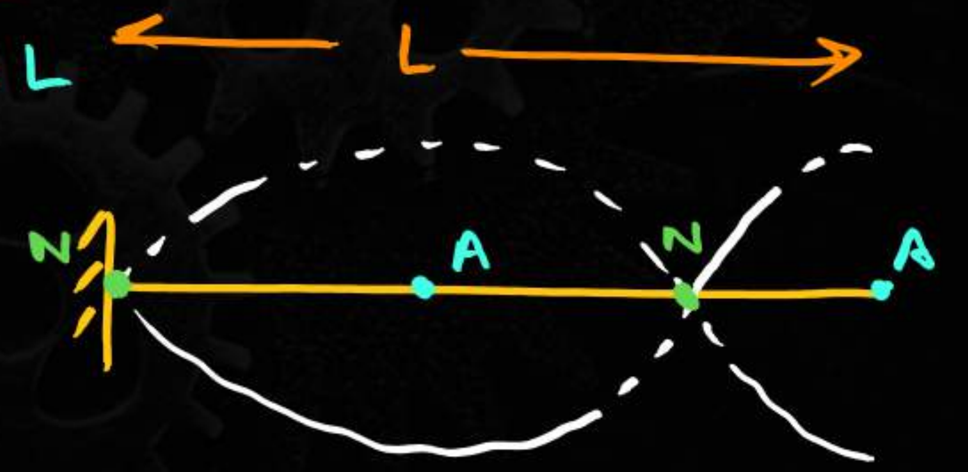
Total no. of nodes = 1

Posn. of nodes = 0

Total no. of anti-nodes = 1

Posn. of antides = L

② Third Harmonic (first overtone)



$L = 3 \times \frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{3}$

$\boxed{f = \frac{v}{\lambda} = \frac{3v}{4L}}$

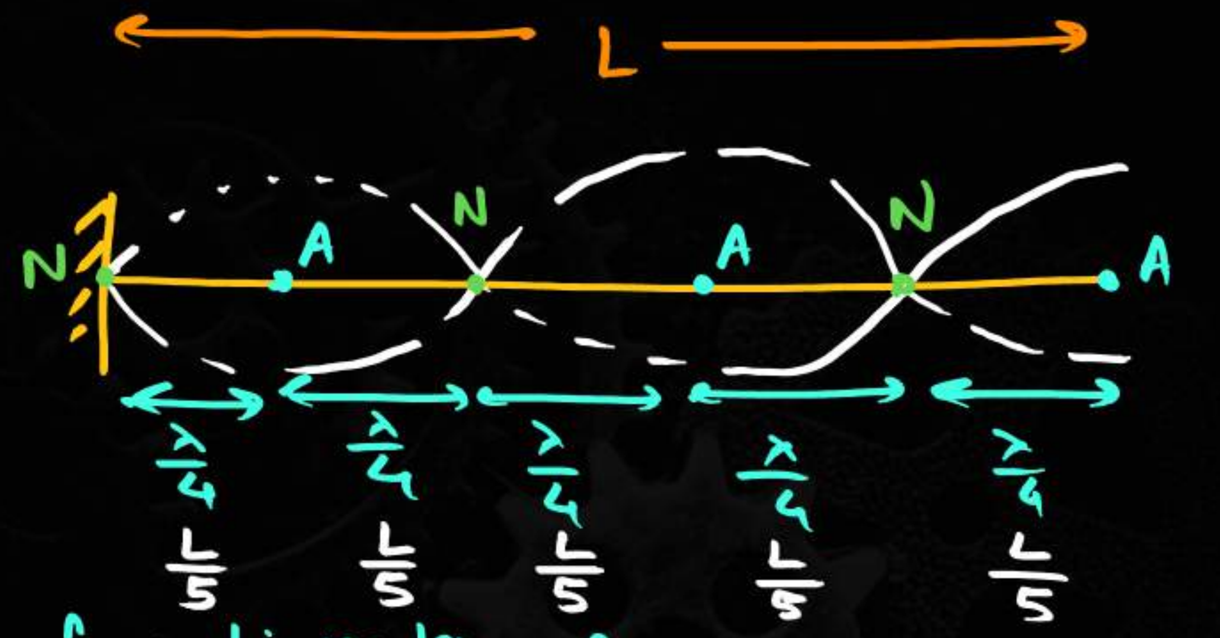
Total no. of nodes = 2

Posn. of nodes = $0, \frac{2L}{3}$

Total no. of anti-nodes = 2

Posn. of antides = $\frac{L}{3}, \frac{3L}{3}$

(3) 5^{th} Harmonic (2^{nd} overtone) ⟶



$$L = 5\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{5}$$

$$f = \frac{v}{\lambda} = \frac{5v}{4L}$$

Total no. of nodes = 3

Posn. of nodes = $0, \frac{2L}{5}, \frac{4L}{5}$

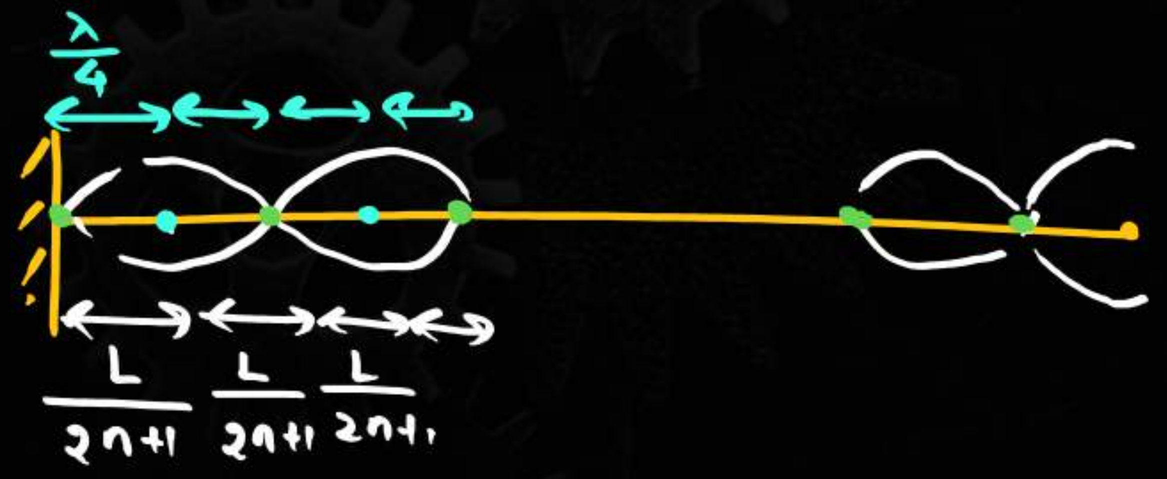
Total no. of anti-nodes = 3

Posn. of anti-nodes = $\frac{L}{5}, \frac{3L}{5}, \frac{5L}{5}$

$n = 0, 1, 2, 3 \ldots$

⋮

O $(2n+1)^{th}$ Harmonic (n^{th} overtone) ⟶



$$(2n+1)\frac{\lambda}{4} = L \Rightarrow \lambda = \frac{4L}{(2n+1)}$$

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{4L}$$

Total no. of nodes = $(n+1)$

Posn. of nodes = $0, \frac{2L}{2n+1}, \frac{4L}{2n+1} \ldots \frac{2nL}{(2n+1)}$

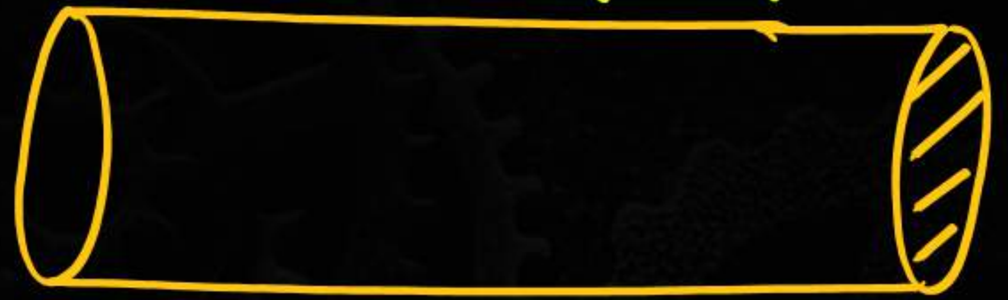
Total no. of antinodes = $(n+1)$

Posn. of anti-nodes = $\frac{L}{2n+1}, \frac{3L}{2n+1} \ldots, \frac{(2n+1)L}{2n+1}$

