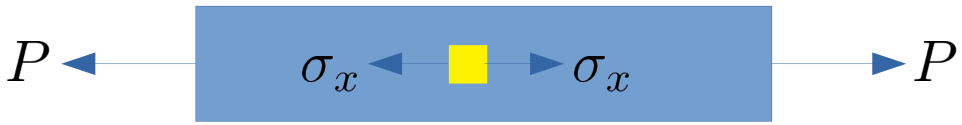
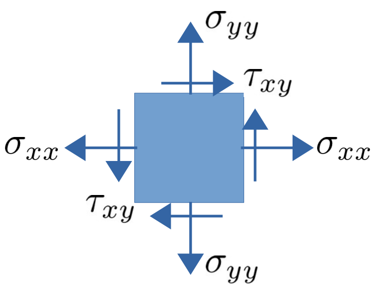
Stress, Strain and Temperature relationship

Theories of failure

Structural elements and machine components, when subjected to uni-axial loading will yield/break at the yield/ultimate stress of the material.



When structural elements and machine components are subjected to multi-axial loading, state of stress is different from the uni-axial condition. Yielding/failure in such cases can not be predicted from uni-axial tests. We need to develop some criteria to predict actual failure considering multiple stress components.



Yield criteria for ductile materials

Maximum shear stress criterion

This criterion is based on the observations that the yield in ductile materials is caused by slippage of material along oblique planes, which is primarily due to shear stresses.

As per this criterion a component is safe when

maximum shear stress in the component	$\leq$	Maximum shear stress in tensile specimen during yield

$$\frac{|\sigma_{\max} - \sigma_{\min}|}{2} \leq \frac{\sigma_y}{2}$$

If principal stresses are $\sigma_1 > \sigma_2 > \sigma_3$, then

$$|\sigma_1 - \sigma_3| \leq \sigma_y$$

Maximum shear stress criterion

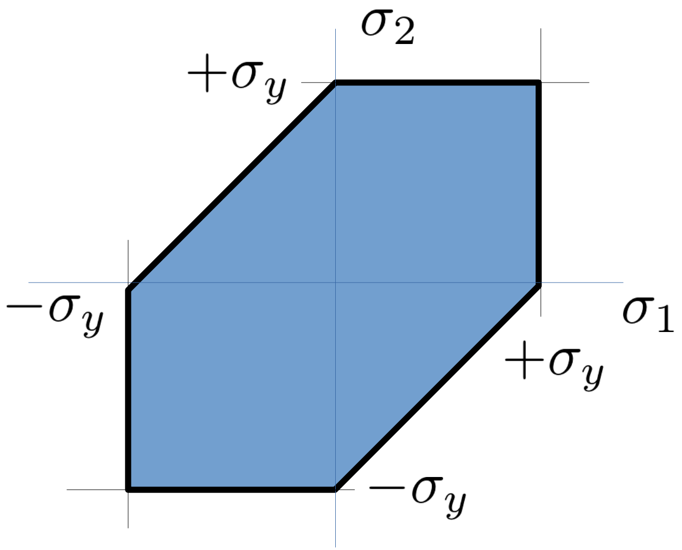
For plane stress when $\sigma_3 = 0$,

$$\text{if } \sigma_1 > 0 \text{ and } \sigma_2 < 0 \text{ then } |\sigma_1 - \sigma_2| \le \sigma_y$$

$$\text{if } \sigma_1 > \sigma_2 > 0, \text{ then } |\sigma_1 - 0| \le \sigma_y \Rightarrow |\sigma_1| \le \sigma_y$$

$$\text{if } \sigma_2 < \sigma_1 < 0, \text{ then } |0 - (-\sigma_2)| \le \sigma_y \Rightarrow |\sigma_2| \le \sigma_y$$

These relations when represented graphically



All state of stresses for which (σ_1, σ_2) point fall inside the shaded area structure remain safe; whereas when the point fall outside it fails.

The hexagon associated with the initiation of yield is known as *Tresca's hexagon* after the French engineer Henri Edouard Tresca and this criteria is aka *Tresca's criterion.*

von Mises yield criterion

This criterion (after the German-American applied mathematician Richard von Mises) suggest that for a three dimensional state of stress, when principal stresses are σ_1, σ_2, σ_3, then yielding occurs when von Mises stress in three dimensional condition is less than or equal to the corresponding von Mises stress in uniaxial stress during yield.

