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Verification of scalable ultra-sensitive detection of heterogeneity in an electronic circuit

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Abstract. We study the impact of small heterogeneity in signals applied to globally coupled nonlinear bistable elements. In the absence of coupling, the collective response is simply the average of all the uncorrelated signals. When the elements are coupled and a bias is applied, we find that even a very small number of different inputs are able to drag the collective response towards the stable state of the minority inputs. In our explicit demonstration we have taken Schmitt triggers as the basic nonlinear bistable elements, and the inputs are encoded as voltages applied to them. The average of output voltages of all the Schmitt triggers corresponds to the global output of the system. We also observe that the minimum heterogeneity that can be detected scales with ratio of threshold voltage to source voltage of the Schmitt triggers, and can be be brought down to the limit of single bit detection.

1 Introduction

Complex interactive systems have been widely used to model spatially extended physical, chemical and biological phenomena. The effect of heterogeneity in the evolution of spatiotemporal patterns has attracted the attention of researchers in recent times. Disorder either in form of static or quenched inhomogenieties, and coherent driving forces, have yielded a host of interesting, often counter-intuitive, behaviours. For instance, stochastic resonance [1] in coupled arrays [2] - [8], diversity induced resonant collective behaviour in ensembles of coupled bistable or excitable systems [9],[10] demonstrated how the response to a sub-threshold input signal is optimized.

In this direction, it was recently shown that the collective response of strongly coupled bistable elements can reflect the presence of very few non-identical inputs in a large array of otherwise identical inputs [11]. Here we verify these findings in an array of globally coupled Schmitt triggers. Schmitt trigger is a simple electronic system that can be easily made from commonly available electronic elements like an opamp, and a few resistors. It has been widely used to model bistable systems and one of the earliest demonstrations of the phenomena of stochastic resonance was realized in this system [12].

2 Coupled bistable elements

We consider N globally coupled Schmitt triggers, where the input voltage to each element n ($n = 1, \ldots N$) is given by:

$$V_i^n = V_a^n + C\langle V_o \rangle + V_b \tag{1}$$

where V_i^n is the input voltage to n^{th} element, V_a^n is the encoded voltage corresponding to input signal a^n applied to the n^{th} element, V_b is the bias and $\langle V_o \rangle$ is the average output of all the elements given by

$$\langle V_o \rangle = \frac{1}{N} \sum_{n=1,N} V_o^n \tag{2}$$

Now, we apply V_i^n to the noninverting terminal of an opamp through a resistance R_1. The feedback voltage is fed into the noninverting terminal through the resistor R_2 as shown in figure 1. The inverting terminal is grounded. In this configuration, the threshold voltages for the schmitt trigger are $V_T = \pm \frac{R_1}{R_2} V_s$ where V_s is the supply voltage. So, if $V_i^n < -V_T$, then $V_o^n = -V_s$ where V_o^n is the output of n^{th} Schmitt trigger. Now as we increase V_i^n, V_o^n will remain $-V_s$ till $V_i^n \leq V_T$. As V_i^n exceeds V_T, V_o^n will become V_s. Now when we decrease V_i^n, the output will remain V_s till $V_i^n \geq -V_T$, at which point it will become $-V_s$. Thus, this element has a hysteresis as shown by its transfer function given in figure 1.

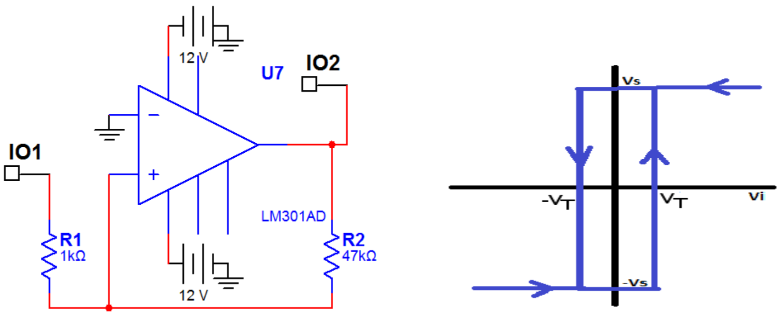

Fig. 1. Individual Schmitt trigger and its transfer function

The heterogeneity in this collection of bistable elements occurs in the diversity of the input signal V_a^n in Eqn. 1. Here we consider that these inputs can have any value in between the two stable states. Without any loss of generality, we can encode one of the states as $-V_a$ and other state as V_a. Thus the inputs can be randomly distributed about zero average, i.e. the inputs are encoded as voltages randomly distributed between $-V_a$ to V_a.

