ME231: Solid Mechanics-I

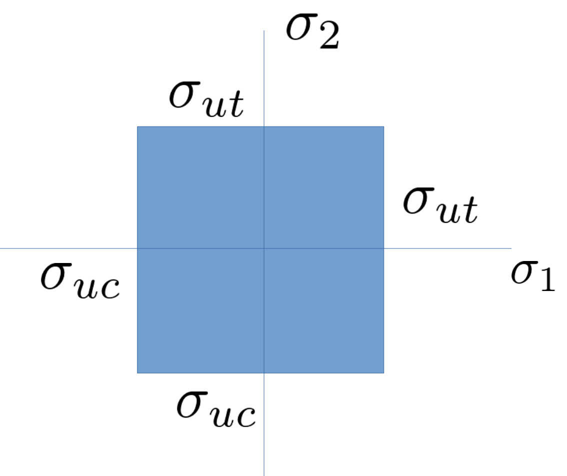
Stress, Strain and Temperature relationship

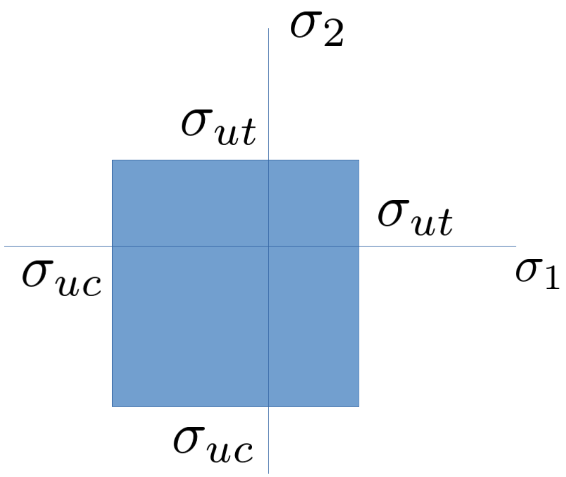
Failure criteria for brittle materials

Maximum normal stress criterion

According to this criterion failure in the brittle material occurs when maximum principal stress exceeds the ultimate strength in tension or when minimum principal stress exceeds the ultimate strength in compression. If the principal stresses are σ_1, σ_2, σ_3, then for plane stress the criterion becomes

$$\sigma_1 \geq \sigma_{ut} \quad \text{or} \quad \sigma_2 \leq \sigma_{uc}$$





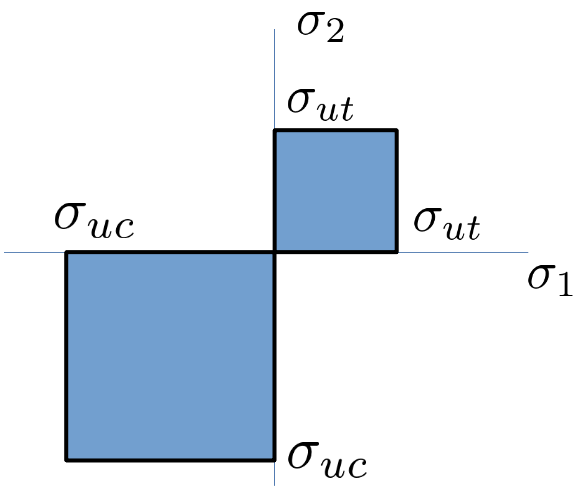
This criterion is also known as *Coulomb's criterion* or *Rankine criterion.*

