Small-world networks exhibit pronounced intermittent synchronization

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We report the phenomenon of temporally intermittently synchronized and desynchronized dynamics in Watts-Strogatz networks of chaotic Rössler oscillators. We consider topologies for which the master stability function (MSF) predicts stable synchronized behaviour, as the rewiring probability (p) is tuned from 0 to 1. MSF essentially utilizes the largest non-zero Lyapunov exponent transversal to the synchronization manifold in making stability considerations, thereby ignoring the other Lyapunov exponents. However, for an N-node networked dynamical system, we observe that the difference in its Lyapunov spectra (corresponding to the $N-1$ directions transversal to the synchronization manifold) is crucial, and serves as an indicator of the presence of intermittently synchronized behaviour. In addition to the linear stability-based(MSF) analysis, we further provide global stability estimate in terms of the fraction of state-space volume shared by the intermittently synchronized state, as p is varied from 0 to 1. This fraction becomes appreciably large in the *small-world* regime, which is surprising, since this limit has been otherwise considered optimal for synchronized dynamics. Finally, we characterize the nature of the observed intermittency and its dominance in state-space as network rewiring probability(p) is varied.

The last few years have witnessed a tremendous amount of research being directed towards the existence and stability of synchronized dynamics on complex networks. Amongst various topologies, *small-world* (SW) networks have been found to be quite conducive for the optimal manifestation of synchronized motion. However, in this work, we present a case which appears to contradict the aforementioned result. In particular we find that, although the synchronized state does occur in SW networks, it seems to occupy only a small fraction of the overall state-space volume. Moreover, a significant fraction of the state-space is actually occupied by *intermittently synchronized* dynamics. Therefore, it becomes crucial to revisit the problem of synchronization in SW networks from the perspective of state-space volumes and identify the reason underlying the emergence of such intermittency in SW topologies, which are otherwise considered optimal for synchronized dynamics.

I. INTRODUCTION

Synchronization of dynamical units coupled on complex networks has been recognized as one of the most significant forms of collective behaviour with implications in population dynamics[1–3], epidemiology[4,5], neural networks[6–9], secure communications[10], power grids[11–14], delay dynamics[15], noise robustness[16], etc. In fact, synchronization between the different components of a complex dynamical system is often critical to its overall functionality[17]. As a result, an extensive body of research has focussed on the identification of network architectures which facilitate robust synchronization[17–24].

In the above context, *small-world* (SW) properties of real-world networks have been found to be particularly conducive towards synchronization[25]. A notable model exhibiting the aforementioned topological structure is the Watts-Strogatz network[26], which has been shown to exhibit more robust synchronization as compared to its completely regular or random counterparts[25,27,28]. Such investigations usually consider the dynamical components (of the network) starting from asynchronous initial conditions, asymptotically reaching the synchronized state and thereby, maintaining synchronized operation. Interestingly however, there have also been reports of chaotic oscillator networks exhibiting dynamics which switches between synchronized and desynchronized behaviour in a temporally intermittent fashion[29–31].

Previous work in this direction, starting with that of Baker et al.[29], subsequently led to explorations of different aspects of intermittent synchronization in coupled dynamical systems[30–33]. The reasons underlying this intermittency in much of these studies were mainly attributed to either *attractor bubbling*[31] or *on-off intermittency*[34]. Further, very recently, the intermittently synchronized dynamics arising out of coupled map networks has been labelled as *Chaotic Griffiths Phase* (CGP), and it has been shown that the number of positive transverse Lyapunov exponents (TLE) in this phase scales anomalously with the power of network size[35]. However, none of the earlier studies have explored the state-space properties of the intermittent state. In this regard, we specifically study the variation of state-space volume shared by the intermittently synchronized dynamical behavior with respect to changes in the network topology.

This temporal coexistence of intermittent dynamics is often undesirable as it hampers the sustained synchronized operation of the respective system. Therefore, it is crucial to investigate the relationship between topological properties of networks and such dynamical behaviour. In this regard, we here make the surprising account that the SW regime is significantly more prone to the existence of such intermittent dynamical behaviour, compared to its regular or random counterparts. We demonstrate this result in Watts-Strogatz networks of chaotic Rössler oscillators, as its parameter of link rewiring probability (p) is tuned from 0 to 1.

In particular, we consider a coupling regime for which the master stability function (MSF) predicts stable synchronized behaviour[36]. MSF essentially utilizes the largest non-zero TLE of the master stability equation in making stability considerations, thereby ignoring the other Lyapunov exponents. We demonstrate in the following, that the largest non-zero TLE fails to capture the occurrence of intermittent bursts in a synchronized dynamics. Further, we show that the complete spectra of Lyapunov exponents emerge as a better indicator of the occurrence of such intermittency.

In addition to the linear stability-based MSF analysis, we also study the variation of the fraction of state-space volume shared by the intermittently synchronized state, as p is tuned from 0 to 1. This fraction becomes appreciably large in the SW regime, which is also surprising and further corroborates our finding. Finally, we characterize the nature of the observed intermittency and the transition from intermittently synchronized dynamics to completely synchronized behaviour.

II. A CASE FOR INTERMITTENT SYNCHRONIZATION

We demonstrate a scenario of observing intermittent synchronization in a SW network of N coupled Rössler oscillators where the dynamics of node i is given by

$$\begin{aligned} \dot{x}_i &= -y_i - z_i + K\sum_{j=1}^{N} L_{ij}x_j, \\ \dot{y}_i &= x_i + ay_i, \\ \dot{z}_i &= b + z_i\left(x_i - c\right), \end{aligned} \tag{1}$$

where K is the coupling constant, $\mathbf{L}$ the Laplacian matrix, and the parameters a, b and c are chosen to be 0.2, 0.2 and 7.0 such that each uncoupled Rössler oscillator exhibits chaotic dynamics and the synchronous state corresponds to the case where all oscillators have identical dynamics (i.e., *complete synchronization*).

