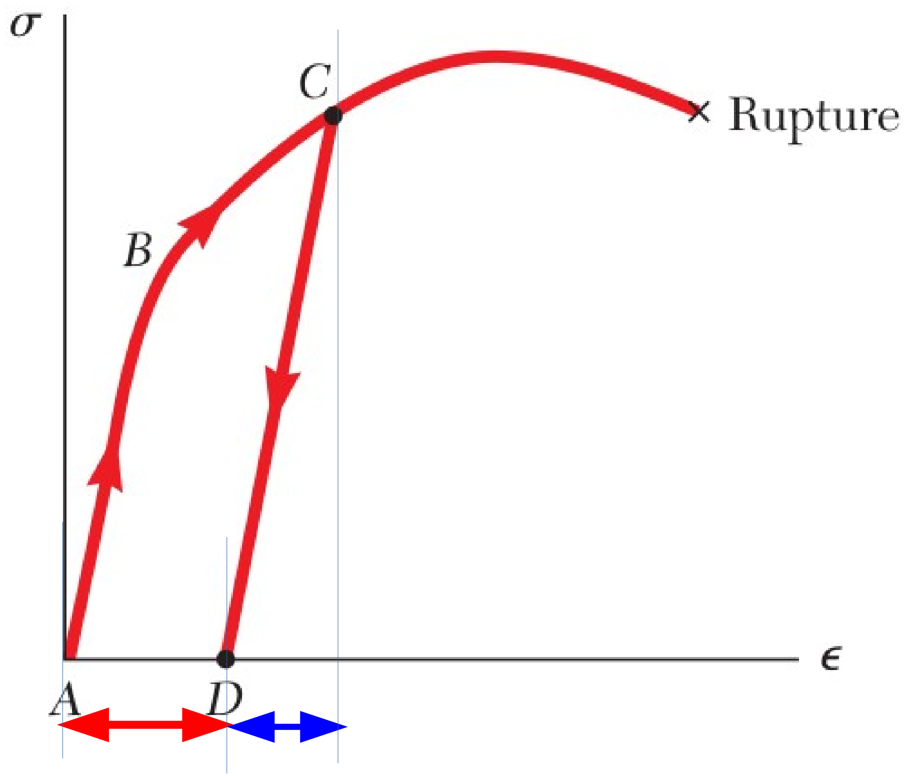
ME231: Solid Mechanics-I

Stress, Strain and Temperature relationship

Elastic vs. Plastic Behaviour: Unloading

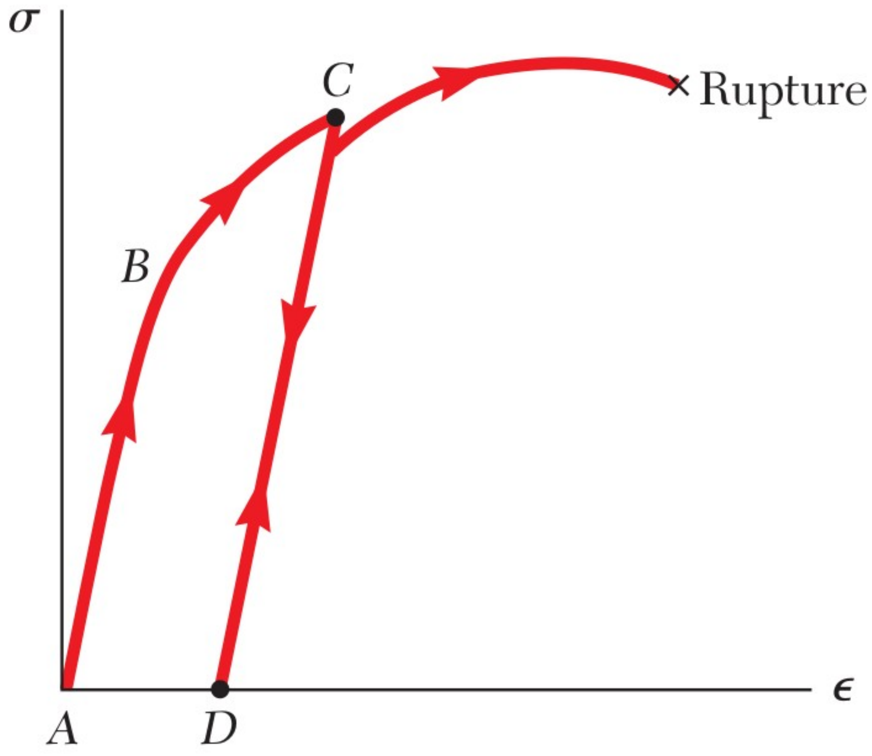


Permanent set or Plastic strain

Elastic strain

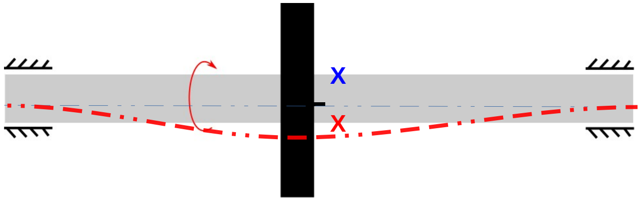
- If a material is subjected beyond its elastic limit (till point C) and then the load is removed; the stress and strain decreases in a linear fashion (path CD). The linear unloading path CD is parallel to the initial loading path AB.
- It should be noted that after complete unloading strain ϵ does not return to zero, which indicates that **a permanent set or plastic deformation** of the material has taken place.
- Strain recovered during the unloading process is the **elastic strain**.

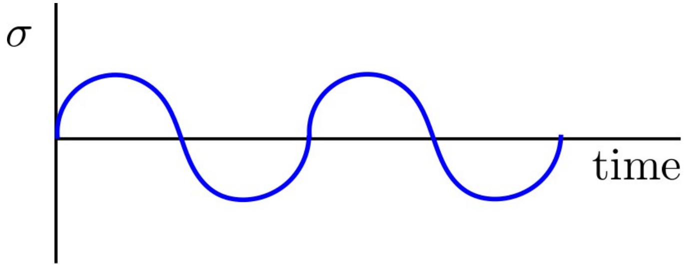
Elastic vs. Plastic Behaviour: Reloading



- If the unloaded test specimen is reloaded under tension, stress-strain curve first follow the path DC, then it will bend to the right and connect with the curved portion of the original stress-strain diagram.
- The straight-line portion of the new loading curve is longer than the corresponding portion of the initial one.
- Thus, the proportional limit and the elastic limit have increased as a result of the strain-hardening that occurred during the earlier loading. However, since the point of rupture remains unchanged, the ductility of the specimen, which should now be measured from point D, has decreased.

Repeated loading and fatigue





- Most engineering components experience repeated or fluctuating load
- For example,
  - A beam supporting an industrial crane can be loaded as many as two million times in 25 years (about 300 loadings per working day)
  - An automobile crankshaft is loaded about half a billion times if the automobile is driven 200,000 miles,
  - An individual turbine blade can be loaded several hundred billion times during its lifetime.