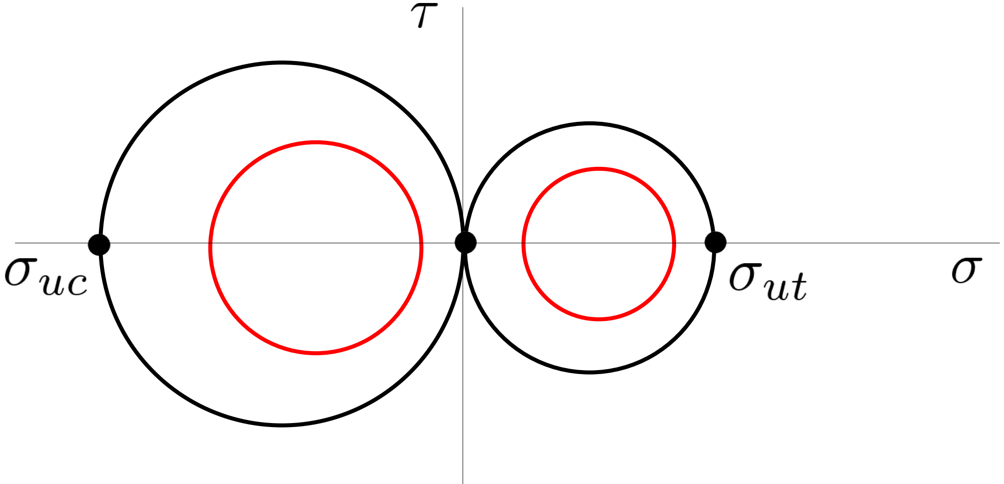
Mohr's criterion

To develop the criterion for plane stress ($\sigma_3=0$) condition, following test results are required.

- Uniaxial tension test
- Uniaxial compression test
- Torsion test

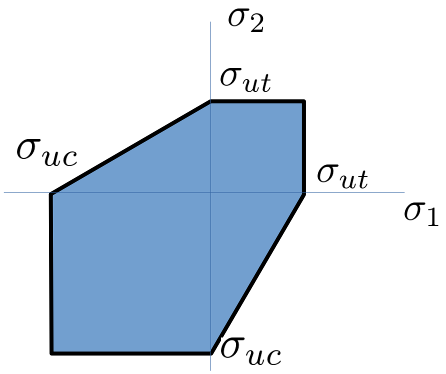
When both the stresses are of same nature (tension/compression)