For the construction of SW networks, we start with a regular ring where each node has k nearest-neighbors on either side. Then, we rewire each link with probability p by removing the connection from either of its vertices and connecting it to some randomly selected node. A network resulting from this process has on average pkN non-local links along with $(1-p)\,kN$ links with nearest-neighbor coupling[37], for large N. We select a network size of $N = 100$ for all the results presented here, where each node has $2k = 8$ nearest-neighbors ($k = 4$ on each side). To begin with, we invoke MSF[36] to obtain the interval of coupling for which synchronization is locally stable. For this purpose, we rewrite the dynamical equations of the system (Eq. (1)) in the following form,

$$\dot{\mathbf{X}}_i = \mathbf{F}\left(\mathbf{X}_i\right) + K\sum_{j=1}^{N} L_{ij}\mathbf{H}\left(\mathbf{X}_j\right), \tag{2}$$

where $\mathbf{X}_i = \left(x_i,\, y_i,\, z_i\right)^{\mathrm{T}}$ is a 3-dimensional state vector of the dynamical variables of node i. $\mathbf{F}$ is the functional form describing the internal dynamics of an isolated node. $\mathbf{H}$ defines the functional form of the coupling relation between the state variables of different nodes. In this study, the oscillators are coupled through linear diffusive coupling in their x-components, so H is simply a 3×3 matrix with $H_{11} = x$ and $H_{ij} = 0$ otherwise.

The synchronization manifold is defined by $N-1$ constraints $\mathbf{X}_1 = \mathbf{X}_2 = \ldots = \mathbf{X}_N$. Further, the variational equation of the system is given by

$$\dot{\xi} = \left[1_N \otimes D\mathbf{F} + K\mathbf{L}\otimes D\mathbf{H}\right]\xi, \tag{3}$$

where $\xi = \left(\xi_1,\, \xi_2,\, \ldots,\, \xi_N\right)^{\mathrm{T}}$ with ξ_i being the perturbation to node i and $D\mathbf{F}$ and $D\mathbf{H}$ being the corresponding Jacobian matrices. Block-diagonalizing this equation gives a 3×3 variational equation, given by

$$\dot{\xi}_k = \left[D\mathbf{F} + K\gamma_k D\mathbf{H}\right]\xi_k, \tag{4}$$

where γ_k $(k = 1,\, 2,\, \ldots,\, N)$ are the eigenvalues of $\mathbf{L}$, corresponding to the transversal perturbations associated with their respective eigenvectors. For the synchronous state to be linearly stable, perturbations along all the transversal directions must decay. This implies that all the Lyapunov exponents of Eq. (4) corresponding to the transversal eigenmodes must be negative for $K > 0$. This is true if $\alpha_1 < K\gamma_k < \alpha_2$. For our model (Eq. (1)), the numerical values of α_1 and α_2 turn out to be 0.1232 and 4.663 respectively[38].

Figure 1 illustrates the range of coupling strength as a function of the link rewiring probability p, for which the synchronized state is locally stable. We emphasize that this interval of coupling has been obtained by using the maximum non-zero TLE from the complete spectrum. Now, we present a case where linear stability analysis predicts stable synchronized behavior, but the actual dynamics can also be intermittent in time depending upon the choice of initial conditions. To capture the transition to synchronization, we

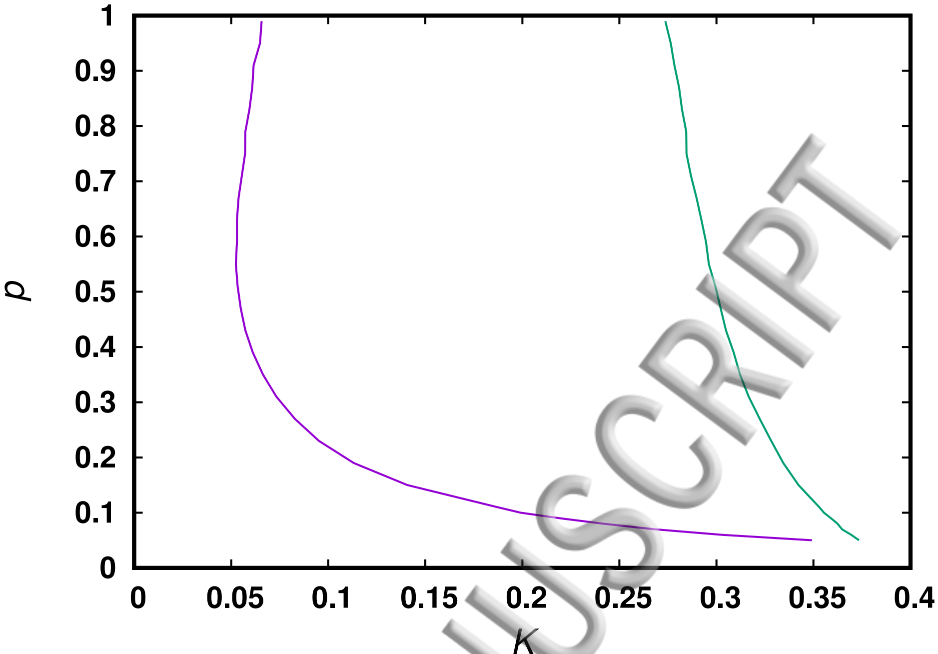

FIG. 1. Plot of minimum (K_{min}, purple) and maximum (K_{max}, green) values of coupling strength for which complete synchronization (Eq. (1)) is locally stable, as a function of the link rewiring probability p.

compute the synchronization error Z_{sync} defined as the mean squared deviation of the dynamical state of the nodes, which is mathematically expressed as

$$Z_{sync} = \frac{1}{N} \sum_{i=1}^{N} \left[x_i(t) - \langle x(t) \rangle \right]^2 , \tag{5}$$

where $\langle x(t) \rangle$ is the mean value of the dynamical state of the system's evolution over time. The above measure going to zero is indicative for the onset of complete synchronization. We observe the transient behavior of this quantity as the system evolves over time. To ensure reproducibility and avoid machine precision-related problems, we set a synchronization error threshold $Z^*_{sync} = 0.0001$ as the value below which the system is considered to have attained complete synchronization[39].

