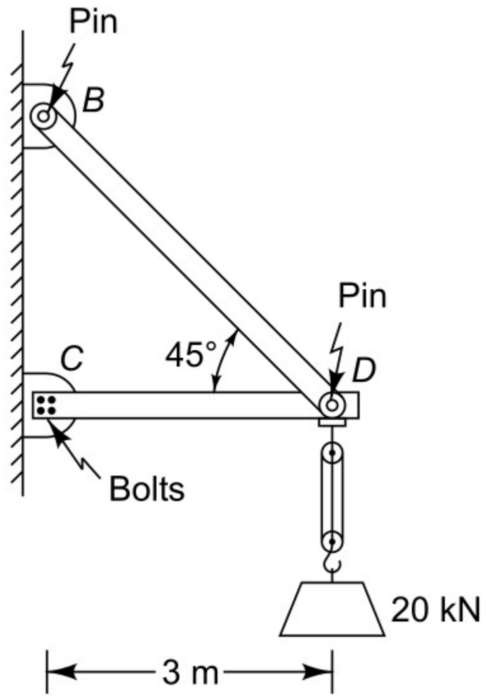
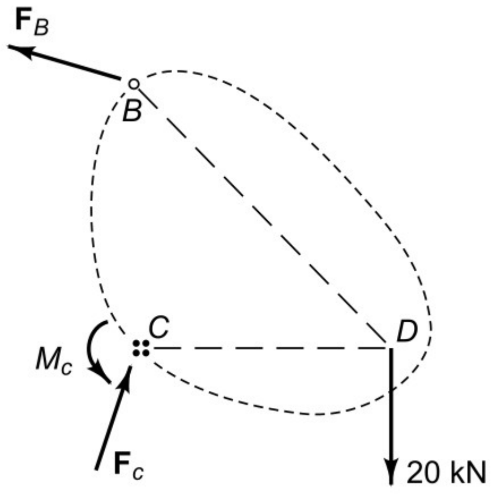
# ME231: Solid Mechanics-I

# Fundamental principles of Mechanics

# Example 1

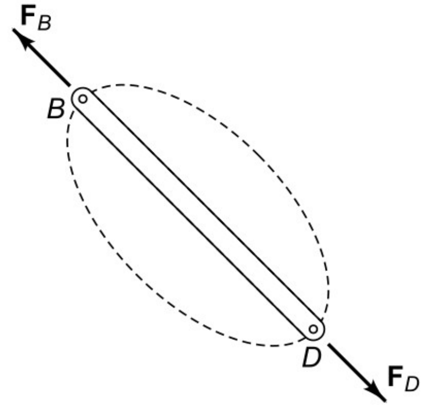
The simple triangular frame shown in figure is used to support a small chain hoist. The chain hoist is supporting its rated capacity of 20 kN. The rod $BD$ is pinned at its ends. The member $CD$ is pinned at $D$ and secured with four bolts at $C$. Predict the forces acting on the wall at $B$ and $C$?

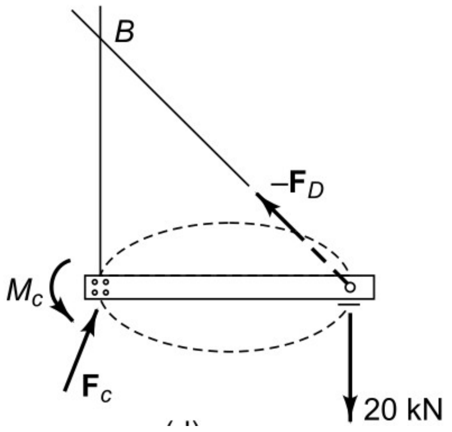





As a very first step isolate the frame from wall support $B$ and $C$, and apply appropriate reactions. Here, we idealize the links to be mass-less.

It can be observed that there are 5 scaler unknowns and only 3 scalar equations are available to solve them. Since, we cannot obtain a complete solution at this stage, we further isolate subsystems (i.e., links) as shown.