Standing wave for sound in a tube closed at one end →

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

(1) Fundamental / First Harmonic →



$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

Displacement wave →

$n = 0, 1, 2, \ldots$

(2) $(2n+1)^{th}$ Harmonic (n^{th} overtone) →

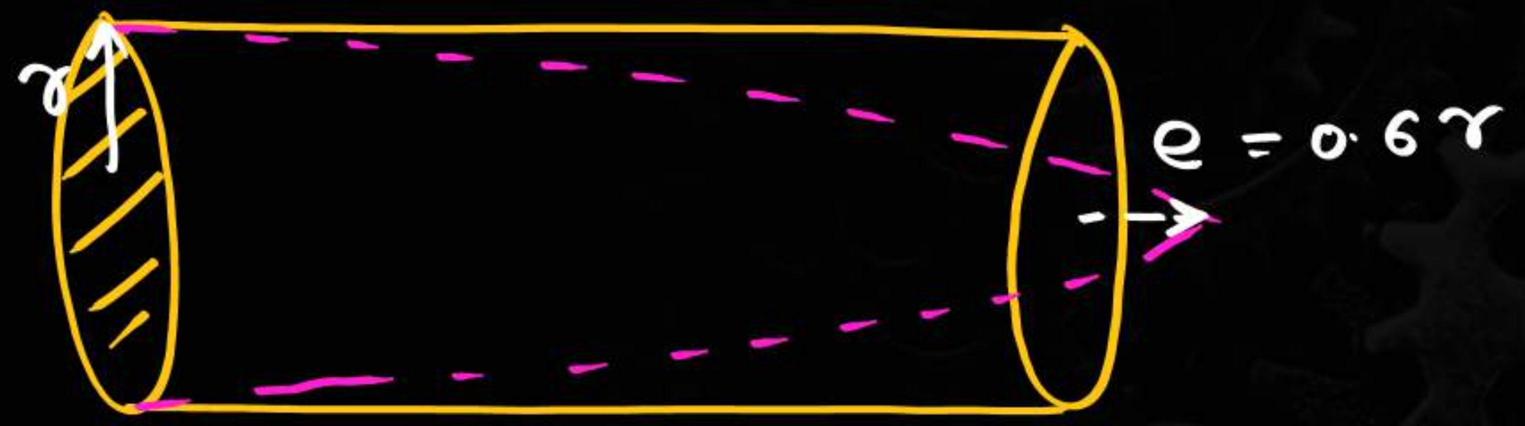
$$L = (2n+1)\frac{\lambda}{4}$$

$$\lambda = \frac{4L}{2n+1}$$

Pressure wave →

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{4L}$$

End correction



$$\frac{\lambda}{4} = L + e$$

Question

The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system?

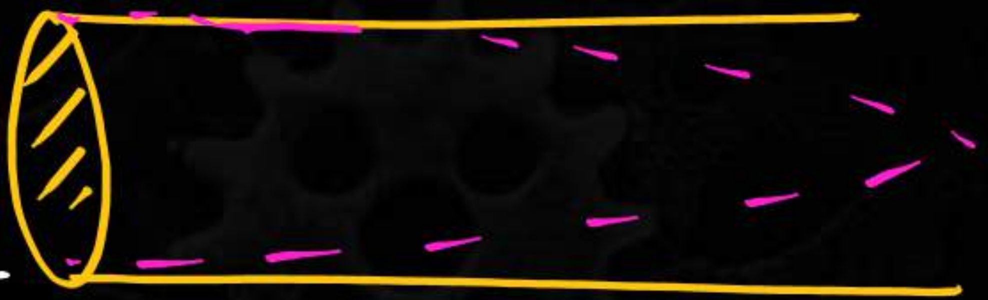
A) 10 Hz

B) 20 Hz

C) 30 Hz

D) 40 Hz

$$f = \frac{(2n+1)v}{4L}$$



$$f_{(2n+1)} - f_{(2n-1)} = \frac{(2n+1)v}{4L} - \frac{(2n-1)v}{4L}$$

$$f_0 = \frac{v}{4L}$$

$$\Rightarrow 260 - 220 = 2 \times \frac{v}{4L}$$

$$\frac{v}{4L} = \frac{40}{2} = 20\ \text{Hz}$$ Ans

Question

If we study the vibration of a pipe open at both ends, which of the following statements is not true?

are

(A) Open end will be antinode

(B) Odd harmonics of the fundamental frequency will be generated

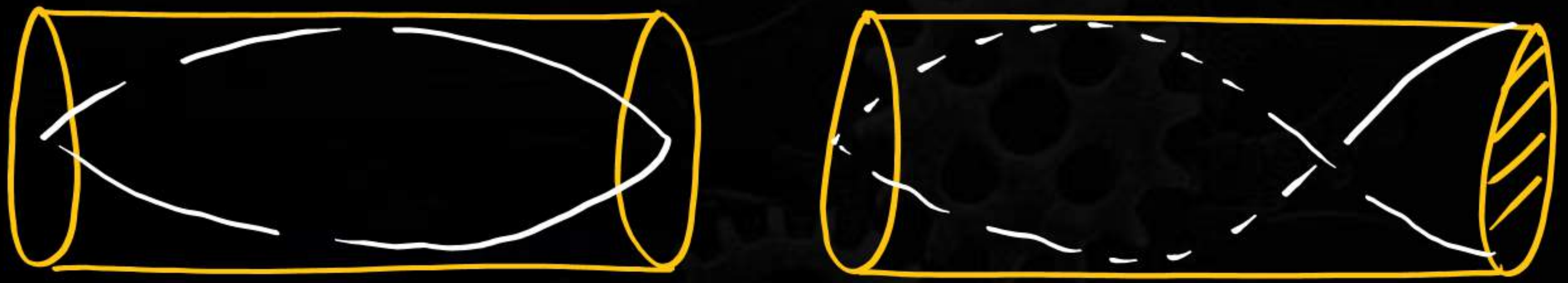
(C) All harmonics of the fundamental frequency will be generated

(D) Pressure change will be maximum at both ends

Question

The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is

l_1 $\quad$ $l_2 = 20\,cm$.



$f_1 = \frac{v}{2l_1}$ $\quad$ $f_3 = \frac{3v}{4l_2}$

$f_1 = f_3$

$\frac{\cancel{v}}{\cancel{2}l_1} = \frac{3\cancel{v}}{\cancel{4}_2 l_2} \Rightarrow l_1 = \frac{2l_2}{3} = \frac{2\times 20}{3} = \frac{40}{3}\,cm$ Ans.

(A) 12.5 cm

(B) 8 cm

(C) 13.3 cm

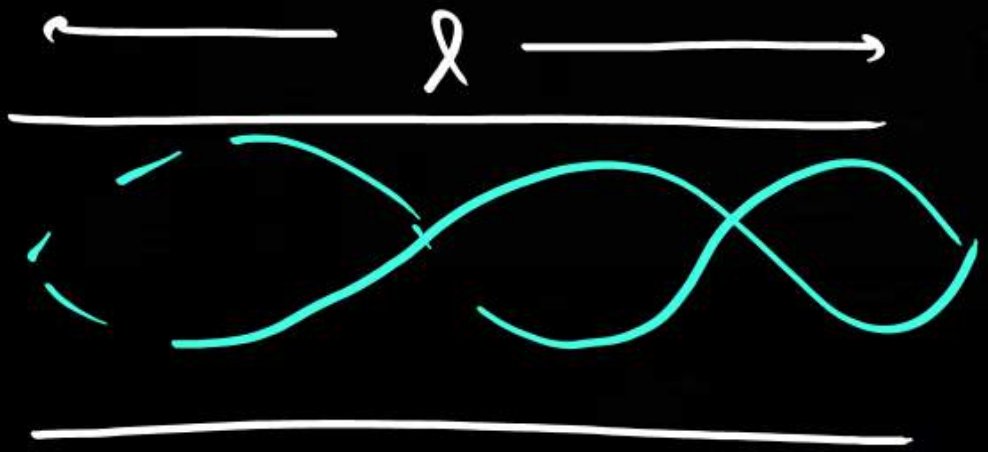
(D) 16 cm

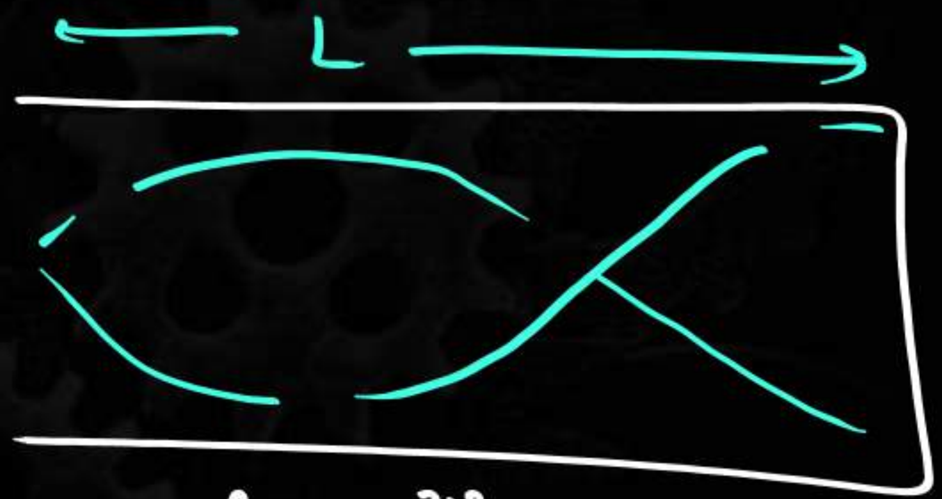
Question

3rd Harmonic

The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be

3rd Harmonic





$f_3 = \frac{3v}{2l}$

$f_3 = \frac{3v}{4L}$

$\frac{\cancel{3v}}{\cancel{2}l} = \frac{\cancel{3v}}{\underset{2}{\cancel{4}}L}$

$\boxed{l = 2L}$ Ans

(A) L

(B) 2L ✓

(C) L/2

(D) 4L

Question

Standing waves are produced in a 10 m long stretched string. If the string vibrates in 5 segments and the wave velocity is 20 m/s, the frequency is

(A) 10 Hz

(B) 5 Hz

(C) 4 Hz

(D) 2 Hz

$$f = \frac{nv}{2L}$$

$$f = \frac{5v}{2L}$$

$$= \frac{\cancel{2}\cancel{0} \times 5}{\cancel{2} \times 1\cancel{0}}$$

$$= 5\ \text{Hz}$$ Ans

$n = 5$

Question

A wave of frequency 100 Hz is sent along a string towards a fixed end. When this wave travels back after reflection, a node is formed at a distance of 10 cm from the fixed end of the string. The speeds of incident (and reflected) wave are

