ME632: Fracture Mechanics

Timings	
Monday	10:00 to 11:20
Thursday	08:30 to 09:50

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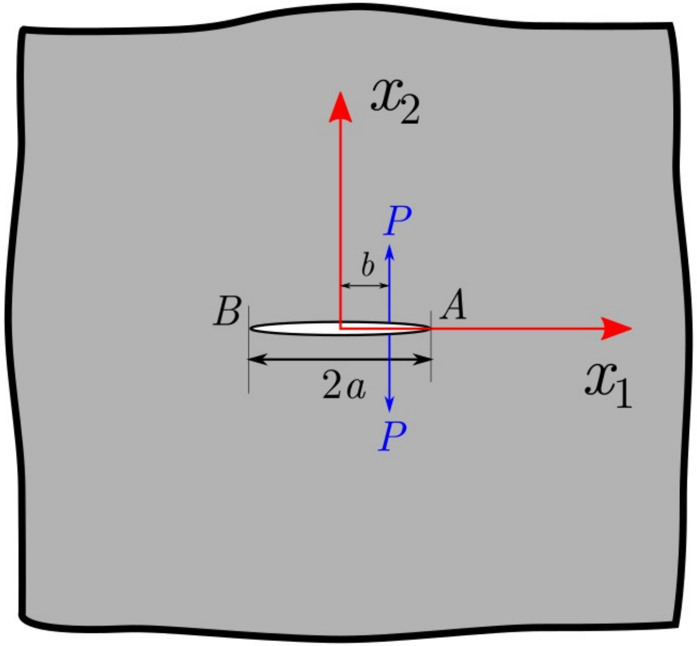
Westergaard function for few more cases

Westergaard function for an infinite plate with a crack of length $2a$ subjected to a pair of forces at $x_1 = b$.

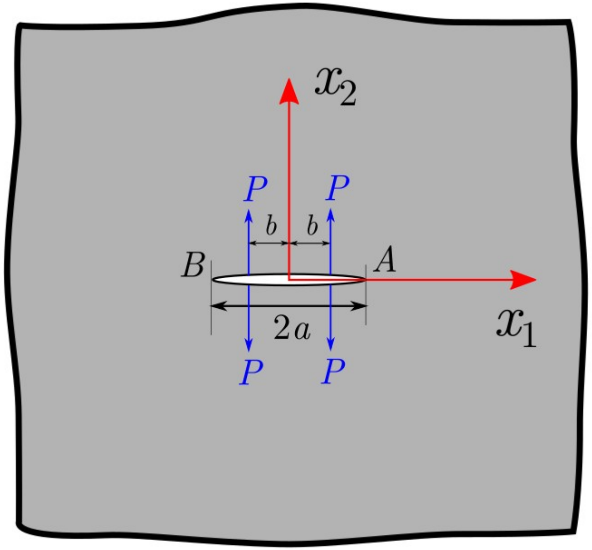


$$Z_I = \frac{P}{\pi(z-b)}\sqrt{\frac{a^2-b^2}{z^2-a^2}} \qquad \cdots\cdots\cdots\cdots\cdots(99)$$

Using this function it can be shown that the SIF at crack tips A and B are

$$K_I^A = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+b}{a-b}}$$

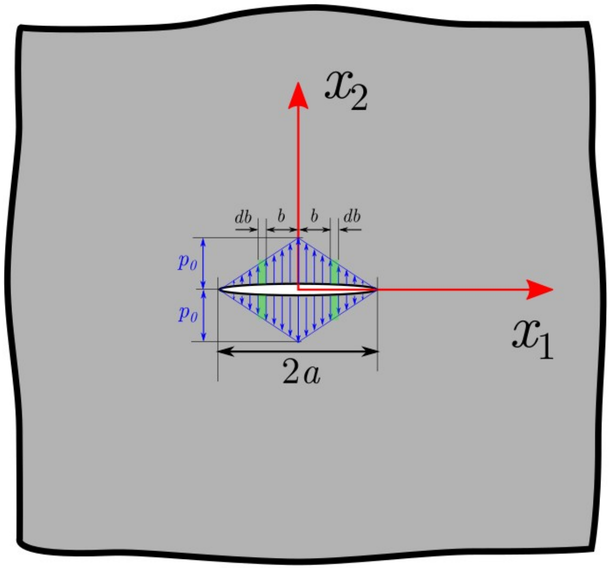
$$K_I^B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-b}{a+b}} \qquad \cdots\cdots\cdots\cdots\cdots(100)$$



