ME232: Dynamics

Vibration

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Basic concepts

- **Space:** Geometric region occupied by bodies.
- **Primary inertial system or Astronomical frame of reference:**
  - The basic frame of reference for the laws of Newtonian mechanics.
  - An imaginary set of rectangular axes assumed to have no translation or rotation in space.
  - The laws of Newtonian mechanics are valid for this reference system as long as any velocities involved are negligible compared with the speed of light (i.e., 300,000 km/s).
  - Measurements made with respect to this reference are said to be absolute, and this reference system may be considered fixed in space.
- A reference frame attached to the surface of the earth has a somewhat complicated motion in the primary system, and a correction to the basic equations of mechanics must be applied for measurements made relative to the reference frame of the earth.

Basic concepts

- **Particle:** A body can be treated as particle if
  - it has negligible dimensions.
  - when the dimensions of a body are irrelevant to the description of its motion or the action of forces on it; e.g., an airplane may be treated as a particle for the description of its flight path.
- **Rigid body:** A body is treated as rigid body when changes in shape are negligible compared with the overall dimensions of the body or with the changes in position of the body as a whole.

Newton's Laws

- **Newton's Laws:**
  - *Law I.* A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.
  - *Law II.* The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.
  - *Law III.* The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.
- The first two laws hold for measurements made in an absolute frame of reference, and they are subject to some correction when the motion is measured relative to a reference system having acceleration, such as one attached to the surface of the earth.
- The correction become insignificant for most engineering problems involving machines and structures which remain on the surface of the earth and in such cases these laws may be applied directly with measurements made relative to the earth.

Newton's Laws

- Newton's second law forms the basis for most of the analysis in dynamics. For a particle of mass m subjected to a resultant force F, the law may be stated as

$$\boldsymbol{F}=m\boldsymbol{a},$$

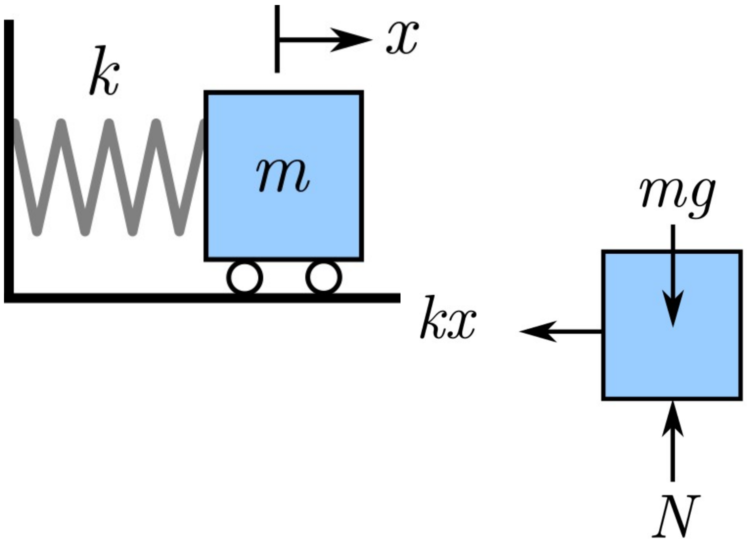
  where $\boldsymbol{a}$ is the resulting acceleration measured in a non-accelerating frame of reference. Newton's first law is a consequence of the second law since there is no acceleration when the force is zero, and so the particle is either at rest or is moving with constant velocity.

Vibration

- Vibration of mechanical systems
  - concerns the linear and angular oscillatory motions of bodies
  - motion in response to applied disturbances in the presence of restoring forces.
- For example:
  - response of an engineering structure to earthquakes,
  - the time response of the plucked string of a musical instrument,
  - the vibration of an unbalanced rotating machine,
  - the wind-induced vibration of power lines,
  - the flutter of aircraft wings.
- In many cases, presence of excessive vibration levels may lead to human discomfort, non-smooth and noisy operation of machines and fatigue, which may ultimately lead to failure.

Free vibration of particles

- When a spring-mounted body is disturbed from its equilibrium position, its ensuing motion in the absence of any imposed external forces is termed **free vibration**.
- In practice, some retarding or damping force always exists which tends to diminish the motion. Common damping forces are those due to mechanical and fluid friction.
- We will first consider the ideal case where the damping forces are small enough to be neglected.



Considering the horizontal vibration of the simple friction-less spring-mass system. The variable x denotes the displacement of the mass from the equilibrium position, which is also the position of zero spring deflection.

Figure shows a plot of the force F_s necessary to deflect the spring vs. the corresponding spring deflection for three types of springs.