(A) 5 m/s

(B) 10 m/s

(C) 20 m/s

(D) 40 m/s

$f = 100\ \text{Hz}.$

$\frac{\lambda}{2} = 10 \times 10^{-2}$

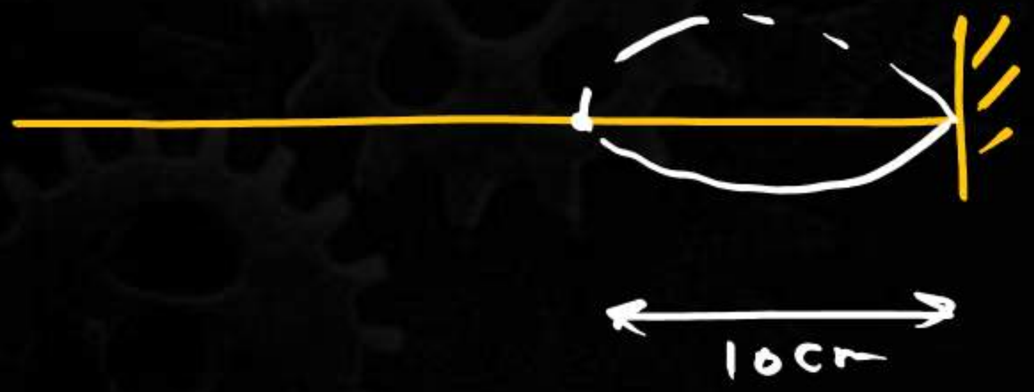
$\lambda = 2 \times 10^{-1}\ \text{m}.$

$f = \frac{v}{\lambda}$

$v = \lambda f$

$= 2 \times 10^{-1} \times 100$

$= 20\ \text{m/s}.$



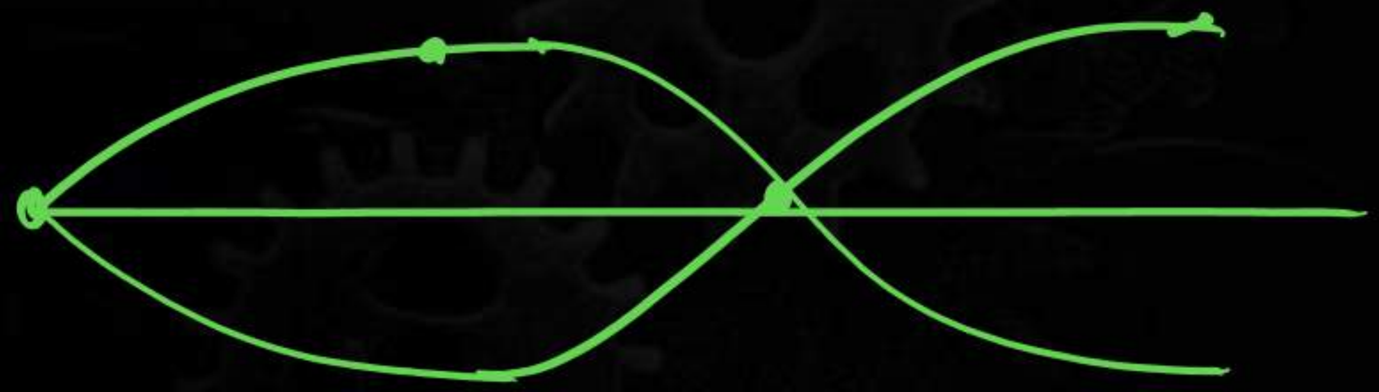
Question

A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has

$(2n+1)$

$(2\times3+1) = 7^{th}$ Harmonic

I ot



A) three noes and three antinodes

B) three nodes and four antinodes

C) four nodes and three antinodes

D) four nodes and four antinodes

Question

H.W

Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is [2002]

(A) $1:2$

(B) $1:4$

(C) $2:1$

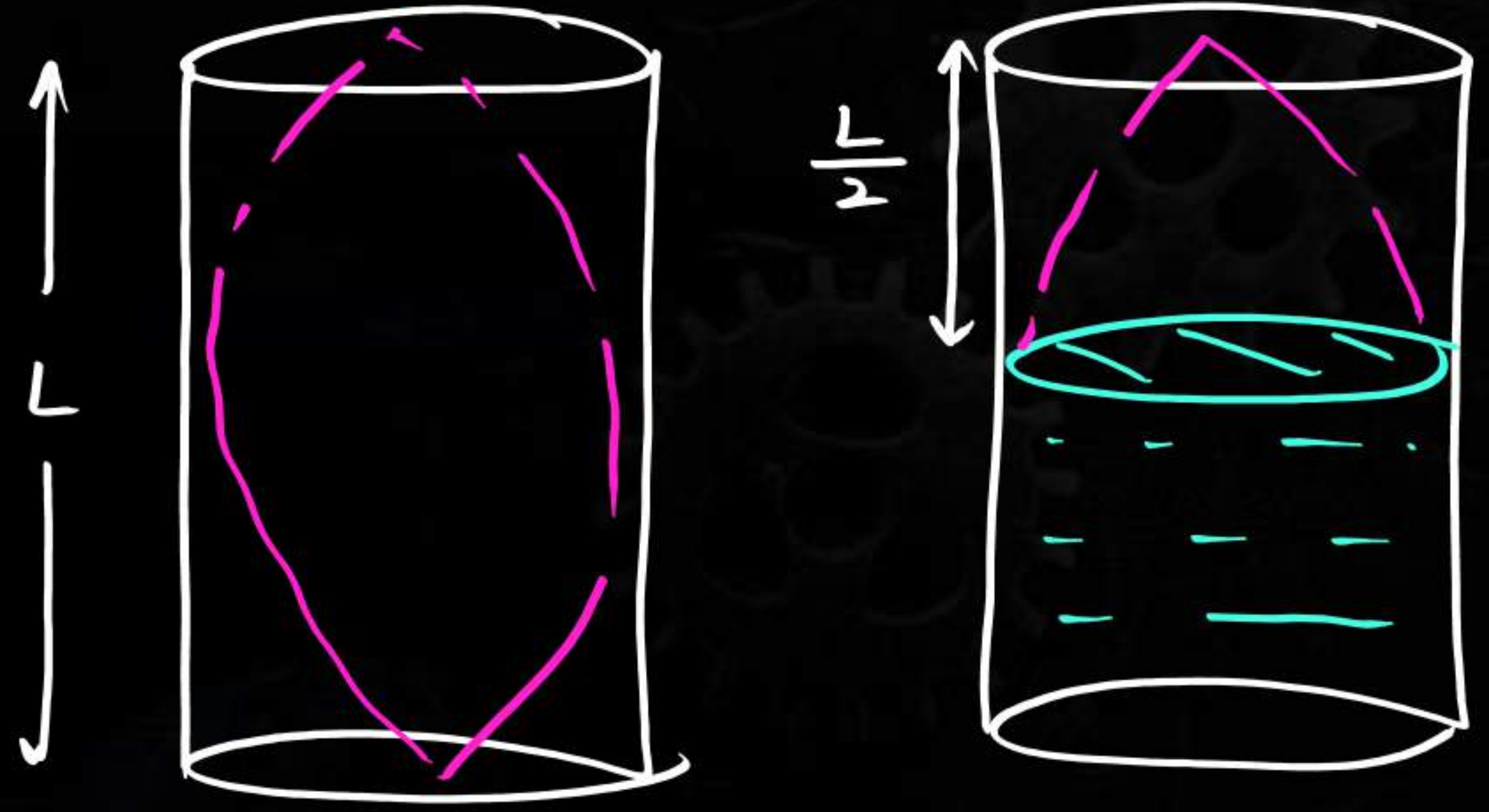
(D) $4:1$

Question

A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now: [2016]

$f = \frac{v}{2L}$

$f' = \frac{v}{4(L/2)} = \frac{2v}{4L} = \frac{v}{2L}$



(A) $2f$

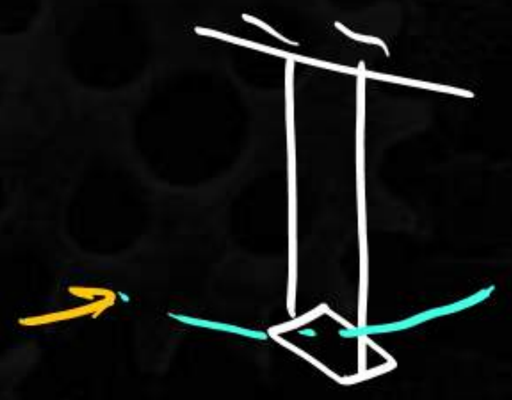
(B) f

(C) $f/2$

(D) $3f/4$

Resonance

freq. of External source = freq. of Standing wave.



