10. Chip has discovered an error in the programming of the game. Before a triangle appeared, a player selected an angle with measure 100° and the computer selected 82° for a different angle measure. Explain how Chip knew there was an error.

11. The measures of the three angles in a triangle are $x°$, $(2x + 4)°$ and $(2x - 9)°$.

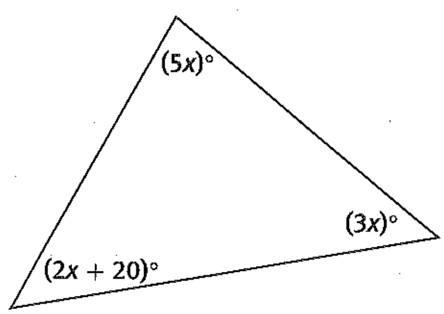
a. Write an equation based on the relationship between the three angle measures and then solve for x.

b. Determine the measures of the three angles of the triangle.

Check Your Understanding

12. If one of the acute angles of a right triangle has a measure of 22°, calculate the measure of the other acute angle.

13. Suppose the measures of the angles in a triangle are given in the figure. Write an equation, solve for x, and determine the measure of each angle.



LESSON 17-1 PRACTICE

In Items 14 and 15, the measures of two angles of a triangle are given. Find the measure of the third angle of the triangle.

14. 23°, 78°

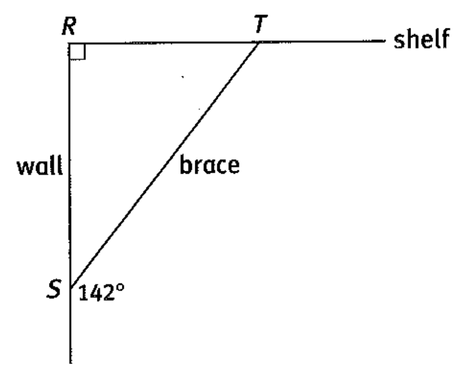
15. 105°, 40°

16. The measures of the three angles in a triangle are $(4x)°$, $(3x - 3)°$, and $(5x + 3)°$. Write an equation, solve for x, and determine the measure of each angle.

17. In $\triangle ABC$, $\angle A$ and $\angle B$ have the same measure. The measure of $\angle C$ is twice the measure of $\angle A$. Find the measures of the angles in the triangle.

18. Eliana claimed that she drew a triangle with two right angles. Draw a sketch of such a triangle or explain why it is not possible.

19. **Model with mathematics.** Brian is building a brace for a shelf. The figure shows the plans for the brace.



a. Based on the information given in the figure, is it possible to determine the three angles of $\triangle RST$? If so, find the measures. If not, explain why not.

b. Brian wants to know the measure of the obtuse angle formed by the brace and the shelf. Explain how he can determine this.

Learning Targets:

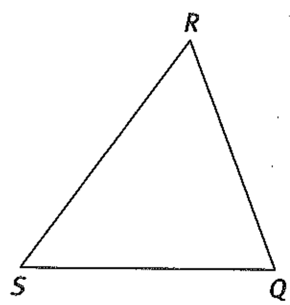
- Describe and apply the relationship between an exterior angle of a triangle and its remote interior angles.
- Describe and apply the relationship among the angles of a quadrilateral.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Look for a Pattern, Visualization, Create Representations, Think-Pair-Share

An ***exterior angle*** of a triangle is formed by extending a side of the triangle. The vertex of the exterior angle is a vertex of the triangle. The sides of the exterior angle are determined by a side of the triangle and the extension of the adjacent side of the triangle at the vertex.

1. Use $\triangle SRQ$ below.

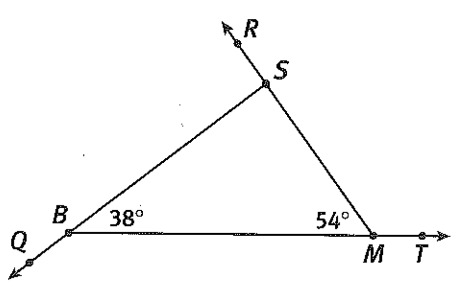
a. Extend side $\overline{SQ}$ of the triangle by drawing $\overrightarrow{SP}$ through point Q to create exterior angle RQP.



b. Describe the relationship between the measures of $\angle RQP$ and $\angle RQS$.

2. An exterior angle has been drawn at each of the three vertices of $\triangle SBM$.

a. Determine the measure of each of the three exterior angles.



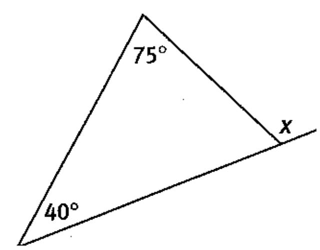
b. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles.** Name the two remote interior angles for each exterior angle of $\triangle SBM$.

Exterior Angle	Two Remote Interior Angles
$\angle SMT$	
$\angle RSB$	
$\angle QBM$	

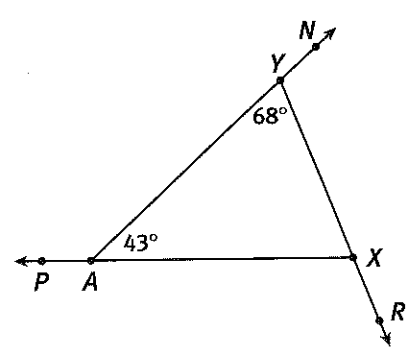
c. Chip claims that there is a relationship between the measure of an exterior angle and its remote interior angles. Examine the measures of the exterior angles and the measures of their corresponding remote interior angles to write a conjecture about their relationship.

Check Your Understanding

3. Determine the value of x.



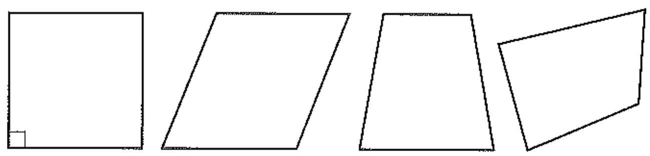
4. Determine the measure of each of the exterior angles of $\triangle YAX$.



5. The measures of two interior angles of a triangle are 75° and 65°. Determine the measure of the largest exterior angle.

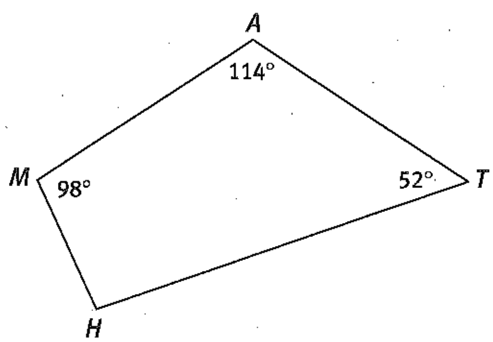
6. Now consider quadrilaterals.

a. Draw a ***diagonal*** from *one* vertex in each quadrilateral.

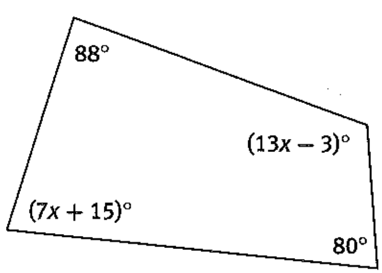


b. Construct viable arguments. What is the sum of the measures of the interior angles in any quadrilateral? Explain your reasoning.

7. Find the unknown angle measure in quadrilateral *MATH*.



8. Determine the value of x in the quadrilateral.

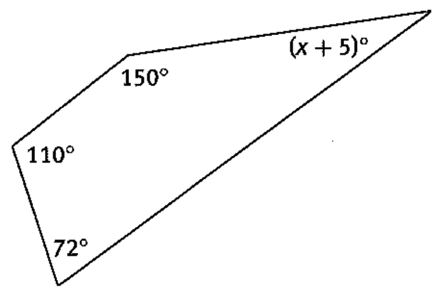


MATH TERMS

A **diagonal** of a polygon is a line segment connecting two nonconsecutive vertices.

Check Your Understanding

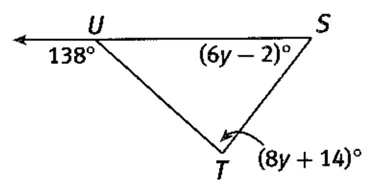
9. Determine the value of x.



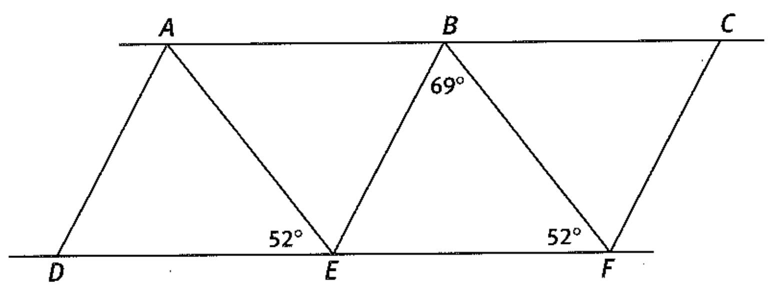
10. In quadrilateral $RSTU$, all of the angles of the quadrilateral are congruent. What can you conclude about the angles? What can you conclude about the quadrilateral?

LESSON 17-2 PRACTICE

11. Determine the value of y. Then find $m\angle S$ and $m\angle T$.



12. A portion of a truss bridge is shown in the figure. Explain how to determine the measure of $\angle AEB$.



13. A quadrilateral contains angles that measure 47°, 102°, and 174°. What is the measure of the fourth angle of the quadrilateral?

14. In quadrilateral $DEFG$, $\angle D$ is a right angle. The measure of $\angle E$ is half the measure of $\angle D$. The measure of $\angle F$ is three times the measure of $\angle E$. Sketch the quadrilateral and label the measure of each angle.

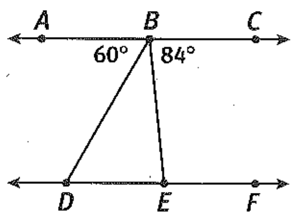
15. **Make use of structure.** Can an exterior angle of a triangle ever be congruent to one of its remote interior angles? Justify your answer.

ACTIVITY 17 PRACTICE

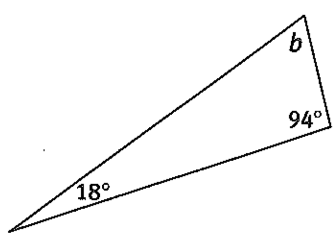
Write your answers on notebook paper. Show your work.

Lesson 17-1

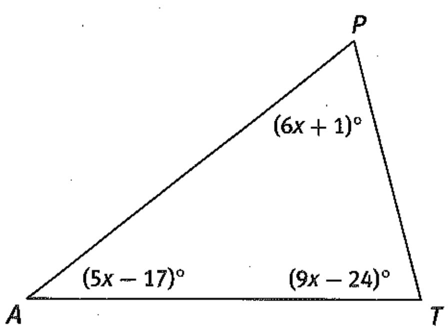
1. Two angles of a triangle measure 32° and 70°. Find the measure of the third angle.
2. In the diagram below, $\overleftrightarrow{AC} \parallel \overleftrightarrow{DF}$. Determine the measure of each of the angles in $\triangle BDE$.



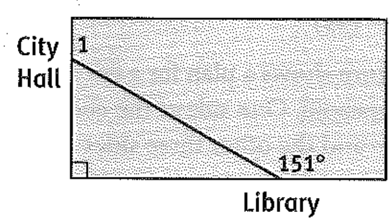
3. Determine the value of b.



4. Write an equation, solve for x, and determine the measure of each angle in $\triangle PAT$.



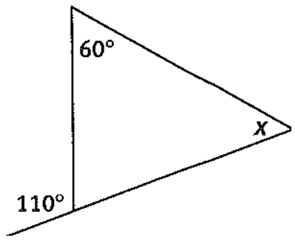
5. The measures of the three interior angles of a triangle are 85°, 20°, and 75°. Determine the measures of the three exterior angles.
6. The measures of two angles of a triangle are 38° and 47°. What is the measure of the third angle?
   A. 85°
   B. 95°
   C. 133°
   D. 142°
7. In $\triangle DEF$, the measure of $\angle D = (3x - 6)°$, the measure of $\angle E = (3x - 6)°$, and the measure of $\angle F = (2x)°$. Which of the following is the measure of $\angle F$?
   A. 24°
   B. 46°
   C. 48°
   D. 66°
8. In $\triangle PQR$, $\angle P$ is an obtuse angle. Which of the following statements about the triangle must be true?
   A. The other two angles must be congruent.
   B. The other two angles must be acute angles.
   C. One of the other two angles could be a right angle.
   D. One of the other two angles could also be an obtuse angle.
9. The figure shows a rectangular lawn at a civic center. Over time, people have cut across the lawn to walk from the library to city hall and made a straight path in the lawn, as shown. What is the measure of $\angle 1$ in the figure?



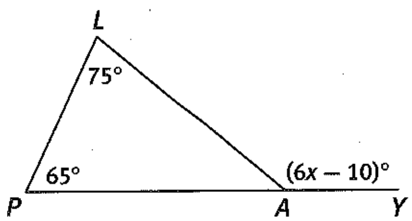
Lesson 17-2

In Items 10–12, determine the value of x.

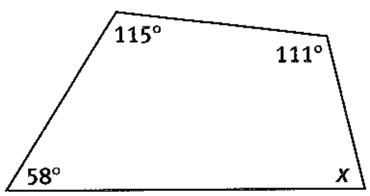
10.



11.

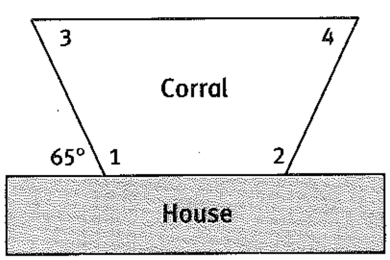


12.



13. Determine the measure of each angle in quadrilateral $DEFG$ with $m\angle D = (12x - 4)°$, $m\angle E = (18x + 4)°$, $m\angle F = (15x + 10)°$, and $m\angle G = (5x)°$.

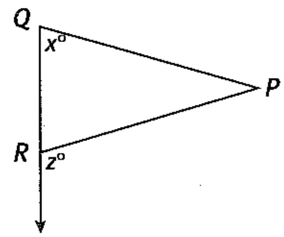
14. The figure shows a plan for a corral in the shape of a trapezoid. One side of the corral is formed by a house and the other three sides are formed by a fence. Given that $\angle 1$ and $\angle 2$ are congruent, and that $\angle 3$ and $\angle 4$ are congruent, find the measures of the four angles.



15. In quadrilateral $ABCD$, $m\angle A = (5x - 5)°$, $m\angle B = (9x)°$, $m\angle C = (12x + 15)°$, and $m\angle D = (15x - 60)°$. Which angle has the greatest measure?

A. $\angle A$ **B.** $\angle B$
C. $\angle C$ **D.** $\angle D$

16. Which expression represents the measure of $\angle P$?

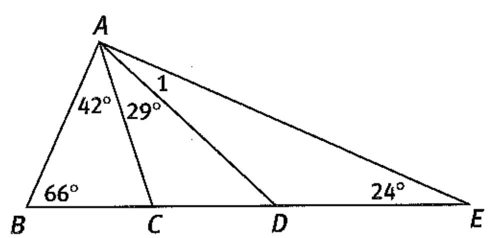


A. $(z + x)°$ **B.** $(z - x)°$
C. $(x - z)°$ **D.** $z°$

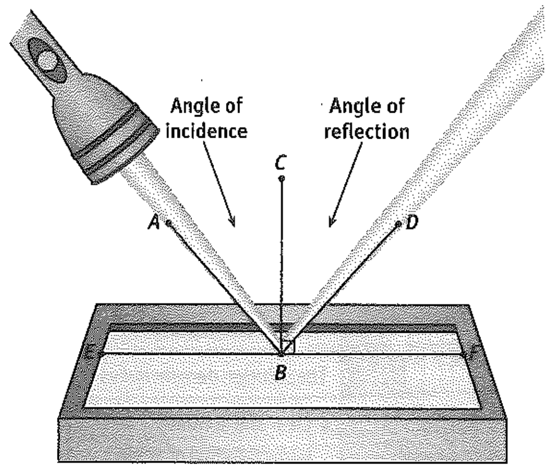
17. Sketch a quadrilateral that contains a 50° angle and a 170° angle. Give possible measures for the other two angles.

MATHEMATICAL PRACTICES
Critique the Reasoning of Others

18. Nick and LaToya are painting a backdrop of a mountain for a stage set. A sketch for the backdrop is shown below. Nick says there is not enough information to determine the measure of $\angle 1$. LaToya says there is enough information to determine this angle measure. Who is correct? Explain.

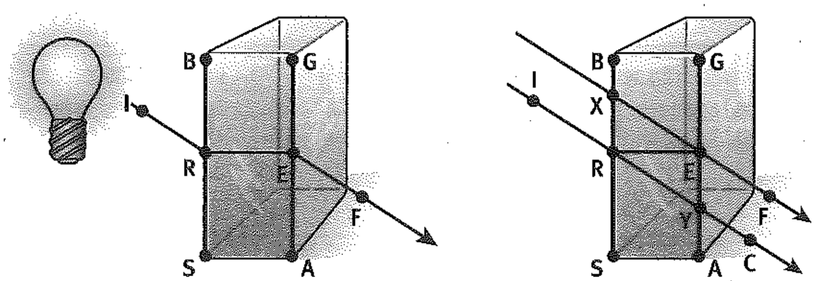


A beam of light and a mirror can be used to study the behavior of light. When light hits the mirror it is reflected so that the angle of incidence and the angle of reflection are congruent.