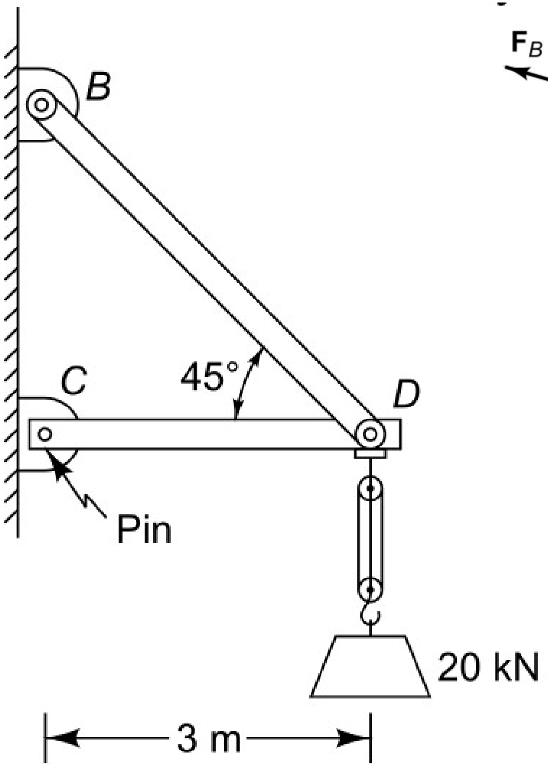

Observe that link BD is a two force member. Hence, both $\boldsymbol{F}_B$ and $\boldsymbol{F}_D$ will act along $BD$, have same magnitude and opposite directions. Thus,

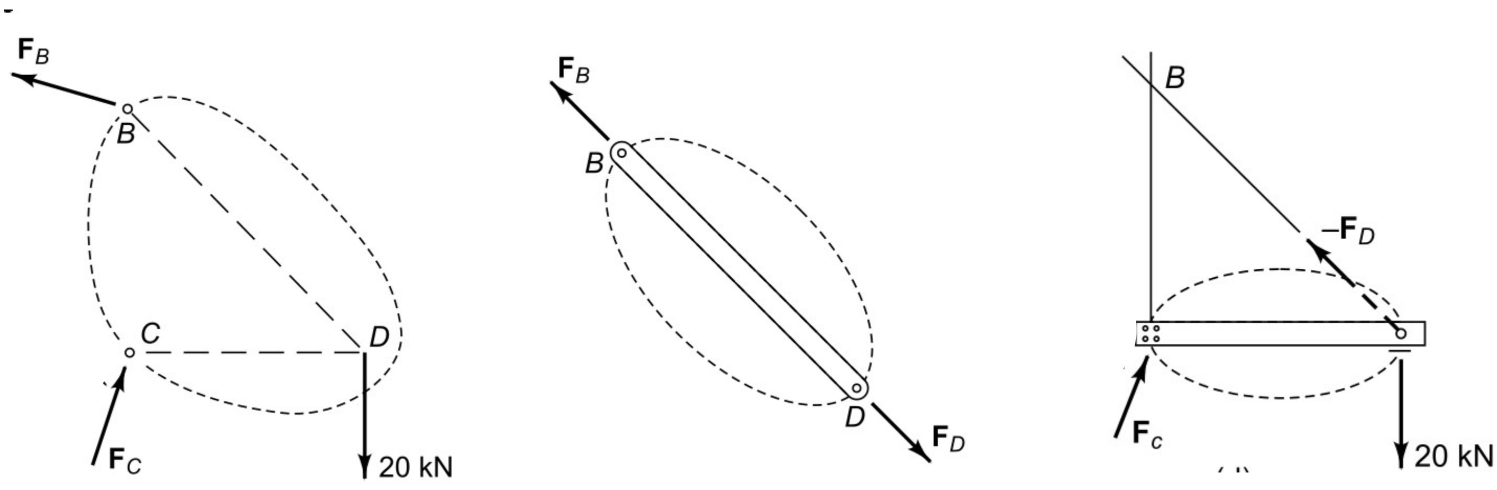
$\boldsymbol{F}_B = -\boldsymbol{F}_D$.

Consider link $CD$ now. Direction of $FD$ is known, however there are still four scalar unknowns with three equilibrium equations. Thus, we can not solve this problem with only equilibrium equations.

This is, in fact, the **statically indeterminate** case.

Now, suppose the joint $C$ to be a pin joint. This avoids the present of moment at point $C$ and we can see that the problem now convert to a statically determinate problem.