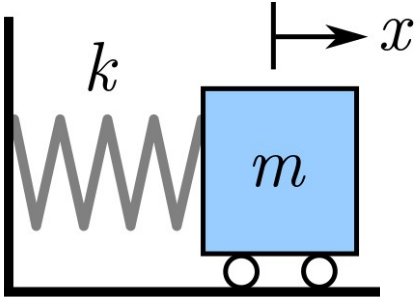
If we consider a linear spring, then the restoring force exerted by the spring on the mass is $-kx$, i.e., when the mass is displaced to the right, the spring force is to the left, and vice versa.

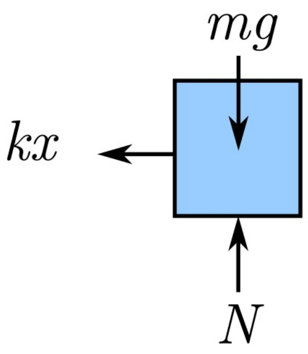
The constant of proportionality k is called the spring constant, modulus, or stiffness and has the units N/m.

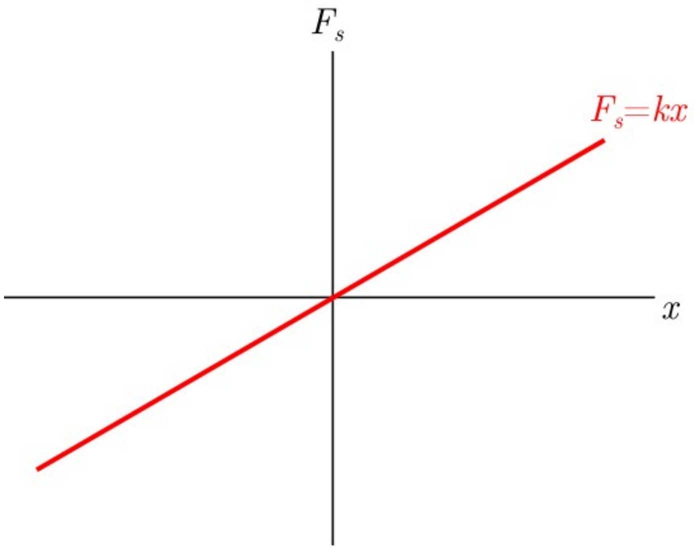
The equation of motion for the body of is obtained by applying Newton's second law,

$$\sum F_x = m\ddot{x} \quad \Rightarrow \quad -kx = m\ddot{x} \quad \text{or} \quad m\ddot{x} + kx = 0.$$

................(1)







The oscillation of a mass subjected to a linear restoring force described by (1) is called simple harmonic motion and is characterized by acceleration which is proportional to the displacement but of opposite sign.

Normally, we write (1) as following:

$$\ddot{x} + \omega_n^2 x = 0, \quad \text{where} \quad \omega_n = \sqrt{k/m}, \qquad \text{......................(2)}$$

Because we anticipate an oscillatory motion, we look for a solution which gives x as a periodic function of time. Thus, a logical choice is

$$x = A\cos\omega_n t + B\sin\omega_n t, \quad \text{or} \qquad \text{......................(3)}$$

$$\text{alternatively} \quad x = C\sin(\omega_n t + \psi). \qquad \text{......................(4)}$$

The constants A and B, or C and ψ, can be determined from knowledge of the initial displacement x_0 and initial velocity $\dot{x}_0$ of the mass. For example, if we work with the solution form of (3) and evaluate x and $\dot{x}$ at time $t = 0$, we obtain,

$$x_0 = A, \qquad \text{and} \qquad \dot{x}_0 = \omega_n B$$

Substitution of these values of A and B into (4) yields

$$x = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t. \qquad \cdots\cdots\cdots\cdots(5)$$

The constants C and ψ of (4) can be determined in terms of given initial conditions in a similar manner. Evaluation of (4) and its first time derivative at $t = 0$ gives