1. Name a pair of nonadjacent complementary angles in the diagram.
2. Name a pair of adjacent supplementary angles in the diagram.
3. In the diagram, $m\angle CBD = (4x)^\circ$ and $m\angle FBD = (3x - 1)^\circ$.
   a. Solve for the value of x.
   b. Determine $m\angle CBD$, $m\angle FBD$, and $m\angle DBE$.

Light rays are bent as they pass through glass. Since a block of glass is a rectangular prism, the opposite sides are parallel and a ray is bent the same amount entering the piece of glass as exiting the glass.

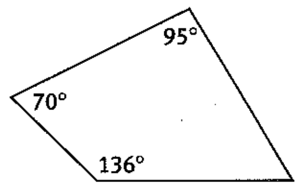


This causes $\overleftrightarrow{XF}$ to be parallel to $\overleftrightarrow{RY}$, as shown.

4. If the measure of $\angle YEX$ is 130°, determine the measure of each of the following angles. Explain how you arrived at your answer.
   a. $\angle BXE$ b. $\angle GEF$ c. $\angle SRY$
5. If $m\angle CYA = (5x)^\circ$ and $m\angle SRY = (6x - 10)^\circ$, then the value of x is _______.
6. If $m\angle XRE = 90^\circ$ and $m\angle REX = 30^\circ$, then $m\angle RXE =$ _______. Explain how you arrived at your answer.

7. The measures of the angles of a triangle are $(2x)^\circ$, $(x + 14)^\circ$, and $(x - 38)^\circ$. Determine the value of x and the measures of each of the three angles.

8. One of the quadrilaterals in a mural design is shown below. Determine the measure of the missing angle.



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	• Clear and accurate understanding of angle relationships, and finding angle measures in a triangle and quadrilateral.	• An understanding of angle relationships and finding angle measures in a triangle and quadrilateral.	• Partial understanding of angle relationships and finding angle measures in a triangle and quadrilateral.	• Little or no understanding of angle relationships and finding angle measures in a triangle and quadrilateral.
Problem Solving (Items 3a-b, 4a-c, 5, 6, 7, 8)	• Interpreting a problem accurately in order to find missing angle measures.	• Interpreting a problem to find missing angle measures.	• Difficulty interpreting a problem to find missing angle measures.	• Incorrect or incomplete interpretation of a problem.
Mathematical Modeling / Representations (Items 1, 2, 3a-b, 4a-c, 5, 6, 7, 8)	• Accurately interpreting figures in order to characterize angle pairs and find angle measures.	• Interpreting figures in order to find angle pairs and find missing angle measures.	• Difficulty interpreting figures in order to find angle pairs and find missing angle measures.	• Incorrectly interpreting figures in order to find angle pairs and find missing angle measures.
Reasoning and Communication (Items 4a-c, 6)	• Precise use of appropriate terms to describe finding angle measures.	• An adequate description of finding of missing angle measures.	• A confusing description of finding missing angle measures.	• An inaccurate description of finding missing angle measures.

Introduction to Transformations

Move It!

Lesson 18-1 What Is a Transformation?

Learning Targets:

- Recognize rotations, reflections, and translations in physical models.
- Explore rigid transformations of figures.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Vocabulary Organizer, Paraphrasing

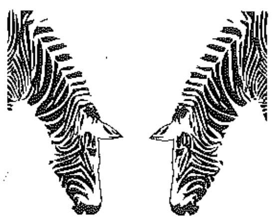
A ***transformation***, such as a flip, slide, or turn, changes the position of a figure. Many graphic artists rely on graphic design software to transform images to create logos or promotional materials.

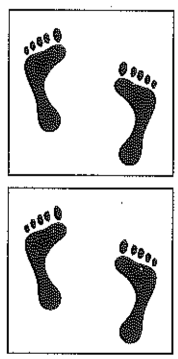
A ***preimage*** is a figure before it has been transformed and the ***image*** is its position after the transformation. You can tell whether a figure has been transformed if the preimage can be moved to coincide with its image.

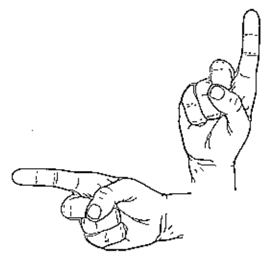
ACADEMIC VOCABULARY

The word ***transform*** means "to change."

1. Each set of pictures below shows the preimage and image of some familiar objects. Use the terms *flip, slide,* and *turn* to describe what transformation will make the two objects coincide.

a. 

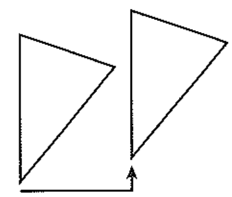
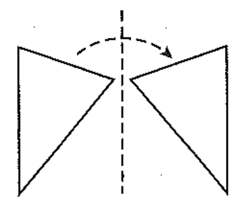
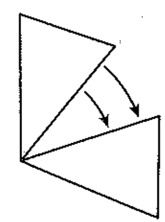
b. 

c. 

2. Make a conjecture about the preimage and image of a transformed object based on your observations of the pictures.

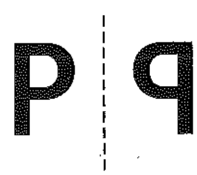
My Notes

3. **Make use of structure.** The table below shows the proper name for transformations and the corresponding definition. Match each transformation with the words *flip, slide,* and *turn.*

Transformation	Definition	Example
Translation	Each point of a figure is moved the same distance in the same direction.	
Reflection	Each point of a figure is reflected over a line, creating a mirror image.	
Rotation	Each point of a figure is rotated through a given angle in a given direction around a fixed point.	

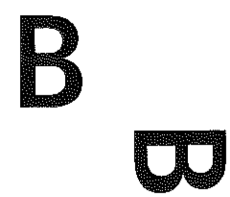
4. For each capital letter shown below, visualize the movement the letter takes while being transformed. Identify the transformation by its proper name.

a.



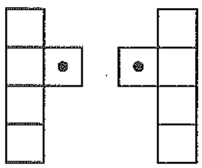
b.

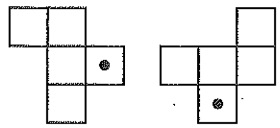
c.

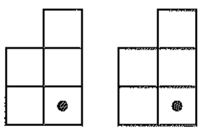


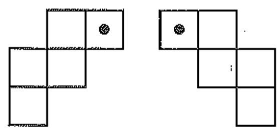
Check Your Understanding

Tell what single transformation, translation, reflection, or rotation will make the figures coincide. Explain how you determined your answers.

5. 

6. 

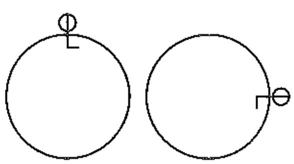
7. 

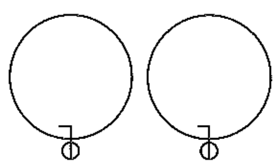
8. 

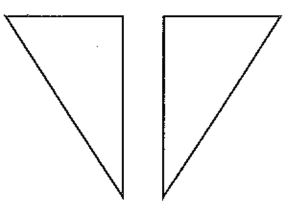
9. **Construct viable arguments.** How do the sides of the image of a triangle after a translation, reflection, or rotation compare with the corresponding sides of the original figure? How do you know?

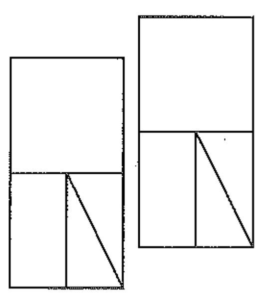
LESSON 18-1 PRACTICE

Each set of figures shows the preimage and image of an object after a single transformation. Describe how the object was transformed using the proper name.

10. 

11. 

12. 

13. 

14. **Reason abstractly.** Which of the three transformations do you most commonly see in the world around you? Give examples to support your answer.

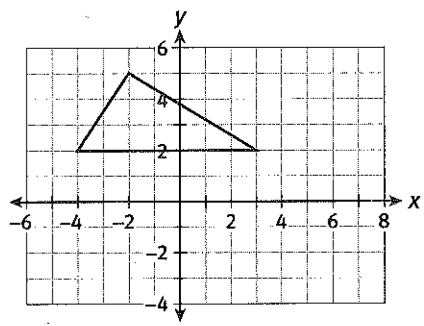
Learning Targets:

- Determine the effect of translations on two-dimensional figures using coordinates.
- Represent and interpret translations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Create Representations, Identify a Subtask, Interactive Word Wall

A ***translation*** changes only a figure's position. A verbal description of a translation includes words such as *right, left, up,* and *down.*

1. Consider the triangle shown on the coordinate plane.



Coordinates of Triangle	Coordinates of Image

a. Record the coordinates of the vertices of the triangle in the table.
b. Translate the triangle down 2 units and right 5 units. Graph the translation.
c. Record the coordinates of the vertices of the ***image*** in the table.

A ***symbolic representation*** of a transformation is an algebraic way to show the changes to the x- and y-coordinates of the vertices of the original figure, or ***preimage***.

d. Make use of structure. Use the information in the table to help you complete the symbolic representation for the translated triangle:

$$(x, y) \rightarrow (x + 5, y - 2)$$

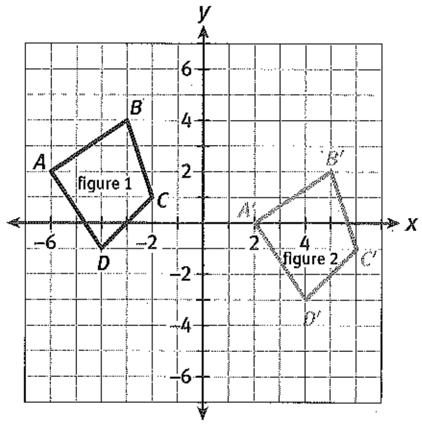
MATH TERMS

A **symbolic representation** of a transformation is an algebraic way to show the changes to the x- and y-coordinates of the vertices of the original figure, or preimage.

A **preimage** is a figure before it has been transformed and the **image** is its position after the transformation.

My Notes

2. Figure 2 is the image of Figure 1 after a translation, as shown in the coordinate plane.



a. Record the coordinates of the vertices of the preimage and image.

Figure 1: Preimage		Figure 2: Image	
A		*A′*	
B		*B′*	
C		*C′*	
D		*D′*	

b. Make sense of problems. Refer to the table and graph.
Was the figure translated up or down? _________ By how much? _________

Was the figure translated to the left or right? _________ By how much? _________

c. Write a verbal description to describe the translation.

d. Describe the translation using a symbolic representation.
symbolic representation: $(x, y) \rightarrow (x + 8, y - 2)$

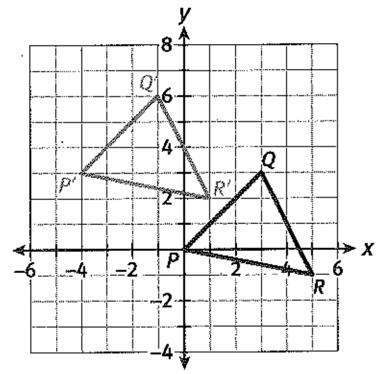
READING MATH

A prime symbol (′) is placed after the letter for the original point to show that the new point is its image.

Example: Point *A′* is the image of point *A*.

My Notes

3. The coordinate plane shows $\triangle P'Q'R'$ after $\triangle PQR$ undergoes a translation.

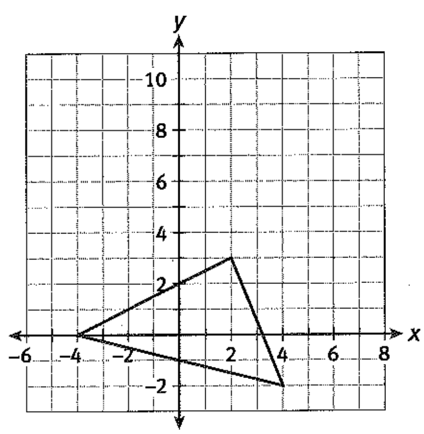


a. Write a verbal description to describe the translation.

b. Describe the translation using a symbolic representation.
symbolic representation: $(x, y) \rightarrow (x - 4, y + 3)$

Check Your Understanding

4. The triangle shown on the coordinate plane is translated according to the following symbolic representation: $(x, y) \rightarrow (x + 1, y + 6)$.



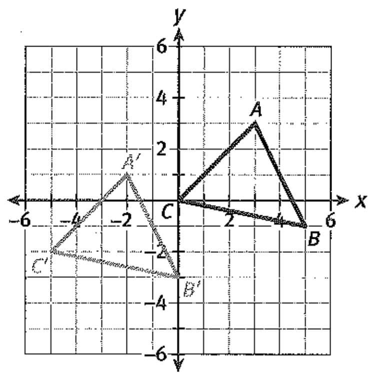
a. Describe how the symbolic representation can be used to determine if the triangle is translated left or right, and up or down.
b. Write a verbal description of the translation.
c. **Attend to precision.** Sketch the image of the triangle according to the symbolic representation.

5. **Construct viable arguments.** Explain how the change in the coordinates of a translated point is related to the symbolic representation.

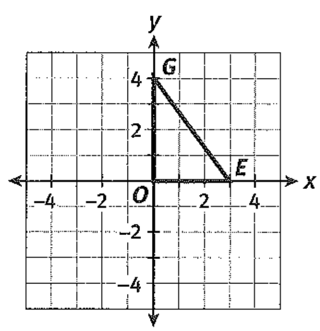
My Notes

LESSON 18-2 PRACTICE

6. Triangle ABC is shown along with its image $\triangle A'B'C'$ on the coordinate plane below.
   a. Write a verbal description of the translation.
   b. Show the translation using symbolic representation.



7. Determine the coordinates of the vertices for each image of $\triangle GEO$ after each of the following translations is performed.



   a. 3 units to the left and 3 units down
   b. $(x, y) \rightarrow (x, y - 4)$
   c. $(x, y) \rightarrow (x - 2, y + 1)$
   d. $(x, y) \rightarrow (x - 4, y)$

8. **Critique the reasoning of others.** Quadrilateral $QRST$ has vertices $Q(0, 0)$, $R(4, 0)$, $S(4, 4)$, and $T(0, 4)$. Eric states that the image of this quadrilateral after a given translation has vertices $Q'(0, 0)$, $R'(2, 0)$, $S'(2, 2)$, and $T'(0, 2)$. Do you agree or disagree with Eric's statement? Justify your reasoning.

My Notes

Learning Targets:

- Determine the effect of reflections on two-dimensional figures using coordinates.
- Represent and interpret reflections involving words, coordinates, and symbols.

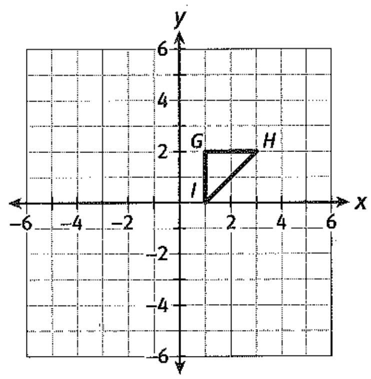
SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Interactive Word Wall, Construct an Argument, Summarizing

To perform a ***reflection***, each point of a preimage is copied on the opposite side of the line of reflection and remains ***equidistant*** from the line.

MATH TERMS

Equidistant means to be the same distance from a given point or line.

1. $\triangle GHI$ is shown on the coordinate plane below.



Coordinates of Triangle	Coordinates of Image

a. Record the coordinates of the vertices of $\triangle GHI$ in the table.

b. Sketch the reflection of $\triangle GHI$ over the x-axis.

c. Record the coordinates of the vertices of the image $\triangle G'H'I'$ in the table.

The symbolic representation for this transformation is $(x, y) \rightarrow (x, -y)$.

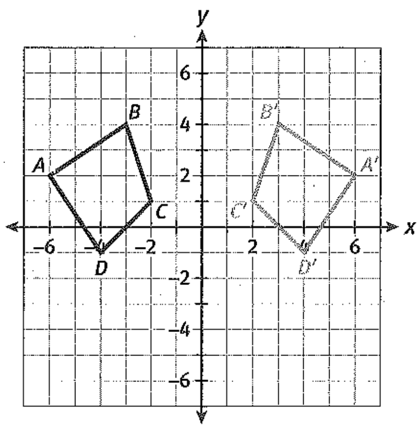
d. Explain how the change in the coordinates of the vertices is related to the symbolic representation for this transformation.

CONNECT TO AP

Translations and reflections of figures in the coordinate plane are preparing you to successfully translate and reflect graphs of functions. This is a helpful tool for visualizing and setting up the graphs for many problems you will solve in calculus.

My Notes

2. Figure 2 is the image of figure 1 after a reflection, as shown in the coordinate plane.



Preimage: Figure 1		Image: Figure 2	
A		*A'*	
B		*B'*	
C		*C'*	
D		*D'*	

a. Record the coordinates of the vertices of the preimage and image.

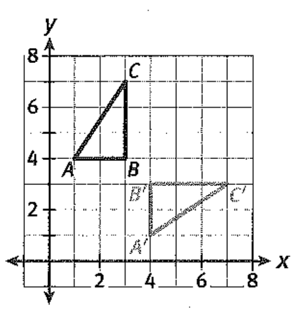
b. The line across which an object is reflected is called the **line of reflection**. Identify the line of reflection in the transformation of Figure 1.

c. A verbal description of a reflection includes the line of reflection. Write a verbal description of the reflection.

d. Describe the reflection using a symbolic representation.
Symbolic Representation: $(x, y) \rightarrow$

Check Your Understanding

3. Triangle *ABC* and its reflected image are shown on the coordinate plane.



Coordinates of $\triangle ABC$		Coordinates of $\triangle A'B'C'$	
A		*A'*	
B		*B'*	
C		*C'*	

a. Complete the table.

b. Identify the line of reflection. Write a verbal description of the transformation.

c. Describe the reflection using symbolic representation.