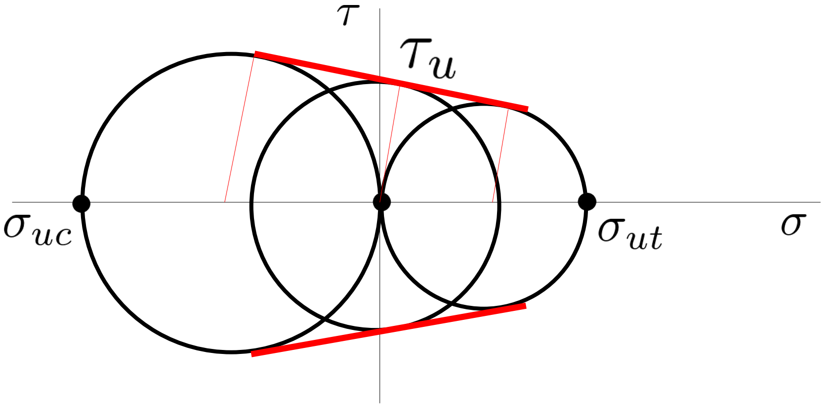




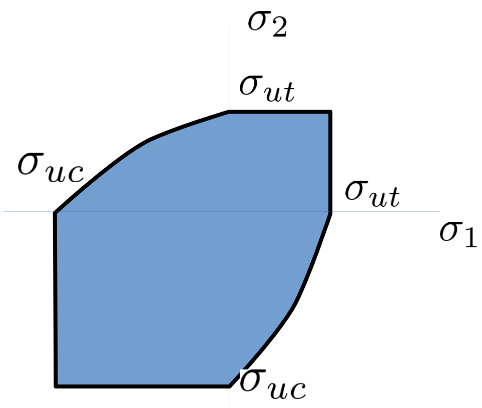
Mohr's criterion

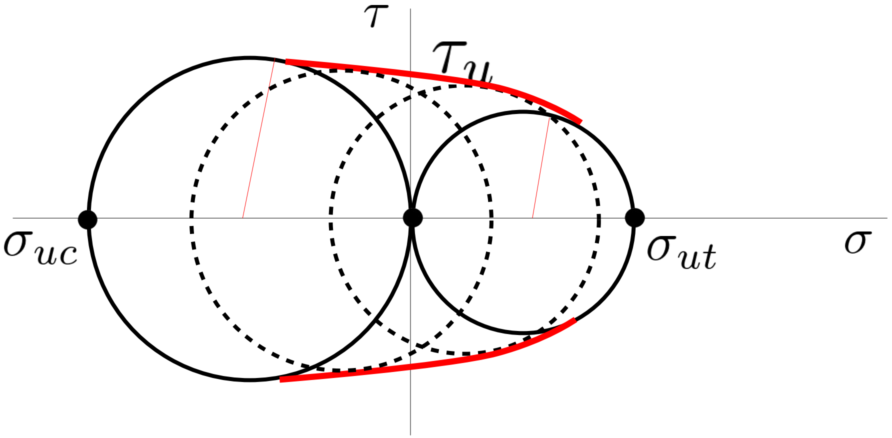
When both the stresses are of different nature



