# Predicting stock return volatility using commodity prices and GARCH models

Anshu Chen*

*Department of Economics, Yale University*

E-mail: a.chen@yale.edu

**Abstract**

This paper builds on Goyal (2000)[1] by comparing six GARCH models' volatility forecasts against an empirical volatility proxy. I replicate three of Goyal's models and compare these to GARCH models that use commodity (gold, copper, oil) prices to predict stock returns. Results suggest that the commodity prices do not significantly improve the GARCH models' volatility forecasts, though the gold price coefficient is consistently and significantly negative.

# Introduction

Researchers have devised myriad GARCH models to forecast stock return volatility. Goyal (2000) tested four GARCH models and their GARCH-in-mean versions against a "proxy of monthly volatility calculated using daily [returns] data" (Goyal 2). He also tested several mean specifications and found them insignificant. Recent literature, however, indicates that commodity prices can predict stock returns in the short term (Driesprong, Jacobsen, Maaat 2008;[2] Jacobsen, Marshall, Visaltanachoti 2008[3]). If this is true, the GARCH models that excluded commodity prices would generate biased results. I add commodity prices (copper, gold, oil) to three of Goyal's GARCH models to compare them with the original, both on their ability to forecast volatility and their model coefficients. The rest of this paper will cover, in order: data, methodology, model evaluation, and conclusion.

## Data

This paper uses monthly and daily data from the CRSP value weighted returns (including dividents) from August 1962 to March 2015. The commodity prices came from the World Bank's GEM Commodities series. I adjust these nominal prices for inflation using the St. Louis Federal Reserve's monthly CPI series: "Consumer Price Index: Total All Items for the United States".

Like Goyal, I estimate monthly volatility using daily return data "where $N_t$ is the number of trading days in month t and $r_{it}$ is the return on the ith day of month t" (Goyal 3).

$$\sigma_t^2 = \sum_{i=1}^{N_t} r_{it}^2 + 2\sum_{i=2}^{N_t} r_{it} r_{(i-1)t}$$

To stationarize the three commodity prices, I take logs before taking the second difference. Taking the first difference proves insufficient to eliminate the stochastic trend. Even the second difference shows volatility clustering (see Fig. 1). Still, I chose second over third difference because the third difference produces perfectly correlated gold, copper, and oil price variables.

## Methodology

I replicate three of Goyal's volatility specifications and add the commodity prices as external regressors. I also replicated EGARCH-M and GJR-GARCH-M, but the results did not significantly differ from the ordinary, non-mean versions reported below.

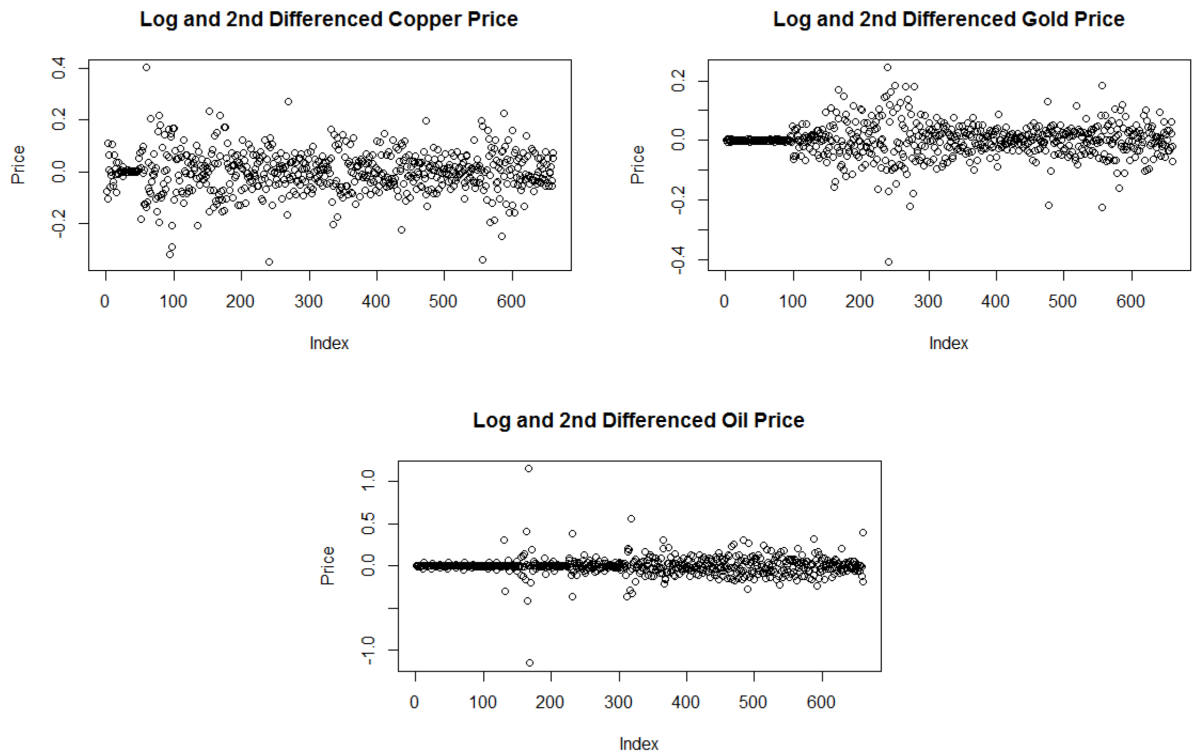


Figure 1: Adjusted prices

## Model specifications

Model notation is that used in Ghalanos'[4] rugarch R package.

