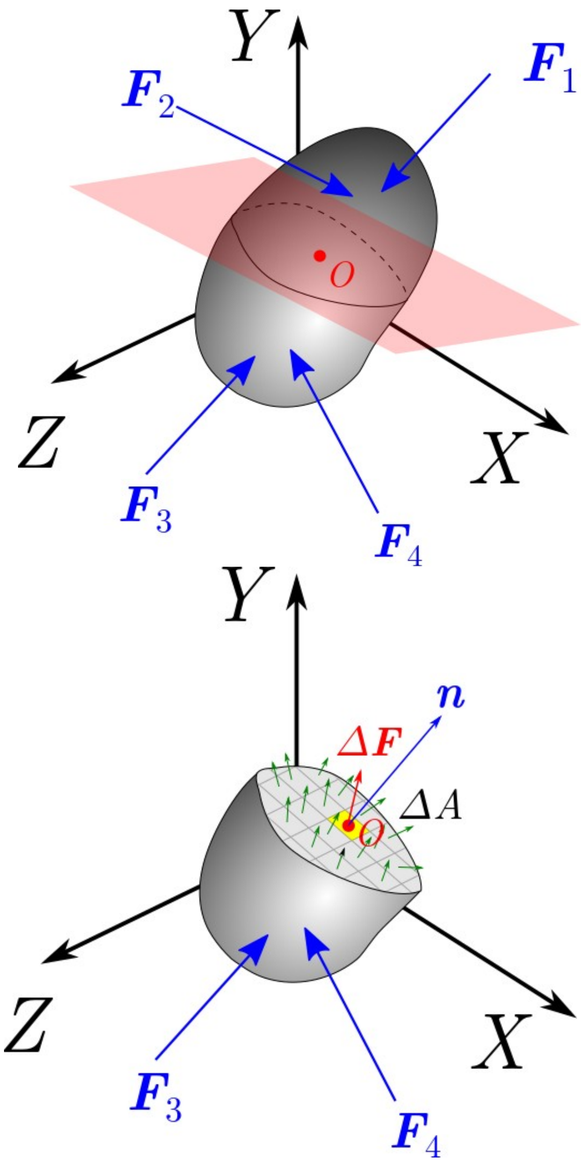
ME231: Solid Mechanics-I

Stress and Strain

Stress



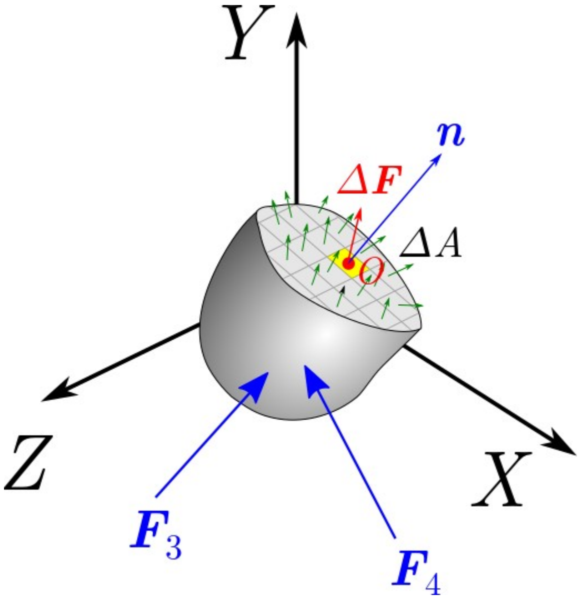
Consider a body acted upon by several external forces.

To determine the internal forces at some point O in the interior of the body, we cut the body by an imaginary plane passing though point O.

For the separate halves of the body to be in equilibrium there must be internal forces transmitted across the cutting plane.

We divide the plane into a number of small areas and measure the force acting on each of these. We will see that these forces in general vary from one area to the other.

Now, consider an elemental area ΔA in the neighborhood of point O. The force on the area is $\Delta \boldsymbol{F}$, and normal to the area is $\boldsymbol{n}$.