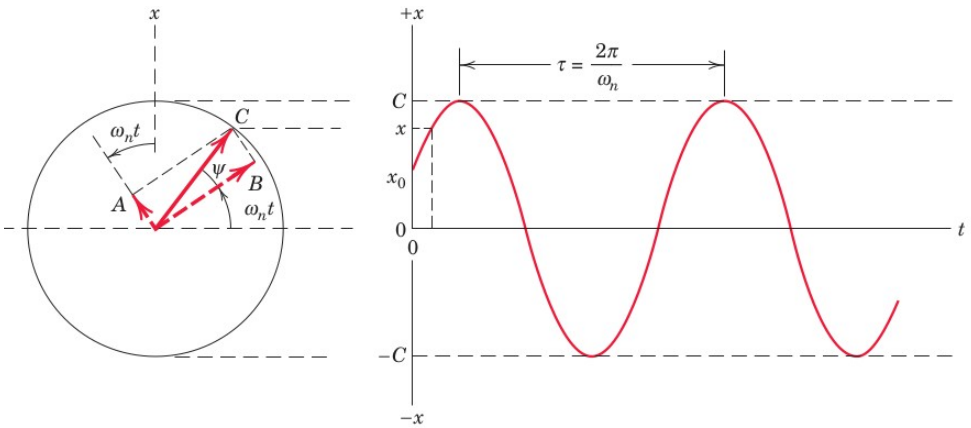
$$x_0 = C \sin \psi, \qquad \text{and} \qquad \dot{x}_0 = C\omega_n \cos \psi. \qquad \cdots\cdots\cdots\cdots(6)$$

Solving for C and ψ yields,

$$C = \sqrt{x_0 + (\dot{x}_0/\omega_n)^2}, \qquad \psi = \tan^{-1}(x_0 \omega_n / \dot{x}_0). \qquad \cdots\cdots\cdots\cdots(7)$$

Equations (3) and (4) represent two different mathematical expressions for the same time-dependent motion.

Graphical representation of motion



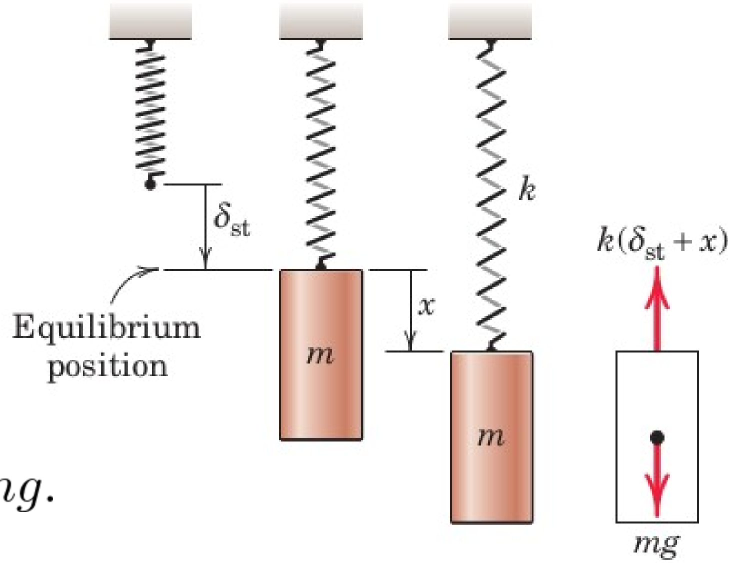
The motion may be represented graphically, where x is seen to be the projection onto a vertical axis of the rotating vector of length C. The vector rotates at the constant angular velocity $\omega_n = \sqrt{(k/m)}$, which is called the natural circular frequency and has the units radians per second.

The number of complete cycles per unit time is the natural frequency $\boldsymbol{f_n = \omega_n / 2\pi}$ and is expressed in hertz (1 hertz (Hz)=1 cycle per second).

The time required for one complete motion cycle (one rotation of the reference vector) is the period of the motion and is given by $\tau = 1/f_n = 2\pi/\omega_n$.

Also note that x is the sum of the projections onto the vertical axis of two perpendicular vectors whose magnitudes are A and B and whose vector sum C is the amplitude. Vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ rotate together with the constant angular velocity ω_n. Thus, as we have already seen, $C = (A^2 + B^2)^{\frac{1}{2}}$ and $\psi = \tan^{-1}(A/B)$.

Now if the motion of mass is vertical rather than horizontal, the equation of motion (and therefore all system properties) is unchanged if we continue to define x as the displacement from the equilibrium position. The equilibrium position now involves a nonzero spring deflection δ_{st}. From the free-body diagram, Newton's second law gives



$$-k(x+\delta_{\text{st}})+mg=m\ddot{x} \qquad \cdots\cdots\cdots\cdots(8)$$

Note that at the equilibrium position (i.e., at $x=0$), $k\delta_{\text{st}}=mg$. Thus (8) becomes,

$$-kx=m\ddot{x} \quad \text{or} \quad m\ddot{x}+kx=0. \qquad \cdots\cdots\cdots\cdots(9)$$

which is same as (1).

The lesson here is that by defining the displacement variable to be zero at equilibrium rather than at the position of zero spring deflection, we may ignore the equal and opposite forces associated with equilibrium.