# ME232: Dynamics

# Vibration

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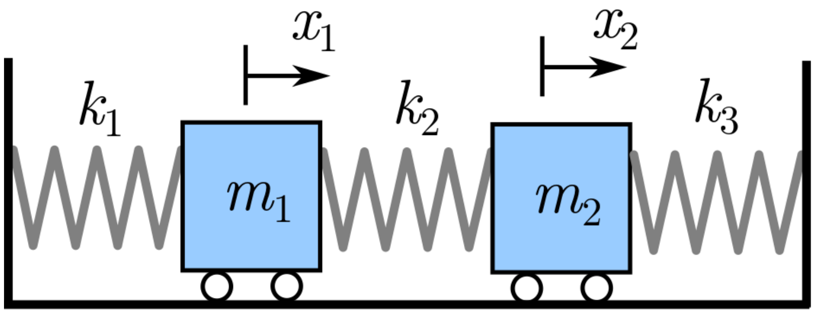
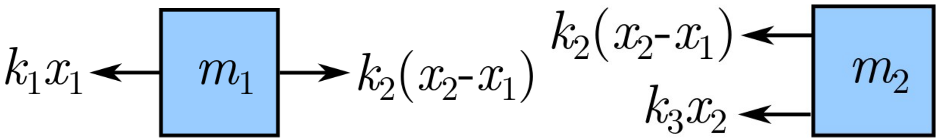
Room # 106

# Two degree of freedom system




Now, consider the system with two masses ($m_1$ and $m_2$) connected with springs as shown. Displacement of both the masses from the equilibrium position is measured as $x_1$ and $x_2$. Similar to previous cases, let us first write the equation of motion for both masses as,

$$\begin{aligned} k_2(x_2 - x_1) - k_1x_1 &= m_1\ddot{x}_1 \\ -k_3x_2 - k_2(x_2 - x_1) &= m_2\ddot{x}_2 \end{aligned} \qquad \text{or} \qquad \begin{aligned} m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 &= 0 \\ m_2\ddot{x}_2 - k_2x_1 + (k_2 + k_3)x_2 &= 0 \end{aligned} \qquad \text{..............(35)}$$

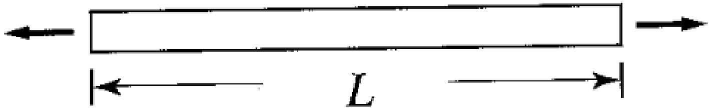
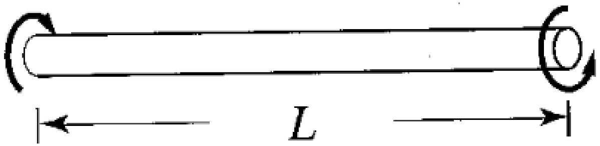
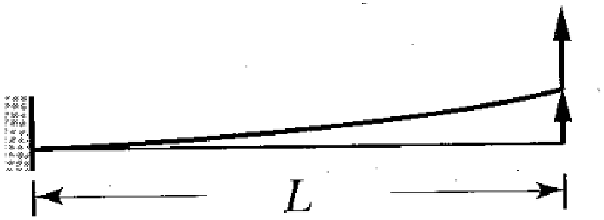
Equations (24) can be written in matrix form as,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}, \quad \text{where,} \quad \boldsymbol{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \text{ and } \boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22}. \end{bmatrix}$$

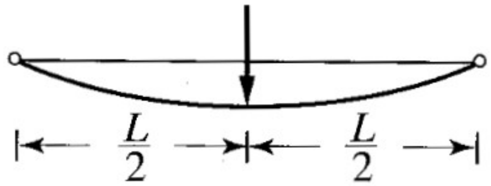
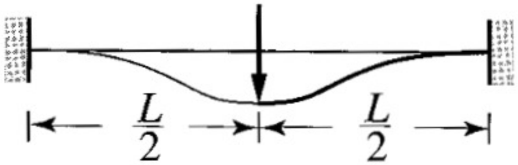
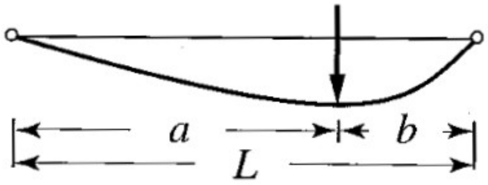
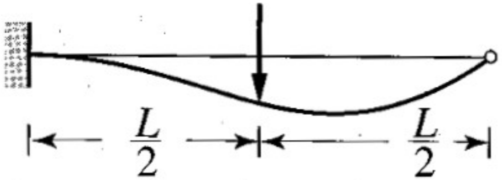
Here $\boldsymbol{M}$ is known as *mass matrix* and $\boldsymbol{K}$ is known as *stiffness matrix.* ..............(36)

- These system of equations can be solved for assumed harmonic response and it can be shown that the system has two natural frequencies.
- Corresponding to each natural frequency there exists a particular type of oscillatory motion which is called mode shape of vibration. So there are two modes of vibration corresponding to two natural frequencies of the system.
- In case of forced vibration resonance occurs when the excitation frequency matches with any of the natural frequency of the system.
- With an appropriate combination of spring stiffness and masses vibration absorbers can be designed.

