Lecture 12/13: Insertion, Faster Sorts

BT 3051 – Data Structures and Algorithms for Biology

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Insertion Sort

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```python
def InsertionSort(a):
    n = len(a)
    for i in range(1,n):
        for j in range(i, 0, -1):
            if a[j] < a[j-1]:
                a[j], a[j-1] = a[j-1], a[j]
            else:
                break
    return a
```

Can we improve upon this?

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    for i in range(1, n):
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        while pos > 0 and a[pos - 1] > currval:
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Properties

- Stable, $O(1)$ extra space
- $O(n^2)$ comparisons and swaps
- Adaptive: $O(n)$ when nearly sorted
- Very low overhead

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- Elementary sorting algorithm with $O(n^2)$ worst-case time; but
- Insertion sort is the algorithm of choice
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  - when the problem size is small (because it has low overhead)
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Overview

Sorting Algorithms

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Bubble Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
Insertion Sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n)$	yes
?	$\Theta(n \lg n)$	$\Theta(n \lg n)$	$\Theta(n \lg n)$	no
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?	$\Theta(n \lg n)$			yes

Divide and Conquer

Algorithm Design: Divide and Conquer

- One of the important algorithm design strategies
- [Unfortunately] named after Roman (then British!) political strategies
  - Divide your enemies (develop distrust)
  - Conquer them individually (easier)
  - Combine the states together(?)

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Divide, Conquer and Combine!

Three major steps

- Split a problem into multiple sub-problems
- Solve individual sub-problems (independently)
- Combine solutions to individual sub-problems

- Huge number computational problems can be solved efficiently using divide-and-conquer
- First avenue of search for an efficient algorithm, once a brute-force solution is understood
- Divide-and-conquer algorithms are typically recursive — since the conquer part involves invoking the same technique on a smaller sub-problem
- Analysis of the running times of recursive programs is ~tricky
- When merging takes less time than solving the two subproblems, we get an efficient algorithm!

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Divide and Conquer:
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- Efficiency
- Parallelism
- Memory access patterns

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Merge Sort
Algorithm

```
def MergeSort(a):
    n=len(a)

    if(len(a)==1):
        return a
    else:
        return merge(MergeSort(a[:n//2]),MergeSort(a[n//
                                                      2:]))
```

Where's all the work happening?

Merge Sort

merge()

```
def merge(a,b):
    c=[None]*(len(a)+len(b))
    i=j=k=0
    while (i<len(a) and j<len(b)):
        if a[i]<b[j]:
            c[k]=a[i]
            i+=1
        else:
            c[k]=b[j]
            j+=1
        k+=1
    if (i<len(a)):
        c[k:]=a[i:]
    else:
        c[k:]=b[j:]
    return c
```

Merge Sort
Time Complexity

$T(n) = 1, \text{if } n = 1$

- Time to `sort` the first half of the array
- Time to `sort` the second half of the array
- Time to `merge` the sorted arrays

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Recurrence Equation

$$T(1) = 1$$
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Can you see a pattern?

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$$T(n)/n = lg\, n + 1$$

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Can you see a pattern?

$$T(n)/n = \lg n + 1 \Rightarrow T(n) \in \Theta(n \lg n)$$

Quick Sort

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- Merge sort is very good, but does not operate in place!
- Quick sort is another divide-and-conquer algorithm
- Jon Bentley's "*Three Beautiful Quicksorts*" :
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Quick Sort

- Central idea: Partition the array
- About a pivot (choice is critical!)
- All elements less than the pivot go into the *left subarray*
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- Pivot fits in between — in its place in the sorted array!
- Quicksort either half and we have the answer …
- What is the `combine()` procedure here?

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Self-assessment Exercise: Read about heaps and heap-sort ...

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Many hybrid algorithms exist, e.g. quick + heap etc.

Why is Quick Sort so popular?

- *"When Quicksort is implemented well, it is typically 2-3 times faster than mergesort or heapsort!"* (Steven Skiena)
- How do we compare two $\Theta(n \lg n)$ algorithms?
- Since the difference between the two programs will be limited to a multiplicative constant factor, details of how you program each algorithm *will make a big difference*!
- Also, operations in the innermost loop are simpler
- Worst-case of quick sort is still $\Theta(n^2)$, but ...
- If you shuffle the input array, "If you give me random input data, quicksort runs in expected $\Theta(n \lg n)$ time."
- Instead, if you pick a pivot at random, "With high probability, randomized quicksort runs in $\Theta(n \lg n)$ time"
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Why is Quick Sort so popular?

- "*When Quicksort is implemented well, it is typically 2-3 times faster than mergesort or heapsort!*" (Steven Skiena)
- How do we compare two $\Theta(n \lg n)$ algorithms?
- Since the difference between the two programs will be limited to a multiplicative constant factor, details of how you program each algorithm *will make a big difference*!
- Also, operations in the innermost loop are simpler
- Worst-case of quick sort is still $\Theta(n^2)$, but ...
- If you shuffle the input array, "If you give me random input data, quicksort runs in expected $\Theta(n \lg n)$ time."
- Instead, if you pick a pivot at random, "With high probability, randomized quicksort runs in $\Theta(n \lg n)$ time"
- Worst-case of quick sort can be conquered by a randomisation!

Quick Sort

Importance of Randomisation (Steven Skiena)

- Since the time bound how does not depend upon your input distribution, this means that unless we are extremely unlucky (as opposed to ill-prepared) we will certainly get good performance
- Randomisation is a general tool to improve algorithms with bad worst-case but good average-case complexity
- The worst-case is still there, but we almost certainly won't see it

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- Bucket sort
- Radix sort

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