EE 309/310:- Term Project

FINAL REPORT

Voltage Stability Assessment in Real-Time (VSI)

Base Paper:- Phasor Measurement Sensor-Assisted Time-to-Collapse Estimation Under Long-Term Voltage Instability of Smart Grids

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CONTENTS

1. Introduction

The increasing complexity of modern smart grids underscores the urgent need for precise and rapid real-time monitoring to ensure reliable operations. Synchrophasor measurement sensors, regarded as the most advanced sensing devices currently available, offer highly accurate phasor data with swift refresh rates. This capability enables operators to gain a better understanding of evolving system dynamics. This report presents a model-free methodology for evaluating long-term voltage stability using real-time phasor data. The approach starts with the optimization of sensor placement to achieve full system observability while minimizing the number of devices required. Phasor measurements from these strategically selected sensors are then processed through a regression-based framework based on the Ornstein–Uhlenbeck process, facilitating the real-time construction of a scaled system Jacobian matrix.

Once the Jacobian is estimated in real-time, modal analysis is conducted to identify critical system modes, and two innovative voltage stability indices are introduced to monitor the system's health. The effectiveness of the proposed methodology is assessed through simulations on standard benchmark networks, including the IEEE 14-bus and New England 39-bus systems, under various scenarios such as dynamic load increases, line outages, and generator failures. Results confirm that this technique can accurately and swiftly predict conditions leading to voltage collapse while requiring minimal measurement infrastructure, thus providing a cost-effective and scalable solution for enhancing the resilience of future smart grids.

2. Objective

- ❖ To utilize synchrophasor measurement sensors for acquiring accurate and real-time phasor data in the power system.
- ❖ To design a model-free framework for assessing long-term voltage stability using only real-time measurements, without relying on detailed offline system models.
- ❖ To ensure system observability with a minimal number of optimally placed measurement devices, enhancing monitoring efficiency.
- ❖ To construct a real-time scaled Jacobian matrix by applying a regression-based approach inspired by the Ornstein–Uhlenbeck process.
- ❖ To perform modal analysis on the measurement-based Jacobian and identify critical system modes influencing voltage stability.
- ❖ To develop and propose two novel voltage stability indices that provide early warning signs of voltage instability or potential collapse.
- ❖ To validate the proposed methodology through extensive simulation studies on the IEEE 14-bus and New England 39-bus test systems under various disturbance scenarios.

3. Methodology

This project adopts a real-time, model-free approach for voltage stability assessment using synchrophasor measurements. The method involves optimal sensor placement, dynamic system estimation through regression, modal analysis for critical mode identification, and computation of stability indices. Corrective actions such as reactive power support and load shedding are triggered automatically upon detecting instability. The detailed steps are outlined below:-

a. <u>Optimal Sensor Placement:-</u>

The load bus network is made completely observable with the optimal number of PMUs to estimate the dynamic loads.

➔ **Integer Linear Programming (ILP)** Formulation

$$\text{Min} \sum_{i=1}^{\eth} x_i C_i$$

Subject to:

$$f_i = \sum_{j=1}^{\eth} B_{ij} x_j \geq d_i \qquad xi \in \{0, 1\}$$

Where:

- $\eth$ = Set of load buses,
- x_i = 1 if PMU is installed at bus i,
- C_i = Cost of PMU at bus i,
- B_{ij} = Bus connectivity matrix (1 if buses connected),
- d_i = 2 (each bus must be observable via 2 PMUs for redundancy).

b. <u>Jacobian Estimation and V-Q sensitivity expression:-</u>

Under steady-state conditions, V and δ, and are expressed as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$$

where ΔP is the change in real power injections, ΔQ is the change in reactive power injections, and $J_{P\delta} = (\partial P/\partial\delta)$, $J_{PV} = (\partial P/\partial V)$, $J_{Q\delta} = (\partial Q/\partial\delta)$ & $J_{QV} = (\partial Q/\partial V)$ are the submatrices of the system Jacobian J. Alternatively, above equation can be rewritten as

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} S_{\delta P} & S_{\delta Q} \\ S_{VP} & S_{VQ} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

where $\begin{bmatrix} S_{\delta P} & S_{\delta Q} \\ S_{VP} & S_{VQ} \end{bmatrix}$ is the inverse of the Jacobian matrix (J) and is also known as the sensitivity matrix.

Due to the strong coupling between reactive power (Q) and V under steady-state, $S_{VQ} = (\Delta V/\Delta Q)$ indicates the voltage sensitivity at dynamic load buses concerning the change in the reactive power changes.