3 Collective response in the presence of heterogeneity

In an uncoupled system, if we take voltages encoding the inputs such that $|V_a| > |V_T|$ then the output state of the element will depend on the value of the input. If the input is in the low state the output of the element will be $-V_s$, and the output will be V_s if the input is in high state. Now when these elements are strongly coupled via average voltage i.e. average voltage of the output of all the Schmitt triggers, $\langle V_o \rangle$, the dynamics of the elements gets correlated.

Consider now the output of this coupled system as $\langle V_o \rangle$. If all inputs are identical, and such that $|V_a^n| > |V_T|$, then every element will go to the state corresponding to that input. However, if $|V_a^n| < |V_T|$ the final state of element will depend on the initial state of that particular element. Given randomly distributed initial conditions, the average voltage will stay close to zero as no element will be able to change its state in this configuration.

If the inputs are not identical, then depending on the distribution of V_a^n, and the initial state of the elements, the elements whose inputs are closer to the low state will tend to move towards $-V_s$, and vice versa. As the states of the elements will also be distributed uniformly, and so in some cases the element will go to lower state whereas in other cases it will go to the upper state. Assuming a uniform random distribution of initial states of the elements we expect that average voltage of the elements will either go to $-V_s$ or V_s. On averaging over different initial configurations the average voltage will again approach zero. If the inputs are not identical and are such that the number of inputs is not distributed uniformly about zero average, then the majority dictates the average voltage. Thus in this case it is not possible to detect the presence of small heterogeneity in the inputs, nor can one infer the number of inputs that were different.

Now consider that the inputs are not distributed over the entire range of voltages between $-V_a$ to V_a, but are two state instead i.e. they take only one of two values. Such systems are very relevant and can, for instance, represent logic 0 and 1. We will show below how to detect the number of nonidentical inputs directly in such systems. Further we will demonstrate that the average voltage of the system approaches the stable state corresponding to the minority population, up to the limit of a single nonidentical input.

To accomplish this, in addition to $\langle V_o \rangle$ we also apply a bias V_b to all the bistable elements. The value of this bias is decided by the type of minority we wish to detect. If the minority corresponds to the low state, we apply a negative bias and vice versa. This bias essentially brings the elements to which majority input is applied, near the tipping point. As the state of a few inputs change, the elements corresponding to those inputs shift their state. This moves the average voltage away from the stable state of the majority input towards the stable state of the minority input. As the elements corresponding to the majority input were already close to the tipping point, this reduction in average voltage causes a few of them to slip over the barrier towards the stable state of the minority input. This further reduces the average voltage and creates a cascading effect so that in a few steps all the elements corresponding to the majority input are dragged to the stable state of the minority input.

Suppose we start with a state in which all inputs are in same state, let us say, in the lower state. Further the encoded voltage V_a satisfies the condtion $|V_a| > |V_T|$. So when inputs are in lower state they are encoded as $-V_a$ which satisfies the condition $-V_a < -V_T$. Thus initially all the elements will go to the lower state. Now we want to find out what is the minimum fraction of elements of a different state necessary such that we can detect the heterogeneity. To do so, we first apply a bias to the system. The value of the bias is such that it alone cannot drive the system. So, in the beginning when all elements are in lower state, the average voltage is $-V_s$. Now on application of bias, the state of the system should not change in absence of heterogeneity. Or the input voltage to bistable elements should remain lower than the threshold voltage i.e. $V_b - CV_s - V_a < V_T$ or $V_b < (CV_s + V_T + V_a)$. Now let us assume that some of the inputs change their state. In this configuration let the number of inputs in the lower state be N_0, and so $N - N_0$ inputs will be in the upper state. This change in state of $N - N_0$ inputs should allow those $N - N_0$ to change their state. So we should have $V_b - CV_s + V_a > V_T$ or $V_b > (CV_s + V_T - V_a)$. From above two conditions we have

$$(CV_s + V_T - V_a) < V_b < (CV_s + V_T + V_a) \tag{3}$$

At this stage N_0 elements go to $-V_s$ and $N - N_0$ elements go to V_s. So the average voltage $\langle V_o \rangle$ is $\frac{(N-2N_0)}{N} V_s$. When $N_0 \sim N$, $\langle V_o \rangle$ will approach $-V_s$ and we will not be able to detect the heterogeneity. Note that the elements are coupled with the global mean field and the individual output state of the elements are not accessible. The only observable quantity is the global mean field. If there is no bias then the majority will always drag the average voltage to its stable point.