In terms of principal stresses von Mises stress is given as

$$\sigma_e = \sqrt{\frac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

In terms of stresses in *xy*-coordinates, von Mises stress is

$$\sigma_e = \sqrt{\frac{1}{3}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{xz}^2\right]}$$

von Mises yield criterion

According to the criterion structure remain safe when,

$$\sqrt{\frac{1}{3}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]} \leq \sqrt{\frac{2}{3}\sigma_Y^2}$$

or

$$\sqrt{\frac{1}{2}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]} \leq \sigma_Y$$

or

$$\sqrt{\frac{1}{3}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+3\tau_{xy}^2+3\tau_{yz}^2+3\tau_{xz}^2\right]} \leq \sqrt{\frac{2}{3}}\sigma_Y$$

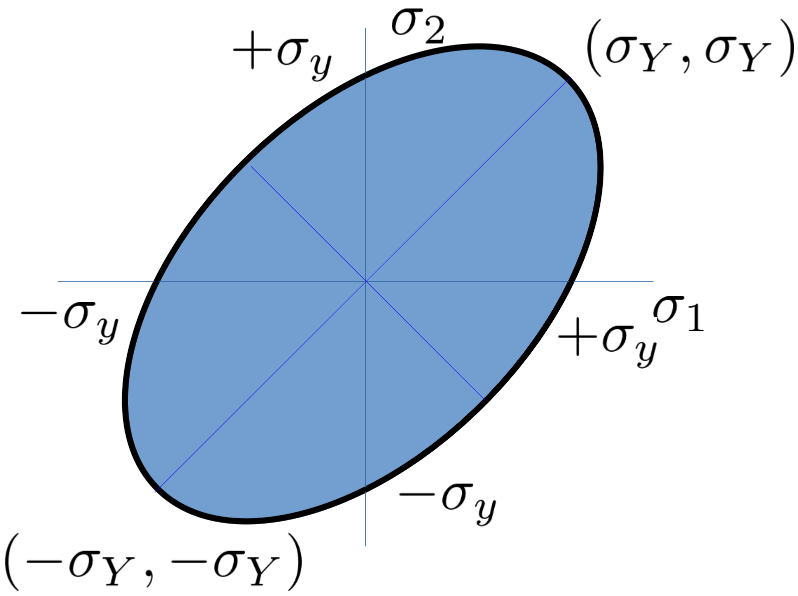
or

$$\sqrt{\frac{1}{2}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+3\tau_{xy}^2+3\tau_{yz}^2+3\tau_{xz}^2\right]} \leq \sigma_Y$$

von Mises yield criterion

For plane stress when σ_3=0, criterion becomes

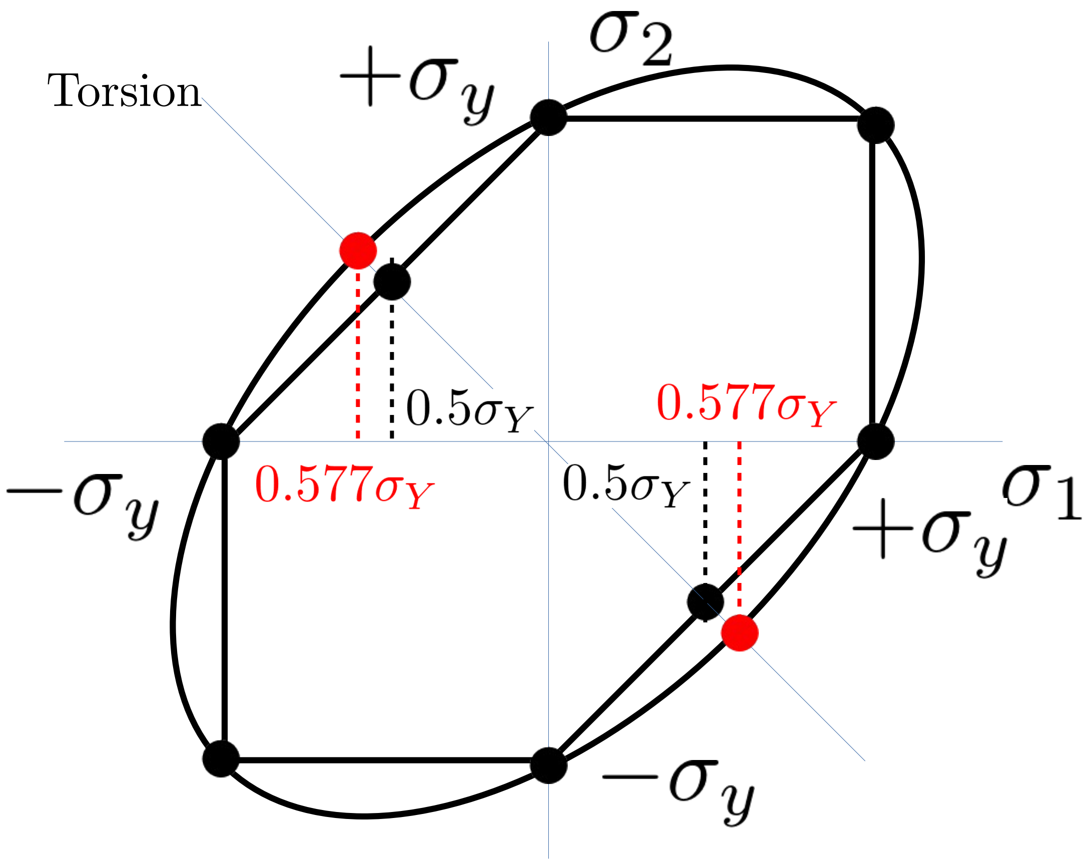
$$\sqrt{\frac{1}{2}\left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)} \leq \sigma_Y \quad \text{or} \quad \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq 2\sigma_Y^2$$