Using the SIF for previous case i.e., equation (100) and principal of superposition, SIF for the case shown is

$$K_I^A = K_I^B = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+b}{a-b}} + \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a-b}{a+b}}$$

$$K_I^A = K_I^B = \frac{2P}{\pi}\sqrt{\frac{\pi a}{a^2-b^2}} \qquad \ldots\ldots\ldots\ldots\ldots(101)$$



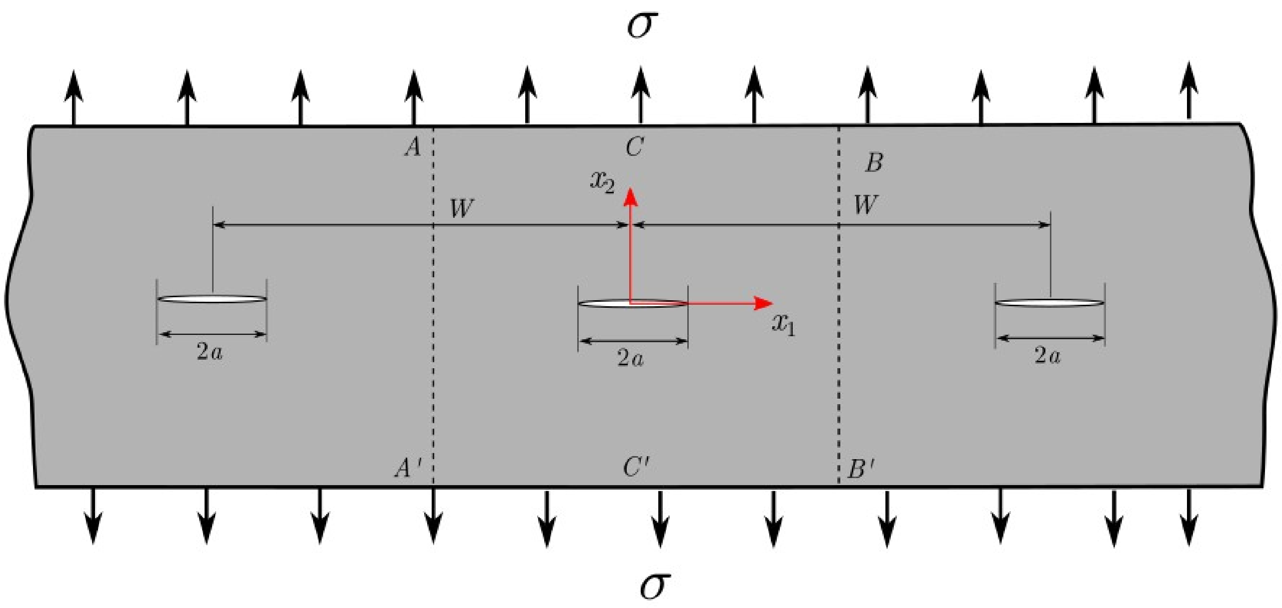
In fact, equation (100) or (101) can be used to determined the SIF for any type of load distribution on the crack face. For e.g., for triangular pressure distribution as shown.

$$dK_I = \frac{2dP}{\pi}\sqrt{\frac{\pi a}{a^2-b^2}} \quad \text{where} \quad dp = p_0 db\frac{(a-b)}{a}.$$

Hence,

$$K_I = \int_0^a dK_I = \int_0^a \frac{p_0}{\pi a}(a-b)\sqrt{\frac{\pi a}{a^2-b^2}}db$$

$$K_I = \left(1-\frac{2}{\pi}\right)p_0\sqrt{\pi a}. \qquad \ldots\ldots\ldots\ldots\ldots(102)$$



Collinear cracks in an infinitely long strip is a classical problem in fracture mechanics. Figure shows identical cracks, each of length $2a$, are separated by a distance W.