We define a **traction (or stress) vector t^n** corresponding to normal $\boldsymbol{n}$ and point O is defined as,

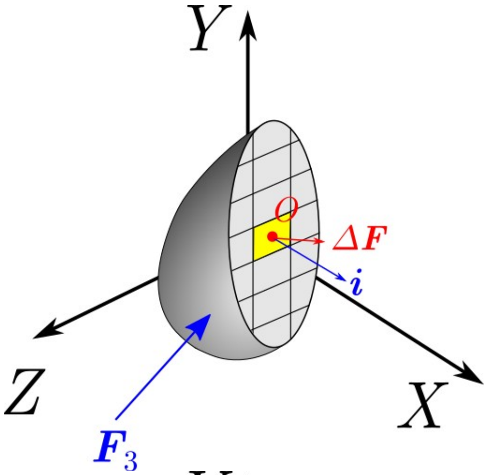
$$\boldsymbol{t^n} = \lim_{\Delta A \to 0} \frac{\Delta \boldsymbol{F}}{\Delta A}. \qquad \text{.......................(1)}$$

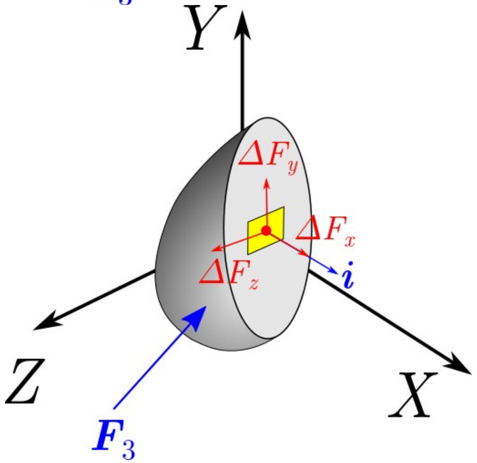
Thus, $\boldsymbol{t^n}$ is the stress vector at point O acting on a plane having normal $\boldsymbol{n}$. This vector in terms of its component along coordinate axes can be written as,

$$\boldsymbol{t^n} = t^n_x\, \boldsymbol{i} + t^n_y\, \boldsymbol{j} + t^n_z\, \boldsymbol{k}. \qquad \text{.......................(2)}$$

It is convenient to express the stress vector $\boldsymbol{t^n}$ in terms of stress vectors on planes perpendicular to coordinate axes.

Once the components are obtained along these plane we can relate it to the components of $\boldsymbol{t^n}$ in (2).





We start by having an imaginary cut on the body by a plane perpendicular to X-axis and passing through point O, as shown in figure.

Now, the stress vector at point O acting on a plane having normal $\boldsymbol{i}$ become,

$$\boldsymbol{t}^{\boldsymbol{i}} = \lim_{\Delta A_x \to 0} \frac{\Delta \boldsymbol{F}}{\Delta A_x}.$$

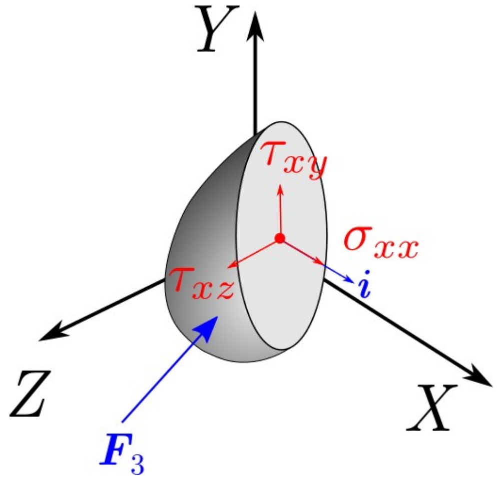
Force $\Delta \boldsymbol{F}$ can further be resolved along the coordinate axes, then,

$$\boldsymbol{t}^{\boldsymbol{i}} = \lim_{\Delta A_x \to 0} \frac{1}{\Delta A_x} \left(\Delta F_x \, \boldsymbol{i} + \Delta F_y \, \boldsymbol{j} + \Delta F_z \, \boldsymbol{k} \right), \quad \text{or}$$