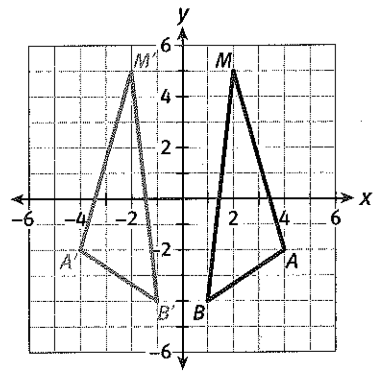
My Notes

4. **Express regularity in repeated reasoning.** Write a summary statement describing which coordinate stays the same when a figure is reflected over the y-axis.

5. Modify your statement in Item 4 describing which coordinate stays the same when a figure is reflected over the x-axis.

LESSON 18-3 PRACTICE

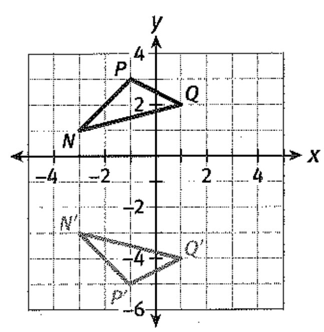
6. Triangle BAM is shown along with its image $\triangle B'A'M'$ on the coordinate plane below.



   a. Write a verbal description of the reflection.
   b. Describe the reflection using symbolic representation.

7. Suppose $\triangle CDF$, whose vertices have coordinates $C(-2, 1)$, $D(4, 5)$, and $F(5, 3)$, is reflected over the x-axis.
   a. Explain a way to determine the coordinates of the vertices of $\triangle C'D'F'$.
   b. Find the coordinates of $\triangle C'D'F'$.

8. **Critique the reasoning of others.** Filip claims $\triangle N'P'Q'$ is a reflection of $\triangle NPQ$ over the x-axis. Is Filip correct? Justify your answer.



My Notes

Learning Targets:

- Determine the effect of rotations on two-dimensional figures using coordinates.
- Represent and interpret rotations involving words, coordinates, and symbols.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Look for a Pattern, Interactive Word Wall

A ***rotation*** is a transformation that describes the motion of a figure about a fixed point. To perform a rotation, each point of the preimage travels along a circle the same number of degrees.

1. The point (3, 1) is rotated in a counterclockwise direction about the origin 90°, 180°, and 270°.

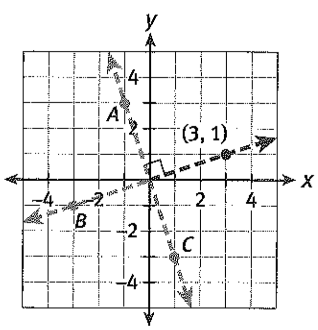


Image Point	Coordinates	Measure of Angle of Rotation
A		
B		
C		

a. Write the coordinates of each image point *A, B,* and *C* in the table.

b. Complete the table by giving the angle of rotation for each image point.

c. **Reason abstractly.** Describe in your own words why the origin is the ***center of rotation*** in this rotation transformation.

d. **Construct viable arguments.** Make a conjecture about the changes of the *x*-and *y*-coordinates when a point is rotated counterclockwise 90°, 180°, and 270° about the origin.

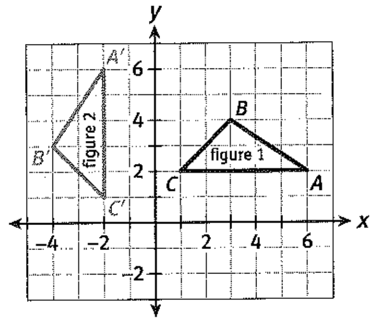
e. What are the coordinates of the point (3,1) after a 360° rotation about the origin? Explain your answer.

MATH TIP

If the direction of a rotation is counterclockwise, the measure of the angle of rotation is given as a positive value. If the direction of a rotation is clockwise, the measure of the angle of rotation is given as a negative value.

My Notes

2. Figure 2 is a 90° counterclockwise rotation about the origin of figure 1.



Determine the coordinates of the vertices for each figure.

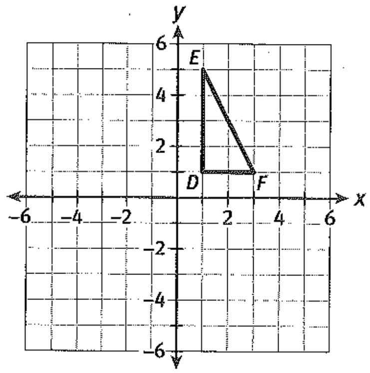
Preimage: Figure 1		**Image: Figure 2**	
A		*A'*	
B		*B'*	
C		*C'*	

3. Make sense of problems. Complete the summary statement:

When a figure in Quadrant I of the coordinate plane is rotated 90° counterclockwise about the origin, its image is located in Quadrant ______.

My Notes

4. Use appropriate tools strategically. Consider $\triangle DEF$ shown on the coordinate plane.



a. Trace $\triangle DEF$ and the positive x-axis on a piece of tracing paper. Label the vertices and the axis.

b. Rotate the triangle 90° counterclockwise by aligning the origin and rotating the tracing paper until the positive x-axis coincides with the positive y-axis.

c. Record the coordinates of the vertices of the image in the table.

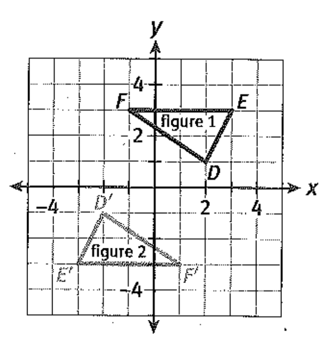
Preimage	$D(1, 1)$	$E(1, 5)$	$F(3, 1)$
Image	$D'(\quad)$	$E'(\quad)$	$F'(\quad)$

d. Sketch $\triangle D'E'F'$ on the coordinate plane above.

Check Your Understanding

5. Make use of structure. Use your results from Items 1, 2, and 3 to write a symbolic representation for a 90° counterclockwise rotation.
$(x, y) \rightarrow (\quad ,\quad)$

6. Critique the reasoning of others. Sven recognized the 180° rotation of $\triangle DEF$ about the origin in the coordinate plane and determined the symbolic representation to be $(x, y) \rightarrow (-x, -y)$.



Determine whether the symbolic representation is correct. Justify your answer.

7. A point with coordinates (x, y) is rotated 360° in a counterclockwise direction about the origin. Write the symbolic representation for this transformation:
$(x, y) \rightarrow (\quad ,\quad)$.

What does the symbolic representation indicate?

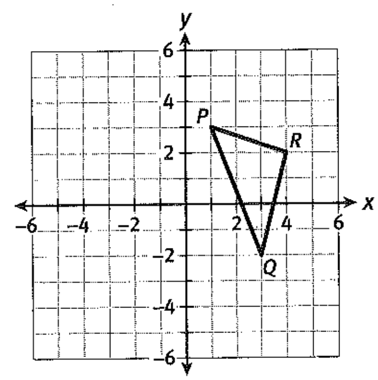
My Notes

LESSON 18-4 PRACTICE

8. Triangle *PQR* with vertices $P(1, 3)$, $Q(3, -2)$, and $R(4, 2)$ is shown on the coordinate plane. Graph each given rotation about the origin.

a. 90° counterclockwise

b. 180° counterclockwise



9. The preimage of point *A* is located at $(-1, 5)$. What are the coordinates of the image, A', after a 270° counterclockwise rotation?

10. Complete the summary statements:

a. When a figure in Quadrant I of the coordinate plane is rotated 180° counterclockwise about the origin, its image is located in Quadrant ______.

b. When a figure in Quadrant I of the coordinate plane is rotated 270° counterclockwise about the origin, its image is located in Quadrant ______.

11. Reason quantitatively. Use your answer from Item 9 to write a conjecture about the symbolic representation for a 270° counterclockwise rotation.

12. Draw a figure on a coordinate plane. Rotate the figure counterclockwise 270° about the origin. How does your drawing confirm your conjecture in Item 11?

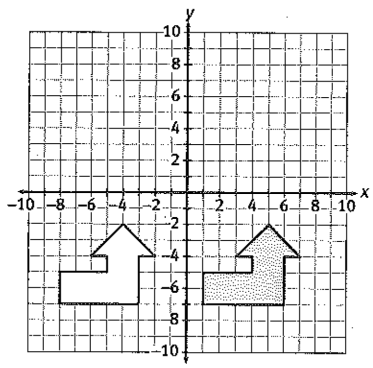
ACTIVITY 18 PRACTICE

Write your answers on notebook paper. Show your work.

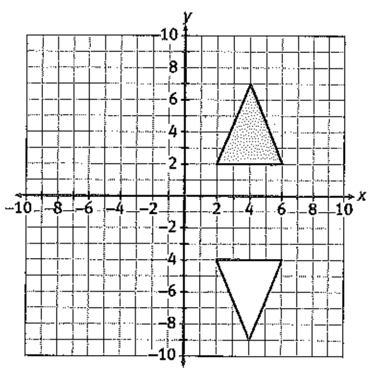
Lesson 18-1

For Items 1–3, the shaded figure is the preimage and the unshaded figure is the image. Identify the single transformation that will make the figures coincide.

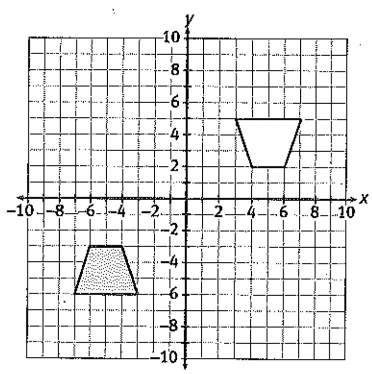
1.



2.

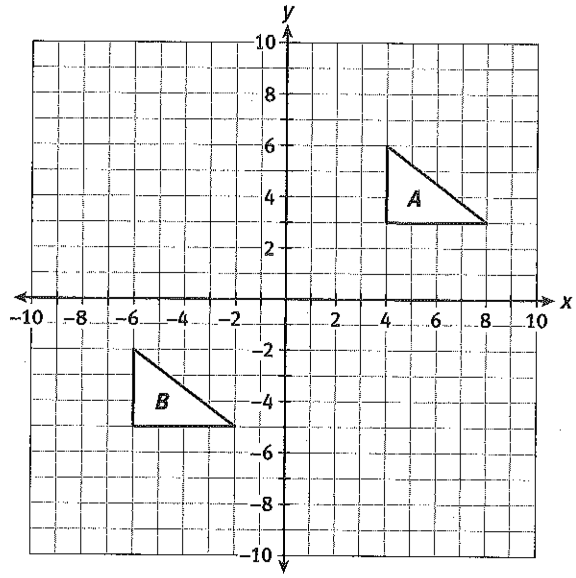


3.



Lesson 18-2

4. Figure B is the image of figure A after a transformation, as shown in the coordinate plane.

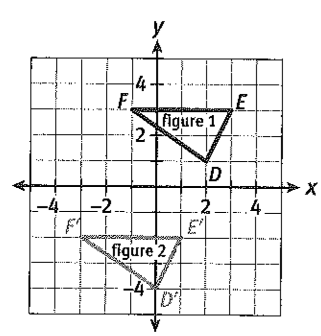


a. Write a verbal description of the transformation.

b. Write a symbolic representation of the transformation.

5. The vertices of $\triangle MOV$ are located at $M(-2, -2)$, $O(4, -2)$, and $V(4, 3)$. Determine the coordinates of the vertices of the image after $\triangle MOV$ is translated 3 units up and 2 units to the right.

6. Which symbolic representation describes the transformation shown on the coordinate plane?



A. $(x, y) \rightarrow (x + 2, y - 5)$

B. $(x, y) \rightarrow (x - 2, y - 5)$

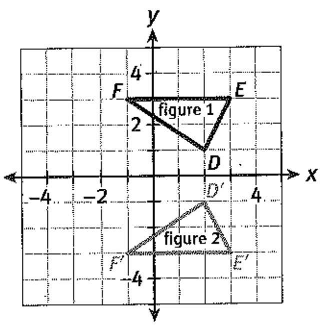
C. $(x, y) \rightarrow (x - 2, y + 5)$

D. $(x, y) \rightarrow (x + 2, y + 5)$

Lesson 18-3

7. The vertices of $\triangle QRS$ are located at $Q(2, 2)$, $R(-4, 2)$, and $S(-4, -4)$. Determine the coordinates of the vertices of each image of $\triangle QRS$ after the following transformations are performed:
   a. $\triangle QRS$ is reflected over the x-axis.
   b. $\triangle QRS$ is reflected over the y-axis.

8. Triangle FED and its transformed image is shown on the coordinate plane.



   a. Identify the line of reflection.
   b. Write a verbal description of the transformation.
   c. Write a symbolic representation of the transformation.

Lesson 18-4

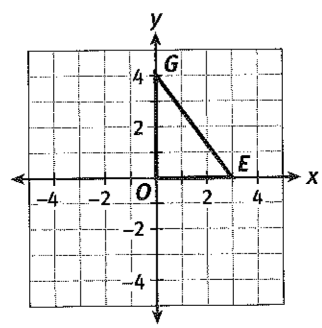
9. The vertices of $\triangle XYZ$ are located at $X(-2, -2)$, $Y(4, -2)$, and $Z(4, 3)$. Determine the coordinates of the vertices of each image of $\triangle XYZ$ after the following transformations are performed:
   a. $\triangle XYZ$ is rotated 90° counterclockwise about the origin.
   b. $\triangle XYZ$ is rotated 180° about the origin.

10. The preimage of point B is located at $(-1, 4)$. Determine the coordinates of the image, B', for each counterclockwise rotation.
   a. 90°
   b. 180°
   c. 270°

11. Triangle ABC has vertices $A(2, 4)$, $B(5, 7)$, and $C(-1, 5)$. If $\triangle ABC$ is rotated 270° counterclockwise about the origin, in what quadrant(s) would you find the image of $\triangle ABC$?
   A. Quadrant I
   B. Quadrant III
   C. Quadrants II and III
   D. Quadrants I and IV

MATHEMATICAL PRACTICES
Make Use of Structure

12. Determine the coordinates of the vertices for each image of $\triangle GEO$ after each of the following transformations is performed.



   a. Translate $\triangle GEO$ 2 units to the left and reflect over the x-axis.
   b. Reflect $\triangle GEO$ over the x-axis and translate 2 units to the left.

13. Does the order in which multiple transformations, such as rotations, reflections, and translations, are performed on a preimage have an effect on the image?

Rigid Transformations and Compositions

All the Right Moves

Lesson 19-1 Properties of Transformations

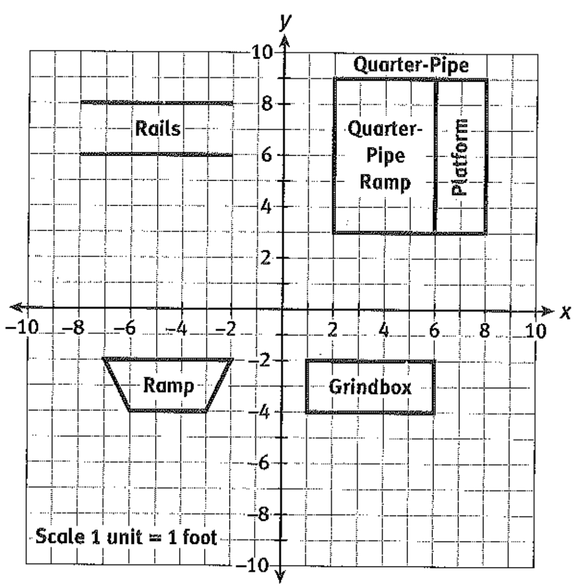
Learning Targets:

- Explore properties of translations, rotations, and reflections on two-dimensional figures.
- Explore congruency of transformed figures.

SUGGESTED LEARNING STRATEGIES: Visualization, Identify a Subtask, Create Representations, Critique Reasoning, Predict and Confirm

Skip and Kate are designing a skateboard park for their neighborhood. They want to include rails, a grindbox, a quarter-pipe, and a ramp. They are deciding where to place the equipment. Kate sketches her plan for the layout on a coordinate plane.

Using the origin as the center of their park, Kate sketched figures to represent the equipment on the coordinate plane, as shown below.



Kate uses the layout on the coordinate plane to determine the dimensions and the area of each figure.

1. **Model with mathematics.** Use the scale on Kate's layout to complete the table of dimensions for each piece of equipment.

Equipment	Base (ft)	Height (ft)	Area (ft^2)
Quarter-Pipe (ramp and platform)			
Ramp	base 1: base 2:		
Grindbox			

MATH TIP

The coordinates of the origin on a coordinate plane are (0, 0).

MATH TIP

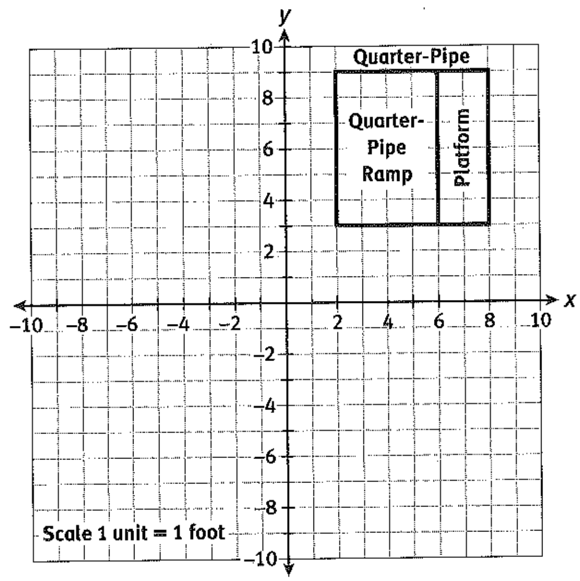
The area of a trapezoid can be found using the formula

Area $= \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

My Notes

Skip reviewed Kate's plan for the skateboarding park. To improve the layout, Skip suggested transformations for each piece of equipment as described.