As an immediate thought this problem may not seem to be practical. However, this is not the case, because the problem acts as a stepping stone to several real life problems dealing with finite size plates.

Subsequently appropriate portions will be cut out from this strip to solve problems encountered in several engineering applications of importance.

The Westergaard function for this problem is

$$Z_I = \frac{\sigma \sin(\pi z/W)}{\sqrt{\sin^2(\pi z/W) - \sin^2(\pi a/W)}}. \quad \text{..............}(103)$$

Transforming axis to the crack-tip by using $z = z_0 + a$ leads to

$$\sin\frac{\pi z}{W} = \sin\frac{\pi z_0}{W}\cos\frac{\pi a}{W} + \cos\frac{\pi z_0}{W}\sin\frac{\pi a}{W}. \quad \text{..............}(104)$$

In the vicinity of the crack-tip, $|z| << a$, hence we can use

$$\sin\frac{\pi z_0}{W} \approx \frac{\pi z_0}{W}, \quad \text{and} \quad \cos\frac{\pi z_0}{W} \approx 1. \quad \text{..............}(105)$$

Using (104) and (105), (103) can be written as.

$$Z_I = \frac{\sigma\left[\frac{\pi z_0}{W}\cos\frac{\pi a}{W} + \sin\frac{\pi a}{W}\right]}{\left[\left(\frac{\pi z_0}{W}\right)^2\cos^2\frac{\pi a}{W} + \frac{2\pi z_0}{W}\cos\frac{\pi a}{W}\sin\frac{\pi a}{W}\right]^{1/2}}$$

Since z_0/W is a small number, we can approximate Z_I as $Z_I = \frac{\sigma\left[\sin\frac{\pi a}{W}\right]^{1/2}}{\left[\frac{2\pi z_0}{W}\cos\frac{\pi a}{W}\right]^{1/2}}$ ....(106)

From (86), one can see another way to determine SIF as

$$K_I = \lim_{z_0 \to 0} \sqrt{2\pi z_0} Z_I(z_0).$$

Thus, K_I from the present case is

$$K_I = \frac{\sigma \left[\sin \frac{\pi a}{W}\right]^{1/2}}{\left[\frac{1}{W} \cos \frac{\pi a}{W}\right]^{1/2}} \qquad \text{or} \quad K_I = \sigma\sqrt{\pi a}\sqrt{\frac{\tan(\pi a/W)}{(\pi a/W)}}. \qquad \cdots\cdots\cdots\cdots\cdots(107)$$

Crack in a plate with finite dimensions

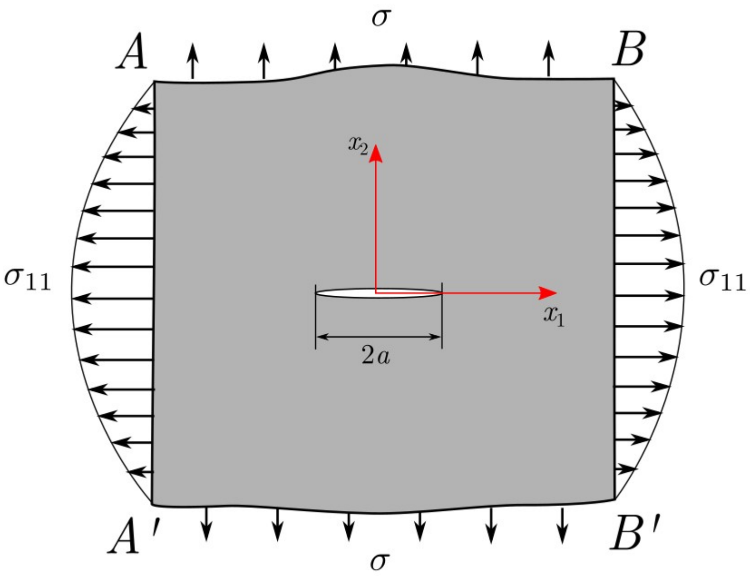
In practical applications edge of a component may be close to the crack tip. Since the edge is traction free, it disturbs the stress field around the crack tip. The edge may have a considerable influence on the stress field in the vicinity of the crack tip and on the SIF and accurate determination of SIF is then required. If the distance of the edge from the crack tip is less than the crack length, or of the order of the crack length, the component is of finite dimensions.

