1) From the notes, for a growing-mode perturbation, $D = At^{2/3}$; when $\Omega_m = 1$, $R = t^{2/3} \Rightarrow D = AR$;
$\delta = D(t)\,e^{-i\vec{k}\cdot\vec{r}} = ARe^{-i\vec{k}\cdot\vec{r}}$; $\nabla_r^2 \phi = 4\pi G R^2 \rho_m \delta$
$= 4\pi G R^2 \rho_m A R e^{-i\vec{k}\cdot\vec{r}}$; The Laplacian won't magically add a time dependence, so in order for ϕ to be time dependent, so must be the RHS. But only R and ρ_m might vary, so $\nabla_r^2 \phi \propto R^3 \rho_m$; But ρ_m goes like $(1+z)^3$, aka R^{-3}, so these time-dep. parts cancel $\Rightarrow$ $\boxed{\phi \text{ time independent}}$ Using the same logic, if the growth is such that $D \propto R^{n<1}$, and $D \to$ constant, then as $t \to \infty$, $\nabla_r^2 \phi \propto R^2 \rho_m$; but since R keeps growing in a low density universe w/o bound, and $\rho_m \propto R^{-3}$, $\nabla^2 \phi \propto R^{-1}$
$\Rightarrow \nabla_r^2 \phi \to 0$, and given suitable boundary conditions, $\boxed{\phi \to 0}$

3) The power spectrum has units of $\boxed{P \equiv [\text{Length}]^3}$ which
a) means that, since denominator dimless, $\boxed{A \equiv [\text{Length}]^4}$
b) $\frac{dP}{dk} = 0 = \frac{-Ak\cdot 2(1+k^2s^2)\,2ks^2 + (1+k^2s^2)^2 A}{''^2} \Rightarrow s^2 4k^2 = (1+k^2s^2) \Rightarrow \boxed{k = \frac{1}{\sqrt{3}s}}$
c) $'' \int_0^\infty '' |\hat{w}(k)|^2 = '' \int_0^K '' |\hat{w}(k)|^2 + '' \int_K^\infty '' |\hat{w}(k)|^2 = '' \int_0^k '' (1)^2 + '' \int_k^\infty '' (0)^2$
$= '' \int_0^K dk\,k^2 \frac{Ak}{(1+k^2s^2)^2}$ Plugging this into an integral calculator, $\sigma^2 = A\left[\frac{(k^2s^2+1)\ln(k^2s^2+1) - k^2s^2}{2s^4\cdot(k^2s^2+1)}\right]\cdot\frac{1}{2\pi^2}$
$\Rightarrow \sigma(K) = \frac{\sqrt{A}}{s^2}\left[\underbrace{\frac{(k^2s^2+1)\ln(k^2s^2+1) - k^2s^2}{2(k^2s^2+1)}}\right]^{1/2}\cdot\frac{1}{\sqrt{2}\pi}$;
$= \frac{'' -1 +1}{''} = \frac{(k^2s^2+1)(\ln(k^2s^2+1) - 1)}{2(k^2s^2+1)} + \frac{1}{2(k^2s^2+1)} = \frac{1}{2}\left[\ln(k^2s^2+1) - 1 + \frac{1}{(k^2s^2+1)}\right]$
$\Rightarrow \boxed{\sigma(k) = \frac{\sqrt{A}}{2\pi s^2}\left[\ln(K^2s^2+1) - 1 + \frac{1}{(k^2s^2+1)}\right]^{1/2}}$ The scale $k = \frac{1}{\sqrt{3}s}$ doesn't appear special here; $Ks \,@\, 8h^{-1}\text{Mpc} = \frac{20\pi}{2\cdot 8} \simeq 3.93$
d) $k = \frac{\pi}{2R} = \frac{\pi}{2(8h^{-1}\text{Mpc})} \Rightarrow 0.9 = \frac{\sqrt{A}}{2\pi}\cdot 2\left[\ln[(3.93)^2+1] - 1 + \frac{1}{(3.93)^2+1}\right]^{1/2}$
$\Rightarrow \sqrt{A} \simeq 1659\ (\text{Mpc}\,h^{-1})^2 \Rightarrow A \simeq 2.75\cdot 10\ (\text{Mpc}\,h^{-1})^4$

For $R = 0.8\,Mpc\,h^{-1}$, $K_s = 39.3 \Rightarrow \boxed{\sigma \simeq 1.66}$

For $R = 80\,Mpc\,h^{-1}$, $K_s = 0.393 \Rightarrow \boxed{\sigma \simeq 0.0654}$

e) $\rho_c \simeq 1.26 \cdot 10^{11}\ \frac{M_\odot}{Mpc^3} \Rightarrow \boxed{M(R = 8h^{-1}Mpc) \simeq 1.17 \cdot 10^{14} M_\odot}$

$\boxed{M(R = 0.8h^{-1}Mpc) \simeq 1.17 \cdot 10^{11} M_\odot}$ All 3 of these length

$\boxed{M(R = 80\,h^{-1}Mpc) \simeq 1.17 \cdot 10^{17} M_\odot}$ scales are quite a

bit larger than galaxies, which have $R \sim 10\,kpc$; whereas $R = 0.8\,h^{-1}Mpc \simeq 1\,Mpc$. This is about the scale of galaxy clusters, but our mass estimate of $1.17 \cdot 10^{11} M_\odot$ is quite lower than the number from the notes, $\sim 2 \cdot 10^{14} h^{-1} M_\odot$. That might be because this is more typical of clusters of $\sim 10\,Mpc$, or $R = 8\,h^{-1}\,Mpc$. The $R = 80\,h^{-1}\,Mpc$ is very large scale structure, which explains why its $\sigma \simeq 0.0654$ suggests it's uncollapsed. Our estimate of $\sim 10^{17} M_\odot$ seems in decent agreement with actual supercluster masses of $\sim 10^{16} M_\odot$.