2. The original placement of the quarter-pipe is shown on the coordinate plane.



a. Reflect the figure representing the quarter-pipe ramp and platform over the y-axis. Label each vertex of the image with an ordered pair.

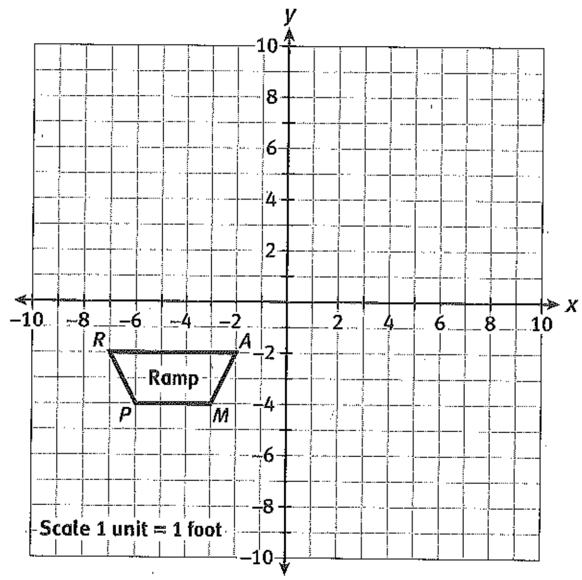
b. Determine the dimensions of the image, in feet.

c. Compare the areas of the original figure and the image.

d. Explain why the image is congruent to the original figure.

My Notes

3. The original placement of the ramp is shown on the coordinate plane.



a. Rotate the figure representing the ramp 90° counterclockwise about the origin. Label the vertices of the image R', A', M', and P'.

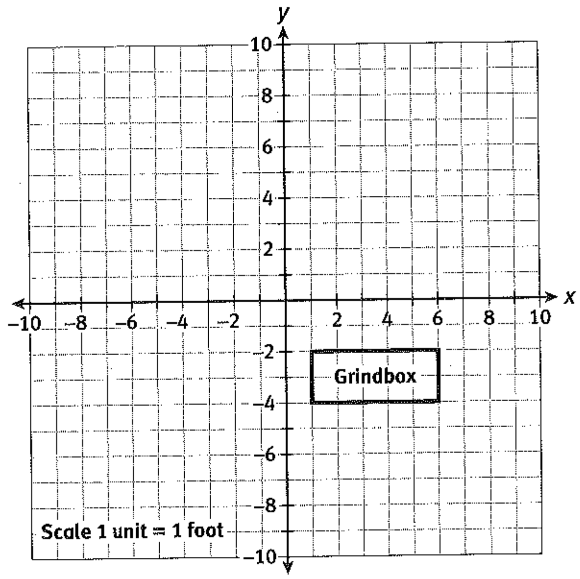
b. **Critique the reasoning of others.** Kate states that this rotation will change the shape and size of the figure. Skip reassures her that the image is congruent to the original figure. With whom do you agree? Justify your reasoning.

Congruent figures have corresponding angles as well as corresponding sides.

c. List the pairs of corresponding angles in trapezoids *RAMP* and $R'A'M'P'$.

d. **Construct viable arguments.** Make a conjecture about the corresponding angles of congruent figures.

4. The original placement of the grindbox is shown on the coordinate plane.



a. Plot the image of this figure using the transformation whose symbolic representation is $(x, y) \rightarrow (x + 2, y + 9)$.

b. Write a verbal description of the transformation.

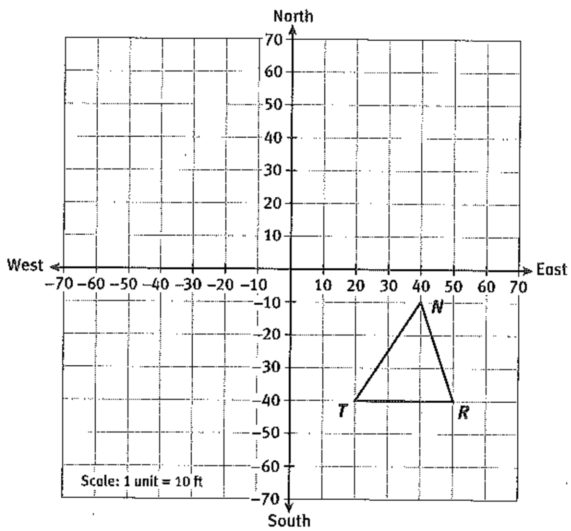
c. Is the image of the grindbox congruent to the preimage of the grindbox? Justify your answer.

5. **Reason abstractly.** After using reflections, rotations, and translations to create images of figures, what can you infer about the preimage and its image under all of these transformations?

My Notes

Check Your Understanding

Consider $\triangle NTR$ shown on the coordinate grid.

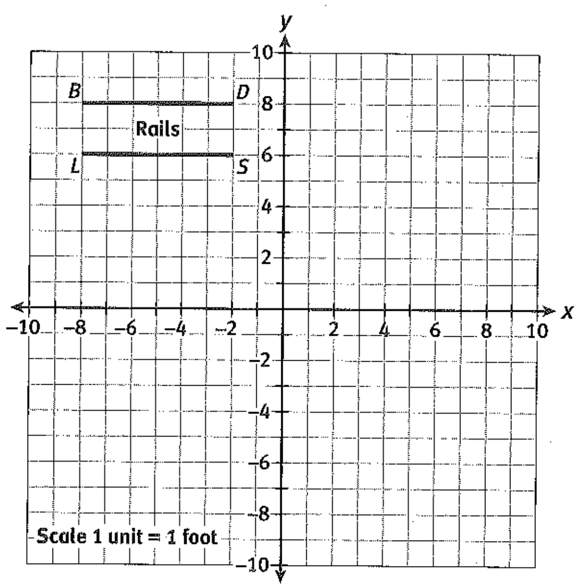


6. Rotate $\triangle NTR$ 180° about the origin. Label the vertices T', R', and N'.
7. Find the area, in square units, of $\triangle NTR$ and $\triangle N'T'R'$. Show the calculations that led to your answer.
8. Write a supporting statement justifying how you know that $\triangle NTR$ and $\triangle N'T'R'$ are congruent triangles.
9. **Express regularity in repeated reasoning.** Could your statement in Item 8 be used to support other types of transformations of $\triangle NTR$? Explain.

MATH TIP

The area of a triangle can be found using the formula
Area $= \frac{1}{2} \times \text{base} \times \text{height}$.

Finally, Skip decides to move the location of the rails. The original placement of the rails is shown on the coordinate plane.



My Notes

MATH TERMS

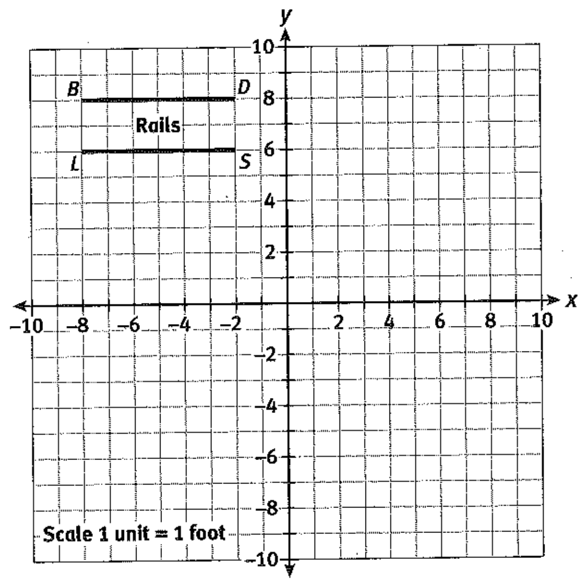
Performing two or more transformations on a figure is called a **composition of transformations**.

READING MATH

A prime symbol (′) is placed after the letter of the original point to show that the new point is its image. Two prime symbols (″) are placed after the letter of the original point to show that the new point has been transformed twice.

10. **Construct viable arguments.** Skip claims that the rails are parallel and that moving them, using a reflection, rotation, or translation, will not affect this relationship. Confirm or contradict Skip's claim. Use examples to justify your answer.

11. Skip decides to move the rails using a ***composition of transformations***.



a. Reflect the graph of each rail, $\overline{BD}$ and $\overline{LS}$, over the x-axis. Label the image points B', D', L', and S'.

b. Then, translate the reflected image 3 feet up and 1 foot left. Label the image points B'', D'', L'', and S''.

My Notes

Check Your Understanding

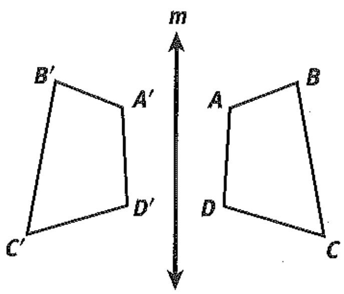
12. Refer to Item 11. Describe a method to determine if $\overline{B''D''}$ and $\overline{L''S''}$ are congruent to $\overline{BD}$ and $\overline{LS}$.

13. Describe how the rails in Item 11 would differ in orientation if the translation in Item 11b was changed to a counterclockwise rotation 90° about the origin.

14. Do you agree with the statement that congruency is preserved under a composition of transformations involving translations, reflections, and rotations? If not, provide a counterexample.

LESSON 19-1 PRACTICE

15. Quadrilateral *ABCD* is reflected across line *m* as shown in the diagram.



a. Name the side that corresponds to $\overline{CD}$ and explain the relationship between the lengths of these two segments.

b. Name the angle that corresponds to angle *C* and explain the relationship between the measures of these two angles.

16. Draw a coordinate plane on grid paper. Create and label a triangle having vertices $D(3, 5)$, $H(0, 8)$, and $G(3, 8)$. Perform each transformation on the coordinate plane.

a. Reflect $\triangle DHG$ across the x-axis.

b. Rotate $\triangle DHG$ 90° counterclockwise about the origin.

c. Translate $\triangle DHG$ 4 units right.

d. Which of the transformed images above are congruent to $\triangle DHG$?

My Notes

Learning Targets:

- Explore composition of transformations.
- Describe the effect of composition of translations, rotations, and reflections on two-dimensional figures using coordinates.

SUGGESTED LEARNING STRATEGIES: Self Revision/Peer Revision, Visualization, Discussion Groups, Create Representations, Close Reading

To explore composition of transformations, you and a partner will play a game called All the Right Moves. Cut out the five All the Right Moves cards on page 259 and two game pieces. You and your partner will use only one set of All the Right Moves game cards to play the game, but you both need a game piece.

DISCUSSION GROUP TIPS

As you read and discuss the rules of All the Right Moves, ask and answer questions to be sure you have a clear understanding of not only all the terminology used, but also how the game is to be played.

All the Right Moves Rules

1. As partners, lay out the 5 All the Right Moves cards face down.
2. Take turns choosing an All the Right Moves card. You will each take 2 cards. The extra card may be used later as a tiebreaker.
3. Working independently, each of you will use your All the Right Moves cards to complete the two game sheets on pages 255 and 256.
4. To complete the first game sheet, follow these steps:
   a. Record the number of one of your All the Right Moves cards on your game sheet. You may use either one first.
   b. Plot and label the points for Position 0 on the grid. Then use those points as the vertices to draw a triangle.
   c. Follow the directions on the All the Right Moves card to find the coordinates of the vertices for Position 1.
   d. Record the new coordinates on your game sheet, plot the new points on the coordinate plane, and draw a triangle. Use your game piece to identify the transformation you made and record its name on your game sheet.
   e. Continue until you have moved the figure to all 5 positions on the All the Right Moves card. Then record the coordinates of the composition of transformations, which is Position 5.
5. Repeat the process with your other All the Right Moves card for the second game sheet.
6. When you and your partner have completed your two cards, exchange game sheets and check each other's work.
7. Score your game sheets: You get 2 points for each transformation you correctly identify and 5 points for the correct coordinates of each composition of transformations.
8. The player with the greater number of points wins the game.

All the Right Moves
Game Sheet

Player: ______________________________

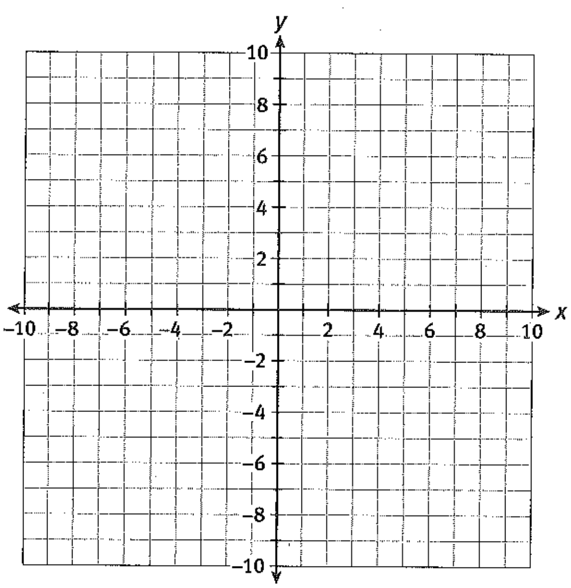
All the Right Moves **Card:** ______________________________

Position 0: $A(\quad), B(\quad), C(\quad)$	**Type of Transformation**
Position 1: $A(\quad), B(\quad), C(\quad)$	
Position 2: $A(\quad), B(\quad), C(\quad)$	
Position 3: $A(\quad), B(\quad), C(\quad)$	
Position 4: $A(\quad), B(\quad), C(\quad)$	
Position 5: $A(\quad), B(\quad), C(\quad)$	

Composition of Transformations:

$A(\quad), B(\quad), C(\quad)$

Points Earned for All the Right Moves **Card:** ______________________________



My Notes

All the Right Moves
Game Sheet

Player: ____________________

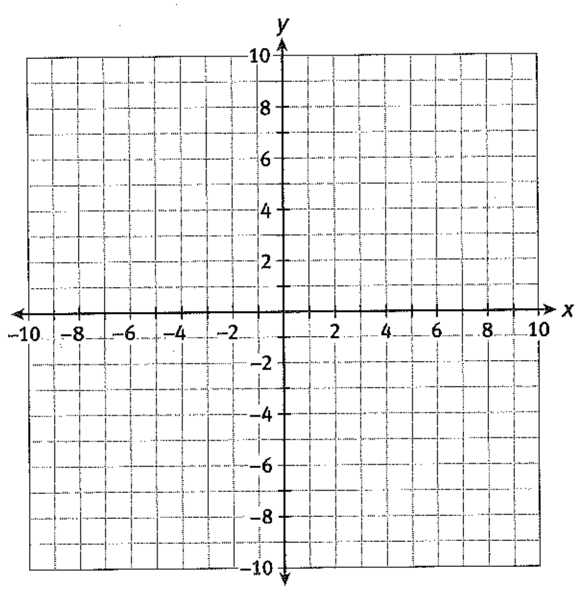
All the Right Moves **Card:** ____________________

Position 0: A( ), B( ), C( )	**Type of Transformation**
Position 1: A( ), B( ), C( )	
Position 2: A( ), B( ), C( )	
Position 3: A( ), B( ), C( )	
Position 4: A( ), B( ), C( )	
Position 5: A( ), B( ), C( )	

Composition of Transformations:

A( ), ***B***( ), ***C***( )

Points Earned for All the Right Moves **Card:** ____________________

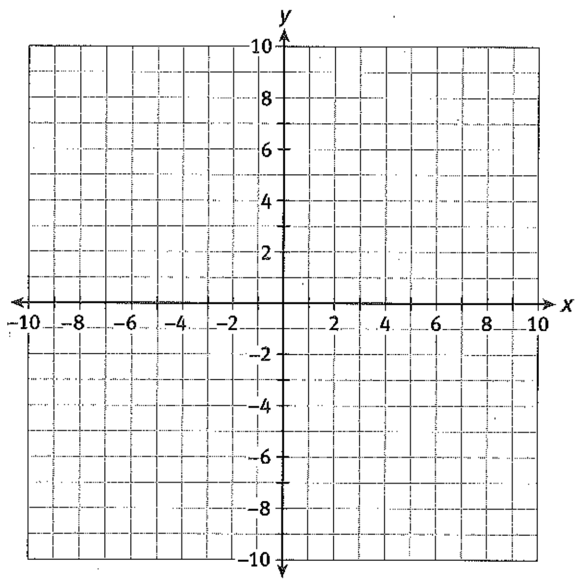


Total Points Earned: ____________________

9. Model with mathematics. Work with your partner to discover a composition of transformations that has the same result as one from the All the Right Moves game but takes fewer transformations.

a. Select one of the All the Right Moves game cards.

b. Follow the instructions on the card and use the coordinate plane below to draw the locations of Position 0 and Position 5.



c. Use what you know about reflections, translations, and rotations to move the game piece from Position 0 to Position 5 in four or fewer steps.

d. Write the directions for the moves you found in Item 9c on a separate sheet of paper. Then trade directions with your partner and follow each other's directions to see whether the new transformation is correct.

My Notes

Check Your Understanding

10. The point $T(5, -1)$ is reflected across the x-axis, then across the y-axis. What are the coordinates of T' and T''?

11. $\triangle ABC$ has vertices $A(-5, 2)$, $B(0, -4)$, and $C(3, 3)$.

a. Determine the coordinates of the image of $\triangle ABC$ after a translation 2 units right and 4 units down followed by a reflection over the y-axis.

b. What are the coordinates of the image of $\triangle ABC$ after a reflection over the y-axis followed by a translation 2 units right and 4 units down?