Question

A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this strings is

[CBSE AIPMT 2015]

(A) 155 Hz

(B) 205 Hz

(C) 10.5 Hz

(D) 105 Hz

$f = \frac{nv}{2L}$

$f_2 = \frac{(n+1)v}{2L} = 315\ Hz$

$f_1 = \frac{nv}{2L} = 420\ Hz$

$f_0 = |f_2 - f_1| = 420 - 315 = 20 + 85$

$= 105\ Hz.$

Question

An air column, closed at one end and open at the other, resonates with a tunning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tunning fork is

A. 100 cm

B. 150 cm

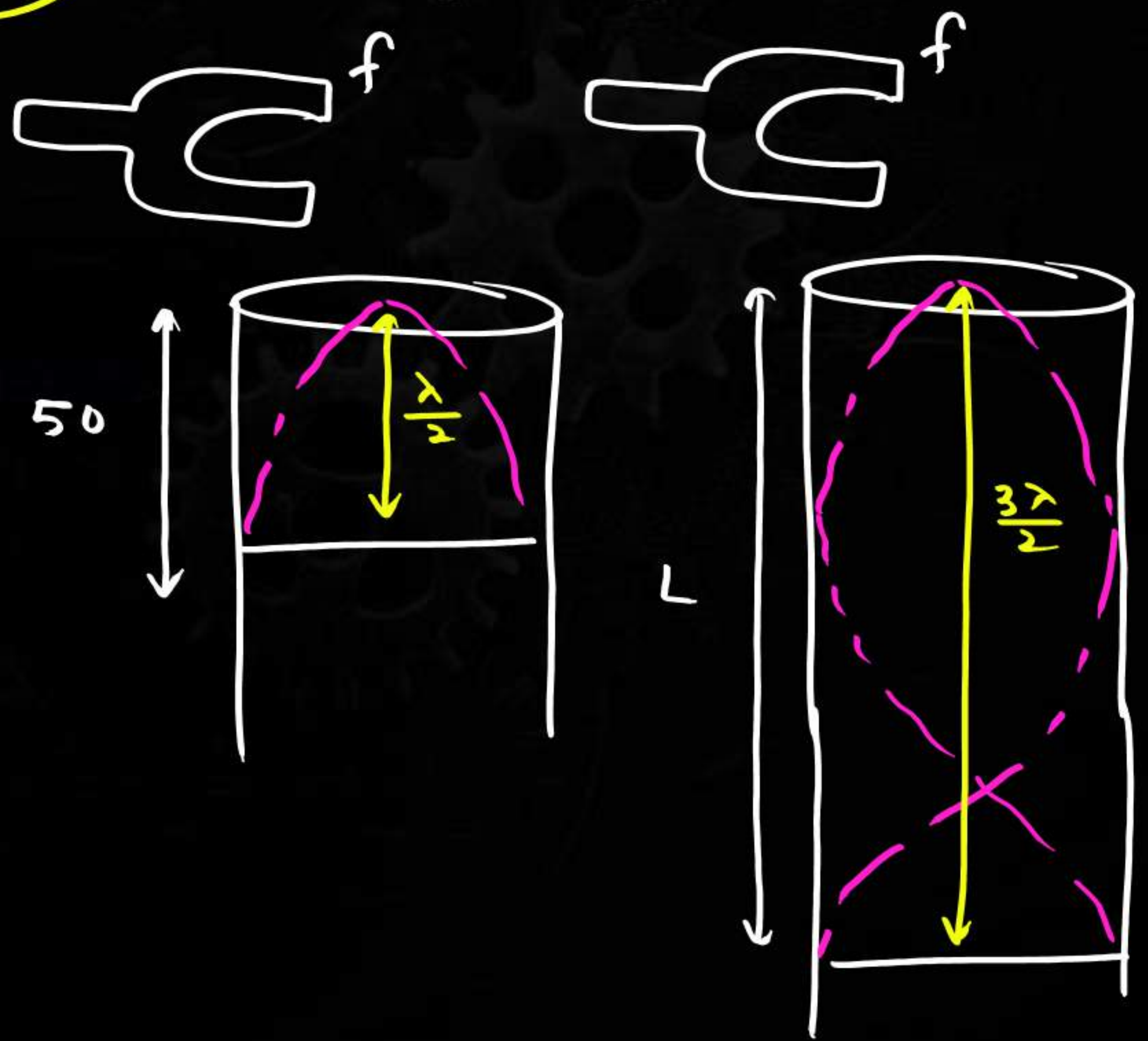
C. 200 cm

D. 66.7 cm

$$\frac{\cancel{v}}{4L_1} = \frac{3\cancel{v}}{4L}$$

$$\frac{1}{\cancel{4}\times 50} = \frac{3}{\cancel{4}\times L}$$

$$\boxed{L = 150\,cm}$$



Question

H.W.

A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of $27^\circ C$, two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at $27^\circ C$ is

A 350 m/s

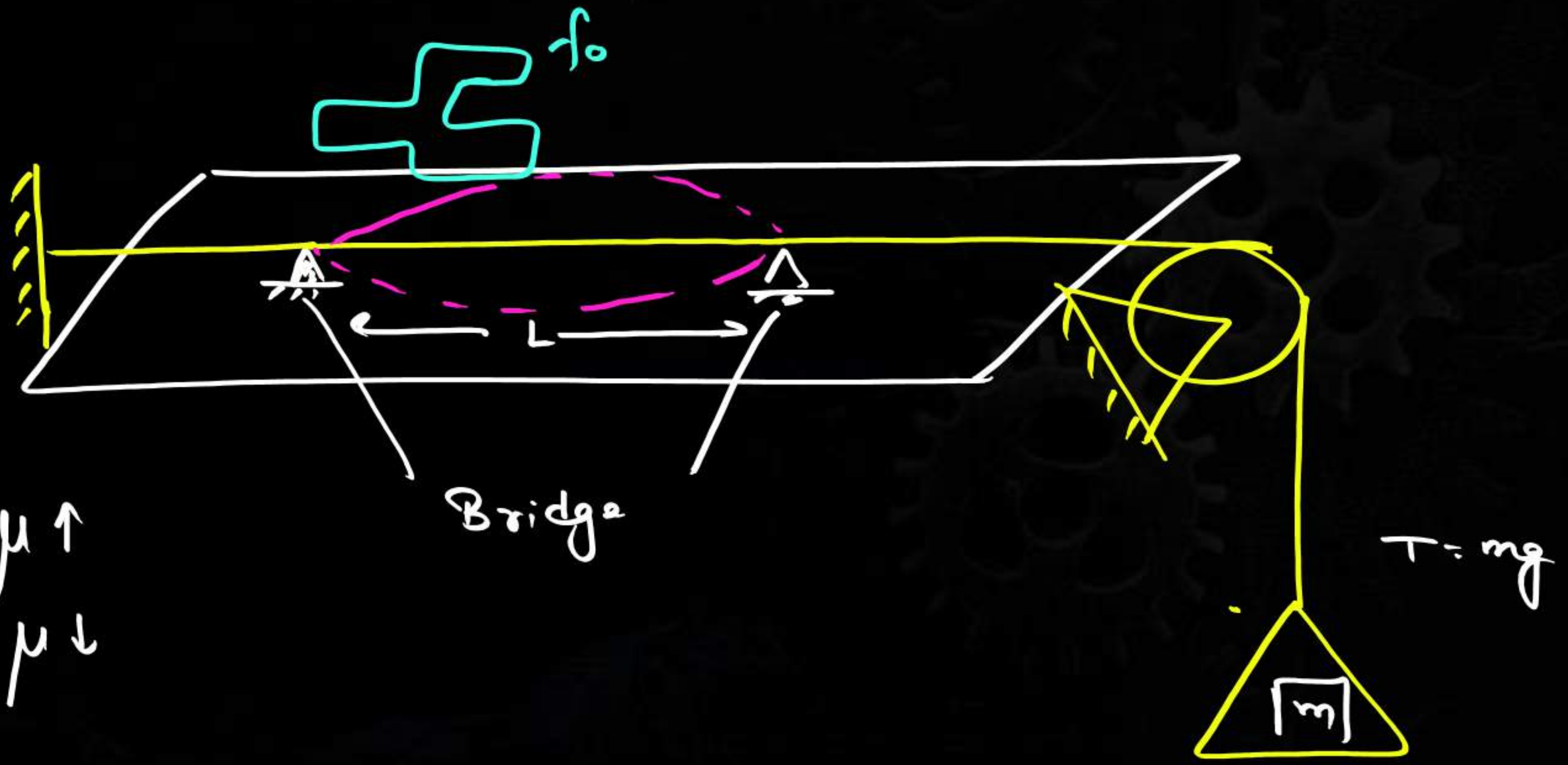
B 339 m/s

C 330 m/s

D 300 m/s

Sonometer

$$f_0 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$



Waxing → $\mu \uparrow$

Waning → $\mu \downarrow$

Question

A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are: 420 Hz and 490 Hz. Then L in meters is:

[9 Jan. 2020 (II)]

$$f = \frac{nv}{2L}$$

$$f_0 = \frac{v}{2L} = f_n - f_{n-1}$$

$$= 490 - 420 = 70\,\text{Hz}$$

$$\frac{v}{2L} = 70 \Rightarrow L = \frac{v}{140}$$

$$= \frac{1}{140}\sqrt{\frac{\cancel{540}^{9}}{\cancel{6}\times 10^{-3}}}$$

$$= \frac{3\times 10^{\cancel{2}}}{14\cancel{0}} = \frac{30}{14} = \frac{15}{7}$$

(A) 2.1 m

(B) 1.1 m

(C) 8.1 m

(D) 5.1 m

Question

HW

The length of the string of a musical instrument is 90 cm and has a fundamental frequency of 120 Hz. Where should it pressed to produce fundamental frequency of 180 Hz?