We provide the first important observation in Fig. 2 (bottom panel), which clearly depicts the intermittent nature of Z_{sync} in time. Notice that the system starts in the asynchronized state and with the progression of time, the synchronization error falls below Z^*_{sync} and stays there for a while. However, it then abruptly makes a jump and crosses the threshold to become asynchronized again and this goes on indefinitely and intermittently. Also, notice that the coupling strength for this case is within the coupling interval as defined in Fig. 1, which predicts stable synchronized behavior. We further characterize this intermittent behavior in section III. Note that the manifestation of this behavior depends on the choice of initial conditions, which will be explored later in section IV.

Further, to inquire the dependence of this intermittent behavior on the SW network's randomness, we extensively investigate the transient behavior of Z_{sync} at various values of p. This reveals that intermittent synchronization manifests itself primarily in the range of $p \in (0.06, 0.2)$, which coincides with the SW limit[26]. For higher values of p, the system exhibits complete synchronization at all times. For instance, Fig. 2 (top panel) is representative of a case where synchronization is maintained at all times after initial transients.

In order to understand the nature of this intermittency, the distribution of laminar length is studied for two specific values of p (for which intermittency is observed), as shown in

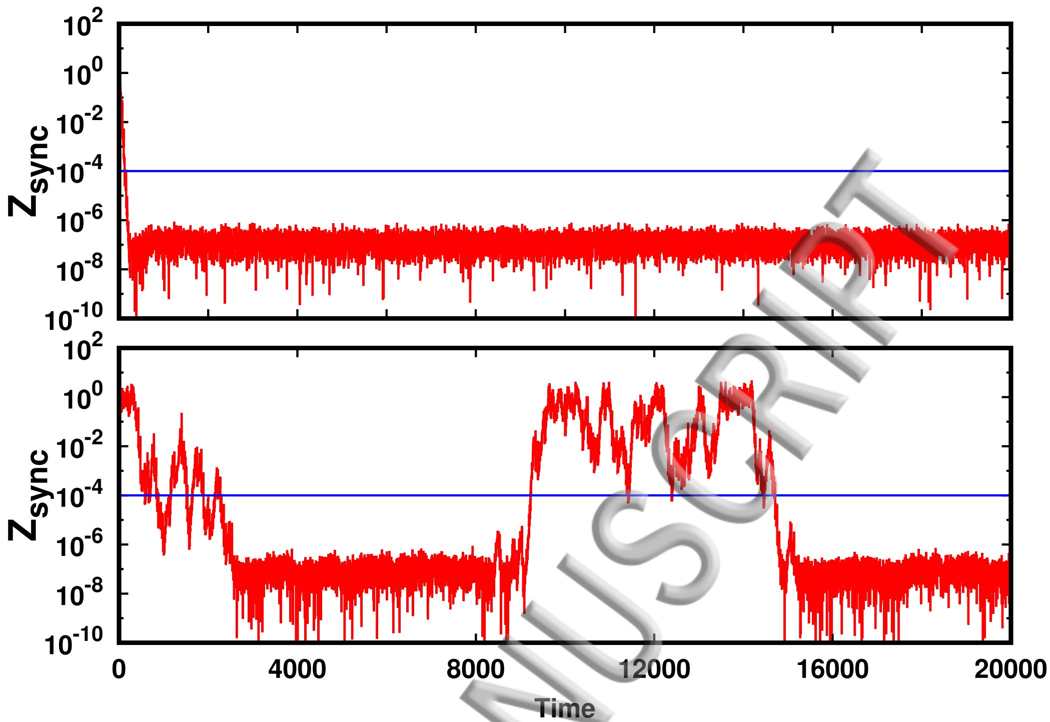

FIG. 2. The variation of synchronization error over time (at $K = 0.28$) for link rewiring probabilities of $p = 0.47$ (top panel) and $p = 0.09$ (bottom panel).

Fig. 3. For each value of p, we have taken 100 different initial conditions and realizations of the system. In each case, we calculate the synchronization error for each time step after transience. If the system stays below the complete synchronization threshold Z^*_{sync}, for the time between t and $t + L$, then we call the time $(t + L) - t = L$ as the laminar length. Therefore, the laminar length can vary from zero (when there is no synchronization) to a maximum of the full run-time under consideration (when there is complete synchronization). For intermittently synchronized cases, we find many such L values as the synchronization error intermittently jumps below and above the sync threshold. We take all these L values for 100 different realizations of the network for a fixed p and draw a normalized histogram of all these values. We plot the frequency from this histogram against the mid-point of the corresponding bin on a log-log scale in Fig. 3. This shows the distribution of different laminar lengths in the trajectories. It is clearly evident from the figure that there is good scaling over four orders of magnitude. Further note that remarkably the data from two different rewiring probabilities collapse on the same curve, i.e. yields the same exponent.