S_{VQ} can thus be used at load buses to analyze the VS as follows:-

1) Positive S_{VQ} means the system is voltage-stable

2) Negative S_{VQ} indicates voltage instability

Under constant active power demand, i.e., with ΔP = 0

$$\Delta Q = \left[J_{QV} - J_{Q\delta} J_{P\delta}^{-1} J_{PV}\right] \Delta V$$

where $[J_{QV} - J_{P\delta} J^{-1}_{P\delta} J_{PV}] = J_R$ is called the reduced Jacobian.

Via eigenvalue decomposition, it can be written as $J_R = \varepsilon\Lambda\eta$, having ε as the right eigenvector matrix of J_R, Λ as the diagonal eigenvalue matrix of J_R, and η as the left eigenvalue matrix of J_R such that $\varepsilon^{-1} = \eta$.

so:

$$\Delta V = \varepsilon \Delta^{-1} \eta \Delta Q$$

and hence the V-Q sensitivity at any bus k is expressed as:-

$$\frac{\Delta V_k}{\Delta Q_k} = \sum_i \frac{\varepsilon_{ki}\eta_{ik}}{\lambda_i}$$

where:

- ε_{ki} = k^{th} element of the i^{th} column of the right eigenvector ε
- η_{ik} = k^{th} element of the i^{th} row of the left eigenvector η
- λ_i = i^{th} diagonal element of the diagonal matrix Λ

Also, the corresponding minimum eigenvalue is given by

$$\lambda_{min} = min(Re(\lambda_i))$$

When the system experiences increasing stress, the minimum eigenvalue gradually declines from a positive value under stable conditions toward zero at the onset of voltage collapse. At the critical voltage point, the right and left eigenvectors associated with the most vulnerable mode provide valuable insight into identifying weak regions within the network and the components that significantly influence that mode.

c. Bus Participation Factor (BPF) Expression:-

The numerator of the V-Q expression determines the contribution of λi to the V–Q sensitivity of bus k which is given by

$$P_{ki} = \varepsilon_{ki}\eta_{ik},$$

which defines the relative participation of the kth bus to mode-i.

★ A larger value of P_{ki} indicates more contribution of the i^{th} mode to the voltage sensitivity at bus–k.

★ For minimum eigenvalues, the bus participation factor (BPF) determines areas close to voltage instability.

$$BPF_k = \frac{\left|\varepsilon_{ki}\eta_{ik}\right|}{\sum_{i=1}^{\eth}\left|\varepsilon_{ki}\eta_{ik}\right|}$$

d. <u>Dynamic Load Modeling:-</u>

Assuming K-load buses in the system are equipped with the following dynamic load model:-

$$\dot{\delta}_m = \frac{1}{\alpha_m}\left(P_{im} - \tilde{P}_{lm}\right)$$
$$\dot{V}_m = \frac{1}{\beta_m}\left(Q_{im} - \tilde{Q}_{lm}\right)$$

Where:

- m ϵ (1, 2, . . . , K), i.e., the set of dynamic load buses,
- δ_m = Voltage angle at the mth bus,
- V_m = Voltage magnitude at the mth bus,
- α_m, β_m = Time constants for the rate of recovery of active and reactive load, respectively,
- $\tilde{P}_{lm}$ and $\tilde{Q}_{lm}$ are the perturbed real and reactive power demands, respectively, at the mth bus which are given as

$$\tilde{P}_{lm} = P_{lm}\left(1 + \sigma_m^P d\epsilon_m^P\right)$$
$$\tilde{Q}_{lm} = Q_{lm}\left(1 + \sigma_m^Q d\epsilon_m^Q\right)$$

here,

- P_{lm}, Q_{lm} = The nominal real and reactive power demand specified at the individual load bus, respectively,
- σ_m^P, σ_m^Q = The standard deviations for the active and reactive power demand, respectively,
- ϵ_m^P, ϵ_m^Q = Random Wiener processes (noise).

The dynamic load model is linearized around the steady-state point expressed in the following compact form:

$$dx = Axdt + Bd\epsilon$$

Where

$$dx = \begin{bmatrix} d\delta_m \\ dV_m \end{bmatrix}_{2K\times 1} \quad d\epsilon = \begin{bmatrix} d\epsilon_m^P \\ d\epsilon_m^Q \end{bmatrix}_{2K\times 1}$$

$$A = \left[\begin{bmatrix} \alpha_m^{-1} & 0 \\ 0 & \beta_m^{-1} \end{bmatrix} \begin{bmatrix} \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial V_m} \\ \frac{\partial Q_m}{\partial \delta_m} & \frac{\partial Q_m}{\partial V_m} \end{bmatrix} \right]_{2K\times 2K}$$