A. 75 cm

B. 60 cm

C. 45 cm

D. 80 cm

Question

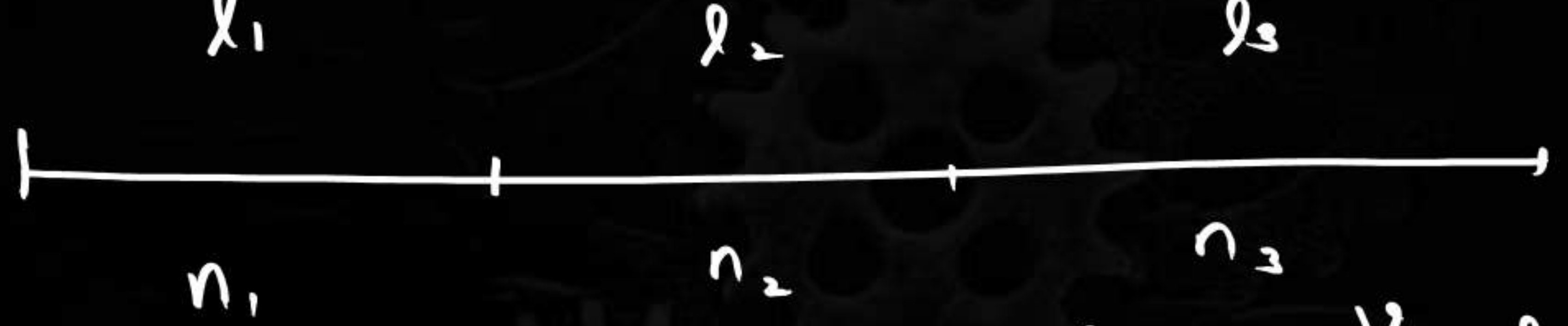
If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by

(A) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

(B) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

(C) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

(D) $n = n_1 + n_2 + n_3$



$n_1 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{2n_1}$ $\quad n_2 = \frac{v}{2l_2} \Rightarrow l_2 = \frac{v}{2n_2}$ $\quad n_3 = \frac{v}{2l_3} \Rightarrow l_3 = \frac{v}{2n_3}$

$$n = \frac{v}{2(l_1+l_2+l_3)} = \frac{v}{\cancel{2}\left(\frac{v}{\cancel{2}n_1} + \frac{v}{\cancel{2}n_2} + \frac{v}{\cancel{2}n_3}\right)}$$

$$\frac{\cancel{v}}{n} = \frac{\cancel{v}}{n_1} + \frac{\cancel{v}}{n_2} + \frac{\cancel{v}}{n_3}$$

Question

H.W

A stretched string resonates with tuning fork of frequency 512 Hz when length of the string is 0.5 m. The length of the string require to vibrate resonantly with a tuning fork of frequency 256 Hz would be

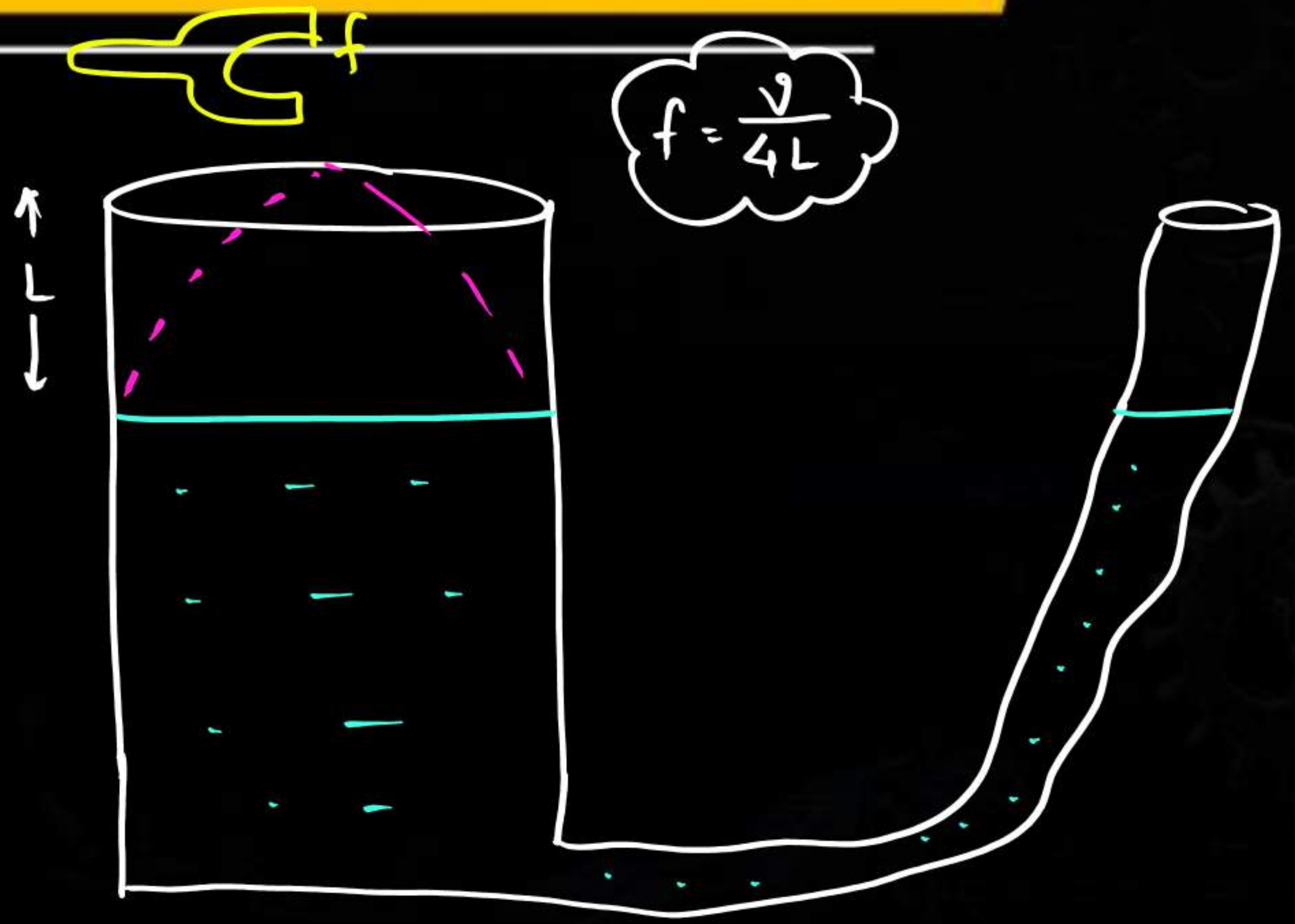
A) 0.25 m

B) 0.5 m

C) 1 m

D) 2 m

Resonance Tube



Question

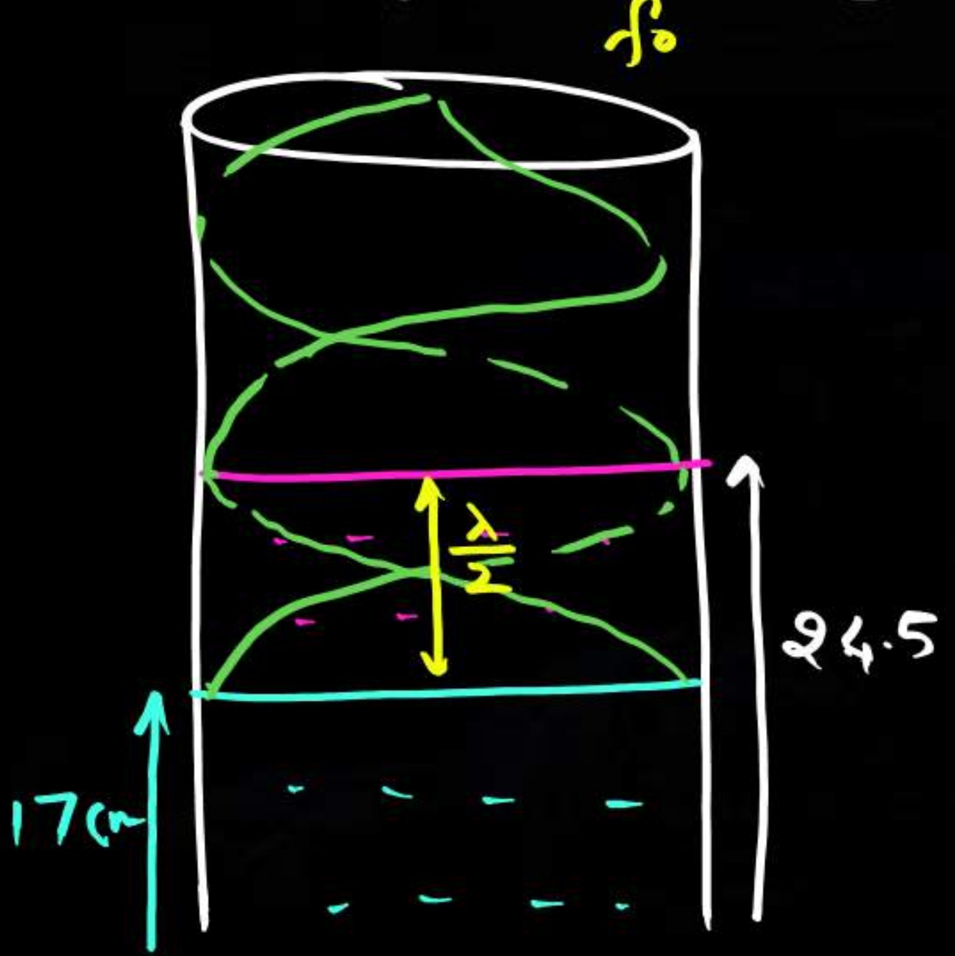
In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is: [Sep. 05, 2020 (I)]