III. LYAPUNOV SPECTRUM ANALYSIS

In the previous section, we established that despite linear stability analysis providing conditions for stability of the completely synchronized state, the behavior turns out to be intermittently synchronized in time for a wide range of the network randomness parameter (p) of the Watts-Strogatz network. In this section, we attempt to unravel the reason underlying the occurrence of intermittent synchronization using information from the whole Lyapunov spectrum. Previous studies have identified the cause of intermittency in the synchronized state as local instabilities in the directions transversal to the synchronization manifold[24,30,31,36,40]. In this regard, Chaté[41] has shed light on the possible link between spatio-temporal intermittency and the complete Lyapunov spectrum. In strong analogy with the aforementioned work, we observe here that there may be a possible correlation be-

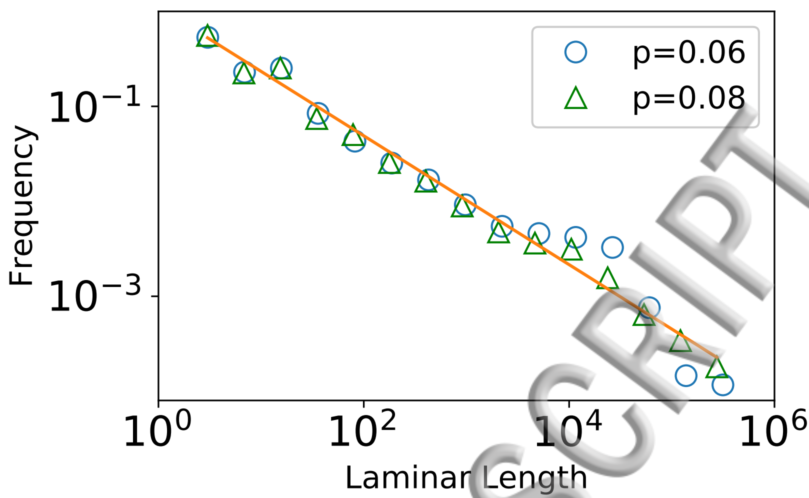

FIG. 3. Distributions of laminar length l (on a log-log scale) for two different link rewiring probabilities, both of them allowing the possibility of intermittent synchronization. Blue colored circles(green triangles) represent the distribution for $p = 0.06(p = 0.08)$. Mustard colored solid line represents the fitted scaling function which varies as a power law. Total run time for this simulation is 6×10^5, out of which 1×10^5 is the transience. Network size and the number of realizations, both are taken to be 100.

tween the structure of the complete Lyapunov spectrum and the occurrence of intermittent synchronization in coupled dynamical systems.

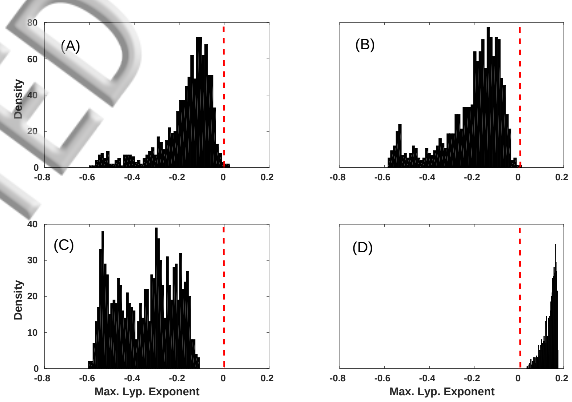

FIG. 4. Density distribution of the maximum Lyapunov exponents along the $N - 1$ directions transversal to the synchronization manifold. (A) $p = 0.1$ and (B) $p = 0.08$ correspond to the cases where intermittent synchronization is observed. In both situations, the coupling is taken to be the mean value from the coupling interval obtained in Fig. 1. (C) $p = 0.95$ corresponding to stable synchronized state with coupling inside the stability interval and (D) $p = 0.95$ corresponding to unstable synchronized state with coupling outside the stability interval.

The system defined by Eq. 1 has a $3 \times N$ dimensional state-space and the synchronization manifold is 3-dimensional. Therefore, transverse to the synchronization manifold there are $3 \times N - 3$ perturbation directions which are further grouped into $N - 1$ 3-dimensional subspaces[36]. We capture the maximum Lyapunov exponents from these $N - 1$ 3-dimensional sub-spaces for different initial conditions and realizations of the network. Figure 4 illustrates the density distribution of the $N-1$ maximal transverse Lyapunov exponents (MTLE) for four representative cases corresponding to regular and intermittent synchronization. Cases (C and D) represent the behavior of distribution of MTLE for $p = 0.95$ which is outside the window of intermittent synchronization. For both cases, the distribution is far away from zero which clearly indicates either a stable (C) or an unstable synchronized state (D). Further, cases (A and B) for which intermittent synchronization is observed, correspond to the distributions obtained for $p = 0.1$ and $p = 0.08$, respectively. Notice that the distribution becomes sharply peaked near zero in the case of intermittent synchronization, which is a possible indicator of intermittency in the synchronized state.

The above feature of intermittent synchronization is reminiscent of spatio-temporal intermittency in spatially extended systems. The increase in density of the Lyapunov spectra near zero is similar to the signature of spatio-temporal intermittency in coupled map lattice models[41,42]. Such trends were also clearly discerned in models of self-organized criticality[43], which also show characteristics of spatio-temporal intermittency[44]. Therefore, in summary, our results lend credence to the conjecture that intermittency in extended systems is signalled by a significant increase in exponents close to zero.

IV. STATE-SPACE VOLUME

In previous sections, we have explored the existence of the intermittently synchronized state for a range of rewiring probability (p) of the Watts-Strogatz network. However, we did not mention the possibility of having other co-existing dynamical states in the state-space. In this regard, it is now important to consider the possibility of encountering the intermittently synchronized state in the aforementioned scenario. For this purpose, we consider the fraction of state-space volume shared by the intermittently synchronized state, which is representative of the probability of the respective state appearing in the evolution of the system, when initiated from a randomly chosen condition. In this section, we explore the variation of this property of the state-space with changes in p.