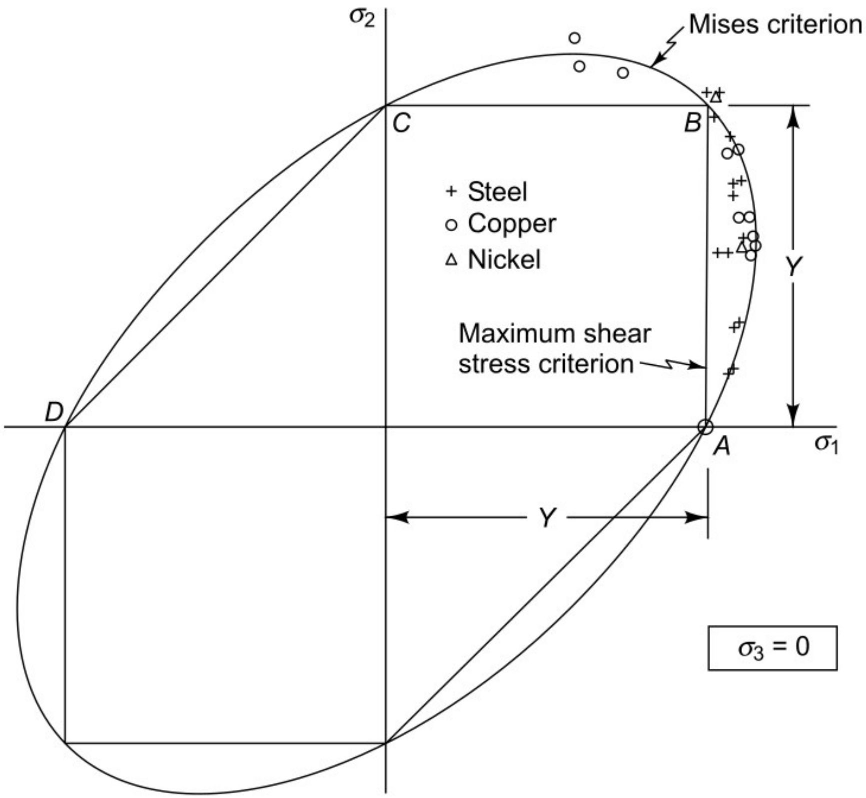


Again, all state of stresses for which (σ_1, σ_2) point fall inside the shaded area structure remain safe; for the point falling outside it fails.

This criterion is also known as *Distortion energy criterion* or *Octahedral shear stress criterion.*