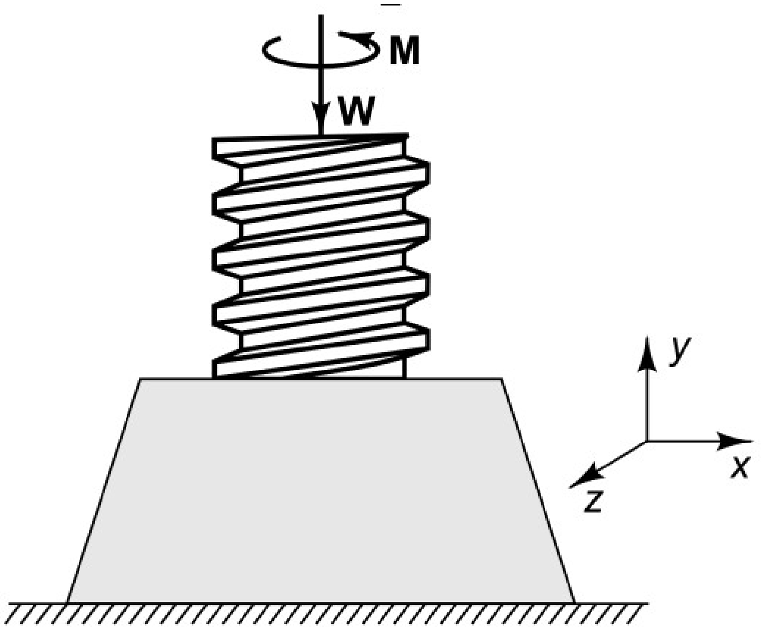



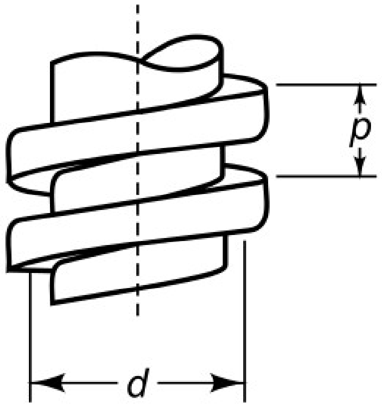

Observe that $CD$ is now three force member. Thus $\boldsymbol{F}_C$, $\boldsymbol{F}_D$ and external load of 20 kN need to intersect at common point.

From this point the problem can be solved in different manners to find all unknowns $\boldsymbol{F}_C$, $\boldsymbol{F}_D$ or $\boldsymbol{F}_B$.

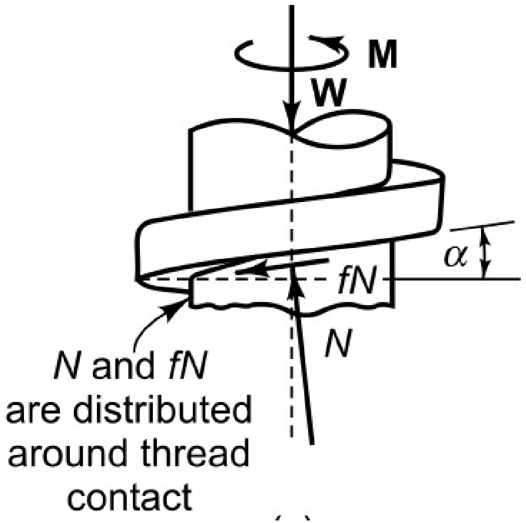
# Example 2

A screw jack, which is frequently used to raise or lower weights, is shown. The screw is characterized by a thread pitch $p$ and diameter $d$. We wish to determine the operating characteristics in the presence of a coefficient of friction $f$ between the screw threads and the jack body. In particular we wish to determine the relationship between the moment necessary to raise and lower the weight $W$ and the frictional and geometrical characteristics of the jack.



Let's isolate the screw. We **idealize** the screw and show the distributed thread loads as acting at one point for convenience of analysis. We see that each portion of the screw must slide up an incline at a helix angle $\alpha$, where

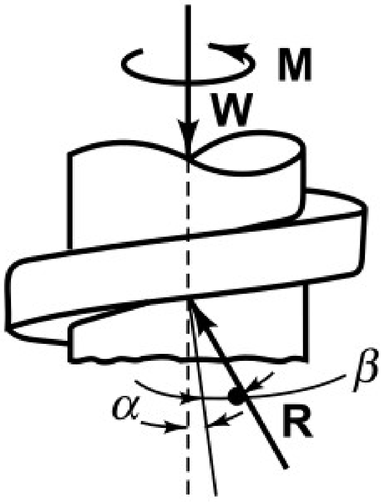



$$\tan\alpha = \frac{p}{\pi d}$$

Above relation can be understood by cutting the thread open and moving over the inclined plane.

Note that friction opposes motion and therefore, the frictional force will flip depending on the direction of the moment applied in trying to rotate the screw.