$$x = \begin{bmatrix} \delta_m \\ V_m \end{bmatrix}_{2K\times 1} \quad B = \begin{bmatrix} -\alpha_m^{-1} P_{lm} \sigma_m^P & 0 \\ 0 & -\beta_m^{-1} Q_{lm} \sigma_m^Q \end{bmatrix}_{2K\times 2K}$$

e. <u>Estimation of System State Matrix:-</u>

Based on the regression theorem of the Ornstein–Uhlenbeck process, state matrix A can be estimated as

$$A = \frac{1}{\tau} log[G(\tau)C^{-1}]$$

where stationary covariance matrix, C, of this process with μ_x as the mean of the process, is defined as

$$C = \{[x(t) - \mu_x][x(t) - \mu_x]^T\} = \begin{bmatrix} C_{\delta\delta} & C_{\delta V} \\ C_{V\delta} & C_{VV} \end{bmatrix}$$

and the τ time lag autocorrelation matrix, G, is given as

$$G = \{[x(t + \tau) - \mu_x][x(t) - \mu_x]^T\}$$

f. Estimation of Phasor-Measurement-Based Time Constants:-

After measuring V, δ, P, and Q using optimally placed PMUs in the system, the dynamic load model is linearised as:

$$\frac{\Delta\delta_m}{\Delta t} = \frac{1}{\alpha_m}[\Delta P_m]$$

$$\frac{\Delta V_m}{\Delta t} = \frac{1}{\beta_m}[\Delta Q_m]$$

where

- $\Delta\delta_m(t_i) = \delta(t_i) - \delta(t_{i-1})$ defines the deviation in angle (in radians) and similarly,
- $\Delta V_m(t_i) = V(t_i) - V(t_{i-1})$ defines the deviation in rms voltage magnitude,
- $\Delta P_m(t_i) = P_m(t_i) - P_{lm}$ denotes steady-state active power deviation at the mth bus,
- $\Delta Q_m(t_i) = Q_m(t_i) - Q_{lm}$ denotes steady-state reactive power deviation at the mth bus.

Using linear regression over PMU data we will evaluate α_m and β_m for each bus.

g. Computation of Stability Indices:-

★ **Voltage Collapse Index (VCI):** This gives an idea about how close the system is to voltage collapse.

$$VCI(t) = \frac{\lambda_{min}(t)}{max(\lambda_{min})},$$

where max(λ_{min}) is the largest available value for the minimum eigenvalue.

★ **Time-to-Collapse (TTC):** The variation in λ_{min} is modeled as a quadratic function of time. Then, estimates time t^* when $\lambda_{min}(t)$ crosses zero. As the system steers toward the voltage collapse, TTC reduces toward zero.

$$TTC(t) = t^* - t$$

h. Alarm Triggering and Corrective Actions:-

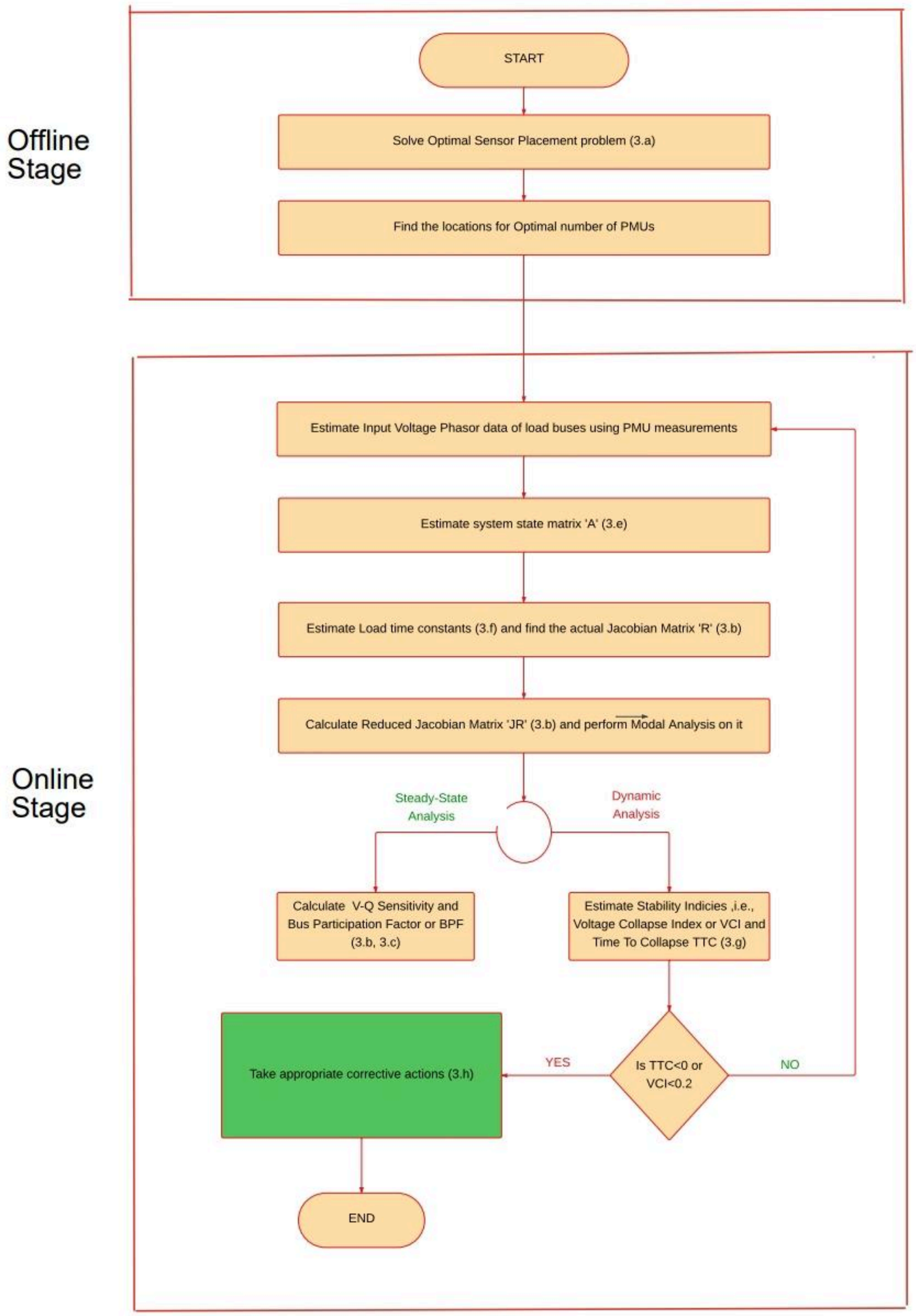
Alarm Criteria:

- If $VCI < 0.2 \; or \; TTC \leq 0$
- Then trigger corrective action.

Corrective Actions:

- Apply **reactive power compensation** (+5% voltage boost).
- Apply **load shedding** (~10% load reduction) at the weak bus (highest BPF).

4. Flowchart


Offline Stage
START
Solve Optimal Sensor Placement problem (3.a)
Find the locations for Optimal number of PMUs
Online Stage
Estimate Input Voltage Phasor data of load buses using PMU measurements
Estimate system state matrix 'A' (3.e)
Estimate Load time constants (3.f) and find the actual Jacobian Matrix 'R' (3.b)
Calculate Reduced Jacobian Matrix 'JR' (3.b) and perform Modal Analysis on it
Steady-State Analysis
Dynamic Analysis
Calculate V-Q Sensitivity and Bus Participation Factor or BPF (3.b, 3.c)
Estimate Stability Indicies ,i.e., Voltage Collapse Index or VCI and Time To Collapse TTC (3.g)
Is TTC<0 or VCI<0.2
YES
NO
Take appropriate corrective actions (3.h)
END

5. Simulation (Code)

i. Optimal Placement Sensor using ILP formulation:-

```
% Total number of buses
N = 14;
C = ones(N, 1);  % Cost vector (1 PMU per bus)

% Define line connections for IEEE 14-bus
lines = [
    1 2;
    1 5;
    2 3;
    2 4;
    2 5;
    3 4;
    4 5;
    4 7;
    4 9;
    5 6;
    6 11;
    6 12;
    6 13;
    7 8;
    7 9;
    9 10;
    9 14;
    10 11;
    12 13;
    13 14
];

% Build adjacency matrix
B = zeros(N);
for k = 1:size(lines, 1)
    i = lines(k, 1);
    j = lines(k, 2);
    B(i,j) = 1;
    B(j,i) = 1;
end
B = B + eye(N);
```

```
% Define critical load buses (excluding bus 7 and 8)
critical_load_buses = [4,5,9,10,11,12,13,14];

% Constraints
A = -B(critical_load_buses, :);
b = -2 * ones(length(critical_load_buses), 1);

% Binary variables
intcon = 1:N;
lb = zeros(N, 1);
ub = ones(N, 1);

% Enforce PMUs at bus 2 and 3
Aeq = zeros(2, N);
Aeq(1,2) = 1; % Bus 2
Aeq(2,3) = 1; % Bus 3
beq = [1; 1];

% Solve
options = optimoptions('intlinprog','Display','off');
[x_opt, fval] = intlinprog(C, intcon, A, b, Aeq, beq, lb, ub, options);

% Results
fprintf('Minimum number of PMUs: %d\n', sum(x_opt));
disp('Place PMUs at buses:');
disp(find(x_opt == 1));
```