Comparison of both yield criteria



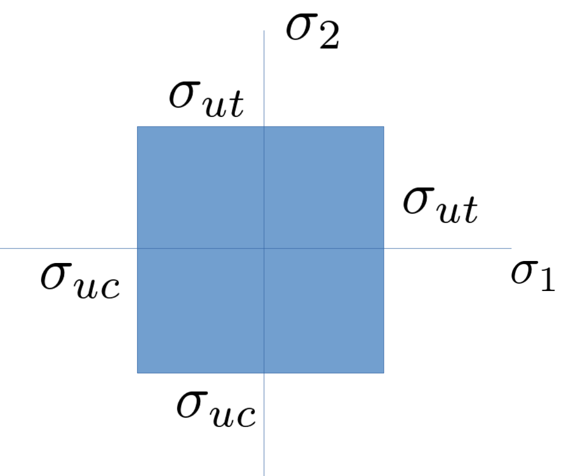


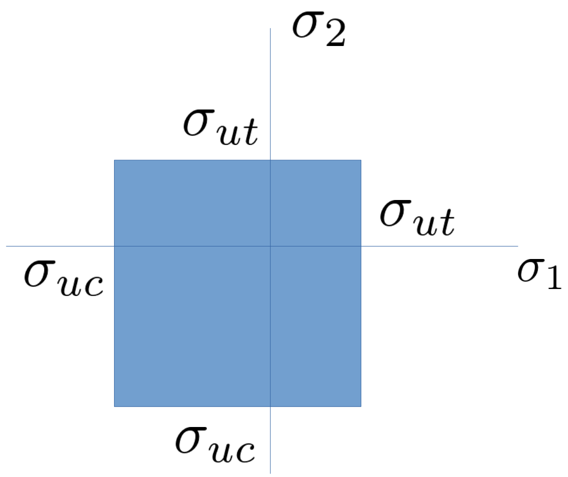
Failure criteria for brittle materials

Maximum normal stress criterion

According to this criterion failure in the brittle material occurs when maximum principal stress exceeds the ultimate strength in tension or when minimum principal stress exceeds the ultimate strength in compression. If the principal stresses are σ_1, σ_2, σ_3, then for plane stress the criterion becomes

$$\sigma_1 \geq \sigma_{ut} \quad \text{or} \quad \sigma_2 \leq \sigma_{uc}$$





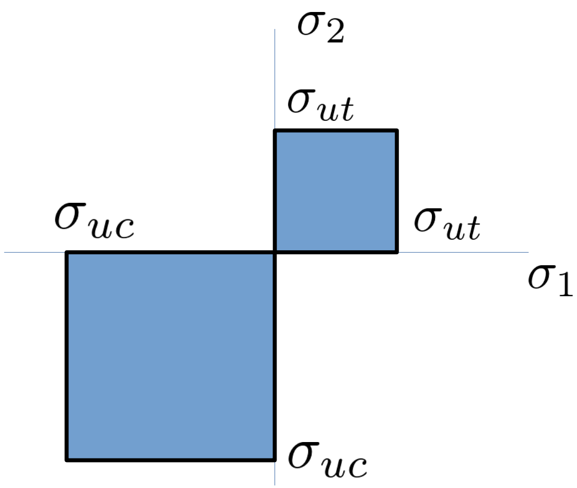
This criterion is also known as *Coulomb's criterion* or *Rankine criterion.*

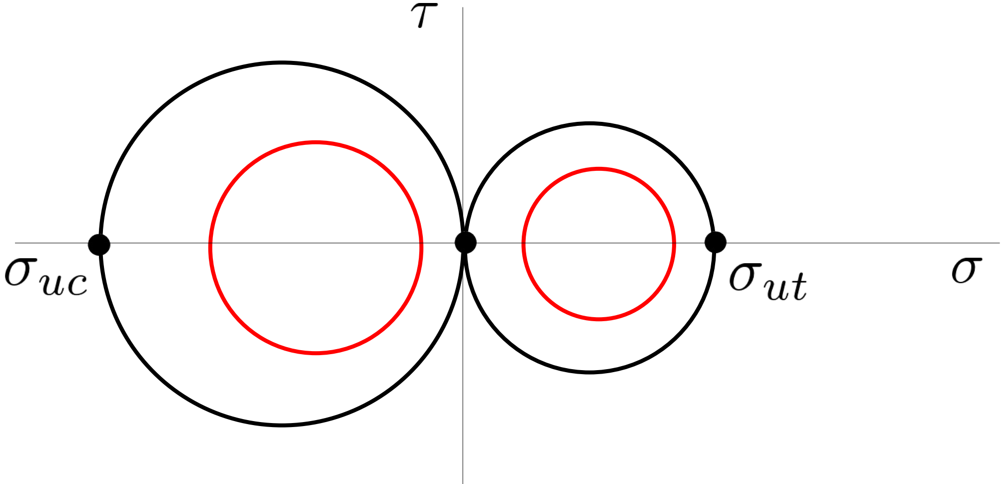
Mohr's criterion

To develop the criterion for plane stress ($\sigma_3 = 0$) condition, following test results are required.