We have broadly observed four types of qualitatively different emergent collective behavior namely, (i) the system exhibits complete synchronization, (ii) it does not synchronize, (iii) it diverges[45], and (iv) synchronizes intermittently in time. To compute the fraction of volume shared by these states, we first initialize all the nodes with random values of: $x \in [-15, 15]$, $y \in [-15, 15]$, and $z \in [-5, 35]$. The state-space volume of an emergent state is computed as the fraction of initial conditions that asymptotically approach the respective state.

Figure 5 illustrates the variation in the state-space volume of the different emergent states with changes in p. Note that the sum of all fractions is always ~ 1, which is to say that the four aforementioned states cover the whole state space volume. Importantly, one can observe that the state-space volume of the intermittently synchronized state is most significant in the range $p \in (0.06, 0.2)$, which is marked by the two vertical black dotted lines. Given that the state-space is shared by four different types of dynamical behavior, the system's response would be quite sensitive to external perturbations for $p \in (0.06, 0.2)$. Further, notice that the state-space volume leading to a synchronized regime, first increases monotonically with p and later decreases gradually for higher values of p. Also, note that the maximal volume corresponding to the completely synchronized and intermittently synchronized regimes do not coincide, but both appear inside the small-world regime.This indicates that the fraction of state space volume previously occupied by intermittent behavior may have become a part of the synchronized state volume, and is suggestive of the onset of complete synchronization via the intermittency route. This behavior is captured in the inset of Fig. 5, which shows the behavior of the scaled average laminar length(namely, the average sum total of laminar time intervals scaled by total time, reflecting the probability of obtaining a laminar region) as p is varied inside the small-world regime, indicating a smooth transition from incoherence to synchronized dynamics via the intermittently synchronized state.

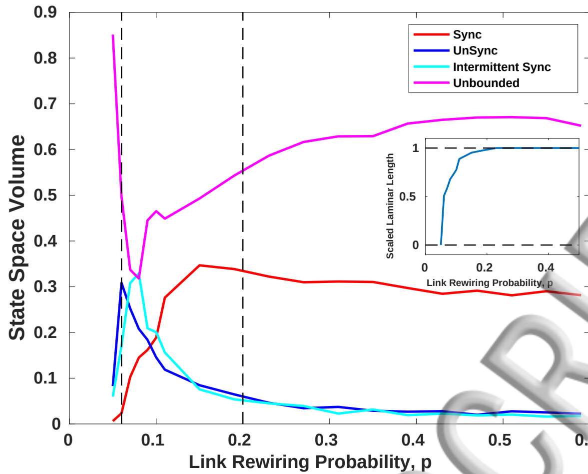

FIG. 5. Fraction of state space volume shared by completely synchronized, unsynchronized, unbounded and intermittently synchronized behavior, as a function of link rewiring probability p. The results are computed over 100 initial conditions. The region bounded by the two vertical black dashed lines, represents the network regime (small-world) where the state-space volume of intermittent synchronization becomes appreciably large. Further, within this regime, the behavior of scaled laminar length as a function of p is shown in the inset. For each value of p, we sample over the coupling strength interval, bounded by K_{min} and K_{max}, where linear-stability analysis yields locally stable synchronization, as displayed in Fig. 1.

V. CONCLUSION

In conclusion, we have reported the existence of intermittent synchronization in Watts-Strogatz networks of chaotic Rössler oscillators. We found the existence of *intermittent synchronization* for network topologies for which linear stability theory predicts the existence of a stable synchronized behaviour. Moreover, our most interesting finding is that these network topologies correspond to the much celebrated *small-world* limit.

We also found that the complete Lyapunov spectrum contains consistent indicators to detect the presence of an intermittently synchronized regime. Further, we have computed the fraction of state-space volume shared by all emergent behaviors, as the network rewiring parameter p is varied. We have observed that the state-space volume of intermittently synchronized behavior becomes appreciably large in the small-world regime, which is surprising, since this limit has been otherwise considered optimal for stable synchronized dynamics. These results thus have important implications in the design of topologies to ensure persistent synchronized operation of dynamical units coupled on them.

However, there are interesting problems which require further attention and can be the topic for future research. For instance, it would be interesting to know how the distribution of the complete Lyapunov spectrum changes as p is varied continuously. Also, one has to check the robustness of this enhanced intermittency for time-varying topologies as well. In addition, we recommend investigating the presence of the intermittently synchronized state when calculating *basin stability*-based measures of the synchronized state in complex oscillators networks[9,14,17,25,46].

