ME232: Dynamics

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Relative acceleration:

Expression for relative acceleration can be obtained by differentiating (11) w.r.t. time as,

$$\dot{\boldsymbol{v}}_A = \dot{\boldsymbol{v}}_B + \dot{\boldsymbol{v}}_{\text{rel}} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times \dot{\boldsymbol{r}}$$

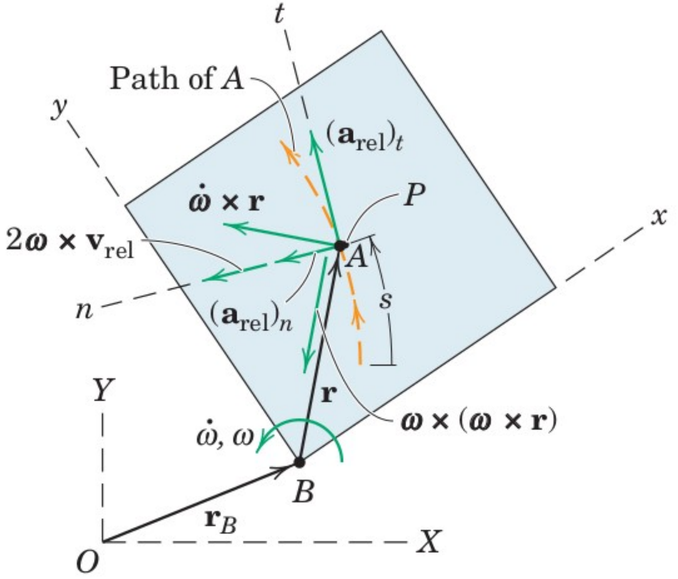
$$\boldsymbol{a}_A = \boldsymbol{a}_B + (\boldsymbol{a}_{\text{rel}} + \boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}}) + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{v}_{\text{rel}} + \boldsymbol{\omega} \times \boldsymbol{r}) \qquad \text{(from 12)}$$

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}} + \boldsymbol{a}_{\text{rel}} \qquad \ldots\ldots\ldots\ldots\ldots(13)$$

Equation (13) is the general vector expression for the absolute acceleration of a particle A in terms of its acceleration $\boldsymbol{a}_{\text{rel}}$ measured relative to a moving coordinate system which rotates with an angular velocity $\boldsymbol{\omega}$ and an angular acceleration $\dot{\boldsymbol{\omega}}$.

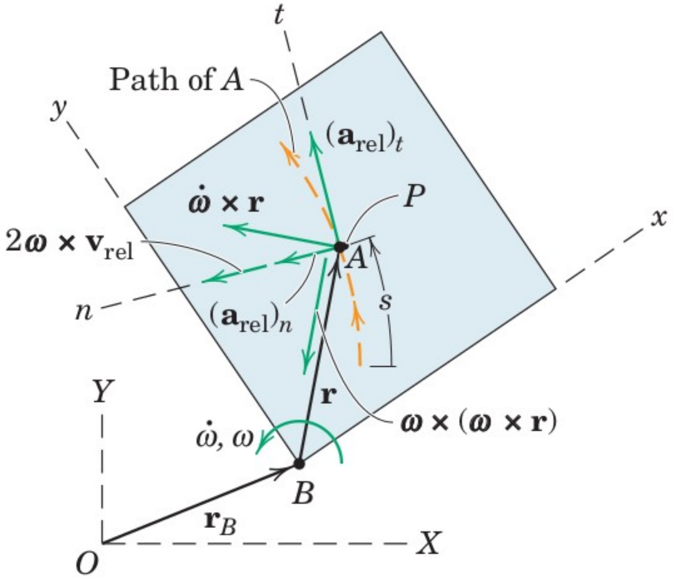
The terms $\dot{\boldsymbol{\omega}} \times \boldsymbol{r}$ and $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ are shown.

They represent, respectively, the tangential and normal components of the acceleration $\boldsymbol{a}_{P/B}$ of the coincident point P in its circular motion with respect to B.