- Uniaxial tension test
- Uniaxial compression test
- Torsion test

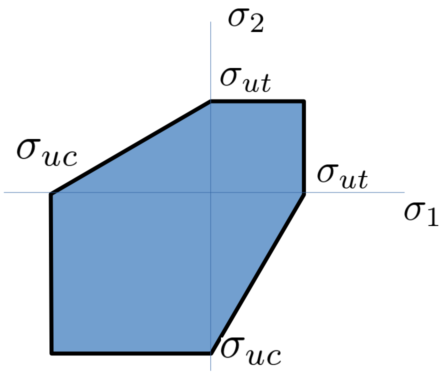
When both the stresses are of same nature (tension/compression)

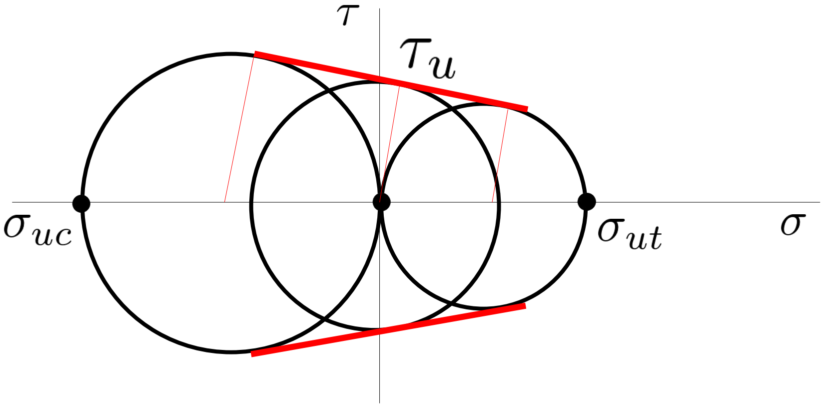




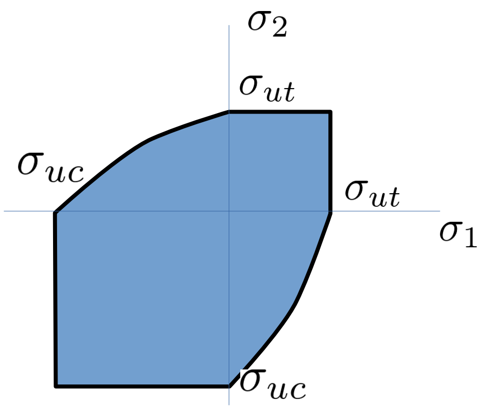
Mohr's criterion

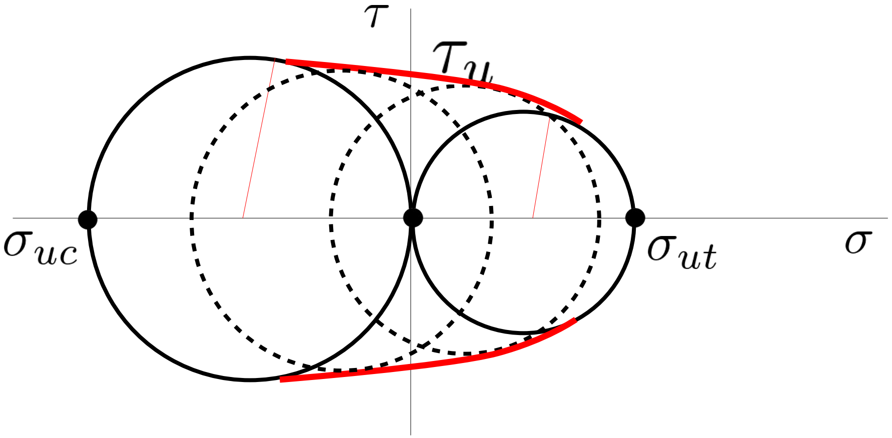
When both the stresses are of different nature