$$\boldsymbol{t}^{\boldsymbol{i}} = \sigma_{xx} \, \boldsymbol{i} + \tau_{xy} \, \boldsymbol{j} + \tau_{xz} \, \boldsymbol{k}, \qquad \cdots\cdots\cdots\cdots(3)$$

where,

$$\sigma_{xx} = \lim_{\Delta A_x \to 0} \frac{\Delta F_x}{\Delta A_x}, \quad \tau_{xy} = \lim_{\Delta A_x \to 0} \frac{\Delta F_y}{\Delta A_x}, \text{ and } \tau_{xz} = \lim_{\Delta A_x \to 0} \frac{\Delta F_z}{\Delta A_x}. \qquad \cdots\cdots\cdots\cdots(4)$$



σ_{xx}, τ_{xy} and τ_{xz} are called stress components associated with the x-face at point O. These stress components are shown in figure.

In similar manner, the body is cut with a plane perpendicular to y-axis and passing through point O to obtain the traction vector,

$$\boldsymbol{t}^{\boldsymbol{j}} = \tau_{yx}\,\boldsymbol{i} + \sigma_{yy}\,\boldsymbol{j} + \tau_{yz}\,\boldsymbol{k},$$

where,

$$\tau_{yx} = \lim_{\Delta A_y \to 0} \frac{\Delta F_x}{\Delta A_y}, \quad \sigma_{yy} = \lim_{\Delta A_y \to 0} \frac{\Delta F_y}{\Delta A_y}, \text{ and } \tau_{yz} = \lim_{\Delta A_y \to 0} \frac{\Delta F_z}{\Delta A_y}. \qquad \cdots\cdots\cdots(5)$$

Cutting the body with a plane perpendicular to z - axis and passing through point O we get the traction vector,

$$\boldsymbol{t}^{\boldsymbol{k}} = \tau_{zx}\,\boldsymbol{i} + \tau_{zy}\,\boldsymbol{j} + \sigma_{zz}\,\boldsymbol{k},$$

where,

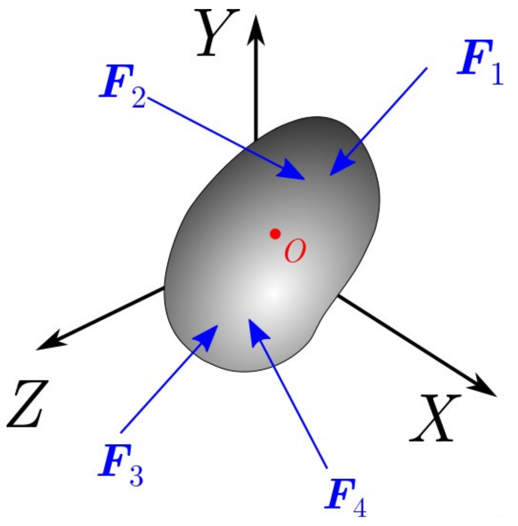
$$\tau_{zx} = \lim_{\Delta A_z \to 0} \frac{\Delta F_x}{\Delta A_z}, \quad \tau_{zy} = \lim_{\Delta A_z \to 0} \frac{\Delta F_y}{\Delta A_z}, \text{ and } \sigma_{zz} = \lim_{\Delta A_z \to 0} \frac{\Delta F_z}{\Delta A_z}. \qquad \cdots\cdots\cdots(6)$$

Now, at point O we see that the state of stress is dependent on nine stress components, which are

$$\begin{matrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}. \end{matrix} \quad \cdots\cdots\cdots(6)$$

Later, we will see how to relate the components of traction vector $\boldsymbol{t}^n$ to these nine components.

Now, think about the overall continuous body again.