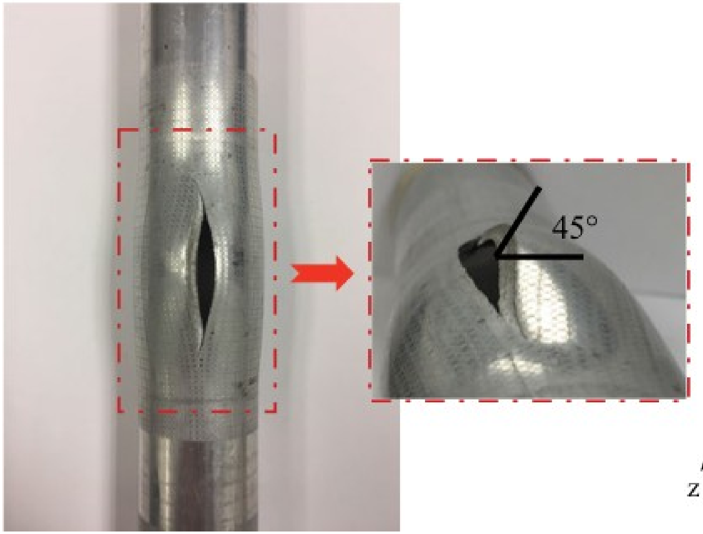


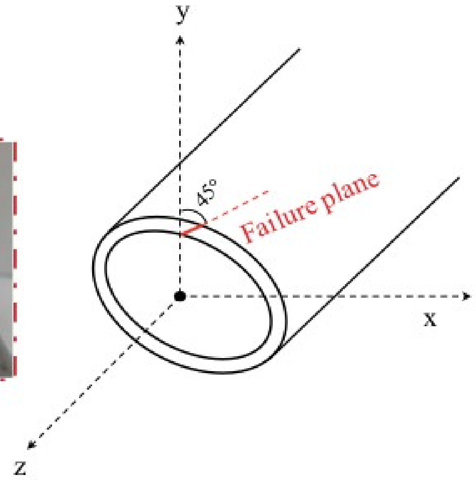
To draw more accurate envelope

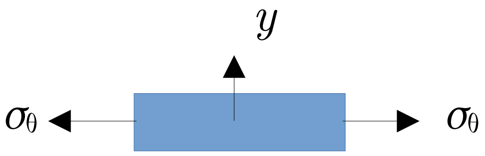


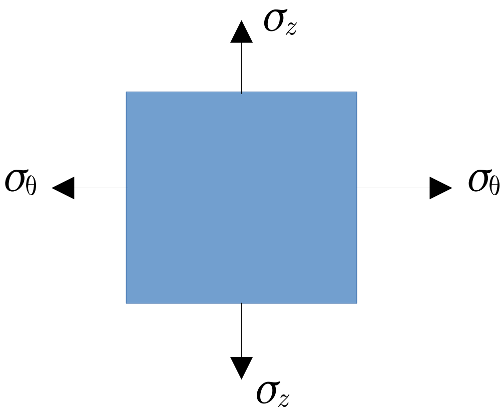


Example: Failure of Pressure Vessel



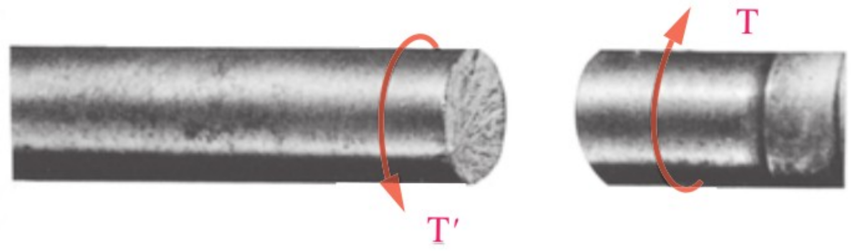




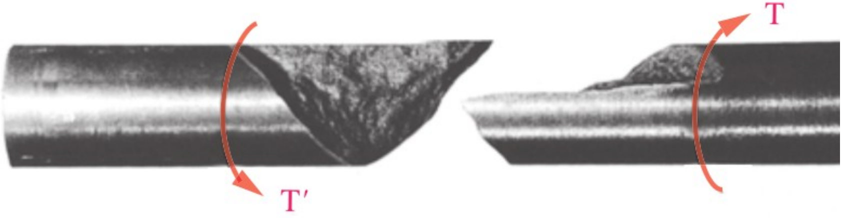


Source: Mechanics of Materials Laboratory Course by Ghatu Subhash and Shannon Ridgeway. Morgan & Claypool publishers

