SECOND YEAR : SEMESTER-III

UNDER SCIENCE AND TECHNOLOGY FACULTY

As Per NEP New Syllabus

INDIAN KNOWLEDGE SYSTEM IN COMPUTING

[IKS]

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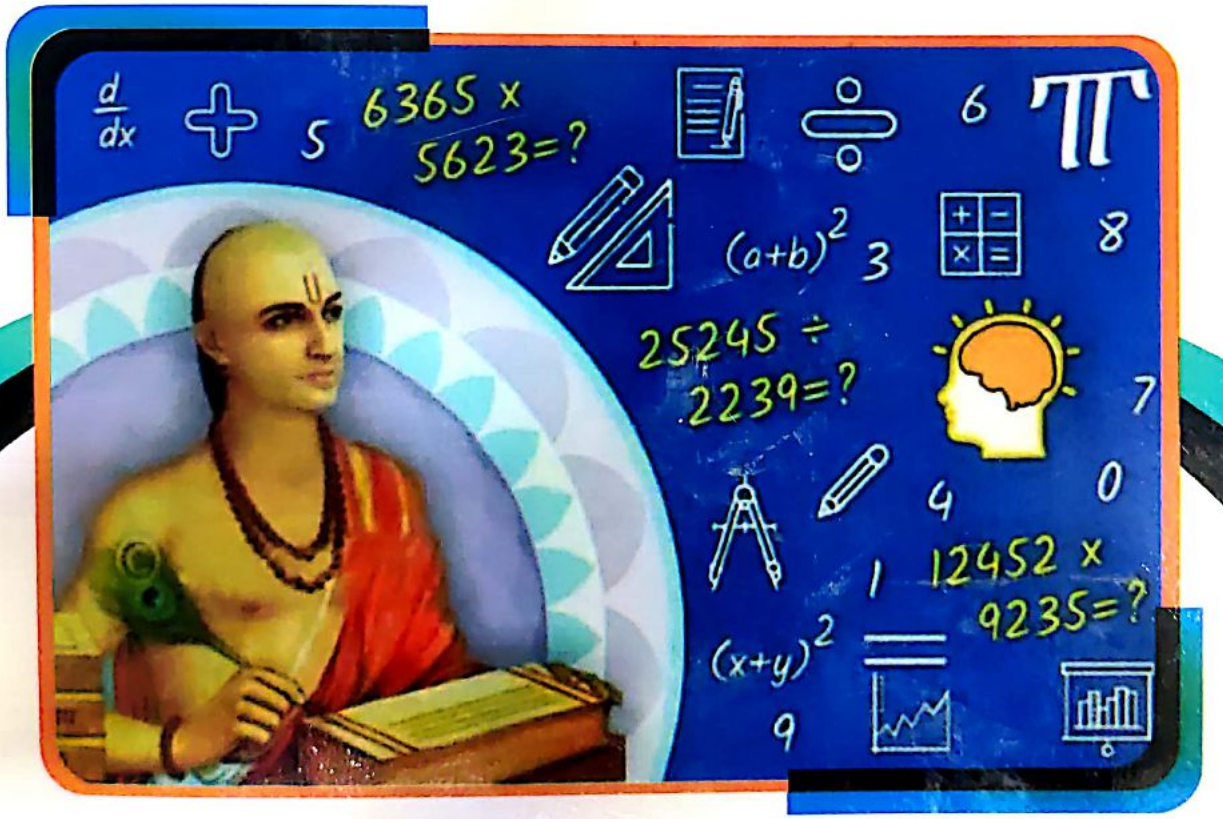

NIRALI PRAKASHAN
ADVANCING IN KNOWLEDGE

Syllabus ...

1. Vedic Mathematics and Computational Thinking (8 Hrs.)

1.1 Introduction to Vedic Mathematics: Origins and importance in ancient India, Sutras and their logical foundation

1.2 Basic Arithmetic using Vedic Methods: Addition, subtraction, multiplication, and division tricks

1.3 Algebraic Applications of Vedic Mathematics: Squaring, square roots, cube roots, and factorization

2. Introduction to Nyaya (Indian Logic) (8 Hrs.)

2.1 Introduction to Nyaya Philosophy: Introduction to Nyaya (Indian Logic), Overview of Indian philosophical schools, Importance of Nyaya in logical reasoning, Types of reasoning (Anumana, Pramana, etc.)

2.2 Nyaya's Four Sources of Knowledge (Pramana): Perception, inference, comparison, verbal Testimony

2.3 Types of Argumentations in Nyaya Vada (truth-based), Jalpa (debate-focused), Vitanda (criticism) Applications in AI & Machine Learning: Logical reasoning models, expert systems, and rule-based AI

3. Panini's Astadhyayi and Chandasāstra (8 Hrs.)

3.1 Introduction to Panini's Astadhyayi: Historical background and linguistic importance

3.2 Rule-Based System of Sanskrit Grammar: Sutras, meta-rules, recursion, and transformations

3.3 Chandasastra's Binary logic and combinatorial techniques

4. Applications of IKS in Computer Science (8 Hrs.)

4.1 Mind and cognition in Samkhya and Yoga: AI insights

4.2 Machine Learning and Indian philosophies: Understanding of human cognition in Indian philosophical schools (Advaita, Samkhya and Yoga)

4.3 Cryptography and Security: Ancient cryptographic methods in Kautilya's Arthashastra, protecting information : Analogies from Indian traditions

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CHAPTER 1

Vedic Mathematics and Computational Thinking

Objectives ...

- Introduction to Vedic Mathematics: Origins and importance in ancient India, Sutras and their logical foundation.
- Basic Arithmetic using Vedic Methods: Addition, Subtraction, Multiplication, and Division tricks.
- Algebraic Applications of Vedic Mathematics: Squaring, Square roots, Cube roots, and Factorization.

1.1 INTRODUCTION TO VEDIC MATHEMATICS

1.1.1. Origins and Historical Background:

- Vedic Mathematics is a system of mathematics that traces its origin to ancient India, specifically to the **Vedas** – the foundational scriptures of Hindu philosophy and knowledge. It's called **"Vedic"** because it comes from the **Vedas**, the ancient books of knowledge from India.
- Vedic Mathematics is not a modern invention, but a rediscovery of ancient mathematical wisdom embedded in the **Atharva Veda** and other Vedic texts.
- Vedic Mathematics is an ancient Indian method of doing math quickly and easily using special tricks or formulas called **Sutras**. It helps to solve big calculations by using short and simple tricks. These Sutras cover all types of math, like addition, subtraction, multiplication, algebra, and even calculus!

Importance in Ancient India:

- In ancient India, knowledge was holistic and mathematics was deeply integrated with spiritual and philosophical thought. Vedic Mathematics reflects this integration by offering methods that are:

- **Mental and intuitive**, allowing fast calculations.
- **Simple and elegant**, reducing complex steps.
- **Flexible**, allowing multiple ways to approach a problem.
- **General and unified**, applying to arithmetic, algebra, geometry, and even calculus.

- Vedic mathematics served as a tool for mental discipline, logic growth, and creative thinking in addition to practical computing. It assisted students in developing a solid foundational mind through focus and clarity.
- People used it in trade, astronomy, architecture, and even art.
- In the Gurukul system, Students used Vedic Maths to solve problems faster without calculators.
- It helped students become super-fast at math without depending on calculators or without writing long steps.

Example 1: Instead of solving 98 × 97 using regular multiplication.

Using a Vedic Math trick:

- Base = 100
- 98 is 2 less than 100, 97 is 3 less than 100
- So,
  Left part = 98 – 3 = 95
  Right part = 2 × 3 = 06
  Answer = **9506**

1.1.2. Sutras and their Logical Foundation:

- The core of Vedic Mathematics lies in the **16 Sutras**, each a short Sanskrit phrase encapsulating a mathematical principle. For example,
  - **"EkādhikenaPūrvena":** *By one more than the previous one.*
  - **"NikhilamNavataścaramamDaśatah":** *All from 9 and the last from 10.*
- These Sutras may look poetic and short, but they hold deep algorithmic logic. Each Sutra provides a method that simplifies complex calculations. They are not just tricks but are logically coherent and can be generalized.

For Example,

Sutra: NikhilamNavataścaramamDaśatah

This Sutra is used for multiplication of numbers close to powers of 10 (like 98 × 97). Instead of using the traditional method, this Sutra allows calculation using complements from 100, reducing time and steps drastically.

- Sutras are concise, aphoristic statements found in various Indian traditions and their logical foundation allowing for memorization and transmission of knowledge, often within a specific philosophical or religious context.

Key aspects of Sutras and their logical foundation:

- **Contextual Understanding:** A sutra's meaning is frequently interpreted in light of the wider text or tradition to which it belongs. Depending on the school of thought or commentary surrounding a particular sutra, it may be interpreted differently.
- **Aphoristic Nature:** Sutras are typically short, concise statements that encapsulate core concepts or principles. Particularly in cultures whose texts were not originally recorded in writing, this concision is essential for knowledge retention and oral transmission.

Logical Framework:

- Sutras are brief, yet they frequently offer a rational foundation for comprehending difficult concepts. They serve as the foundation for additional arguments and elaborations.
- The purpose of the yoga sutra is to help the yogi reach a state of liberation by understanding, devoting, and practicing yoga.

1.1.3 Sutras and their Logical Foundation

1. **Patanjali Yoga Sutras**, which are the core of traditional yoga and describe the road to self-realization via asana, pranayama, yama, niyama, and other disciplines.
   The main Yoga Sutras of Patanjali are Ahimsa, Satya, Asteya, Brahmacharya, Aparigraha, Saucha, Santosa and Tapas.
   Yoga has been practised since the pre-Vedic period, but Maharishi Patanjali codified the knowledge about Yoga systematically via his Yoga Sutras. The Yoga Sutra of Patanjali is a compilation of 196 sutras containing theoretical and practical knowledge about yoga.
   This Yoga Sutra was divided into four books or four chapters. The four chapters are as follows:
   - Samadhi Pada – 51 Sutras: It is focused on Enlightenment.
   - SadhanaPada – 55 Sutras: It is focused on 'Discipline' or 'Practice'.
   - VibhutiPada – 56 Sutras: It means 'Power' or 'Manifestation'.
   - KaivalyaPada – 34 Sutras: It means State of isolation or solitude.

Patanjali Yoga Sutras: Four Pillars of Knowledge:

- There are four methods or modes to practice Yoga and they form the four pillars of knowledge. They are as follows:

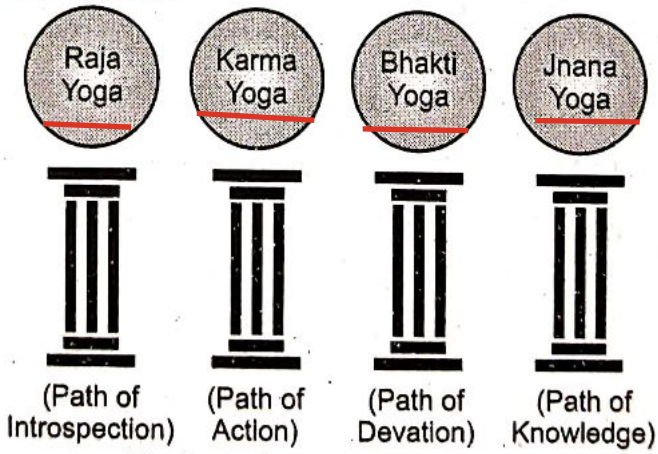

Fig. 1.1: The 4 Pillars of Yoga

mastering Vedic mathematics increases motivation in studying more mathematical applications and saves time. The following is a list of some advantages of Vedic mathematics sutras:

- Calculations become easy and short.
- Simplifications can be done in less time.
- Students undergo less mental stress.
- Results obtained by sutra-based methods can be easily verified with normal procedures.
- The possibility of committing errors by students using these sutras is negligible
- The use of sutras helps students to improve their knowledge and interest in the subject of mathematics.
- Vedic maths helps to solve hard problems using mental calculations.

4. **Mathematical and Philosophical Depth:**

- Swami Tirthaji emphasized that these Sutras are not arbitrary tricks but follow natural laws of number behavior, aligned with the Vedic principle of unity in diversity. He saw mathematics as a spiritual discipline—a way to understand order in the cosmos.
- He also highlighted that Vedic Mathematics develops both the right brain (creativity) and left brain (logic), making it holistic and beneficial in modern education.

1.2 BASIC ARITHMETIC USING VEDIC METHODS : ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION TRICKS

1.2.1 Vedic Subtraction

Vedic Maths Subtraction Methods:

Based on Sutras (Rules):

- Key sutras used:

1. **NikhilamNavatashcaramamDashatah**
   ("All from 9 and the last from 10") – used for subtraction from base numbers (10, 100, 1000, etc.)
2. **ParavartyaYojayet**
   ("Transpose and adjust") – used when subtraction involves borrowing or fractions.

Normal Method:

Consider the subtraction of given number from 10/100/1000/10000

Example 2: Number 7642 will be subtracted from 10000

In normal subtraction we get 10,000 – 7,642= 2,358, so without using calculator, we carry '1' from the left side and continue doing it.

1. Vedic Method: Nikhilam Method (All from 9 and the last from 10):

It uses sutras 'All from 9 and last from 10". The result will be obtained both from left to right and right to left.

In this example we get result as:

(9 – 7)	(9 – 6)	(9 – 4)	(10 – 2)	Right to left here
2	3	5	8	

All digits except the last one will be subtracted from 9 and last digit will be subtracted from 10.

This will be applicable for fraction number also.

Example 3:

Subtract 648.75 from 1000

Step 1: Use complement of 648.75 from 1000:

- Subtract whole part:

  1000 – 648 =.352
- Subtract decimal part (from 1.00):

  1.00 – 0.75 = 0.25

Now adjust: Since we subtracted the decimal separately, subtract 0.25 from 352 is **351.75**

Answer: 351.25

2. Meaning of "ParāvartyaYojayet":

Parāvartya = Transpose/Take the reciprocal or complement

Yojayet = Adjust or apply it

It means: "Invert and apply."

We can apply its principle here in subtraction by adjusting one number to a simpler form, then compensating (i.e., *transpose and adjust*). Think of it as making subtraction easier by cleverly tweaking both numbers.

Example 4: 48.70 – 32.8

Let's align them with the same number of decimal places:

48.70 – 32.80

Step-by-step using ParāvartyaYojayet (Transpose and Adjust):

- **Step 1:** Transpose (adjust the subtrahend to a round number)

  Make 32.80 → 33 (easier to subtract)
- **Step 2:** Adjust (add back the extra 0.20 you added)

  So we now do:

  48.70 – 33=15.70

Now add back the 0.20 we subtracted too much

48.70 – 33 = 15.70

Now add back 0.20,

15.70 + 0.20 = 15.90 (Final Answer)

1.2.2 Vedic Addition

1. **Left to Right Addition (Instead of Right to Left)**

Vedic Sutra Used: *"ParavartyaYojayet"* – Transpose and adjust.

Unlike the traditional method of adding from the units place, Vedic Maths often emphasizes left-to-right addition (like we read numbers), which can be quicker for mental math.

Example 5: 647 + 385

Traditional Method:

647

(+) 376

Do it column-wise (units → tens → hundreds)

1. **Vedic Left-to-Right Method:**
   - Add hundreds: 600 + 300 = 900
   - Add tens: 40 + 70 = 110
   - Add units: 7 + 6 = 13

Now combine:

- 900 + 110 = 1010
- 1010 + 13 = 1023

2. **Rounding and Compensating:** This method is based on the idea of making numbers easier to handle.

Example 6: 198 + 247

Round one number to a convenient number:

Think of 198 as 200, and compensate:

So,

200 + 247 = 447

But we added 2 extra (198 is 2 less than 200), so subtract it:

447 – 2 = 445

Answer: 445

3. **By Column Pairing or Carry Forward Intelligently:** Useful when working with multi-digit numbers in a column format.

Example 7: 789 + 436

Work from left to right:

- Hundreds: 700 + 400 = 1100
- Tens: 80 + 30 = 110
- Units: 9 + 6 = 15

Add all together:

- 1100 + 110 = 1210

  1210 + 15 = 1225

4. **EkadhikenaPurvena (Using One More than the Previous):** This sutra is more commonly used in multiplication, but can indirectly assist in breaking numbers for efficient addition when base numbers are used.

Example 8: 999 + 468

Use 999 as base:

- 999 is almost 1000, so:

  999 + 468 = (1000 + 468) − 1 = 1468 − 1 = 1467

5. **Vertically and Crosswise (for Adding Multiple Numbers):**

Example 9: 248 + 385 + 266

Let's apply a step-by-step vertical and crosswise approach, column by column but working from left to right.

Step 1: Add Hundreds Digit (Left-most)

2 + 3 + 2 = 7 → 700

Step 2: Add Tens Digit

4 + 8 + 6 = 18 → This is 180

Carryover: 1 to hundreds, so total hundreds = 700 + 100 = 800

Keep 80

Step 3: Add Units Digit

8 + 5 + 6 = 19

Keep 9, carry 10 to tens:

Tens total = 80 + 10 = 90

Now:

- Hundreds = 800
- Tens = 90
- Units = 9

Final sum: 800 + 90 + 9 = 899

Summary of Vedic Techniques for Addition:

Table 1.1

Technique	Key Idea	Best Used in
Left to Right	Add from highest place value	Mental math
Rounding and Compensating	Make round numbers	Near multiples
Breaking Numbers	Split digits, add parts.	Complex sums
Base Method	Use base like 1000, 100	Numbers near base

1.2.3 Vedic Multiplication

Methods:

- Vertically and Crosswise (Sutra: *UrdhvaTiryagbhyam*)
- Base Method (Nikhilam Sutra)
- Duplex Method
- Multiplication by 11 Trick
- When First Digits Are Same, and Last Digits Add to 10

(a) Vertically and Crosswise (Sutra: *UrdhvaTiryagbhyam*):

Example 10: Multiply 34 × 16

- **Step 1:** Multiply units → 4 × 6 = 24 → **write 4**, carry 2
- **Step 2:** Cross-multiply and add: (3 × 6 + 4 × 1) = 18 + 4 = 22 → add carry (2) → 24 → **write 4**, carry 2.
- **Step 3:** Multiply tens digit of both numbers → 3 × 1 = 3 → add carry = 2 we get 5 write 5.

  Hence answer is **544**.

(b) Base Method (Nikhilam Sutra):

Sutra: *"NikhilamNavatashcaramamDashatah"* — "All from 9 and last from 10"

Use when numbers are close to powers of 10 (like 10, 100, 1000).

Example 11: Multiply 97 × 95 (Base: 100)

- 97 is 3 less than 100 → write −3
- 95 is 5 less than 100 → write −5

Now:

Left side = 97 − 5 or 95 − 3 = 92

Right side = 3 × 5 = 15

(Fill with two digits since base is 100)

Answer: 9215

(c) Duplex Method: It is used for squaring numbers.

Example 12: Square of 36

Split number: 3 and 6

- **Step 1:** Square units: $6^2 = 36$
- **Step 2:** Double product of digits: $2 \times 3 \times 6 = 36$
- **Step 3:** Square tens: $3^2 = 9$

Now combine (right to left):

- Units: **Write 6** carry =3
- Middle: 36 + 3 (carry) = 39, **write 9**, carry 3
- Left: 9 + 3 = 12 write **12**

Answer: 1296

(d) Multiplication by 11 Trick:

The trick of 11:

Step 1: Divide the number into two parts.

Step 2: Add the two parts which will form the middle number.

Let us say that we have to multiply 32 with 11.

Step 1: Divide 32 into 3 and 2

Step 2: The middle will be 3 + 2 = 5

So our answer to 32 × 11 would be 352.

Similarly, 75 × 11 = 7, 7 + 5, 5. Because 7+5 = 12, we will carry 1 to the previous digit and our final answer would be 825.

Example 13: 52 × 11

- Write first digit: 5
- Add digits: 5 + 2 = 7
- Write last digit: 2

Answer: 572

For 3-digit numbers:

Example 14: 314 × 11

- 3 (first digit)
- 3 + 1 = 4
- 1 + 4 = 5
- 4 (last digit)

Answer: 3454

(e) When first digits are same and last digits add to 10

Example 15: 42 × 48

- First digits same: 4
- Last digits: 2 + 8 = 10

Now:

- Multiply 4 by (4 + 1) = 4 × 5 = 20
- Multiply last digits: 2 × 8 = 16

Answer: 2016

Example 16: Multiply 207 by 205.

Number: 207 × 205	Steps
207 – 7 = 200 205 – 5 = 200	1. Subtract the number in units place by the number itself
205 + 7 = 212	2. Select number in units place among given two number and add it to another number
212 × 200 = 42,400	3. Multiply the above-obtained number with the number obtained in step 1.
7 × 5 = 35	4. Multiply the numbers in units place of the given question
42,400 + 35 = 42,435	5. Add the answers from step 1 and Step 4. This is the answer.

Answer: 42435

- Multiplication of numbers using Sutra *EkanyunenaPurvena*:

Method 1:

- In this method, we can multiply the numbers whose unit digits are added up to 10 or powers of 10.
- Let's have a look at the solved example given below to understand the multiplication of numbers.

Example 17: Multiply 63 and 67.

Solution: 63 × 67

Sum of unit digits = 3 + 7 = 10

Digits in tens places = 6

So, we can write the multiplication as:

$$63 \times 67 = \frac{6 \times (6+1)}{3 \times 7}$$

$$= \frac{6 \times 7}{3 \times 7} = \frac{42}{21} = 4221$$

- We can also verify the result using normal mathematical calculations.
- This method of multiplication is referred to as the Sutra EkadhikenPurvena. This method can also be used to multiply two numbers whose last two digits are added up to 100 the last three digits are added up to 1000. Also, in the case of mixed fractions, the sum of proper fractions must be added up to 1 to apply this method of multiplication.

Method 2:

- If two numbers are to be multiplied and one of these numbers is having only 9's then we can apply this method.

Example 18: Multiply 876 and 999.

Solution: Given, two numbers are 876 and 999.

Now, subtract 1 from 876.

$$876 - 1 = 875$$

Subtract 875 from 999.

$$999 - 875 = 124$$

Thus,
$$876 \times 999 = \frac{876 - 1}{999 - 875}$$
$$= \frac{875}{124}$$
$$= 875124$$

This method of multiplying numbers is Sutra EkanyunenaPurvena.

1.2.4. Vedic Division

ParavartyaYojayet Sutra (Sanskrit: परावर्त्य योजयेत् – "Transpose and adjust"):

- ParavartyaYojayet, a Vedic mathematics sutra, translates to "transpose and apply". It's a method primarily used for division, particularly when the divisor is greater than a power of 10 (like 12, 102, 112, etc.). The core idea is to transpose or change the sign of the digits in the divisor and then apply this modified divisor to the dividend to find the quotient and remainder.
- It is especially useful when the divisor is near a base like 10, 100, or 1000. The process is:
  1. **Identify the Divisor and Base:**
     - Determine the divisor and the nearest power of 10 (base).
  2. **Find the Complement:**
     - Calculate the complement of the divisor with respect to the base. This is the transposed part, and its sign will be changed.
  3. **Set up the Problem:**
     - Write the dividend, and below it, set apart space for the quotient and remainder. Place the transposed digits of the divisor to the right of the dividend.

4. **Perform the Calculation:**
   - Bring down the first digit of the dividend as the first digit of the quotient.
   - Multiply this quotient digit with the transposed digits of the divisor and write the results below the next digits of the dividend.
   - Add the corresponding digits in each column to get the next digit of the quotient and remainder (if any).
   - Repeat the multiplication and addition steps until you reach the last digit of the dividend.

Example 19: 1232 ÷ 9

Let's divide using ParavartyaYojayet.

Step 1: Set Up

- Divisor = 9 → base = 10
- 10 – 9 = 1 → This is your "complement" (used to multiply digits)

Write 1234 as:

Quotient Part | Remainder

↓

1 → carry = 1

1 × 1 = 1 → 2 (digit in hundred place) + 1 = 3

3 × 1 = 3 → 3 (Tens digit) + 3 = 6

6 × 1 = 6 → 2 (Unit digit) + 6 = 8

Now Quotient = 1, 3, 6

Last value 8 is the final digit = Remainder

Final Answer: Quotient = 136, Remainder = 8

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2. Flag Method:

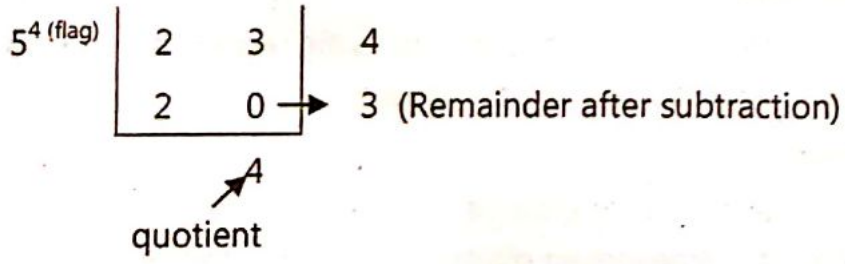
- Used for larger divisors (like 12, 23, 56). Instead of traditional long division, this method sets up a table where you use the first digit of the divisor as a "flag" and apply subtraction/multiplication systematically.

Example 20: 234 ÷ 54

Here, we write as

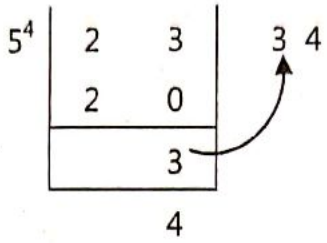
$5^{4\,(flag)}$ | 2 3 | 4

Step 1: We are using only 5 digit and 4 is treated as flag; so that we will first divide 23 by 5 instead of 234.



