# Forces and Moments Transmitted by Slender Members

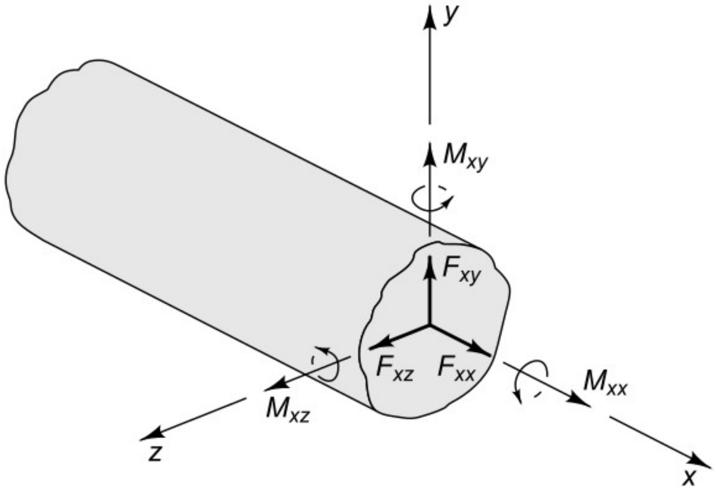
A large number of engineering structures has **slender members** as load carrying-members.

A slender member has a **relatively higher length** (at least five time or more) than the other two (cross-sectional) dimensions.

Beams, columns, shafts, rods, struts, and links can be classified as slender members. A hoop or coil is also considered as slender structure as they are form by a long thin rod.

In general, a slender member can have axial, torsion or bending loads.

If a slender member is in equilibrium, **all the sub-systems isolated by assuming a hypothetical cuts/sections** at the point of interest will also be in equilibrium. Considering equilibrium of these sub-systems, forces and moments at each cross-section will be determined.



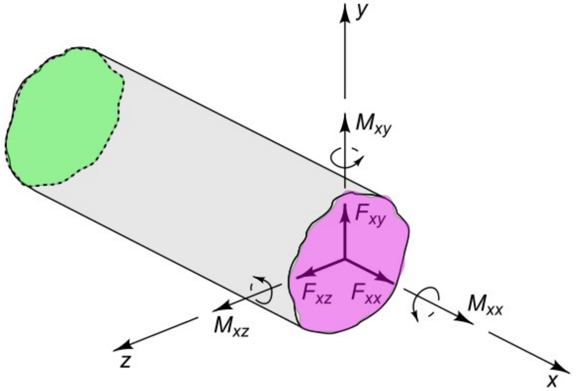

$F_{xx}$ – Axial force in $x$-direction (also denoted as $F_x$),

$F_{xy}, F_{xz}$ – Shear forces at $x$-plane (also denoted as $V_y, V_z$),

$M_{xx}$ – Twisting moment (also denoted as $M_t$ or $T$),

$M_{xy}, M_{xz}$ – Bending moment (also denoted as $M_{by}, M_{bz}$).

# Sign conventions

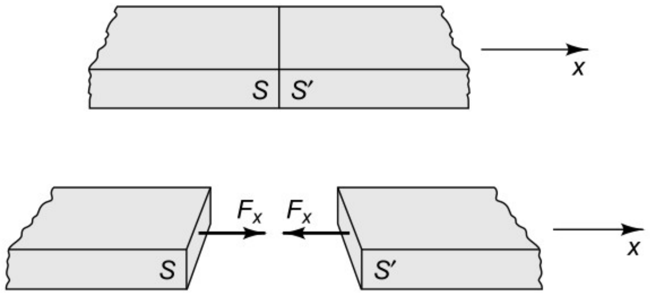



The cross-sectional face will be called

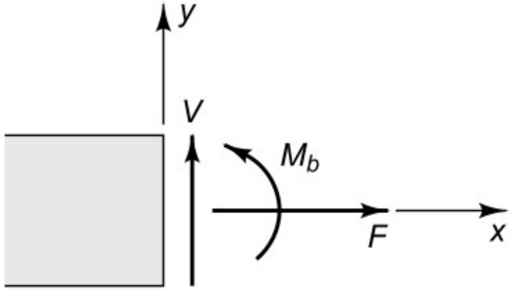
- **positive** when the outward normal points in the positive direction (magenta colored face)
- **negative** when the outward normal points in the negative direction (green colored face)

The force or moment component are positive when

- acting on a positive face in a positive direction
- acting on a negative face in a negative direction