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ii. Dynamic Load Modelling:-

```
>> fs = 1;                  % Sampling rate: 1 sample/sec
T_total = 300;            % Total simulation time (seconds)
N = T_total * fs;         % Total number of samples
t = (0:N-1)/fs;           % Time vector
% Dynamic Load Model Parameters
alpha = 30; % Active power recovery time constant (s)
beta = 30;  % Reactive power recovery time constant (s)

sigma_P = 0.005; % 0.5% noise for better stability
sigma_Q = 0.005; % 0.5% noise for better stability
for k = 1:N
    mpc = mpc_original;

    % Apply exponential load increase + noise
    noise_P = Pd0 .* sigma_P .* randn(size(Pd0));
    noise_Q = Qd0 .* sigma_Q .* randn(size(Qd0));

    Pd_target = Pd0 .* load_growth(k) + noise_P;
    Qd_target = Qd0 .* load_growth(k) + noise_Q;
% Dynamic load update (Euler Integration)
    Pd_dyn(load_buses) = Pd_dyn(load_buses) + (Pd_target(load_buses) - Pd_dyn(load_buses)) / alpha;
    Qd_dyn(load_buses) = Qd_dyn(load_buses) + (Qd_target(load_buses) - Qd_dyn(load_buses)) / beta;
X = [delta_pmu V_pmu]';
M = N;
mu_X = mean(X,2);

C = (X - mu_X*ones(1,M))*(X - mu_X*ones(1,M))'/(M-1); ...
```

iii. MATRIX Calculation:-

```
C = (X - mu_X*ones(1,M))*(X - mu_X*ones(1,M))'/(M-1);
G = (X(:,2:end) - mu_X*ones(1,M-1))*(X(:,1:end-1) - mu_X*ones(1,M-1))'/(M-1);
dt = 1/fs;
A = (1/dt) * logm(G / C);
% Find eigenvectors and eigenvalues
[Vr, Dr] = eig(A);   % Right eigenvectors and eigenvalues
Vl = inv(Vr);        % Left eigenvectors (inverse of right eigenvectors)
```

iv. Modal Analysis Code:-

```
>> %% Step 6: Modal Analysis: Track Minimum Eigenvalue
window_size = 50; % Sliding window (seconds)
eigenvalues_min = zeros(N,1);

for k = 1:N-window_size
    X_window = X(:,k:k+window_size-1);
    C_w = (X_window - mean(X_window,2)ones(1,window_size))(X_window - mean(X_window,2)*ones(1,window_size))'/(window_size-1);
    G_w = (X_window(:,2:end) - mean(X_window,2)ones(1,window_size-1))(X_window(:,1:end-1) - mean(X_window,2)*ones(1,window_size-1))'/(window_size-1);
    A_w = (1/dt) * logm(G_w / C_w);
    lambda_window = real(eig(A_w));
    eigenvalues_min(k+window_size-1) = min(lambda_window);
end
```

v. VCI code:-

```
>> %% Step 7: Voltage Collapse Index (VCI)
VCI = eigenvalues_min / max(eigenvalues_min);
```

vi. Line Outage Code:-

```
% Line outage simulation at t=100 sec
 if k == 100
     branch_idx = find((mpc.branch(:,1)==9 & mpc.branch(:,2)==14) | (mpc.branch(:,1)==14 & mpc.branch(:,2)==9));
     if ~isempty(branch_idx)
         mpc.branch(branch_idx,11) = 0; % set status = 0 (outage)
     end
 end
```

6. Result & Discussion

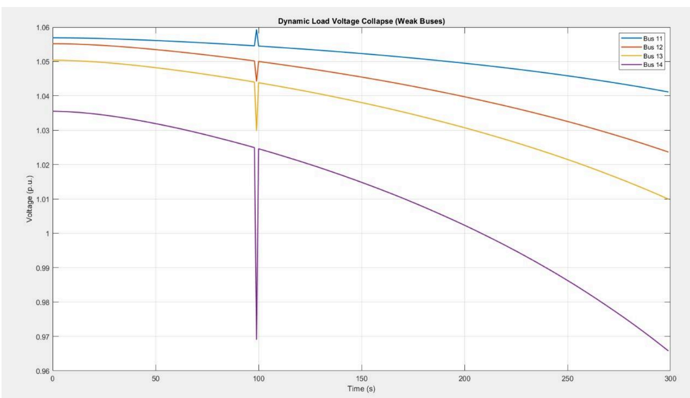
i. Optimal Placement of PMU sensors

```
Minimum number of PMUs: 6
Place PMUs at buses:
     2
     3
     6
     9
    10
    13
```

This clearly shows we need 6PMUs at specified buses 2,3,6,9,10,13 for our system to be completely observable.