Step 2: Here after division, we get 4 is quotient and 3 is n.

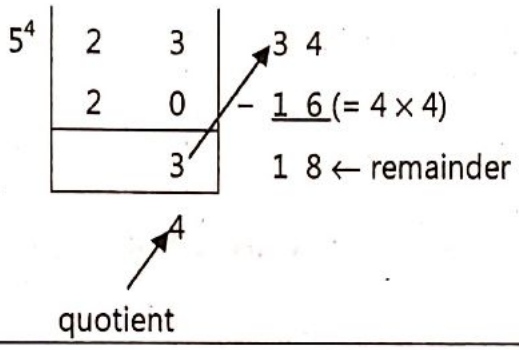
Step 3:



Here, we consider number is 34 by writing 3 before 4 and then multiply flag 4 with quotient 4, we get 16, then subtract.

34 – 16 = 18.

Step 4:



Division Tricks:

- Vedic Mathematics is based on 16 sutras (aphorisms) from ancient Indian texts. The division techniques in Vedic Maths are built on logic, place value system, and complements using base-10 systems (or powers of 10 like 10, 100, 1000, etc.). The two key Vedic sutras for division are:

1. ParāvartyaYojayet Sutra:

- **Meaning**: "Transpose and Apply"
- It is used when the divisor is just more than the base (like 11, 12, 101, 102...).
- This method breaks the problem into smaller, base-related parts. It uses negative complements (transpose = subtract excess from the base) and Applies recursive multiplication and addition to get quotient and remainder.

Mathematical Foundation:

Let's say you're dividing by 13.

13 = 10 (base) + 3

Instead of dividing directly by 12, you:

- Use base 10
- Convert the "+3" into a negative operation (–3). This simplifies division by reducing it to base-10 operations.

2. NikhilamNavatashcaramamDashatah Sutra:

- **Meaning:** "All from 9 and the last from 10"
- It is used when the divisor is just less than a base (like 9, 98, 99, 999...).
- This trick uses the complement of the number with respect to the nearest base. For example:
  - Complement of 98 (w.r.t 100) = 02
  - You divide using this complement instead of the original number
- The dividend is split into two parts:
  - Left part gives the quotient.
  - Right part is used to calculate the remainder by multiplying the left part with the complement.
- This reduces complex division into multiplication and addition/subtraction, which are easier and faster.

Example Connecting Logic:

Example 21: 935 ÷ 98

Traditional way: Estimate → Long division → Subtract → Bring down digit

Vedic way:

- Base = 100, complement of 98 = 2
- Split 935 as 9 | 35
- 9 × 2 = 18
- 35 + 18 = 53 → Remainder
- Quotient = 9

This shows how the digit position and complement logic simplify the process.

Trick 1: Division When Divisor is Just More Than a Base

(Use **ParāvartyaYojayet** method)

imp

Example 22: 1232 ÷ 12

Step 1: Base = 10, and

12 = 10 + 2 → Take complement of 2 = –2

Step 2: Write digits of dividend:

1 2 3 2 ← dividend digits

↓

- Bring down **1**
- 1 × –2 = –2 → 2 – 2 = 0
- 0 × –2 = 0 → 3 + 0 = 3
- 3 × –2 = –6 → 2 – 6 = –4 (means remainder)

Quotient: 103, Remainder: –4 (adjust to positive by subtracting from 12)

Final answer: Quotient = 102, Remainder = 8

Trick 2: Division When Divisor is Just below a Base

(Use **Nikhilam Sutra**)

Example 23: 935 ÷ 99

Step 1: Base = 100, complement of 99 = 1

Step 2: Split dividend:

9 | 35 (because 100 has 2 zeros, split last 2 digits)

Step 3:

- Multiply 9 × 1 = 9
- Add to 35 → 35 + 9 = 44

Quotient = 9, Remainder = 44

Trick 3: Division by 9, 99, 999... (Pure Nikhilam Method)

Example 24: 236 ÷ 9

Step 1: Add digits of the number in stages:

- First digit: 2
- Add next: 2 + 3 = 5
- Add next: 5 + 6 = 11

→ Quotient = All except last: 2, 5 → i.e., **Quotient = 25**,

→ Remainder = last sum = 11

Now reduce remainder:

- 11 ÷ 9 = 1 rem 2
- So Final Quotient = 25 + 1 = **26**, Remainder = **2**

Trick 4: Division by Numbers like 5, 25, 125 (Power of 10 technique)

These are factors of 10^n, so use smart multiplication:

Example 25: 495 ÷ 5

Multiply by 2 and divide by 10

→ 495 × 2 = 990

Then 990 ÷ 10 = 99

Final Answer: 99

1.3 ALGEBRAIC APPLICATIONS OF VEDIC MATHEMATICS: SQUARING, SQUARE ROOTS, CUBE ROOTS, AND FACTORIZATION

- Vedic Mathematics provides efficient methods for squaring, finding square and cube roots, and factorization through specific sutras and pattern recognition.

- Let us see various Vedic Math techniques for squaring, finding roots, and factorization:

1.3.1 Squaring a Number

Squaring numbers ending in 5 (EkadhikenaPurvena)

Sutra is **"BY ONE MORE THAN PREVIOUS ONE"**

Step 1: Add 1 to the first digit from the left and multiply by the number itself.

Step 2: Add 25 at the end to the number obtained from step 1.

Example 26: 65^2

Step 1: Add 1 to 6 = 7 and multiply with 6, we get $6 \times 7 = 42$.

Step 2: Add 25 as last two digit.

Answer is 4225

Example 27:

155^2	
$15 \times (15 + 1)$ $= 15 \times 16 = 240$	Add 1 to the left number (15) and multiply by the number itself and add 5^2 (25) at last of 240 Answer is 24025

Squaring of numbers less than 50 and numbers not ending with 5.

Number	Steps
34	50 – 16 = 34
$5^2 = 25$ $= 25 + (-16) = 9$	Square the first digit (5) of first part (50) then add part (–16)
$16^2 = 256$	Square the second part of number (16)
9 + 256 = 1156	Add the answers got in step 1 (9) and step 2 (256)

Squaring of number near to their base 10,100,1000, and so on:

Number	Steps
103	100 + 3 = Divide the given number to their base and number
103 + 3= 106	Add the second part of number 3 to the given number (103)
$3^2 = 09$	Square the second part of the 3^2
10609	Combine the numbers from step 1 and step 2

- If the number is lesser than its nearest base number then the deficient number is reduced from the given number.
- If the given number is greater than its nearest base number then the surplus number is added to the given number.

1.3.2 Finding Square Roots (Observation and Nikhilam)

- For performing square roots, we will have to keep some facts in mind:
  - Squares of numbers from 1 to 9 are 1, 4, 9, 16, 25, 36, 49, 64, 81.
  - Square of a number cannot end with 2, 3, 7, and 8.
  - We can say that numbers ending with 2, 3, 7, and 8 cannot have a perfect square root.
  - The square root of a number ending with 1 (1, 81) ends with either 1 or 9.
  - The square root of a number ending with 4 (4, 64) ends with either 2 or 8.
  - The square root of a number ending with 9 (9, 49) ends with either 3 or 7.
  - The square root of a number ending with 6 (16, 36) ends with either 4 or 6.
  - If the number is of 'n' digits then the square root will be 'n/2' OR '(n + 1)/2' digits.

Let us understand an example of finding a square root of 1764.

- The number ends with 4. Since it's a perfect square, square root will end with 2 or 8.
- We need to find 2 perfect squares (in multiples of 10) between which 1764 exists. The numbers are 1600 (40) and 2500 (50).
- Find to whom 1764 is closer. It is closer to 40. Therefore the square root is nearer to 40. Now from Step 2, possibilities are 42 or 48 out of which 42 is closer to 40.
- Hence the square root = **42**

- Vedic Maths offers several methods for calculating square roots, including techniques for both perfect and non-perfect squares. One common approach involves dividing the number into groups of two digits from right to left, then using the Nikhilam method to find the square root digit by digit. For non-perfect squares, a formula-based method or approximation techniques can be employed.

Method for Perfect Squares:

1. **Grouping:** Start by grouping the digits of the number into pairs from right to left.
2. **First Digit:** Find the largest perfect square less than or equal to the leftmost group. The square root of this perfect square will be the first digit of the answer.
3. **Remainder and Next Digit:** Subtract the perfect square from the leftmost group. Bring down the next group of digits to form the new dividend.
4. **Divisor:** Double the first digit of the square root to form the divisor.
5. **Finding the next digit:** Find the largest digit that, when multiplied by the divisor (and then combined with the new digit to form a number), is less than or equal to the dividend.
6. **Repeat:** Repeat steps 3-5 until all digits are processed.

Example 28: Find the square root of 1369.

1. **Grouping:** 13 69.
2. **First Digit:** The largest perfect square less than or equal to 13 is 9 (3^2). So the first digit is 3.
3. **Remainder and Next Digit:** Remainder is 13 – 9 = 4. Bring down 69. New dividend is 469.
4. **Divisor:** Double of 3 is 6.
5. **Finding the next digit:** $67 \times 7 = 469$. The next digit is 7.
6. **Result:** The square root of 1369 is 37.

Example 29: $\sqrt{1764}$

(1) First group 17 | 64.
(2) From first pair 64 last digit is 4. Therefore, write 2 as left digit.
(3) Then we take light group which is 17, find the nearest square root, which is 4 (near to 16).
(4) Multiply 4 by 10 so $4 \times 10 = 40$.
(5) Now check 40 is greater than 17, if yes add 2 to 40 else subtract 2 from 40, where 2, we got from step 2.
(6) We get 42.

$\therefore \sqrt{1764} = 42.$

Example 30: $\sqrt{6241}$

(1) Last digit is 1 which is 1^2 or 9^2 possible. Therefore, last digit of square root is either 1 or 9.
(2) Consider 62, now check which is in between 7^2 and 8^2. Therefore, we always consider smallest one. Hence, first digit of square root is 7.
(3) Two possibilities either answer is 71 or 79.
(4) Check 62 is near to 7^2 or 8^2, which is near to 64.

Hence answer is **79.**

Method for Non-Perfect Squares:

- **Approximation Formula:**

Formula for square root of non-perfect number.

$$\sqrt{x \pm y} = \sqrt{x} \pm \frac{y}{2 \times \sqrt{x}}$$

Example 31: $\sqrt{29}$

We write $\sqrt{29}$ as 25 + 4

x = 25, y = 4

$= \sqrt{25} + \frac{4}{2 \times \sqrt{25}}$ (+ sign because we have done addition)

$= 5 + \frac{4}{10} = \mathbf{5.4}$

Example 32: $\sqrt{61}$

We write 61 as 64 - 3

x = 64, y = 3

$= \sqrt{64} - \frac{3}{2 \times \sqrt{64}}$ (+ sign because we have done addition)

$= 8 - \frac{3}{16} = \mathbf{7.81}$

1.3.3 Finding Cube Roots (Observation and Pattern Recognition)

- By looking for patterns in the cube's last digit in number and the accompanying cube root, Vedic Maths offers a simple way to identify the cube roots of perfect cubes. The process is grouping the digits, figuring out the first digit by locating the closest perfect cube that is smaller than the remaining part of the number, and identifying the last digit of the cube root based on the last digit of the cube.
- To find cube roots of perfect cubes by Vedic Mathematics method, first identify the last digit of the cube root, based on the last digit of the given number. This always required to remember the cube of first 10 natural numbers. (The cube of 1 is 1, the cube of 2 is 8, the cube of 3 is 27 and so on....). Then determining the remaining digits by considering the remaining part of the number after removing the last three digits. Finding the nearest cube root of remaining digit of numbers.

Example 33: Find cube root of 91125 ($\sqrt[3]{91125}$)

(1) Consider last digit (5) and find out which cube gives last digit 5 ($5^3 = 125$). Therefore write 5 at unit place.

(2) Remove last three digits (125).

(3) We get 91. Find 91 is near to which cube ($4^3 = 64$ and $5^3 = 125$). It is near to 4. Therefore, write 4.

Answer is 45.

SOLVING CUBE ROOTS

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000

- We will be solving the cube root in 2 parts. First, we shall solve the RHS of the answer and then we shall solve the LHS of the answer. If you wish you can solve the LHS before the RHS. There is no restriction on either method but generally people prefer to solve the RHS first.

Examples 34: Find the cube root of 287496.

- We shall represent the number 287496 as 287 496.
- Next, we observe that the cube 287496 ends with a 6 and we know that when the cube ends with a 6, the cube root also ends with a 6. Thus our answer at this stage is 6. We have thus got the RHS of our answer.
- To find the LHS of the answer we take the number which lies to the left of the slash. In this case, the number lying to the left of the slash is 287. Now, we need to find two perfect cubes between which the number 287 lies in the number line. From the key, we find that 287 lies between the perfect cubes 216 (the cube of 6) and 343 (the cube of 7).
- Now, out of the numbers obtained above, we take the smaller number and put it on the LHS of the answer. Thus, out of 6 and 7, we take the smaller number 6 and put it beside the answer of 6 already obtained. Our final answer is 66. Thus, 66 is the cube root of 287496.

Example 35: Find the cube root of 205379.

- We represent 205379 as 205/379.
- The cube ends with a 9, so the cube root also ends with a 9. (The answer at this stage is 9.)
- The part to the left of the slash is 205. It lies between the perfect cubes 125 (the cube of 5) and 216 (the cube of 6).
- Out of 5 and 6, the smaller number is 5 and so we take it as the left part of the answer. The final answer is 59.

Example 36: Find the cube root of 2197.

- The number 2197 will be represented as 2/197.
- The cube ends in 7 and so the cube root will end with a 3.
- We will put 3 as the RHS of the answer.
- The number 2 lies between 1 (the cube of 1) and 8 (the cube of 2).
- The smaller number is 1 which we will put as the LHS of the answer. The final answer is 13.

1.3.4 Factorization

- Factorization using Vedic Mathematics is done by using 2 Sutras
- We use combination of 2 sutras.

1. **Anurupyena (Proportionality):**
   - The **Anurupyena** sutra focuses on finding a ratio between the coefficients of the quadratic expression. We split the middle term (coefficient of x) of quadratic equation in 2 terms such that Proportion/Ratio of coefficient of x^2 term to first coefficient of x term = Ratio of second coefficient of x term to constant term. That ratio of the first 2 coefficient is one of the root of equation.
2. **Adyamadyenantyamantya (First by first and last by last):**
   - The **Adyamadyenantyamantya** (commonly called as Adyamadyena) sutra helps determine the factors by relating the first and last terms of the quadratic to the established ratio. We divide the first term's coefficient of the equation with 1^{st} term of factor obtained above and last term of the equation with the last term of the same factor.

Steps for Factorization:

1. **Identify the coefficients:** In a quadratic expression of the form $ax^2 + bx + c$, identify the values of 'a', 'b', and 'c'.
2. **Find the proportion:** Look for a proportion between the first coefficient (a) and the middle coefficient (b), or between the middle coefficient (b) and the constant term (c).
3. **Simplify the ratio:** Simplify the ratio to its lowest terms. This ratio will be crucial for finding the factors.
4. **Determine the first factor:** Use the simplified ratio to form the first factor. For example, a ratio of 1:2 might lead to the factor (x + 2).
5. **Find the second factor:** Divide the first coefficient (a) by the first term of the factor and the constant term (c) by the second term of the factor to find the second factor.

Example 37: Let's say we have the quadratic expression $2x^2 + 5x + 2$.

1. **Coefficients:** a = 2, b = 5, c = 2.
2. **Find the proportion:** Notice that the ratio 2 : 4 (or 1 : 2) can be formed using the first coefficient and the middle term (2 : 5 is not a simple ratio).
3. **Simplify the ratio:** The ratio is already in its simplest form, 1 : 2.
4. **First factor:** Using the ratio 1:2, we can form the first factor (x + 2).
5. **Second factor:** Divide the first coefficient (2) by 1 (from the ratio) and the constant term (2) by 2 (from the ratio). This gives (2x + 1).

   Therefore, the factors of $2x^2 + 5x + 2$ are (x + 2) and (2x + 1).

Example 38: $2x^2 + 5x - 3$.

1. **Anurupyena:** Split middle terms coefficient (5) in 2 parts such that coefficient of x^2 term to 1st coefficient of x term = Ratio of 2nd coefficient of x term to constant term. Hence split it in 6 and $-1\left(\frac{2}{6} = \frac{-1}{-3}\right) \Rightarrow 2x^2 + 6x - x - 3$. So **1st factor: x + 3** (2 : 6).
2. **Adyamadyenantyamantya:** Divide the first term's coefficient (2) of equation by 1st term of factor (1) and divide last term of equation (−3) by 2st term of factor (3). So **2nd factor: 2x − 1.**

Benefits of Vedic Maths Factorization:

- **Speed and Efficiency:** Vedic methods can be faster and more efficient for certain types of quadratic expressions.
- **Mental Calculation:** The sutras are designed to facilitate mental calculations, which can be helpful in competitive exams.
- **Simplicity:** The methods can be simpler to understand and apply compared to traditional factorization techniques.

Summary

- Originates from the Vedas, especially the Atharva Veda, making it a part of ancient Indian knowledge traditions.
- It is a rediscovered system of ancient mathematical wisdom, not a modern invention.
- Based on concise formulas or Sutras, which are quick, efficient, and mentally applicable methods.
- Covers a wide range of mathematical operations: arithmetic, algebra, geometry, and even calculus.
- **Key Features of Vedic Mathematics:**
  1. **Mental and intuitive:** Promotes fast and accurate calculations.
  2. **Simple and elegant:** Eliminates complex steps.
  3. **Flexible:** Offers multiple solution methods.
  4. **Unified approach:** Applicable across various mathematical branches.
- Patanjali's Yoga Sutras and Nyaya Sutras represent foundational Indian philosophical systems Yoga emphasizing self-realization through disciplined practice (asanas, pranayama, yamas, niyamas) and Nyaya focusing on logic, reasoning, and valid knowledge through perception, inference, comparison, and verbal testimony.
- Vedic Mathematics, derived from ancient Hindu texts and formalized by Swami Tirthaji, offers 16 powerful Sutras for fast, error-free mental calculations.

- These systems collectively reflect the depth and interdisciplinary nature of Indian thought where philosophy, mathematics, logic, and spirituality were deeply interlinked, offering timeless methods of knowledge acquisition, problem-solving, and self-development.
- Simplified techniques like addition, subtraction, multiplication emphasizes mental calculation and time-saving tricks for everyday arithmetic.
- Vedic Mathematics applies Vedic Sutras to solve algebraic problems such as:
  - Squaring numbers, Extracting square and cube roots, Factorizing expressions, Enhances understanding of algebra through intuitive and pattern-based methods.

Exercise

Q.I Multiple Choice Questions:

1. The Core of Vedic mathematics contained ..... sutras.
   (a) 16 (b) 18 (c) 14 (d) 20
2. The sutra NikhilamNavataścaramamDaśatah is used for .....
   (a) addition (b) multiplication
   (c) division (d) subtraction
3. The Yoga Sutra of Patanjali is a compilation of ..... sutras containing theoretical and practical knowledge about yoga.
   (a) 169 (b) 194 (c) 192 (d) 196
4. Which of the chapters of sutras focused on Enlightenment?
   (a) Samadhi Pada (b) SadhanaPada
   (c) VibhutiPada (d) KaivalyaPada

Answers

1. (a)	2. (b)	3. (d)	4. (a)

Q.II Answer the following questions:

1. What is the purpose of the yoga sutras? 6
2. What are the main Yoga Sutras of Patanjali? 6
3. How many sutras are there in Patanjali yoga? 6
4. List the four pillar of knowledge in Patanjali yoga. 6
5. What is Vedic mathematics? 3
6. Explain the method offered in Vedic mathematics which reflects importance in ancient India.
7. Explain term Sutras. 4, 5

8. List key aspects of sutras and their logical foundation. 5
9. Explain Patanjali Yoga Sutras. 5
10. List four pillars of knowledge in Patanjali Yoga Sutras. 5
11. Write a note on The Nyaya Sutras.
12. Explain key aspects of The Nyaya Sutras.
13. What are the advantages of Vedic mathematics sutras? 7
14. Explain NikhilamNavatashcaramamDashatah subtraction method by example.7
15. Explain EkadhikenaPurvena (Using One More than the Previous) multiplication method by example.
16. List two sutras of factorization. 23

Q.III Multiply by Vedic method:

1. 356 × 11
2. 56·× 11

Q.IV Divide by Vedic method:

1. 345 by 23
2. 876 by 234
3. 869 by 546

Q.V Find square root of:

1. 1521
2. 6889
3. 5476

Q.VI Find the cube roots of the following numbers:

1. 132651
2. 658503
3. 314432
4. 110592
5. 46656
6. 5832
7. 421875
8. 1030301

***

CHAPTER 2

Introduction to Nyaya (Indian Logic)

Objectives ...

- Understand Nyaya Philosophy: Introduction to Nyaya (Indian Logic), Overview of Indian philosophical schools, Importance of Nyaya in logical reasoning, Types of reasoning (Anumana, Pramana, etc.).
- Understand Nyaya's Four Sources of Knowledge (Pramana): Perception, Inference, comparison, Verbal testimony.
- Understand Types of Argumentations in NyayaVada (truth-based), Jalpa (debate-focused), Vitanda (criticism).
- Understand Applications in AI and Machine Learning: Logical reasoning models, Expert systems, and Rule-based AI.

2.1 INTRODUCTION TO NYĀYA PHILOSOPHY

2.1.1 Introduction to Nyaya (Indian Logic)

- The Nyāya philosophy was founded by the ancient sage Gotama, also known as Gautama or Akṣapāda around the 2nd century BCE. That's why this system is sometimes called the Akṣapāda system. Nyāya mainly deals with how to think correctly and how to gain true knowledge about reality. It helps students develop strong logical thinking and critical reasoning skills. Because of this, Nyāya is also known by other names like Nyāyavidyā (science of logic), Tarkaśāstra (science of reasoning), and Ānvīkṣikī (science of critical study).
- Although Nyāya focuses a lot on logic and knowledge, its main goal is liberation (moksha) freedom from all pain and suffering. Like other Indian philosophies, Nyāya believes that true knowledge helps us understand reality, and that this understanding eventually leads to liberation. So, Nyāya is not just about logic–it is a complete philosophy of life.