VI. ACKNOWLEDGEMENTS

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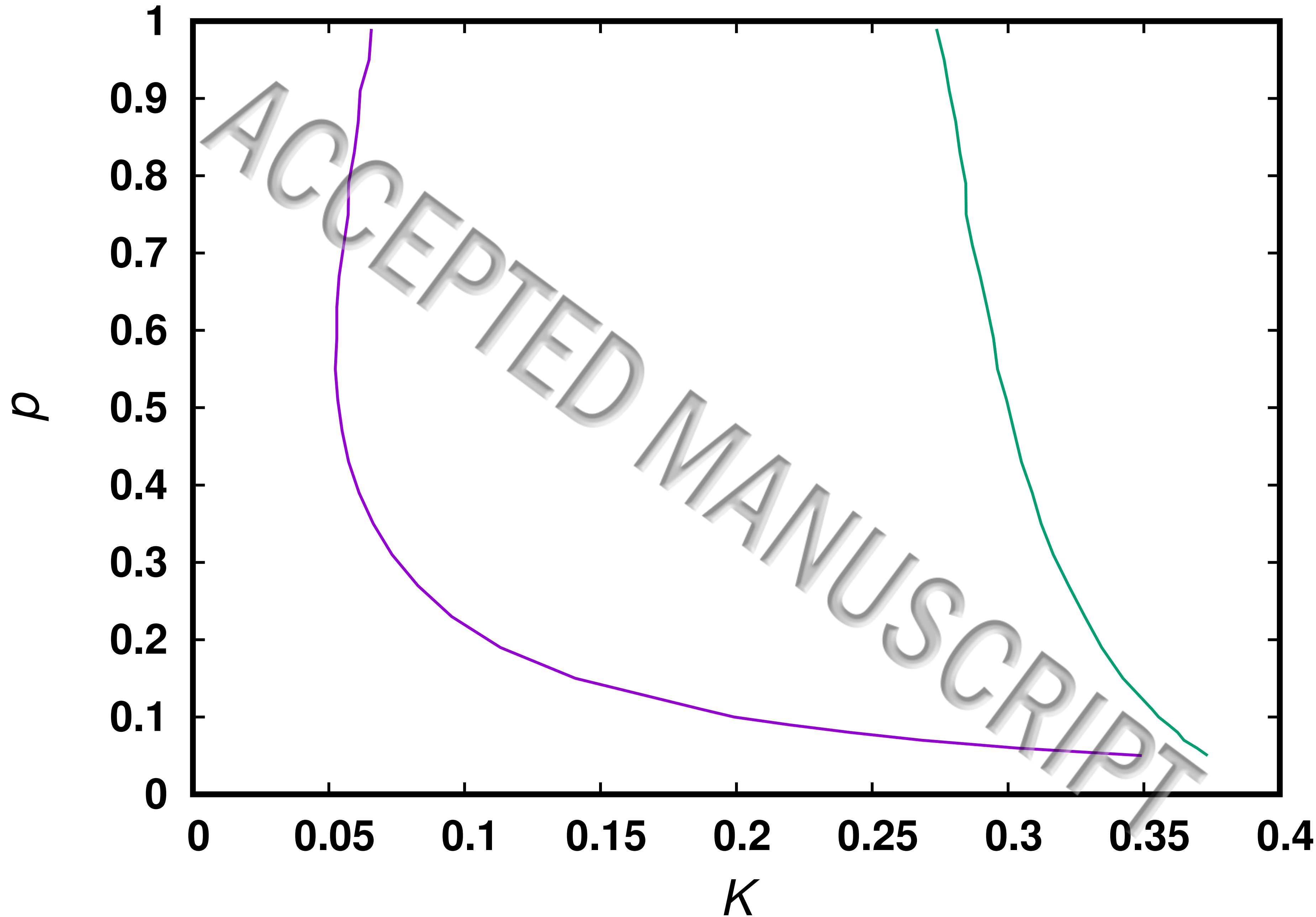