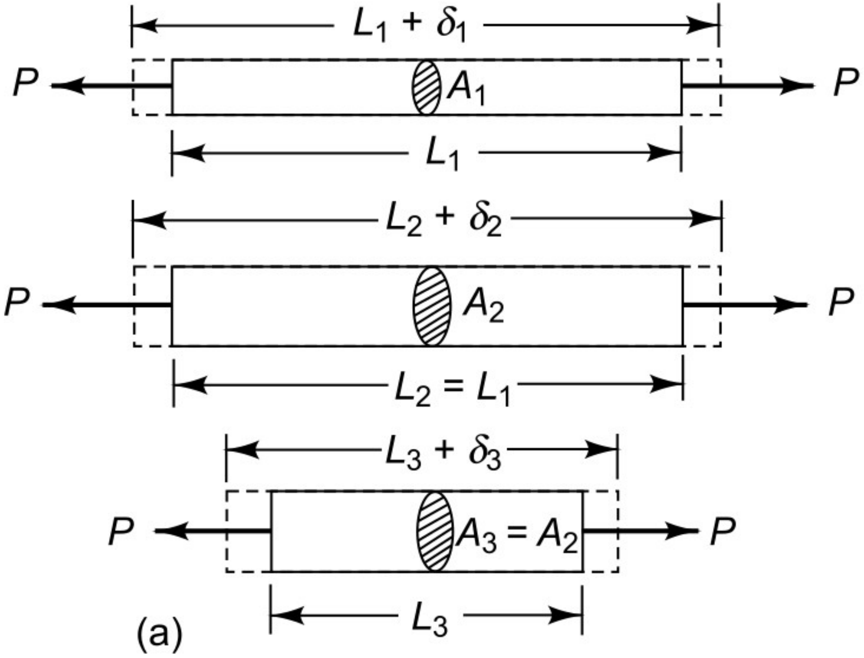

For a two-dimensional case positive components of force and moments are shown



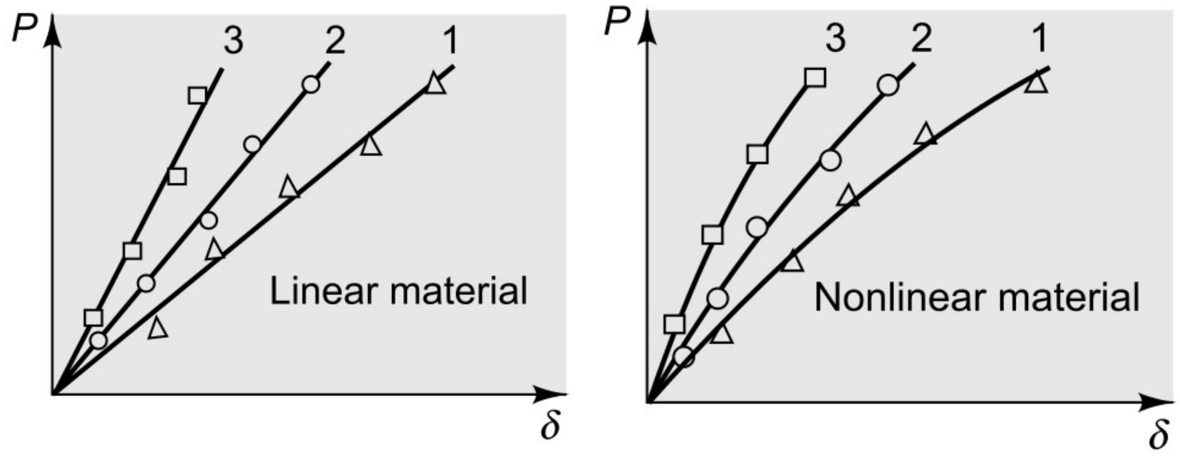
# Axial Loading

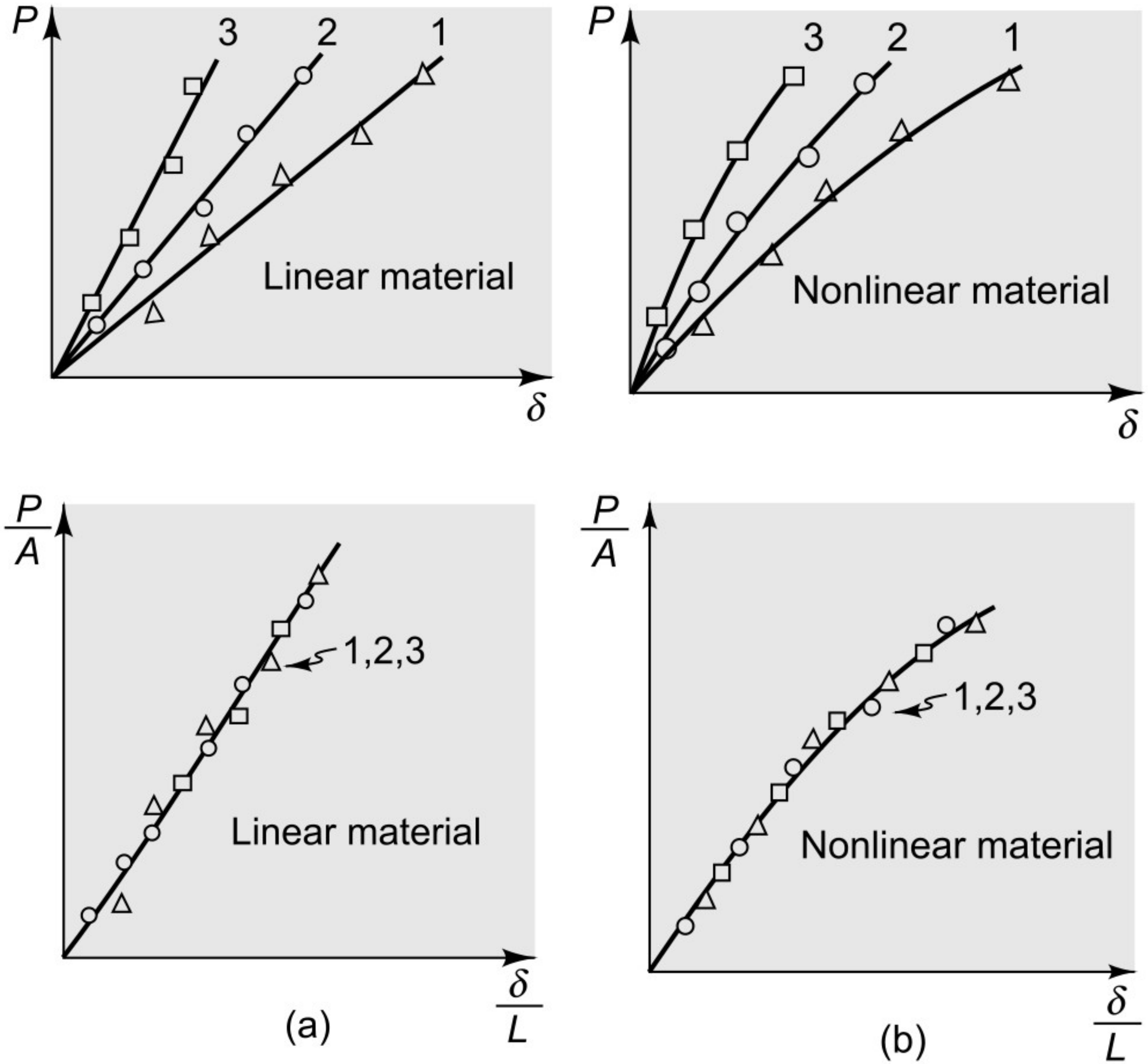
# Uniaxial loading and deformation

One of the most basic loading, which can be considered on structures like bar or rod is the load applied along their axis. For such a loading load vs. deformation curve for a given specimen can be plotted.




**Load – deformation behaviour**






When the load ($P$) - deformation ($\delta$) curves are re-plotted as ($P/A$) - ($\delta/L$) curves, then it is observed that curves obtained for specimens of different dimensions overlap.

This suggests that the new curves are **independent of specimen dimensions.**

Thus, they represent **load-elongation characteristics of a particular material.**

For materials having a **linear** uni-axial load-deformation characteristics, we define **modulus of elasticity** as the slope of load-deformation curve, which is generally denoted as $E$.