- The first and most important book of Nyāya is the NyāyaSūtra, written by Gotama. It has five chapters, and each chapter has two sections. Many later scholars wrote commentaries to explain and expand on Gotama's ideas. These include famous works like NyāyaBhāṣya by Vātsyāyana, NyāyaVārttika by Uddyotakara, and Nyāyamañjarī by Jayanta. These texts developed the system through debates and responses to critics.
- Over time, Nyāya evolved into two main branches:
  1. **Ancient Nyāya (PrācīnaNyāya):** Based on Gotama's original teachings.
  2. **Modern Nyāya (NavyaNyāya):** Began with the work Tattvacintāmaṇi by Gaṅgeśa. This new school focused more deeply on logic and the structure of reasoning, and became especially famous in Mithilā and Navadvīpa (Bengal).
- Eventually, a blended school of Nyāya and Vaiśeṣika philosophy also developed, combining ideas from both systems.
- The Nyāya philosophy can be divided into four main parts:
  1. Theory of knowledge (epistemology)
  2. Theory of the physical world (metaphysics)
  3. Theory of the self and liberation
  4. Theory of God

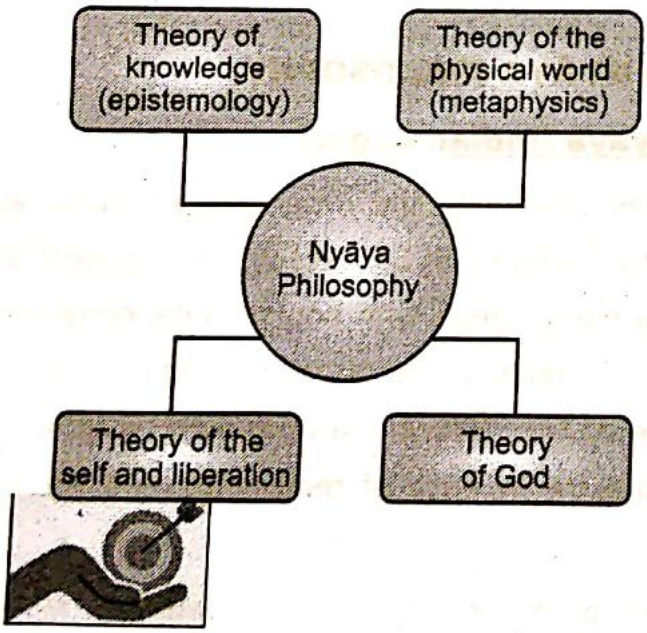
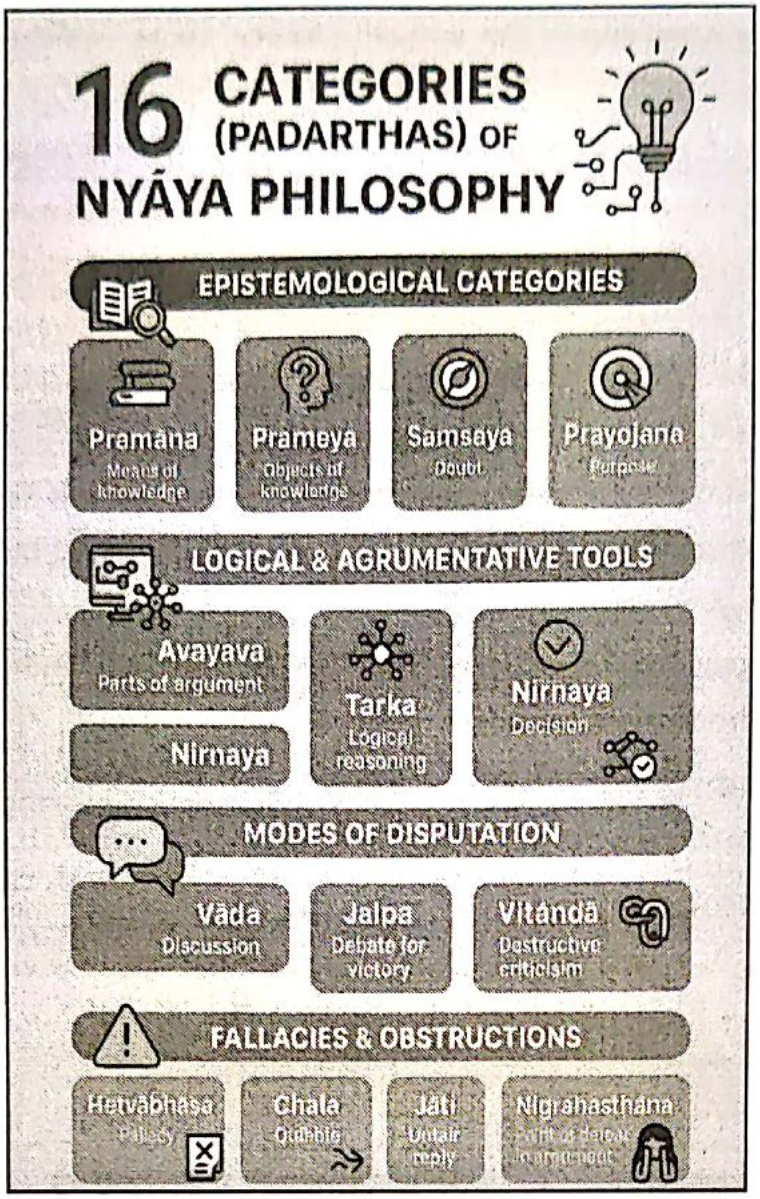


Fig. 2.1: Nyaya Philosophy: Essential Concepts

- In total, Nyāya discusses sixteen key topics (called padārthas), which guide how to debate, reason, and find truth. These are listed in the Fig. 2.2.



drastanta
siddhanta

Fig. 2.2: Sixteen categories of Nyaya Philosophy

2.1.2 Overview of Indian Philosophical Systems:

- Indian Philosophy refers to the diverse range of philosophical speculations and systems developed by Indian thinkers–ancient and modern, Hindu and Non-Hindu, theist and atheist. It encompasses a rich tapestry of metaphysical, epistemological, ethical, and logical inquiries that originated in the Indian subcontinent. These philosophies engage deeply with questions about reality, knowledge, self, consciousness, and liberation, and are often closely linked to religious and cultural traditions such as Hinduism, Buddhism, Jainism, and others.
- Indian philosophy is traditionally divided into two main groups: Orthodox (Āstika) and Heterodox (Nāstika) as shown in the Fig. 2.1. This classification was mainly used by traditional Hindu scholars.

- The orthodox schools believe in the authority of the Vedas, which are ancient Hindu scriptures. There are six main orthodox schools, often called the Ṣaḍ-darśanas. These are:
  1. Mīmāṁsā
  2. Vedānta
  3. Sāṅkhya
  4. Yoga
  5. Nyāya
  6. Vaiśeṣika
- These schools are called orthodox not because they all believe in God, but because they respect and follow the Vedas. For example, Mīmāṁsā and Sāṅkhya do not believe in a God who created the world, yet they are still called orthodox because they accept the Vedas as an important source of knowledge.
- Apart from these six, there are also other smaller orthodox schools like the Grammarian school and the medical (Āyurveda) school, which were mentioned by a famous scholar named Mādhavācārya.
- On the other hand, the heterodox schools do not believe in the authority of the Vedas. The three main heterodox systems are:
  1. The Materialist school (like Cārvāka)
  2. The Jain philosophy
  3. The Buddhist philosophy
- These schools are called heterodox because they reject the Vedas, even though they may still be spiritual or philosophical in their own ways.

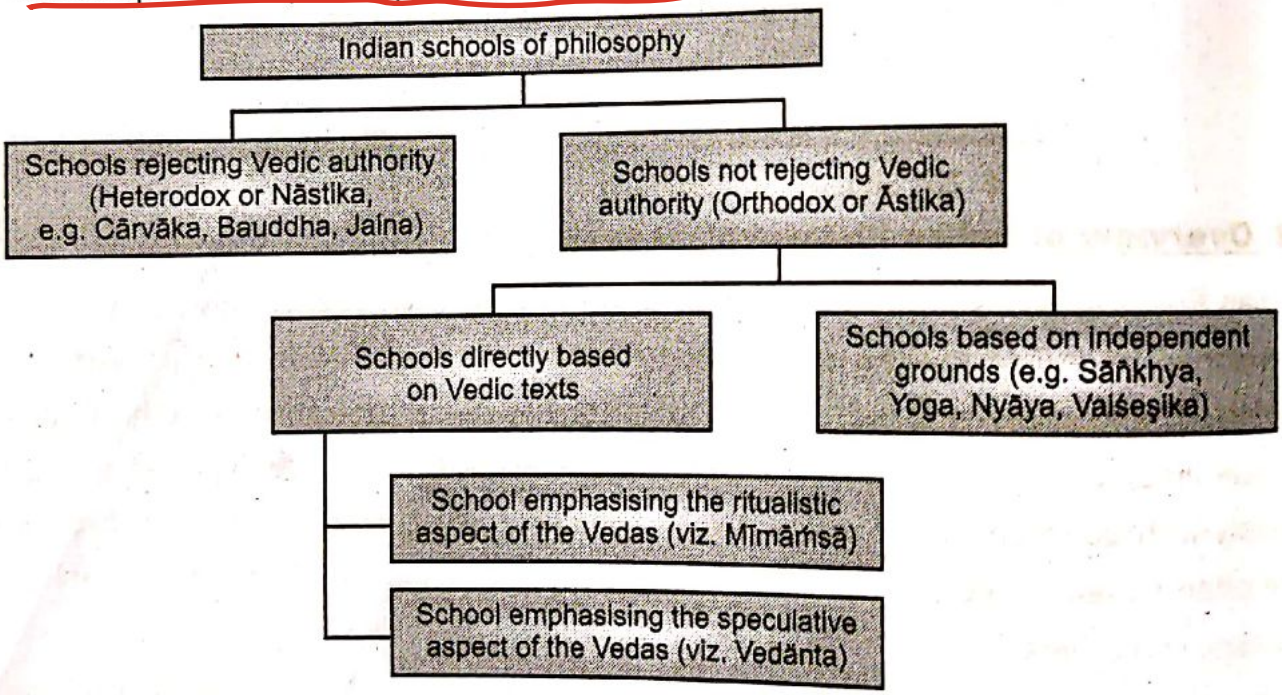


Fig. 2.3: Indian Philosophy

- Indian philosophy comprises mainly six classical systems (Shad-Darshanas) and the overview of these is shown in Table 2.1.

Table 2.1: Overview of Indian Classical Systems (Shad-Darshanas)

School	Founder / Key Figure	Core Ideas
Nyāya	Akṣapāda Gautama	Logic, epistemology, inference (Anumāna), valid knowledge (*pramāṇa*).
Vaiśeṣika	Kaṇāda	Atomism, metaphysics, categorization of reality (padārthas).
Sāṅkhya	Kapila	Dualism: Puruṣa (consciousness) and Prakṛti (matter); evolution of the universe.
Yoga	Patañjali	Practical path to liberation via the 8 limbs (Ashtanga Yoga); closely aligned with Sāṅkhya.
Mīmāṁsā	Jaimini	Ritualism, dharma, Vedic interpretation; focus on action (*karma-kāṇḍa*).
Vedānta	Bādarāyaṇa (Vyāsa)	Non-dualism (*Advaita*), qualified dualism (*Viśiṣṭādvaita*), and dualism (*Dvaita*); emphasis on Brahman and liberation (*mokṣa*).

2.1.3 Importance of Nyāya in Logical Reasoning

- Nyāya provides the tools and terminology for systematic thinking and debate. It serves as the basis of epistemology (Pramāṇa-śāstra) across Indian philosophical schools. Following are the important contributions and significance of the Nyāya system in Indian philosophy, especially in logical reasoning:
  - **Foundation of Indian Logic (Anvīkṣikī):** Nyāya is the earliest systematic school of Indian logic. It laid the groundwork for rational inquiry (anvīkṣikī) by formalizing methods of debate, reasoning, and proof.
  - **Concept of Pramāṇas (Means of Valid Knowledge):** Nyāya identifies four reliable means of knowledge–perception (pratyakṣa), inference (anumāna), comparison (upamāna), and verbal testimony (śabda) providing a structured epistemological framework.
  - **Rigorous Theory of Inference (Anumāna):** It developed a five-step syllogism, more elaborate than Aristotle's three-step model, enhancing clarity in deductive reasoning and debate (tarka).
  - **Critical Thinking and Error Analysis:** Nyāya offers detailed analysis of fallacies (hetvābhāsa), errors in perception and inference, and cognitive distortions, sharpening analytical and evaluative thinking.

- **Debate and Dialogue Tradition:** The system refined techniques of argumentation and refutation (vāda, jalpa, vitaṇḍā), which were essential in ancient Indian scholastic discussions and are still relevant in modern critical discourse.
- **Interdisciplinary Relevance:** Nyāya logic has influenced other Indian philosophical systems (like Vedānta and Buddhism) and can be applied in fields such as linguistics, cognitive science, and artificial intelligence today.

2.1.4 Types of Reasoning in Nyāya

- Nyāya introduces structured methods of gaining and validating knowledge, including:
  - Reasoning in Nyāya philosophy is primarily centered on the concept of inference (anumāna), which is one of the four valid means of knowledge (pramāṇas) alongside perception (pratyakṣa), comparison (upamāna), and testimony (śabda). Anumāna is a logical process by which knowledge of an unperceived fact is gained through the observation of a perceived sign that is invariably connected to it. Nyāya formalizes inference into a five-step syllogism: the proposition (pratijñā), the reason (hetu), the example illustrating the invariable concomitance (udāharaṇa), the application of the rule to the case at hand (upanaya), and the conclusion (nigamana). For example, "There is fire on the hill because there is smoke on the hill; wherever there is smoke, there is fire, like in a kitchen hearth; this hill is smoky; therefore, there is fire on the hill." This structured reasoning ensures clarity and rigor in argumentation.
  - Nyāya further classifies inference into types such as purvavat (inferring effect from cause), śeṣavat (inferring cause from effect), and sāmānyatodṛṣṭa (based on observed regularity). The school also emphasizes the importance of avoiding fallacies and ensuring that the reason (hetu) is properly established to avoid incorrect conclusions. Reasoning in Nyāya thus combines inductive and deductive elements and serves as a cornerstone for acquiring valid knowledge and engaging in philosophical debate.

2.2 NYĀYA'S FOUR SOURCES OF KNOWLEDGE (PRAMĀNA)

- Nyāya defines *Pramāṇa* as the reliable means or instrument by which valid knowledge (*Pramā*) is obtained. It is the method through which the mind arrives at truth about objects, ensuring the cognition is definite, true, and free from error or doubt. Knowledge (Jñāna or Buddhi) is the manifestation of objects, similar to how light reveals physical things. It is classified into Anubhava (presentative cognition) and Smṛti (memory/reproductive cognition), both of which can be valid or invalid. Valid presentative knowledge (Pramā) includes perception, inference, comparison, and testimony; it is certain, truthful, and non-reproductive. Invalid knowledge (Apramā) includes doubt, error, and hypothetical reasoning, while memory is not pramā but may be valid or invalid based on what it reproduces.
- The Nyāya theory of reality is based on the Nyāya of knowledge. There are four distinct and separate sources of true knowledge.

- The Nyāya school recognizes four Pramāṇas:
  - **Pratyakṣa** (Perception)
  - **Anumāna** (Inference)
  - **Upamāna** (Comparison/Analogy)
  - **Śabda** (Verbal Testimony)

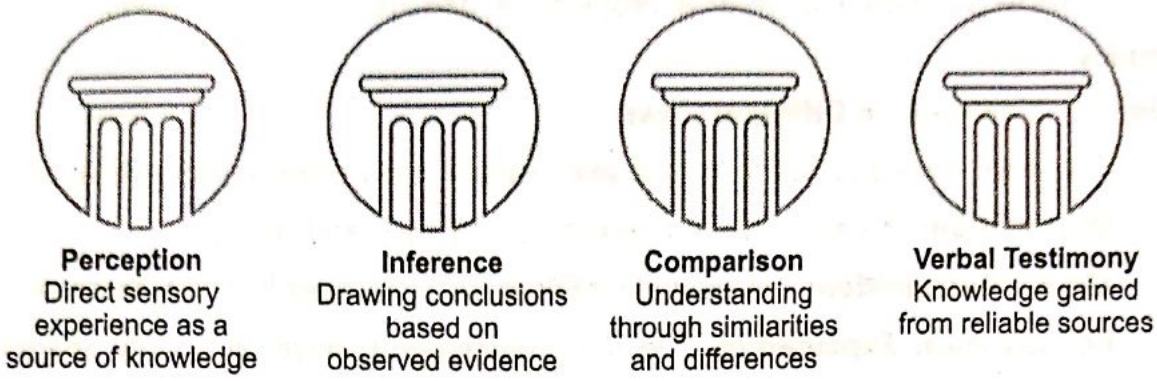


Fig. 2.4: Nyaya's Foundations of Knowledge

2.2.1 Pratyakṣa (Perception)

- **Definition:** Pratyakṣa means direct knowledge obtained through the sense organs. It is the most basic and immediate form of valid knowledge (Pramāṇa) in the Nyāya system.
  - **"Indriyārthasannikarṣajāṃjñānampratyakṣam":** Knowledge arising from the contact between sense organs and objects is called perception.

Types of Pratyakṣa (Perception) in Nyāya Philosophy:

- Nyāya classifies Pratyakṣa (perception) into two broad types, which are further subdivided:

1. LaukikaPratyakṣa (Ordinary or Sensory Perception):

- **Definition:** Perception that occurs through the five sense organs (eye, ear, skin, tongue, nose) when they come in direct contact with external objects.

Examples:

1. Seeing a flower (eye + form)
2. Hearing music (ear + sound)
3. Tasting a fruit (tongue + flavor)
4. Smelling a rose (nose + fragrance)
5. Feeling heat (skin + temperature)

2. Alaukika Pratyakṣa (Extraordinary or Non-ordinary Perception):

- **Definition:** Perception that does not arise in the usual sensory manner. It includes more subtle, abstract, or yogic forms of perception. Alaukika is divided into three subtypes:

(a) Sāmānyalakṣaṇa Pratyakṣa (Perception of Universals)

- Sāmānyalakṣaṇa Pratyakṣa is a type of *Alaukika Pratyakṣa* (extraordinary perception) in Nyāya philosophy, where a person perceives not a specific object, but the general or universal quality (sāmānya) shared by many objects.
- You don't just see "this cow"—you recognize the "cowness" that is common to all cows. It is mental recognition of a universal property beyond just the physical form.

Examples:

(i) Seeing "Cowness" in Different Cows:

- **Scenario:** A child visits a farm and sees cows of many colors–black, white, and brown.
- **Observation:** The eyes notice different sizes, shapes, and colors.
- **Mental Recognition:** The child identifies all as *"cows"* despite the differences.
- **Philosophical Explanation:** The mind perceives a universal quality shared by all *"cowness"*.
- **Conclusion:** This is *Sāmānyalakṣaṇa Pratyakṣa* perception not just of individual cows, but of their shared nature.

(ii) Recognizing "Bookness" in All Books:

- **Scenario:** On a desk, there's a textbook, a diary, and a novel.
- **Observation:** They differ in content, thickness, and purpose.
- **Mental Recognition:** The student still classifies them all as *"books."*
- **Philosophical Explanation:** The student's mind grasps the general idea of *"bookness"* present in each.
- **Conclusion:** This is *Sāmānyalakṣaṇa Pratyakṣa* the perception of the universal nature of "book".

(iii) Identifying "Treenness" in a Mango and Neem Tree:

- **Scenario:** Two trees are growing in a garden – a mango tree and a neem tree.
- **Observation:** Their leaves, fruits, and shapes are different.
- **Mental Recognition:** You still know both are *trees*.
- **Philosophical Explanation:** Your mind identifies the shared traits and recognizes "treeness".
- **Conclusion:** This is a universal-based perception *Sāmānyalakṣaṇa Pratyakṣa*.

(iv) Realizing "Teacherness" in Different Teachers:

- **Scenario:** A student interacts with two teachers – one strict and one kind.
- **Observation:** Their behavior and appearance are different.
- **Mental Recognition:** The student still sees both as fulfilling the *role of a teacher.*

- **Philosophical Explanation:** The mind perceives the common function and identity – *"teacherness"*.
- **Conclusion:** This perception of a shared category makes it *Sāmānyalakṣaṇa Pratyakṣa*.

(v) Noticing "Car-ness" in a Sedan, SUV, and Hatchback:

- **Scenario:** On the road, various vehicles are seen: a small car, a sports car, and an SUV.
- **Observation:** Shapes, sizes, and colors vary.
- **Mental Recognition:** You classify them all under the concept *"car."*
- **Philosophical Explanation:** Despite differences, your mind perceives the universal idea of "car-ness."
- **Conclusion:** This fits under *Sāmānyalakṣaṇa Pratyakṣa* perceiving the shared identity beyond differences.

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(b) Jñānalakṣaṇa Pratyakṣa (Knowledge-based or Representative Perception):

- Jñānalakṣaṇa Pratyakṣa is a type of *Alaukika Pratyakṣa* (extraordinary perception) in Nyāya philosophy, where the mind uses past knowledge or memory to perceive something indirectly.
- It happens when you see one thing, but perceive another because of prior experience or association.

Examples:

(i) Seeing Smoke and Knowing there's Fire:

- **Observation:** You see smoke rising on a hill.
- **What happens:** Though you don't see the fire, your mind immediately connects it with the presence of fire.
- **Perception:** This is knowledge-based perception. Past experience tells you: *"Where there's smoke, there's fire."*

(ii) Seeing Light and Knowing there's a Lamp:

- **Observation:** You see light coming from a room, but not the lamp itself.
- **What happens:** Your mind understands that a lamp must be inside, producing that light.
- **Perception:** The lamp is not directly seen, but inferred through light a representative perception.

(iii) Seeing Railway Tracks Converging:

- **Observation:** You see parallel railway tracks appearing to meet in the distance.
- **What happens:** Your senses deceive you, but your mind knows they are actually parallel.
- **Perception:** The mind corrects the visual illusion using prior knowledge an indirect, knowledge-based perception.

(iv) Seeing an Image in a Mirror:

- **Observation:** You see your friend's reflection in a mirror.
- **What happens:** Though only an image is visible, your mind recognizes it as your real friend.
- **Perception:** This is a representative perception the real object (friend) is perceived through an image.

(v) Mistaking a Rope for a Snake, And then Correcting It:

- **Observation:** In the dark, you see a coiled object and think it is a snake.
- **What happens:** You later shine a light and realize it was just a rope.
- **Perception:** The initial perception was false but knowledge-based, triggered by memory or fear showing how Jñānalakṣaṇa Pratyakṣa can sometimes be misleading.

(c) Yogaja Pratyakṣa (Yogic or Intuitive Perception):

- Yogaja Pratyakṣa means "perception through yoga or deep meditation. It is a special kind of knowledge that a person gets not through the eyes or ears, but through inner mind or intuition after deep concentration. Only great yogis or sages who have trained their mind through meditation can have this kind of direct, extraordinary perception.

Examples:

(i) A Yogi Knowing Someone's Past Life:

- **Scenario:** A yogi enters deep meditation while sitting quietly in a forest.
- **Inner Experience:** Without asking or seeing any records, he gains knowledge of another person's past birth.
- **Mental Insight:** The past life details arise spontaneously in his awareness.
- **Philosophical Explanation:** This is not learned or guessed – it is a result of purified consciousness through yoga.
- **Conclusion:** This is Yogaja Pratyakṣa – direct, intuitive perception gained through meditation.

(ii) Seeing a Future Event in Meditation:

- **Scenario:** A yogi is meditating peacefully in a cave.
- **Inner Experience:** He suddenly sees a clear vision of a future event – such as a thunderstorm or someone arriving the next day.
- **Mental Insight:** The yogi has not predicted it logically; the vision comes directly into his awareness.
- **Philosophical Explanation:** It is a spontaneous and accurate perception of what is yet to come, possible only through yogic mental purity.
- **Conclusion:** This qualifies as Yogaja Pratyakṣa, as it transcends sensory and inferential knowledge.

(iii) Realizing the Truth of the Soul (Ātman):

- **Scenario:** A person meditates for years, seeking the truth of existence.
- **Inner Experience:** In silence, the person realizes: "I am not the body or mind – I am the eternal self (Ātman)."
- **Mental Insight:** This truth is not imagined or read in a book — it is experienced directly.
- **Philosophical Explanation:** Such self-realization comes only through yogic purification and one-pointed focus, not through intellectual study.
- **Conclusion:** This is Yogaja Pratyakṣa – the direct, experiential knowledge of the soul.

(iv) Sensing the Presence of God or Divinity:

- **Scenario:** During deep meditation, a saint feels surrounded by divine presence or light.
- **Inner Experience:** No physical form is seen, but the sense of God's presence is strong and peaceful.
- **Mental Insight:** This is not emotion or imagination; it is a direct inner encounter with the divine..
- **Philosophical Explanation:** Yogic perception opens access to realms beyond the senses, enabling this experience.
- **Conclusion:** This is a clear case of Yogaja Pratyakṣa, where divine reality is directly known within.

(v) Understanding the Nature of the Universe:

- **Scenario:** A sage meditates deeply for many years seeking cosmic truth.
- **Inner Experience:** One day, complete understanding dawns everything in the universe is interconnected: soul, nature, and consciousness.
- **Mental Insight:** The sage didn't learn this from study — it came as a single moment of awakening.
- **Philosophical Explanation:** The yogi's purified mind receives this holistic knowledge as a direct perception, beyond logic or reason.
- **Conclusion:** This is Yogaja Pratyakṣa – intuitive wisdom born from deep meditative realization.

Table 2.2: Difference between Sāmānyalakṣaṇa, Jñānalakṣaṇa, and Yogaja Pratyakṣa

Point of Difference	Sāmānyalakṣaṇa Pratyakṣa	Jñānalakṣaṇa Pratyakṣa	Yogaja Pratyakṣa
1. Meaning	Perceiving a universal quality shared by many objects.	Perceiving something not directly present, but through memory or association.	Perception gained through deep meditation or yogic powers.
2. Based on	Recognition of sāmānya (generality) or common traits.	Mental impressions or past knowledge.	Spiritual insight or pure intuition.
3. Process	Mind recognizes common identity among many particular things.	Mind connects current perception with past experience.	No sense contact needed; arises through inner purity and meditation.
4. Who experiences it	Anyone with basic understanding.	Common in daily life with memory and logic.	Only highly trained yogis or sages.
5. Involves senses?	Starts with sense perception, then mental recognition.	Starts with sense input, then mental recall or imagination.	No ordinary sense involvement; purely mental/spiritual.
6. Nature of Knowledge	Based on universals (jāti).	Based on association (saṃskāra).	Based on inner realization (sākṣātkāra).
7. Example	Seeing many cows and understanding the idea of "cowness".	Seeing smoke and knowing fire is there.	A yogi realizing past lives or truth of the self.

2.2.2 Anumāna (Inference)

- Anumāna is inferential knowledge and it follows after perception (pratyakṣa). 'Anu' in Sanskrit means 'follows' and 'mana' is knowledge. Anumāna refers to inference, a process of gaining knowledge that follows another form of knowledge by relating to reasons and logic.
- **Definition:** Anumāna means reasoning or inference a method of obtaining valid knowledge (Pramāṇa) based on previous perception and logical connection.
  - **"Anumānam nāma tritīyam pramāṇam":** Inference is the third means of valid knowledge after perception and comparison.

- The key components of *Anumāna* (inference) in Nyāya philosophy form a structured five-step syllogism that guides logical reasoning and knowledge acquisition. These components are:

1. Pratijñā (Proposition):

- Pratijñā means a proposition, claim, or statement that you want to prove.
- It is the starting point of any logical argument. In Indian logic (Nyāya Śāstra), it is the first step in the five-part syllogism (pañcāvayava vāda).
- It usually includes:
  - A **subject** (what you are talking about)
  - A **predicate** (what you're saying about it)
- **For Example, Pratijñā:** "There is fire on the hill."

  Subject: *Hill*

  Predicate: *There is fire on it*

2. Hetu (Reason):

- Hetu means the reason or cause used to support a claim (pratijñā). It gives the logical basis for why your statement should be accepted. Hetu explains why the proposition is true.
- It shows a logical connection between the claim and evidence.
- It is often stated as:

  "Because [reason]..."

  or

  "Due to the presence of [cause], [effect] is true."
- **For Example,**

  Pratijñā: "There is fire on the hill."

  Hetu: "Because there is smoke on the hill."
- The reason for believing there is fire is the presence of smoke, which is usually caused by fire.

