ME632: Problem Set-2

Q.1: The stress intensity factor for a penny-shaped crack of radius a subjected to a load P acting along the circumference of a circle of radius c (Fig. 1a) is given by

$$K_I = \frac{P}{(\pi a)^{3/2}\sqrt{1-(c/a)^2}}.$$

Determine the stress intensity factor for this crack when it is subjected to a uniform stress of magnitude σ distributed over a concentric circular area of radius $b(b < a)$ (Fig. 1b)

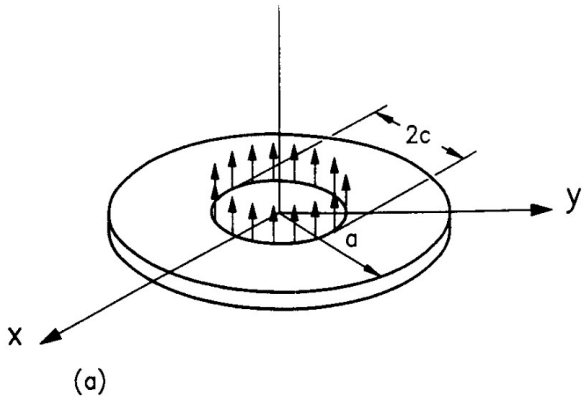

Fig. 1(a)

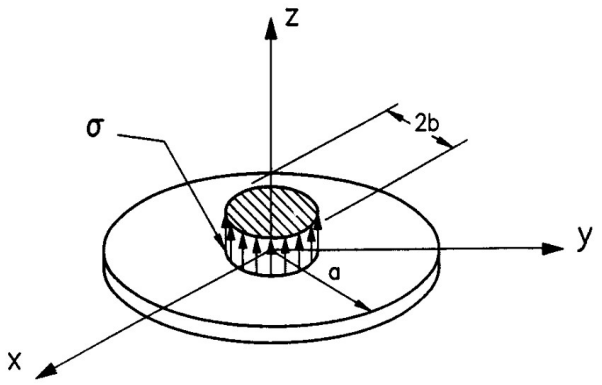

Fig. 1(b)

Q.2: Show that the stress intensity factor for an edge crack of length a in a semi-infinite solid subjected to a uniform out-of-plane shear stress τ at infinity is the same as for a crack of length $2a$ in a full solid. Think why it is so.

Q.3: Consider a central crack of length $2a$ in an infinite plate subjected to uniaxial stress σ at infinity perpendicular to the crack plane. According to the Irwin model, the effective crack is larger than the actual crack by the length of plastic zone. Show that the stress intensity factor corresponding to the effective crack, called effective stress intensity factor K_{eff}, for conditions of plane stress, is given by

$$K_{\text{eff}} = \frac{\sigma\sqrt{\pi a}}{\sqrt{1-0.5(\sigma/\sigma_Y)^2}}.$$

Q.4: Consider a crack of length $2a$ in an infinite plate subjected to a uniform stress a at infinity that makes an angle β with the crack axis (Fig. 2). Show that the stress intensity factors K_I and K_{II} are given by

$$K_I = \sigma\sqrt{\pi a}\sin^2\beta \text{ and } K_{II} = \sigma\sqrt{\pi a}\sin\beta\cos\beta.$$

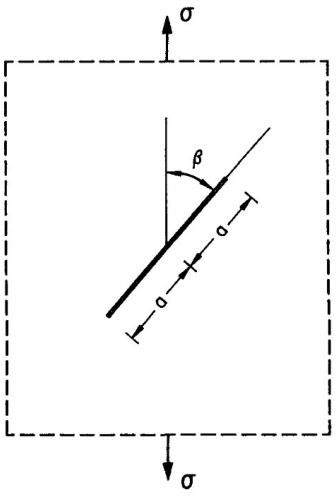

Fig. 2

Q.5: Consider a short crack of length a0 emanating from an elliptical hole with axes $2a$ and $2b$ in an infinite plate subjected to a stress u perpendicular to the major axis of the hole (Fig. 3). Determine the stress intensity factor. Note that the hoop stress at the end point of the major axis of the ellipse is given by

$$\sigma_A = \sigma\left(1 + \frac{2a}{b}\right).$$

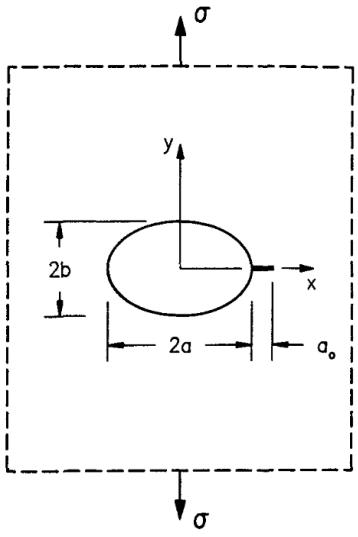

Fig. 3

Q.6: Determine the minimum plate thickness required for plane strain conditions to prevail at the crack tip according to the ASTM specifications for the following steels:

(a) 4340, with $K_{IC} = 100\ \text{MPa}\sqrt{\text{m}}$ and $a = 860$ MPa.
(b) A533, with $K_{IC} = 180\ \text{MPa}\sqrt{\text{m}}$ and $a = 350$ MPa.

Think about the results.

Q.7: Show that the radius of the plastic zone surrounding the tip of a mode-II crack is given by

$$r_p(\theta) = \frac{1}{8\pi}\left(\frac{K_{II}}{\sigma_Y}\right)^2 \left(14 - 2\cos\theta - 9\sin^2\theta\right)$$

for plane stress, and by

$$r_p(\theta) = \frac{1}{8\pi}\left(\frac{K_{II}}{\sigma_Y}\right)^2 \left[12 + 2(1-\nu)(1-\cos\theta) - 9\sin^2\theta\right]$$

for plane strain.

Q.8: Determine the J-Integral for a double cantilever beam (DCB) specimen, if each cantilever is pulled by a distributed load P, as shown in Fig. 4

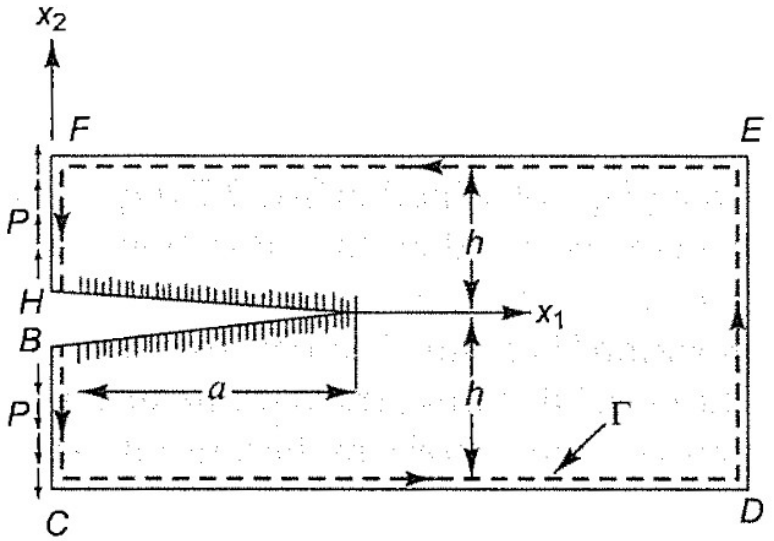

Fig. 4