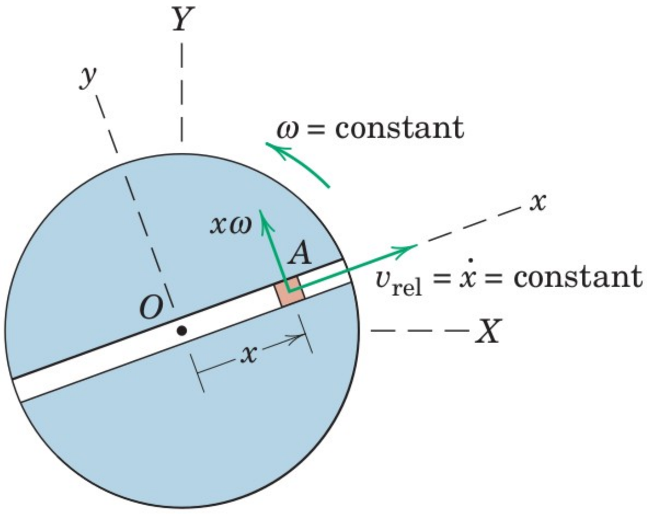
This motion would be observed from a set of nonrotating axes moving with B. The magnitude of $\dot{\boldsymbol{\omega}} \times \boldsymbol{r}$ is $r\ddot{\theta}$ and its direction is tangent to the circle. The magnitude of $\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$ is $r\omega^2$ and its direction is from P to B along the normal to the circle.

The acceleration of A relative to the plate along the path, $\boldsymbol{a}_{\text{rel}}$, may be expressed in rectangular, normal and tangential, or polar coordinates in the rotating system. Frequently, n- and t-components are used, and these components are shown in figure. The tangential component has the magnitude $(a_{\text{rel}})_t = \ddot{s}$, where s is the distance measured along the path to A. The normal component has the magnitude $(a_{\text{rel}})_n = v^2_{\text{rel}}/\rho$, where ρ is the radius of curvature of the path as measured in x-y. The sense of this vector is always toward the center of curvature.

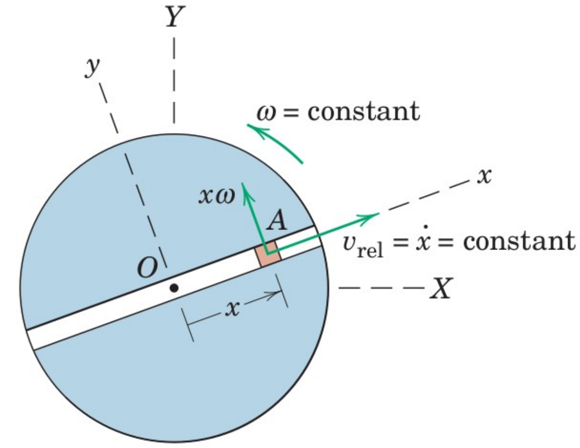


The term $2\boldsymbol{\omega}\times\boldsymbol{v}_{\text{rel}}$, is called the **Coriolis acceleration**. It represents the **difference between the acceleration of A relative to P as measured from nonrotating axes and from rotating axes.** The direction is always normal to the vector $\boldsymbol{v}_{\text{rel}}$, and the sense is established by the right-hand rule for the cross product.

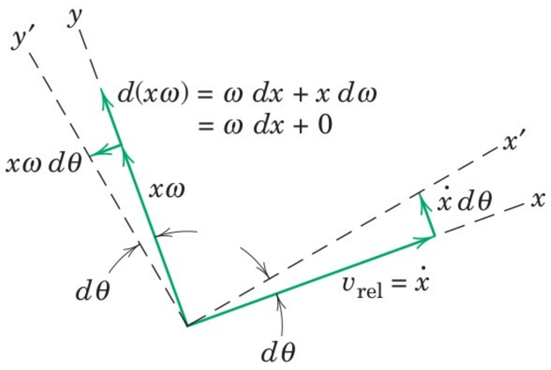
The Coriolis acceleration $\boldsymbol{a}_{\text{Cor}} = 2\boldsymbol{\omega}\times\boldsymbol{v}_{\text{rel}}$ is difficult to visualize because it is composed of two separate physical effects. To help with this visualization, we will consider the simplest possible motion in which this term appears. Consider a rotating disk with a radial slot in which a small particle A is confined to slide. Let the disk turn with a constant angular velocity $\boldsymbol{\omega}$ and let the particle move along the slot with a constant speed $\boldsymbol{v}_{\text{rel}} = \dot{\boldsymbol{x}}$ relative to the slot. The velocity of A has the two components (a) $\dot{\boldsymbol{x}}$ due to motion along the slot, and (b) $\boldsymbol{x}\boldsymbol{\omega}$ due to the rotation of the slot.