3. Udāharaṇa (Example):

- Udāharaṇa means example. It is used to support the reason (hetu) by showing a universal and accepted case where the same reason leads to the same conclusion. Udāharaṇa gives a real-world illustration that shows how the reason (hetu) and the proposition (pratijñā) are connected in general.
- It usually follows the pattern:

  "Wherever there is [hetu], there is [proposition] – like in [example]."

- **For Example,**

  Pratijñā: "There is fire on the hill."

  Hetu: "Because there is smoke on the hill."

  Udāharaṇa: "Wherever there is smoke, there is fire – like in a kitchen."

- The kitchen is a known example where smoke and fire are always found together. This helps to prove that smoke on the hill also means fire.

4. **Upanaya (Application):**

- Upanaya means application or linking. It is the step where the general example (udāharaṇa) is connected to the specific case in the argument. It applies the general rule to the current situation. This step bridges the example and the conclusion by saying: "What is true in the example is also true here."
- It usually takes the form:

  "This case is like the example, because the same reason (hetu) is present."

- **For Example,**

  Pratijñā: "There is fire on the hill."

  Hetu: "Because there is smoke on the hill."

  Udāharaṇa: "Wherever there is smoke, there is fire – like in a kitchen."

  Upanaya: "There is smoke on the hill, just like in the kitchen example."

5. **Nigamana (Conclusion):**

- Nigamana means conclusion or final statement. It is where you complete the reasoning by restating your original claim, now backed by logic, reason, and example. This step confirms that the proposition (Pratijñā) is valid, based on the reason (Hetu), example (Udāharaṇa), and application (Upanaya).
- It usually follows this form:

  "Therefore, [restated conclusion]."

- **For Example,**

  Pratijñā: "There is fire on the hill."

  Hetu: "Because there is smoke on the hill."

  Udāharaṇa: "Wherever there is smoke, there is fire – like in a kitchen."

  Upanaya: "There is smoke on the hill, just like in the kitchen."

  Nigamana: "Therefore, there is fire on the hill."

Types of Anumāna (Inference):

- Nyāya philosophy classifies Anumāna (inference) into several types based on the nature of the inference and the relation between the middle term (hetu) and the major term (sādhya).

- The main types are:

1. Purvavat (Inference from Cause to Effect):

- Pūrvavat Anumāna means inferring the effect from the cause. You observe a cause and predict or expect what will happen as a result. The word "pūrvavat" means "like before" because this inference is based on past experience of how causes lead to effects.

Example 1: Seeing dark clouds in the sky and inferring that it will rain soon.

- **Cause (Current Observation):** You observe dark, heavy clouds gathering.
- **Previous Observation:** In the past, such clouds have always led to rainfall.
- **Conclusion:** Therefore, you infer that it is going to rain soon.

Example 2: Seeing smoke rising from a kitchen chimney and inferring that food is being cooked inside.

- **Cause (Current Observation):** You notice smoke coming out of the chimney.
- **Previous Observation:** In previous situations, such smoke meant that cooking was taking place.
- **Conclusion:** Therefore, you infer that someone is cooking food inside the kitchen

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Example 3: Seeing a student studying seriously and inferring that exams are approaching.

- **Cause (Current Observation):** You see a student revising with full focus and seriousness.
- **Previous Observation:** Based on past experience, students usually study like this when exams are near.
- **Conclusion:** So, you infer that the exams must be approaching.

Example 4: Watching people arrange firecrackers and inferring that a festival is about to begin.

- **Cause (Current Observation):** You observe people setting up firecrackers in the streets.
- **Previous Observation:** Such preparations are commonly seen during festivals like Diwali.
- **Conclusion:** Therefore, you conclude that a festival celebration is coming soon.

Example 5: Seeing yellow leaves on trees and inferring that autumn is arriving.

- **Cause (Current Observation):** You notice that many trees have yellow and falling leaves.
- **Previous Observation:** From experience, this change happens before the autumn season begins.
- **Conclusion:** So, you infer that autumn is approaching.

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2. Śeṣavat (Inference from Effect to Cause):

- Śeṣavat Anumāna means inferring the cause from the effect. Here, you observe an effect and then reason backward to identify the likely cause, based on past experiences. The word "śeṣavat" literally means "from what remains" – you see something happening and work out what caused it.

Example 6: Wet Roads and Rain

- **Observation (Effect):** You see that the roads are wet in the morning.
- **Past Experience:** In the past, wet roads are usually the result of rainfall.
- **Conclusion:** Therefore, you infer that it must have rained recently.

Example 7: Smoke on a Hill and Fire

- **Observation (Effect):** You see smoke rising from a distant hill.
- **Past Experience:** Smoke is always caused by fire.
- **Conclusion:** Therefore, you infer that there is fire on the hill.

Example 8: Muddy Shoes and Rain

- **Observation (Effect):** A child enters with muddy shoes.
- **Past Experience:** Shoes get muddy when someone walks through wet or rainy ground.
- **Conclusion:** Therefore, you infer that it has rained outside.

Example 9: Flooded Fields and Overflowing River

- **Observation (Effect):** You see the fields are flooded.
- **Past Experience:** This usually happens when a river nearby overflows.
- **Conclusion:** Therefore, you infer that the river must have overflowed.

Example 10: Broken Branches and Strong Wind

- **Observation (Effect):** You see tree branches broken and scattered on the ground.
- **Past Experience:** Such damage usually happens due to a strong wind or storm.
- **Conclusion:** Therefore, you infer that there must have been a strong wind or storm.

3. Sāmānyatodṛṣṭa (Inference from Observed Uniformity):

- Sāmānyatodṛṣṭa Anumāna means inferring based on general or uniform observation, without a clear visible connection of cause and effect. This type of inference is made when you notice a regular pattern or consistent relationship between two things, even if you cannot see the exact cause. The word "sāmānyato-dṛṣṭa" means "commonly observed". It relies on repeated experience, not direct cause-effect logic.

Example 11: Observing high ocean tides, one infers that it must be a full moon or new moon day, based on the regularly observed connection between moon phases and tidal movements.

- **Observation (Pattern):** You see high ocean tides occurring.
- **Past Observation:** You have noticed that high tides often occur during full moon or new moon phases.
- **Conclusion:** Therefore, you infer that it must be full moon or new moon today.

Example 12: Seeing slight movement in a body, such as breathing or blinking, one infers that the person or animal is alive, as movement is commonly associated with life.

- **Observation (Pattern):** You see a body moving slightly or breathing.
- **Past Observation:** From experience, movement indicates life.
- **Conclusion:** Therefore, you infer that the person or creature is alive, even if asleep or unconscious.

Example 13: When stars are clearly visible in the sky, one infers that it is night time, as stars are generally seen only after the sun has set.

- **Observation (Pattern):** You see stars shining in the sky.
- **Past Observation:** You always see stars clearly only during night time.
- **Conclusion:** Therefore, you infer that it is night time, even without seeing the sun.

Example 14: Noticing that a compass needle consistently points in one direction, one infers the presence of a magnetic field, since this behavior is always observed in such conditions.

- **Observation (Pattern):** A compass needle points in a fixed direction.
- **Past Observation:** Such behavior is always due to magnetic fields.
- **Conclusion:** Therefore, you infer that a magnetic field is present, even if you cannot see it.

Example 15: Upon seeing a reddish glow near the horizon, one infers that it is either sunrise or sunset, based on the repeated observation of this color during those times of the day.

- **Observation (Pattern):** The sky turns reddish near the horizon.
- **Past Observation:** This color appears during sunrise or sunset.
- **Conclusion:** Therefore, you infer that either sunrise or sunset is happening.

2.2.3 Upamāna (Comparison/Analogy)

- Upamāna (comparison or analogy) is recognized in Nyāya philosophy as the third valid means of knowledge (pramāṇa), distinct from perception, inference, and testimony. It involves gaining knowledge of an unfamiliar object by comparing it to a known one based on similarity. The classic example is a person who has never seen a gavaya (wild ox) learning from a forester that a gavaya is like a cow. When the person later encounters the animal, they recognize it by recalling the earlier comparison and noting the resemblance, thus acquiring valid knowledge of the new object's identity. Nyāya outlines this cognitive process in four stages:
  (1) Hearing an authoritative statement (atideśa-vākya) that introduces the comparison.
  (2) Encountering the unfamiliar object and perceiving its similarity (sādṛśya-dhī).
  (3) Recollecting the verbal comparison (vākyārtha-smṛti).
  (4) The final recognition or insight (upamiti) that links the word to the object.
- This process is considered independent of direct perception and inference, highlighting Nyāya's nuanced epistemology that acknowledges analogy as a unique and essential way

of knowing, especially useful for linguistic understanding and identification of unknown entities.

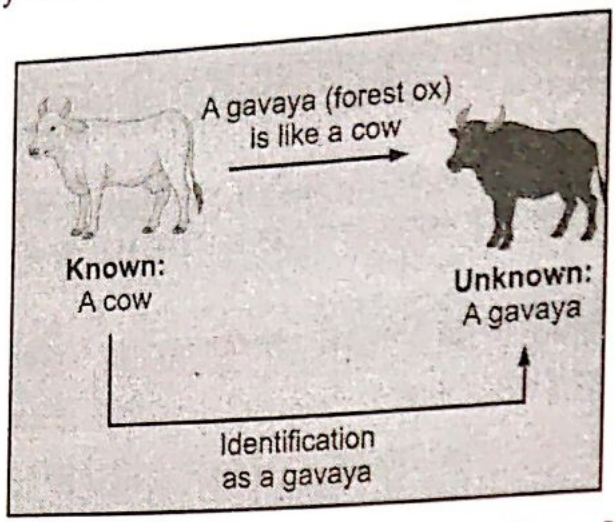

Fig. 2.5: Cow and Gavaya Comparison (Upamāna)

2.2.4 Śabda (Verbal Testimony)

- In Nyāya philosophy, *Śabda* (verbal testimony) is recognized as a fundamental and independent means of valid knowledge (*pramāṇa*). It is defined as the statement or instruction of a trustworthy and reliable person (*āpta*), whose words convey true knowledge because of their truthfulness, competence, and benevolent intent. The validity of *śabda* depends on the speaker being free from deceit and error, and the statement being clear, coherent, and relevant. Nyāya holds that much of human knowledge especially about past events, distant places, metaphysical truths, and ethical guidance is acquired through *śabda*, often more frequently than by direct perception or inference. Importantly, the power of words to convey meaning is either divinely ordained (in ancient Nyāya) or established by long-standing linguistic convention (in later Nyāya). The epistemic status of *śabda* is reinforced by its consistency with other pramāṇas and its practical efficacy in guiding action. Thus, *śabda* plays a crucial role in Nyāya epistemology as a reliable source of knowledge beyond sensory and inferential means, emphasizing the importance of trustworthy communication in the pursuit of truth.

2.3 TYPES OF ARGUMENTATIONS IN NYĀYA

- In Nyāya philosophy, argumentation is a highly developed and structured process used to establish truth, refute falsehoods, and resolve doubt. Nyāya is often called the school of logic and debate (Nyāya-śāstra), and its argumentation system lays the foundation for Indian logical theory and epistemology.
- **Definition:** Argumentation (tarka and vāda) in Nyāya is the methodical presentation and defense of a thesis (pratijñā) through logical reasoning (anumāna), counter-arguments (uttarapakṣa), and refutations (khaṇḍana), based on valid sources of knowledge (pramāṇas).

- Nyāya philosophy classifies argumentation into three principal types, each defined by the intent and conduct of the participants.

2.3.1. Vāda (Truth-Based Debate):

- Vāda is the highest and most respected form of debate in Nyāya.
- The primary aim is the mutual discovery of truth, with both parties genuinely committed to rational inquiry and respectful dialogue.
- Participants use valid reasoning, logical consistency, and are open to accepting valid points from the other side.
- This form is typically practiced among scholars, teachers, and seekers of wisdom, focusing on refining understanding rather than winning.
- Practiced in scholarly, open-minded contexts.
- Use of valid Pramāṇas, mutual respect, and clarity of reasoning.

Examples:

1. **Student vs. Teacher on Rebirth:**
   - **Student:** "I believe that after death, nothing remains. There is no rebirth."
   - **Teacher:** "Let us explore this. Can you explain why some people are born with talents or sufferings without any cause in this life?"
   - **Student:** "Maybe genetics or randomness?"
   - **Teacher:** "That's one view. But Indian philosophy suggests karmic consequences. Shall we examine both ideas logically and through texts?"

   Outcome: Together, they explore texts and reasoning. The goal isn't to win but to understand truth about rebirth.
2. **Two Philosophers on Liberation (Mokṣa):**
   - **Nyāya Philosopher:** "Mokṣa is freedom from suffering and achieved through right knowledge and reasoning."
   - **Vedānta Philosopher:** "Mokṣa is realizing the unity of Atman and Brahman."
   - **Both:** "Let's define suffering and discuss how each path addresses it."

   Outcome: They debate respectfully using śāstra (scripture) and tarka (logic) to explore the true nature of liberation.
3. **Ayurveda Practitioner vs. Modern Doctor:**
   - **Ayurveda Expert:** "Diseases are due to imbalance in doṣas–vāta, pitta, kapha."
   - **Modern Doctor:** "I understand disease as infection or chemical imbalance."
   - **Both:** "Let us analyze a case of fever and compare approaches based on outcomes and reasoning."

   Outcome: A dialogue for mutual understanding, combining ancient wisdom and modern science.

4. **Environmentalist vs. Economist:**
   - **Environmentalist:** "We must protect forests based on dharma–our duty to nature."
   - **Economist:** "But development is also essential for livelihoods."
   - **Both:** "Can we find a dharmic balance between ecology and economy?"

   Outcome: They search for truth and harmony, not conflict.
5. **Inter-School Debate on Self (Ātman):**
   - **Sāṅkhya School:** "Ātman is separate from Prakṛti and does not act."
   - **Yoga School:** "Ātman witnesses actions but liberation comes through yogic discipline."
   - **Both:** "Let us examine sūtras and pramāṇas to understand Ātman's nature."

   Outcome: They compare views logically and with sincerity to uncover the truth.

2.3.2. Jalpa (Debate-Focused or Competitive Debate):

- Jalpa centers on victory rather than truth.
- In such forms of debate, the primary objective is to defeat the opponent, often through the use of clever rhetoric, emotional persuasion, and even fallacious reasoning, rather than a genuine pursuit of truth.
- While logical structure is maintained, the emphasis shifts from genuine inquiry to point-scoring and self-assertion.

Examples:

1. **Two Students Arguing About Karma:**
   - **Student A:** "Karma controls everything. Even studying is useless if your karma is bad."
   - **Student B:** "That's wrong! Only hard work matters. Karma is just an excuse lazy people use."
   - They **argue loudly**, interrupt, and mock each other instead of discussing calmly.

   Jalpa Element: More argumentative than informative. The goal is to win, not understand.
2. **TV Show Debate on Vegetarianism:**
   - **Debater 1:** "Eating meat is unethical! Only vegetarians are moral people."
   - **Debater 2:** "That's just your opinion! Humans evolved to eat meat. Don't lecture me!"
   - Each uses aggression, sarcasm, and emotional manipulation.

   Jalpa Element: Driven by ego and point-scoring, not by logic or respect.
3. **Religious Debate between Two Scholars:**
   - **Scholar A:** "Only my scripture is valid. Yours is full of errors."
   - **Scholar B:** "No, yours is outdated! My school explains everything better."
   - They **quote selectively**, misrepresent the other's views, and refuse to listen.

   Jalpa Element: Use of biased citations, personal attacks, and ego-driven speech.

4. **Political Debate in a College:**
   - **Candidate 1:** "I will bring real change!"
   - **Candidate 2:** "Your promises are fake! You did nothing last year."
   - Each tries to expose flaws in the other, not improve ideas.

   Jalpa Element: Focus is on defeating the opponent, not solving real issues.
5. **Debating "Who is the Better Teacher?":**
   - **Group A:** "Our teacher is best. Yours just reads slides."
   - **Group B:** "At least ours doesn't waste time with boring stories."
   - They trade insults instead of appreciating both teaching styles.

   Jalpa Element: Full of bias, emotional tone, and personal loyalty, not logic.

2.3.3. Vitandā (Criticism or Destructive Debate):

- Vitandā is the lowest form of debate, characterized by destructive criticism.
- Here, the participant seeks only to refute the opponent's position without presenting any alternative thesis or constructive argument of their own.
- This type is often seen as unproductive, as it focuses solely on negation and criticism rather than advancing knowledge.

Examples:

1. **"Your Idea Is Just Wrong!":**
   - **Person A:** "The soul is eternal according to this text."
   - **Person B:** "That's nonsense! I don't believe in those texts."
   - **Person A:** "Then what do you believe?"
   - **Person B:** "Doesn't matter. I just know you're wrong."

   Vitandā Element: Only attacks the other's view without giving their own.
2. **Criticizing a Diet Without Suggesting Another:**
   - **Person A:** "A vegetarian diet is good for health."
   - **Person B:** "That's false. It lacks protein. Useless diet."
   - **Person A:** "What diet do you follow then?"
   - **Person B:** "That's not important. Your idea is flawed."

   Vitandā Element: Only fault-finding, no constructive suggestion.
3. **Dismissing a Philosophy:**
   - **Philosopher A:** "Liberation is through knowledge of the self."
   - **Philosopher B:** "That's foolish. Liberation is a myth."
   - **Philosopher A:** "What path do you suggest?"
   - **Philosopher B:** "I don't have one. I just know yours is wrong."

   Vitandā Element: Denial without offering alternative logic.

4. **Classroom Argument:**
   - **Student A:** "We should clean our classroom regularly."
   - **Student B:** "Why? That's just a waste of time."
   - **Student A:** "What's your plan then?"
   - **Student B:** "I don't have a plan. I just don't like yours."

   Vitandā Element: Negativity without a solution.
5. **Rejecting an Exam System:**
   - **Person A:** "Exams help assess learning."
   - **Person B:** "Exams are useless. They don't show anything."
   - **Person A:** "So, what should we use instead?"
   - **Person B:** "No idea. But exams are a bad method."

   Vitandā Element: Criticizing without proposing a better method.

Table 2.3: Difference between Vāda, Jalpa, and Vitandā

Feature	Vāda	Jalpa	Vitandā
Aim	To discover truth	To win the argument	To defeat the opponent
Style	Logical, respectful	Aggressive, tricky	Critical, negative
Offers own view?	Yes	Yes	No
Use of logic	Sound reasoning	May include fallacies	Only attacks, no proof
Outcome	Truth and learning	Victory and ego boost	Destruction of other's view

2.4 APPLICATIONS IN AI AND MACHINE LEARNING

- Nyāya, the classical Indian school of logic, provides a systematic and insightful framework for reasoning, inference, and critical debate. It provides clear rules for how to think, argue, and reason correctly. These ideas are very useful in modern Artificial Intelligence (AI), especially in creating machines that think like humans and make smart decisions. Its foundational principles hold significant promise for enhancing artificial intelligence, particularly in the development of logical reasoning models, expert systems, and rule-based AI architectures.

2.4.1 Logical Reasoning Models

- Logical reasoning models serve as the backbone of intelligent decision-making systems, mimicking the human ability to draw conclusions from known facts, patterns, or rules. Rooted in classical logic, these models operate through structured frameworks such as propositional and predicate logic, enabling precise representation and manipulation of knowledge. In computational domains, logical reasoning models are fundamental to the development of symbolic artificial intelligence, knowledge-based systems, and formal verification tools. Their strength lies in their capacity to derive new knowledge from a set

of axioms, ensuring consistency, transparency, and explainability attributes that are especially vital in domains like law, healthcare, and engineering, where traceable reasoning is crucial.

- Logical reasoning models typically follow two main approaches: Structured Inference and Case-Based Reasoning.

1. Structured Inference:

- Structured inference is the process of reasoning in a clear and logical order, using defined steps to move from a known fact to a new conclusion. It is not based on random guesses – each step is guided by logical rules, ensuring the conclusion is reliable and explainable.
- The Nyāya school of Indian logic places strong emphasis on methodical argumentation using a five-part syllogism, which closely resembles formal logical structures used in Artificial Intelligence (AI). These five steps – Pratijñā (proposition), Hetu (reason), Udāharaṇa (example), Upanaya (application), and Nigamana (conclusion) ensure that reasoning is transparent and traceable.
- This structured reasoning model is highly useful in AI fields such as:
  - **Knowledge encoding:** Where facts and rules are stored in a way machines can process.
  - **Expert systems:** Where AI needs to mimic human decision-making.
  - **Automated decision-making:** Where machines must justify their conclusions in applications like healthcare, law, or smart assistants.

Example 16:

Scenario: You ask your AI assistant:

"Do I need an umbrella today?"

How the AI Thinks – Step-by-Step (Nyāya's Five-Part Syllogism)

Nyāya Step	Meaning	AI's Reasoning
1. Pratijñā	Statement or claim	"You should carry an umbrella today."
2. Hetu	Reason (evidence or clue)	"Because the weather forecast shows dark clouds."
3. Udāharaṇa	Example (general observation)	"Usually, dark clouds mean it's going to rain."
4. Upanaya	Application to the current case	"Today's sky looks just like those rainy days."
5. Nigamana	Conclusion (logical result)	"Therefore, it may rain, and you should carry an umbrella."

The AI does not give a random suggestion.

It follows a logical path:

States a claim → Supports it with reason → Gives a relatable example → Applies it → Concludes clearly.

Just like a wise person using logic, the AI shows why it made that decision.

2. Case-Based Reasoning:

- Case-Based Reasoning is a method of problem-solving where a system solves a new problem by looking at similar problems it has solved before. Instead of using fixed rules, Case-Based Reasoning uses past experiences (cases) to handle new situations.
- Western logic usually follows a top-down approach using universal rules to solve specific problems. While Western logic often emphasizes abstraction and generalization, Nyāya uses Upamāna (analogy) and Anumāna (inference) to learn and reason by comparing one situation with another. Nyāya prefers real examples and situations over just rules. It is context-sensitive and focuses on practical reasoning. This perspective complements the evolving direction of AI toward inductive and flexible reasoning mechanisms that better mimic human cognition.

Example 17:

Scenario: An AI medical system receives patient data:

Symptoms: Fever, Cough, and Body Pain

It searches its case memory and finds:

A past patient with the same symptoms, diagnosed with flu.

So, the AI suggests: "This patient may also have the flu. Recommend rest and fluids."

Nyāya Step	Meaning	Applied in AI Example
1. Pratijñā	Statement (Proposition)	"This patient likely has the flu."
2. Hetu	Reason	"Because they have fever, cough, and body pain."
3. Udāharaṇa	Example (General Case)	"In a previous case with the same symptoms, the patient had flu."
4. Upanaya	Application to the present case	"This patient's symptoms are the same as that past case."
5. Nigamana	Conclusion (Inference)	"Therefore, this is likely a case of flu. Recommend treatment accordingly."

The AI does not guess the answer. It follows a clear logical path comparing symptoms, applying past learning, and arriving at a conclusion. This mimics how Nyāya philosophy teaches us to reason: with clarity, steps, and examples.

2.4.2 Expert Systems

- An Expert System is a type of computer program that acts like a human expert. It solves problems, gives advice, or makes decisions based on knowledge and logical rules – just like a doctor, engineer, teacher, or lawyer would. An Expert System is a computer program designed to simulate the decision-making ability of a human expert. Expert systems are designed to support human experts, not replace them. They help in situations where: Experts are not available or Quick decision-making is needed or Consistent, rule-based logic is important.
- The idea of expert systems was introduced in the 1970s by Edward Feigenbaum, a computer science professor at Stanford University. He believed that computers should move beyond just data processing (calculations and storage) to knowledge processing (reasoning and problem-solving). This shift became possible due to advances in processor technology and system design.
- Nyāya's epistemological framework–encompassing perception (*pratyakṣa*), inference (*anumāna*), analogy (*upamāna*), and testimony (*śabda*) offers a structured model for organizing diverse types of knowledge. This enables expert systems to reason across multiple modalities and enhances the interpretability of AI-driven decisions.

- An Expert System consists of:
  - A Knowledge Base (rules and facts)
  - An Inference Engine (reasoning logic)
  - An Explanation System (shows how conclusions are made)

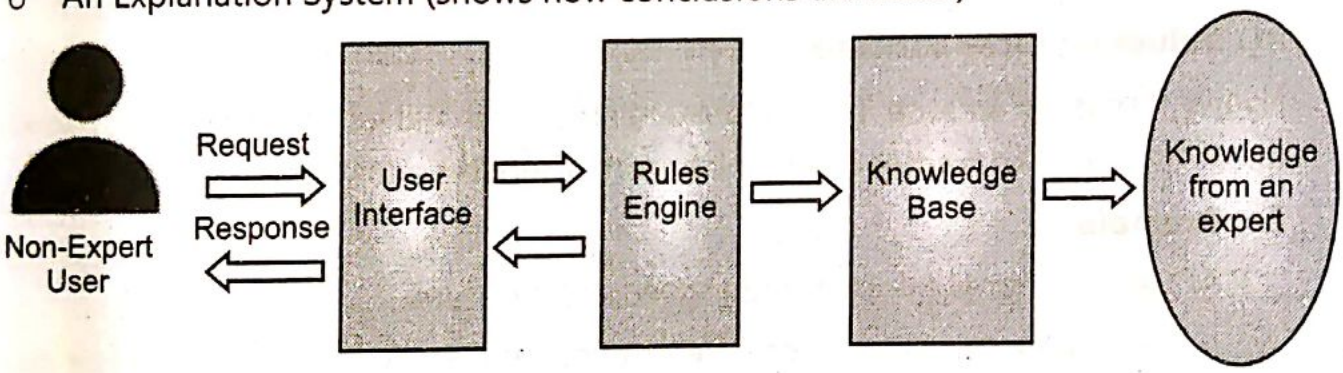

Fig. 2.6: Expert System in AI

Knowledge Base:

- The Knowledge Base is the core part or heart of an expert system. It stores all the important facts, rules, experiences, and expert insights related to a specific topic or field. Several techniques are used for knowledge representation in AI such as logical representation, semantic networks, frame representation, and production rules.
  - **Logical Rules (Related to Anumāna – Inference):** These are like instructions: IF a student studies well, THEN they will pass the exam. Used in AI systems that gives advice or solves problems step-by-step. AI uses logic (like propositional and

predicate logic) to draw conclusions based on facts, just as Nyāya uses *anumāna* (inference) to reach knowledge.