This result matches the base paper thoroughly.

ii. Voltage Collapse with Time:-



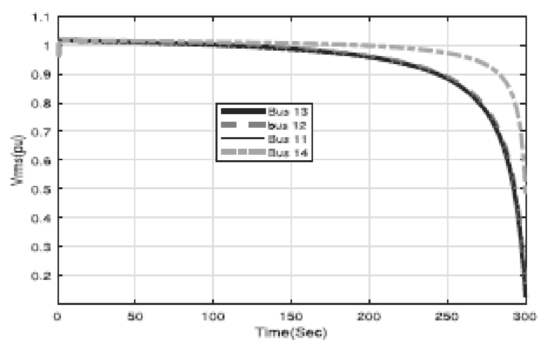
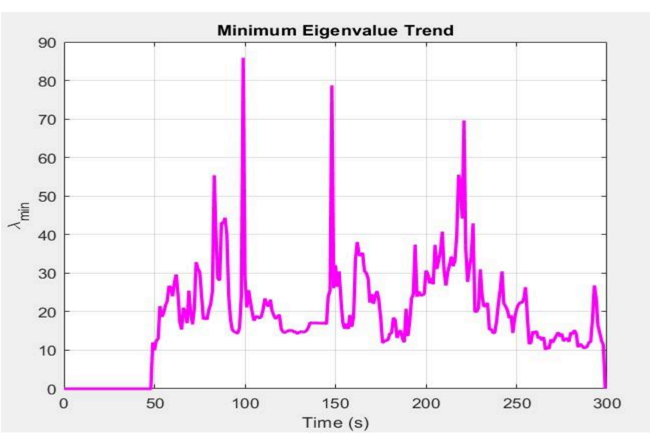


Fig. 3. Voltage profile at weak buses in the IEEE 14-bus system.

Fig.: Voltage Collapse with time for weak buses in IEEE14 bus system (Simulated and base results from left to right)

iii. Min. Eigen Value Trend:-



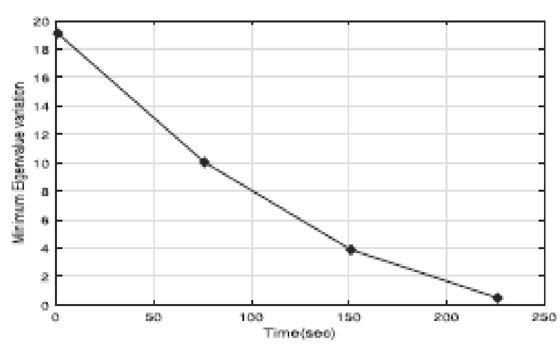
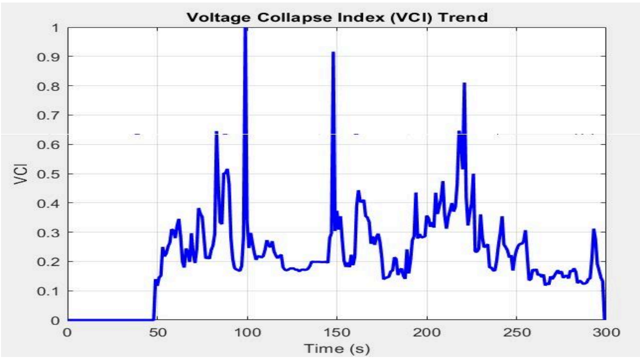


Fig. 8. Variation in λ_{min} with respect to time.

Fig.: Min. Eigen Value Trend (Simulated and base paper results from left to right)

iv. Voltage Collapse Index (VCI) trend:-



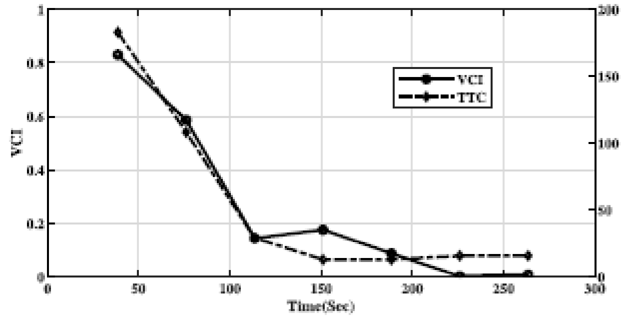
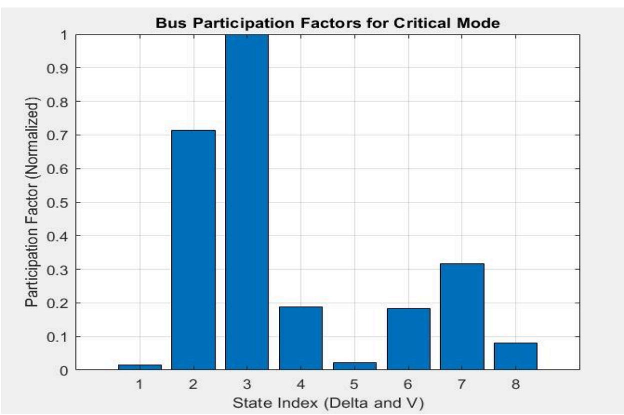


Fig. 4. Proposed VCI and TTC for IEEE 14 bus system.

