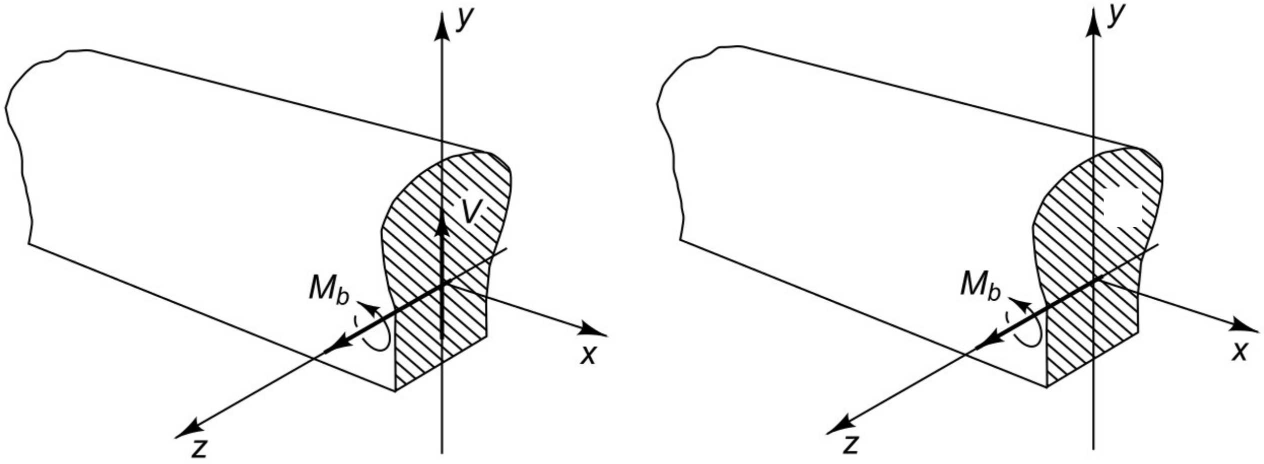
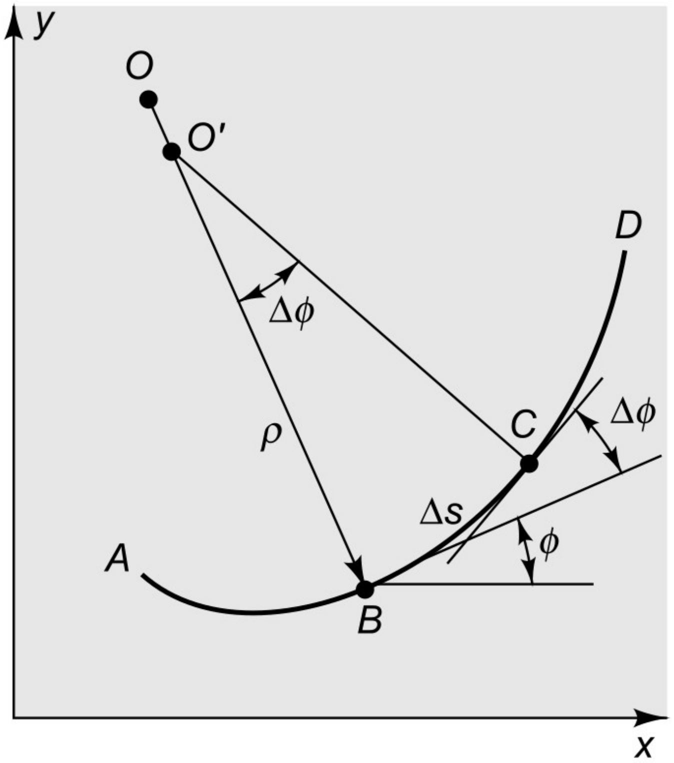
ME231: Solid Mechanics-I