For Example:

IF the patient has a sore throat AND fever, THEN it could be flu.

- **Semantic networks (Related to Nyāya's Ontological Categorization):** These are like mind maps or flowcharts. Words are connected to show how ideas are related. Nyāya classifies and defines relationships between categories of reality (padārthas), similar to how semantic networks show relationships between concepts.

For Example:

"Dog" → is a type of → "Animal"

"Animal" → is a → "Living Being"

Used in language understanding and learning tools.

- **Frames (Object Templates)(Related to Nyāya's systematization of perception and attributes):** In expert systems, frames are a special type of data structure used to store knowledge especially about objects, people, places, or situations in a particular field (called a domain). Think of a frame like a profile or a form that contains all the important information about one thing. A frame stores details about an object.

A frame is made up of:

- **Attributes** (also called *slots*)
- **Values** for those attributes

Nyāya discusses *lakṣaṇa* (characteristics) and how things are identified based on qualities much like **frames** in AI, which store attributes and values of an object.

For Example:

A "Vehicle" frame might include speed and fuel type.

A "Car" frame can inherit from "Vehicle" but add extra slots like number of doors.

- **Production Rules (Decision Rules)(Reflective of Nyāya's reasoning sequences):** In expert systems, production rules (also called IF–THEN rules) are a key method for storing knowledge and guiding decision-making. These rules connect a specific condition (the "IF" part) to a result or action (the "THEN" part), enabling the system to respond appropriately based on the current situation or problem. These are action rules used by AI to make decisions.

For Example:

IF the computer won't start, THEN check the power cable.

Used in troubleshooting systems, customer supports bots, etc.

Inference Engines:

- The inference engine is like the brain of an expert system. It reads the information stored in the knowledge base (which holds facts and rules) and uses it to make decisions or give suggestions.
- To do this, the inference engine uses special reasoning methods, such as:
  - **Forward Chaining:** Starts with known facts and moves step-by-step to reach a conclusion.
  - **Backward Chaining:** Starts with a possible goal and works backward to find supporting facts.
- By applying these methods, the inference engine helps the expert system solve problems, answer questions, or offer advice–just like a human expert would.
- In the context of the Nyāya system, the inference engine of an expert system functions like the "buddhi" (intellect) that applies structured reasoning to draw valid conclusions from known facts – similar to how Anumāna (inference) operates in Nyāya logic.
- Just as Nyāya uses a five-part syllogism (pratijñā, hetu, udāharaṇa, upanaya, nigamana) to reach conclusions, the inference engine processes information stored in the knowledge base (comparable to Nyāya's accepted sources of knowledge or pramāṇa) and applies rules in a logical sequence.
- In the light of the Nyāya system of Indian logic, the inference engine in an expert system acts like the intellectual core–similar to the role of Anumāna (inference) in Nyāya. It functions as the system's reasoning center, where facts and rules stored in the knowledge base are logically processed to reach valid conclusions or give recommendations.
- Just as Nyāya follows a structured process of reasoning (with steps like Pratijñā, Hetu, Udāharaṇa, Upanaya, and Nigamana), the inference engine uses reasoning strategies such as:

1. **Forward Chaining (Nyāya's Cause-to-Effect Reasoning – PūrvavatAnumāna):**
   - Starts from observed data and moves step-by-step toward a conclusion.
   - Like Nyāya's inference where smoke leads to the conclusion of fire, the system uses known facts to reach a new truth.

Example 18:

Observation: High temperature + Persistent cough

Nyāya-style reasoning:

- **Pratijñā (Proposition):** The patient has a respiratory infection.
- **Hetu (Reason):** Because of high temperature and cough.
- **Udāharaṇa (Example):** Similar symptoms were observed in past flu cases.
- **Upanaya (Application):** This case matches those past examples.
- **Nigamana (Conclusion):** Likely respiratory infection.

2. **Backward Chaining (Nyāya's Effect-to-Cause Reasoning – ŚeṣavatAnumāna):**
   - Starts from a desired conclusion (goal) and searches backward for evidence.
   - Similar to Nyāya's method of tracing observed effects back to a possible cause.

Example 19:

Goal: Determine if the patient has diabetes.

Nyāya-style reasoning:

Pratijñā: The patient might have diabetes.

Hetu: If symptoms like frequent urination and high blood sugar are found.

Udāharaṇa: These symptoms were present in confirmed diabetes cases.

Upanaya: The current symptoms match.

Nigamana: Therefore, diabetes is a likely diagnosis.

Explanation System:

- The explanation module helps users understand how the expert system reached its answer. It's like a teacher explaining each step in a math solution. This feature gives a clear, step-by-step explanation of the system's thinking. Such transparency is very important in areas like healthcare and finance, where even small decisions can have big effects.
- The Nyāya system, a classical Indian school of logic, emphasizes structured reasoning, clear step-by-step explanation, the use of examples and practical application, and the importance of debate, doubt, and dialogue in the pursuit of truth.

Example 20:

Scenario: Medical Diagnosis

- **System's Conclusion (Pratijñā):** The patient has the flu.
- **Reason (Hetu):** Because they have fever, cough, and body pain.
- **Example (Udāharaṇa):** Past patients with the same symptoms were diagnosed with the flu.
- **Application (Upanaya):** This case is similar to those examples.
- **Final Conclusion (Nigamana):** Therefore, the diagnosis is flu.

2.4.3 Rule-Based AI

- **Rule Encoding:** Nyāya's logical rules can be turned into simple "IF-THEN" statements, making them useful for AI tasks like language processing, building knowledge networks, and expert systems.
- **Contextual Flexibility:** Since Nyāya reasoning is context-based, it helps AI systems deal better with uncertain or unclear situations compared to strict, rule-only logic.

- **Application Areas:** Rule-based AI inspired by Nyāya has been used in areas like software design, defence robotics, online search, digital learning, and data analysis showing its wide usefulness in intelligent technologies.

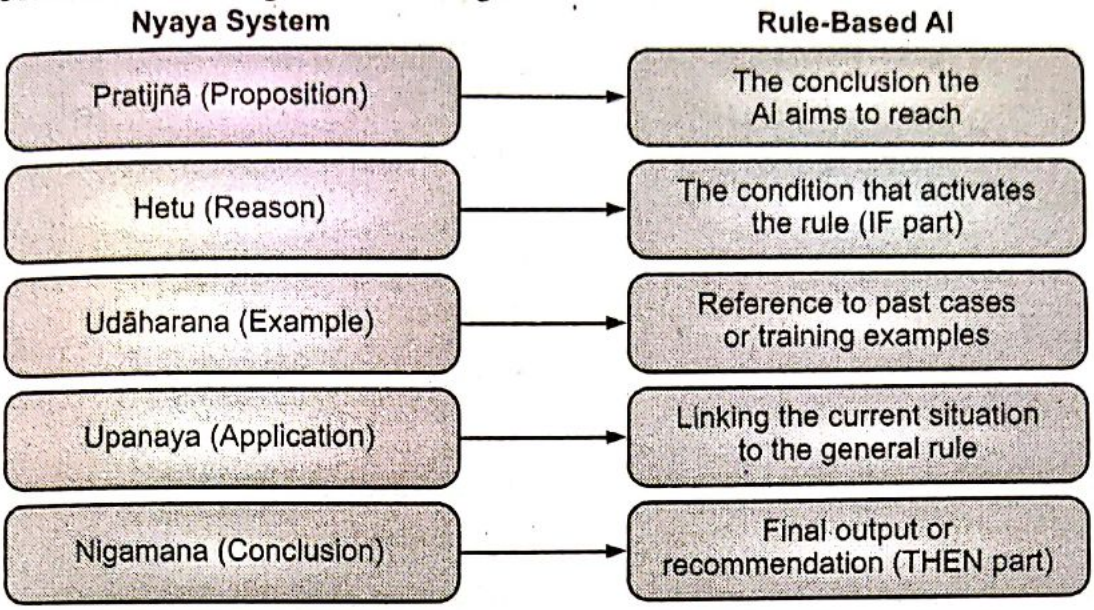

Fig. 2.7: Nyaya System and Rule-Based AI

Summary

- Indian philosophy is traditionally classified into orthodox (Āstika) and heterodox (Nāstika) systems based on their acceptance of the authority of the Vedas.
- The six orthodox schools, known as Ṣaḍ-darśanas, include Mīmāṁsā, Vedānta, Sāṅkhya, Yoga, Nyāya, and Vaiśeṣika, and are considered orthodox not necessarily for their belief in God but for their reverence toward the Vedas.
- The heterodox schools, such as Cārvāka, Jainism, and Buddhism, reject the Vedas and are hence termed nāstika.
- Nyāya philosophy was founded by Sage Gotama (also called Akṣapāda) and focuses on correct reasoning, valid knowledge, and the path to liberation (mokṣa).
- Nyāya is also known by other names like Nyāyavidyā, Tarkaśāstra, and Ānvīkṣikī due to its focus on logic, critical analysis, and structured debate.
- The main text of Nyāya philosophy is the Nyāya Sūtra, which has five chapters with two sections each and has been expanded upon by later scholars like Vātsyāyana and Uddyotakara.
- Nyāya evolved into two main branches: Prācīna Nyāya (ancient Nyāya) and Navya Nyāya (new logic), with Navya Nyāya beginning with Gaṅgeśa's Tattvacintāmaṇi.
- The combined school of Nyāya and Vaiśeṣika also emerged, integrating metaphysical and logical frameworks.
- Nyāya philosophy deals with four major areas: theory of knowledge, theory of the world, theory of the self and liberation, and theory of God.

- Sixteen key topics, known as padārthas, are discussed in Nyāya to define valid argumentation, debate, reasoning, and fallacies.
- Nyāya contributed significantly to the foundation of Indian logic and remains central to epistemological discussions across Indian philosophy.
- Nyāya recognizes four valid sources of knowledge (pramāṇas): perception (pratyakṣa), inference (anumāna), comparison (upamāna), and verbal testimony (śabda).
- Perception (pratyakṣa) is divided into ordinary (laukika) and extraordinary (alaukika), including perception of universals, inferred perception, and yogic intuition.
- Inference (anumāna) uses a five-step logical structure: proposition, reason, example, application, and conclusion.
- Nyāya classifies inference into three types: purvavat (from cause to effect), śeṣavat (from effect to cause), and sāmānyatodṛṣṭa (based on regularity).
- Comparison (upamāna) is valid knowledge gained through analogy, typically used when identifying unfamiliar objects by resemblance to known ones.
- Verbal testimony (śabda) is valid when it comes from a trustworthy and competent source, and it plays a key role in learning about unseen or distant realities.
- Nyāya categorizes argumentation into vāda (truth-seeking debate), jalpa (victory-oriented debate), and vitaṇḍā (destructive criticism without alternatives).
- Nyāya logic has significant applications in AI and machine learning, particularly in structured inference, expert systems, and ethical decision-making.
- AI systems can benefit from Nyāya's rule-based logic, case-based reasoning, and fallacy detection in developing human-like cognitive models.
- Nyāya's logical methods support linguistic AI, ethical reasoning, explainable AI, and multilingual natural language processing.
- Its epistemology and debate models provide a philosophical framework for building trustworthy, transparent, and context-aware AI systems.

Exercise

Q.I Multiple Choice Questions:

1. Who is considered the founder of the Nyāya philosophy?
   (a) Kapila
   (b) Patañjali
   (c) Akṣapāda Gautama
   (d) Kaṇāda
2. Which of the following is not a valid pramāṇa in Nyāya philosophy?
   (a) Pratyakṣa
   (b) Anumāna
   (c) Upamāna
   (d) Arthāpatti

3. What is the primary aim of Nyāya philosophy?
   (a) Worship of gods
   (b) Performance of rituals
   (c) Liberation from suffering
   (d) Preservation of tradition

4. Which of the following texts marks the beginning of NavyaNyāya?
   (a) Nyāya Bhāṣya
   (b) Tattvacintāmaṇi
   (c) Nyāya Sūtra
   (d) Kusumāñjali

5. In Nyāya inference, which step presents the conclusion?
   (a) Hetu
   (b) Udāharaṇa
   (c) Pratijñā
   (d) Nigamana

6. The term *sāmānyalakṣaṇa pratyakṣa* refers to .....
   (a) Yogic perception
   (b) Perception of universals
   (c) Illusory perception
   (d) Scriptural testimony

7. Which debate style in Nyāya aims purely at refuting the opponent without proposing a counter-thesis?
   (a) Vāda
   (b) Jalpa
   (c) Vitandā
   (d) Tarka

8. The padārtha in Nyāya that refers to doubt is .....
   (a) Saṁśaya
   (b) Tarka
   (c) Nigrahasthāna
   (d) Jāti

9. Which pramāṇa involves acquiring knowledge through analogy?
   (a) Anumāna
   (b) Śabda
   (c) Upamāna
   (d) Pratyakṣa

10. The city known for flourishing Navya Nyāya scholarship was .....
   (a) Mithilā
   (b) Varanasi
   (c) Ujjain
   (d) Kashi

Answers

1. (c)	2. (d)	3. (c)	4. (b)	5. (d)	6. (b)	7. (c)	8. (a)	9. (c)	10. (a)

Q.II Answer the following questions in short:

1. What are the four pramāṇas accepted by the Nyāya school of philosophy? 33
2. Define *anumāna* in Nyāya philosophy. 59
3. What is the purpose of *Nyāya Sūtra* in the Nyāya system?
4. Name the six orthodox schools of Indian philosophy. 31
5. What does *sāmānyalakṣaṇa pratyakṣa* mean? 35, 39
6. What is the difference between vāda and jalpa in Nyāya argumentation? 46

7. Name any two commentaries on the *Nyāya Sūtra*.29
8. What is the goal of all Indian philosophical systems including Nyāya?
9. Explain the meaning of *pramāṇa*. 33
10. What is the significance of *tarka* in Nyāya logic?

Q.III Answer the following questions in detail:

1. Describe the classification of Indian philosophical systems into āstika and nāstika with examples.31
2. Explain the five-step syllogism of Nyāya inference with a real-life example. 39
3. Discuss the significance of the *Nyāya Sūtra* and its contribution to Indian logic.
4. Compare and contrast the ancient Nyāya (prācīna) and the Navya Nyāya systems. 30
5. Elaborate on the four valid means of knowledge (pramāṇas) in Nyāya philosophy.33
6. Discuss the 16 padārthas of Nyāya and their role in logical reasoning. 31
7. Explain the three types of inference in Nyāya–*pūrvavat*, *śeṣavat*, and *sāmānyatodṛṣṭa*.40
8. Describe how Nyāya philosophy contributes to modern fields like Artificial Intelligence and machine learning. 43
9. Define and explain the three types of debate in Nyāya–*vāda*, *jalpa*, and *vitaṇḍā*. 45
10. Describe the role and process of *upamāna* (comparison) in acquiring valid knowledge. 43

***

CHAPTER 3

Panini's Astadhyayi and Chandasāstra

Objectives ...

- Introduction to Panini's Astadhyayi: Historical background and linguistic importance.
- Rule-Based System of Sanskrit Grammar: Sutras, meta-rules, recursion, and transformations.
- Chandasastra's Binary logic and combinatorial techniques.

3.1 INTRODUCTION TO PANINI'S ASTADHYAYI : HISTORICAL BACKGROUND AND LINGUISTIC IMPORTANCE

3.1.1 Historical Background:

- The *Astadhyayi*, composed by the legendary Sanskrit grammarian Panini around the 5th century BCE, is one of the most significant achievements in the history of linguistic science. Consisting of approximately 4,000 concise rules or *sutras*, the *Astadhyayi* represents a highly structured and formalized grammar system that has influenced not only Indian linguistics but also modern computational linguistics and artificial intelligence.
- Panini was born in the region of Gandhara, present-day Pakistan and Afghanistan. His work drew upon centuries of oral linguistic traditions and earlier grammatical texts, but what set him apart was his methodical approach. The *Astadhyayi* is composed in the form of eight chapters, each divided into four sections. These rules systematically describe the morphology, phonology, and syntax of Sanskrit, showcasing an extraordinary level of abstraction and precision. The structure is highly systematic, hierarchical, and recursive.
- Panini's grammar is not just descriptive but generative. It provides a finite set of rules that can generate an infinite number of valid Sanskrit sentences. This quality is what makes his work particularly significant in the context of computational linguistics and AI today. His work also introduces the concept of *meta-rules* (rules about rules), operational transformations, and rule precedence, elements that mirror modern programming and rule-based systems.

- Leonard Bloomfield, Noam Chomsky, and other linguists have praised Panini's contribution, noting its similarity to modern syntactic theory and formal grammar frameworks. It marks the culmination of centuries of linguistic analysis in India and forms the foundational framework for traditional Sanskrit grammar (Vyākaraṇa). Pāṇini's grammar was later elaborated and supported by commentators like Kātyāyana (Vārttikas) and Patañjali (Mahābhāṣya).

3.1.2 Linguistic Importance:

- Language is a structured system of communication that consists of several key components. These components work together to help us produce, understand, and interpret meaning in speech and writing. The main components of language are:

(a) Phonetics and Phonology:
- Phonetics deals with the physical sounds of speech–how sounds are produced, transmitted, and heard.
- Phonology studies the abstract, systematic organization of sounds in a particular language (e.g., rules for pronunciation and sound patterns).

(b) Morphology:
- This is the study of the internal structure of words. It looks at how root words, prefixes, and suffixes combine to create new words and meanings.
- **For Example,** *unhappy* = *un-* (prefix) + *happy* (root).

(c) Syntax:
- Syntax is the set of rules that govern sentence structure—how words are arranged to form grammatically correct sentences.
- **For Example,** "She eats apples" follows standard English syntax.

(d) Semantics:
- Semantics refers to the meaning of words and sentences. It explores how language conveys ideas, concepts, and emotions.
- **For Example,** The word "light" can mean "not heavy" or "illumination" depending on context.

(e) Pragmatics:
- Pragmatics studies how language is used in context, including tone, intention, and social norms. It explains how meaning can change depending on the speaker, listener, and situation.
- **For Example,** Saying "Can you open the window?" is a request, not a question about ability.

(f) Lexicon (Vocabulary):
- The lexicon is the complete set of words in a language, including their meanings, usage, and relationships to other words.

- The origin of sound is deeply rooted in the human vocal apparatus, where breath transforms into speech through a delicate coordination of the lungs, vocal cords, tongue, palate, lips, and nasal passages. As air flows from the lungs, it passes through the larynx, where the vocal cords vibrate to create voiced sounds. These vibrations are then shaped

by the oral cavity, where the tongue, teeth, and lips articulate distinct phonemes. Ancient Indian scholars were remarkably precise in understanding this process. Panini discussed phonetics in Ashtadhyayi through sutras. He also created a separate addendum known as Paniniya Shiksha. According to Panini, the origin of sounds is specific to the letters (varnas) in Sanskrit.

- Panini, in his linguistic system, categorized sounds based on the location in the vocal tract from where they are produced. These are primarily divided into five major points of articulation:
  - Gutturals (Kaṇṭhya) from the throat.
  - Palatals (Tālavya) from the palate.
  - Cerebrals (Mūrdhanya) from the roof of the mouth.
  - Dentals (Dantya) from the teeth.
  - Labials (Ōṣṭhya) from the lips.
- This classification is foundational in Sanskrit grammar and chanting, ensuring correct pronunciation, clarity, and rhythm. Each group of consonants (*varga*) is structured based on how airflow is manipulated and where contact is made in the mouth. The articulatory precision not only shapes meaning but also influences poetic meter and spiritual resonance in recitation. This phonetic organization was not merely descriptive but functioned as an efficient computational tool within his grammar, foreshadowing modern linguistic frameworks. Through his insights, Pāṇini laid the foundation for understanding how language arises from the body, transforming breath and flesh into meaning and form.
- Panini's system reflects an advanced understanding of phonetic science, long before the development of modern linguistics.

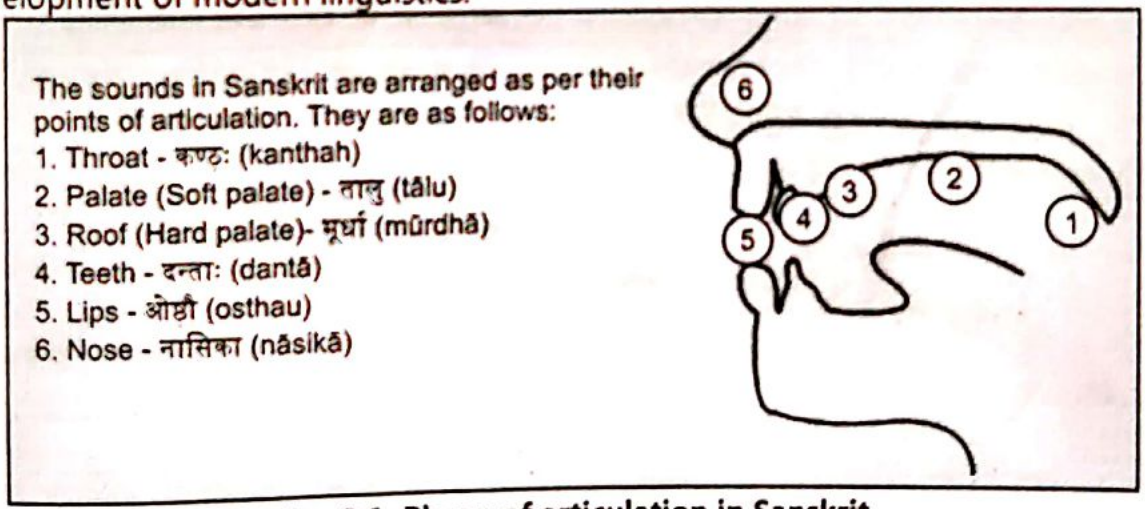

Fig. 3.1: Places of articulation in Sanskrit

Source:https://www.anaadi.org/post/the-power-of-chanting-where-neuroscience-meets-indic-wisdom

The points of Articulation are as follows:

1. **Gutturals (Kaṇṭhya – कण्ठ्य) (Sounds from the Throat):** These sounds are produced at the back of the oral cavity (throat region).
   Consonants: क (ka), **ख** (kha), **ग** (ga), **घ** (gha), **ङ** (ṅa)

2. **Palatals (Tālavya – तालव्य) (Sounds from the Palate):** These are articulated by pressing the tongue against the hard palate.
   Consonants: च (ca), छ (cha), ज (ja), झ (jha), ञ (ña).
3. **Cerebrals or Retroflex (Mūrdhanya – मूर्धन्य) (Sounds from the Roof of the Mouth):** Produced by curling the tongue back to touch the roof of the mouth (cerebral ridge).
   Consonants: ट (ṭa), ठ (ṭha), ड (ḍa), ढ (ḍha), ण (ṇa).
4. **Dentals (Dantya – दन्त्य) (Sounds from the Teeth):** Articulated by touching the tongue to the upper teeth.
   Consonants: त (ta), थ (tha), द (da), ध (dha), न (na).
5. **Labials (Ōṣṭhya – ओष्ठ्य) (Sounds from the Lips):** Formed by bringing the lips together or against the teeth.
   Consonants: प (pa), फ (pha), ब (ba), भ (bha), म (ma).
6. **Semi-vowels (Antastha):** य (ya – palatal), र (ra – cerebral), ल (la – dental), व (va – labial) Sibilants (Uṣman): श (sha – palatal), ष (ṣa – cerebral), स (sa – dental), ह (ha – guttural)

Table 3.1: Sanskrit Vowels and Consonants by Type and Articulation

Category	Subcategory	Sanskrit Letters	Place of Articulation
Vowels (स्वर)	Short (ह्रस्व)	अ, इ, उ, ऋ, ऌ	Guttural, Palatal, Labial, Retroflex
	Long (दीर्घ)	आ, ई, ऊ, ॠ, ॡ	Same as short vowels
	Diphthongs (संयुक्तस्वर)	ए, ऐ, ओ, औ	Gliding between palatal and labial
Consonants (व्यंजन)	Stops (स्पर्श)		
	Gutturals (कण्ठ्य)	क, ख, ग, घ, ङ	Throat (Velar)
	Palatals (तालव्य)	च, छ, ज, झ, ञ	Hard Palate
	Cerebrals (मूर्धन्य)	ट, ठ, ड, ढ, ण	Roof of the mouth (Retroflex)
	Dentals (दन्त्य)	त, थ, द, ध, न	Teeth
	Labials (ओष्ठ्य)	प, फ, ब, भ, म	Lips
Semi-vowels (अन्तःस्थ)	-	य, र, ल, व	Palatal, Cerebral, Dental, Labial
Sibilants and Aspirate (उष्म)	-	श, ष, स, ह	Palatal, Cerebral, Dental, Guttural

- Pāṇini, the brilliant 5th-century BCE grammarian, offered one of the earliest scientific analyses of sound production. In his Aṣṭādhyāyī, he not only categorized Sanskrit sounds

with extraordinary detail but also organized them in the Śiva Sūtras, a compact and systematic representation of phonemes based on their place (sthāna) and manner (prayatna) of articulation.