Even under the laboratory conditions meeting the conditions of infinite plate is difficult because of practical difficulties. For e.g., to save money on the material of the specimen, to reduce machining charges, to use a test-machine of low load capacity and to minimize material handling problems.

For specimen of finite dimension SIF is expressed in the following form,

$$K_I = \sigma\sqrt{\pi a} f(a/W),$$

where, W is the width of plate and the function f depends on a/W. For most cases, $f(a/W)$ is written as a series of ratio a/W. For most of the laboratory specimen this function f is obtained from numerical simulations



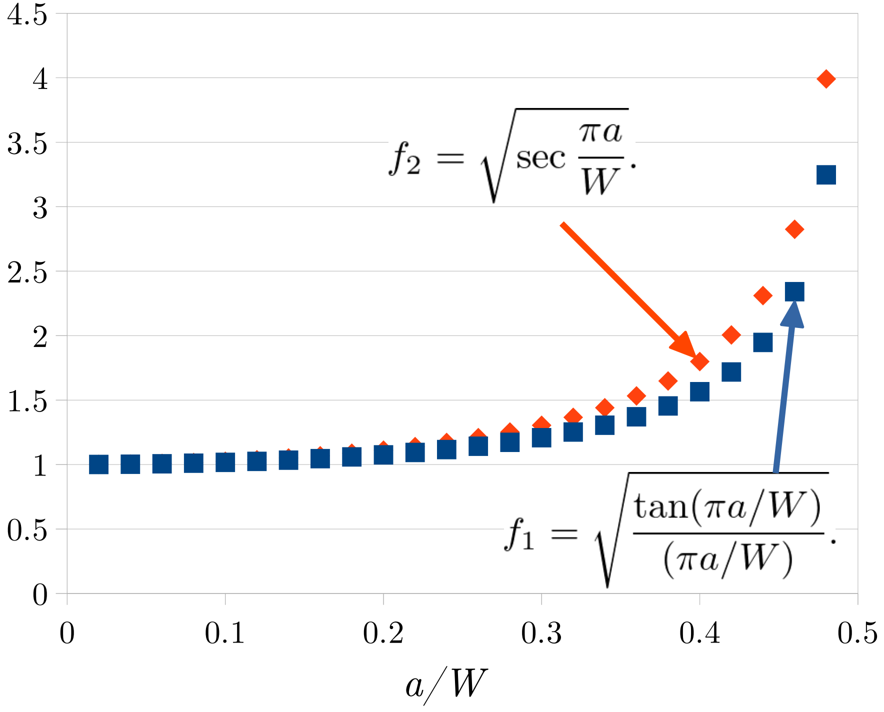
For a center-cracked plate of finite dimensions under Mode-I loading, SIF is estimated using the results of collinear cracks in an infinite strip. A portion is cut from the strip at AA' and BB', which leads to traction at both the faces. Since AA' and BB' are the planes of symmetry, shear stress on them is zero. The stress component σ_{11} has some distribution, shown qualitatively in the figure.