Stresses due to bending

Introduction



Curvature



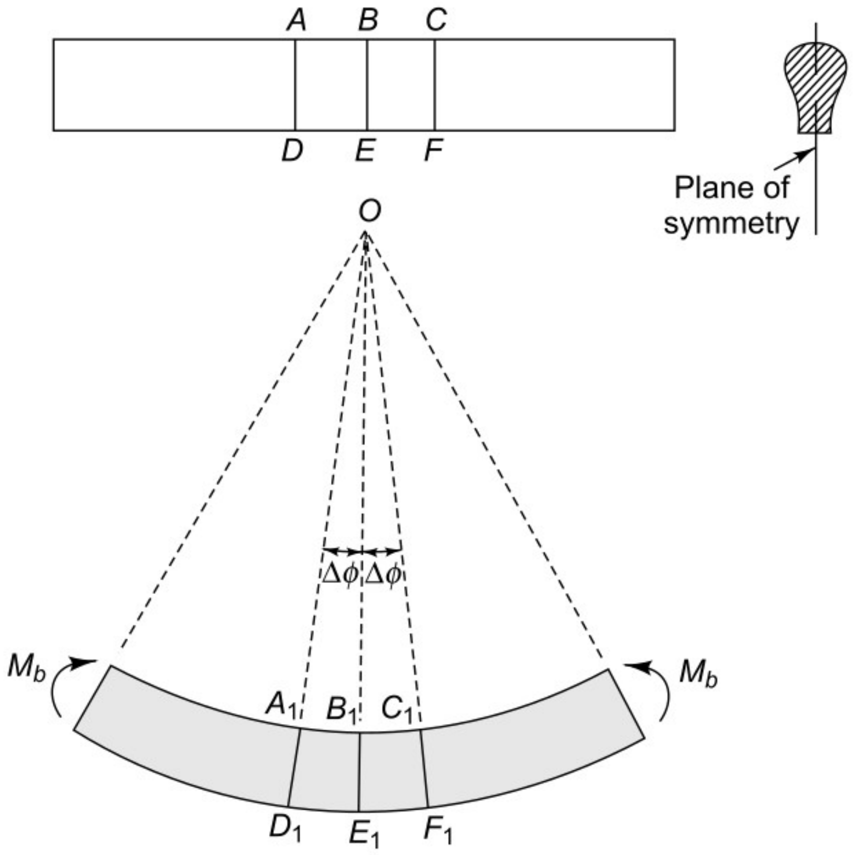
Curvature is the rate of change of *slope angle* of the curve w.r.t. the *distance along the curve*