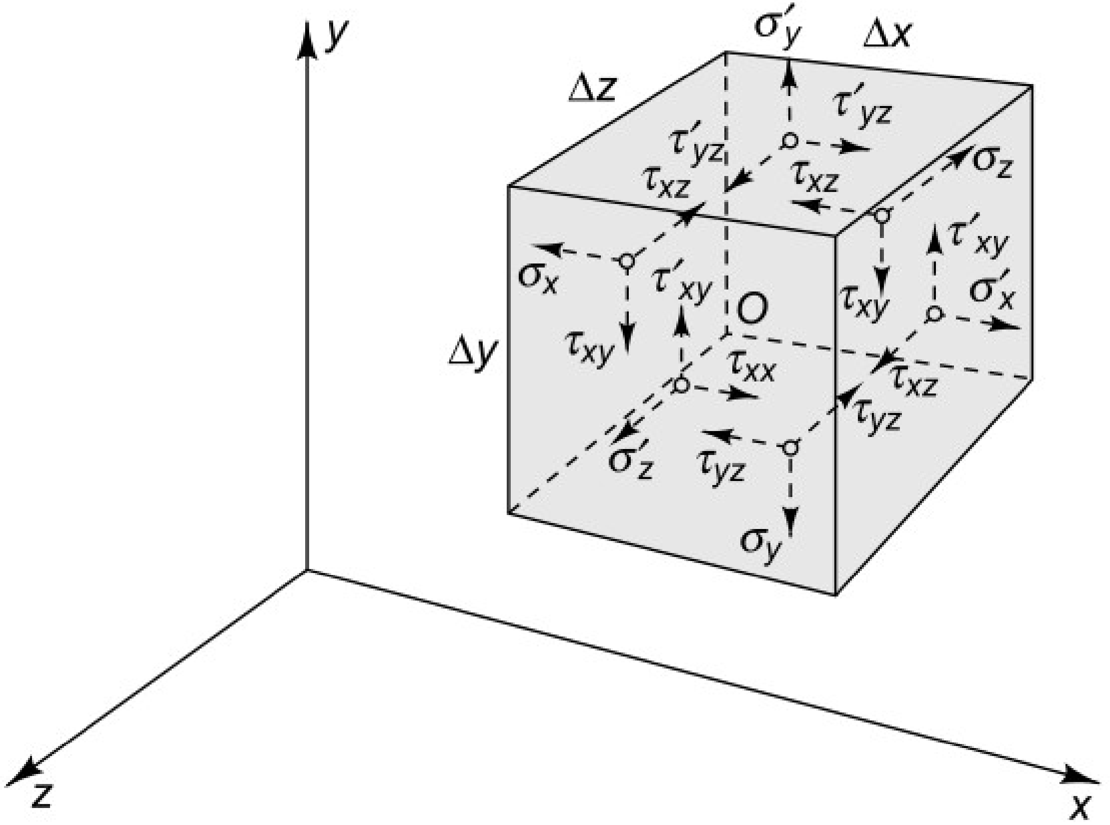
Along with the plane perpendicular to x-axis through O, we pass another plane perpendicular to x-axis at a distance Δx from O in the x-direction.

Similarly. along with the plane perpendicular to y-axis through O, we pass another plane perpendicular to y-axis at a distance Δy from O in the y-direction.

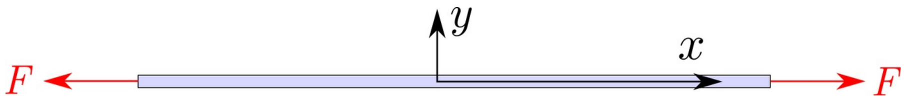
And along with the plane perpendicular to z-axis through O, we pass another plane perpendicular to z-axis at a distance Δz from O in the z-direction.