LESSON 19-2 PRACTICE

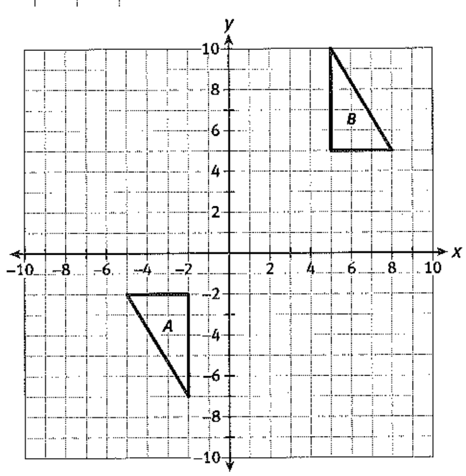
12. The point $(1, 3)$ is rotated 90° about the origin and then reflected across the y-axis. What are the coordinates of the image?

13. **Attend to precision.** Find a single transformation that has the same effect as the composition of translations $(x, y) \rightarrow (x - 2, y + 1)$ followed by $(x, y) \rightarrow (x + 1, y + 3)$. Use at least three ordered pairs to confirm your answer.

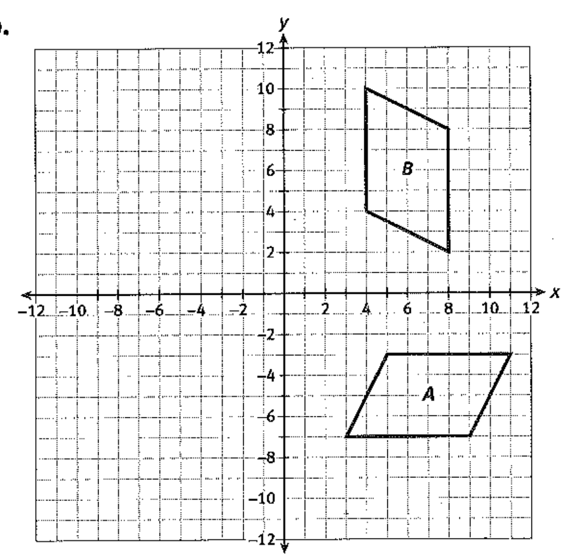
14. **Reason abstractly.** Describe a single transformation that has the same effect as the composition of transformations reflecting over the x-axis followed by reflecting over the y-axis. Use at least three ordered pairs to confirm your answer.

15. Write a composition of transformations that moves figure A so that it coincides with figure B.

a.



b.



My Notes

All the Right Moves Game Cards

All the Right Moves Card 1	All the Right Moves Card 2	All the Right Moves Card 3
Position 0: $A(3, 4), B(3, 1), C(7, 1)$ **Position 1:** $(x, y) \rightarrow (-x, y)$ **Position 2:** $(x, y) \rightarrow (x + 3, y + 4)$ **Position 3:** $(x, y) \rightarrow (x, -y)$ **Position 4:** $(x, y) \rightarrow (x - 1, y)$ **Position 5:** $(x, y) \rightarrow (-y, x)$	**Position 0:** $A(1, 1), B(1, -2), C(5, -2)$ **Position 1:** $(x, y) \rightarrow (x + 2, y)$ **Position 2:** $(x, y) \rightarrow (-x, y)$ **Position 3:** $(x, y) \rightarrow (x, 4 - y)$ **Position 4:** $(x, y) \rightarrow (-y, x)$ **Position 5:** $(x, y) \rightarrow (x - 1, y)$	**Position 0:** $A(-5, 0), B(-5, -3), C(-1, -3)$ **Position 1:** $(x, y) \rightarrow (-y, x)$ **Position 2:** $(x, y) \rightarrow (-x, y)$ **Position 3:** $(x, y) \rightarrow (-6 - x, y)$ **Position 4:** $(x, y) \rightarrow (x + 3, y + 4)$ **Position 5:** $(x, y) \rightarrow (-y, x)$
All the Right Moves Card 4	**All the Right Moves Card 5**	**Game Pieces**
Position 0: $A(-3, -4), B(-3, -7), C(1, -7)$ **Position 1:** $(x, y) \rightarrow (-x, y)$ **Position 2:** $(x, y) \rightarrow (x + 5, y)$ **Position 3:** $(x, y) \rightarrow (x + 2, y - 1)$ **Position 4:** $(x, y) \rightarrow (4 - x, y)$ **Position 5:** $(x, y) \rightarrow (y, -x)$	**Position 0:** $A(0, -1), B(0, -4), C(4, -4)$ **Position 1:** $(x, y) \rightarrow (y, -x)$ **Position 2:** $(x, y) \rightarrow (x, 2 - y)$ **Position 3:** $(x, y) \rightarrow (x, -y)$ **Position 4:** $(x, y) \rightarrow (x + 2, y)$ **Position 5:** $(x, y) \rightarrow (-x, -y)$	Cut out one game piece for each partner.

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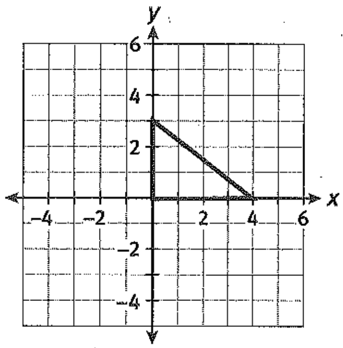
ACTIVITY 19 PRACTICE

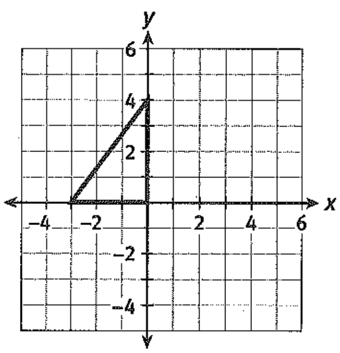
Write your answers on notebook paper. Show your work.

Lesson 19-1

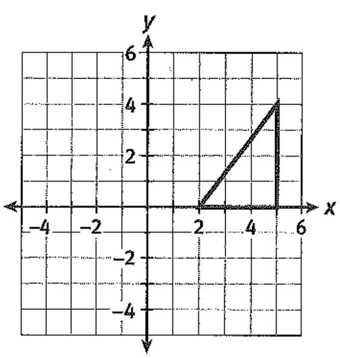
Each figure in Items 1–4 is an image of the figure shown on the coordinate plane below. Describe the transformations that were performed to obtain each image.



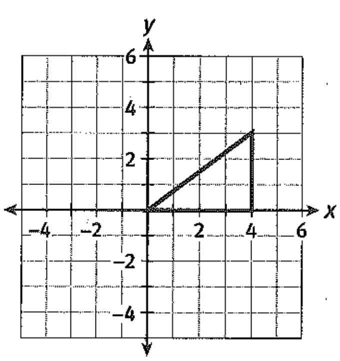
1.



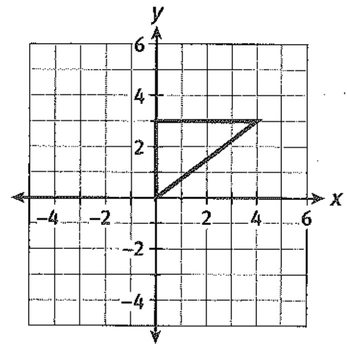
2.



3.



4.



5. Compare the figures in Items 1–4.

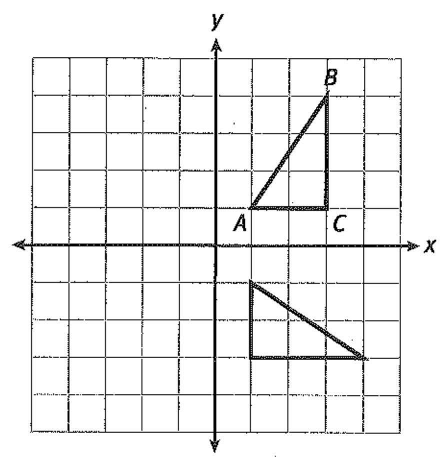
a. How do the areas of each figure compare to the area of the original figure?

b. What can you determine about the corresponding sides of each figure?

c. What can you determine about the corresponding angles of each figure?

d. Can you determine if the images of each figure are congruent to the original figure? Provide reasoning for your answer.

6. The coordinate plane below shows $\triangle ABC$ and a 90° clockwise rotation of $\triangle ABC$ about the origin.



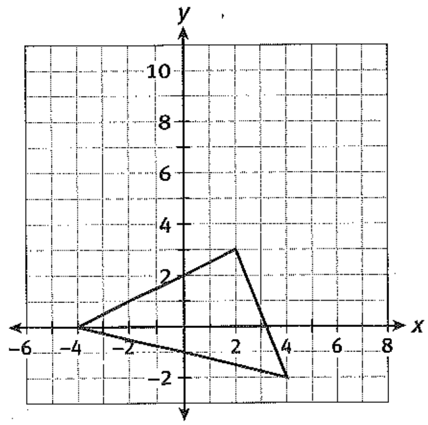
a. Sketch the 180° clockwise rotation of $\triangle ABC$.

b. Sketch the 90° counterclockwise rotation of $\triangle ABC$.

c. How do the images compare with $\triangle ABC$?

Lesson 19-2

7. The preimage of a triangle is shown on the coordinate plane.

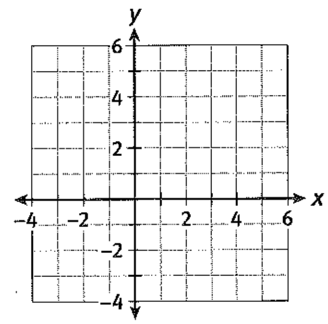


Which of the following types of transformation results in an image where corresponding angles and sides are NOT congruent?
A. reflection
B. rotation
C. translation
D. none of the above

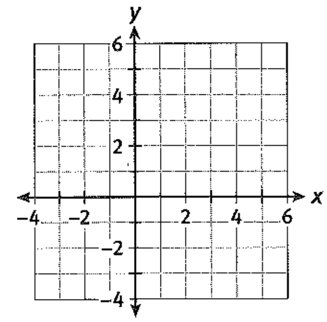
8. To create a logo, Henry transforms a quadrilateral by reflecting it over the x-axis, translating it 4 units up and then rotating the image 270° counterclockwise about the origin. Does the order in which Henry performs the transformations on the preimage change the size or shape of the image? Explain.

9. Figure 1 shows the preimage of a figure.

Figure 1



Image



Which of the following transformation(s) have been performed on Figure 1 to obtain the image?
A. Rotate 180°.
B. Shift down 2 units and reflect over the line $y = 2$.
C. Reflect over the x-axis and shift up 4 units.
D. Reflect over the y-axis and shift up 4 units.

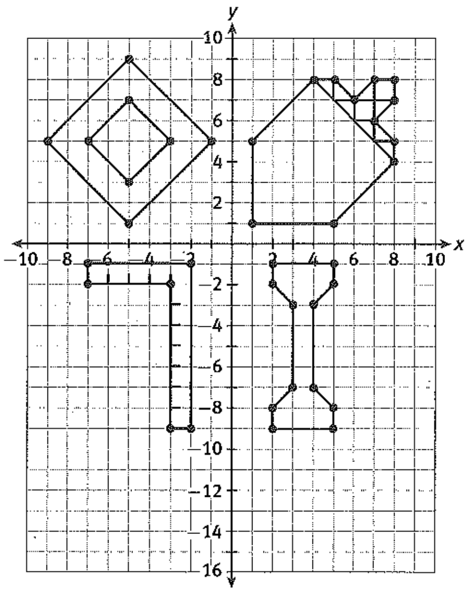
10. List two transformations and then name one transformation that gives the same result as the two transformations.

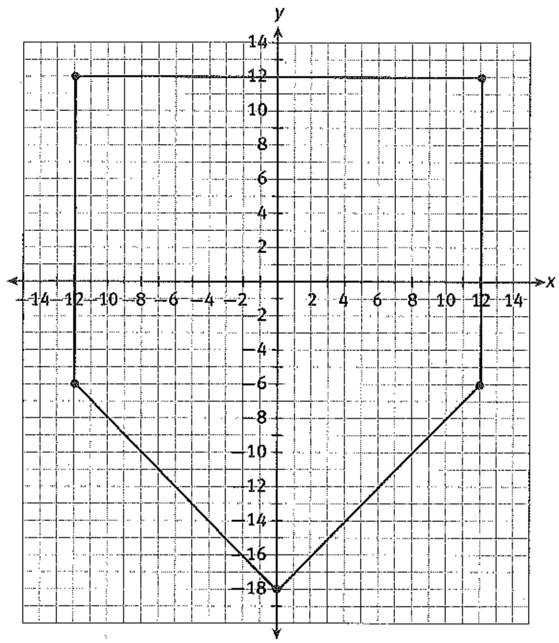
11. Find a translation that has the same effect as the composition of translations $(x, y) \rightarrow (x + 7, y - 2)$ followed by $(x, y) \rightarrow (x - 3, y + 2)$.

MATHEMATICAL PRACTICES
Reason Abstractly

12. How many and what types of reflections would have to be performed on a preimage to get the same image as a 180° rotation?

In medieval times, a person was rewarded with a coat of arms in recognition of noble acts. In honor of your noble acts so far in this course, you are being rewarded with a coat of arms. Each symbol on the coordinate plane below represents a special meaning in the history of heraldry.





Transform the figures from their original positions to their intended positions on the shield above using the following descriptions.

1. The **acorn** in Quadrant I stands for antiquity and strength and is also the icon used in the SpringBoard logo.
   a. Reflect the acorn over the x-axis. Sketch the image of the acorn on the shield.
   b. Write the symbolic representation of this transformation.
2. The **mascle** in Quadrant II represents the persuasiveness you have exhibited in justifying your answers.
   a. Rotate the mascle 270° counterclockwise about the origin. Sketch the image of the mascle on the shield.
   b. Write the symbolic representation of this transformation.
3. The **carpenter's square** in Quadrant III represents your compliance with the laws of right and equity. The location of the carpenter's square is determined by a composition of transformations. Rotate the carpenter's square 90° counterclockwise about the origin followed by a reflection over the y-axis.
   a. Copy and complete the table by listing the coordinates of the image after the carpenter's square is rotated 90° counterclockwise about the origin.
   b. Sketch the image of the carpenter's square after the composition of transformations described.

Preimage	Image
(−7, −1)	
(−7, −2)	
(−2, −1)	
(−3, −2)	
(−3, −9)	
(−2, −9)	

4. Finally, the **column** in Quadrant IV represents the determination and steadiness you've shown throughout your work in this course.
   a. Sketch the column using the transformation given by the symbolic representation $(x, y) \rightarrow (x - 9, y + 10)$.
   b. Write a verbal description of the transformation.

5. Explain why each of the symbols on your coat of arms is congruent to the preimage of the symbol on the original coordinate plane.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-b, 2a-b, 3a-b, 4a-b, 5)	• Clear and accurate understanding of reflections, rotations, and translations in the coordinate plane.	• An understanding of reflections, rotations, and translations in the coordinate plane with few errors.	• Partial understanding of reflections, rotations, and translations in the coordinate plane.	• Incorrect understanding of reflections, rotations, and translations in the coordinate plane.
Problem Solving (Items 1a-b, 2a-b, 3a-b, 4a-b)	• Interpreting a problem accurately in order to carry out a transformation.	• Interpreting a problem to carry out a transformation.	• Difficulty interpreting a problem to carry out a transformation.	• Incorrect or incomplete interpretation of a transformation situation.
Mathematical Modeling / Representations (Items 1a, 2a, 3a, 4a)	• Accurately transforming pre-images and drawing the images.	• Transforming pre-images and drawing the images with few, if any, errors.	• Difficulty transforming pre-images and drawing the images.	• Incorrectly transforming pre-images and drawing the images.
Reasoning and Communication (Items 4b, 5)	• A precise explanation of congruent transformations.	• An understanding of transformations that retain congruence.	• A confusing explanation of congruent transformations.	• An inaccurate explanation of congruent transformations.

Similar Triangles

Mirrors and Shadows

Lesson 20-1 Exploring Similarity

My Notes

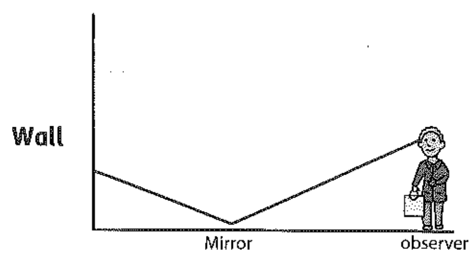
Learning Targets:

- Identify similar triangles.
- Identify corresponding sides and angles in similar triangles.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Visualization, Create Representations, Group Discussion

Thales of Miletus was a Greek philosopher, mathematician, and scientist who lived in 600 B.C.E. Two thousand six hundred years ago, he wondered about the height of the Great Pyramid in Egypt. Thales noticed that the sun's shadows fell from every object in the desert at the same angle, creating similar triangles from every object. Thales's research allowed him to use similar triangles to measure the height of the pyramids of Egypt and the distance to a ship at sea.

Thales used shadows in his work; however, a mirror placed on the floor can also be used to determine measures indirectly. When the mirror is placed at a particular distance from the wall, the distance that an observer stands from the mirror determines the reflection that the observer sees in the mirror.



1. Use the table on the next page to record results for each of the steps below.
   - Find a spot on the floor 20 feet away from one of the walls of your classroom.
   - Place a mirror on the floor 4 feet from that wall.
   - Each group member should take a turn standing on the spot 20 feet from the wall and look into the mirror. Other group members should help the observer locate the point on the wall that the observer sees in the mirror and then measure the height of this point above the floor.
   - Before moving the mirror, each group member should take a turn as the observer.
   - Repeat the same process by moving the mirror to locations that are 8 feet and 10 feet away from the wall.

