ME632: Fracture Mechanics

Timings	
Monday	10:00 to 11:20
Thursday	08:30 to 09:50

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Room No. # 106

Course Plan

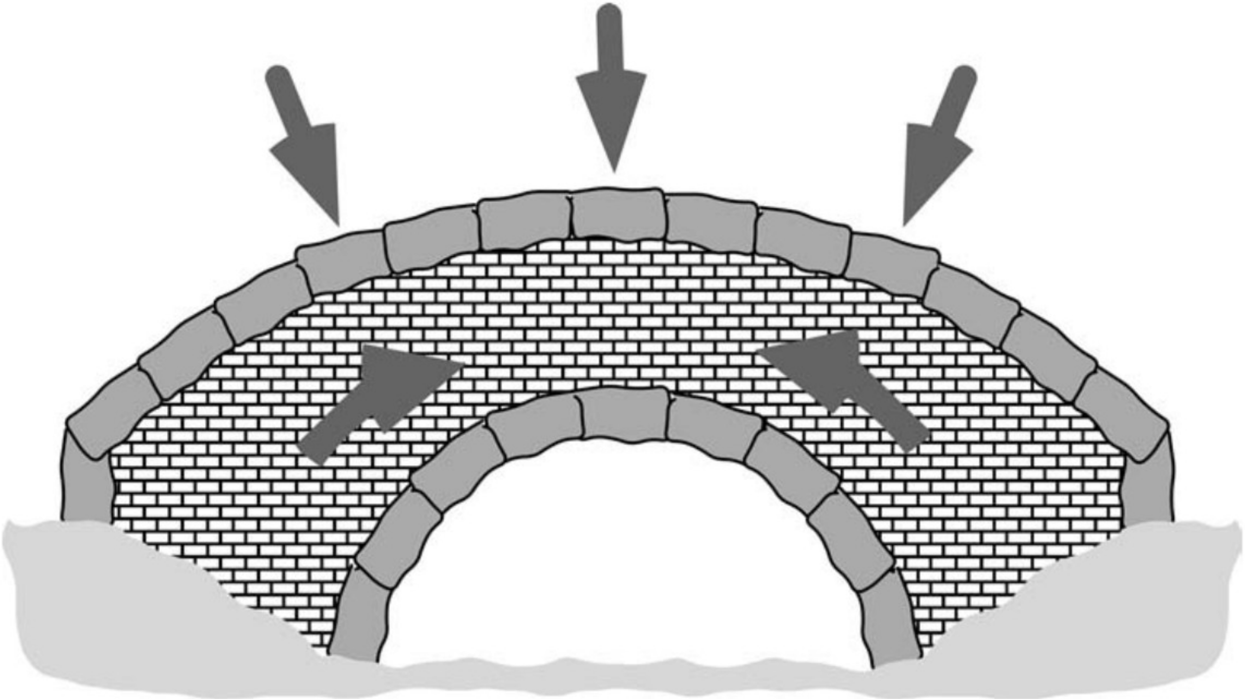
- **Tierce exam: 25% + 25% + 25% = 75%**
  - Open book exams. Use of laptop/mobile is permitted.
  - Anyone found to be using unfair means (in any form) during the exam will be penalized with 100% negative marks. In case of coping both the students, the one who has copied and from whom he/she has copied, will be penalized without seeking any clarification/ justification.
  - Use of laptop and/or mobile is permitted only for accessing study material in soft form. Copying solutions from internet will be treated as usage of unfair means.
- **Term paper: 25%**
  - It will consist of application experiments, simulation etc. Problems will be given after basic concepts are covered in the course (i.e., approximately half the semester).
  - However, you are encouraged to explore problems for term paper. You need to get it approved by me before you start working on it.

Books

- *Fracture Mechanics: Fundamentals and Applications* by T. L. Anderson. Taylor and Francis.
- *Elements of Fracture Mechanics* by Prashant Kumar. McGrawHill.
- *Fracture Mechanics: Fundamentals and Applications* by S. K. Maiti. Cambridge University Press.
- *Elementary Engineering Fracture Mechanics* by David Broek. Kluwer Academic Publication.
- *Fracture Mechanics* by E. E. Gdoutos. Springer.