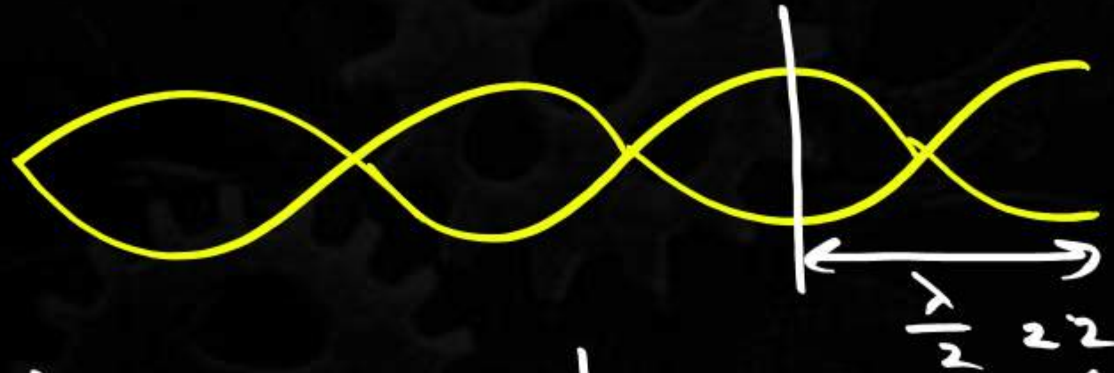
(A) 2200 Hz

(B) 550 Hz

(C) 1100 Hz

(D) 3300 Hz





$$\frac{\lambda}{2} = 24.5 - 17$$

$$\frac{\lambda}{2} = 7.5$$

$$\lambda = 15\text{ cm}.$$

$$f_0 = \frac{v}{\lambda} = \frac{\cancel{330}^{\,22}}{\cancel{15} \times 10^{-2}}$$

$$= 2200\text{ Hz}.$$

Question

Repeat

A cylindrical resonance tube open a both ends, has a fundamental frequency f, in air. If half of the length is dipped vertically in water, the fundamental frequency of the air column will be

(A) $2f$

(B) $3f/2$

(C) f

(D) $f/2$

Question

H.W

The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (velocity of sound = 340 m^{-1})

$$\frac{(2n+1)v}{4L} \leq 1250$$

(A) 4

(B) 5

(C) 7

(D) 6

Question

A tuning fork with frequency 800 Hz produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at length $\underbrace{9.75 \text{ cm}, 31.25}_{\Delta l = \frac{\lambda}{2}}$ $\underbrace{\text{cm and } 52}_{\Delta l = \frac{\lambda}{2}}.75$ cm. The speed of sound in air is

(A) 500 m/s

(B) 156 m/s

(C) 344 m/s ✓

(D) 172 m/s

$$\frac{\lambda}{2} = 31.25 - 9.75$$

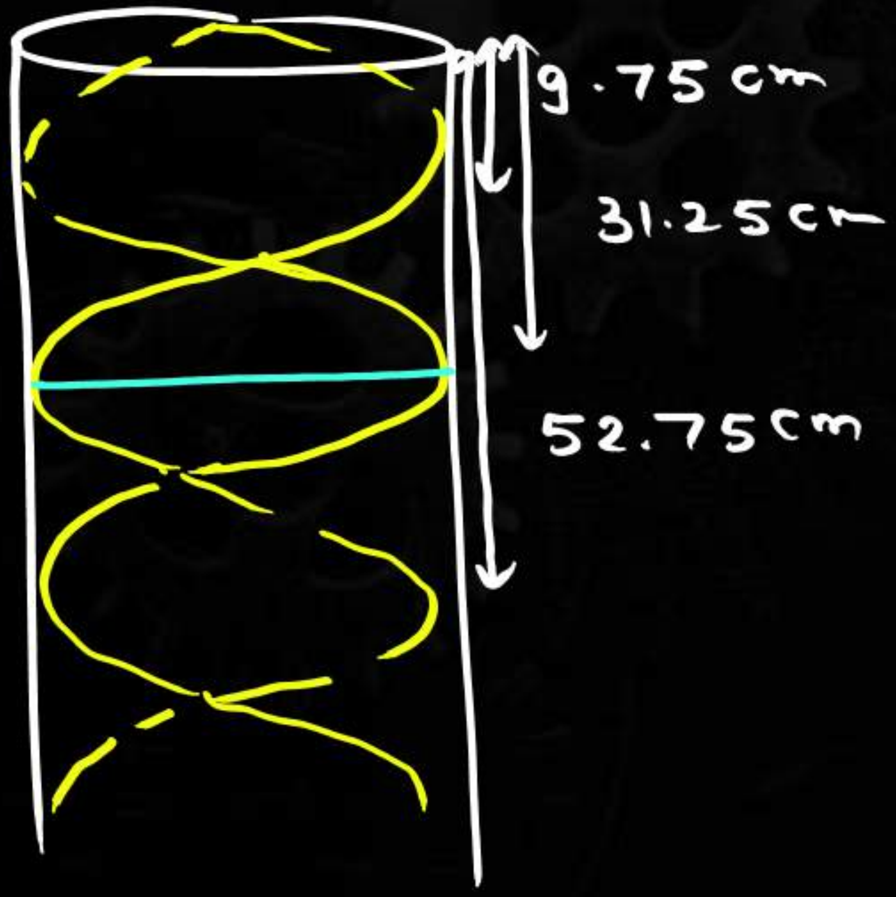
$$\frac{\lambda}{2} = 21.5$$

$$\lambda = 43 \text{ cm}$$

$$v = \lambda \cdot f$$

$$= 43 \times \frac{1}{\cancel{100}} \times 8\cancel{00}$$

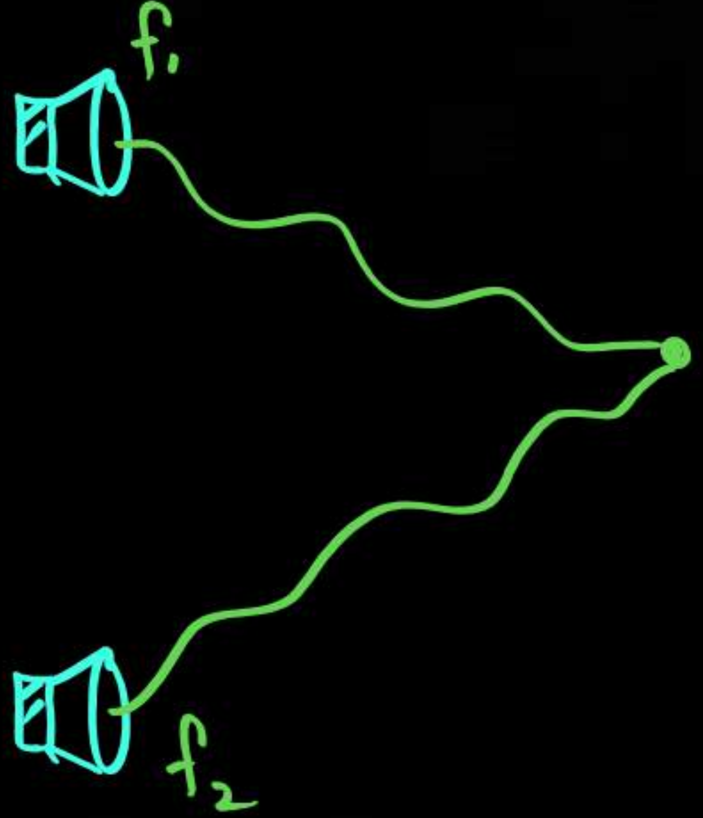
$$= 344 \text{ m/s}$$



Beats

generally - (Sound)

3.) Interference of waves of slightly different freq.



$$y_1 = A\sin(\omega_1 t)$$

$$y_2 = A\sin(\omega_2 t)$$

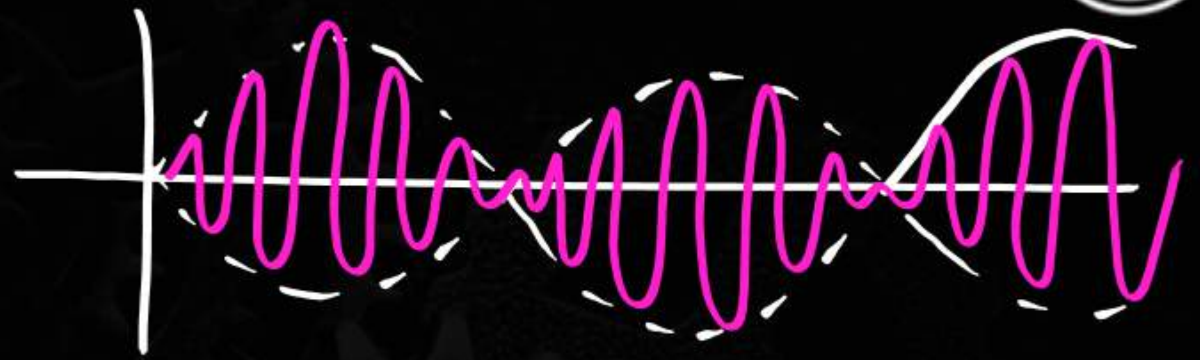
$$y = y_1 + y_2 = A\left[\sin(\omega_1 t) + \sin(\omega_2 t)\right]$$

$$= A\,2\sin\left(\frac{\omega_1 t + \omega_2 t}{2}\right)\cos\left(\frac{(\omega_1 - \omega_2)t}{2}\right)$$

$$\left\{\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right\}$$

$$= \underbrace{\left\{2A\cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\right\}}_{\text{Time varying amplitude}}\sin\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right]$$

$$\text{freq. of amplitude} = \frac{\omega}{2\pi} = \frac{\frac{\omega_1 - \omega_2}{2}}{2\pi}$$



$$\text{freq. of beat (loudness)} = 2 \times f_{amplitude}.$$

$$= \cancel{2} \times \frac{\omega_1 - \omega_2}{\cancel{2} \times 2\pi}$$

$$f_{beat} = \frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi}$$

$$\boxed{f_{beat} = f_1 - f_2}$$

Question

Two sound waves with wavelengths 5 m and 5.5 m respectively, each propagate in a gas with velocity 330 m/s. We expect the following number of beat per second