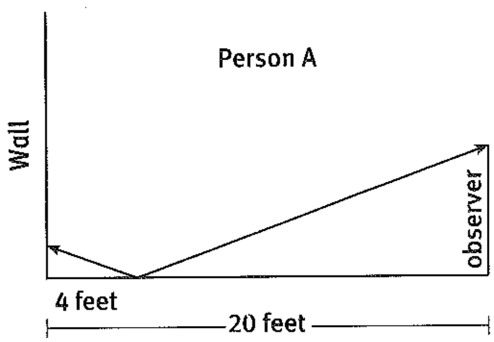
Distance from the Wall to the Mirror (in feet)	Height of the Point on the Wall Reflected in the Mirror (in feet)			
	Person A	Person B	Person C	Person D
4				
8				
10				

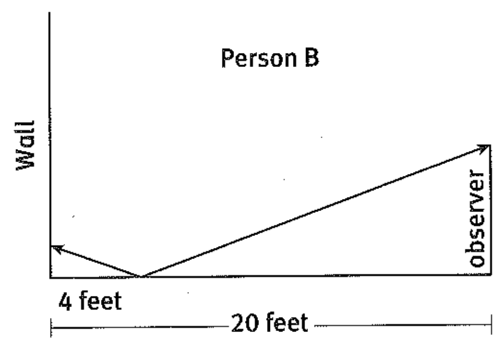
2. Measure the eye-level height for each member of the group and record it in the table below.

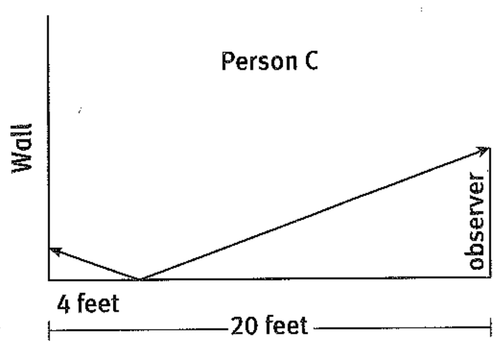
Eye-Level Height for Each Group Member			
Person A	Person B	Person C	Person D

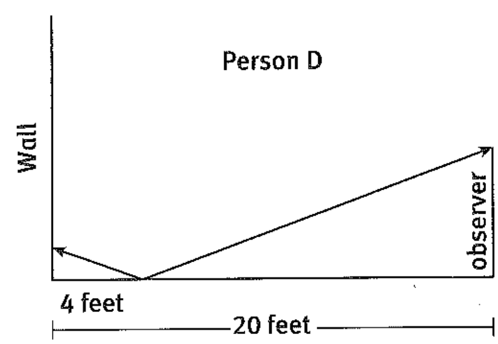
3. Consider the data collected when the mirror was 4 feet from the wall.

a. On the diagrams below, label the height of each group member and the height of the point on the wall determined by the group member.









My Notes

b. For each person in the group, determine the ratio of the height of the point on the wall to the eye-level height of the observer.

		Person A	Person B	Person C	Person D
Ratio of height of the point on the wall to eye level of observer	Ratio as a fraction				
	Ratio as a decimal				

c. Express regularity in repeated reasoning. What appears to be true about the ratios you found?

4. If the eye-level height of a five-year-old observer is 3.6 feet, what height can you predict for the point on the wall? Explain your reasoning.

5. Consider the data collected when the mirror was 8 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?

6. Consider the data collected when the mirror was 10 feet from the wall. For each group member, determine the ratio of the height of the point on the wall to the eye-level height of the observer. What appears to be true?

My Notes

MATH TERMS

Similar polygons are polygons in which the lengths of the corresponding sides are in proportion, and the corresponding angles are congruent.

MATH TERMS

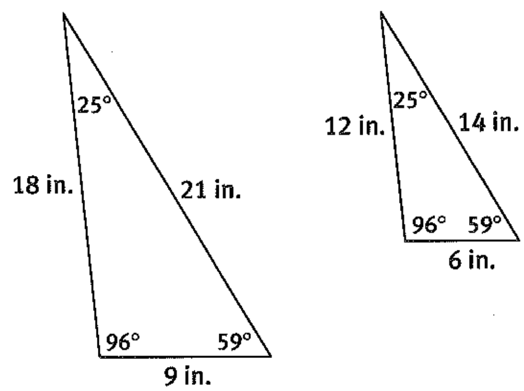
When two ratios are equivalent, then they form a **proportion**. For example, the ratios $\frac{3}{7}$ and $\frac{12}{28}$ are equivalent. Setting these ratios equal generates the proportion $\frac{3}{7} = \frac{12}{28}$.

WRITING MATH

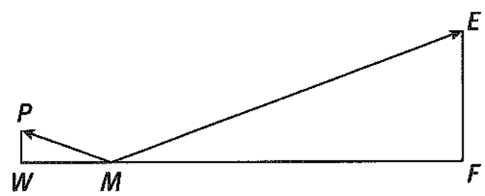
The symbol $\sim$ is used to denote two similar figures.

Similar polygons are polygons in which the lengths of the corresponding sides are in ***proportion***, and the corresponding angles are congruent.

For example, in the following triangles, the corresponding angles are congruent, and the corresponding sides are in proportion. Therefore, the triangles are similar.

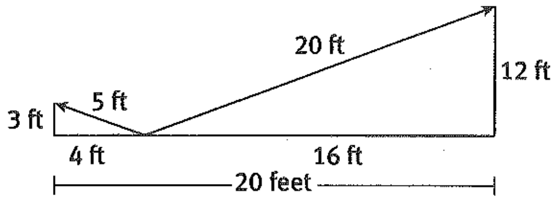


A similarity statement for the triangles below is $\triangle PWM \sim \triangle EFM$. A ***similarity statement*** indicates that the corresponding angles are congruent, and the corresponding sides are proportional.



My Notes

7. The diagram below shows two similar triangles like the triangles you worked with in Item 3.

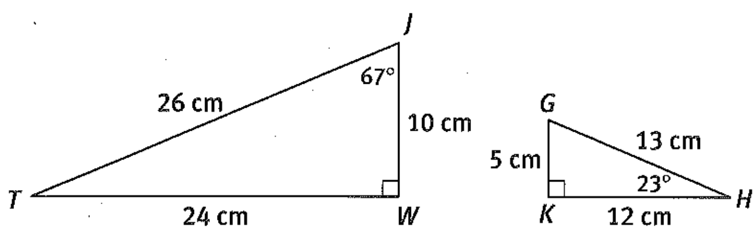


a. Use the lengths of the three pairs of corresponding sides to create three ratios in the form $\frac{\text{side length in small triangle}}{\text{corresponding length in large triangle}}$

b. Compare the ratios written in part a. Then explain how these ratios relate to the ratios you created in Item 3.

Check Your Understanding

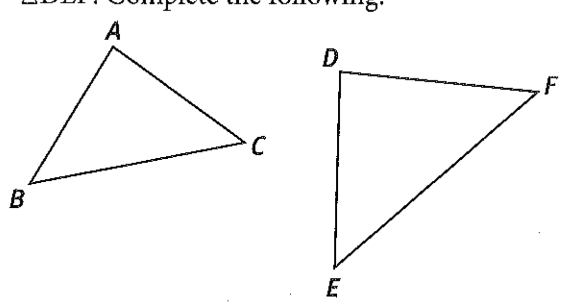
8. Are the triangles shown below similar? If so, explain why and write a similarity statement. If not, explain why not.



9. In the figure, $\triangle ABC \sim \triangle DEF$. Complete the following.

a. $m\angle F =$ _______

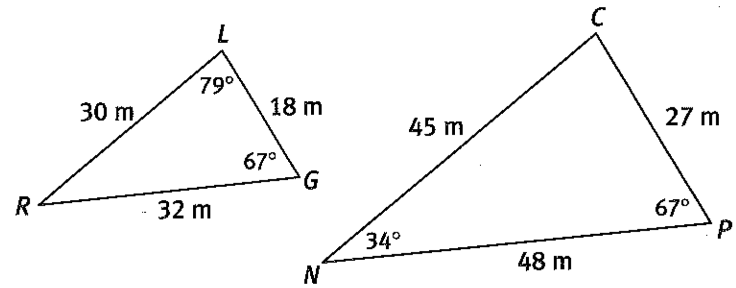
b. $\frac{AB}{DE} = \frac{\square}{DF}$



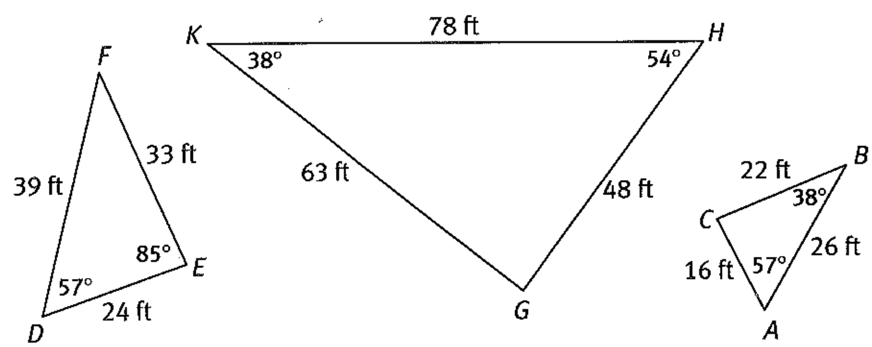
My Notes

LESSON 20-1 PRACTICE

10. Are the triangles below similar? If so, explain why and write a similarity statement. If not, explain why not.



Use the figure below for Items 11–13.



11. Identify the pair of similar triangles in the figure. Explain your answer.

12. Write a similarity statement for the triangles you identified in Item 11. Is there more than one correct way to write the statement?

13. What are the pairs of corresponding sides in the triangles you identified in Item 11?

14. **Construct viable arguments.** Malia is a jewelry designer. She created two silver triangles that she would like to use as earrings, but she is not sure if the two triangles are similar. One triangle has angles that measure 51° and 36°. The other triangle has angles that measure 36° and 95°. Is it possible to determine whether or not the triangles are similar? Justify your answer.

Learning Targets:

- Determine whether triangles are similar given side lengths or angle measures.
- Calculate unknown side lengths in similar triangles.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Identify a Subtask, Visualization

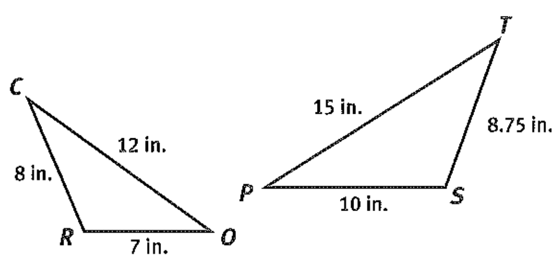
Two figures are similar if the corresponding angles are congruent and the corresponding sides are proportional. However, only one of these conditions is necessary in order to conclude that figures are congruent.

> **Necessary Condition for Similarity**
> When two triangles satisfy at least one of the following conditions, then they are similar.
> (1) The corresponding angles are congruent.
> (2) The corresponding sides are proportional.

The ratio of two corresponding sides of similar triangles is called the ***scale factor***.

1. What appears to be the scale factor for the similar triangles you created In Lesson 20-1 using the data collected when the mirror was 8 feet from the wall? Support your answer using corresponding sides for the similar triangles.

2. The triangles shown here are similar.



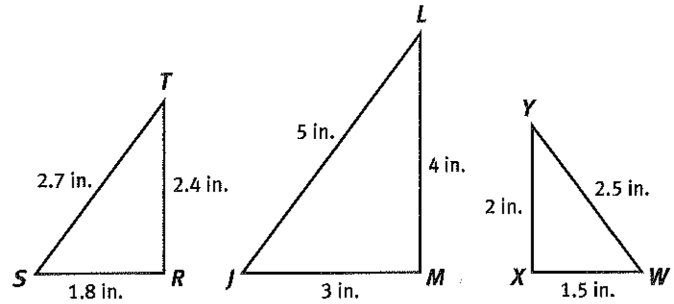
a. Name the transformation that can help you identify the corresponding parts of the triangles.

b. Write a similarity statement for the triangles.

c. Determine the scale factor for the two similar triangles. Show your calculations.

My Notes

3. Consider the three triangles below.



a. Compare ratios to identify any similar triangles.

b. Write a similarity statement to identify the similar triangles.

c. State the scale factor for the similar triangles.

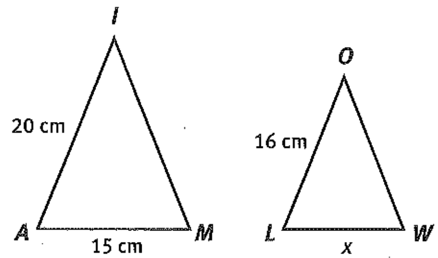
d. What are the pairs of corresponding angles of the similar triangles?

My Notes

The scale factor can be used to determine an unknown side length in similar figures.

Example A

Solve for x if $\triangle AIM \sim \triangle LOW$.



Step 1: Find the scale factor using known corresponding lengths.
The scale factor is $\frac{20\text{ cm}}{16\text{ cm}}$ or $\frac{5}{4}$.

Step 2: Write a proportion using the scale factor.

$$\frac{5}{4} = \frac{15\text{ cm}}{x}$$

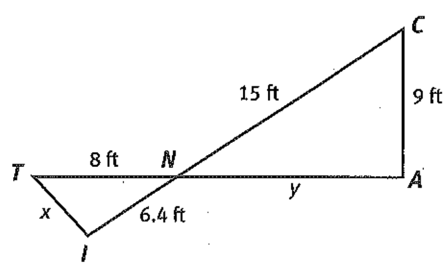
Step 3: Solve the proportion.

$$5x = 60.$$
$$x = 12$$

Solution: $x = 12$ cm

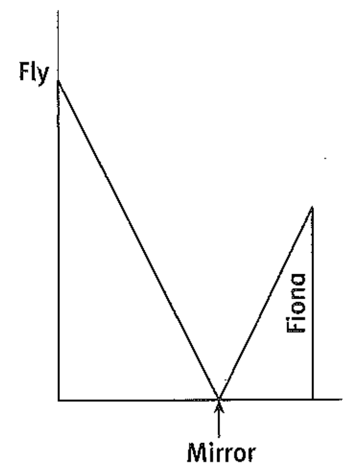
Try These A

Given $\triangle TIN \sim \triangle CAN$.



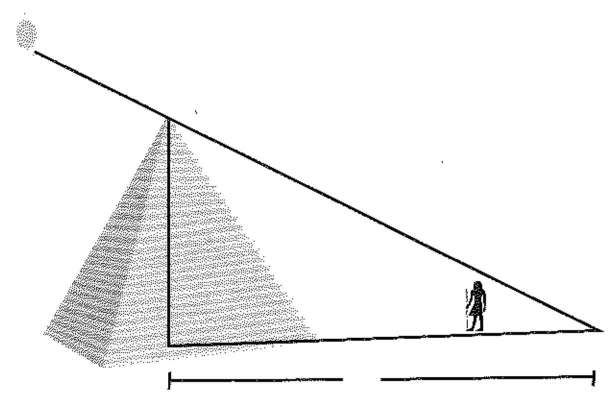
a. Determine the scale factor.
b. Solve for x and y.

4. Suppose that a fly has landed on the wall and a mirror is lying on the floor 5 feet from the base of the wall. Fiona, whose eye-level height is 6 feet, is standing 3 feet away from the mirror and 8 feet away from the wall. She can see the fly reflected in the mirror.
   a. Use the information provided to label the distances on the diagram.



   b. Show how to use the properties of similar triangles to calculate the distance from the floor to the observed fly.

5. **Model with mathematics.** In his research, Thales determined that the height of the Great Pyramid could easily be calculated by the using the length of its shadow relative to the length of Thales's own shadow. Assume Thales was 6 feet tall and the shadow of the pyramid was 264 feet at the same time the shadow of Thales was 3.5 feet.
   a. Using these data, label the distances on the diagram.



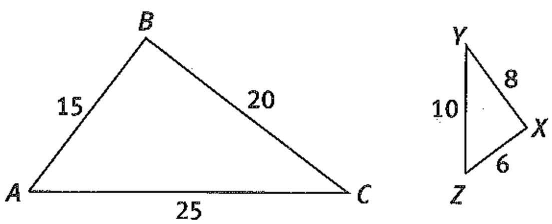
   b. Determine the height of the Great Pyramid. Round your answer to the nearest tenth.

6. In $\triangle JKL$, $m\angle J = 32°$ and $m\angle K = 67°$. In $\triangle PQR$, $m\angle P = 32°$ and $m\angle Q = 67°$. Is $\triangle JKL \sim \triangle PQR$? Explain.

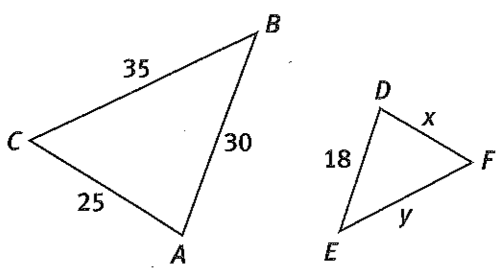
My Notes

Check Your Understanding

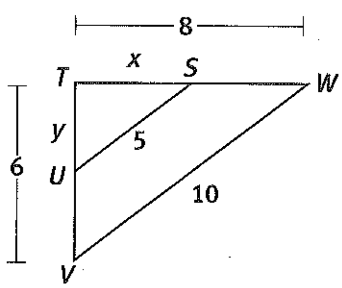
7. Are the two triangles shown below similar? If so, write a similarity statement and determine the scale factor. If not, explain why not.