Applying equilibrium equation for two different directions of rotation, the appropriate moments necessary to start the movement up or down of the screw.




The resultant $\boldsymbol{R}$ of the normal component $\boldsymbol{N}$ and the frictional component $fN$ acts at an angle $\beta$ to the normal to the screw thread. Thus, $\boldsymbol{R}$ acts at a $\alpha \pm \beta$ to the vertical, depending on whether the jack is being raised, or is being lowered. If we now sum forces along the $y$-axis and moments about the $y$-axis, we have

$$\sum F_y = R\cos(\alpha \pm \beta) - W = 0$$

$$\sum M_y = M - d/R\sin(\alpha \pm \beta) = 0$$

Thus, $M = \dfrac{Wd}{2}\tan(\alpha \pm \beta)$,

where plus sign appears for the moment necessary to move the screw upward and the minus sign for the moment necessary to lower or unwind the screw.

Note that, when $\beta = \alpha$, the moment $M$ vanishes for equilibrium, and the screw will support the weight $W$ without unwinding. If $\beta > \alpha$, a negative $M$ is required to lower the weight. Thus, a jack that has $\beta \geq \alpha$ is said to be self-locking, a desirable property for a jack to have.

For a system of this type an efficiency $\eta$ can be defined as the ratio of work input to useful work output (the difference being due to wasted frictional heating).

Here, the work input per revolution is $2pM$, while the useful work of raising the weight is $pW$; thus

$$\eta = \frac{pW}{2\pi M} = \frac{\tan\alpha}{\tan(\alpha+\beta)}.$$

For small value of α and $\beta$, efficiency  is approximately

$$\eta \approx \frac{\alpha}{\alpha+\beta}.$$

And thus for a self-locking device, $\beta \geq \alpha$, the efficiency cannot surpass 50%.

# Introduction to Mechanics of Deformable Bodies

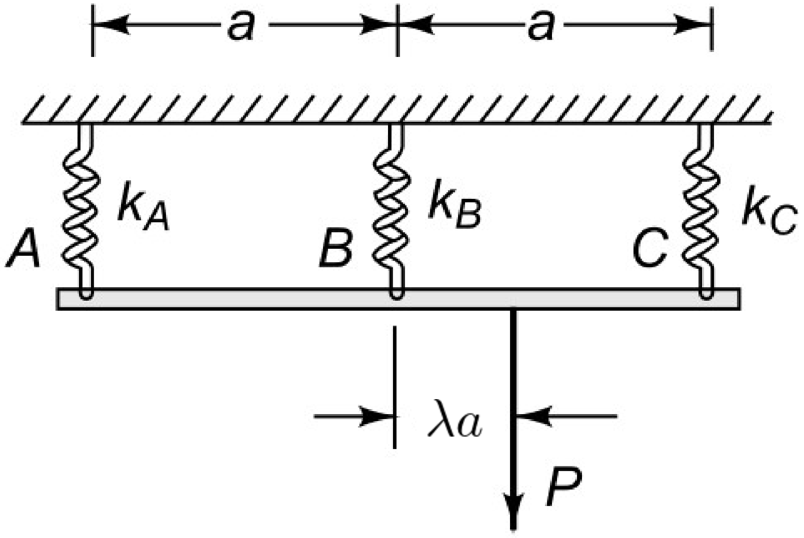
We have already discussed the following steps involved in the analysis based on the principles of mechanics:

1. Study of forces and equilibrium requirements
2. Study of deformation and conditions of geometric fit
3. Applications of force-deformation relations

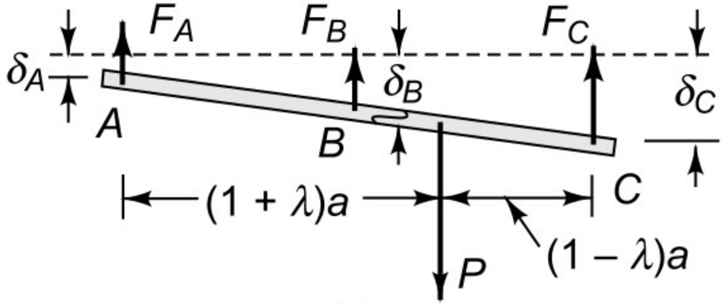
We saw examples where step 1 is sufficient to solve the problems. Let us look at some cases now where inclusion of step 2 and 3 become important.