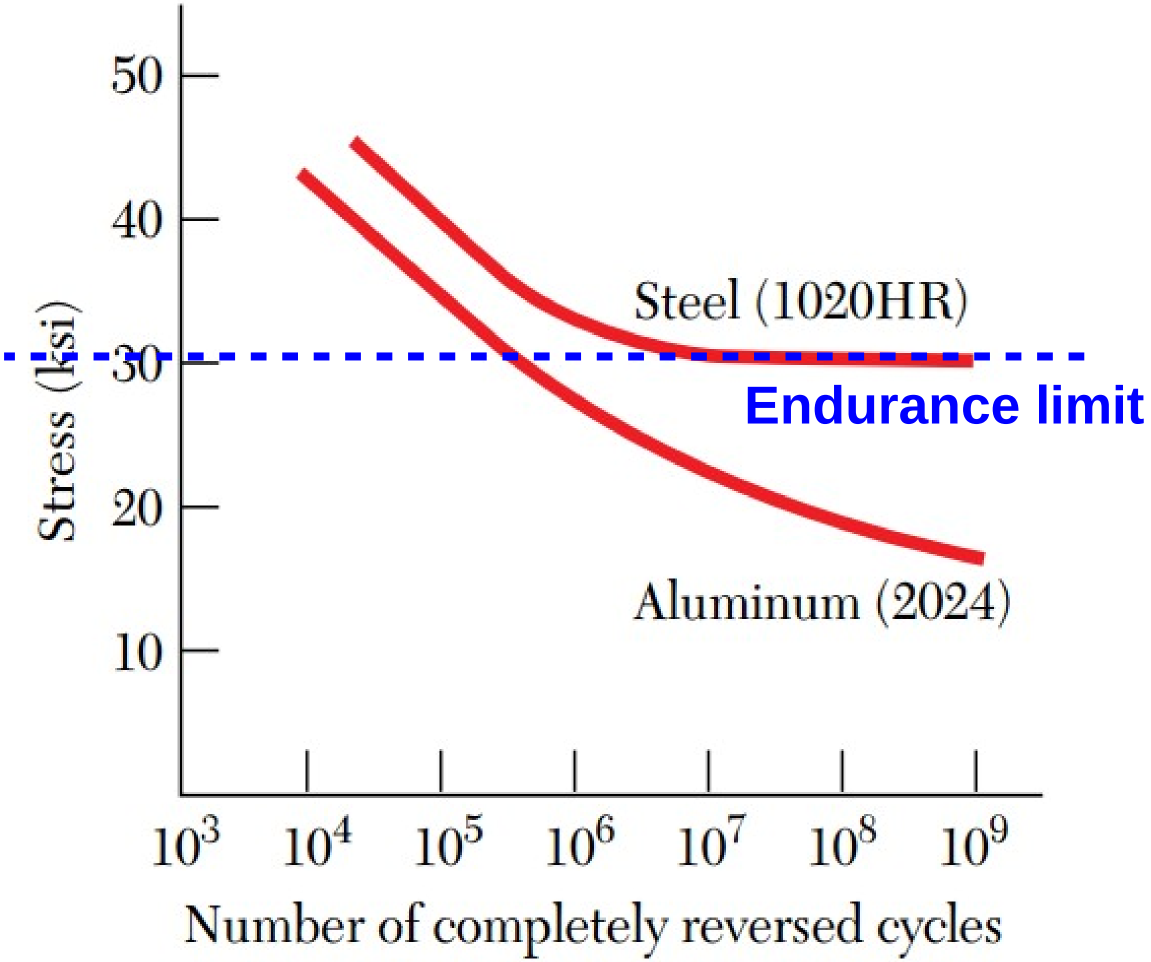
- When loadings are repeated thousands or millions of times, then the rupture can occur at a stress much lower than the static breaking strength; this phenomenon is known as **fatigue**.
- A fatigue failure is of a **brittle nature**, even for materials that are normally ductile.