When a bias voltage V_b is applied, then in order that the average voltage of the coupled system switches to the stable point of minority, we need that this reduction in average voltage should be sufficient to drive the other elements to the stable state of the minority. So let us consider an element on which the input signal corresponding to majority input is applied. So we require that the voltage acting on this element should exceed the threshold voltage. Namely,

$$-V_a + V_b + CV_s(1 - \frac{2N_0}{N}) > V_T \tag{4}$$

This is satisfied if

$$N - N_0 > \frac{N}{2}(1 - \frac{V_b - (V_T + V_a)}{CV_s}) \tag{5}$$

This gives the minimum number of inputs of the minority type needed for successful detection. In the limiting case when we require that a single different input should be detected, the following condition must be satisfied

$$\frac{V_b - (V_T + V_a)}{CV_s} > \frac{N-2}{N} \tag{6}$$

4 Explicit demonstration of ultrasensitivity

In this section we will explicitly show the detection of a single heterogeneous input in an array of 20 inputs. Initially all the 20 inputs are zero. Then after sometime one input changes its state and we will show how the collective response of the whole system changes its state in response to this change.

As explained in previous sections, we use Schmitt trigger as the bistable element. The voltage applied to n^{th} schmitt trigger is given by equation 1. We apply voltage V_i^n to the non inverting terminal of a comparator through a resistance $R1$. The feedback voltage is fed into the noninverting terminal through the resistor $R2$. The inverting terminal is grounded. Thus the threshold voltages for this schmitt trigger are $\pm\frac{R_1}{R_2}V_s$ where V_s is the supply voltage. We take $R_1 = 1k\Omega$ and $R_2 = 47k\Omega$ and thus the threshold voltages are $\pm 0.255V$. The output voltage obtained with this schmitt trigger is $\pm 10V$.

With no loss of generality, we encode the inputs as $\pm V_a$ where $V_a = 1.5V$ i.e. if input is in low state we apply $-1.5V$ and if it is in high state we apply $1.5V$. Note that we can also encode the inputs as $0V$ and V_aV. In that case, we just have to shift V_T by applying a bias.

In this case, as we have minority of low states and a single heterogeneity of high state, so we apply a positive bias to our system. Specifically we choose $V_b = 11V$.

Note that these values of the variables satisfy the criterion given by equations 3 and 6. Thus the detection of even a single heterogeneity is possible. The schematic diagram of the same is shown in figure 2.

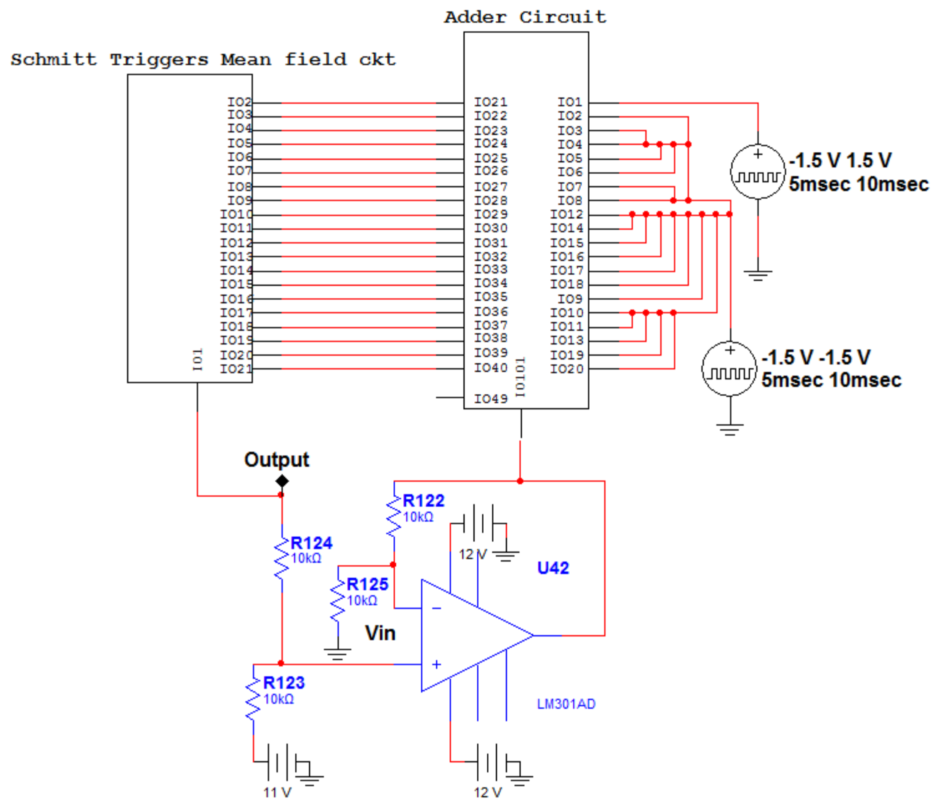

Fig. 2. Schematic of the electronic circuit model

Here the block on left side has all the bistable elements, namely the Schmitt triggers. The internal diagram of this block is shown in figure 3. Each Schmitt trigger is the same as that shown in figure 1. This block receives V_i^n from the block on the right and these are fed to the Schmitt triggers.