The changes in these two velocity components due to the rotation of the disk are shown for the interval dt, during which the x-y axes rotate with the disk through the angle $d\theta$ to x'-y'.



The velocity increment due to the change in direction of $\boldsymbol{v}_{rel}$ is $\dot{x}\ d\theta$ and that due to the change in magnitude of $x\omega$ is ωdx, both being in the y-direction normal to the slot. Dividing each increment by dt and adding give the sum $\omega\dot{x} + \dot{x}\omega = 2\dot{x}\omega$, which is the magnitude of the Coriolis acceleration $2\boldsymbol{\omega} \times \boldsymbol{v}_{rel}$.



Dividing the remaining velocity increment $x\omega d\theta$ due to the change in direction of $x\omega$ by dt gives $x\omega\dot{\theta}$ or $x\omega^2$, which is the acceleration of a point P fixed to the slot and momentarily coincident with the particle A.

Now applying (13) for the same motion, we see that

$$\boldsymbol{a}_A = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}}$$

Replacing $\boldsymbol{r}$ by $x\boldsymbol{i}$, $\boldsymbol{\omega}$ by $\omega\boldsymbol{k}$, and $\boldsymbol{v}_{\text{rel}}$ by $\dot{x}\boldsymbol{i}$ gives

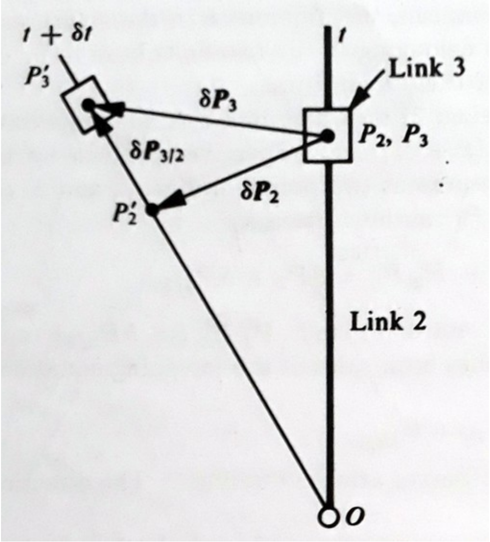
$$\boldsymbol{a}_A = \omega^2 x\boldsymbol{i} + 2\omega\dot{x}\boldsymbol{j}$$

Motion difference between two instantaneously coincident points:

$$\boldsymbol{OP}_3 = \boldsymbol{OP}_2 + \boldsymbol{P}_2\boldsymbol{P}_3$$

$$\boldsymbol{v}_{P_3} = \boldsymbol{v}_{P_2} + \boldsymbol{\omega} \times \boldsymbol{P}_3\boldsymbol{P}_2 + \boldsymbol{v}_{P_3/P_2}$$

$$\boldsymbol{a}_{P_3} = \boldsymbol{a}_{P_2} + \dot{\boldsymbol{\omega}} \times \boldsymbol{P}_3\boldsymbol{P}_2 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{P}_3\boldsymbol{P}_2) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{P_3/P_2} + \boldsymbol{a}_{P_3/P_2}$$



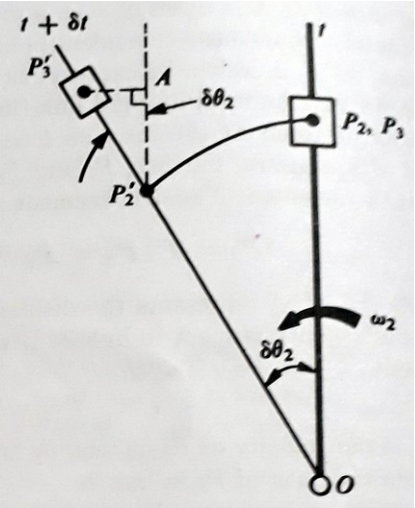
Substitute $\boldsymbol{P}_2\boldsymbol{P}_3$=0 now,

$$\boldsymbol{v}_{P_3} = \boldsymbol{v}_{P_2} + \boldsymbol{v}_{P_3/P_2}$$

$$\boldsymbol{a}_{P_3} = \boldsymbol{a}_{P_2} + 2\boldsymbol{\omega} \times \boldsymbol{v}_{P_3/P_2} + \boldsymbol{a}_{P_3/P_2}$$