Thus,

$$E = \frac{P/A}{\delta/L}. \quad \cdots\cdots\cdots\cdots\cdots(1)$$

Dimensions of $E$ is same as that of $P/A$, (denominator is unit-less), which is $N/m^2$ (Pa) in SI Units.

| *Material* | *E, psi* | *E, kN/m²* |
|---|---|---|
| Tungsten carbide | $60–100 \times 10^6$ | $410–690 \times 10^6$ |
| Tungsten | $58 \times 10^6$ | $400 \times 10^6$ |
| Molybdenum | $40 \times 10^6$ | $275 \times 10^6$ |
| Aluminum oxide | $47 \times 10^6$ | $325 \times 10^6$ |
| Steel and iron | $28–30 \times 10^6$ | $194–205 \times 10^6$ |
| Brass | $15 \times 10^6$ | $103 \times 10^6$ |
| Aluminum | $10 \times 10^6$ | $69 \times 10^6$ |
| Glass | $10 \times 10^6$ | $69 \times 10^6$ |
| Cast iron | $10–20 \times 10^6$ | $69–138 \times 10^6$ |
| Wood | $1–2 \times 10^6$ | $6.9–13.8 \times 10^6$ |
| Nylon, epoxy, etc. | $4–8 \times 10^4$ | $27.5–55 \times 10^4$ |
| Collagen | $2–15 \times 10^3$ | $13.8–103 \times 10^3$ |
| Soft rubber | $2–8 \times 10^2$ | $13.8–55 \times 10^2$ |
| Smooth muscle | 2–150 | 13.8–1034 |
| Elastin | 50–100 | 345–690 |