The output of all Schmitt triggers is averaged and this is fed to the opamp shown in the figure. The opamp adds the bias to this averaged value and the output is sent to the block on the right. The block on the right has 20 adder circuits which add the input received from the opamp and V_a^n and send the 20 different outputs so obtained to the block on the left. The internal block diagram of this block is shown in figure 4. Figure 5 shows the circuit diagram of an individual adder.

We display the following representative result in Fig. 6: initially we apply $-V_a$ voltage to all the inputs (i.e. there is no heterogeneity in the system, and all elements are identical). Then after some time, one element is changed, i.e. we have a system with 19 identical elements (with the majority having input $-V_a$) and *only one* different element with input changed to V_a. We find that the average voltage of the circuit is $-V_s$ initially, as expected, when the system has no diversity. However, after

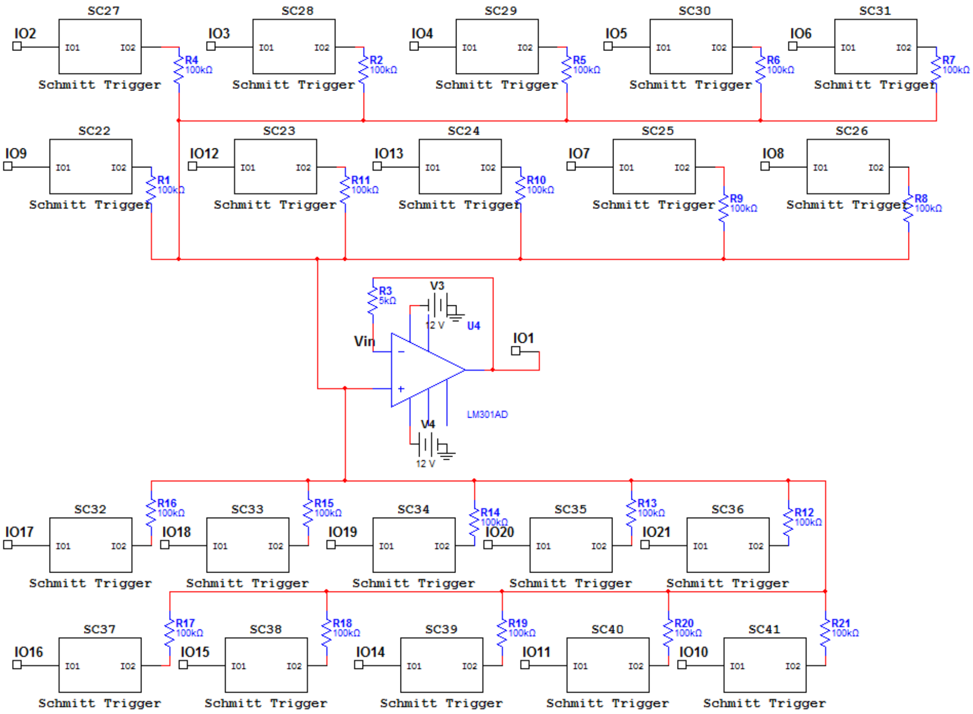

Fig. 3. Array of Schmitt triggers and the averaging circuit represented by left block in figure 2

one element changes, the average jumps to V_s. This clearly indicates the extreme sensitivity to diversity in the coupled system.

Further note, that the circuit can be reset at any time by applying a pulse of high magnitude negative voltage. Also, importantly, by varying the bias we can infer the number of elements that differ from the majority, as evident through equation 5.

5 Conclusion

We have studied the impact of small heterogeneity in signals applied to globally coupled nonlinear bistable elements. In the absence of coupling, the output of the nonlinear elements swings towards the signal applied to that element and the collective response mirrors the state corresponding to average of all the signals. When the elements are coupled and a bias is applied, we find that even a very small number of different inputs are able to drag the collective response towards the stable state of the minority inputs. In our explicit demonstration we have taken Schmitt triggers as the basic nonlinear bistable elements, and the inputs are encoded as voltages applied to them. The average of output voltages of all the Schmitt triggers corresponds to the global output of the system. We also observe that the minimum heterogeneity that can be detected scales with ratio of threshold voltage to source voltage of the Schmitt triggers, and can be be brought down to the limit of single bit detection.