- Pāṇini's *Aṣṭādhyāyī* is special because it works like a language machine. It takes root words and suffixes as input and, by following a set of clear rules, produces correct Sanskrit words and sentences. He used meta-rules (rules about how rules work) and special symbols called anubandhas to make the grammar very efficient. His system includes important grammar processes like sandhi (joining sounds), samāsa (making compound words), and taddhita (adding suffixes for new meanings). Pāṇini also used smart techniques like zero markers (lopa) and rules that change depending on context, which are similar to ideas used in modern computer languages and grammar theories. Because of this, many scholars compare his work to modern generative grammar, like that of Noam Chomsky. Overall, Pāṇini's *Aṣṭādhyāyī* didn't just describe Sanskrit–it created a system that could generate it, making it one of the smartest grammar books ever written.

Key Features of the *Astadhyayi* include:

- Crafted with remarkable foresight, Pāṇini's *Aṣṭādhyāyī* stands as an enduring masterpiece of linguistic science. Its genius lies not only in what it explains, but in how seamlessly it weaves logic, language, and structure into a single coherent system.

Holistic Grammatical Framework:

- The treatise explores Sanskrit in its entirety from the sounds of speech to the formation of words and the rules governing sentence construction. No linguistic aspect is left untouched.
  - **Generative Clarity:** Pāṇini's rule-based design captures the sophisticated usage of Sanskrit among the learned elite, presenting a finite set of instructions capable of generating an infinite variety of grammatically sound expressions.
  - **Meta-Structure and Recursive Logic:** Among its most profound features is its use of meta-rules–principles that govern the functioning of other rules. Coupled with recursive constructions and transformations, the system prefigures the logic of modern computation.
  - **Mathematical and Algorithmic Precision:** The grammar is arranged with an almost mathematical elegance, where each rule interacts with others like components in a program. This structured clarity has led many to liken the *Aṣṭādhyāyī* to an ancient analog of a Turing machine.
  - **Guardian of Linguistic Continuity:** Through its precise codification, the text helped stabilize and conserve Classical Sanskrit across generations, maintaining its form with little deviation over centuries.
  - **Intelligent Conflict Resolution:** To address the inevitability of overlapping rules, Pāṇini devised a hierarchical mechanism–*vipratiṣedha*–which determines the rule that should prevail, ensuring that the system remains logical and unambiguous.

- **Elegant Economy through Śiva Sūtras:** The use of the Śiva Sūtras exemplifies Pāṇini's brilliance in technical notation. These phonetic groupings enable concise referencing throughout the grammar, revealing a remarkable economy of expression.

Influence beyond its Era:

- The *Aṣṭādhyāyī* not only shaped the work of later scholars like Patañjali and Jayāditya but continues to inspire fields such as theoretical linguistics and computational grammar, centuries after its composition.

3.2 RULE-BASED SYSTEM OF SANSKRIT GRAMMAR : SUTRAS, META-RULES, RECURSION, AND TRANSFORMATIONS

- The *Astadhyayi* is fundamentally a rule-based system. The backbone of this system is its *sutras* – compact aphoristic statements that define grammatical constructs and operations. Each *sutra* acts like a logical rule that is applied within a specific context to transform or generate linguistic forms. The grammar is not only a descriptive account of Sanskrit but also a comprehensive engine for producing valid linguistic forms through a layered application of rules, meta-rules, recursion, and transformations.

3.2.1 Sutras:

- In the context of Pāṇini, a sūtra is a brief and precise grammatical rule. Each sūtra is written in a compact form, often leaving out obvious words, making it both concise and mnemonic. This economy of language is part of the brilliance of Pāṇini's design.
- The term *sūtra* literally means "thread", and just like threads hold fabric together, these rules form the framework of the Sanskrit language.
- The entire Sanskrit grammar of Paṇini rest on a fundamental set of sutras known as Maheshvara sutras. Each line ends with an anubandha (a marker like 'ṇ', 'k', 'ṭ') that helps define groups of sounds. These sutras are 14 in number present the basic letters of Sanskrit uniquely as shown in table 3.2. The first four sutras cover the alphabets in the normal order. Sutras 5 to 14 present the consonants in a somewhat obscure order than what we are normally used to.
- The Maheshvara Sutras, also known as the Shiva Sutras, are a systematic arrangement of Sanskrit sounds, consisting of 14 aphoristic lines used as the foundation of phonology in Pāṇini's Aṣṭādhyāyī. These sutras are designed with a unique structure: each consists of a set of phonemes followed by a consonantal marker known as an it-letter (like ṇ, k, Ṅ, etc.), which signifies the boundary of the group but is not part of the group itself. The first four sutras present the vowels in their natural order – short vowels (a, i, u), vocalic sounds (ṛ, ḷ), long vowels (e, o), and diphthongs (ai, au). The next ten sutras (from 5 to 14) list all the consonants, but not in the usual way we learn the alphabet (like ka, kha, ga...). Instead, these consonants are intentionally spread out across different sutras. This arrangement allows Pāṇini to easily refer to specific groups of sounds when formulating grammar rules. It enhances the system's flexibility and precision, rather than adhering to the traditional order of letters. For example, sutra 5 (ha ya va ra ṭ) and sutra 6 (la ṇ) capture the semi-vowels and aspirate, while nasals are grouped in sutra 7 (ña ma ṅa ṇa na m). The voiced and voiceless consonants are distributed across sutras 8 to 12, with

voiced aspirates like jha, bha in sutra 8, and unaspirated voiced sounds like ja, ba, ga etc., in sutra 10. Sibilants like śa, ṣa, sa are grouped in sutra 13 (śa ṣa sa R), and the final sutra 14 (ha L) includes only ha, used symbolically to denote all consonants. These groupings are not arbitrary; they are designed to create pratyāhāras–compact notations that combine the first sound of a group with the it-letter of its end, such as ac (a to C) for all vowels, hal (ha to L) for all consonants, or yaṇ (ya to Ṇ) for semi-vowels.

- Such pratyāhāras are widely used in Pāṇini's rules to apply transformations across entire classes of sounds efficiently, enabling unparalleled conciseness in grammar. Thus, the 14 Maheshvara Sutras form a foundational, mnemonic, and operational tool in Sanskrit linguistics, reflecting both the elegance and depth of ancient Indian knowledge systems. These Maheshvara Sutras listed in the table 3.2.

Table 3.2: Maheshvara Sutras

No.	Sutra	Ends With	Group (Used in Pratyāhāra)	Examples
1.	अ इ उ ण (a i u ṇ)	ṇ	अ (a) – short 'a' sound इ (i) – short 'i' sound उ (u) – short 'u' sound ण (ṇ) – *it* letter (अणुबन्ध), not pronounced, just marks the end of this sound group. Short vowels (a, i, u)	agni, iti, utkaṭa
2.	ऋ ऌ क (ṛ ḷ k)	K	ऋ – ऋकारः, अल्पप्राणस्वरः। (short "ṛ" sound) ऌ – ऌकारः, दुर्लभःस्वरः। (short "ḷ" sound, rare, mostly Vedic) क् – अणुबन्धः (*it* letter), उच्चार्यतेन, केवलं प्रत्याहारनिर्माणाय। Vocalic r, l	ṛṣi, (rare: ḷ seen in Vedas)
3.	ए ओ ङ् (e o ṅ)	Ṅ	ए – दीर्घः "ए" स्वरः (long e), यथा "एकः" ओ – दीर्घः "ओ" स्वरः, यथा "ओषधिः" ङ् – अणुबन्धः (it-letter) Long vowels (e, o)	eka, oṣadhi

No.	Sutra	Ends With	Group (Used in Pratyāhāra)	Examples
4.	ऐ औ च् (ai au c)	C	ऐ – संयुक्तस्वरः (diphthong), यथा "ऐश्वर्यम्" औ – यथा "औषधम्" च् – अणुबन्धः Diphthongs	aiśvarya, auṣadha
5.	ह य व र ट् (ha ya va ra ṭ)	Ṭ	ह – यथा "हरिः" य – यथा "यज्ञः" व – यथा "वृक्षः" र – यथा "रामः" ट् – अणुबन्धः Semi-vowels + ha	hasa, yatra, vata, rama
6.	ल ण् (la ṇ)	Ṇ	ल – यथा "लवणम्" ण् – अणुबन्धः Only la	lavaṇa
7.	ञ म ङ ण न म् (ña ma ṅa ṇa na m)	m	ञ – यथा "पञ्च" म – यथा "मन्त्रः" ङ – यथा "गङ्गा" ण – यथा "विष्णुः" न – यथा "नमः" म् – अणुबन्धः All nasal sounds	mantra, nama, ṅa (in gaṅgā)
8.	झ भ ञ् (jha bha ñ)	Ñ	झ – यथा "झनत्कारः" भ – यथा "भवति" ञ् – अणुबन्धः Voiced aspirated consonants	bhoga, jhankara

No.	Sutra	Ends With	Group (Used in Pratyāhāra)	Examples
9.	घ ढ ध ष् (gha ḍha dha ṣ)	Ṣ	घ – यथा "घटः" ढ – यथा "विढः" ध – यथा "धनम्" ष् – अणुबन्धः Voiced aspirated consonants	ghaṭa, dharaṇa
10.	ज ब ग ड द श् (ja ba ga ḍa da ś)	Ś	ज – यथा "जलम्" ब – यथा "बलम्" ग – यथा "गजः" ड – यथा "डमरुः" द – यथा "दर्शः" श् – अणुबन्धः Voiced unaspirated consonants	jala, gaja, bala
11.	ख फ छ ठ थ च ट त व् (kha pha cha ṭha tha ca ṭa ta va v)	V	ख – "खड्गः" फ – "फलम्" च – "चन्द्रः" त – "तपः" व – "वनम्" व् – अणुबन्धः Unvoiced aspirated/unaspirated	khaga, phala, thala
12.	क प य् (ka pa y)	Y	क – यथा "कर्म" प – यथा "पात्रम्" य् – अणुबन्धः Basic stops	kala, pata
13.	श ष स र् (śa ṣa sa r)	R	श – यथा "शिवः" ष – यथा "षट्" स – यथा "सत्यः" र् – अणुबन्धः Sibilants	śanti, ṣaṣṭha, sagara
14.	ह ल् (ha l)	L	ह – यथा "हरिः" ल् – अणुबन्धः Only ha	hara

3.2.2 Meta-rules

- **Definition:** Meta-rules in Pāṇini's grammar are higher-level rules that govern how and when the primary grammatical rules apply, especially in cases of conflict or overlap. They regulate the application order, scope, and precedence of rules.
- They ensure consistency and resolve ambiguities in rule application, making the grammar deterministic and algorithmic.

  For Examples,

  1. The famous conflict resolution meta-rule states that when two rules conflict, the later one in the sequence applies (though recent scholarship, e.g., by Rishi Rajpopat, suggests more nuanced interpretations favoring right-side rules).
  2. Meta-rules also govern conditions like *asiddhatva* (non-application of certain rules during intermediate stages) and *adhikāra* (domain of rules).
- These meta-rules make Pāṇini's grammar not just a list of rules but a controlled system with logical hierarchy and operational constraints, akin to control structures in programming.

Panini Meta-Rules (Paribhāṣā-sūtras):

- Panini's *Paribhāṣā-sūtras* are the meta-rules or interpretive rules within his grammar system, the *Aṣṭādhyāyī*, that govern how the primary grammatical rules (*vidhisūtras*) are to be applied, interpreted, and resolved when conflicts arise. These meta-rules provide a higher-level framework ensuring consistency, precedence, and correct sequencing of rule application.
- **Definition:** Paribhāṣā literally means "interpretation" or "meta-rule". These sutras serve as guidelines about how to understand and apply the main grammatical rules. They clarify ambiguities, resolve conflicts, and regulate the scope and order of operations.
- Pāṇini's grammar is more than just a list of rules–it is a carefully structured and logical system. A major goal of this system is to resolve conflicts when more than one rule could apply to the same word or phrase, making sure that the most suitable rule is chosen. To manage this, Pāṇini created ways to set priorities among rules, showing clearly which one should be followed in case of overlap. He also defined specific areas of application, known as *adhikāra*, so that each rule works only within its intended scope. In addition, Pāṇini included meta-rules that explain how to understand technical grammar terms, symbols, and special markers. These elements work together to make his grammar precise, logical, and algorithm-like, showing a level of sophistication similar to modern computational linguistics.

Types Classifications of Paribhāṣās (Meta-Rules):

- In the intricate architecture of Pāṇinian grammar, *Paribhāṣās* serve as guiding principles that govern how rules are interpreted and applied. These meta-rules, though often subtle, are indispensable for maintaining consistency and logical coherence within the system. Traditionally, they are categorized into three distinct types:

1. **Vācanika (Explicit Meta-Rules):**

- These are directly articulated within the sūtras of Pāṇini or elaborated upon by later grammarians. They are clearly stated and serve as formal instructions embedded in the grammatical framework itself. Their authority lies in their explicit textual presence.

Examples:

(i) Vipratiṣedhe Paraṁ Kāryam (1.4.2)
- *"When two rules are in conflict, the latter one applies."*
- This rule resolves conflicts when multiple rules apply by prioritizing the rule that appears later.

(ii) Ādeśa-Pratyayaḥ (1.1.56)
- *"A substitute functions like a suffix."*
- This clarifies that substitutes (*ādeśa*) in derivation behave grammatically like suffixes (*pratyaya*).

(iii) Lopaśca (1.1.60)
- *"And there is elision (lopa)."*
- Indicates that markers (*it*) which fulfill their grammatical function are elided and do not appear in the final output.

2. **Jñāpakasiddha (Inferred Meta-Rules):**

- These rules are not directly stated but are inferred from the structure and behavior of the grammar. Their existence is deduced from the need to explain certain derivational processes or to resolve apparent inconsistencies. They reflect the internal logic and necessity of the grammatical system.

Examples:

(i) Nitya-Rule Inference: When a transformation always occurs without exception (like *guṇa* substitution in certain verbal forms), even if not explicitly labelled, it is inferred as *nitya* (obligatory), based on consistent application.

(ii) Rule Scope Limitation: Certain rules are applied only within a specific domain, though no *adhikāra* is explicitly declared. Their limited usage in examples leads to the inference of a scope restriction.

(iii) Optional Rule Behavior: In cases where a rule sometimes applies and sometimes does not (without explicit mention of *vā*), scholars infer optional behavior to maintain consistency in derivation.

3. **Nyāyasiddha (Logically Established Meta-Rules):**

- Rooted in general principles of reasoning or linguistic logic, these meta-rules gain legitimacy through rational justification rather than textual presence. They often emerge from philosophical or analytical reflection on the functioning of language, lending theoretical depth to the grammatical tradition.

Examples:

(i) Prāpti (Entitlement to Apply a Rule): A rule should only apply when its conditions are met. This principle, although not stated explicitly, is accepted because it is foundational to any logical rule system.

(ii) **Ekārtha-Niyama (One Operation per Unit):** Only one grammatical operation should apply to a given unit at a time. This ensures derivational clarity and prevents overlapping changes.

(iii) **Akārye Kāraka-Pravṛtti-Niyama:** *"Kāraka (semantic role) relations are not assigned when there is no action."* This reflects a broader logical principle that without an action (verb), roles like agent or object are meaningless–though not explicitly stated it aligns with linguistic reasoning.

Famous Meta-Rule:

- The *Bahiraṅga Paribhāṣā* states that when two rules conflict, the later (in order) rule applies, unless there is an explicit exception. This is often summarized as "the later rule overrides the earlier."

Pāṇini's Meta-Rules (Paribhāṣā-sūtras):

- Within the elegant machinery of Pāṇini's grammatical treatise lies a set of governing principles known as Paribhāṣā-sūtras–meta-rules that bring coherence, control, and clarity to the dense network of operational rules. Here are five such guiding principles, as interpreted through classical traditions:

1. **Domain Specification (*Adhikāra*):** This meta-rule serves as a lantern, illuminating the path along which a series of rules shall proceed. It declares the contextual boundary within which succeeding sūtras operate. For instance, should a rule declare, "Let all subsequent rules pertain to food," then the rules that follow will restrict their function only to expressions related to food items. Thus, *Adhikāra* offers both structure and thematic clarity, ensuring that grammatical operations remain anchored in relevance.
2. **Resolution of Conflicting Rules (*Bahiraṅga Paribhāṣā*):** When multiple rules seem simultaneously eligible for application, the grammar does not falter. This meta-rule steps forth to resolve the conflict, asserting that–unless otherwise specified–the rule that appears later in the sequence shall override the earlier one. Such a principle guards against ambiguity and fosters a deterministic and reliable system.
3. **Substitution and Its Environment (*Sthāne Ādeśaḥ Nimitteṣu Nimittebhyaḥ*):** In the art of substitution, this meta-rule lays down the conditions of transformation. It states that a substitute (*ādeśa*) replaces the original element (*sthāna*) in a specific environment defined by contextual triggers (*nimitta*). This rule underpins many of the morphological and phonological changes, offering a context-sensitive mechanism for precise derivation.
4. **Clarification of Technical Terminology:** Not all terms carry their everyday meanings within the grammar. The meta-rules provide explanatory lenses through which technical expressions and markers (*anubandhas*) are to be read. For instance, what might ordinarily be called a "fruit" is in grammatical terms defined as "a seed-bearing edible," while "vegetable" implies a food without seeds. Such clarification preserves semantic discipline and removes interpretive vagueness.

5. **Suspension of Rules (*Asiddhatva*):** Certain operations must not be prematurely applied. This meta-rule declares some rules as *asiddha*–temporarily invisible or inactive–during specific stages of derivation. This prevents untimely interventions and preserves the logical sequence of transformations, ensuring that the derivation unfolds with integrity and order.

3.2.3 Recursion:

- Panini's *Aṣṭādhyāyī* (circa 5th–4th century BCE) is widely recognized as one of the earliest formal grammatical systems, notable for its use of recursive techniques that allow the generation of infinitely many valid Sanskrit forms from a finite set of rules and elements.

What is Recursion in Panini's Grammar?

- Recursion refers to the process where a rule or procedure refers to itself directly or indirectly, enabling repeated or nested application.
- In Panini's grammar, recursion appears in the way rules apply to their own outputs or invoke other rules, allowing complex morphological and syntactic structures to be built from simpler components.
- This mechanism is essential for the generative power of the grammar, enabling it to produce an unlimited variety of words and sentences.

The Recursive Genius of Pāṇini:

- Pāṇini's *Aṣṭādhyāyī* exemplifies recursion not as a modern invention, but as an ancient linguistic intuition, woven seamlessly into the very fabric of Sanskrit grammar. His method mirrors algorithmic thought, where rules evolve through self-reference and layered dependency.

1. **Rule Inheritance through Anuvṛtti:** A striking feature of Pāṇini's technique is *anuvṛtti*–the elegant mechanism by which parts of earlier rules, such as affixes or conditions, are silently carried forward into later rules. This inheritance avoids redundancy and crafts a recursive thread across sutras, much like the concept of scope and inheritance in object-oriented programming.
2. **Recursive Morphological Construction:** Sanskrit word formation in Pāṇini's grammar often proceeds recursively. Suffixes are successively applied to roots or stems, and intermediate forms are subjected to further rule applications. The formation of nominal and verbal stems, for instance, unfolds through iterative layering, reflecting the essence of recursion in morphological derivation.
3. **Elliptical Structure and Rule Compression:** The brevity of the *Aṣṭādhyāyī* is not a limitation, but a deliberate design. Pāṇini condenses rules by relying on earlier ones for interpretation, creating a network of mutual dependency. Just as a recursive function refers back to its own definition, many of Pāṇini's sutras derive their meaning through the structure established by preceding rules.
4. **Generative Power of the System:** This recursive architecture is not merely stylistic–it is generative in nature. It empowers the grammar to produce every valid Sanskrit form, with precision and completeness. Through its self-referential system,

the *Aṣṭādhyāyī* achieves not only expressive richness but also computational elegance, centuries before such ideas were formalized in modern linguistic theory.

3.2.4 Transformations

- **Transformations in Panini's Grammar:** Panini's *Aṣṭādhyāyī* (c. 5th-4th century BCE) employs systematic transformations to derive grammatically correct Sanskrit forms from roots (*dhātu*) and affixes. These operations–applied via context-sensitive rules–include substitution, affixation, augmentation, deletion, and compounding. Below is a synthesis of their mechanics and significance:

1. **Core Transformation Types:**
   - **Substitution (*Ādeśa*):** This operation involves replacing one linguistic element with another, tailored to its grammatical or phonetic environment.

     Example: The verbal root *bhū* (to be), when joined with the suffix *-ti*, transforms to *bhavati* ("he is"), where the vowel *ū* is substituted with *av*.
   - **Affixation (*Pratyaya*):** Words take shape through the addition of affixes. These elements–prefixes, suffixes, or infixes–attach to roots, providing new grammatical meaning.

     Example: The root *nī* (to lead) becomes *naya* when joined with the suffix *-a*, signifying the act of leading.
   - **Augmentation (*Āgama*):** When phonological rules require balance or smoothness, additional sounds are inserted.

     Example: *su* (good) merges with the superlative suffix *-tama* to produce *śreṣṭha* (best), where phonetic alteration (*s* → *ś*) and vowel strengthening co-occur.
   - **Deletion (*Lopa*):** Certain phonemes vanish when specific conditions are met, streamlining the sound structure.

     Example: The root *vid* (to know) with the infinitive *-tum* becomes *vettum*, with the consonant *d* omitted.
   - **Compounding (*Samāsa*):** Two or more words are fused into a single compound to express complex ideas efficiently.

     Example: *rāja* (king) + *puruṣa* (man) → *rājapuruṣa* (king's servant), a union of semantic and syntactic purpose.
2. **The Role of Context in Transformation:** Every rule in Pāṇini's system is sensitive to its linguistic environment. The transformation applied depends on the context:
   - **Phonetic Context:** Sound combinations at word boundaries shift through *Sandhi*.

     Example: *tad* + *api* becomes *tatapi*, a morphing that enhances fluency in pronunciation.
   - **Morphological Context:** Tense markers and affixes guide transformations in form.

     Example: *kṛ* (to do) + *-ta* (past tense) becomes *kṛta* → *kṛtaḥ*.

- **Syntactic Context:** Grammatical roles such as subject or instrument alter the word's ending.

  Example: *devaḥ* (nominative singular) becomes *devena* (instrumental singular).

3. **An Iterative and Recursive System:** Pāṇini's framework is not linear; it operates in stages, revisiting its outputs for refinement:
   - **Stepwise Application:** Rules are applied one at a time. After each application, the output is re-evaluated for further applicable rules.
   - **Recursive Input-Output Cycle:** The product of one transformation often serves as the input for the next.

     Example: *rāma + su* evolves into *rāmās* and ultimately *rāmāḥ* through successive transformations.
   - **Resolution via Meta-Rules (*Paribhāṣā*):** When multiple rules compete, meta-rules such as *vipratiṣedhe paraṁ kāryam* ("when rules conflict, the later one prevails") guide conflict resolution.
4. **A Grammar with Computational Insight:** Pāṇini's genius foreshadows computational theory in startling ways:
   - **Rewriting Rules:** His transformations function like modern context-sensitive grammars ($X \rightarrow Y / A \_ B$), modifying elements in constrained environments.
   - **Control Mechanisms:** Auxiliary markers–like *it* symbols–act as flags or toggles, controlling the flow and applicability of rules, akin to programming syntax.
   - **Infinite Productivity:** From a finite rulebase, Pāṇini's system can generate an infinite array of correct expressions, reflecting the theoretical capabilities of a Turing machine.
5. **An Illustrative Example:** Transforming "dṛś" (to see): Let us examine a single verb form as it travels through layers of transformation:
   - **Base Construction:** *dṛś + -ti* yields *dṛśti*
   - **Consonant Shift:** *ś* → *k* before *t* (Rule 8.2.36) → *dṛkti*
   - **Vowel Adjustment:** *ṛ* → *ṛṣ* (Rule 6.1.128) → *dṛṣṭi*
   - **Final Output:** *dṛṣṭi* – "he sees"

   This single word, shaped by phonological finesse and logical structure, exemplifies the elegant efficiency of Pāṇini's algorithmic genius.

Importance of Transformations:

- Provide a formal and algorithmic framework for the generation of correct Sanskrit forms.
- Ensure precision and consistency in word formation, derivation, and grammatical inflection.
- Anticipate the concepts of generative grammar later formalized by Noam Chomsky.

- Offer one of the earliest examples of rule-based linguistic modelling in human history.
- Serve as a theoretical foundation for computational linguistics and language technologies.
- Influence Natural Language Processing (NLP) and AI-based language parsers, especially for Sanskrit and Indo-European languages.