p
K
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.05
0.1
0.15
0.2
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0.35
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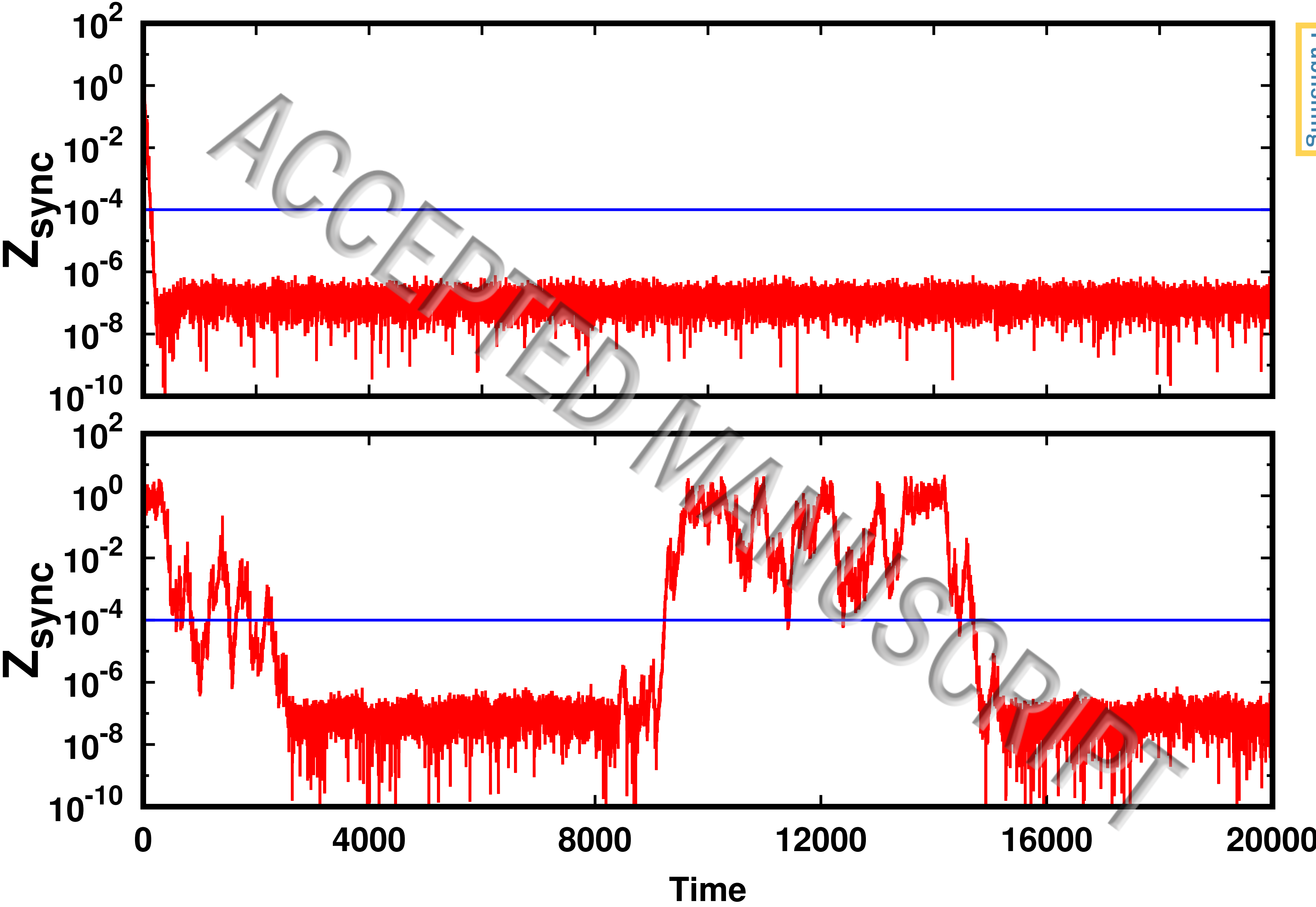
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
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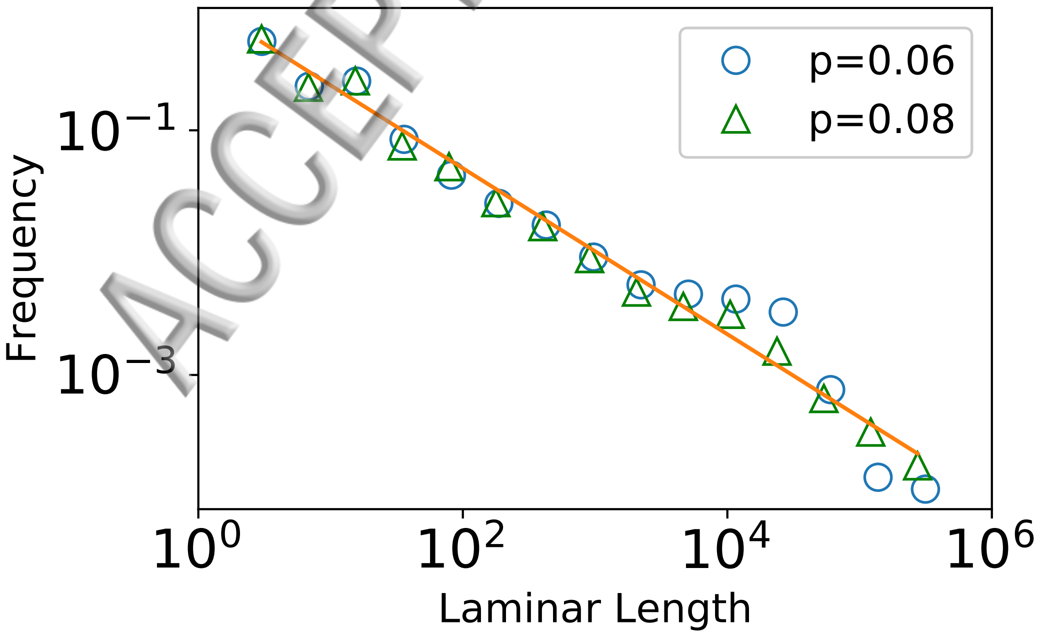
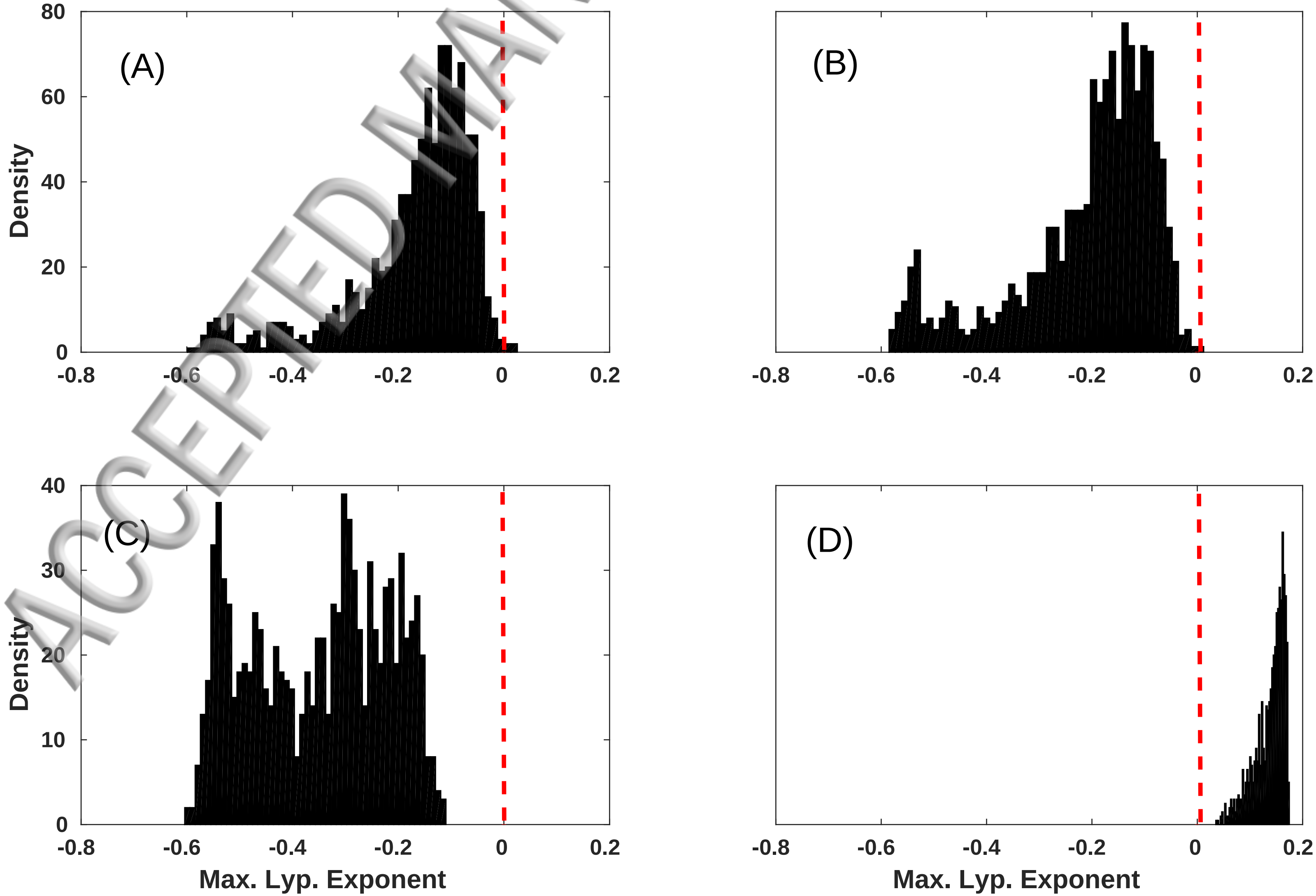
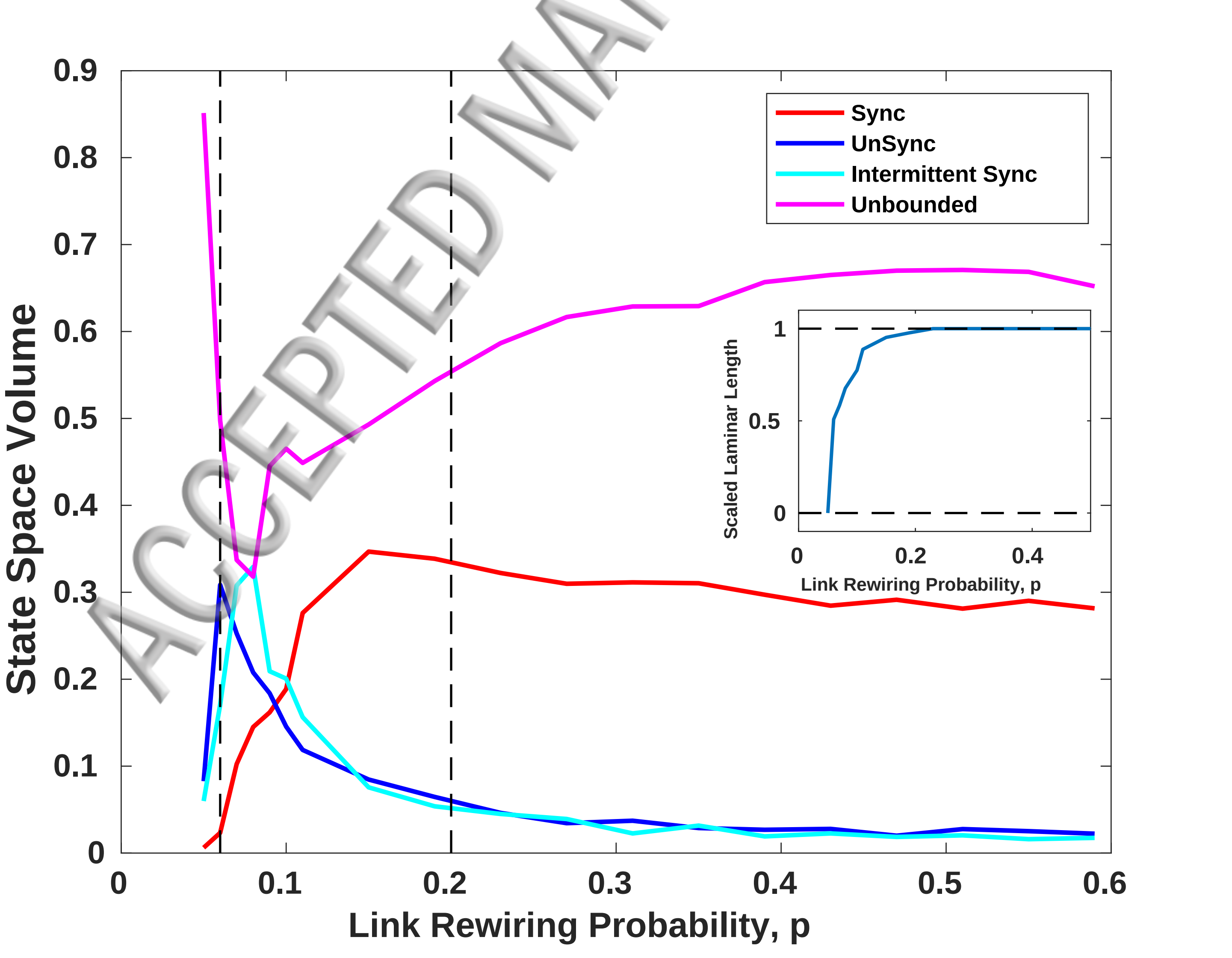
Z_{sync}
0
4000
8000
12000
16000
20000
Time



p=0.06
p=0.08
Frequency
10^{-1}
10^{-3}
10^{0}
10^{2}
10^{4}
10^{6}
Laminar Length



(A)
(B)
(C)
(D)
Density
Density
80
60
40
20
0
40
30
20
10
0
-0.8
-0.6
-0.4
-0.2
0
0.2
Max. Lyp. Exponent
Max. Lyp. Exponent



Sync
UnSync
Intermittent Sync
Unbounded
State Space Volume
Link Rewiring Probability, p
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Scaled Laminar Length
1
0.5
Link Rewiring Probability, p