Construction of brick structures

Old brick structures were designed such that the loads are transmitted as compressive stresses through the structures.



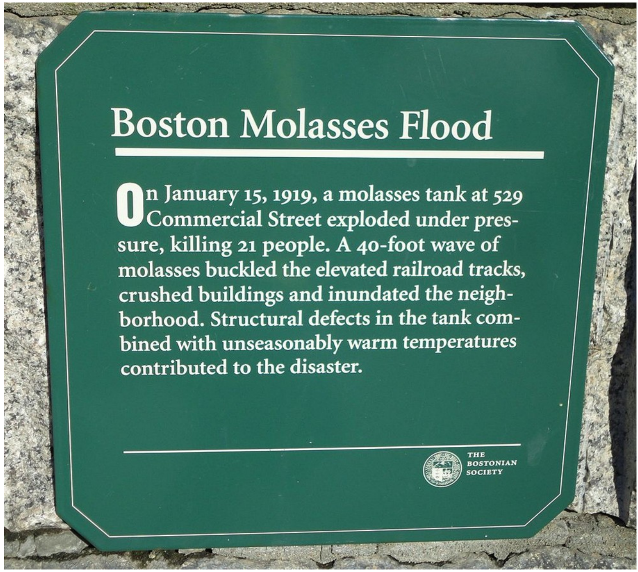
Source: Fracture Mechanics by T. L. Anderson

Early design of steel structures

- After industrial revolution steel became popular as structural materials.
- Such structures were largely subjected to tensile stresses and subjected to unexpected failures many times.
- Design of steel structures were based on strength of material concept. Hence, failure of such structures were also analyzed by theories of failure.
- Many times causes of failure could not be completely explained by failure theories.
- High safety factors (upto 10) were used to avoid such random failure.

Boston Molasses Flood

Tank hight: 50 foot
Volume: 8700 m^3,
Weight of the molasses: 12000 tons

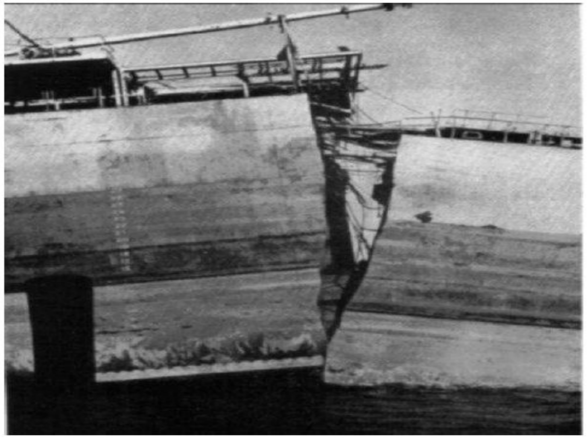


The cause of the failure of the molasses tank was largely a mystery at the time.

Source: https://en.wikipedia.org/wiki/Great_Molasses_Flood

Liberty ships failure

During WW-II several ships broke completely in two pieces while sailing in the North Pacific.



Reasons for failure

The welds contained crack-like flaws.

Most of the fractures initiated on the deck at square hatch corners, where there was a local stress concentration.

The steel from which the Liberty ships were made had poor toughness, as measured by Charpy impact tests.

Corrections

The remaining ships were retrofitted with rounded reinforcements at the hatch corners.

High toughness steel crack-arrester plates were riveted to the deck at strategic locations.