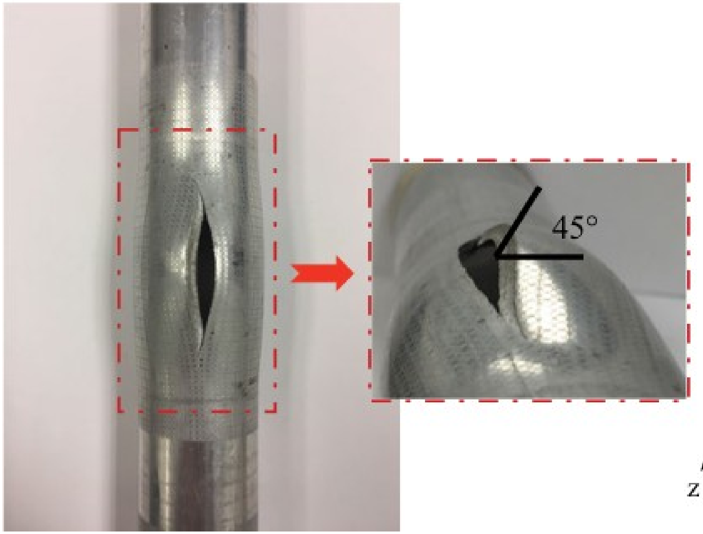


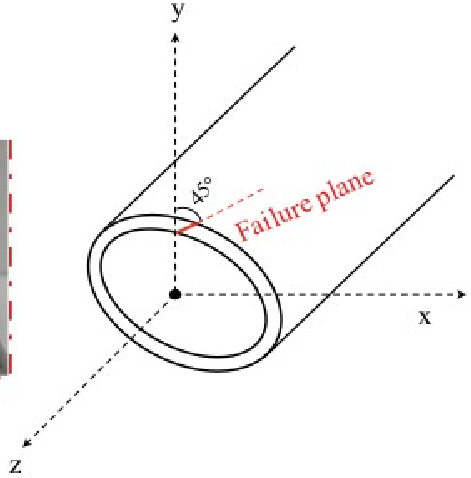
To draw more accurate envelope

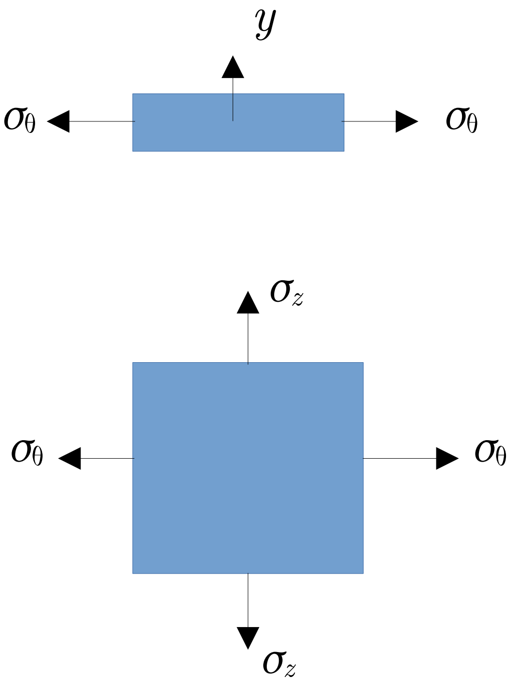




Example: Failure of Pressure Vessel

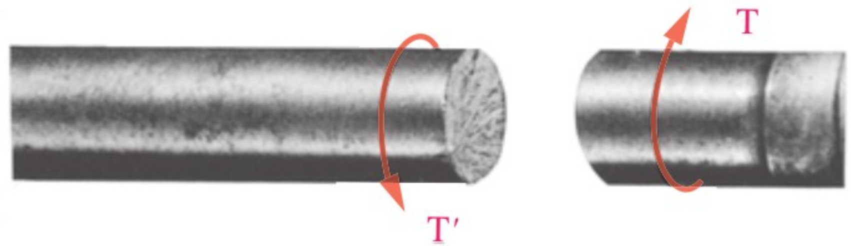




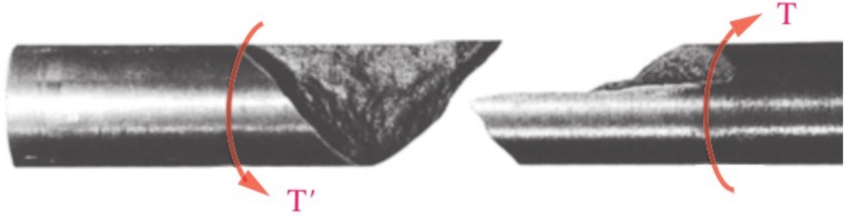


Source: Mechanics of Materials Laboratory Course by Ghatu Subhash and Shannon Ridgeway. Morgan & Claypool publishers