Fig.: VCI trend (Simulated and Base paper Results from left to right)

v. Bus Participation Factor (BPF) for load buses under critical conditions:-



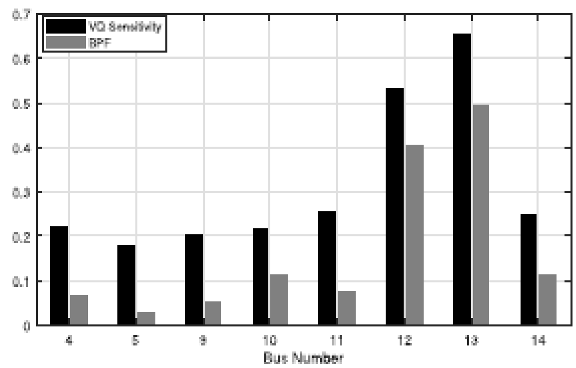
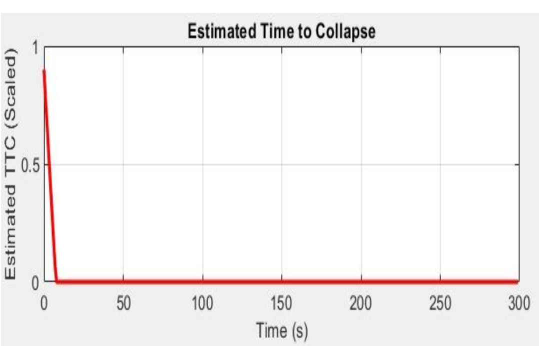


Fig.: BPF for load buses (Simulated and Base paper Results from left to right)

vi. Time To Collapse Estimation:-



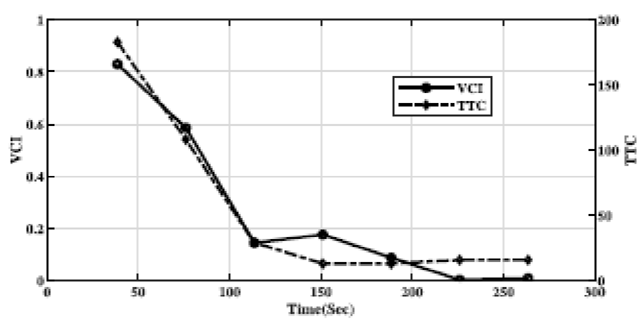
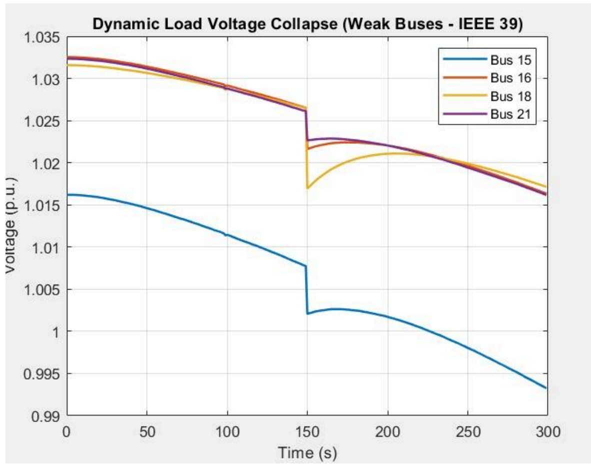
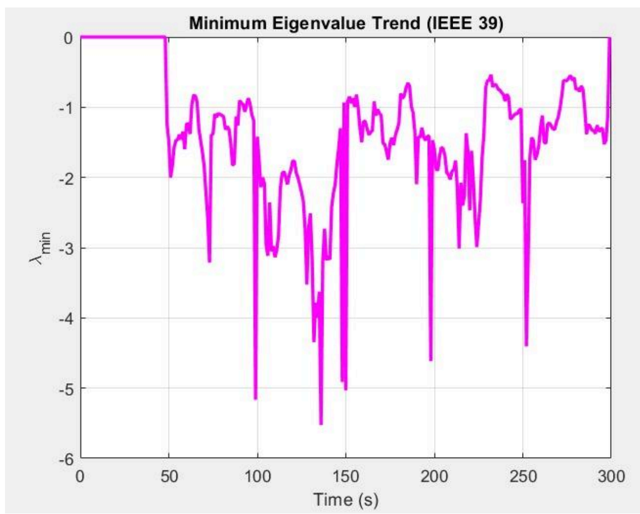
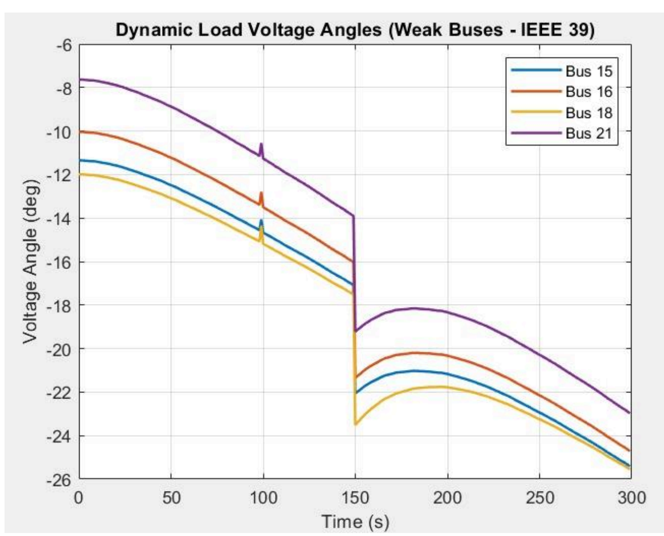