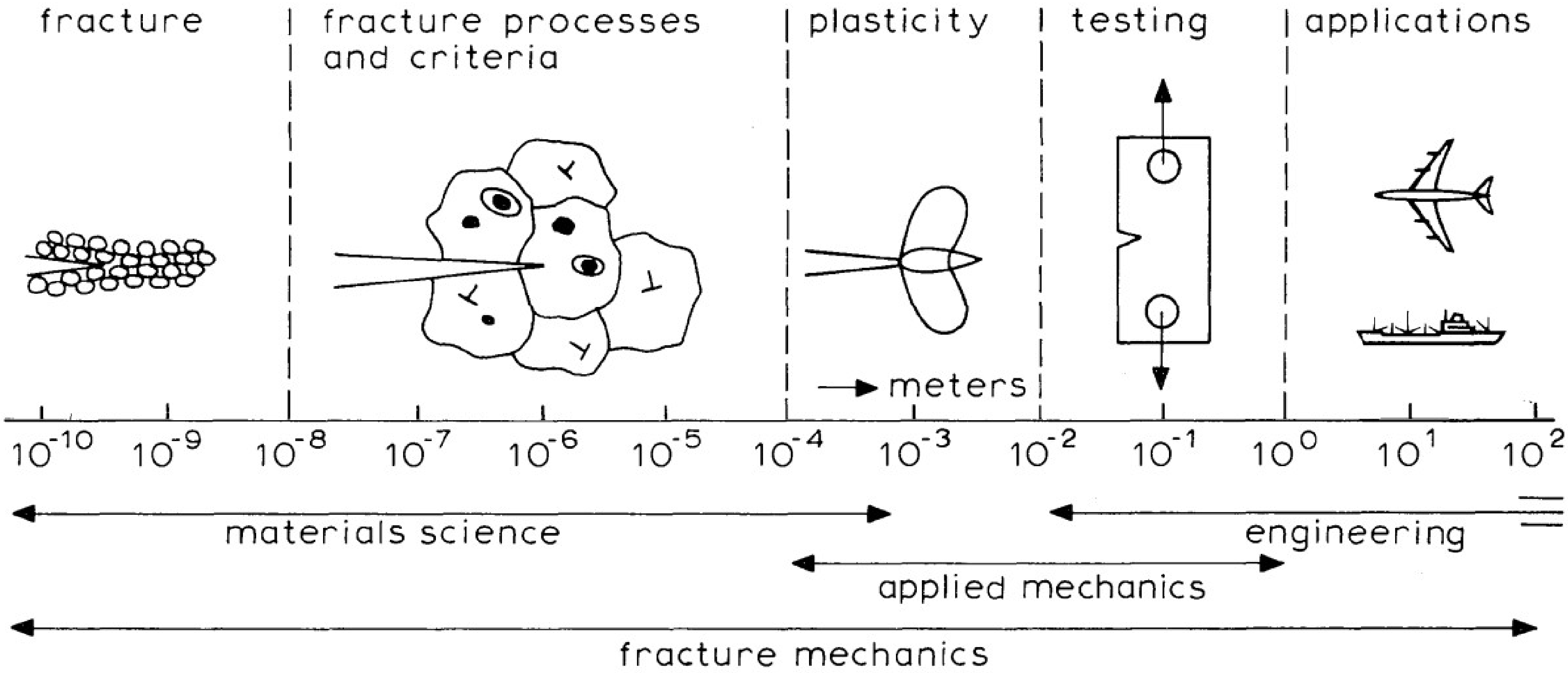
Development of high toughness steels

Development of weld quality control standards

Fracture Mechanics

- Fracture mechanics starts with the assumption that there **exists a crack like defect of known size** in the material.
- How valid this assumption is?
    - All materials have inherent defects such as dislocations, microvoids, microcracks etc.
    - Defect may be occur during manufacturing or processing.
    - Defects may also occur on the components during service (such as scractches, corrosion, pitting etc.)
- In fracture mechanics we try to answer the following:
    - when will a crack grow? (at what time time? or at what load?),
    - how will a crack grow? (direction, speed)
    - Is a structure safe with an existing crack?
    - Can we delay or stop growing defects/cracks?
    - Can we design materials with better fracture resistance?