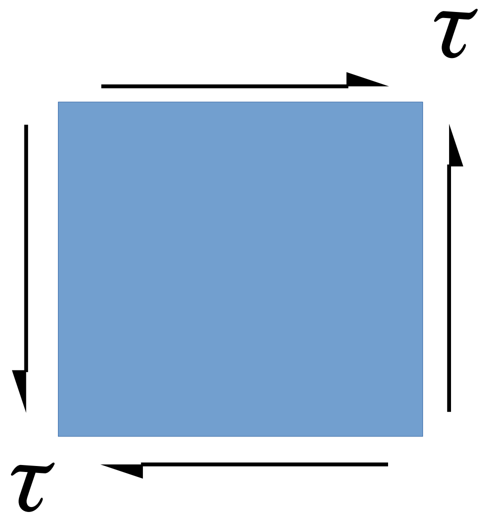
Example: Failure of shaft under torsion



Shaft made of ductile material

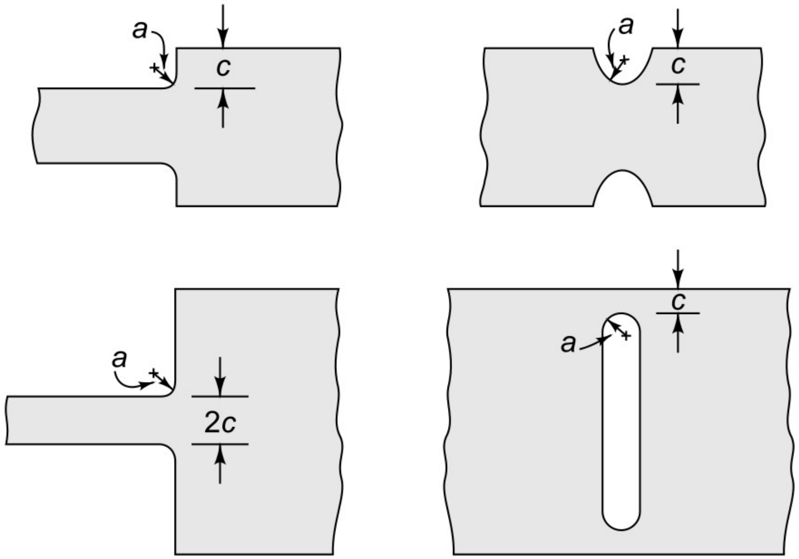


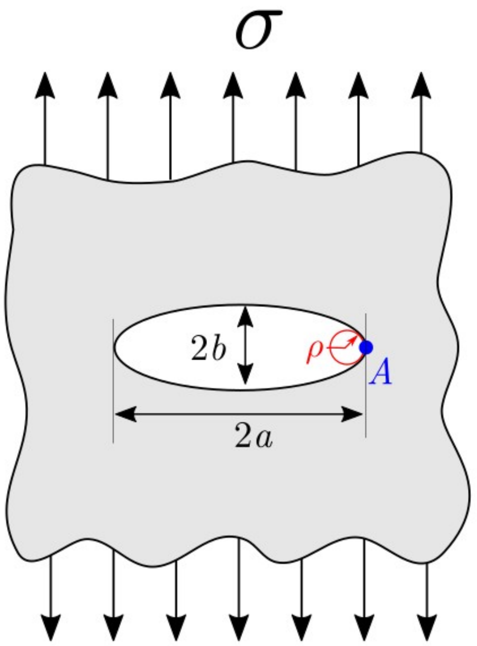
Shaft made of brittle material



Stress Concentration

- All engineering structures have local geometrical irregularities in the form of an oil hole, a keyway, a fillet or a notch.
- When such structures are stressed, there is a localized variation in the stress state in the immediate neighborhood of the irregularity. The maximum stress levels at the irregularity may be several times larger than the nominal stress levels in the bulk of the body. This increase in stress caused by the irregularity in geometry is called a stress concentration.
- Where the stress concentration cannot be avoided by a change in design, it is important to base the design on the local value of the stress rather than on an average value.



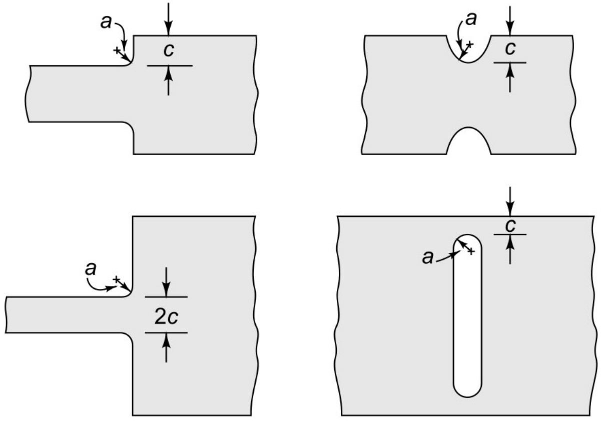


Analytical solutions is available for an elliptical hold in an infinite load under tensile loading. Stress at point A is given as,

$$\sigma_A = \sigma\left(1 + \frac{2a}{b}\right).$$

When the major axis, a, increases relative to b, the elliptical hole begins to take on the appearance of a sharp notch (or a crack). For this case, *Inglis* (1913) found it more convenient to express the above expression in terms of the radius of curvature ρ as,