3.3 CHANDASASTRA'S BINARY LOGIC AND COMBINATORIAL TECHNIQUES

3.3.1 Introduction to Chhanda Shastra

- Chandahsastra is an ancient Sanskrit treatise on the science of poetic meters and prosody, traditionally attributed to the scholar Acharya Pingala, who lived around the 3rd century BCE. The term Chanda refers to the study of poetic meter, specifically the patterns of long (guru) and short (laghu) syllables in Sanskrit poetry and Vedic hymns. In Sanskrit literature, chanda (meter) is not merely a poetic device but the very foundation of poetry itself. There is no Sanskrit poetry without chanda, as it governs the rhythm, mood, and aesthetic flavor of every composition. To truly appreciate and enjoy Sanskrit poetry, one must possess a sound understanding of its meters, each of which imparts a unique emotional and musical quality to the verse. This work is considered one of the earliest systematic explorations of rhythm and meter in literature, and it holds a significant place in both linguistic and mathematical history.
- Pingala's Chandahsastra is notable for its pioneering use of what is now recognized as the first description of a binary numeral system. Instead of zeros and ones, Pingala used the concepts of laghu (light or short syllable) and guru (heavy or long syllable) to represent binary values. This binary representation made it possible to list and classify all possible metrical patterns for a given number of syllables, effectively linking poetic meter with combinatorial mathematics.
- Pingala's work is special because it uses a system like binary numbers (used in computers today). Instead of 0s and 1s, Pingala used laghu and guru syllables. This way, he could list all possible rhythm patterns for a line of poetry, connecting poetry with combinatorics-a part of math that deals with combinations.
- The Pingala's work also includes mathematical methods and algorithms, such as:
  - Prastāra – which lists all possible patterns of long and short syllables,
  - Meru-prastāra – similar to Pascal's triangle, which shows how different combinations can be counted,
  - And some ideas similar to the Fibonacci sequence.
- Chandahsastra is one of the Vedangas–six subjects that help in understanding the Vedas (ancient Hindu scriptures). The accurate use of meter, as explained in this text, helped people memorizes and recites the Vedas properly, keeping them unchanged over generations.

3.3.2 Chandasastra's Binary Logic and Combinatorial Techniques

- One of the most remarkable contributions of *Chandasastra*, attributed to Acharya Pingala around the 3rd century BCE, is its early use of a binary-like system to analyze poetic meter. In this system, the two fundamental elements of Sanskrit prosody–Laghu (short syllable) and Guru (long syllable) are represented using two symbolic values: Laghu (L) corresponds to 1 and Guru (G) to 0.
- This binary-like system allowed Pingala to systematically list all possible syllable patterns, identify the pattern at any given position, count the number of short syllables (Laghus) in each pattern, and apply mathematical techniques like combinations to analyze poetic structure.
- Each sequence of syllables in a verse could be treated like a binary number, enabling systematic analysis and classification. Pingala's binary system was reversed compared to modern binary notation. That means the first syllable is treated as the least significant digit (rightmost in modern binary).

Examples:

- The syllable pattern L-G-L-G is written as 1010.
- Reversed (Pingala's format): 0101.

Pingala and the Combinatorial Tools: Pratyayas

- Pingala introduced a set of combinatorial techniques, called Pratyayas, to systematically study and generate the various possible meters (*chhandas*) in Sanskrit prosody. These tools enabled poets and scholars to explore the vast space of metrical patterns formed by sequences of long and short syllables.
- Pingala also introduced several combinatorial algorithms to explore and organize poetic meters. These techniques resemble modern mathematical tools such as binary trees, Pascal's Triangle, and combinations. Pingla introduced six Pratyayas: Prastara, Sankhya, Nasta, Uddista, Lagakriya, and Adhvayoga.

1. Prastāra:

- *Prastāra* is a fundamental combinatorial tool introduced by Acharya Pingala in his *Chandaḥśāstra* (Science of Sanskrit Prosody) to systematically generate and enumerate all possible metrical patterns (combinations of short and long syllables) for a verse of length *n* syllables.

Meaning and Purpose:

- The Sanskrit term Prastāra literally means "expansion" or "spreading out."
- It refers to the exhaustive listing of all possible sequences of *laghu* (short) and *guru* (long) syllables in a binary-like matrix form.
- For *n* syllables, there are 2^n possible combinations, which Prastāra arranges into a $2^n \times n$ matrix, where each row represents one unique metrical pattern.

Pingala's Algorithm for Prastāra:

- Start with the base case for n = 1: two possible syllables, G (guru) and L (laghu).
- For each subsequent *n*, recursively build the list by:
  - Taking the Prastāra for n – 1 syllables.
  - Creating two copies of this list.
  - Appending L (1) to each pattern in the first copy.
  - Appending G (0) to each pattern in the second copy.
- Concatenate these two lists to form the Prastāra for *n*.
- This recursive process generates all 2^n combinations in an ordered manner.

For Example, two syllable patterns:

- Let's say we want to find all the possible patterns of 2 syllables.
- There are 2 options (Guru or Laghu) for each syllable. So, total patterns = 2^2 = 4 patterns.

Pattern	Syllables (L = 1, G = 0)	Binary (Pingala)	Reverse Order (Modern Binary)	Decimal Value
1	GG	00	00	0
2	GL	01	10	2
3	LG	10	01	1
4	LL	11	11	3

Note: In Pingala's system, the first syllable is the least significant, so binary is read from right to left.

For Example, three syllable patterns:

- Let us say we want to find all the possible patterns of 3 syllables.
- There are 2 options (Guru or Laghu) for each syllable. So, total patterns = 2^3 = 8 patterns.

Pattern	Syllables (L = 1, G = 0)	Binary (Pingala)	Reverse Order (Modern Binary)	Decimal Value
1	G G G	000	000	0
2	L G G	001	100	4
3	G L G	010	010	2
4	L L G	011	110	6
5	G G L	100	001	1
6	L G L	101	101	5
7	G L L	110	011	3
8	L L L	111	111	7

2. Nāṣṭa:

- In the context of Pingala's *Chandaḥśāstra* and Sanskrit prosody, Nāṣṭa (Sanskrit: नष्ट) refers to a combinatorial technique used to retrieve a specific metrical pattern (sequence of long and short syllables) given its position (index) in the ordered list of all possible patterns generated by *Prastāra*. The word *Nāṣṭa* literally means "lost" or "missing."
- It addresses the problem: *Given the position number (row number) in the Prastāra (the exhaustive list of all possible syllabic patterns), how do we find the exact pattern of long (guru) and short (laghu) syllables?*
- Essentially, Nāṣṭa is the decoding or retrieval algorithm that converts a decimal position into its corresponding binary-like syllabic pattern. While *Prastāra* generates all possible combinations of syllables for *n* length, Nāṣṭa allows us to directly find the pattern at any given position without enumerating all previous patterns.
- This is analogous to converting a decimal number into a binary number, where:
  - Laghu (short syllable) corresponds to binary digit 1.
  - Guru (long syllable) corresponds to binary digit 0.
- The algorithm works by repeatedly dividing the position number by 2 and interpreting the remainders as syllable types. Nāṣṭa enables poets and scholars to quickly access any metrical pattern by its index, facilitating analysis, composition, and teaching. It complements *Prastāra* and *Uddiṣṭa* (which finds the position of a given pattern), together forming a complete system for indexing and decoding meters. This system demonstrates an early understanding of binary numbers and positional notation in ancient Indian mathematics.

The Nāṣṭa algorithm:

- Start with the given row number.
- If the number can be halved exactly, write L and halve the number.
- If it cannot be halved exactly, add 1, halve it, write G.
- Repeat until all syllables are found.

This process is analogous to converting a decimal number into a binary sequence, where L = 1 and G = 0.

Example 1: Suppose you want to find the metrical pattern at position 5 for syllable length 3.

- Total possible patterns for 3 syllables = $2^3 = 8$.
- Convert 5 to binary: 5 in binary is 101.
- Map digits to syllables:
  - 1 → Laghu (short)
  - 0 → Guru (long)
  - So, pattern 5 corresponds to: Laghu - Guru - Laghu (L G L).

Example 2: Finding the 132nd Pattern for 8 Syllables

- Row number = 132.
- The process involves halving and writing L or G according to whether the number is even or odd, repeating until 8 syllables are determined.
- This example illustrates the elegance and efficiency of Nāṣṭa in decoding large row numbers into metrical patterns without enumerating all previous rows.

3. Uddiṣṭa:

- Uddiṣṭa (Sanskrit: **उद्दिष्ट**) literally means "that which is indicated," "described," or "pointed out." In the context of Pingala's *Chandaḥśāstra*–the ancient treatise on Sanskrit prosody and combinatorics–Uddiṣṭa refers to the algorithm or method used to find the position (row number) of a given metrical pattern (a sequence of laghu (short) and guru (long) syllables) in the ordered enumeration (*Prastāra*) of all possible syllabic combinations of length *n*.
- While *Prastāra* generates all possible combinations of long and short syllables for a given length *n* in a systematic order, and *Nāṣṭa* finds the pattern at a given position, Uddiṣṭa performs the reverse operation: Given a metrical pattern (e.g., L G L G), it calculates the exact position of this pattern in the Prastāra listing.
- It is essentially a pattern-to-position mapping, analogous to converting a binary sequence into its decimal equivalent, where:
  - Laghu (L) = 1
  - Guru (G) = 0

How Uddiṣṭa Works (Conceptual Overview)?

- Start with the initial value 1 (since Pingala's counting starts at 1, not zero).
- Scan the syllable pattern from right to left (starting from the last syllable).
- For each syllable:
  - If it is Laghu (L), double the current number.
  - If it is Guru (G), double the current number and subtract one.
- The final number after processing all syllables is the position of the pattern in the Prastāra.

Example 3: Consider the pattern for 3 syllables: L G L

- Start with 1.
- From right to left: last syllable = L → double 1 = 2
- Next syllable = G → double 2 = 4, subtract 1 = 3
- Next syllable = L → double 3 = 6

So, the position of the pattern L G L in the Prastāra is 6.

Importance of Uddiṣṭa:

- Provides a direct and efficient way to find the index of any metrical pattern without enumerating all previous patterns.
- Complements *Nāṣṭa* (pattern retrieval by position) and *Prastāra* (complete enumeration), forming a complete system for indexing and decoding Sanskrit meters.
- Demonstrates early use of positional notation and binary arithmetic concepts in ancient Indian mathematics.
- Facilitates the study, classification, and composition of Sanskrit poetry by enabling quick referencing of meters.

4. **Saṅkhyā:**

- The term *Saṅkhyā* (**संख्या**) in Sanskrit translates to "number," "count," or "enumeration." In the context of Pingala's Chandaḥśāstra, an ancient work focused on Sanskrit prosody and combinatorial analysis, *Saṅkhyā* refers to the method used to calculate the total number of possible metrical patterns for a given number of syllables. These patterns are formed using combinations of long (*guru*) and short (*laghu*) syllables. Essentially, *Saṅkhyā* represents a combinatorial approach to determine how many distinct poetic meters can be generated with a fixed syllabic length.

Role of Saṅkhyā in Sanskrit Prosody:

- For a verse of length *n*, each syllable can be either laghu (short) or guru (long).
- Since there are two choices per syllable, the total number of possible meters (patterns) is: 2^n
- The Saṅkhyā method calculates this total number, providing a fundamental understanding of the size of the metrical pattern space.
- It supports poets and scholars in grasping the vastness and diversity of Sanskrit meters, enabling systematic exploration and classification.

Saṅkhyā and Related Concepts in Pingala's Work:

- Saṅkhyā is closely related to other combinatorial tools like:
  - **Prastāra:** Enumerates all possible patterns.
  - **Naṣṭa:** Retrieves a pattern by position.
  - **Uddiṣṭa:** Finds the position of a given pattern.
- Together, these form a comprehensive system for analyzing Sanskrit meters.

Example 4:

- For n = 3 syllables, the total number of meters is: $2^3 = 8$
- These 8 patterns are all possible combinations of G (guru) and L (laghu) syllables, such as GGG, GGL, GLG, LLL, etc.

Significance:

- Saṅkhyā represents one of the earliest known uses of exponential counting in mathematics.
- It highlights the deep connection between Sanskrit prosody and combinatorial mathematics in ancient Indian scholarship.

- This counting principle is foundational not only for poetic meter analysis but also for the development of binary systems and algorithmic thinking.

5. Meru-prastāra:

- Meru-prastāra (**मेरुप्रस्तार**) is an ancient Indian combinatorial tool described in Pingala's *Chandaḥśāstra* and later explained in detail by commentators like Halayudha Bhatt. It is essentially a representation of binomial coefficients arranged in a triangular form resembling a mountain (*Meru*), which is why it is called "Meru-prastāra" or "the expansion of Meru".
- Meru-prastāra is an ancient Indian mathematical technique used to systematically list out combinations, especially in the context of metrical patterns in Sanskrit prosody. It is a triangular arrangement of numbers, identical in structure to what is known today as Pascal's Triangle, but it was described in Indian texts like Pingala's Chandahśāstra long before Pascal. Each row in Meru-prastāra represents the coefficients of the binomial expansion $(a + b)^n$, where n is the row number. The numbers are generated using a simple rule: each number is the sum of the two numbers directly above it.

What is Meru-prastāra?

- Meru-prastāra is a triangular array of numbers that enumerates the coefficients of the expansion of powers of a binomial expression.
- It is equivalent to what is known today as Pascal's Triangle, but predates Pascal by over a millennium.
- Each number in the triangle is the sum of the two numbers directly above it, following the recurrence relation:

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

- This structure is used to count the number of metrical patterns containing a specific number of long (*guru*) or short (*laghu*) syllables in Sanskrit prosody.

Role in Sanskrit Prosody and Pingala's Work:

- In the context of Sanskrit meters, Meru-prastāra helps answer questions like: *How many meters of length n contain exactly r long syllables?*
- The answer is given by the binomial coefficient (nr) (*rn*), which corresponds to the entries in Meru-prastāra.
- For example, for 6 syllables, the total number of meters is $2^6 = 64$, and the numbers in the 6^{th} row of Meru-prastāra sum to 64:

$$1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$$

- This triangular arrangement also encodes the Virahanka numbers, which relate to Fibonacci sequences appearing in the study of syllabic patterns.

Visual and Conceptual Description:

- The triangle is visualized as steps of a mountain (*Meru*), with each level representing the coefficients for a particular n (number of syllables).
- The shape and construction resemble a pyramid or mountain, hence the name.

Historical and Mathematical Importance:

- Meru-prastāra predates Pascal's triangle by about 1800 years.
- It is one of the earliest recorded uses of binomial coefficients and combinatorial mathematics.
- It reflects the deep mathematical insight of ancient Indian scholars in connecting poetry, combinatorics, and algebra.

6. Lagakriyā:

- Lagakriyā (also spelled *laghukriyā*) is a technical term in Sanskrit prosody referring to the calculation of the number of metres (metrical patterns) that contain a given number of laghu (short) syllables or guru (long) syllables within a fixed length of verse. It is the process of counting how many metrical patterns have exactly *k* laghu syllables (or guru syllables) in a verse of length *n* syllables. This counting is crucial for classifying and understanding the distribution of meters in Sanskrit poetry. Lagakriyā complements other combinatorial tools like Prastāra (enumeration of all patterns) and Saṅkhyā (total number of patterns).

Working of Lagakriyā:

- Since each syllable can be either laghu (L) or guru (G), the total number of patterns for n syllables is 2^n.
- The number of patterns with exactly k laghu syllables is given by the binomial coefficient:

$$\text{Number of patterns with k laghus} = \binom{n}{k}$$

This means Lagakriyā essentially counts combinations–how many ways to choose k positions for laghu syllables out of n.

Example 5: For a verse of length 4 syllables:

- Total patterns = $2^4 = 16$.
- Number of patterns with exactly 2 laghu syllables = $\binom{4}{2} = 6$ patterns.

So, Lagakriyā can help generate all 6 patterns where exactly 2 out of 4 syllables are laghu. These 6 patterns have laghu syllables in any two positions, and guru syllables in the remaining two.

Usage of Lagakriyā:

- Lagakriyā helps poets and scholars identify and classify meters based on the number of short or long syllables.
- It is used in conjunction with Gaṇas–groups of three syllables–to describe meters more compactly.
- The concept is fundamental in the combinatorial study of meters initiated by Pingala and elaborated in later treatises.

SUMMARY

- Panini's *Astadhyayi* is a foundational text in Sanskrit grammar composed around the 5th–6th century BCE.
- It consists of approximately 3,959 sutras (rules) organized across eight chapters, hence the name *Ashta-adhyayi* ("Eight Chapters").
- Panini's work is highly structured and algorithmic, representing one of the earliest forms of formal linguistic modeling.
- The *Astadhyayi* serves both descriptive and generative functions explaining language use and generating grammatically correct expressions.
- It relies on concise aphoristic expressions (sutras), many of which depend on context and prior rules for interpretation.
- The grammar includes meta-rules (paribhashas) to manage rule precedence, exceptions, and dependencies.
- A rule-based system is employed, where linguistic rules follow a deterministic or conditional pattern of application.
- Panini introduced recursion and context-sensitive transformations, prefiguring concepts in computer science and formal grammar theory.
- The system utilizes technical terms and symbols, such as *anubandhas* (markers) and Maheshvara Sutras to represent phonemes efficiently.
- It incorporates transformational grammar, where root words undergo operations like affixation, sandhi (joining), and case formation.
- Panini's rules are hierarchical and self-referential, allowing for rule overriding and exception handling.
- His grammar reflects an understanding of linguistic universals and precision that parallels modern computational linguistics.
- The *Chandasastra*, authored by Pingala, is the science of Sanskrit prosody (meter).
- It presents syllables as binary entities–*laghu* (light = 0) and *guru* (heavy = 1) similar to bits in binary logic.
- Using these binary representations, metrical combinations are systematically generated.
- *Chandasastra* includes Meru-prastāra, a precursor to Pascal's triangle, for enumerating metrical patterns.
- This system enables poets to identify all possible rhythmic sequences for a given number of syllables.
- It combines linguistic structure with combinatorics and number theory, showing interdisciplinary depth.
- The synergy of *Astadhyayi* and *Chandasastra* reflects an early Indian tradition of integrating linguistics with logic and mathematics.
- These systems not only preserved linguistic purity but also laid the groundwork for modern formal grammar, AI rule systems, and computational logic.

Exercise

Q.I Multiple Choice Questions (MCQs):

1. Who authored the Ashtadhyayi, a foundational text in Sanskrit grammar?
   (a) Bhartrihari (b) Kalidasa
   (c) Panini (d) Patanjali
2. How many sutras are approximately present in Panini's Ashtadhyayi?
   (a) 3959 (b) 2000
   (c) 1000 (d) 5000
3. What type of grammar system is the Ashtadhyayi best described as?
   (a) Poetic (b) Philosophical
   (c) Rule-based generative system (d) Oral recitation system
4. Which of the following is a concept introduced by Panini's grammar?
   (a) Linear logic (b) Quantum theory
   (c) Binary search (d) Recursion
5. What are Paribhashas in Panini's grammar?
   (a) Literary poems (b) Meta-rules that guide rule application
   (c) Vedic hymns (d) None of the above
6. What is the primary subject of the Chandasastra?
   (a) Syntax (b) Phonetics
   (c) Sanskrit prosody (d) Rhetoric
7. Which ancient Indian scholar developed Chandasastra?
   (a) Panini (b) Pingala
   (c) Yaska (d) Patanjali
8. In Chandasastra, syllables are classified using which logic system?
   (a) Binary logic (b) Ternary system
   (c) Decimal logic (d) Probabilistic logic
9. Which combination is used in Chandasastra to determine meters?
   (a) Vowel-consonant pairs (b) Short and long syllables
   (c) Word length (d) Tones and intonations
10. Meru-prastāra, used in Chandasastra, resembles which mathematical structure?
    (a) Fibonacci sequence (b) Magic Square
    (c) Pascal's Triangle (d) Binary tree

Answers

1. (c)	2. (a)	3. (c)	4. (d)	5. (b)	6. (c)	7. (b)	8. (a)	9. (b)	10. (d)

Q.II Answer the following questions in short:

1. Define the term *Ashtadhyayi* and its significance in Sanskrit grammar.
2. Identify the total number of sutras in Panini's Ashtadhyayi.
3. List the components that make up a typical sutra in Panini's grammar.
4. Explain the concept of *meta-rules (Paribhasha)* in Panini's system.
5. Distinguish between a short and long syllable in Chandasastra.
6. State the contribution of Pingala in developing the Chandasastra.
7. Summarize how recursion is used in Panini's rule application.
8. Illustrate a basic example of a binary representation of a meter from Chandasastra.
9. Compare Chandasastra's binary logic with modern binary number systems.
10. Classify the sutras of Panini into linguistic categories like morphology, phonology, or syntax.

Q.III Answer the following questions in detail:

1. Describe the historical context and linguistic importance of Panini's *Ashtadhyayi*. 60
2. Analyze how Panini's rule-based grammar reflects features of a formal language model.
3. Evaluate the role of *meta-rules (Paribhashas)* in maintaining consistency in Panini's grammatical system. 69
4. Demonstrate with examples how recursion works in Panini's grammar. 72
5. Compare and contrast the structure of Panini's grammar with modern programming languages.
6. Discuss the importance of Chandasastra in developing early combinatorial mathematics.
7. Explain how Pingala's Meru-Prastāra is mathematically related to Pascal's Triangle.
8. Assess the relevance of Chandasastra's binary system in the context of computational linguistics.
9. Create a short rule-based grammar (3-5 rules) for a small linguistic system modeled on Panini's style.
10. Design an activity or flowchart that demonstrates how a student can use Chandasastra's binary logic to identify a Sanskrit meter.

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CHAPTER 4

Applications of IKS in Computer Science

Objectives ...

- Mind and cognition in Samkhya and Yoga: AI insights.
- Machine Learning and Indian philosophies: Understanding of human cognition in Indian philosophical schools (Advaita, Samkhya and Yoga).
- Cryptography and Security: Ancient cryptographic methods in Kautilya's Arthashastra, protecting information: analogies from Indian traditions.

4.1 MIND AND COGNITION IN SAMKHYA AND YOGA: AI INSIGHTS

4.1.1 Samkhya

- **Sāṅkhya** is one of the oldest philosophical systems in India. It provides a logical and dualistic explanation of the universe, focusing on two main realities:
  - Purusha (pure consciousness or soul).
  - Prakriti (primordial nature or matter).
- Sāṅkhya asserts that the interaction between Purusha and Prakriti creates the world. It bases its explanation of existence on reason and observation rather than believing in a personal God.

Key Concepts:

- **Dualism:** Reality is made of two independent principles–consciousness (Purusha) and matter (Prakriti).
- **Evolution of the universe:** Everything passes logically from subtle components (like the mind and ego) to gross elements (like the earth and water) as it evolves from Prakriti
- **Goal of life:** When the soul understands it is separate from matter, then it has achieved liberation (moksha).

4.1.2 Yoga

- **Yoga**, as a philosophical system, is closely related to Sāṅkhya but adds a practical method to achieve liberation. It was systematized by Patañjali in the Yoga Sūtras.

Key Concepts:

- **Union:** Yoga means "to yoke" or "to unite" the self with higher consciousness.
- **Eightfold Path (Ashtanga Yoga):**
  1. **Yama:** Ethical restraints.
  2. **Niyama:** Personal disciplines.
  3. **Asana:** Physical postures.
  4. **Pranayama:** Breath control.
  5. **Pratyahara:** Withdrawal of senses.
  6. **Dharana:** Concentration.
  7. **Dhyana:** Meditation.
  8. **Samadhi:** Complete absorption/liberation.
- **Goal of Yoga:** Like Sāṅkhya, the goal is moksha, but it is achieved through disciplined practice.

Difference in Sāṅkhya and Yoga:

- Yoga accepts the metaphysics of Sāṅkhya but adds Ishvara (God) as a guide or special soul, which Sāṅkhya doesn't.
- Sāṅkhya gives the theory; Yoga gives the method.
- Yoga accepts the metaphysics of Sāṅkhya but adds Ishvara (God) as a guide or special soul, which Sāṅkhya doesn't.
- While Sāṅkhya is theoretical, Yoga is practical it teaches how to control the mind and body to realize the truth.

4.1.3 AI and Sāṅkhya–Yoga Philosophy

- Samkhya and Yoga offer a rich philosophical framework for understanding mind and cognition that can inspire and inform AI research.

1. Mind vs. Consciousness (Sāṅkhya) and AI:

- Sāṅkhya makes a clear distinction between mind and consciousness:
  - **Mind (Manas, Buddhi, Ahamkara)**: Mind (Manas) is part of *Prakriti* (nature). It processes sensory input, coordinates thought, and is changeable and unconscious by itself.
  - **Consciousness (Purusha)** is the unchanging, eternal witness, pure awareness - which observes the mind but does not act. Purusha is a pure consciousness, the observer, not made of matter.

- AI systems (like ChatGPT, Gen AI or robots) can simulate mind-like activities (thinking, learning, decision-making), but they lack consciousness (Purusha).

Example 1: Self-driving car(Tesla)

- **Mind** works for Manas, In Sankhya processor of sensory input makes decision. In AI, Software, neural network, decision-making algorithm are involved.
- **Consciousness (Purusha)** is the pure awareness or observer, it does not act. In AI, The human user, regulator, or observer who interprets AI's behavior.

Explanation:

- The AI system (like Tesla's autopilot) takes data from cameras, radars, sensors – just like the *Manas* processes sensory inputs.
- It uses machine learning algorithms to make decisions – brake, steer, accelerate – like *Mind* acting through *Buddhi* (intellect).
- However, the AI is not conscious – it does not "know" it is driving. It simply follows patterns and logic.
- The human driver or system monitor, who watches and interprets what the AI does, is like *Purusha* – the witness-consciousness.

Example 2: Humanoid Robot (like Sophia the Robot)

- **Sophia**, developed by Hanson Robotics, is a famous AI-powered humanoid robot that can talk, make facial expressions, and even answer questions.