$f_1 = \frac{v}{\lambda_1}$ $f_2 = \frac{v}{\lambda_2}$

$f_{beat} = |f_1 - f_2|$

$= v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$= 330\left[\frac{1}{5} - \frac{1}{5.5}\right]$

$= 330 \times \left[\frac{5.5-5}{5.5 \times 5}\right]$

$= 12 \times 0.5 = 6$

A) 12

B) zero

C) 1

D) 6

Question

Two vibrating tuning forks produce progressive waves given by

$y_1 = 4 \sin 500\pi t$ and

$y_2 = 2 \sin 506\pi t$. Number of beat produced per minute is

$f_1 = \frac{\omega_1}{2\pi} = \frac{500\pi}{2\pi} = 250\,Hz.$

$f_2 = \frac{506\pi}{2\pi} = 253\,Hz.$

$f_{beat} = f_2 - f_1 = 3\,Hz$

A 360

B 180

C 3

D 60

Question

H.W

Two sources of sound placed closed to each other, are emitting progressive wave given by $y_1 = 4 \sin 600\pi t$ and $y_2 = 5 \sin 608\pi t$. An observer located near these two sources of sound will hear

$A_{max} = 9 \quad I_{max} = 81$

$A_{min} = 1 \quad I_{min} = 1$

(A) 4 beat/s with intensity ratio 25 : 16 between waxing and waning

(B) 8 beat/s with intensity ratio 25 : 16 between waxing and waning

(C) 8 beat/s with intensity ratio 81 : 1 between waxing and waning

(D) 4 beat/s with intensity ratio 81 : 1 between waxing and waning

Question

H.W

Two waves of wavelength 50 cm and 51 cm produce 12 beat/s. The speed of sound is

A 306 m/s

B 331 m/s

C 340 m/s

D 360 m/s

Question

f_0

A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound of air is 340 ms^{-1})

[2018]

A 190 cm

B 180 cm

C 220 cm

D 200 cm

$$257 \text{ or } 255 = \frac{3v}{2L}$$

$$255 = \frac{3 \times 340}{2 \times L}$$

$$L = \frac{3 \times \cancel{340}\ \cancel{170}\ 34}{\cancel{2} \times \cancel{255}\ 51}$$

$$= \frac{\cancel{3} \times \cancel{34}\ 2}{\cancel{51}\ \cancel{3}} = 2\,m$$

Question

5 beats/ second are heard when a turning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95m or 1m. The frequency of the fork will be: [April 15, 2018]

(A) 195 Hz

(B) 251 Hz

(C) 150 Hz

(D) 300 Hz

f_0

$f_0 - 5$ $\quad$ $f_0 + 5$

$f \downarrow = \frac{v}{2L\uparrow}$

$f_0 - 5 = \frac{v}{2 \times 1}$ $\quad$ $f_0 + 5 = \frac{v}{2 \times 0.95}$

$f_0 + 5 = \frac{v}{2 \times 0.95}$

$-f_0 \mp 5 = -\frac{v}{2 \times 1}$

$10 = \frac{v}{1.90} - \frac{v}{2}$

$\frac{v}{1.9} - \frac{v}{2} = 10$

$v = $

Doppler's Effect

$f_0 = \frac{c}{\lambda_0}$

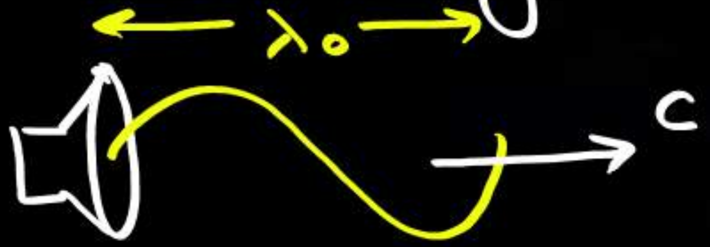
S

f_0 , T_0

O

f_0

Case I :- Observer is moving towards source →



$(c + v_0) T = \lambda_0$

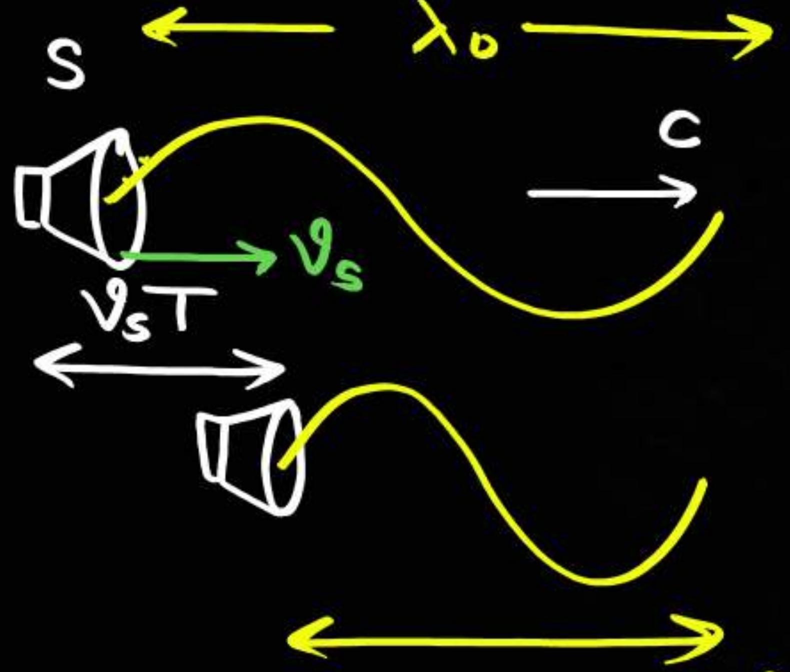
$T = \frac{\lambda_0}{c + v_0}$

$f = \frac{c + v_0}{\lambda_0}$

$f = \frac{c}{\lambda_0}\left[1 + \frac{v_0}{c}\right]$

$f = f_0\left[\frac{c + v_0}{c}\right]$

Case II :-



$$\lambda = \lambda_0 - v_s T$$

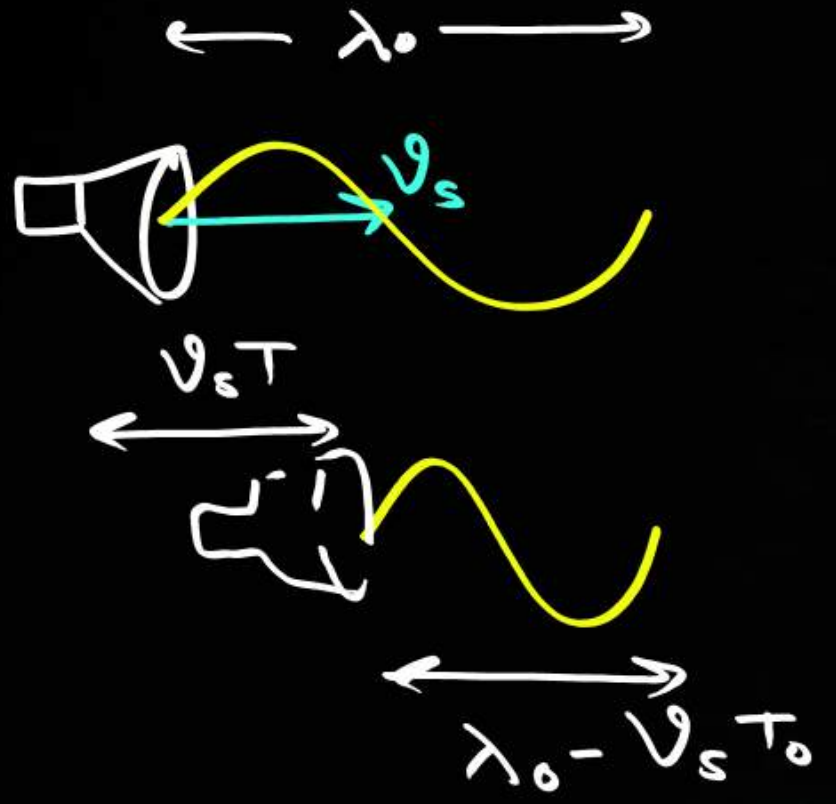
$$f = \frac{c}{\lambda} = \frac{c}{\lambda_0 - v_s T}$$

$$= \frac{c}{cT - v_s T}$$

$$= \frac{c}{T(c - v_s)}$$

$$= f_0\left[\frac{c}{c - v_s}\right]$$

Case III →



$$T = \frac{\lambda_0 - v_s T}{c + v_0}$$

$$f = \frac{1}{T} = \frac{c + v_0}{\lambda_0 - v_s T_0}$$

$$= \frac{c + v_0}{c \times T_0 - v_s T_0} = f_0 \left[\frac{c + v_0}{c - v_s}\right]$$