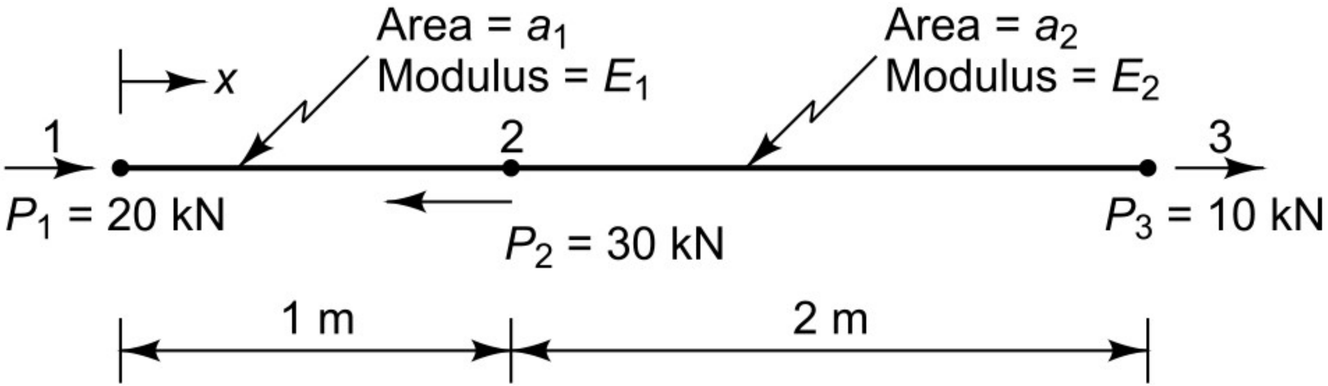
1 $N/m^2$ = pascal (Pa)

Materials having nonlinear uni-axial load-deformation characteristics can not be characterized by just one constant; in-fact actual curve is required to specify the load-deformation behaviour. Therefore for non-linear materials analytical work become complicated and materials showing small amount of non-linearity are **approximated as linear materials**.

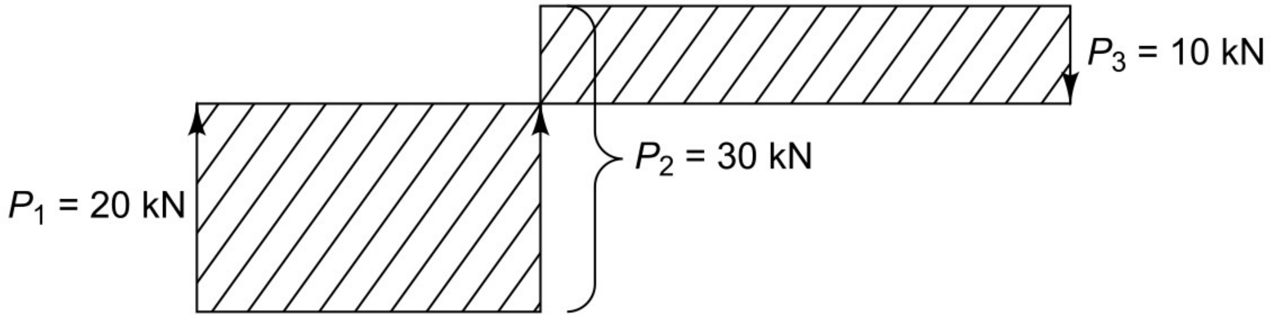
For most materials, elongation under uni-axial tensile load is same as the shortening or compression under the compressive load. Hence, (1) is assumed to be applicable under compression also.

# Example 1

For the below rod, let us determine the various quantities such as axial force and axial deflection with respect to the left end 1, in terms of $x$. Also draw a diagram showing the distribution of axial force in the member with respect to $x$ (axial force diagram).



*AFD*:



# Example 2

A triangular frame supporting a load of 20 kN. Our aim is to estimate the displacement at the point $D$ due to the 20-kN load carried by the chain hoist.