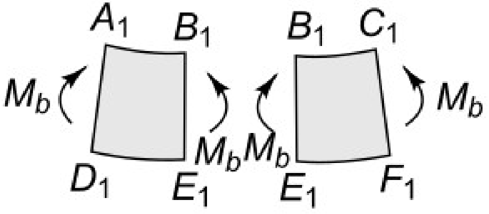
Curvature at point B is defined as

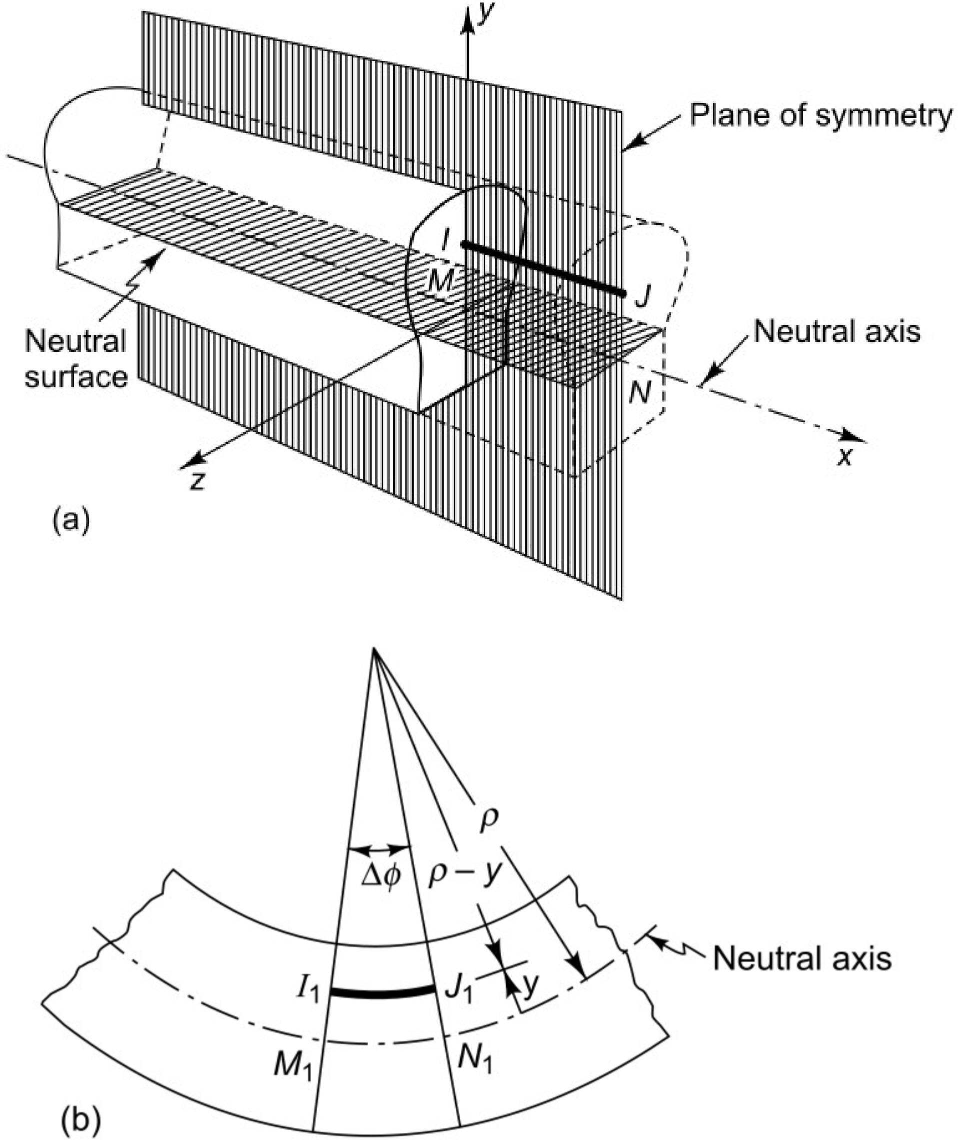
$$\frac{d\phi}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\phi}{\Delta s} = \lim_{\Delta s \to 0} \frac{1}{O'B} = \frac{1}{\rho}$$

$\rho = OB$ is the *radius of curvature* at point B.

Deformation







Assume that, deformation within the planes are sufficiently small

$$IJ = MN = M_1N_1$$

Strain of I_1J_1 is

$$\epsilon_x = \frac{I_1J_1 - IJ}{IJ} = \frac{I_1J_1 - M_1N_1}{M_1N_1}$$

$$\epsilon_x = \frac{(\rho - y)\Delta\phi - \rho\Delta\phi}{\rho\Delta\phi} = -\frac{y}{\rho} = -\frac{d\phi}{ds}y$$

- Strain is linearly proportional to the distance from the neural axis
- Derivation is strictly applicable to the plane of symmetry, however we assume that the longitudinal strain at all points in the c.s. of the beam is given by the same equation.
- As plane sections remain plane

$$\gamma_{xy} = \gamma_{xz} = 0 \Rightarrow \tau_{xy} = \tau_{xz} = 0.$$

- No quantitative statement about $\varepsilon_y, \varepsilon_z$ and γ_{yz} beyond the remark that they must be symmetrical w.r.t to the xy-plane.

Stresses from stress-strain relation

$$\epsilon_x = \frac{1}{E}[\sigma_x - v(\sigma_y + \sigma_z)] = -\frac{y}{\rho}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} = 0$$

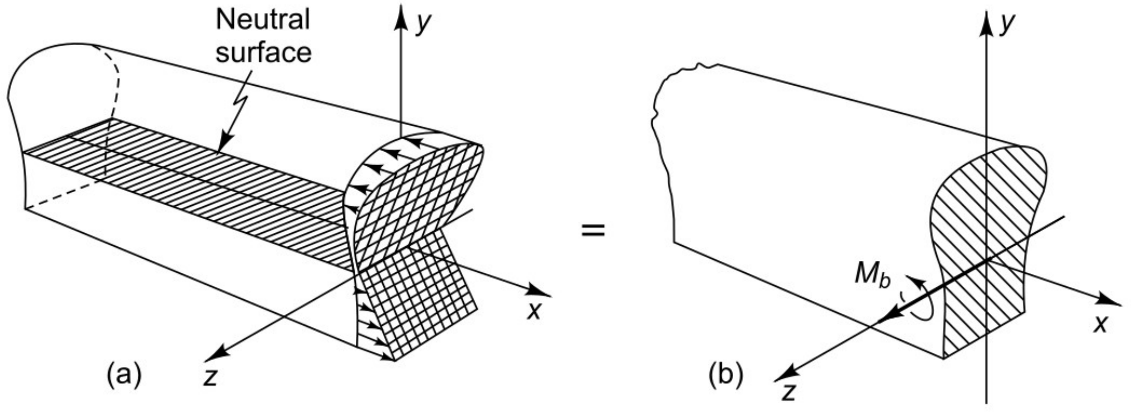
Thus the shear-stress components τ_{xy} and τ_{xz} must vanish in pure bending.