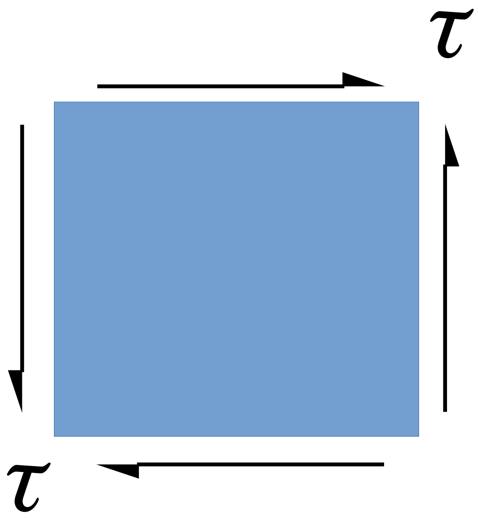
Example: Failure of shaft under torsion



Shaft made of ductile material

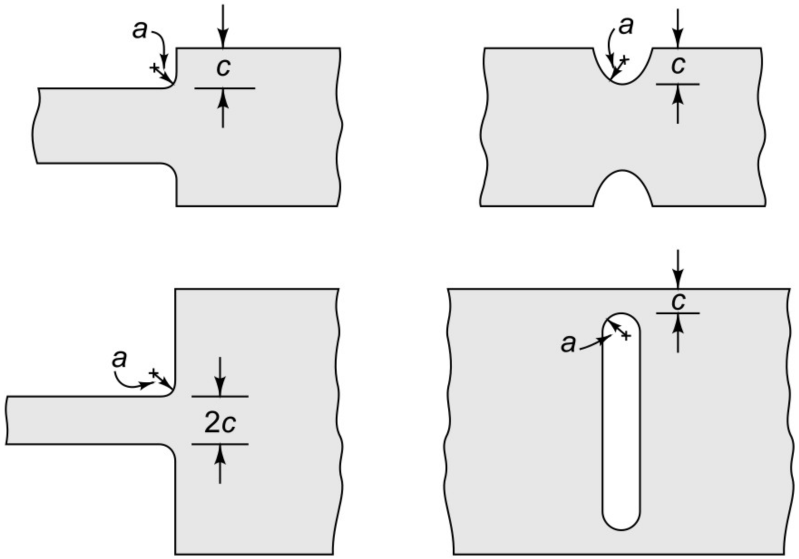


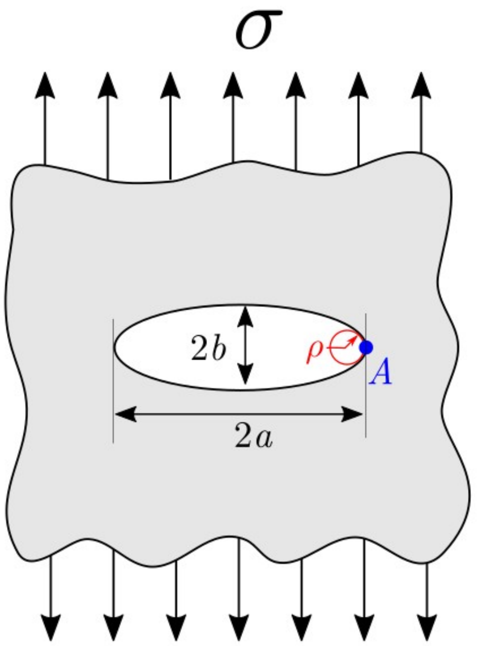
Shaft made of brittle material

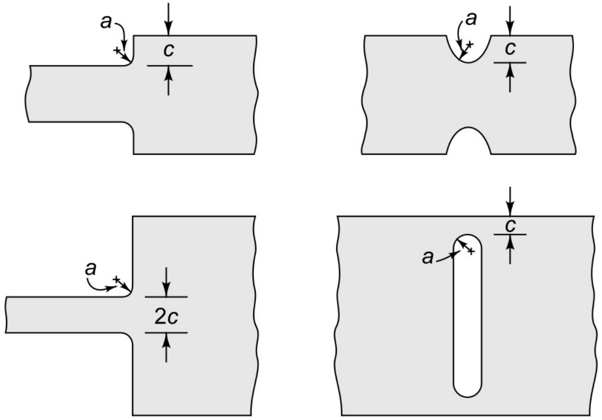


Stress Concentration

- All engineering structures have local geometrical irregularities in the form of an oil hole, a keyway, a fillet or a notch.
- When such structures are stressed, there is a localized variation in the stress state in the immediate neighborhood of the irregularity. The maximum stress levels at the irregularity may be several times larger than the nominal stress levels in the bulk of the body. This increase in stress caused by the irregularity in geometry is called a stress concentration.
- Where the stress concentration cannot be avoided by a change in design, it is important to base the design on the local value of the stress rather than on an average value.







Analytical solutions is available for an elliptical hold in an infinite load under tensile loading. Stress at point A is given as,