# Example 1

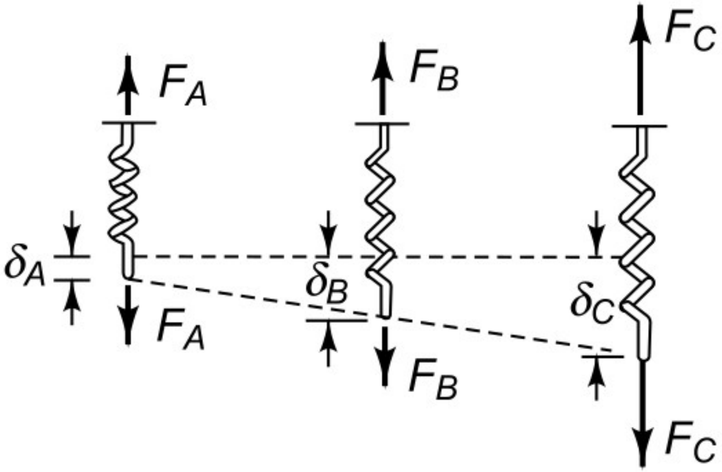
A light rigid bar ABC is supported by three springs, as shown in figure. Before the load $P$ is applied, the bar is horizontal. The distance from the center spring to the point of application of $P$ is $\lambda a$, where $\lambda$ is a dimensionless parameter which can vary between $\lambda = -1$ and $\lambda = 1$. The problem is to determine the deflections in the three springs as functions of the load position parameter $\lambda$. We shall obtain a general solution for arbitrary values of the spring constants and display the results for the particular set $k_A = (1/2)k$, $k_B = k$ and $k_C = (3/2)k$.



**Idealization:** The bar is relatively rigid and massless.




(c)




FBD of the bar as well as springs are shown.

**Application of equilibrium conditions:**
For bar:

$$\sum F = F_A + F_B + F_C - P = 0, \quad \cdots\cdots\cdots\text{(a)}$$

$$\sum M_A = F_B \cdot a + F_C \cdot 2a - P \cdot (1 + \lambda a) = 0, \quad \cdots\cdots\cdots\text{(b)}$$

It can be observed that the problem is statically indeterminate.

**Study of deformation conditions and geometric fit:**
If springs has to be connected with the bar, they have to follow specific deflections, as shown in the figure. Relation between the deflections can be written as,

$$2\delta_B = \delta_A + \delta_C. \quad \cdots\cdots\cdots\text{(c)}$$

**Application of force-displacement relations:**
Force in each spring can be related to corresponding deflection of spring as,

$$\delta_A = \frac{F_A}{k_A}, \quad \delta_B = \frac{F_B}{k_B}, \quad \text{and} \quad \delta_C = \frac{F_C}{k_C}. \qquad \text{...............(d)}$$

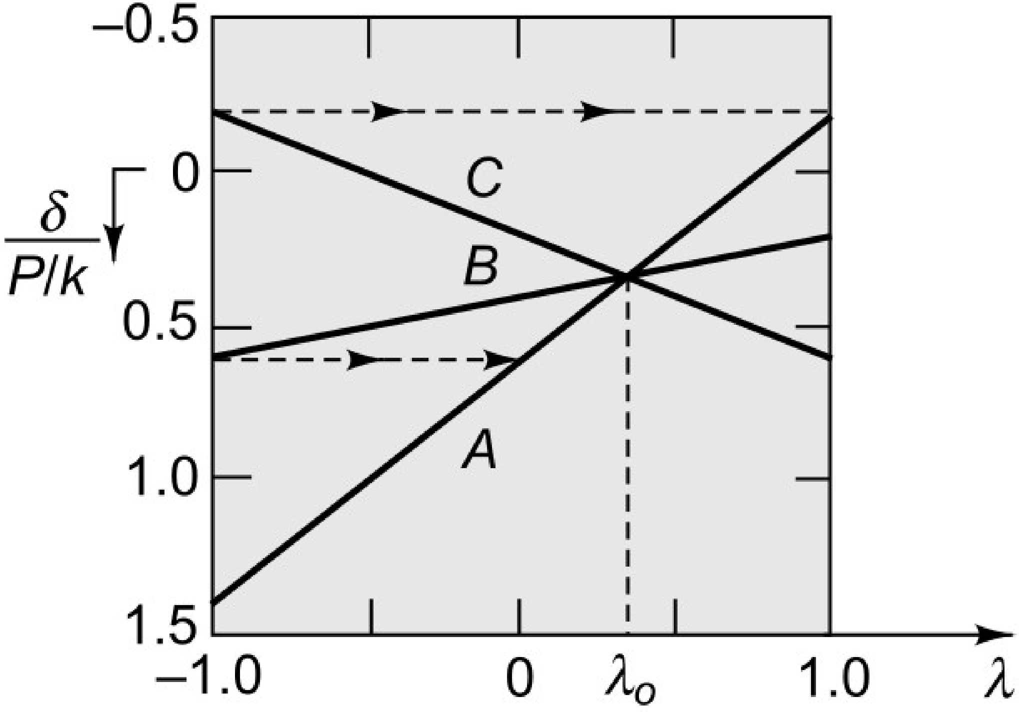
Equations (a)-(d) are six independent equations in six unknowns (three forces and three deflections). These equations can be solved very easily for all six unknowns.
Deflections are found to be,

