DS Assignment

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IT

A1 $O(n^2)$

$t = 5$ sec

$n = 10$

$$T(n) = O(n^2)$$

$$t \propto n^2$$

$$\frac{5}{t_2} = \left(\frac{10}{50}\right)^2$$

$$\boxed{t_2 = 125 \text{ sec}}$$

A2 $T_A(n) = n^3$

$T_B(n) = 2n^2$

For Break Pt.

$$\therefore \quad n^3 = 2n^2$$

$$\boxed{n = 2}$$

A3 $n \times 2^n = O(4^n)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \Rightarrow \lim_{n\to\infty} \frac{n \times 2^n}{4^n} \Rightarrow 0 = \text{Const}$$

A4 Log func grow at slow rate but not slowest

Let n^α

$$\lim_{n\to\infty} \frac{\log n}{n^\alpha} \Rightarrow \frac{1}{n\alpha n^{\alpha-1}} = 0$$

so lets take $\log_a n$, $\log_e n$

$$\lim_{n\to\infty} \frac{\log_a n}{\ln n} \Rightarrow \frac{1}{n}$$

$\therefore$ we can say all log are equal

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A5 θ : avg case O = worst case

θ

asymptotically bounded by upper side

O

asymptotically bounded by func from above & below.

Everything that is θ is also O but converse to not true

$O \rightarrow$ gives avg case best time for algo or it is more informative

A6 $n^4 + \log n + 17$ is $O(n^4)$

n	n^4	$\log n$	17	$f(n)$
1	1	0	17	18
2	8	0.6	17	25.6
100	$(100)^4$	4.6	17	$100^4 + 21.6$
10000	$(10000)^4$	9.21	17	$(10000)^4 + 26.21$

So, we can see as ~~n~~ n^4 becomes more dominate so we can denote $f(n)$ as $O(n^4)$

A7 a) k = 1
while (k<n)
k = k+1
End while

No. of steps = n+1
Total = 2 (n+1)

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b) for (i = 1 to n-1)
for (j = i+1 to n)
swap
End for
End for

Total steps $= 1 + n + (n-1) + (n-1) + \frac{(n+1)n}{2}$

$\Rightarrow 2(n-1) + \frac{n}{2} + \frac{3n^2}{2}$

A $\lim_{n \to \infty} \frac{T_A}{T_B} \to \lim_{n \to \infty} \frac{100^n}{n^4}$

L' Hospital Rule $\Rightarrow \lim_{n \to \infty} \frac{n(n-1)(n-2)(n-3)100^{n-4}}{84} \to \infty$

$\therefore n^4$ grows faster than 100^n

An $\lim_{n \to \infty} \frac{\log(n!)}{n \log n}$

$\Rightarrow \lim_{n \to \infty} \frac{n \ln n - n - 1}{n \ln n}$

$\Rightarrow \boxed{1}$

A. $T_A = n^2$ $T_B = n+2$ Break pt :-

$n^2 = n+2$

$n = 2, -1$ $n \geq 0$

$\boxed{\therefore n = 2}$

A

$2^{n-1} + 4^{n+1}$

θ is avg case

Let 2^{n-1} & 4^{n+1} be two func.

$f = 2^{n-1}$ $g = 4^{n+1}$

~~$f\theta$~~ $f = 2^{n-1}$ $g = 2^{2n+2}$

$\boxed{\theta = 2^k}$ $k \in (n-1, 2n+2)$

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