8. Given $\triangle ABC \sim \triangle DEF$. Determine the value of x and y.



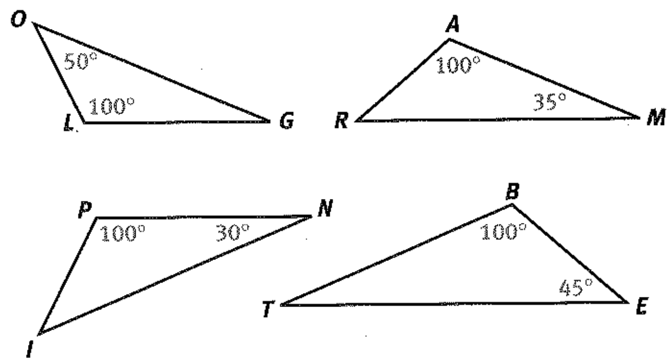
9. Given $\triangle TUS \sim \triangle TVW$. Determine the value of x and y.



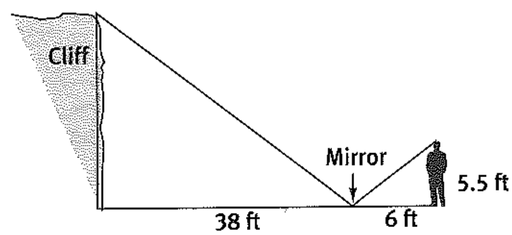
My Notes

LESSON 20-2 PRACTICE

10. Write similarity statements to show which triangles are similar.



11. Before rock climbing to the top of a cliff, Chen wants to know how high he will climb. He places a mirror on the ground and walks backward until he sees the top of the cliff in the mirror, as shown in the figure. What is the height of the cliff?



12. Given: $\triangle ABC \sim \triangle RST$
$AB = 44$ in., $BC = 33$ in., and $AC = 22$ in.
$RS = 20$ in. and $ST = 15$ in.
Find RT.

13. If two triangles are similar, how does the ratio of their perimeters compare to the scale factor? Use an example to justify your answer.

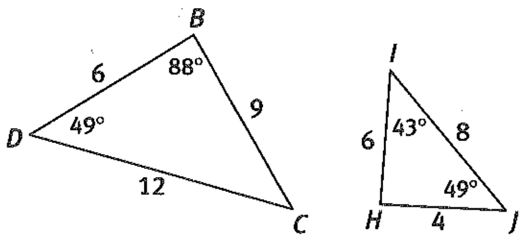
14. **Critique the reasoning of others.** Lucas claims, "If triangles have two pairs of congruent corresponding angles, then the third angles must also be congruent and the triangles must be similar." Is Lucas correct? Justify your answer.

ACTIVITY 20 PRACTICE

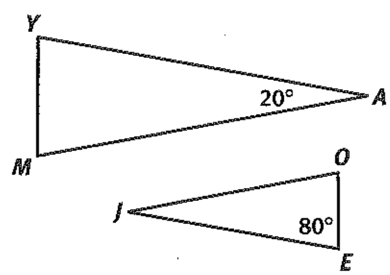
Write your answers on notebook paper. Show your work.

Lesson 20-1

1. Determine whether the triangles are similar. If so, write a similarity statement. If not, explain why not.



2. If $\triangle JOE \sim \triangle AMY$, find the measure of each of the following angles.



   a. $m\angle J$ b. $m\angle O$
   c. $m\angle Y$ d. $m\angle M$

3. $\triangle ABC$ has side lengths 15 cm, 20 cm, and 25 cm. What could be the side lengths of a triangle similar to $\triangle ABC$?
   A. 7 m, 8 m, and 9 m
   B. 6 m, 8 m, and 10 m
   C. 5 cm, 10 cm, and 15 cm
   D. 30 mm, 40 mm, and 55 mm

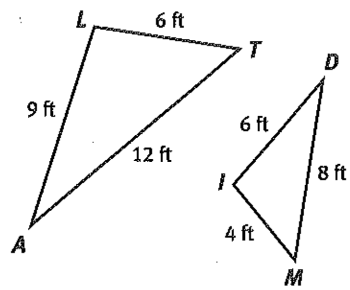
4. In $\triangle PQR$, $m\angle P = 27°$ and $m\angle R = 61°$. In $\triangle XYZ$, $m\angle Y = 92°$.
   a. Is it possible for $\triangle PQR$ to be similar to $\triangle XYZ$? Explain your reasoning.
   b. Can you conclude that $\triangle PQR$ is similar to $\triangle XYZ$? Why or why not?

5. Given that $\triangle ABC$ is similar to $\triangle GHJ$, which of the following statements must be true?
   A. Both triangles have the same side lengths.
   B. If $\triangle ABC$ has a right angle, then $\triangle GHJ$ has a right angle.
   C. The perimeter of $\triangle ABC$ is greater than the perimeter of $\triangle GHJ$.
   D. If $\triangle ABC$ has a side of length 2 cm, then $\triangle GHJ$ has a side of length 2 cm.

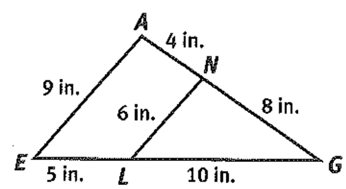
Lesson 20-2

For Items 6 and 7, determine whether the triangles shown are similar. If so, write a similarity statement for the triangles and determine the scale factor. If not, explain why not.

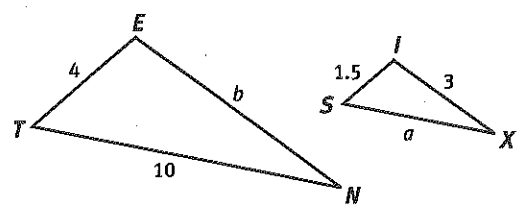
6.



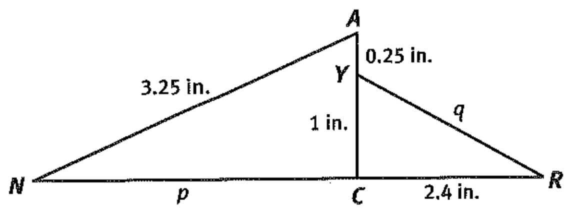
7.



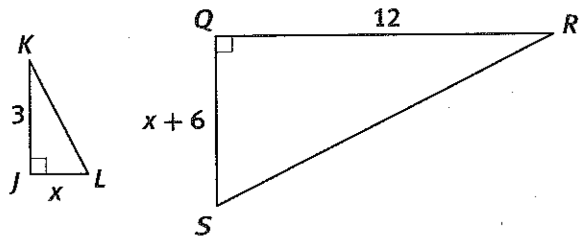
8. Given $\triangle SIX \sim \triangle TEN$, find a and b.



9. Given $\triangle CAN \sim \triangle CYR$, find p and q.



10. Given: $\triangle JKL \sim \triangle QRS$. Determine the value of x.



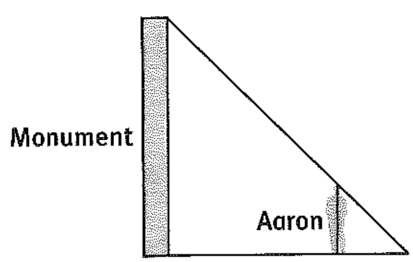
11. $\triangle MON \sim \triangle WED$, $m\angle M = 37°$, and $m\angle E = 82°$. Find the measure of each of the following angles.

a. $\angle O$ **b.** $\angle W$
c. $\angle N$ **d.** $\angle D$

12. Tell the measure of each angle of $\triangle ABC$ and $\triangle PQR$ if $\triangle ABC \sim \triangle PQR$, $m\angle A = 90°$, and $m\angle B = 56°$.

13. Aaron is 6.25 ft tall, and he casts a shadow that is 5 ft long. At the same time, a nearby monument casts a shadow that is 25 ft long.

a. Copy the figure and label the dimensions on the figure.



b. Determine the height of the monument.

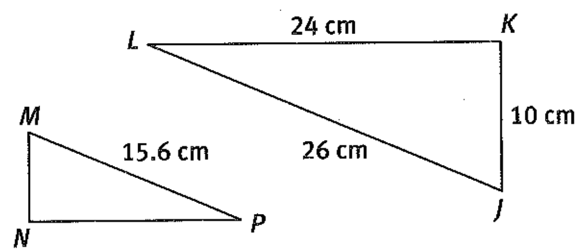
14. $\triangle ABC \sim \triangle DEF$ and the scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{4}{3}$. If $AB = 60$, what is DE?

15. Sonia is 124 centimeters tall and casts a shadow that is 93 centimeters long. She is standing next to a tree that casts a shadow that is 135 meters long. How tall is the tree?

16. $\triangle STU \sim \triangle XYZ$, $ST = 6$, $SU = 8$, $XZ = 12$, and $YZ = 15$. What is the scale factor of $\triangle STU$ to $\triangle XYZ$?

A. $\frac{2}{5}$ **B.** $\frac{1}{2}$
C. $\frac{8}{15}$ **D.** $\frac{2}{3}$

17. In the figure, $\triangle JKL \sim \triangle MNP$. What is the perimeter of $\triangle MNP$?



18. $\triangle ABC \sim \triangle DEF$. $AB = 12$, $AC = 16$, $DE = 30$, and $DF = x + 5$. What is the value of x?

A. 30 **B.** 35
C. 40 **D.** 45

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

19. An equiangular triangle is a triangle with three congruent angles. Explain why all equiangular triangles are similar.

Dilations

Alice's Adventures in Shrinking and Growing

Lesson 21-1 Stretching and Shrinking Geometric Figures

My Notes

Learning Targets:

- Investigate the effect of dilations on two-dimensional figures.
- Explore the relationship of dilated figures on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Predict and Confirm, Create Representations, Visualization

In the story *Alice's Adventures in Wonderland* written by Lewis Carroll, Alice spends a lot of time shrinking and growing in height. The height changes occur when she drinks a potion or eats a cake.

1. Complete the table by finding Alice's new height after she eats each bite of cake or drinks each potion.

Starting Height (inches)	Change in Height	New Height (inches)
56	$\frac{1}{8}$ times as tall	
60	$\frac{2}{5}$ times as tall	
60	1.5 times as tall	
24	$\frac{5}{3}$ times as tall	
30	2.2 times as tall	

2. Each change in height resulted in a decrease or increase to Alice's starting height.

a. Alice's starting height decreased when it was multiplied by which two factors?

CONNECT TO LITERATURE

Alice's Adventures in Wonderland is a novel published in 1865. It is the story of a young girl named Alice who wants to escape being bored by adulthood. In a dream she follows a white rabbit and falls down a deep tunnel and the adventure begins.

b. Write a conjecture regarding the number you multiply by to decrease Alice's height.

c. Confirm your conjecture by providing two additional examples that show that Alice's starting height decreases.

d. Write a conjecture regarding the number you multiply by to increase Alice's starting height.

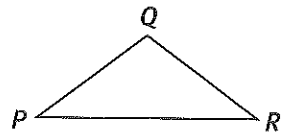
e. Confirm your conjecture regarding Alice's increase in height by providing two additional examples that show that Alice's starting height increases.

Alice's height changes—shrinking and growing—are a type of transformation known as a dilation.

My Notes

A ***dilation*** is a transformation where the image is *similar* to the preimage; the size of the image changes but the shape stays the same.

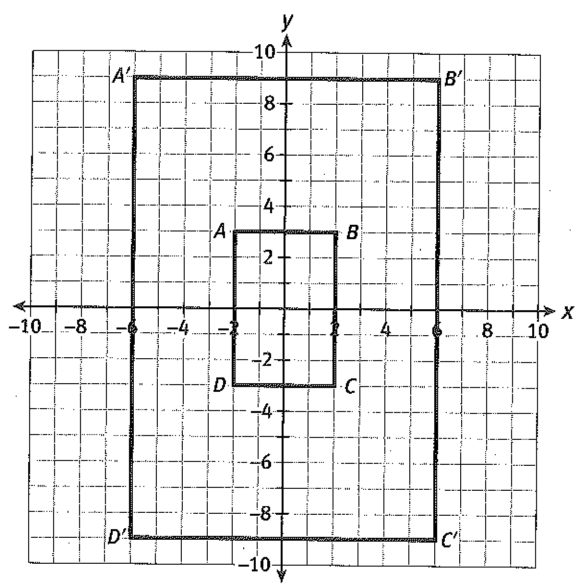
3. Use appropriate tools strategically. Given the preimage of $\triangle PQR$ below, use a ruler to draw the image of $\triangle PQR$ if it is dilated:



a. by a factor of 2

b. by a factor of $\frac{1}{2}$

4. Rectangles $ABCD$ and $A'B'C'D'$ are shown on the coordinate plane with the ***center of dilation*** at the origin, O.



MATH TERMS

A **dilation** is a transformation that changes the size but not the shape of an object.

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move.

My Notes

a. Determine the length of each side of rectangles $ABCD$ and $A'B'C'D'$.

Side	Length (in units)
$\overline{AB}$	
$\overline{BC}$	
$\overline{CD}$	
$\overline{AD}$	

Side	Length (in units)
$\overline{A'B'}$	
$\overline{B'C'}$	
$\overline{C'D'}$	
$\overline{A'D'}$	

b. Describe the relationship between the side lengths of rectangle $ABCD$ and rectangle $A'B'C'D'$.

c. Determine the coordinates of each of the vertices of both rectangles.

Rectangle $ABCD$		Rectangle $A'B'C'D'$	
A		A'	
B		B'	
C		C'	
D		D'	

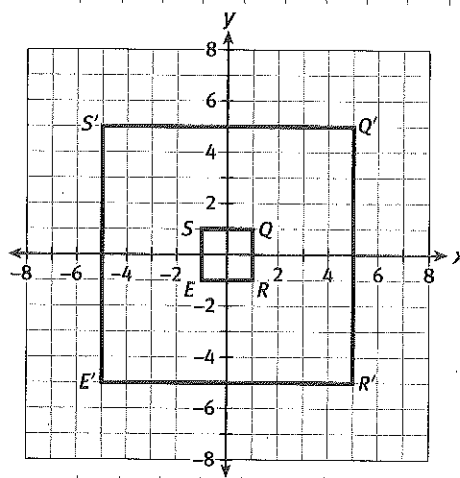
d. Describe the relationship between the coordinates of the vertices of $ABCD$ and the coordinates of the vertices of $A'B'C'D'$.

e. The point $\left(\frac{1}{3}, -3\right)$ is a point on rectangle $ABCD$. What are the coordinates of the image of the point on $A'B'C'D'$? Explain how you determined your answer.

My Notes

Example A

Quadrilateral $SQRE$ is dilated to quadrilateral $S'Q'R'E'$ as shown on the coordinate plane. What is the relationship between the side lengths, perimeter, and area of the two figures?



Step 1: Compare the side lengths of corresponding sides of quadrilateral $S'Q'R'E'$ to quadrilateral $SQRE$.

$$\frac{S'Q'}{SQ} = \frac{10}{2} = \frac{5}{1};\ \frac{S'E'}{SE} = \frac{10}{2} = \frac{5}{1}$$

$$\frac{E'R'}{ER} = \frac{10}{2} = \frac{5}{1};\ \frac{R'Q'}{RQ} = \frac{10}{2} = \frac{5}{1}$$

The side lengths of quadrilateral $S'Q'R'E'$ are 5 times as great as the side lengths of quadrilateral $SQRE$.

Step 2: Find the perimeter of each quadrilateral. Then write the ratio of the perimeter of quadrilateral $S'Q'R'E'$ to the perimeter of quadrilateral $SQRE$.

Perimeter of quadrilateral $SQRE = 8$ units
Perimeter of quadrilateral $S'Q'R'E' = 40$ units

$$\text{ratio}: \frac{\text{Perimeter of } S'Q'R'E'}{\text{Perimeter of } SQRE} = \frac{40}{8} = \frac{5}{1}$$

Solution: The perimeter of quadrilateral $S'Q'R'E'$ is 5 times as great as that of quadrilateral $SQRE$.

Step 3: Find the area of each quadrilateral. Then write the ratio of the area of quadrilateral $S'Q'R'E'$ to the area of quadrilateral $SQRE$.

Area of quadrilateral $SQRE = 4$ square units
Area of quadrilateral $S'Q'R'E' = 100$ square units

$$\text{ratio}: \frac{\text{Area of } S'Q'R'E'}{\text{Area of } SQRE} = \frac{100}{4} = \frac{25}{1}$$

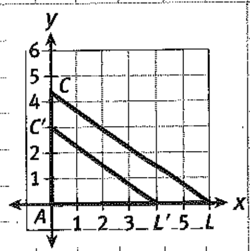
Solution: The area of quadrilateral $S'Q'R'E'$ is 25 times as great as that of quadrilateral $SQRE$.

READING MATH

The fraction bar in a ratio is read aloud as "to." For example, the ratio $\frac{4}{1}$ is read as "4 to 1."

As you discuss Example A, make notes about the notation and vocabulary used so you can review them later to aid your understanding of dilating geometric figures.

My Notes



MATH TIP

The area of a triangle can be found using the formula
Area $= \frac{1}{2}$ base × height.

In a right triangle, the legs can be used as the base and height.

Try These A

Triangle ALC is dilated to $\triangle AL'C'$ as shown on the coordinate plane. Triangle ALC has vertices $A(0, 0)$, $L(6, 0)$, $C(0, 4\frac{1}{2})$. The length of $\overline{C'L'}$ is 5 units.

a. Substitute known values into the proportion to find the length of $\overline{CL}$.

$$\frac{LA}{L'A} = \frac{CL}{C'L'}$$

b. Determine the ratio of the perimeter of $\triangle AL'C'$ to the perimeter of $\triangle ALC$.

c. Determine the ratio of the area of $\triangle AL'C'$ to the area of $\triangle ALC$.