In fact, the problem is somewhat similar to the biaxial loading, where it was argued that σ_{11} does not change the SIF significantly. On the same lines, the effect of σ_{11} on cut faces may be ignored. Then, the SIF of a plate with a center-crack is approximated to be same as that of the case of collinear cracks in an infinite strip, i.e.,

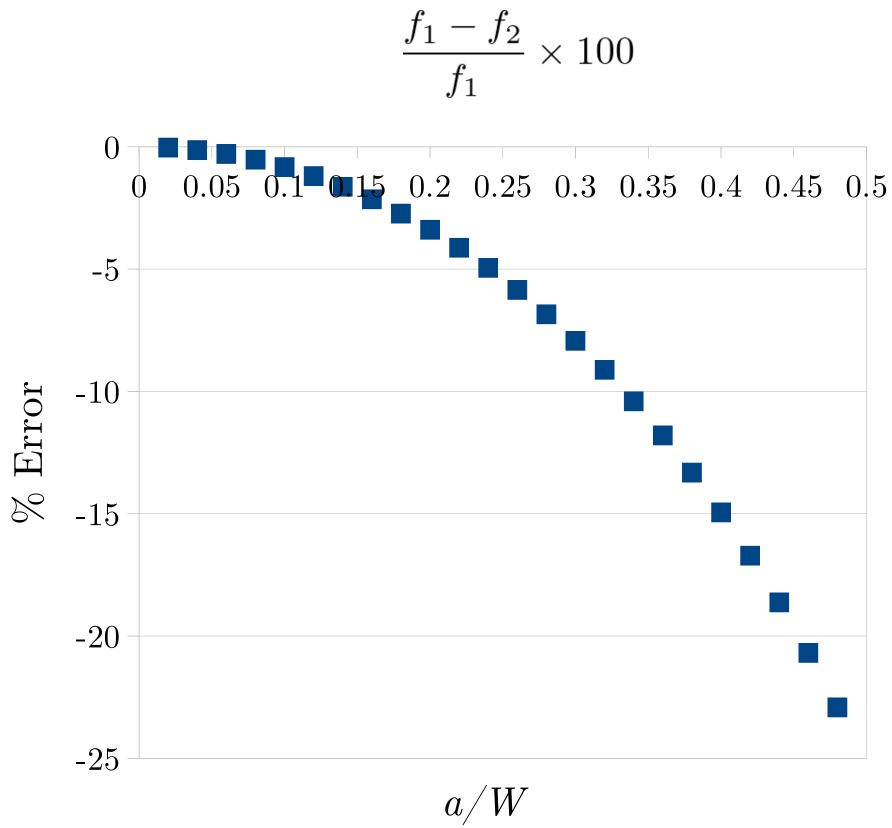
$$K_I = \sigma\sqrt{\pi a}\sqrt{\frac{\tan(\pi a/W)}{(\pi a/W)}}. \quad \cdots\cdots\cdots\cdots\cdots(108)$$

However, exact solution of this problem obtained through advanced mathematical method and numerical solutions is more close to

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec\frac{\pi a}{W}}. \quad \cdots\cdots\cdots\cdots\cdots(109)$$

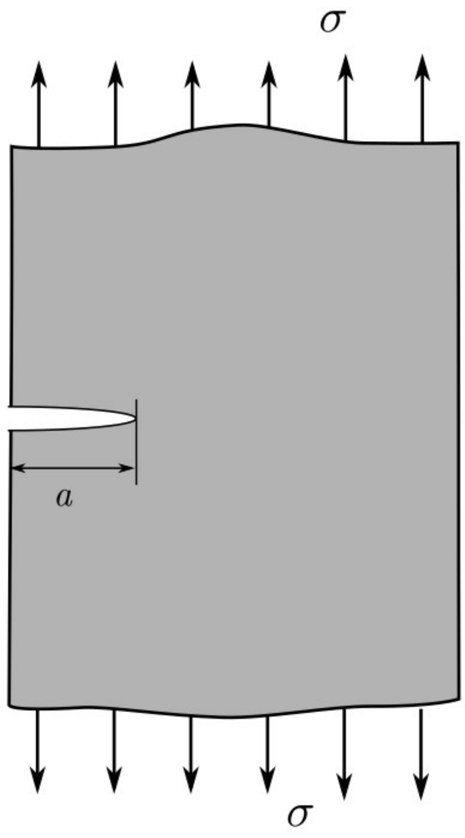


$f_2 = \sqrt{\sec \frac{\pi a}{W}}.$
$f_1 = \sqrt{\frac{\tan(\pi a/W)}{(\pi a/W)}}.$
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
a/W