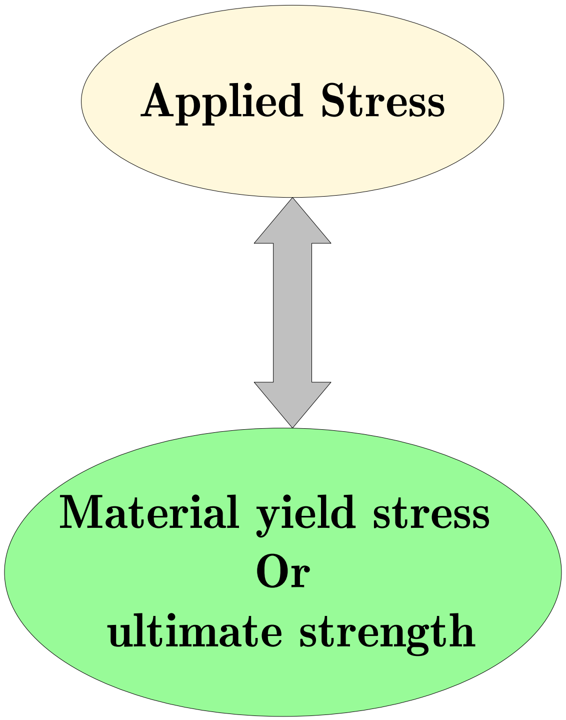
Fracture mechanics at different scales



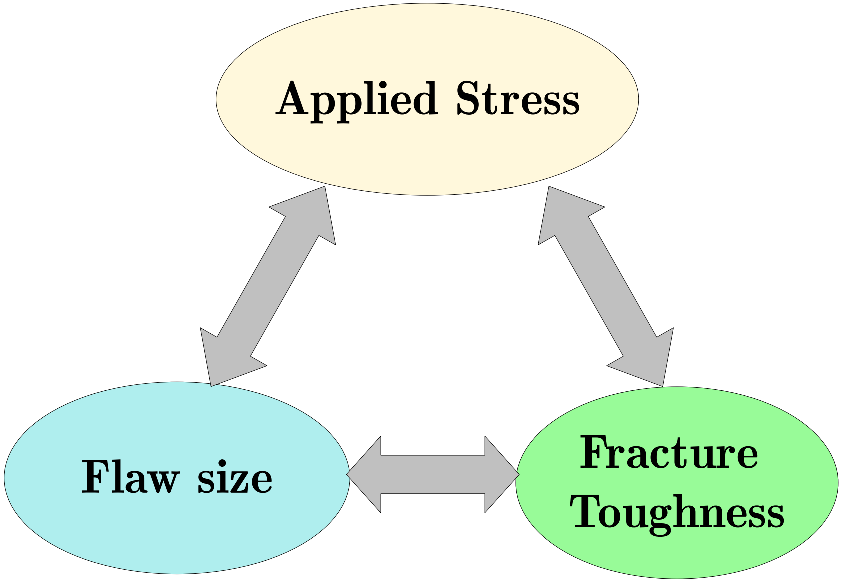
Source: Elementary Engineering Fracture Mechanics by David Broek

Design philosophy

Mechanics of Solid based



Fracture Mechanics based



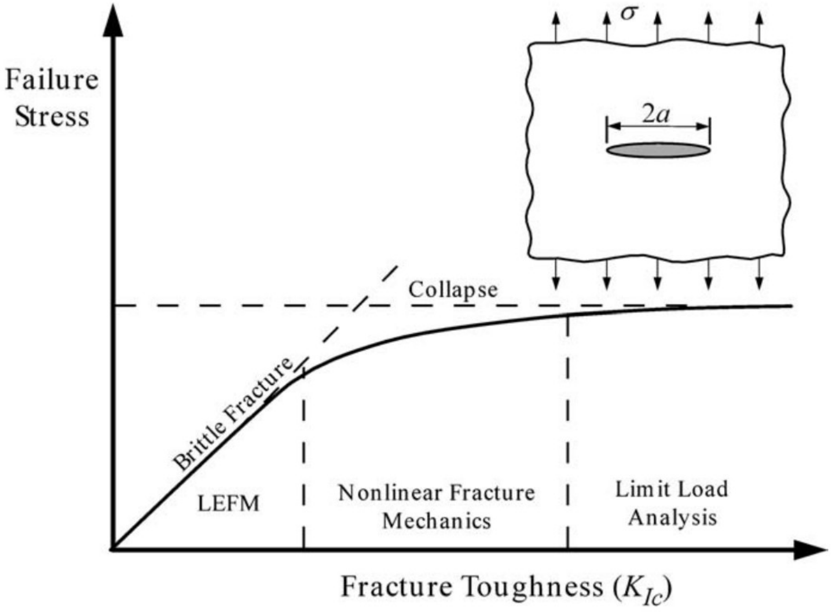
Effect of material properties on fracture