$$\delta_A = P\frac{2k_C - \lambda(k_B + 2k_C)}{k_A k_B + 4k_A k_C + k_B k_C}, \qquad \delta_B = P\frac{k_A + k_C + \lambda(k_A - k_C)}{k_A k_B + 4k_A k_C + k_B k_C}, \quad \text{and}$$

$$\delta_C = P\frac{2k_A + \lambda(k_B + 2k_A)}{k_A k_B + 4k_A k_C + k_B k_C}. \qquad \text{...............(e)}$$

Note that deflections of all springs are proportional to P and linear function of $\lambda$.

Let us try to understand more about the deformation. We plot all deflections as function of $\lambda$ for the given stiffness values.




$\lambda_0$ can be determined as,

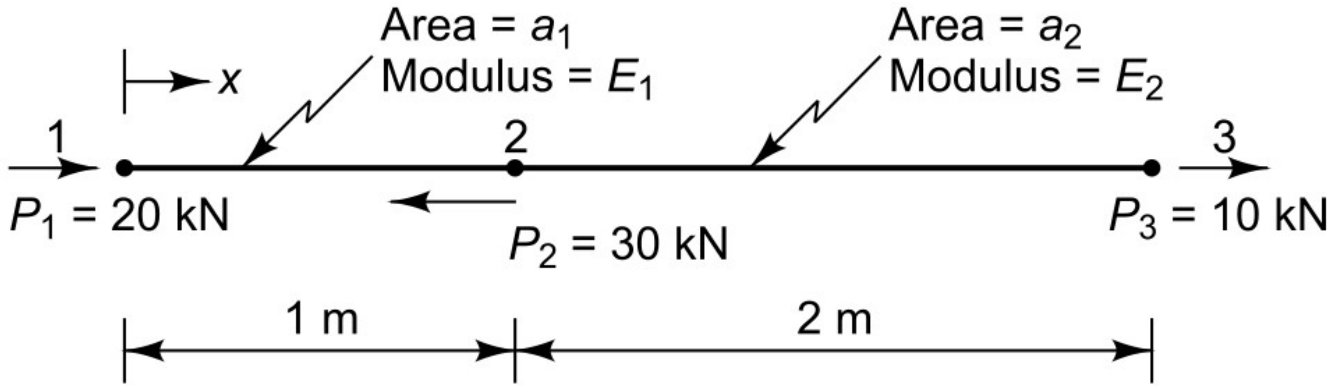
$$\lambda_0 = \frac{k_C - k_A}{k_A + k_B + k_C},$$

and deflection of all points correction to $\lambda_0$ is

$$\delta(\lambda_0) = \frac{P}{k_A + k_B + k_C}.$$

# Example 2

For the below rod, let us determine the various quantities such as axial force and axial deflection with respect to the left end 1, in terms of $x$. Also draw a diagram showing the distribution of axial force in the member with respect to $x$ (axial force diagram).