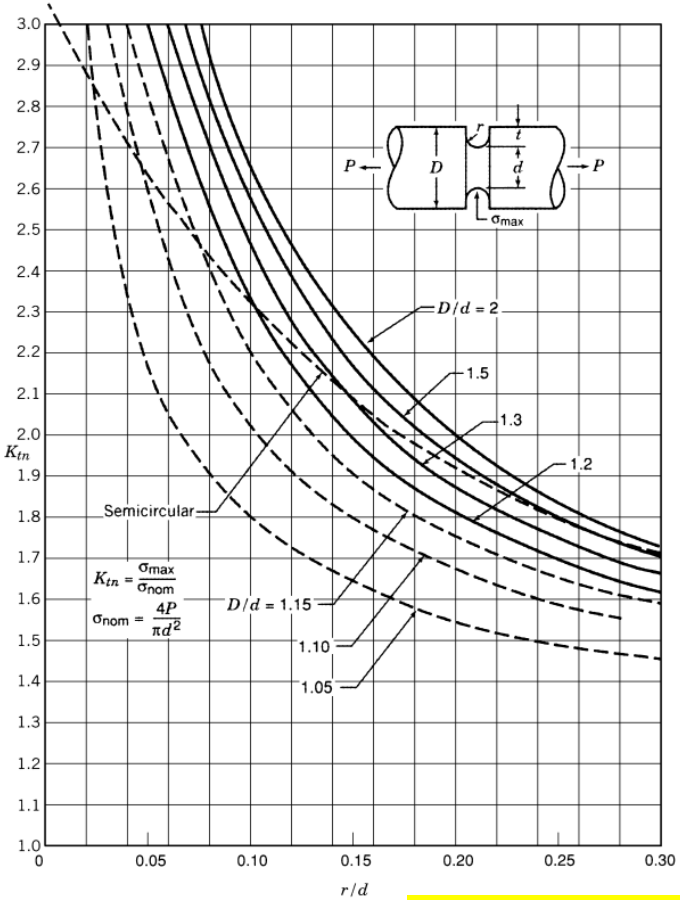
$$\sigma_A = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right), \quad \text{where} \quad \rho = b^2/a.$$

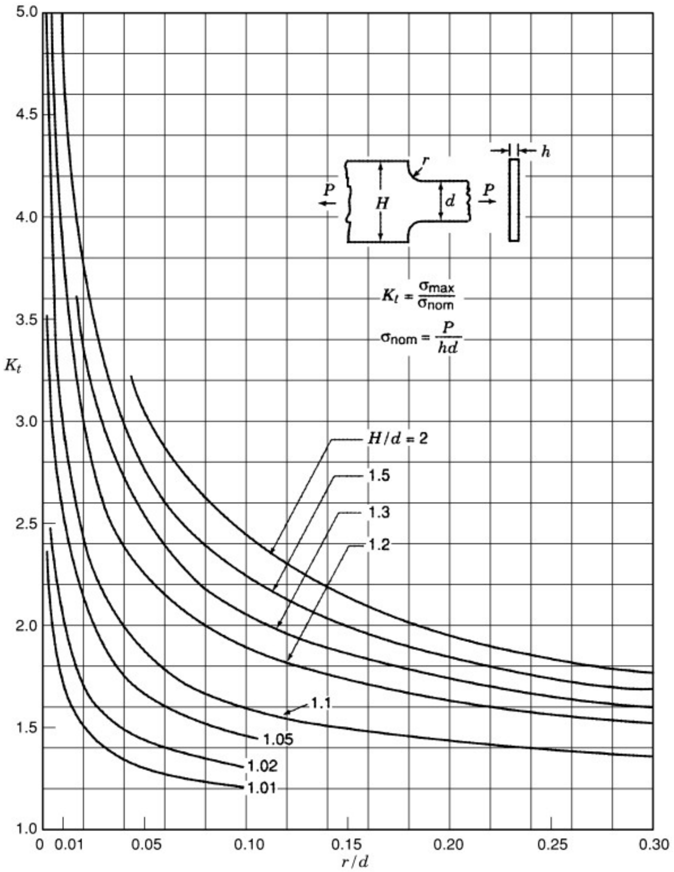


For the cases, where analytical solutions are not available following empirical relationship gives a good estimate of the stress concentration factor.

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}} \approx 1 + (0.3 \text{ to } 2)\sqrt{\frac{c}{a}}.$$

For practical design purpose design curves are available in handbooks for different types of geometrical irregularities.





Peterson's Stress Concentration Factors by
Walter D. Pilkey & Deborah F. Pilkey & Zhuming Bi. John Wiley & Sons.