### GARCH-M

The ordinary GARCH-M model, without commodity prices

$$r_t = \mu + \delta\sigma_t^2 + \epsilon_t \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

$$\epsilon_t = \sqrt{h_t} e_t \tag{3}$$

Where $e_t \sim$ WN(0,1). Adding commodity prices:

$$r_t = \mu + \gamma_1 c_t + \gamma_2 g_t + \gamma_3 o_t + \epsilon_t \tag{4}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{5}$$

$$\epsilon_t = \sqrt{h_t}e_t \tag{6}$$

Where $c_t$, $g_t$, and $o_t$ are the adjusted copper, gold, and crude oil prices respectively.

**EGARCH**

Nelson (1991)'s exponential GARCH model. z represents standardized innovation.

$$r_t = \mu + \epsilon_t \tag{7}$$

$$ln(\sigma_t^2) = \omega + \alpha z_{t-1}^2 + \theta(|z_{t-1}| - E(|z_{t-1}|)) + \beta ln(\sigma_{t-1}^2) \tag{8}$$

$$\epsilon_t = \sqrt{h_t}e_t \tag{9}$$

Adding commodity prices:

$$r_t = \mu + \gamma_1 c_t + \gamma_2 g_t + \gamma_3 o_t + \epsilon_t \tag{10}$$

$$ln(\sigma_t^2) = \omega + \alpha z_{t-1}^2 + \theta(|z_{t-1}| - E(|z_{t-1}|)) + \beta ln(\sigma_{t-1}^2) \tag{11}$$

$$\epsilon_t = \sqrt{h_t}e_t \tag{12}$$

### GJR-GARCH

Glosten et al. (1993)'s model, which describes asymmetric variance responses to positive and negative shocks. I is an indicator function that takes value 1 for $\epsilon \leq 0$ and 0 otherwise.

$$r_t = \mu + \epsilon_t \tag{13}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \theta I - t - 1\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{14}$$

$$\epsilon_t = \sqrt{h_t} e_t \tag{15}$$

Adding commodity prices:

$$r_t = \mu + \gamma_1 c_t + \gamma_2 g_t + \gamma_3 o_t + \epsilon_t \tag{16}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \theta I - t - 1\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{17}$$

$$\epsilon_t = \sqrt{h_t} e_t \tag{18}$$

## Extended time horizons

I fit two sets of the above models, one using Goyal's timeframe (August 1962 to December 1998) and one using more recent data (August 1962 to March 2015). I expect that the additional observations should generate more accurate estimates.

# Results

To evaluate how well the GARCH models' predicted volatility ($h_t$) forecasts actual volatility ($\sigma_t^2$), I regress the proxy volatility on the GARCH predicted volatility (Goyal 8).

$$\sigma_t^2 = a + bh_t + \mu_t \tag{19}$$

Table 1 presents the results. An asterisk indicates that the model includes commodity prices. The parenthesis contain heteroskedasticity-robust standard errors.

Evidently, the commodity prices do not improve the original models' volatility forecasts.

Table 1: OLS results, 1962-1998

| Model | a | b | $R^2$ |
|---|---|---|---|
| GARCH-M | -0.002<br>(0) | 0.838<br>(0.113) | 0.025 |
| GARCH-M* | -0.001<br>(0) | 0.826<br>(0.116) | 0.023 |
| EGARCH | -0.002<br>(0) | 0.859<br>(0.141) | 0.033 |
| EGARCH* | -0.002<br>(0) | 0.858<br>(0.143) | 0.033 |
| GJR-GARCH | -0.001<br>(0) | 0.648<br>(0.105) | 0.025 |
| GJR-GARCH* | -0.001<br>(0) | 0.629<br>(0.109) | 0.024 |

The a-estimates remain close to zero and the b-estimates close to 1, but $R^2$ never rises above four percent for any of the models–it seems the GARCH model simply cannot capture most of the variation in returns volatility.

I limited the above models to Goyal's original timespan, but I fit the same models to new data up to 2015. B-estimates decrease for all models, suggesting lower predictive power–if $h_t$ were a perfect predictor and $\sigma_t$ a perfect volatility measure, b would equal 1. Conversely, $R^2$ increased for all models after adding new data. This may be a spurious result arising from a larger sample. Or, since the new data contains the financial crisis, it may showcase how commodity price movements precede major market changes. Interestingly, EGARCH* performed slightly better than EGARCH with the new data. See Table 2.

But are the commodity-price models really indistinguishable from the ordinary ones? Upon examining the GARCH coefficients, I found that the gold price's coefficient was always significantly less than zero (See tables 3-5). So gold does seem to predict stock returns, but its absence from the non-commodity models does not significantly bias the volatility estimates.