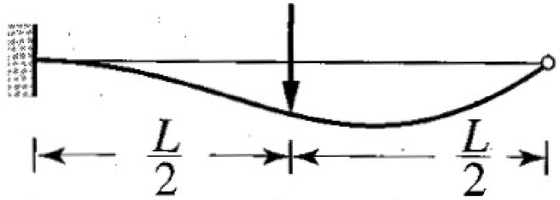
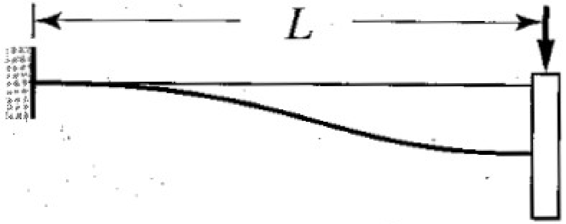
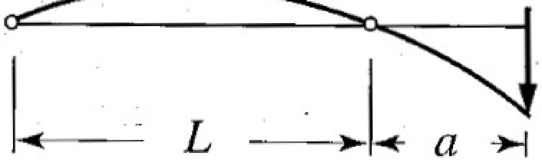
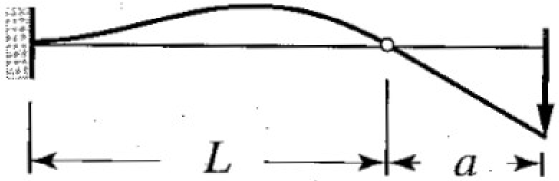
# Equivalent spring constants

| | | |
|---|---|---|
|  | Rod in axial deformation | $\frac{EA}{L}$ |
|  | Shaft in torsion | $\frac{GJ}{L}$ |
|  | Cantilever beam with a moment at the tip | $\frac{EI}{L}$ |
|  | Cantilever beam with a force at the tip | $\frac{3EI}{L^3}$ |