Fig.: TTC trend (Simulated and Base paper Results fro left to right)

NE39 BUS system Simulation Results







7. Conclusion

- ➔ A real-time, model-free framework was developed for long-term voltage stability assessment using synchrophasor measurements.
- ➔ Optimal PMU placement was used to achieve system observability with minimal sensors, reducing cost and complexity.
- ➔ A scaled Jacobian matrix was estimated in real time using a regression process based on phasor data.
- ➔ The modal analysis identified critical modes and weak buses by analyzing eigenvalues and eigenvectors.
- ➔ Two stability indices, VCI and TTC, were proposed for early detection of voltage collapse risk.
- ➔ The method was validated on IEEE 14-bus and New England 39-bus systems under dynamic load increase, line, and generator outages.

8. Future Research and Advancements

I. Extend the proposed framework to larger and more complex power systems, such as full regional or national grids.
II. Incorporate dynamic modeling of renewable energy sources like solar and wind into the voltage stability assessment.
III. Develop adaptive PMU placement strategies that can adjust sensor locations dynamically based on system changes.
IV. Integrate machine learning techniques for predictive stability monitoring and faster collapse estimation.
V. Improve noise handling by applying advanced filtering techniques to further enhance the accuracy of real-time Jacobian estimation.
VI. Implement decentralized stability monitoring schemes using distributed PMU networks for improved scalability.

9. Challenges Faced

A. Accurate PMU Data Handling

➔ Challenge:- PMU data may have some errors or noise that can propagate small errors and cause wrong Jacobian estimation or instability detection.
➔ Way(s) to improve:- Apply noise filtering and missing data handling techniques carefully.

B. Dynamic Load Modeling Assumptions

➔ Challenge:- In the simulation, dynamic load growth is assumed (exponential voltage decay), but in real systems, load behavior can be unpredictable and non-uniform.
➔ Way(s) to improve:- Realistic load model or adapt the method to uncertain behaviors.

C. Eigenvalue Tracking and Noise Sensitivity

➔ Challenge:- Eigenvalues can jump uncontrollably if measurement noise is too high. Tracking the minimum eigenvalue (λ_{min}) smoothly is essential for accurate TTC prediction.
➔ Way(s) to improve:- Use smoothing techniques or moving averages when plotting eigenvalue evolution.

D. PMU Placement Practicality

➔ Challenge:- Practical deployment of optimal PMUs may face difficulties like site issues or high installation costs, etc.
➔ Ways to improve:- Consider practical placement limitations during system design.

E. Corrective Actions Timeliness

➔ Challenge:- In real systems, reactive power support and load shedding must happen very fast after alarms.
➔ Delays in detection → delays in action → instability.
➔ Way(s) to overcome:- Test corrective action logic thoroughly to ensure it's triggered immediately upon alarm.

10. References

- <u>Base paper</u>:- M. Pandit and R. Sodhi, "Phasor Measurement Sensor-Assisted Time-to-Collapse Estimation Under Long-Term Voltage Instability of Smart Grids," in IEEE Sensors Journal, vol. 24, no. 5, pp. 6523-6531, March 1, 2024, doi: 10.1109/JSEN.2024.3349595.

  Link:- Base Paper

- <u>Softwares Used:</u>- MATLAB/ Simulink/ PSCAD/ Google Docs