- Linear Elastic Fracture Mechanics (LEFM) – (Linear time-independent materials)
- Elastic-plastic Fracture Mechanics (EPFM) – (Non-linear time-independent materials)
- Viscoelastic/Viscoplastic Fracture Mechanics (Linear/Nonlinear time-dependent materials)
- Dynamic Mechanics (Time-dependent fracture for all material)

Typical Fracture Behavior of Selected Materials[a]

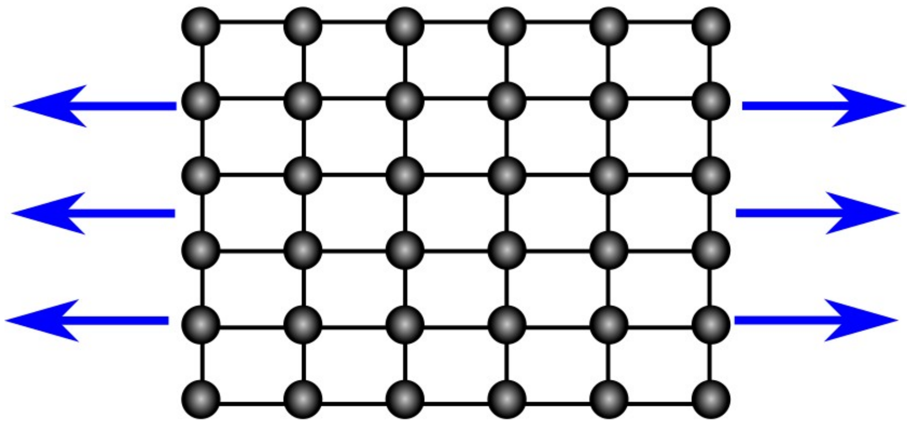
Material	Typical Fracture Behavior
High strength steel	Linear elastic
Low- and medium-strength steel	Elastic-plastic/fully plastic
Austenitic stainless steel	Fully plastic
Precipitation-hardened aluminum	Linear elastic
Metals at high temperatures	Viscoplastic
Metals at high strain rates	Dynamic/viscoplastic
Polymers (below T_g)[b]	Linear elastic/viscoelastic
Polymers (above T_g)[b]	Viscoelastic
Monolithic ceramics	Linear elastic
Ceramic composites	Linear elastic
Ceramics at high temperatures	Viscoplastic

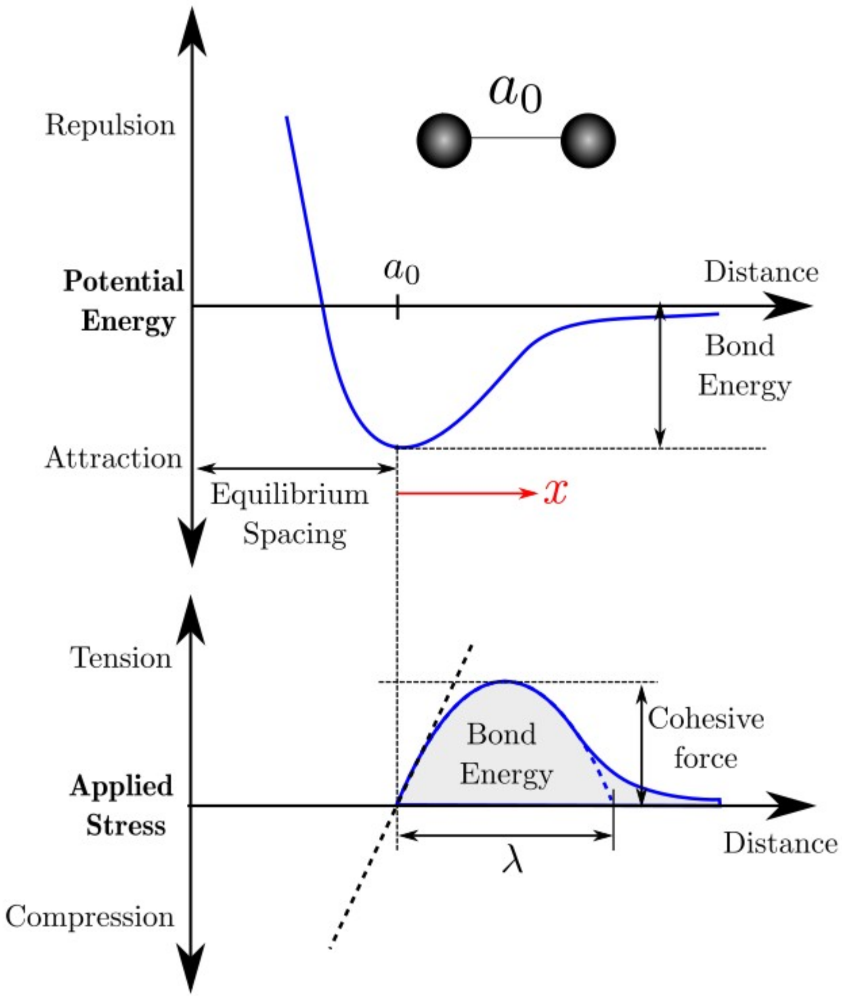
[a] Temperature is ambient unless otherwise specified.
[b] T_g—Glass transition temperature.