$\frac{f_1 - f_2}{f_1} \times 100$
% Error
0
-5
-10
-15
-20
-25
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
a/W

Edge crack



We have already discussed that a free edge close to the crack influences the stress field near the crack tip. In the case of an edge

crack, the free edge is not only close to the crack, but it intersects the crack. Edge cracks are very commonly encountered in day-to-day life. Since they are more dangerous, special attention is required to deal with them.

Consider an edge crack in a semi-infinite plate which is loaded by a far field stress. The stress intensity factor for this case is known $1.12\sigma\sqrt{a}$. When the edge crack is compared with one half of the overall length of an interior crack, the value of the SIF is about 12% more, as the ends of cracked faces at the free edge tend to open up more easily.

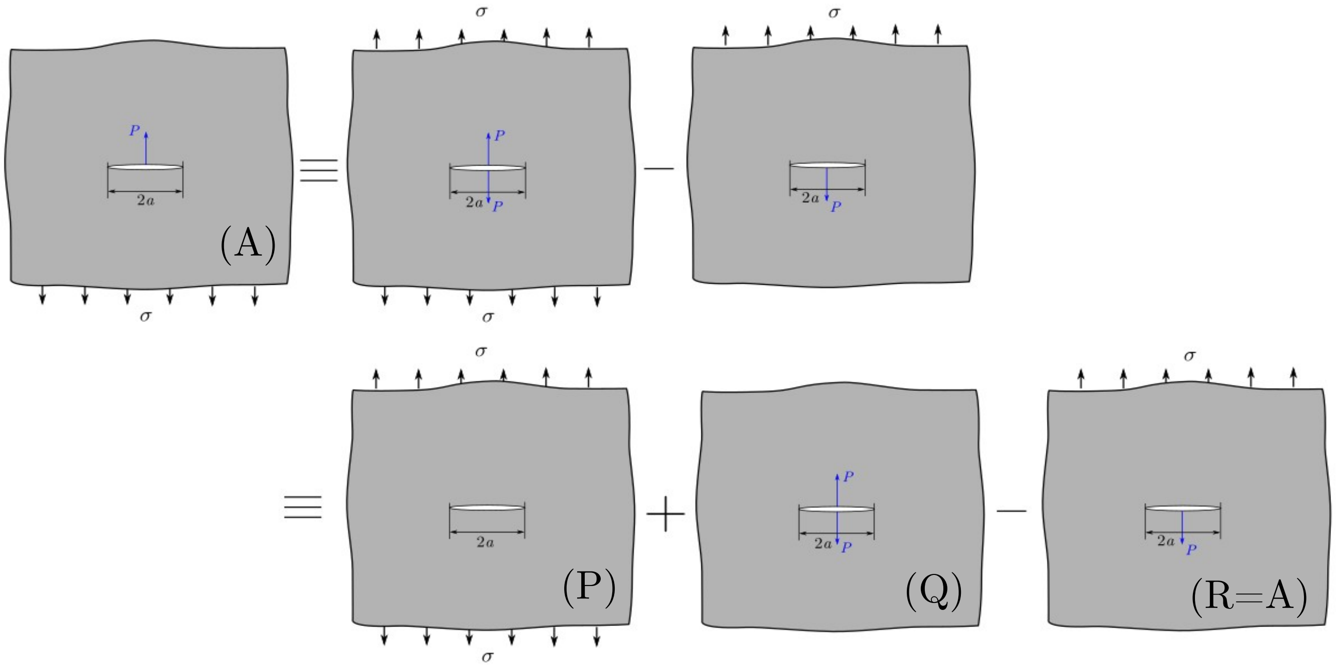
The problem of edge crack can be solved by separating a portion $CBB'C'$ from the strip of collinear cracks and invoking the principle of superposition to make the traction zero on section. However, the solution is quite complex for the same.