The Coriolis component of acceleration can be understood as follows:

$$AP_3' = P_2'P_3' \cdot \delta\theta_2 = V_{P_3/P_2}\delta t \cdot \omega_2 \delta t = V_{P_3/P_2} \cdot \omega_2 \delta t^2$$

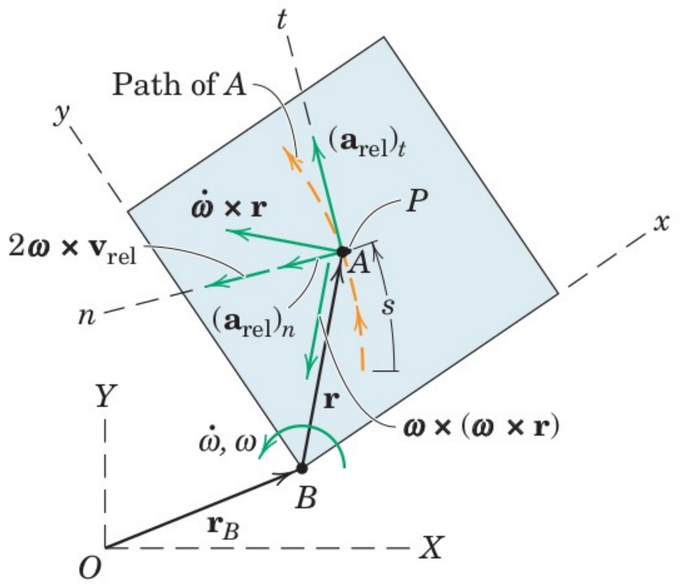


This displacement term is proportional to the square of time elapsed. Therefore this displacement must be due to an additional acceleration of P_3 in transverse direction. If the magnitude of this additional acceleration is a_c, then

$$\frac{1}{2}a_c\delta t^2 = V_{P_3/P_2} \cdot \omega_2 \cdot \delta t^2 \Rightarrow a_c = 2V_{P_3/P_2}\omega_2.$$

Rotating vs. Nonrotating system

The following comparison will help to establish the equivalence of, and clarify the differences between, the relative-acceleration equations written for rotating and nonrotating reference axes:



$$\boldsymbol{a}_A = \boldsymbol{a}_B + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{\mathrm{rel}} + \boldsymbol{a}_{\mathrm{rel}}$$

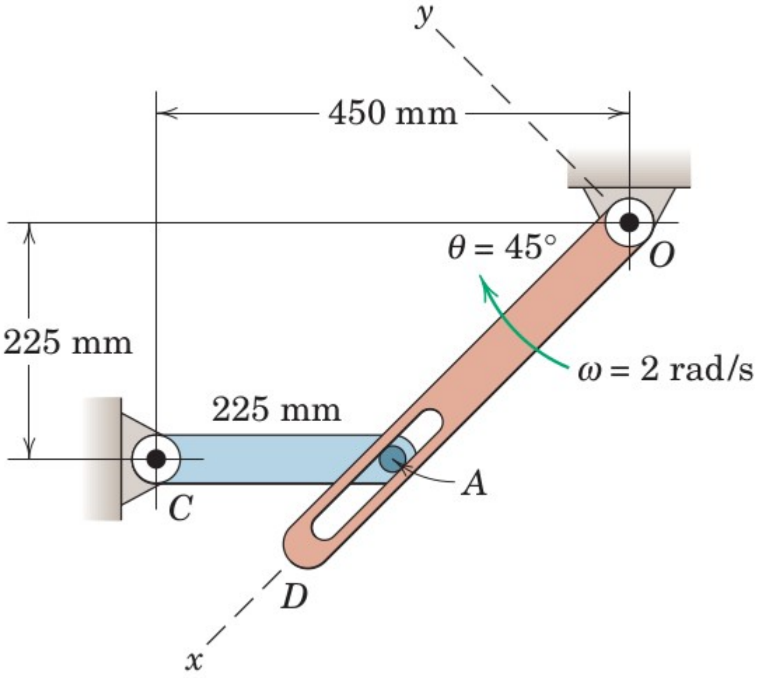
$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{P/B} + \boldsymbol{a}_{A/P}$$