$$\sigma_A = \sigma \left(1 + \frac{2a}{b}\right).$$

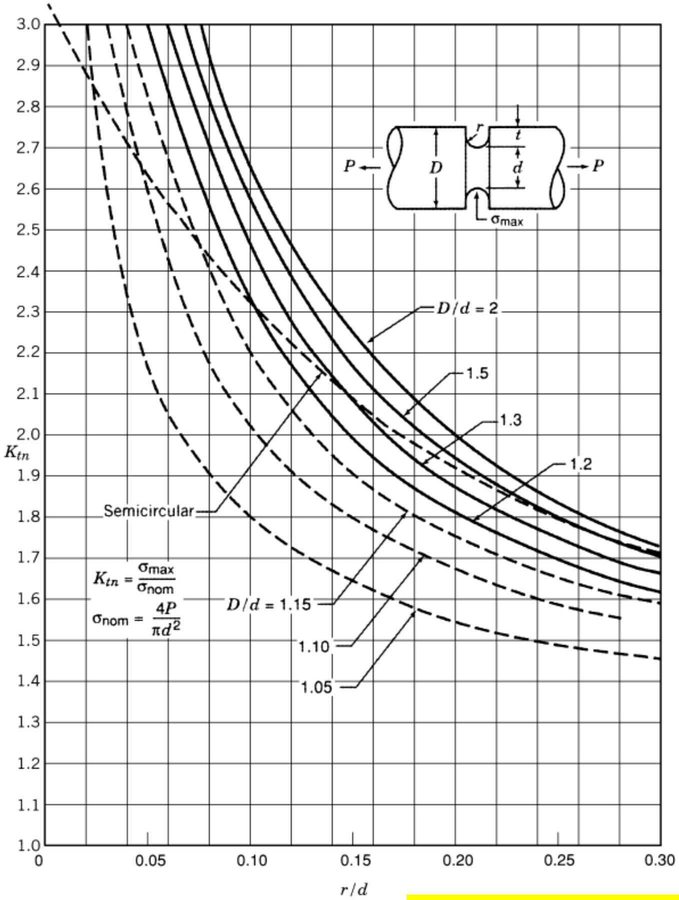
When the major axis, a, increases relative to b, the elliptical hole begins to take on the appearance of a sharp notch (or a crack). For this case, *Inglis* (1913) found it more convenient to express the above expression in terms of the radius of curvature ρ as,

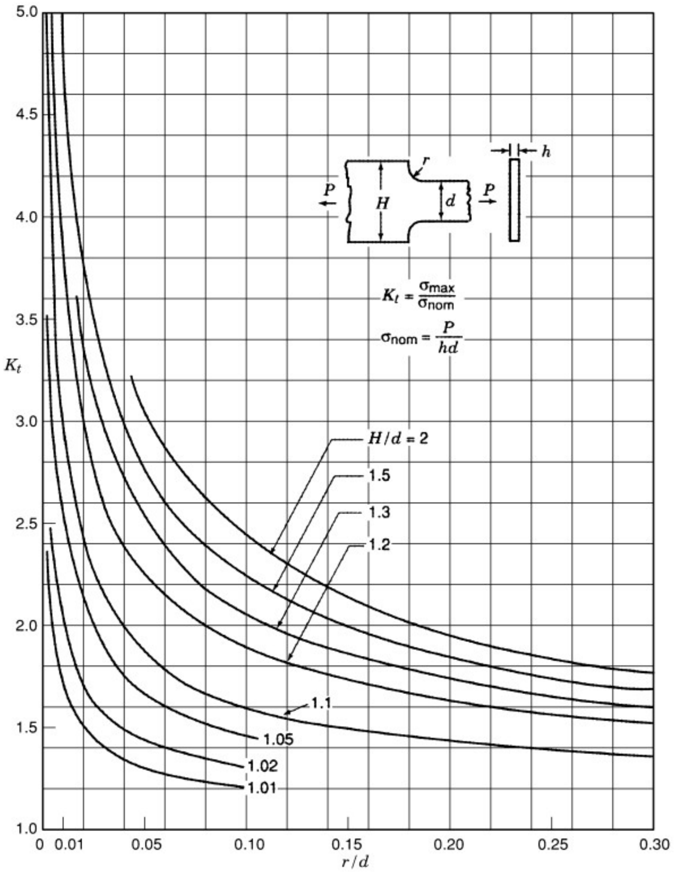
$$\sigma_A = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}}\right), \quad \text{where} \quad \rho = b^2/a.$$

For the cases, where analytical solutions are not available following empirical relationship gives a good estimate of the stress concentration factor.

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}} \approx 1 + (0.3 \text{ to } 2)\sqrt{\frac{c}{a}}.$$

For practical design purpose design curves are available in handbooks for different types of geometrical irregularities.





Peterson's Stress Concentration Factors by
Walter D. Pilkey & Deborah F. Pilkey & Zhuming Bi. John Wiley & Sons.