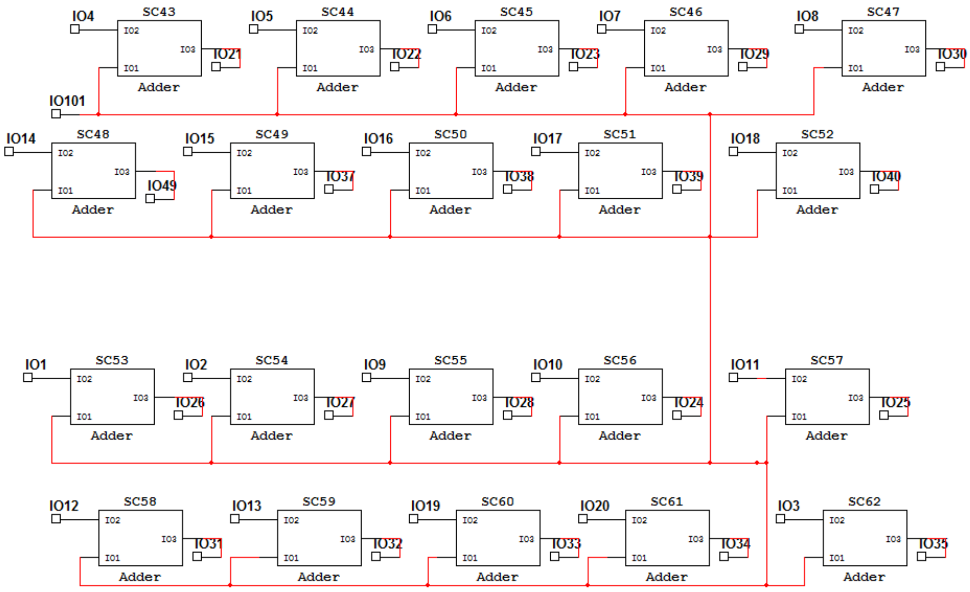

Fig. 4. Array of adder circuits for each input represented by right block in figure 2

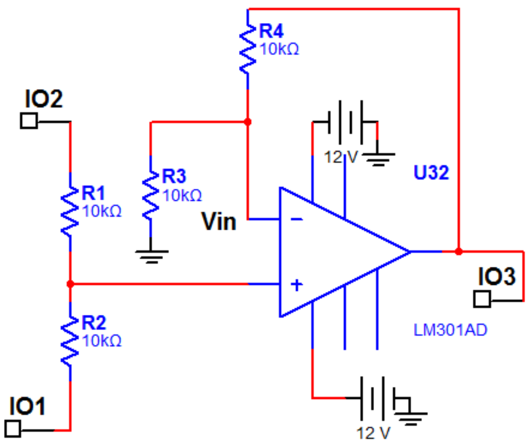

Fig. 5. Circuit diagram of a single adder

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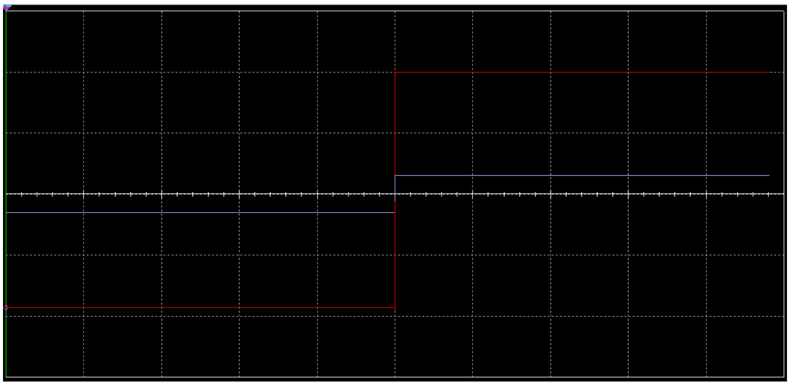

Fig. 6. Time series of the input to one element (light blue), and the average of output voltage of 20 coupled elements (red). The input to other 19 elements is held constant at the same value as was applied to first input in first half of time series. We observe that as the input changes for just this one particular element, the average voltage, which is the collective response of the whole system, jumps to the high state. Y axis shows the voltage with each grid element representing 2V and X axis represents time with each grid element representing 1ms.

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