$$\boldsymbol{a}_A = \boldsymbol{a}_P + \boldsymbol{a}_{A/P}$$

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B}$$

Example 8

The pin A of the hinged link AC is confined to move in the rotating slot of link OD. The angular velocity of OD is $\omega = 2$ rad /s clockwise and is constant for the interval of motion concerned. For the position where $\theta = 45°$ with AC horizontal, determine
(a) the velocity of pin A and the velocity of A relative to the rotating slot in OD,
(b) determine the angular acceleration of AC and the acceleration of A relative to the rotating slot in arm OD.



Pin A always moves along the slot, hence we use a rotating coordinate system x-y attached to arm OD. The expression of velocity of A, with the origin at the fixed point O as,

$$\boldsymbol{v}_A = \boldsymbol{\omega}_{OD} \times \boldsymbol{r} + \boldsymbol{v}_{\text{rel}}.$$

Point A also moves in a circular path about C, hence

$$\boldsymbol{v}_A = \boldsymbol{\omega}_{CA} \times \boldsymbol{r}_{CA} = \omega_{CA}\, \boldsymbol{k} \times (225/\sqrt{2})(-\boldsymbol{i} - \boldsymbol{j}) = (225/\sqrt{2})\omega_{CA}\, (\boldsymbol{i} - \boldsymbol{j})$$

where ω_{CA} is assumed to be clockwise arbitrarily.

The vector from the origin to the point P on OD coincident with A is

$$\boldsymbol{r} = OP\, \boldsymbol{i} = 225\sqrt{2}\, \boldsymbol{i} \text{ mm}.$$

Thus, $\boldsymbol{\omega}_{OD} \times \boldsymbol{r} = 450\sqrt{2}\, \boldsymbol{j}$ mm/s.

Finally, the relative-velocity term $\boldsymbol{v}_{\text{rel}}$ is the velocity measured by an observer attached to the rotating reference frame and is $\boldsymbol{v}_{\text{rel}} = \dot{x}\, \boldsymbol{i}$.

Substituting into the relative-velocity equation and solving it gives, $\boldsymbol{\omega}_{CA} = -4$ rad/s and $\dot{\boldsymbol{x}} = \boldsymbol{v}_{\text{rel}} = -450\sqrt{2}$ mm/s. Now $\boldsymbol{v}_A$ can also be calculated.

The expression for acceleration of point A with the origin at the fixed point O,

$$\boldsymbol{a}_A = \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_{\text{rel}} + \boldsymbol{a}_{\text{rel}}$$

Acceleration of point A can also be written as,

$$\boldsymbol{a}_A = \dot{\boldsymbol{\omega}}_{CA} \times \boldsymbol{r}_{CA} + \boldsymbol{\omega}_{CA} \times (\boldsymbol{\omega}_{CA} \times \boldsymbol{r}_{CA})$$

Substituting these values from solution of (a),

$$\boldsymbol{a}_A = \dot{\omega}_{CA}\boldsymbol{k} \times \frac{225}{\sqrt{2}}(-\boldsymbol{i} - \boldsymbol{j}) - 4\boldsymbol{k} \times \left(-4\boldsymbol{k} \times \frac{225}{\sqrt{2}}[-\boldsymbol{i} - \boldsymbol{j}]\right)$$

As $\boldsymbol{\omega}$ is constant, $\dot{\boldsymbol{\omega}} \times \boldsymbol{r} = 0$.

Other terms can also be calculated except, $\boldsymbol{a}_{\text{rel}} = \ddot{x}\boldsymbol{i}$.

Substituting all values in the first expression $\dot{\boldsymbol{\omega}}_{CA}$ and $\ddot{\boldsymbol{x}}$, and then $\boldsymbol{a}_A$ can be calculated.