Algorithm 3.1 Algorithm for finding nodes reachable from X given \boldsymbol{Z} via active trails

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Procedure Reachable (
                 // Bayesian network graph
         X
                 // Source variable
         \boldsymbol{Z}
                // Observations
1
            // Phase I: Insert all ancestors of Z into A
2
         m{L} \leftarrow m{Z}
                        // Nodes to be visited
3
         A \leftarrow \emptyset
                       // Ancestors of Z
4
         while L \neq \emptyset
5
            Select some Y from L
            L \leftarrow L - \{Y\}
6
7
            if Y \notin A then
8
               L \leftarrow L \cup Pa_Y // Y's parents need to be visited
9
            A \leftarrow A \cup \{Y\} // Y is ancestor of evidence
10
11
            // Phase II: traverse active trails starting from X
12
         \boldsymbol{L} \leftarrow \{(X,\uparrow)\}
                               // (Node, direction) to be visited
13
         V \leftarrow \emptyset // (Node, direction) marked as visited
         R \leftarrow \emptyset
14
                       // Nodes reachable via active trail
15
         while L \neq \emptyset
16
            Select some (Y, d) from L
            \boldsymbol{L} \leftarrow \boldsymbol{L} - \{(Y,d)\}
17
            if (Y, d) \notin V then
18
              if Y \notin \mathbb{Z} then
19
20
                  R \leftarrow R \cup \{Y\} // Y is reachable
               V \leftarrow V \cup \{(Y,d)\} // Mark (Y,d) as visited
21
22
               if d = \uparrow and Y \notin \mathbf{Z} then // Trail up through Y active if Y not in \mathbf{Z}
23
                  for each Z \in Pa_V
24
                    L \leftarrow L \cup \{(Z,\uparrow)\} // Y's parents to be visited from bottom
                  for each Z \in Ch_Y
25
                     L \leftarrow L \cup \{(Z,\downarrow)\} // Y's children to be visited from top
26
27
               else if d = \downarrow then // Trails down through Y
28
                  if Y \notin \mathbb{Z} then
29
                        // Downward trails to Y's children are active
30
                     for each Z \in Ch_V
                       \boldsymbol{L} \leftarrow \boldsymbol{L} \cup \{(Z,\downarrow)\}
31
                                                  // Y's children to be visited from top
                  if Y \in A then // v-structure trails are active
32
33
                     for each Z \in Pa_V
                       L \leftarrow L \cup \{(Z,\uparrow)\} // Y's parents to be visited from bottom
34
35
      return R
```