Source: Fracture Mechanics by T. L. Anderson

Theoretical strength of a solid





A tensile force is required to increase the separation distance from the equilibrium value and this force must exceed the cohesive force to break the bond completely. The bond energy is then,

$$E_b = \int_{a_0}^{\infty} P dx, \qquad \cdots\cdots\cdots\cdots\cdots(1)$$

where P is the applied force.

For simplifying the calculations we idealize the interatomic force-displacement relationship as one half of the period of a sine wave, so that

$$P = P_c \sin\left(\frac{\pi x}{\lambda}\right). \qquad \cdots\cdots\cdots\cdots(2)$$

For small displacements, the force-displacement relationship is linear as $P = P_c \frac{\pi x}{\lambda}$, $\cdots\cdots\cdots(3)$

and the bond stiffness is given as $\frac{P}{x} = k = P_c \frac{\pi}{\lambda}$. $\cdots\cdots\cdots\cdots(4)$

Equation (3) can be written in terms of stress as, $\sigma = \sigma_c \frac{\pi x}{\lambda}$, where stress is the load divided by the number of bonds per unit area.

Using Hooke's law as $\sigma = E\varepsilon = E\frac{x}{a_0} = \sigma_c \frac{\pi x}{\lambda}$, $\cdots\cdots\cdots\cdots(5)$

Hence,

$$\sigma_c = \frac{E\lambda}{\pi a_0}, \qquad \cdots\cdots\cdots\cdots(6)$$

or

$$\sigma_c \approx \frac{E}{\pi}. \quad (\text{if } \lambda \approx a_0 \text{ is assumed}) \qquad \cdots\cdots\cdots\cdots(7)$$

Separation will cause the create of two new surfaces. So if surface energy per unit area is γ_s then it can be estimated as,

$$2\gamma_s = \int_0^\lambda \sigma_c \sin\frac{\pi x}{\lambda}dx = \frac{2\sigma_c\lambda}{x} \quad \text{or} \quad \gamma_s = \frac{\sigma_c\lambda}{\pi}, \qquad \text{...............(8)}$$

Using (6) we can write γ_s as,

$$\gamma_s = \frac{\sigma_c^2 a_0}{E} \approx \frac{Ea_0}{\pi^2}, \qquad \text{...............(9)}$$

or

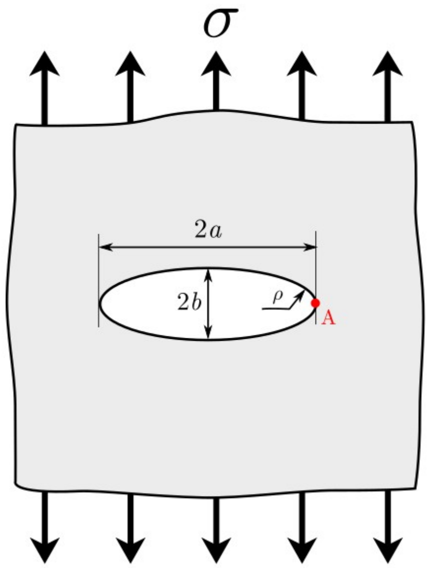
$$\sigma_c = \sqrt{\frac{\gamma_s E}{a_0}}. \qquad \text{...............(10)}$$