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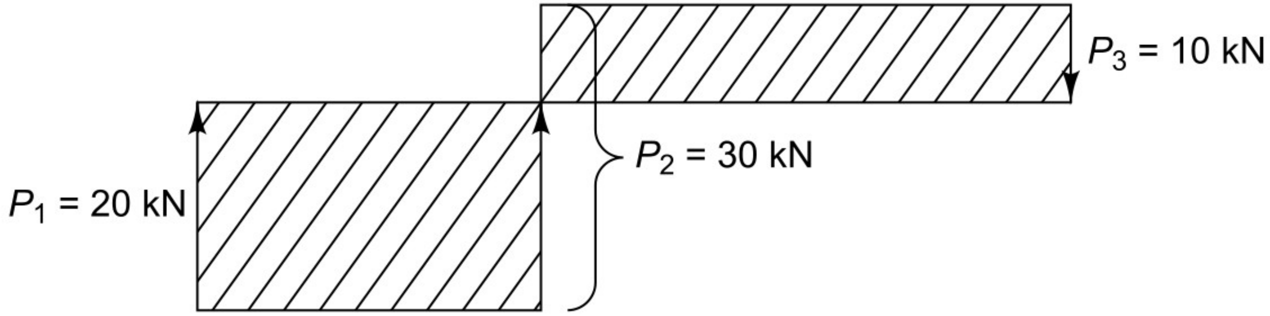
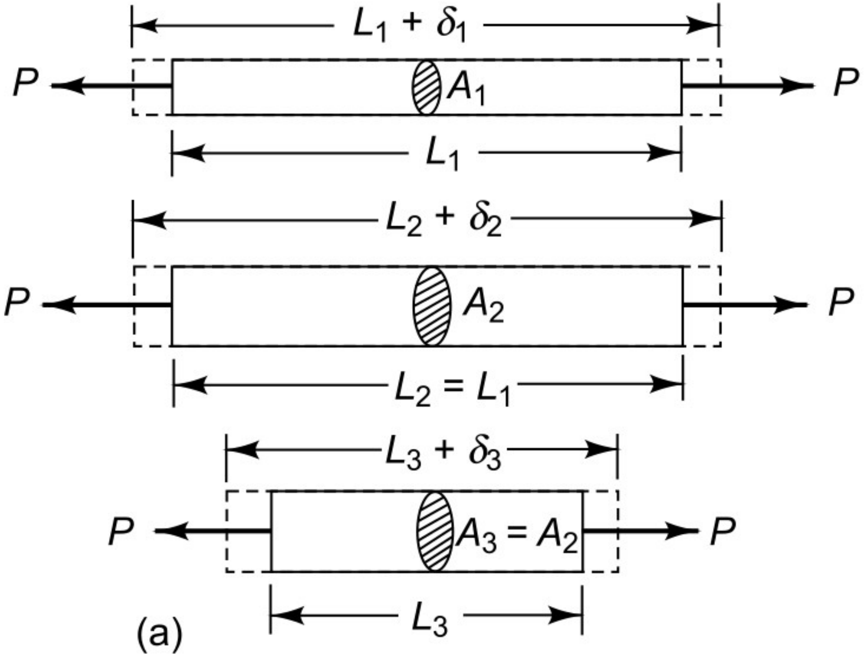
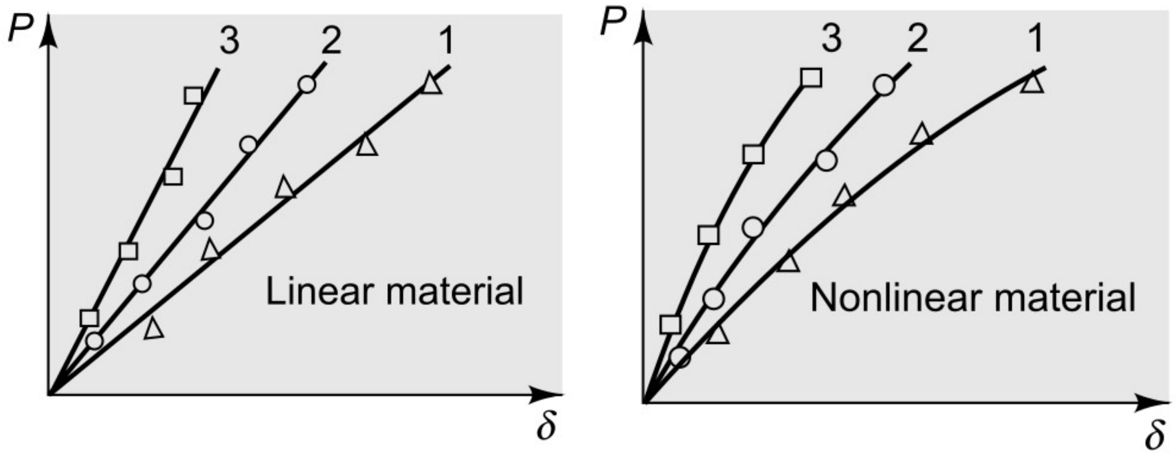