Principal of superposition for determining SIF

In many cases, principal of superposition can be exploited to find SIF for complex loading by considering it as a combination of simple loading cases.

$$\begin{aligned} K_I^A &= K_I^P + K_I^Q - K_I^R \\ &= K_I^P + K_I^Q - K_I^A \end{aligned}$$

or $\quad K_I^A = \frac{1}{2}\left(K_I^P + K_I^Q\right)$



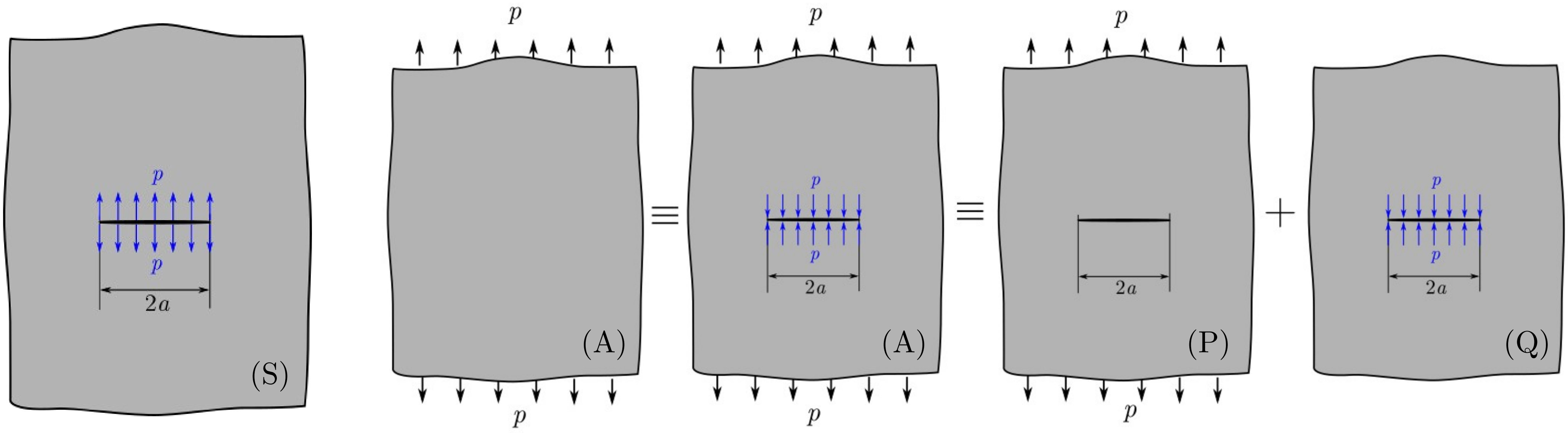
Thus,

$$K_I^A = \frac{1}{2}\left[\sigma\sqrt{\pi a} + \frac{P}{\sqrt{\pi a}}\right]$$

If width of plate is W, then from equilibrium of configuration (A) we can write $P = \sigma W$ and thus,

$$K_I^A = \frac{1}{2}\left[\sigma\sqrt{\pi a} + \frac{\sigma W}{\sqrt{\pi a}}\right]$$

Crack with internal pressure



The problem of a plate with center crack can be solved using the Green's function as discussed earlier. However, we will also see an application of the method of superposition to determine the SIF for this case.

$$K_I^A = K_I^P + K_I^Q = K_I^P - K_I^S = 0$$

Thus, $\quad K_I^S = K_I^P = p\sqrt{\pi a}$ ....(110)