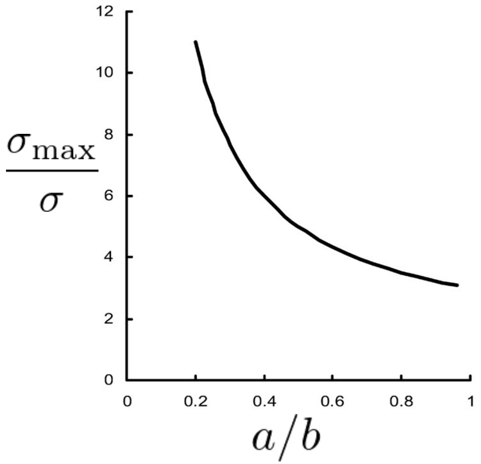
Let's apply relations (7) to iron or steel. We use $E = 210$ GPa and $a_0 = 2.5$ Å.

We obtain $\sigma_c \approx 70$ GPa and $\gamma_c = 5$ J/m^2.

These values are significantly higher than the experimentally obtained strength or surface energy for bulk iron or steel.

Stress concentration due to a flaw





Inglis (1913) gave analytical solution for stress near a two dimensional elliptical hole in an infinite plate. He showed that the stress at the tip of major axis (i.e., point A) is given as

$$\sigma_A = \sigma \left(1 + \frac{2a}{b} \right). \qquad \text{...............(11)}$$

As major axis, a, increases relative to b the hole begin to take the shape of a sharp crack. In this case it is more convenient to express (11) in terms of radius of curvature at the tip as

$$\sigma_A = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}} \right). \quad (\text{where, } \rho = b^2/a) \qquad \text{...............(12)}$$

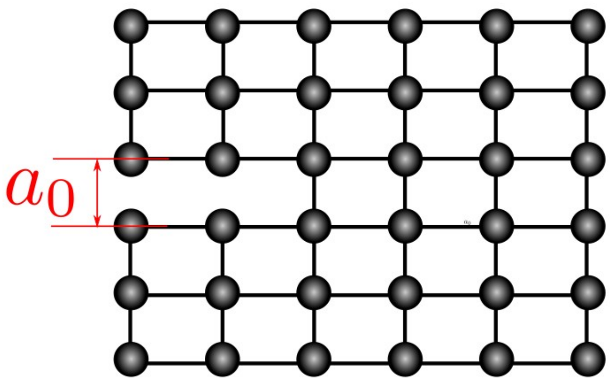
When $a \gg b$ then,

$$\sigma_A = 2\sigma \sqrt{\frac{a}{\rho}} \qquad \text{...............(13)}$$

Equation (13) gives an infinite stress at the tip of an infinitely sharp crack (i.e., ρ=0).

As no material is capable of withstanding infinite stress this result was a caused a concern at first. A material that contains a sharp crack should theoretically fail upon the application of an infinitesimal load, which is not observed in practice.

Real materials are made of atoms. For an elastic material the minimum radius a crack tip can have is on the order of the equilibrium distance between molecules.



To obtain a rough estimate of failure stress for continuum materials, we apply (13) at the atomic level (though it is valid for a continuum) and use $\rho = a_0$.

$$\sigma_A = 2\sigma\sqrt{\frac{a}{a_0}}. \qquad \text{.............. (14)}$$

Now, if we assume that the failure happens when $\sigma_A = \sigma_c$, then

$$\sigma_A = \sigma_c = 2\sigma_f\sqrt{\frac{a}{a_0}}, \qquad \text{.............. (15)}$$

where σ_f is the applied remote stress at failure. Using (10),

$$\sigma_c = \sqrt{\frac{\gamma_s E}{a_0}} = 2\sigma_f\sqrt{\frac{a}{a_0}} \quad \Rightarrow \sigma_f = \sqrt{\frac{E\gamma_s}{4a}}. \qquad \text{.............. (16)}$$

Similar results have been obtained from numerical simulation of a crack in a two-dimensional lattice (Gehlen and Kanninen, 1970) as,

$$\sigma_f = \alpha\sqrt{\frac{E\gamma_s}{a}}, \qquad \text{...............(17)}$$

where α is a constant, on the order of unity, which depends slightly on the assumed atomic force-displacement law.