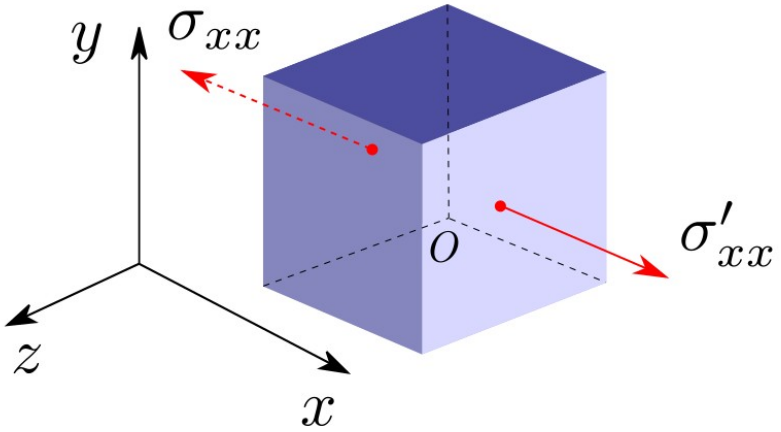
By doing this we get a cuboid having length Δx, Δy, and Δz in x-, y- and z- directions respectively. Corresponding to every face of the cuboid stress components are shown.

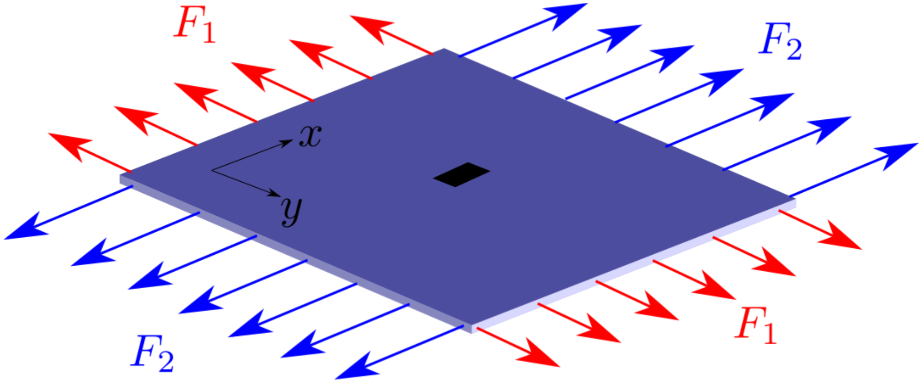
Note the direction of stress. Sign convention for faces and stress components follows same rules as that of the force components discussed earlier.

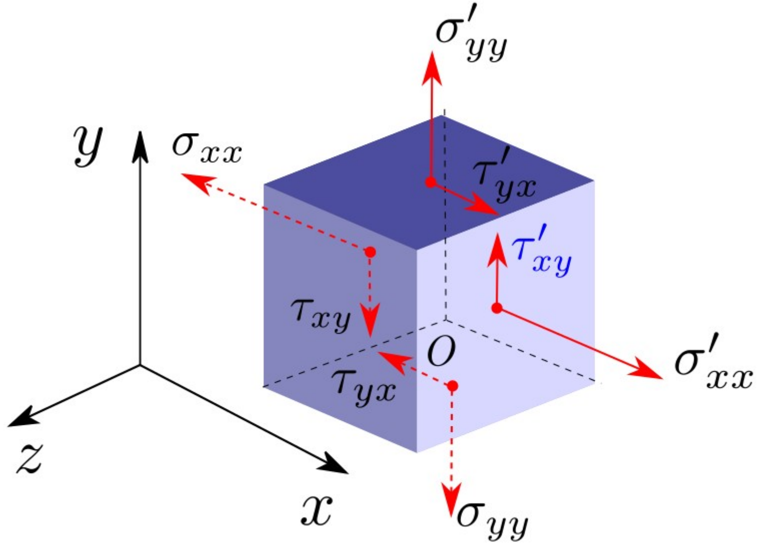


Some simpler stress situations









For a thin sheet there is no force in the z-direction. Hence stress components at the z-faces are zero. As the thickness is small the z-direction stress components are almost constant throughout the thickness and hence it can be assumed these stress components are zero throughout the thickness. For such case state of stress at a point is given as

$$\begin{matrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{matrix} \qquad \cdots\cdots\cdots\cdot(8)$$

where stress components are only functions of **only $\boldsymbol{x}$ and $\boldsymbol{y}$**. This combination of stress components is called **plane stress** in the xy plane.

Plane stress condition is of practical importance in several applications, which will be discussed later.