Endurance limit

$\sigma - n \ curve$

or

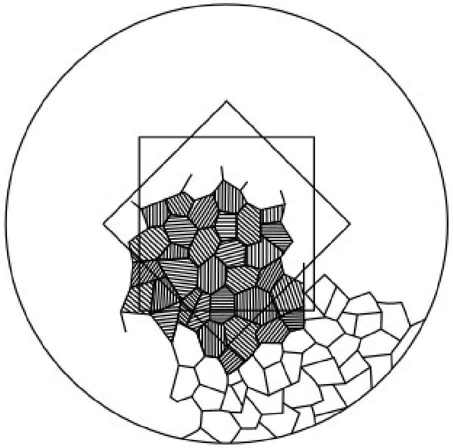
$s - n \ curve$



Elastic stress-strain relations

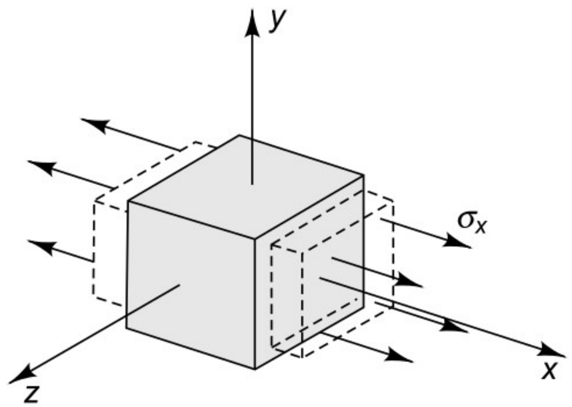
- We already developed stress-strain relationship as Hooke's law for special case of one dimensional loading.
- We will now generalize the elastic behaviour and establish the relationship between six components of stress and six components of elastic strain.
- We will restrict our self to all **linearly elastic materials**. We will also restrict our self to definitions of **strain for small deformations**.

- We will also assume materials to be **homogeneous isotropic**. An isotropic material is defined as one whose properties are **independent of orientation.**
- Materials made up of randomly oriented structural elements may be thought of as being **statistically isotropic**.
- Homogeneity implied that material properties are **independent of position.**



Statistically isotropic material

Consider an element on which only one component of normal stress is acting. This normal component of stress will produce a corresponding normal component of strain. Relation between the normal stress and normal strain produced is,



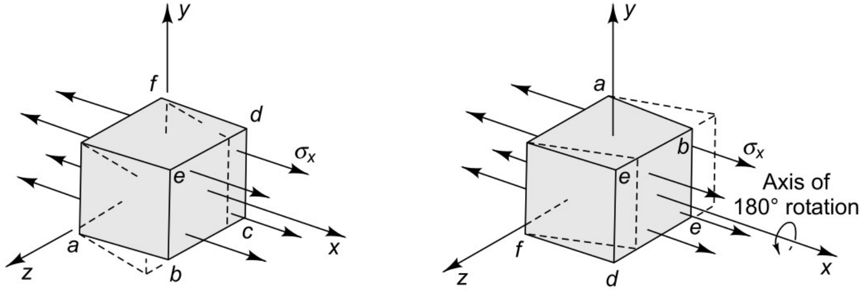
$$\epsilon_x = \frac{\sigma_x}{E} \qquad \text{............(6)}$$

From the measurement made during the uniaxial tensile test, it is observed that there are deformations in the lateral directions also. It is found that lateral strain is a fixed fraction of the normal strain. This fixed fracture is called **Poisson's ratio** and is denoted by the symbol ν. Thus, lateral strain can be defined as,

$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -\nu\frac{\sigma_x}{E}. \qquad \text{............(7)}$$

The possibility of occurrence of shear strain because of normal stress σ_x can be discarded because of isotropy.

Thus normal stress will produce only normal strains.



(a) (b)

After 180° rotation

Now, if normal stress σ_y is considered then, normal strain in y-direction will be

$$\epsilon_y = \frac{\sigma_y}{E}, \qquad \cdots\cdots\cdots\cdots(8)$$

and corresponding lateral strains will be, $\epsilon_x = \epsilon_z = -\nu\epsilon_y = -\nu\dfrac{\sigma_y}{E}$. $\cdots\cdots\cdots\cdots(9)$

Similarly for normal stress σ_z corresponding strains are,

$$\epsilon_z = \frac{\sigma_z}{E}, \quad \text{and} \quad \epsilon_x = \epsilon_y = -\nu\epsilon_z = -\nu\frac{\sigma_z}{E}. \qquad \cdots\cdots\cdots\cdots(10)$$