Check Your Understanding

5. Triangle ABC is dilated to $\triangle A'B'C'$. The ratio of the perimeter of $\triangle A'B'C'$ to the perimeter of $\triangle ABC$ is $\frac{4}{1}$. Explain how you can use this information to determine if the image has a larger or smaller perimeter than the preimage.

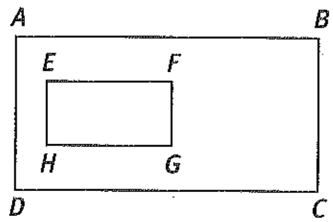
6. Square $TUVW$ is enlarged to form square $T'U'V'W'$. What must be true about the relationship between corresponding sides for the enlargement to be considered a dilation?

7. **Reason abstractly.** Bradley states that in theory circles with different diameters are all dilations of each other. Susan states that in theory rectangles with different side lengths are all dilations of each other. Do you agree with either, both, or neither statement? Explain your reasoning.

My Notes

LESSON 21-1 PRACTICE

8. Rectangle *ABCD* is dilated to the rectangle *EFGH*. It is given that $AB = 48$ ft, $BC = 24$ ft, and $FG = 10$ ft.



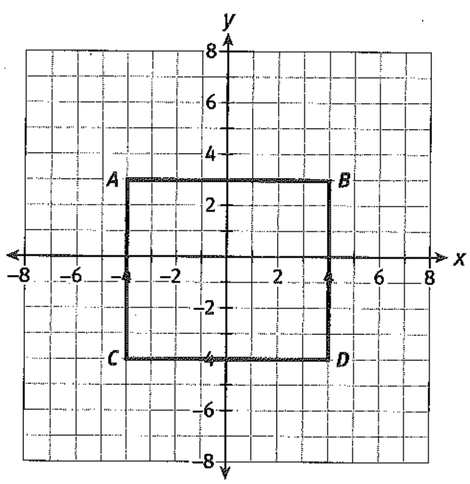
a. Determine the ratio between corresponding side lengths.

b. Explain how knowing the ratio of corresponding side lengths helps you to determine the length of $\overline{EF}$.

c. Find the length of $\overline{EF}$.

9. A right triangle has vertices $A(0, 0)$, $B(10, 0)$, and $C(10, 24)$. The triangle is dilated so that the ratio between corresponding side lengths of the preimage to the image is $\frac{3}{1}$. Explain the effect on the area and perimeter of the dilated triangle.

10. Reason quantitatively. Figure *ABCD* is shown on the coordinate plane. Suppose a graphic designer wants to dilate the figure so that the resulting image has a smaller area than figure *ABCD*. Describe a way the designer can achieve this type of dilation.



11. Construct viable arguments. Alice's teacher explains that all circles are similar and asks the class to investigate relationships between a circle with radius 4 cm and a circle with radius 6 cm. Dante claims that the ratio of the areas of the circles is $\frac{4}{9}$, while Louisa claims that the ratio of the areas is 2.25 to 1. Who is correct? Give evidence to support the claim.

My Notes

Learning Targets:

- Determine the effect of the value of the scale factor on a dilation.
- Explore how scale factor affects two-dimensional figures on a coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Graphic Organizer, Create Representations

In the story *Alice's Adventures in Wonderland*, when Alice drinks a potion or eats a cake, she physically becomes taller or shorter, depending on a given factor. When this height change occurs, Alice changes size, but she does not change shape. Each dimension of her body is proportionally larger or smaller than her original self.

The factor by which Alice's height is changed, or dilated, is known as a **scale factor**.

The ***scale factor of dilation***, typically represented by the variable k, determines the size of the image of a dilated figure.

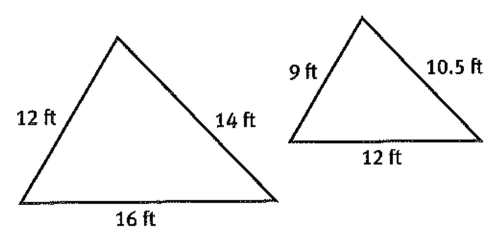
If $0 < k < 1$, then the image will be smaller than the original figure. In this case, the dilation is called a **reduction**.

If $k > 1$, then the image will be larger than the original figure, and dilation is called an **enlargement**.

MATH TERMS

The **scale factor of dilation** is the factor by which each linear measure of the figure is multiplied.

1. Consider the similar triangles shown.



a. By what scale factor is the smaller triangle enlarged? Explain why the factor given must result in an enlargement.

b. By what scale factor is the larger triangle reduced? Explain why the factor given must result in a reduction.

c. What is the relationship between the two scale factors?

My Notes

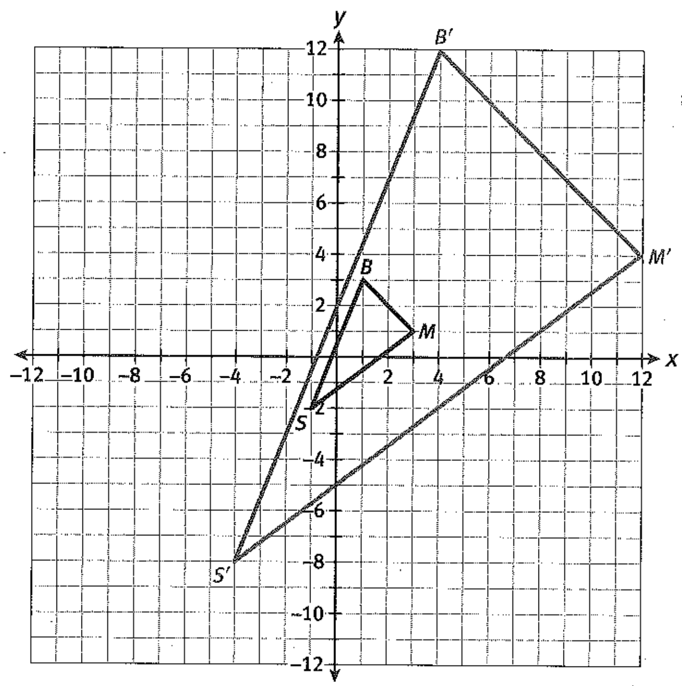
2. Suppose a point with coordinates (x, y) is a vertex of a geometric figure and that figure is dilated by a scale factor of k with the ***center of dilation*** at the origin.
   a. Create an ordered pair to represent the coordinates of the corresponding point on the image.
   b. Predict the size of the image as it compares to the preimage if k is 10.
   c. Predict the size of the image as it compares to the preimage if k is 0.5.

MATH TERMS

The **center of dilation** is a fixed point in the plane about which all points are expanded or reduced. It is the only point under a dilation that does not move. The center of dilation determines the location of the image.

Example A

Triangle $S'B'M'$ is a dilation of $\triangle SBM$ with a scale factor of 4. Using the coordinates of the vertices of $\triangle SBM$, determine the coordinates of the vertices of $\triangle S'B'M'$. Then plot $\triangle S'B'M'$ on the coordinate plane.



Step 1: Determine if the dilation is a reduction or enlargement.
Since the scale factor is 4 and $4 > 1$, the dilation is an enlargement.

Step 2: Multiply the coordinates of the vertices of $\triangle SBM$ by the scale factor.

$\triangle SBM$: $S(-1, -2)$, $B(1, 3)$, $M(3, 1)$
Multiply each coordinate by 4.
$\triangle S'B'M'$: $S'(-4, -8)$, $B'(4, 12)$, $M'(12, 4)$

Step 3: Plot the coordinates of the vertices of $\triangle S'B'M'$ on the coordinate plane.

My Notes

Try These A

a. Suppose the scale factor of dilation of $\triangle SBM$ in Example A is $\frac{1}{2}$. Determine if the resulting image, $\triangle S'B'M'$, will be a reduction or an enlargement of $\triangle SBM$. Then, determine the coordinates of $\triangle S'B'M'$.

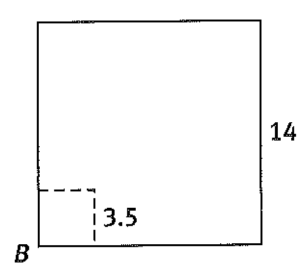
b. Figure $A'B'C'D'$ is a dilation of figure $ABCD$ with a scale factor of 5. Given the coordinates of the vertices of $A(0, 0)$, $B(0, 2)$, $C(-2, -2)$, $D(-2, 0)$, determine the coordinates of the vertices of figure $A'B'C'D'$.

Check Your Understanding

3. Compare the ratio of the side lengths of figure $A'B'C'D'$ and figure $ABCD$ to the scale factor in Try These part b. Make a conjecture about the ratio of side lengths of dilated figures and the scale factor of dilation.
4. Triangle $P'Q'R'$ is a dilation image of $\triangle PQR$. The scale factor for the dilation is 0.12. Is the dilation an enlargement or a reduction? Explain.
5. **Make use of structure.** A geometric figure contains the point (0, 0) and is dilated by a factor of m with the center at the origin. What changes will occur to the point (0, 0)?

The scale factor of dilation describes the size change from the original figure to the image. The scale factor can be determined by comparing the ratio of corresponding side lengths.

6. The solid line figure shown is a dilation of the figure formed by the dashed lines. Describe a method for determining the scale factor used to dilate the figure.



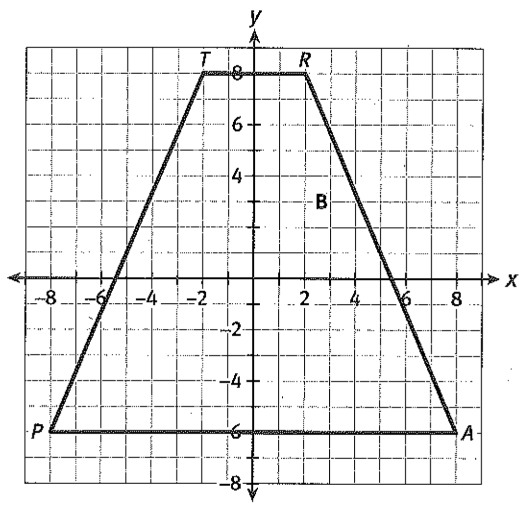
My Notes

7. Critique the reasoning of others. Josie found the scale factor in Item 6 to be $\frac{1}{4}$. Explain why Josie got the wrong scale factor.

There exists a relationship between the area of dilated figures and the perimeter of dilated figures.

8. Make a prediction about the effect of the scale of dilation on the area and perimeter of two figures.

9. Trapezoid *TRAP*, shown on the coordinate plane, has vertices $(-2, 8)$, $(2, 8)$, $(8, -6)$, $(-8, -6)$. Suppose trapezoid TRAP is dilated by a scale factor of $\frac{1}{4}$.



a. Plot and label the vertices of the image $T'R'A'P'$.

b. Determine the area of trapezoids *TRAP* and $T'R'A'P'$.

c. What is the ratio of the area of *TRAP* to the area of $T'R'A'P'$?

d. Reason quantitatively. Make a conjecture about the relationship between scale factor of dilation and the area of dilated figures.

MATH TIP

The area of a trapezoid can be found using the formula Area $= \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases.

My Notes

Check Your Understanding

10. Suppose a polygon is dilated by a scale factor of k. Write an expression for the ratio of the perimeters. Then, write an expression to represent the ratio of the areas.

11. A triangle is dilated by a scale factor of $\frac{2}{5}$.

a. What is the ratio of the perimeters?

b. What is the ratio of the areas?

12. **Construct viable arguments.** Suppose that a dilation is executed with a scale factor of 1. How would the preimage relate to the image? Using an example, justify your answer.

LESSON 21-2 PRACTICE

13. A rectangle has a perimeter of 24 ft. Following a dilation, the new perimeter of the rectangle is 36 ft.

a. Determine the scale factor of dilation.

b. What is the ratio of the areas?

14. A triangle has an area of 40 cm^2. Following a dilation, the new area of the triangle is 360 cm^2. What is the scale factor of dilation?

15. The vertices of trapezoid $ABCD$ are $A(-1, -1)$, $B(-1, 1)$, $C(2, 2)$, and $D(2, -1)$.

a. Draw the trapezoid and its dilation image for a dilation with center (0, 0) and scale factor 3.

b. Determine the ratio of the perimeter.

c. Determine the ratio of the areas.

16. **Make sense of problems.** Eye doctors dilate patients' pupils to get a better view inside the eye. If a patient's pupil had a 3.6-mm diameter before dilation and 8.4-mm diameter after dilation, determine the scale factor used to dilate the pupil. Explain why this created an enlargement.

MATH TIP

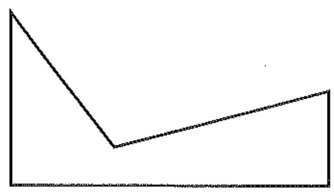
The area of a circle can be found using the formula Area $= \pi r^2$, where r is the radius of the circle.

ACTIVITY 21 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 21-1

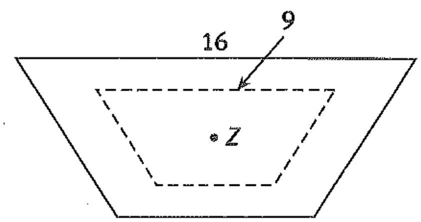
1. **Use appropriate tools strategically.** Sketch the dilation of the image of the figure below using a scale factor of $\frac{2}{3}$.



2. Does the size of a preimage increase or decrease when
   a. dilated by a factor greater than 1?
   b. dilated by a factor between 0 and 1?

3. The ratio of the area of $\triangle X'Y'Z'$ to the area of $\triangle XYZ$ is $\frac{2}{9}$. Explain how you can use this information to determine if the image is greater or smaller in area than the preimage.

4. The solid line figure is a dilation of the dashed line figure. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.



5. Explain how dilations are different from other types of transformations you have studied.

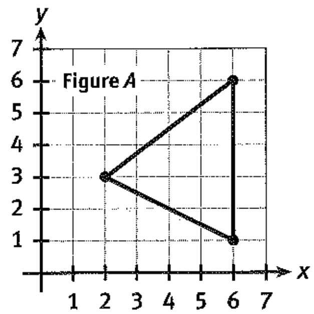
6. If the radius of a circle is 24 ft, how many circles can be the dilations of this circle? Why?

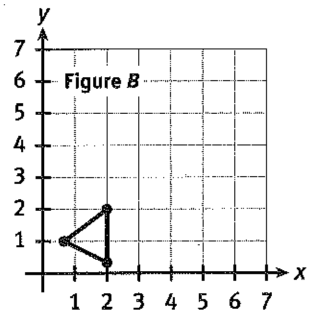
Lesson 21-2

7. A dilation has a center (0, 0) and scale factor 1.5. What is the image of the point (−3, 2)?

8. A triangle has vertices (−1, 1), (6, −2), and (3, 5). If the triangle is dilated with a scale factor of 3, which of the following are the vertices of the image?
   A. (−3, 3), (18, −6), (9, 15)
   B. (3, 3), (18, 6), (9, 15)
   C. (−3, 3), (18, 6), (9, 15)
   D. (3, 3), (18, −6), (9, 15)

9. Figure B is the result of a dilation of Figure A.



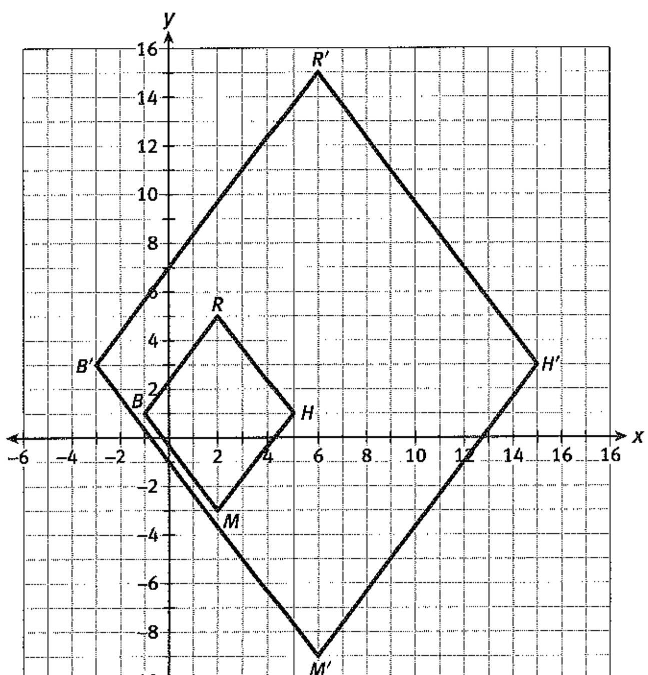


What is the scale factor of dilation?

A. 3 **B.** 2

C. $\frac{1}{3}$ **D.** $\frac{1}{2}$

10. Rhombus $RHMB$ has vertices (2, 5), (5, 1), (2, −3), and (−1, 1). This figure has been dilated to rhombus $R'H'M'B'$, as shown on the coordinate plane.



The area of rhombus $RHMB$ is 24 square units. Which of the following is the area of rhombus $R'H'M'B'$?

A. 216 square units
B. 72 square units
C. 8 square units
D. 2.7 square units

11. The diagonals of rhombus $ABCD$ are 6 ft and 8 ft. Rhombus $ABCD$ is dilated to rhombus $RSTU$ with the scale factor 8. What is the perimeter of rhombus $RSTU$?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

12. The endpoints of $\overline{AB}$ are $A(78, 52)$ and $B(26, -52)$. $\overline{AB}$ is dilated to $\overline{GH}$ with endpoints at $G(30, 20)$ and $H(10, -20)$. Then, $\overline{GH}$ is dilated to $\overline{PQ}$ with endpoints at $P(42, 28)$ and $Q(14, -28)$. If $\overline{AB}$ is dilated directly to $\overline{PQ}$, what will be the scale factor?