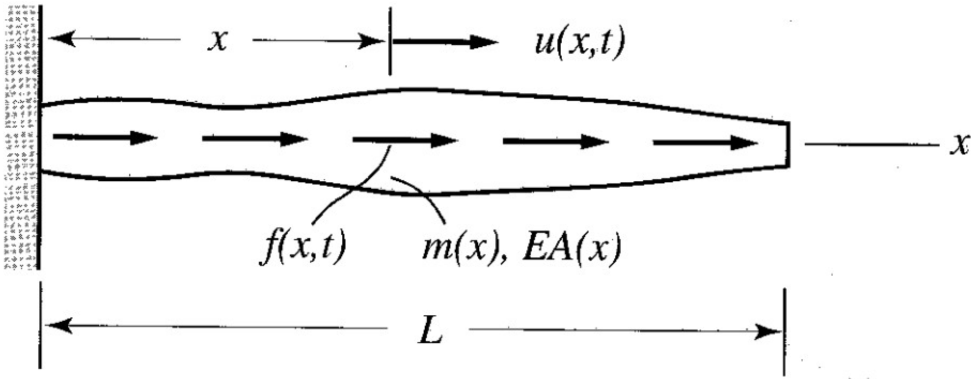
# Equivalent spring constants

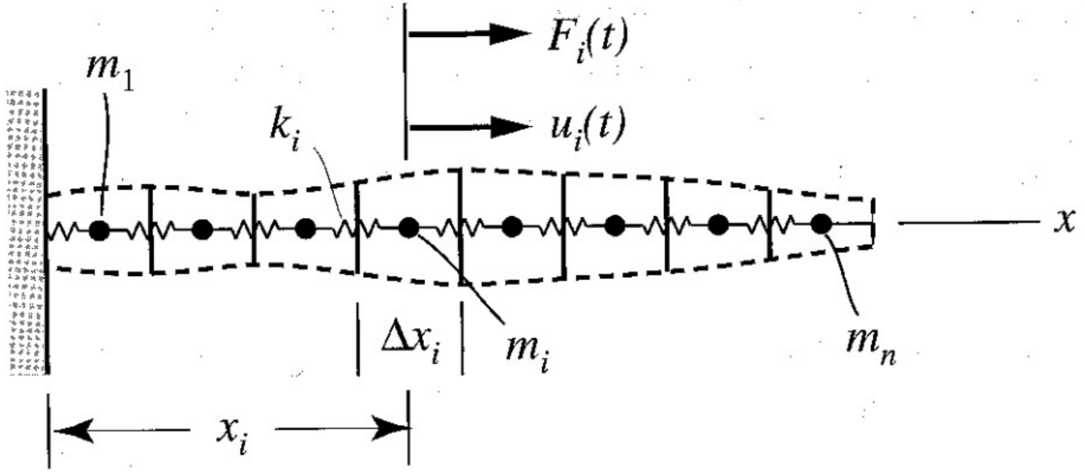
| Component Sketch and Description | | $k_{eq}$ |
|---|---|---|
|  | Pinned-pinned beam with a force at midspan | $\frac{48EI}{L^3}$ |
|  | Clamped-clamped beam with a force at midspan | $\frac{192EI}{L^3}$ |
|  | Pinned-pinned beam with an off-center force | $\frac{3EIL}{a^2b^2}$ |
|  | Clamped-pinned beam with a force at midspan | $\frac{768EI}{7L^3}$ |

# Equivalent spring constants

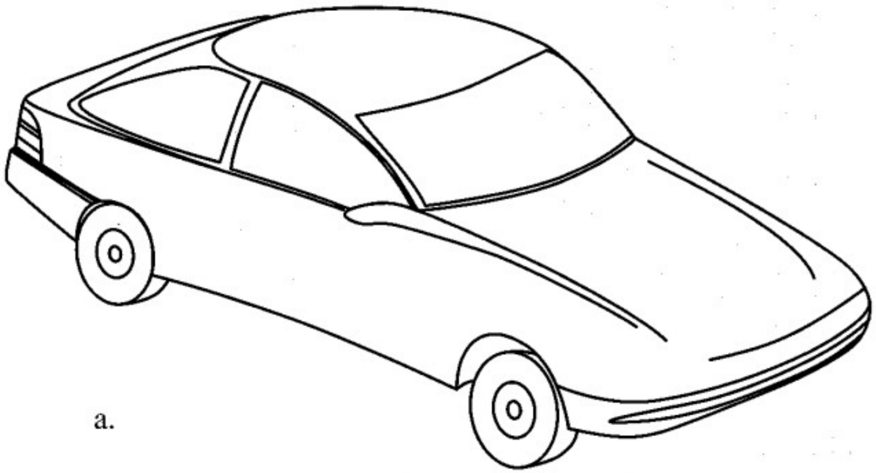
| | | |
|---|---|---|
|  | Clamped-pinned beam with a force at midspan | $\frac{768EI}{7L^3}$ |
|  | Clamped-clamped beam with one end sagging under a force | $\frac{12EI}{L^3}$ |
|  | Pinned-pinned beam with an overhang and a force at the tip | $\frac{3EI}{a^2(L+a)}$ |
|  | Clamped-pinned beam with an overhang and a force at the tip | $\frac{12EI}{a^2(3L+4a)}$ |

# Modeling of continuous systems as discrete systems

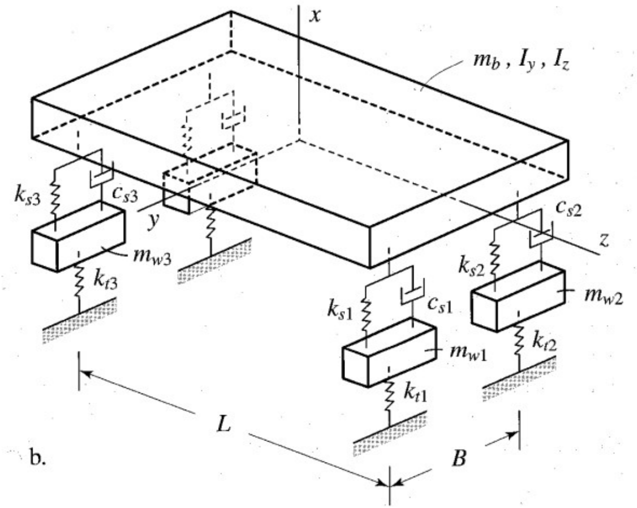


# Different models of automobile for vibration analysis



a.




b.