FIM 548: Monte Carlo Methods in Finance

Subject: Credit Suisse-Barrier Reverse Convertible Pricing

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Report for individual research project for FIM 548
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Introduction

The contract in question is a Credit Suisse complex product namely Barrier Reverse Convertible. The objective here is to price the contract using standard monte carlo methods, apply variance reduction technique and observe value of contract as a function of the coupon rate.
The contract has a length of one year and pays a coupon of 11%(according to the termsheet) with a barrier level of 55% on the underlying assets. The underlying assets for the contract in question are -:

1) Zoom Video Communications Inc. -A Registered Share(NYSE)
2) Netflix Inc. Registered Share (NASDAQ)

Contract Specifics

Now that we have a general overview of what the contract is and what the underlying assets are we can dive in the specifics of the contract. The table below shows the key specific terms relevant to the contract-:

Reference Share	**Initial Level**	**Barrier**	**Conversion Ratio**
Zoom Video Communications Inc.	USD 331.28	USD 182.204	3.0186
Netflix Inc.	USD 554.58	USD 305.019	1.8032

Coupon	11% p.a. paid semi-annually
Initial Fixing Date	8 Apr, 2021
Payment Date	15 Apr,2021
Final Fixing Date	12 Apr, 2022
Redemption Date	15 April 2022
Barrier	55% of initial level, continuously observed
Currency/Denomination	USD 1,000

The fixed coupon paid by the contract is 11% which is paid semi-annually and will be paid under all/any circumstances. The initial level mentioned for the stock prices is the price for each respective underlying reference share recorded on the initial fixing date i.e. Apr 8, 2021. It is from this date that the contract can be traded. The Payment Date is Apr 15, 2021 which is when the issue price is paid and the contract is issued. The last trading date and final fixing date i.e Apr 12, 2021 is the last date of trading for the contract and additionally, on this day the closing price of the underlying reference shares will be set as the final level of stock price. The barrier for the stock prices is set to 55% of their level which is mentioned in the first table. The conversion ratio is necessary for the payout and we will get to its use in the next section. Lastly, it is important to note that the currency/denomination is USD 1,000. So for our pricing problem we will assume a single contract of initial value 1000 for the ease of calculation.

Contract Scenarios Analysis at Redemption

So the contract pays a fixed coupon rate which we take to be 11% for us. Each denomination is returned in full if the stock's prices never drop below it's respective barrier throughout the lifetime of the contract. If the barrier is breached then the invested capital is converted into the worst-performing reference share. Lets look at it in terms of the specific scenario:

Scenario (a): None of the Reference Shares reach their respective Barrier OR all Reference Shares close above their respective Initial Level
In this case,

- Investors receive the Coupon payment.
- Investors receive 100% of the Denomination.

Scenario (b): At least one Reference Share closes below it's Initial Level AND at least one Barrier has reached during the lifetime of the note.
In this case,

- Investors receive the Coupon payment.
- Investor's invested capital will be converted into the worst performing Reference Share using the Conversion Ratio.
- fraction shares are settled in cash but for the sake of making our calculations easy when pricing we assume the settlement in cash entirely.

Equation for Pricing of Contract

The value of contract will be the sum of two components-:

1) Coupon Rate)
The coupon payments paid semi-annually will be priced as follows:

$$P_{Coupon} = (e^{(-r\frac{T}{2})}\frac{C}{2} + e^{(-rT)}\frac{C}{2}) \times \$1000$$

where,
C = Coupon Rate
T = Time at Maturity(in years)
r = risk-free rate of interest

2) Payoff at maturity
The calculation of payoff would be different to each scenario so lets go over each of them case by case:

Scenario (a)
The payoff here is 100% denomination paid back to investor i.e USD 1,000. We can show that using the following expression:

$$Payoff_{(a)} = I(S_{Z_i} > B_Z) \cdot I((S_{N_i}) > B_N) \times \$1000$$

where,
S_{Z_i} = Daily stock price levels of Zoom for the length of Contract.
S_{N_i} = Daily stock price levels of Netflix for the length of Contract.
B_Z = Barrier Level Price for Zoom.
B_N = Barrier Level Price for Netflix.

Scenario (b)
The payoff in this case will be a conversion of capital to the worst performing reference share using its respective conversion ratio and fractional shares to be settled in cash. For the sake of making calculations easier we assume instant liquidation of reference shares which are converted thus making the entire settlement in cash. We can show this payoff using the following equation:

$$Payoff_{(b)} = min\{(S_{Z_T} \times CR_Z), (S_{N_T} \times CR_N)\}$$

where,
S_{Z_T} = Stock Price of Zoom at maturity.
S_{N_T} = Stock Price of Netflix at maturity.
CR_Z = Conversion Ratio of Zoom.
CR_N = Conversion Ratio of Netflix.

The final equation to price the contract can thus be made by combining payoff and coupon payment will be given by the following equation below:

$$P_{Contract} = P_{Coupon} + e^{-rT} Payoff_{[(a)|(b)]}$$

where,
$Payoff_{[(a)|(b)]} = Payoff_{(a)}$ or $Payoff_{(b)}$ depending on which scenario occurs.

Applying Standard Monte Carlo method

We can price the contract using the standard Monte Carlo method
By running a large number of simulations of stock prices of Zoom and Netflix and then calculating Payoff for each scenario and then taking the mean of payoffs for each simulations. By law of large numbers, we observe the mean of payoffs will converge to the expected value of payoffs and thus we can price the contract effectively.

$$P_{Contract} = P_{Coupon} + e^{-rT}\,\mathbb{E}[Payoff_{[(a)|(b)]}]$$

Of course to do so we need to model stock prices of the Reference Shares and to do so we take some assumptions and calculate some parameters for conducting our simulations as well. These are explored in the next section.