Under the most general loading condition, **shear stresses does not affect the normal strains directly when deformations are small**. **Also shear stresses in a direction does not affect shear strains in other directions**. Hence, Hooke's law for shear stresses is

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}, \quad \text{and} \quad \gamma_{yz} = \frac{\tau_{yz}}{G}. \qquad \cdots\cdots\cdots\cdots(11)$$

where G is called the **shear modulus.**

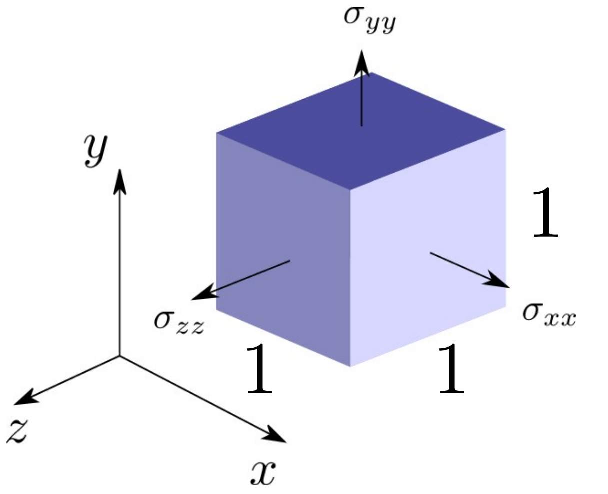
Multi-axial loading: Generalized Hooke's Law

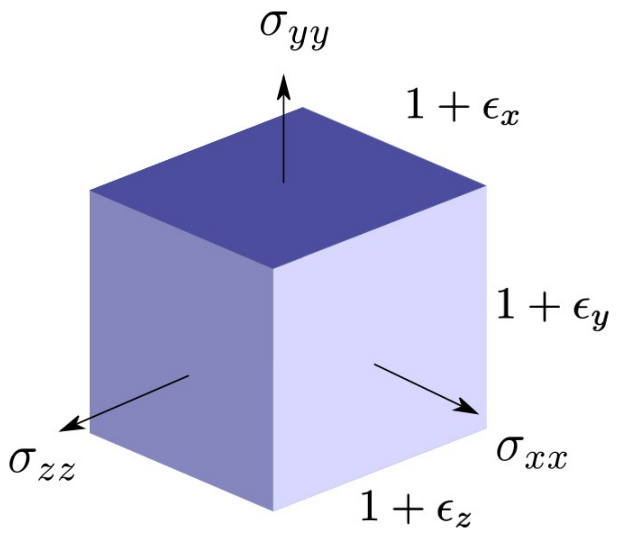
Consider a case where all stress components are $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}$ and τ_{yz} acting simultaneously, then **within the limits of linear elasticity and small deformations** stresses and strains can be related as,

$$
\begin{aligned}
\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E}, & \gamma_{xy} &= \frac{\tau_{xy}}{G}, \\
\epsilon_{yy} &= \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{zz}}{E}, & \gamma_{xz} &= \frac{\tau_{xz}}{G}, \qquad \cdots\cdots\cdots\cdots(12) \\
\epsilon_{zz} &= \frac{\sigma_{zz}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E}, & \gamma_{yz} &= \frac{\tau_{yz}}{G}.
\end{aligned}
$$

These equations are known as the **generalized Hooke's law.** These equations involves three constants E, G and ν.