Table 2: OLS results, 1962-2015

| Model | a | b | $R^2$ |
|---|---|---|---|
| GARCH-M | -0.001<br>(0) | 0.665<br>(0.113) | 0.032 |
| GARCH-M* | -0.001<br>(0) | 0.658<br>(0.080) | 0.031 |
| EGARCH | -0.001<br>(0) | 0.771<br>(0.120) | 0.046 |
| EGARCH* | -0.001<br>(0) | 0.754<br>(0.117) | 0.047 |
| GJR-GARCH | -0.001<br>(0) | 0.599<br>(0.091) | 0.038 |
| GJR-GARCH* | -0.001<br>(0) | 0.583<br>(0.089) | 0.036 |

This may be because return volatility does not follow a GARCH process, so the models are misspecified in any case.

It is interesting that gold was the only significant commodity. Since correlation between the three commodities never rises above 14%, this is likely because gold is widely considered a money equivalent. The public tends to buy gold when anticipating an economic downturn, whereas crude oil is more difficult to transport and store.

Table 3: GARCH-M* coefficients

| Variable | Estimate | Standard error | p-value |
|---|---|---|---|
| $\mu$ | 0.005927 | 0.003474 | 0.087951 |
| $\delta$ | 2.725723 | 1.973489 | 0.167227 |
| $\gamma_1$ | 0.030646 | 0.025254 | 0.224938 |
| $\gamma_2$ | **-0.061512** | **0.030428** | **0.043223** |
| $\gamma_3$ | 0.011667 | 0.018629 | 0.531149 |
| $\omega$ | 0.000074 | 0.000040 | 0.068276 |
| $\alpha$ | 0.125711 | 0.037471 | 0.000794 |
| $\beta$ | 0.840834 | 0.033759 | 0.000000 |

Table 4: EGARCH* coefficients

| Variable | Estimate | Standard error | p-value |
|---|---|---|---|
| $\mu$ | 0.008717 | 0.001500 | 0.000000 |
| $\gamma_1$ | 0.023785 | 0.022134 | 0.282550 |
| $\gamma_2$ | **-0.062319** | **0.027136** | **0.021647** |
| $\gamma_3$ | 0.017683 | 0.015662 | 0.258877 |
| $\omega$ | -0.521980 | 0.254761 | 0.040472 |
| $\alpha$ | -0.119641 | 0.054437 | 0.027963 |
| $\beta$ | 0.918225 | 0.039378 | 0.000000 |
| $\theta$ | 0.231200 | 0.038422 | 0.000000 |

Table 5: GJR-GARCH* coefficients

| Variable | Estimate | Standard error | p-value |
|---|---|---|---|
| $\mu$ | 0.008853 | 0.001513 | 0.000000 |
| $\gamma_1$ | 0.029521 | 0.024142 | 0.221412 |
| $\gamma_2$ | **-0.066164** | **0.031416** | **0.035203** |
| $\gamma_3$ | 0.013493 | 0.020042 | 0.500788 |
| $\omega$ | 0.000099 | 0.000092 | 0.281690 |
| $\alpha$ | 0.048710 | 0.056102 | 0.385260 |
| $\beta$ | 0.827270 | 0.054114 | 0.000000 |
| $\theta$ | 0.148476 | 0.103904 | 0.153010 |

# Areas for future research

This paper reinforces Goyal's conclusion that GARCH models do not accurately forecast stock return volatility. To extend the study, I would have attempted a stochastic volatility estimator and measured its performance against these GARCH models. I would also consider using gold as an external regressor for variance rather than mean–if investors perceive gold as "safe" in uncertain times, increasing gold prices should forecast increasing volatility. By the same principle, Treasury yields might also predict volatility.

## Conclusion

This paper attempts to determine whether GARCH models containing commodity price (gold, copper, oil) regressors accurately forecast equity return volatility. I also test three models from Goyal (2000) on new data to evaluate whether the paper's results remain accurate.

I use Goyal's empirical volatility estimate to evaluate each model, specifically how well each model could predict the estimate. I replicate three of Goyal's models and compare them to the same models with commodity price regressors. I find that the prices do not significantly improve the model forecasts. This may be because GARCH models do not forecast returns volatility well; none of the models had $R^2$ above five percent. So, all of the models are likely misspecified.

Interestingly, every GARCH model containing price regressors generates a significant nonzero coefficient for gold. This supports the hypothesis that GARCH does not predict returns volatility, since adding gold should increase model accuracy. Gold, rather than copper or oil, could be significant for a number of reasons. Since people have historically perceived gold as a safe harbor from economic upheaval, it may play a bigger role in predicting equity returns than either of the two other commodities.

## References


(1) Goyal, A. *Working Paper.* **2000**,

(2) Driedsprong, G.; Jacobsen, B.; Maat, B. *EFA 2005 Moscow Meetings Paper.* **2007**,

(3) Jacobsen, B.; Marshall, B.; Visaltanachoti, N. *EFA 2007 Ljubljana Meetings Paper.* **2007**,

(4) Ghalanos, A. rugarch: Univariate GARCH models. 2014; R package version 1.3-5.