Model Parameters for Simulations

We make the assumption that stock prices follow geometric brownian motion. We calculate the model paramters for the GBM trajectories of reference shares using historical data from April 22,2019(since Zoom Video Communications Inc. has only been listed since) April, 8 2021.

- We take the risk free rate of interest as the yield on 1 year US treasury which is 0.07%.

- Neither Netflix Inc. nor Zoom Video Communications Inc. release any dividends so we assume dividend yield rates to be 0.

- For volatility we use the standard deviation of the log returns on individual stocks to calculate daily volatility σ. We find annualized volatility by:

$$\sigma_A = \sqrt{252} \times \sigma$$

- correlation between the two stocks is calculated by the np.corrcoef built-in function in numpy in Python which calculates correlation using the following equation:

$$\rho(zoom, netflix) = \frac{Cov(zoom, netflix)}{\sqrt{\sigma_{zoom}\sigma_{netflix}}}$$

[1]

[1] NOTE: The values of paramters here are calculated in the python file named-*Model_Parameters_CreditSuisse.py*

Implementing Simulations

In the Matlab implementation of this approach outlined above we use 10^6 number of simulations to price the contract. We have two objectives as a part of this study. Firstly, to price the contract using standard Monte Carlo method for an arbitrary coupon rate(which we have taken as 11%) and secondly to apply variance reduction technique.

Standard Monte Carlo Method: Results

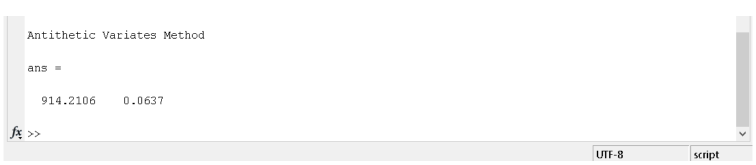
We apply standard Monte Carlo using the pricing equation given on page 4. We get the price of contract returning a fixed coupon of 11% to 914.24 with a variance of 0.0891. Given below is a screenshot of the output window in MATLAB showing the same.

Variance Reduction Technique: Antithetic Variates

Antithetic Variates is a variance reduction technique where introduce negative dependence between pairs of replications. We can do this by generating samples that are negatively correlated. We do this in our simulations through the standard normal random numbers we generate.
Say, we generate N standard random variables and call it Z for standard MC method. In antithetic Variates, we generate $\frac{N}{2}$ random variables say, Z^+ and $\frac{N}{2}$ random variables of Z^- such that:

$$Z^+ = Z^-$$

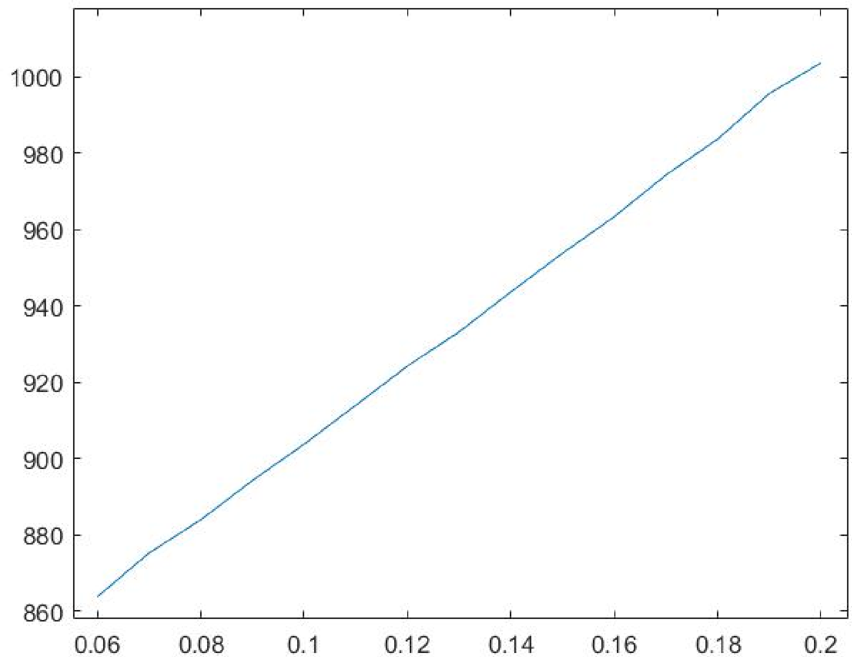
Thus by inducing negative covariance in our samples when we take average of payoffs generated by using Z^+ and Z^- in our GBM paths for Reference Shares we actually reduce the variance. And since normal distribution is symmetric we can generate 2 perfectly anticorrelated samples by just inverting the signs.
When applying antithetic variates for pricing the contract at a fixed coupon of 11% we get the price to 914.21 and variance of estimator at 0.0637 as shown in the screenshot of output window below:



Price of Contract v/s Coupon Rates

For the final part of our study we are required to use standard Monte Carlo method to price the Contract as a function of Coupon Rates

We price the contract using standard Monte Carlo method but for coupon rates starting from 6% upto 20%. The graph below shows the relationship between the price(y-axis) of the contract estimated by standard MC method and fixed coupon rate(x-axis).



The graph appears to show a linear relationship. Intuitively it makes sense the relationship would be linear as it is the fixed coupon that keeps increasing. Over a large number of simulations the payoffs will be converging to expected value so the only change in our pricing equation on page 4 is coming from the change in the coupon rate.

[2] [3]

[2]The code for Standard MC pricing and antithetic variates implementation is in the file named *RBC_CreditSuisse*

[3]The code for the plotting of price of contract as a function of coupon rate is in the files *RBC_CS* and *RBC_CreditSuisse_MultiCoupons*