Dilatation and Bulk Modulus





Consider a cubic material element having unit volume shown in its unstressed state. Under the stresses σ_{xx}, σ_{yy} and σ_{zz} it deforms into a rectangular parallelepiped of volume v, where

$$v = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z).$$

As strains are smaller than unity, we can write,

$$v \approx 1 + \epsilon_x + \epsilon_y + \epsilon_z.$$

Now the change in volume is

$$e = v - 1 = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} \qquad \text{............(13)}$$

Here, e represents **the change in volume per unit volume** which is called **dilatation** of the material. Using (12) we can rewrite (13) as,

$$e = \frac{1-2\nu}{E}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right). \qquad \text{............(14)}$$

If a body is subjected to uniform hydrostatic pressure, i.e., $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p$, then (14) yields

$$e = -\frac{3p(1-2\nu)}{E} = -\frac{p}{k}, \qquad \cdots\cdots\cdots\cdots(15)$$

where $k = \dfrac{E}{3(1-2\nu)}$ is a material constant, known as **bulk modulus** of the material.

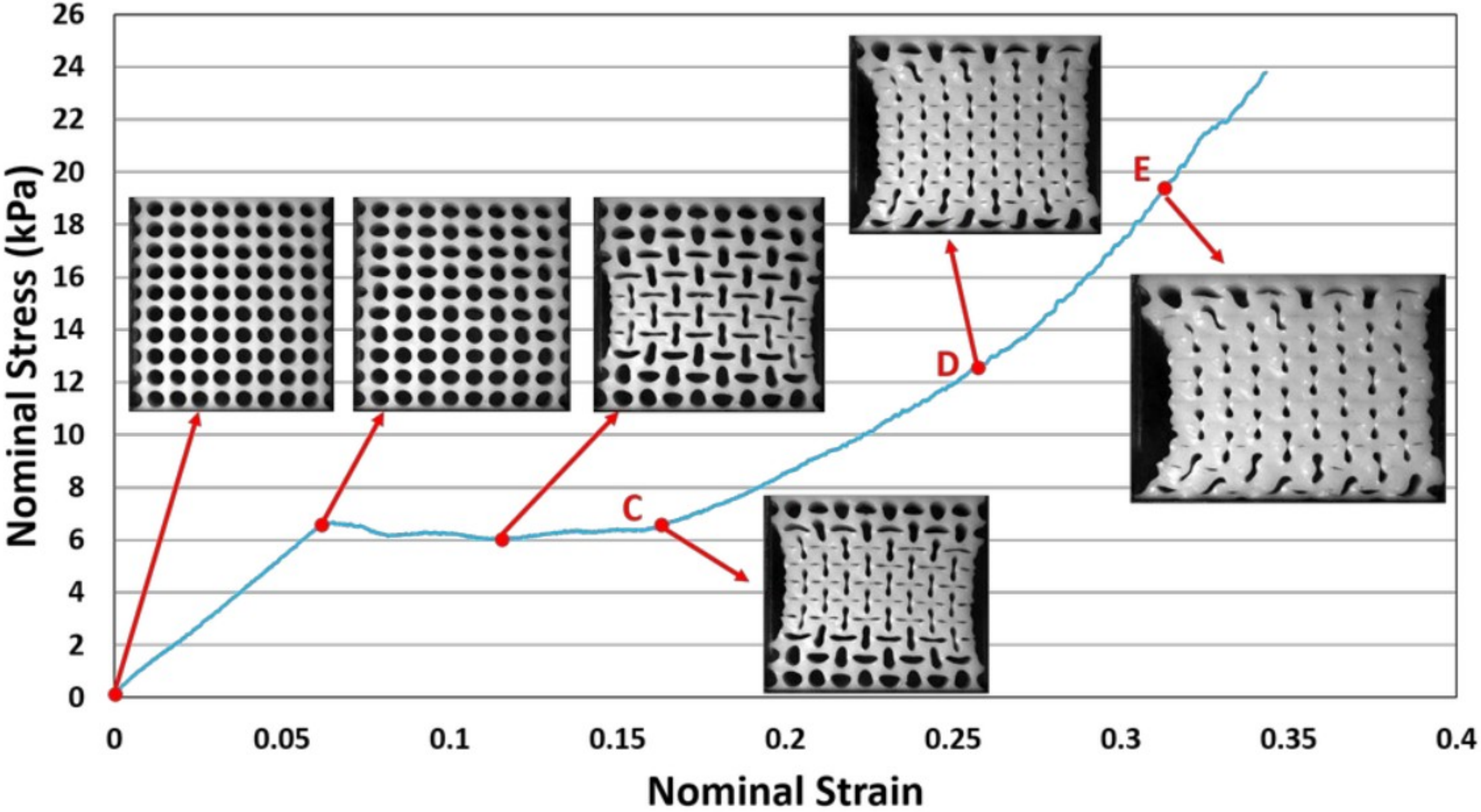
Bulk modulus is defined as the ratio of pressure to dilatation/volumetric strain (e). Note that k is always positive, as hydrostatic pressure will always decrease the volume.