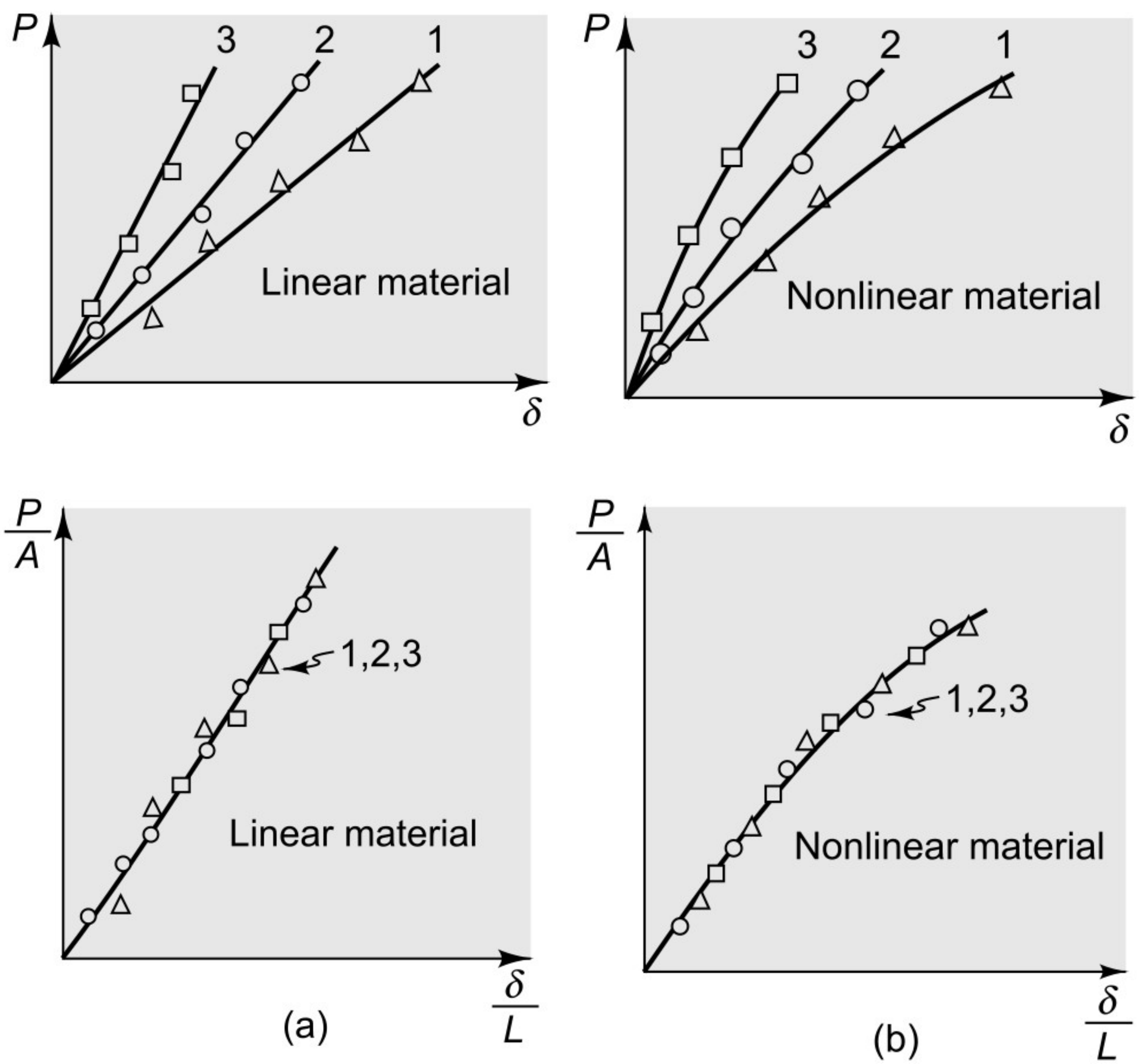
# Uniaxial loading and deformation

One of the most basic loading, which can be considered on structures like bar or rod is the load applied along their axis. For such a loading load vs. deformation curve for a given specimen can be plotted.




**Load – deformation behaviour**






When the load ($P$) - deformation ($\delta$) curves are re-plotted as ($P/A$) - ($\delta/L$) curves, then it is observed that curves obtained for specimens of different dimensions overlap.

This suggests that the new curves obtained now are **independent of specimen dimensions.**

Thus, they represent **load-elongation characteristics of a particular material.**