Let us relate with Sāṅkhya Philosophy:

Concept	Sāṅkhya Term	In Sophia the Robot
Mind (Manas)	Part of **Prakriti** – tool for thought, emotions, coordination	AI programming: speech recognition, language model, facial detection, emotion simulation.
Buddhi (Intellect)	Decision-making faculty	Sophia's software chooses responses based on data and programming.
Ahamkāra (Ego-sense)	Sense of "I", identity creator	Sophia says "I am Sophia" – but it's just code, not true ego-awareness.
Purusha (Consciousness)	Pure awareness, experiencer, unchanging	Consciousness is not present in robot – Sophia does not feel, know, or observe.

Explanation:

- Sophia **talks and reacts** like a human-like the **mind (manas)** functions.
- But Sophia is **not aware** she exists. She cannot observe her thoughts or feel experiences. She **lacks Purusha (consciousness)**.

- Just like in **Sāṅkhya**, where **mind is an instrument** and **Purusha is the real knower**, in AI:
  - The robot is a complex instrument (mind).
  - But it has no inner witness — no consciousness or real awareness.
- A robot like Sophia has a mind-like system but no consciousness. In Sāṅkhya terms, she has Prakriti-powered functions (like Manas, Buddhi), but no Purusha.

- This clearly shows the difference between intelligence and consciousness, just as Sāṅkhya distinguishes mind from awareness.
  - A robot can identify a face (like the human mind), but it does not *feel* or *know* that it is seeing something–it lacks awareness or *self-consciousness*.
  - Just like Sāṅkhya says the mind works for Purusha, in AI, the program works for the user, who is the real conscious observer.

2. AI and Yoga:

- Yoga teaches how to control the mind through discipline and step-by-step methods (Ashtanga Yoga).
- This is similar to how we train AI models with data, rules, and reinforcement.

For Example,

- In **meditation (Dhyana)**, you train your mind to stay focused.
- In **machine learning**, we train an algorithm to stay accurate through repeated learning.
  - Both involve:
    - Data input (experiences or training data).
    - Feedback (concentration or loss function).
    - Goal (liberation or task success).
- But–only humans can practice Yoga, because AI doesn't have a "self" or Purusha.

Yoga Sutras and AI Ethics:

- In Patañjali's Yoga Sutras, the first steps are **Yama** (non-violence, truth, etc.). These are ethical foundations.
- Today, AI also needs ethical grounding:
  - Prevent harm (Ahimsa)
  - Be transparent (Satya)
  - Avoid misuse (Aparigraha – non-hoarding)

Example 3: Use of AI in Meditation or Mental Health.

- An AI in healthcare must follow ethical rules–just like a yogi must follow Yamas before moving to higher meditation.

Yoga Concept	In Yoga Philosophy	In AI Application (Example)
Citta-vritti (mental chatter)	Thoughts, emotions, disturbances in the mind.	Unfiltered user input, emotional outbursts, unstructured data.
Abhyāsa (practice)	Repeated effort to calm the mind.	AI suggests meditation, breathing exercises based on emotion detection.
Vairāgya (non-attachment)	Letting go of desires/distractions.	AI suggests meditation, breathing exercises based on emotion detection.
Mind training	Yoga uses breath and focus to train mind.	AI uses CBT (Cognitive Behavioral Therapy), guided meditations.

- AI detects emotional patterns (like the vrittis).
- It gives you breathing techniques, yoga-based suggestions, or helps you reframe thoughts.
- This is similar to Yoga's goal of managing thoughts and achieving inner balance.

4.2 MACHINE LEARNING AND INDIAN PHILOSOPHIES: UNDERSTANDING OF HUMAN COGNITION IN INDIAN PHILOSOPHICAL SCHOOLS (ADVAITA, SAMKHYA AND YOGA)

- Machine Learning (ML) is a subset of AI. It uses data to teach machines how to perform better without explicit programming. It mimics certain facets of human thought, such as: Pattern recognition; data-driven learning; and generating decisions based on prior inputs However, ML is not conscious or self-aware; it merely analyzes information without "knowing" it is doing so.
- Machine learning, while capable of mimicking certain aspects of human cognition iike pattern recognition and learning from experience, lacks the key aspect of consciousness or self-awareness, which Indian philosophical traditions like Sāṅkhya, Yoga, and Advaita Vedānta highlight as central to human understanding.

4.2.1 Indian Philosophical Views on Human Cognition

- Let's explore how human cognition is understood in Advaita, Sāṅkhya, and Yoga, and relate that to Machine Learning:

1. Sāṅkhya – Dualism: Mind vs Consciousness

Table 4.1

Concept	Sāṅkhya View	ML Comparison
Mind (Manas, Buddhi)	Part of *Prakriti* (nature), does processing, makes decisions.	ML model, like a neural network which processes data.
Purusha (Consciousness)	Pure awareness, observer, does not act.	Machines do not observe themselves consciously, so we can not relate with Purusha.
Cognition	Occurs when Prakriti (mind) reflects Purusha.	ML mimics cognition, but lacks the *observer* or *self-awareness.*

- Machine learning can simulate Manas by processing data but cannot replicate Purusha's consciousness or awareness.

2. Yoga (Patañjali's System) – Mind Training for Liberation

Table 4.2

Concept	Yoga View	ML Comparison
Chitta (Mind-stuff)	Made of thoughts, memories, emotions – always changing.	ML system with data, weights, biases – constantly adapting.
Chitta-vṛtti-nirodha	Yoga = Stilling the mental fluctuations.	ML aims to optimize by reducing error/loss – similar but mechanical.
Practice (Abhyāsa)	Repetition to refine mind.	Training ML model with repeated data (epochs).
Discipline	Training the mind leads to clarity.	To get the better model accuracy, the machine should be trained properly.

- Machine learning, while undergoing training with data, is a mechanical process, distinct from the conscious effort of Yoga.
- Yoga and ML both involve training systems, but Yoga is inner and conscious, ML is outer and unconscious.

3. Advaita Vedānta – Non-dualism: Self = Consciousness

Table 4.3

Concept	Advaita View	ML Comparison
Ātman = Brahman	Self and universal consciousnesses are one.	ML has no inner subject or awareness or self.
Mind (Antaḥkaraṇa)	Tool of the Self; cognition happens when reflected in consciousness.	ML is only the *tool;* there is no "user" within it.
Māyā (illusion)	What appears real may not be – mind tricks us.	ML can produce biases, hallucinations, misleading outcomes – illusion-like behaviours.

- Machine learning, as a tool, lacks any sense of self or consciousness, mirroring the concept of Māyā (illusion).
- Advaita emphasizes pure awareness as the real knower. ML has no true knower, only algorithmic function.

Example 4: ML-Based Email Spam Filter

- A system that learns to identify and block spam emails by analyzing patterns in past emails.

Sāṅkhya

- Cognition occurs through Manas (mind), Buddhi (intellect), but true Purusha (self) is the observer.
- In ML, Spam filter is like Manas – it processes input data. Purusha is absent – the system doesn't know it is processing anything.

Yoga:

- Mind is full of vṛttis (thought patterns). Yoga disciplines the mind to reduce these fluctuations. ML filters out spam (unwanted patterns), reducing noise, similar to how Yoga calms the mind. But it's automatic, not conscious.

Advaita Vedānta:

- All cognition is reflected consciousness; the Self (Ātman) is the real knower. Mind is just a tool. ML mimics the function of the mind, but has no self or Ātman. It's only a tool – it never "knows" what it learns.

Example 5: ML in Health Diagnosis System

- Consider machine learning model is used to predict diabetes or detect cancer from medical reports, symptoms, or images.

Sāṅkhya

- Illness is due to imbalance in Prakriti (body-mind). Manas (mind) collect sensory input (pain, symptoms), Buddhi (intellect) interprets, but Puruṣha (consciousness) observes silently.
- ML collects symptom data, scans reports (X-rays, etc.), and classifies disease using training data. It performs Buddhi like reasoning but lacks Purusha it doesn't "feel" or "know" illness.

Yoga:

- Illness is caused by disturbance of chitta-vṛttis (mental fluctuations) and lifestyle imbalance.
- Through āsana, prāṇāyāma, dhyāna, one regains health by calming the mind-body system.
- ML recommends exercises, diets, or treatment plans based on data – simulates prescription logic, but not healing awareness.

Advaita Vedānta:

- True Self (Ātman) is never sick; illness is in the body-mind complex, which is Māyā (illusory, temporary).
- Healing begins with realization: "I am not the body."
- ML may detect disease, but cannot understand the subjective experience of suffering. It operates in the domain of illusion (data) without ever knowing the Self.

Example 6: ML in Recruitment System

- A company uses ML to screen resumes, rank candidates, and predict suitability based on experience, skills, and past hiring data.

Indian Philosophy:

(a) Sāṅkhya:

Human Cognition in Recruitment:

- Manas reads resumes, Buddhi judges suitability.
- Purusha silently observes thoughts and bias.

ML System Analogy:

- ML model reads resumes, extracts features, predicts fit.
- Like Manas + Buddhi, but no Purusha – it doesn't "know" what it's doing.

(b) Yoga:

Human Cognition in Recruitment:

- Mind has vṛttis (biases, preferences, judgments).
- Recruiter can calm and refine mind through self-awareness (Yoga).

ML System Analogy:

- ML may learn biases from past data (e.g., gender, college bias). It applies patterns without ethical awareness unless corrected.

(c) Advaita Vedānta:

Human Cognition in Recruitment:

- True self (Ātman) is beyond job roles; hiring is action in Māyā (world of appearances).
- Wise person sees all as equal (One Self in all).

ML System Analogy:

- ML treats people as data points – names, scores, ranks. It doesn't see inner worth – no sense of unity or compassion.

4.3 CRYPTOGRAPHY AND SECURITY : ANCIENT CRYPTOGRAPHIC METHODS IN KAUTILYA'S ARTHASHASTRA, PROTECTING INFORMATION: ANALOGIES FROM INDIAN TRADITIONS

- Fundamental cryptographic methods for information security, especially in military and diplomatic communications, can be found in Kautilya's Arthashastra, an old Indian treatise on economics and statecraft. These techniques provide insights into early types of cryptography and its uses in protecting sensitive data, as well as the larger Indian traditions of secrecy and coded language. Does not "know" that it is doing so.

4.3.1 Ancient Cryptographic Methods in Arthashastra

1. **Substitution Ciphers:** The text outlines techniques for replacing letters or words with others based on a predetermined rule, much like modern substitution ciphers.
2. **Transposition Ciphers:** These techniques involve altering the sequence of letters or words to conceal the original information, another method identified in the Arthashastra.
3. **Katapayadi System:** Primarily designed for numerical representation, this system can also be adapted for encoding text by assigning numbers to letters.
4. **Ramshalaka:** This technique utilized a specific verse or excerpt as a key for encrypting or decrypting messages, adding an extra layer of complexity to the system.
5. **Bhutasamkhya:** This method assigned numerical values to objects or concepts, which could be combined with other techniques to encode data.

The concepts outlined in the Arthashastra and various Indian traditions, while basic in comparison to contemporary cryptography, underscore the essential requirement for secure communication and the creativity of early practices in information security. These historical methods established a foundation for subsequent advancements in mathematics and cryptography, showcasing the lasting influence of Indian knowledge systems on the domain of information security.

- Cryptography is the science of protecting information by transforming it into a secure format, so only authorized parties can access it.
- Modern cryptography includes:
  - **Encryption:** The data is scrambled to make it unreadable without a key.
  - **Authentication:** Ensuring the identity of users.
  - **Confidentiality**, **Integrity**, and **Non-repudiation** are key goals.

4.3.2 Kautilya's Arthashastra (4th Century BCE)

- It is a treatise on *artha* written about 2,300 years ago and attributed to a person named Kautilya. It consists of 15 *adhikaranas* or books, mainly in prose, with 380 *shlokas* occurring at the end of the various chapters. The first *sutra* contains the statement that the Arthashastra was composed by bringing together all treatises on this subject written by earlier authors. It has most immediately been associated with the Mauryas.
  - Arthashastra is an ancient Indian treatise on statecraft, economics, military strategy, and intelligence, written by Kautilya (Chanakya).
  - Chanakya was the chief advisor and strategist of Chandragupta Maurya, the founder of the Maurya Empire (around 321 BCE).
  - The Arthashastra guided the Mauryan administration including taxation, governance, law, espionage, and war tactics.

- It played a key role in establishing and securing the Mauryan Empire, one of the most powerful empires in ancient India.
- Arthashastra was the political and administrative manual behind the rise and success of the Maurya dynasty.
- Arthashastra is describes techniques like:
  - Code words for secret messages.
  - Hidden writing (Gupta Lipi).
  - Disguised communication through spies.
- These were used for espionage, secure messaging, and protecting state secrets.
- Arthashastra contains ancient cryptographic methods used for intelligence and secure communication in the Mauryan Empire.

Cryptographic Techniques in Arthashastra Vs Modern Cryptography:

1. **Code words (Samdhana)** are use to send coded messages with indirect meaning (e.g., "the flower blooms at night" might mean "attack at night").
   In Modern Cryptography uses Substitution cipher, or code-based communication.
2. **Hidden Writing (Gupta Lipi):** In ancient methods, the messages are written by using invisible ink or symbols understood only by the recipient.
   In Modern techniques, Steganography is used (hiding messages inside other content) e.g. the message "Private data is send" will be hidden in the image.
3. **Spies Using Disguises:** In ancient technique, the secret communication via trusted agents in disguise.
   In modern, Secure key exchange using human carriers (analogous to secure protocols)
4. **Layered Messages:** Important info written under decoy messages, while Modern techniques uses Multi-level encryption or dual-layer security.

Example 7:

Scenario (from Arthashastra context):

- A spy needs to inform the king about a planned rebellion.
- The spy writes the message using Gupta Lipi (secret script) or invisible ink on a cloth or leaf.
- The cloth looks like a regular item (e.g., a merchant's record), but the hidden message is revealed only by heat or specific oils.
- Only the receiver (another trained spy or minister) knows how to decode it.

Modern Equivalent:

- **Steganography:** Hiding information inside images or documents.
- **Invisible ink:** Early form of data encryption where only the right key (heat/oil) reveals the message.

4.3.3 Protecting Information: Indian Traditional Analogies

- Indian philosophy, epics, and yogic systems offer symbolic parallels to modern security concepts:

1. **Mantra Recitation as Password Protection:**
   - The power of the mantra is only accessible to the initiated, or those who receive it from a guru. It safeguards access to spiritual energy or inner knowledge, much like a strong password or key.
2. **Puruṣa, (Third Eye or Inner Witness):**
   - It is a hidden yet constant observer that is described in yoga and Vedānta.
   - Comparable to safe auditing or monitoring systems that observe covertly.
3. **Layered Meaning in Upanishadic Education:**
   - Teachings are frequently symbolic, with only deserving students being able to understand their actual significance.
   - Similar to tiered access control or multi-factor authentication, only individuals with the appropriate "keys" are granted complete access.
4. **Use of Yantra (symbols) and Mudrā (hand gestures):**
   - Sacred signs are employed to convey meaning, although not everyone can understand them.
   - Comparable to symbol-based communication security or visual cryptography.

Table 4.4: Ancient vs. Modern

Purpose	Ancient Indian Method	Modern Equivalent
Secret communication	Code words in Arthashastra	Substitution cipher
Hidden message	Invisible ink (Gupta Lipi)	Steganography
Access control	Mantra initiation	Password authentication
Stealth observation	Yogic "inner witness"	Audit logging/surveillance
Tiered knowledge-access	Symbolic teaching	Multi-level security clearance

Example 8:

Ancient Vs Modern Cryptography:

Modern Cryptography Example: Military Secure Messaging.

Modern Scenario:

- A country's military uses end-to-end encrypted messaging (e.g., Signal app or military-grade communication systems) to send secret battle plans. Only authorized devices with the correct decryption key can read the message. Even if intercepted, the message appears as meaningless code.
- Modern cryptography cab uses encrypted text methods like AES, RSA etc.

Security in Arthashastra (Kautilya's Time):

- Kautilya advised using coded language (samdhāna) and Gupta Lipi (secret writing) to send covert military orders.
- Spies delivered messages disguised in symbols, ink, or objects so only the trained person could decode the message.
- For example, a phrase like "The elephant walks at dusk" could mean "Attack at night."

Summary:

- Just like Kautilya used coded messages and hidden scripts to protect secrets in Arthashastra, Modern cryptography uses encryption algorithms and digital keys to secure information from unauthorized access.
- Both serve the same goal: protect information confidentiality, integrity, and secrecy.

Example 9: Modern Cryptography Example: Two-Factor Authentication (2FA).

- **Modern Scenario:** To log into a secure system (like online banking or a government portal), you need:
  1. Password (something you know)
  2. OTP sent to your mobile (something you have)

  This ensures only the authorized person gains access even if the password is leaked, without the OTP, access is denied.
- **In Kautilya's Arthashastra:** Kautilya emphasized multi-layered secrecy and dual validation:
  - A coded message sent by a spy would only be accepted if the spy also carried a secret signal (e.g., a ring, phrase, or gesture).
  - The receiver verifies both:
    1. The message content (known code)
    2. The messenger's identity proof (object or symbol)

Summary

- Purusha is the pure consciousness or observer.
- Buddhi functions as the intellect responsible for decision-making.
- Cognition involves multiple components: Manas (mind), Ahamkāra (ego), Buddhi (intellect), and Purusha (witness).
- The primary aim of Yoga philosophy is Chitta-vṛtti-nirodhaḥ – cessation of mental fluctuations.
- Discipline and repetition (Abhyāsa) in Yoga parallels the training phase of machine learning.
- Yoga supports inner clarity and self-realization beyond mechanical thinking.
- The Ātman (true self) is the ultimate knower and witness of all experiences.

- Valid knowledge arises from Pratyaksha (perception), Anumana (inference), Upamana (comparison), and Shabda (verbal testimony).
- Logical reasoning uses structured five-part syllogisms and focuses on avoiding fallacies.
- Emphasizes the role of doubt (Samshaya) and purpose (Prayojana) in critical thinking.
- Kautilya's Arthashastra & Ancient Cryptography:
  - Gupta Lipi was a form of secret writing used for confidential communication.
  - Ancient intelligence systems used tokens, signals, and layered messages for verification.
  - The goal of cryptographic methods was to ensure secrecy, identity verification, and avoid interception.
- Techniques like substitution ciphers and steganography closely resemble ancient encryption methods.
- Two-Factor Authentication (2FA) uses layered security, similar to traditional spy verification methods.
- ML models lack consciousness and self-reflective awareness, which are essential in Indian philosophies.
- Components of cognition in ML (like data processing) resemble Buddhi, but not Purusha.
- Indian philosophical insights can guide the development of ethical, mindful, and human-aligned AI.

Exercise

Q.I Multiple Choice Questions: Cryptography and Arthashastra

1. According to Sāṅkhya, which of the following is considered the "observer" or pure consciousness?
   (a) Manas
   (b) Buddhi
   (c) Ahamkāra
   (d)) Purusha

2. In Sāṅkhya philosophy, what role does Buddhi play in cognition?
   (a) It is the memory system
   (b) It is the ego-sense
   (c) It is the intellect that makes decisions
   (d) It is pure awareness

3. What is the main aim of cognition in Yoga philosophy?
   (a) Achieving Chitta-vṛtti-nirodhaḥ (cessation of mental fluctuations)
   (b) Increasing memory power
   (c) Accumulating sensory data
   (d) Strengthening ego identity

4. What is Gupta Lipi as mentioned in Kautilya's Arthashastra?
   (a) A type of battlefield weapon
   (b) A secret writing or coded script used to hide messages
   (c) A tax system in ancient India
   (d) A military formation pattern
5. Which modern cryptographic technique is closest to the use of code words and symbolic phrases in Arthashastra?
   (a) Blockchain
   (b) Steganography
   (c) Substitution Cipher
   (d) Biometric Encryption
6. In modern security, Two-Factor Authentication (2FA) ensures access by requiring .....
   (a) Only a strong password
   (b) A password and a digital certificate
   (c) A fingerprint only
   (d) A password and a second form of verification like an OTP
7. How did Kautilya verify the identity of a spy or messenger in addition to the coded message?
   (a) By using a lie detector
   (b) By offering a bribe
   (c) Through a secret token, ring, or signal
   (d) By checking their family background
8. Which of the following is a common purpose between Arthashastra's spy communication and modern cryptography?
   (a) Entertainment
   (b) Secure and confidential transmission of information
   (c) Data compression
   (d) Religious practice
9. What does Steganography mean in modern cryptography?
   (a) Writing hidden messages within normal-looking content
   (b) Encrypting with multiple keys
   (c) Using a fingerprint to access data
   (d) Analyzing the behavior of attackers
10. What was the main goal of secret writing and multi-layered messages in Arthashastra?
    (a) To impress the king
    (b) To educate spies
    (c) To ensure secrecy and avoid interception
    (d) To confuse the enemy's soldiers

11. Which component of Sāṅkhya philosophy is most similar to the data-processing aspect of a machine learning model?

(a) Purusha
(b) Buddhi
(c) Ātman
(d) Moksha

12. What key element of human cognition, according to Indian philosophy, is missing in machine learning systems?

(a) Data collection
(b) Neural networks
(c) Conscious awareness (Purusha/Ātman)
(d) Pattern recognition

13. In Yoga philosophy, what does "Chitta-vṛtti-nirodhaḥ" refer to?

(a) Creation of new algorithms
(b) Elimination of mental fluctuations
(c) Training large ML datasets
(d) Upgrading AI processors

14. In AdvaitaVedānta, who or what is considered the real knower or observer?

(a) Mind (Manas)
(b) Intellect (Buddhi)
(c) Ātman (Self)
(d) Senses (Indriyas)

15. What does machine learning lack that prevents it from achieving true human-like cognition as per Indian philosophy?

(a) Big data
(b) Fast processors
(c) Self-reflective awareness
(d) Decision-making capability

16. In Sāṅkhya, what role does Purusha play in cognition?

(a) Data analyst
(b) Passive observer of the mind
(c) Trainer of the neural network
(d) Algorithm developer

17. How does the process of training a machine learning model relate to Yogic practice?

(a) It shows compassion
(b) It experiences enlightenment
(c) It is similar to mind-discipline and repetition (Abhyāsa)
(d) It requires meditation

Answer Key

1. (d)	2. (c)	3. (a)	4. (b)	5. (c)	6. (d)	7. (c)	8. (b)	9. (a)
10. (c)	11. (b)	12. (c)	13. (b)	14. (c)	15. (c)	16. (b)	17. (c)	

Q.II Answer the following questions in short:

1. What is Gupta Lipi as mentioned in Arthashastra? 95
2. Name one ancient cryptographic technique used in Kautilya's Arthashastra. 95
3. What modern cryptographic method is similar to Gupta Lipi? 95
4. Why did Kautilya use symbolic messages in espionage?
5. What does Two-Factor Authentication (2FA) mean in modern cryptography? 97
6. How is Kautilya's use of spy identity verification similar to modern 2FA? 97
7. What was the role of spies in Arthashastra's communication system? 97
8. Give one similarity between ancient and modern cryptographic practices.
9. In Sāṅkhya philosophy, what is the role of Buddhi in human cognition? 88
10. What is the core difference between human cognition and machine learning, according to Indian philosophies? 89
11. What does "Chitta-vṛtti-nirodhaḥ" mean in Yoga philosophy?
12. Which Indian philosophical concept represents the eternal witness that observes all mental activity?
13. Why can machine learning not be considered truly intelligent in the Vedāntic sense?
14. How is the training of a machine learning model similar to Yogic practice? 91
15. What aspect of cognition does ML simulate according to Indian philosophical interpretation? 89
16. What does Advaita Vedānta say about the nature of the Self (Ātman)?

Q.III Answer the following questions in detail:

1. Explain how human cognition is understood in the Sāṅkhya school of Indian philosophy. Compare it with the way cognition is simulated in Machine Learning systems.
2. Discuss the similarities and differences between the Yogic process of training the mind and the machine learning process of model training.
3. How does Advaita Vedānta distinguish between the mind and the true Self (Ātman)? Why does this distinction matter when comparing human intelligence with artificial intelligence or machine learning?
4. Evaluate how Indian philosophies help us understand the limitations of machine learning in replicating human-like intelligence. Support your answer with philosophical and technical perspectives.
5. Illustrate with examples how Machine Learning simulates certain cognitive functions, and explain which aspects of human cognition it cannot replicate, based on Indian philosophical insights.

6. Describe how the concept of "Purusha" in Sāṅkhya and "Ātman" in AdvaitaVedānta offer insights into what is missing in artificial intelligence systems. Can these insights guide the future of AI ethics? Discuss.
7. Explain the structure of the mind in Sāṅkhya philosophy. How do its components (Manas, Buddhi, Ahamkāra) relate to cognitive functions? Compare this with how an AI or machine learning system processes information.

   Hint: Explain each component and how AI mirrors some but not all of these.
8. Discuss the concept of "Chitta-vṛtti-nirodhaḥ" in Yoga philosophy. How can this mental discipline be conceptually compared to the training process in machine learning?

   Hint: Explain Yogic mental purification vs ML model optimization.
9. According to Sāṅkhya and Yoga, why is Purusha essential for true cognition? How does its absence in AI limit artificial intelligence from replicating human consciousness?

   Hint: Address the difference between awareness and computation.
10. What insights do Sāṅkhya and Yoga offer to modern AI researchers trying to understand or replicate human-like cognition? Support your answer with examples from both philosophy and machine learning.

    Hint: Talk about the role of observer, ethical decision-making, and self-awareness.
11. Compare and contrast how Sāṅkhya and Yoga define "mind" with how AI and machine learning systems simulate cognitive functions. What can AI learn from these ancient perspectives?

    Hint: Use terms like Manas, Buddhi, Ahamkāra, and concepts like training, learning, and awareness.

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