Hence, $(1\text{-}2\nu)>0$ or $\nu < 0.5$. ν is also positive, hence for any engineering material

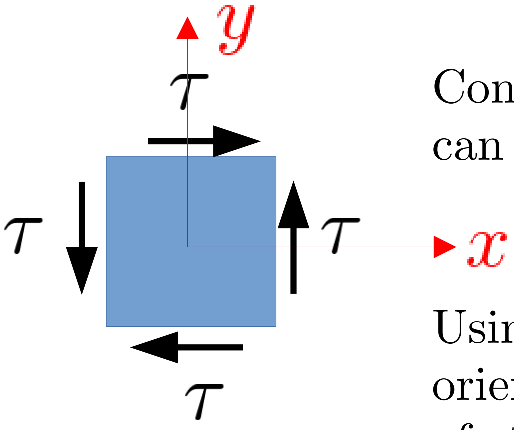
$$0 < \nu < 0.5.$$

- $\nu=0$: Stretching is one directional without contraction in lateral direction.
- $\nu=0.5$, i.e., $k=\infty$, which means, zero dilatation or no change is volume when pressure is applied. i.e., perfectly incompressible materials.

Structures with negative Possion's ratio



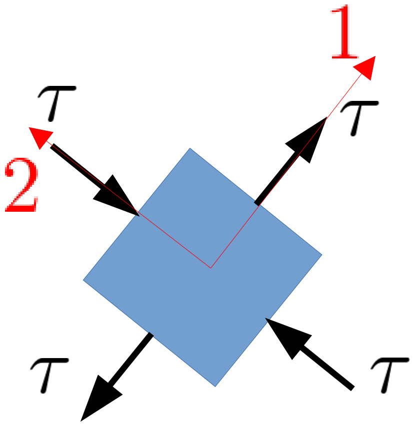
Relationship between E, ν and G



Consider a material element in pure shear loading. Using Hooke's law, we can write shear strain in the element as,

$$\gamma_{xy} = \frac{\tau}{G}. \qquad \cdots\cdots\cdots\cdots(16)$$

Using stress transformation, let us determine the state of stress at angle orientation of 45°. We already did this as exercise and shown that the state of stress at 45° orientation of the element will be as follows.



For this element, applying generalized Hooke's law yields,

$$\epsilon_1 = \frac{\tau}{E} - \nu\frac{-\tau}{E} = \frac{(1+\nu)\tau}{E}$$

$$\epsilon_2 = \frac{-\tau}{E} - \nu\frac{\tau}{E} = -\frac{(1+\nu)\tau}{E} \qquad \cdots\cdots\cdots\cdots(17)$$

Maximum shear strain is nothing but γ_{xy}, which can be determined as

$$\gamma_{xy} = \epsilon_1 - \epsilon_2 = \frac{2(1+\nu)}{E}\tau. \qquad \cdots\cdots\cdots\cdots(18)$$

Now equating (16) and (18) we can write,

$$G = \frac{E}{2(1+\nu)}. \qquad \cdots\cdots\cdots\cdots(19)$$

Thus for an isotropic elastic material there are just **two independent elastic constants**.