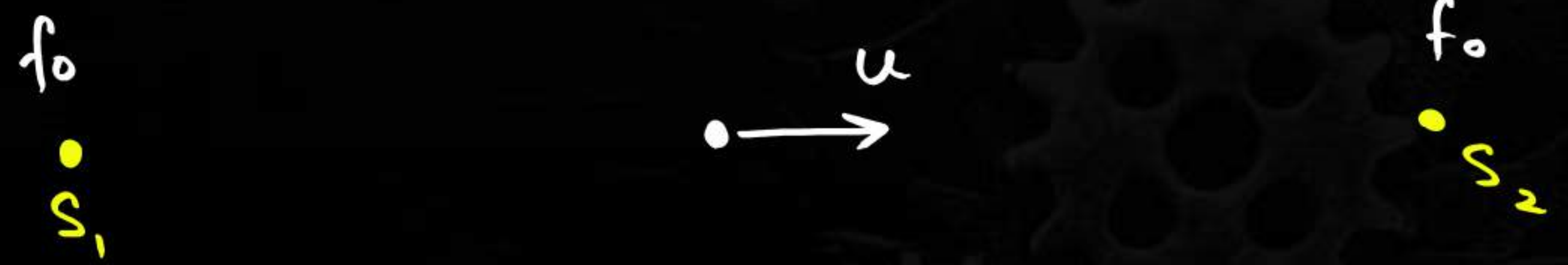
DOPPLERS EFFECT

OS → Operating System

$$f = f_0 \left[\frac{c + v_0}{c - v_s}\right]$$

Question

Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then u equals: **[12 April 2019 II]**



(A) 5.5 m/s

(B) 15.0 m/s

(C) 2.5 m/s

(D) 10.0 m/s

$$f_2 = f_0\left[\frac{c+u}{c}\right]$$

$$f_1 = f_0\left[\frac{c-u}{c}\right]$$

$$f_2 - f_1 = 10$$

$$\Rightarrow \frac{f_0}{c}\left[(\cancel{c}+u)-(\cancel{c}-u)\right] = 10$$

$$\frac{2u\times f_0}{c} = 10$$

$$u = \frac{\cancel{10}^{5}\times \cancel{330}}{2\times \cancel{660}} = \frac{5}{2}.$$

Question

Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a home having a frequency 400 Hz. The frequency heard by the driver of the second car is [velocity of sound 340 m/s]

A. 350 Hz

B. 361 Hz

C. 411 Hz

D. 448 Hz

$$f = f_0\left[\frac{c+v_0}{c-v_s}\right]$$

$$= 400\left[\frac{340+16.5}{340-20}\right] = 400 \times \frac{356.5}{320}$$

Question

A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15 ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (Take, velocity of sound in air = $330 ms^{-1}$)

(A) 800 Hz

(B) 838 Hz

(C) 885 Hz

(D) 765 Hz

$$f_1 = f_0 \left[\frac{c+0}{c-v_s}\right]$$

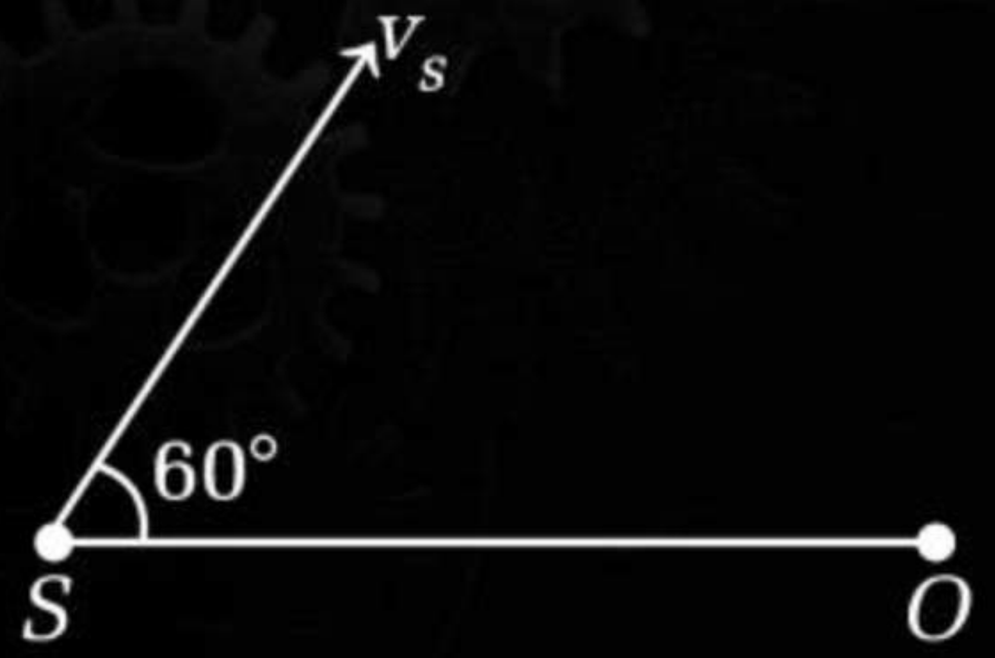
$$f_1 = 800\left[\frac{330}{330-15}\right]$$

$$= 800 \times \frac{\cancel{330}^{\cancel{66}\,22}}{\cancel{315}_{\cancel{63}\,21}} = \frac{800 \times 22}{21}$$

Question

H.W

A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air in 330 ms^{-1}), is



(A) 100 Hz

(B) 103 Hz

(C) 106 Hz

(D) 97 Hz

Question

H.W

A speeding motorcyclist sees traffic jam ahead of him. He slows down to 36 km/h. He finds that traffic has eased and a car moving ahead of him at 18 km/h is honking at a frequency of 1392 Hz. If the speed of sound is 343 m/s, the frequency of the honk as heard by him will be

(A) 1332 Hz

(B) 1372 Hz

(C) 1412 Hz

(D) 1454 Hz

Question

H.W

Two sources are at a finite distance apart. They emit sounds of wavelength *l*. An observer situated between them on line joining approaches one source with speed u. Then, the number of beat heard/second by observer will be

A) $2\mu/\lambda$

B) μ/λ

C) $\mu/2\lambda$

D) λ/μ

Question

HW

A car is moving towards a high cliff. The car driver sounds a horn of frequency f. The reflected sound heard by the driver has a frequency $2f$. If v be the velocity of sound, then the velocity of the car, in the same velocity units, will be

A. $v/\sqrt{2}$

B. $v/3$

C. $v/4$

D. $v/2$

Sound Properties

- Pitch → frequency.
  ↘ पतलापन in sound

Audible freq. range = (20 - 20,000) Hz.

- Loudness →

$$\boxed{L = 10 \log_{10} \frac{I}{I_0}}$$ decibels.

$$I_0 = 10^{-12} \frac{W}{m^2}$$

Question

A small speaker delivers 2W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as $10^{-12} W/m^2$]

[12 April 2019 II]

(A) 40 cm

(B) 20 cm

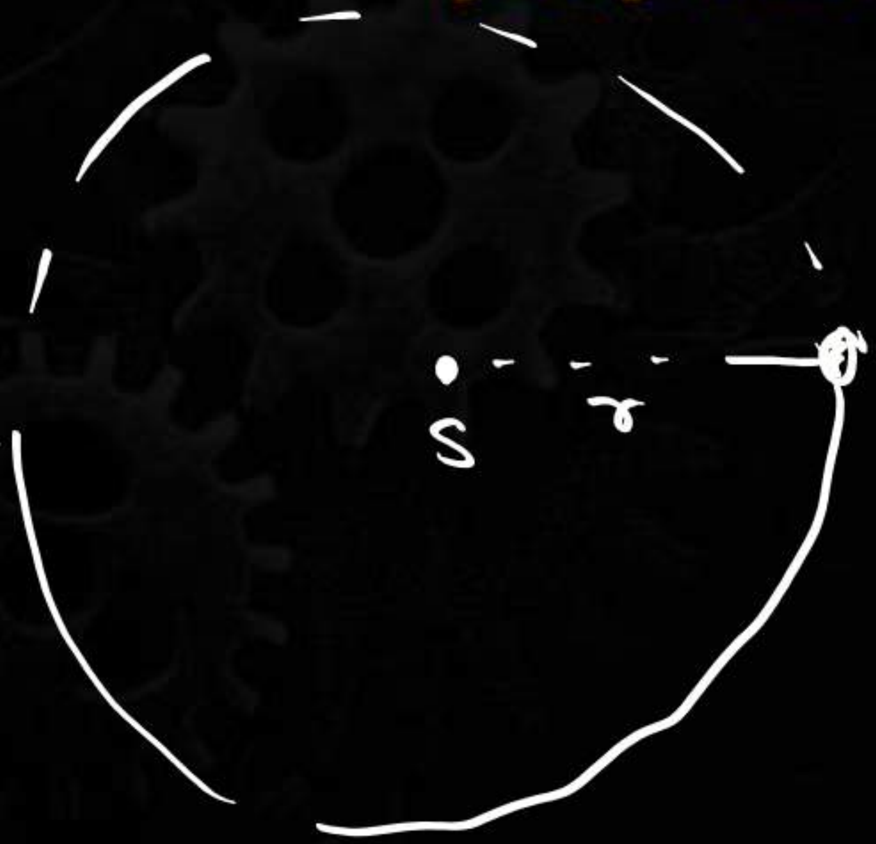
(C) 10 cm

(D) 30 cm

$$120 = 10 \log_{10} \frac{I}{I_0}$$

$$I = 10^{12} I_0$$

$$I = 10^{12} \times 10^{-12} = 1 \frac{W}{m^2}$$



$$I = \frac{E}{A} = \frac{2}{4\pi r^2}$$

$$1 = \frac{2}{4\pi r^2}$$

$$r^2 = \frac{1}{2\pi} \Rightarrow \boxed{r = \frac{1}{\sqrt{2\pi}} m}$$

HOMEWORK

Module + Class Q.


Thank You