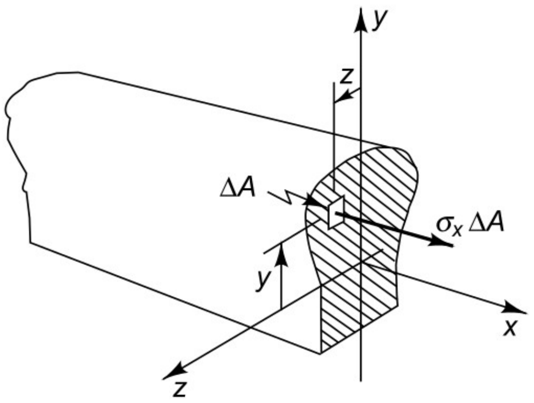
Equilibrium requirements

$$\sum F_x = \int_A \sigma_x dA = 0$$

$$\sum M_y = \int_A z\sigma_x dA = 0$$

$$\sum M_z = -\int_A y\sigma_x dA = M_b$$

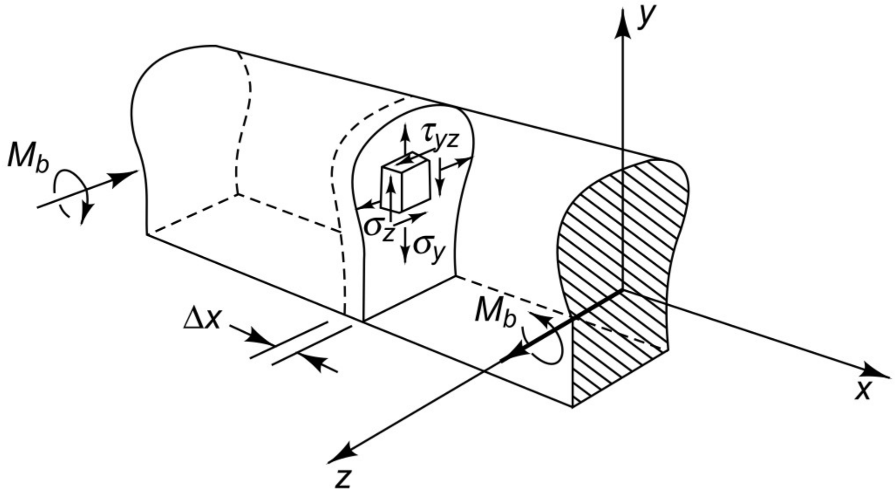




We can not say anything about ε_y and ε_z at this stage; hence also for σ_y and σ_z.

Hence, we make some assumptions about the transverse behaviour.

Basis of the assumption comes from the slenderness of the beam.



Slenderness of the beam suggests a plausibility of $\sigma_y = \sigma_z = \tau_{yz} = 0.$

With this assumption, now we are ready to find the following relations.

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{\sigma_x}{E} = -\frac{y}{\rho}$$

$$\sigma_x = -E\frac{y}{\rho} = -E\frac{d\phi}{ds}y$$

First moment of c.s. area

$$\sum F_x = \int_A \sigma_x dA = -\int_A E\frac{y}{\rho}dA = -\frac{E}{\rho}\int_A ydA = 0$$

- First moment of cross-sectional area about the neutral surface must be zero.
- The neutral surface must pass through the centroid of the cross-sectional area

$$\sum M_y = \int_A z\sigma_x dA = -\int_A E\frac{y}{\rho}zdA = -\frac{E}{\rho}\int_A yzdA = 0$$

- Symmetry of the cross-section about xy-plane will ensure $\int_A yzdA = 0$.