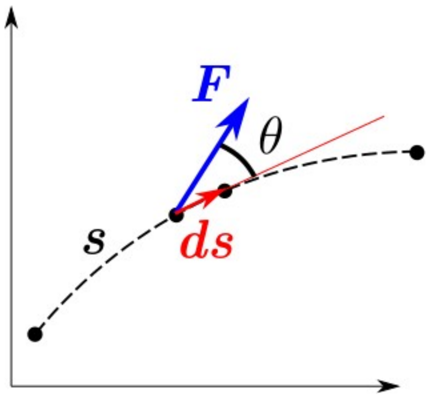
ME231: Solid Mechanics-I

Introduction to Mechanics of Deformable Bodies

Elastic energy; Castigliano's theorem

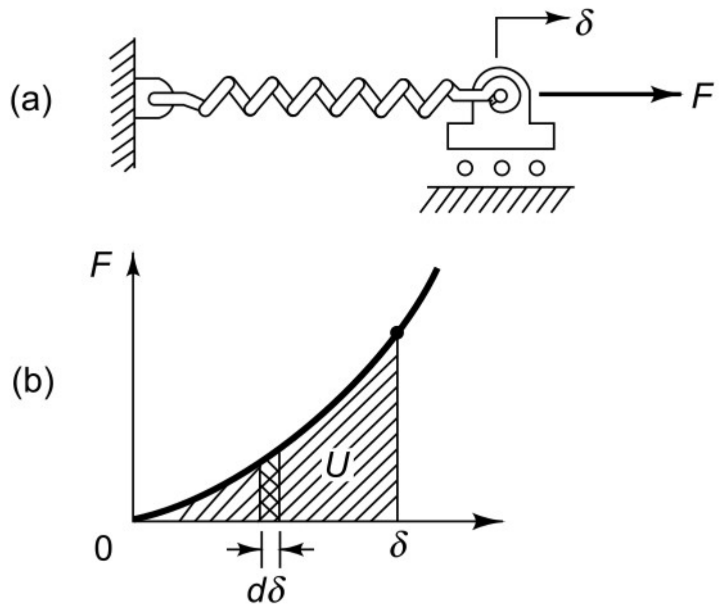


For elastic systems, concept of energy can be a very powerful tool for deflection analysis.
Total work done by a force $\boldsymbol{F}$ when the point of application travels a distance s, is given as

$$W = \int_s \boldsymbol{F} \cdot d\boldsymbol{s}.$$

When work is done by an external force on certain systems, their internal geometric states are altered in such a way that they have the potential to give back equal amounts of work whenever they returned to their original configurations. Such systems are called *conservative*, and the work done on them is said to be stored in the form of **potential energy**.

For example the work done in lifting a weight is said to be stored as *gravitational potential energy*. The work done in deforming an elastic spring is said to be stored as *elastic potential energy*.



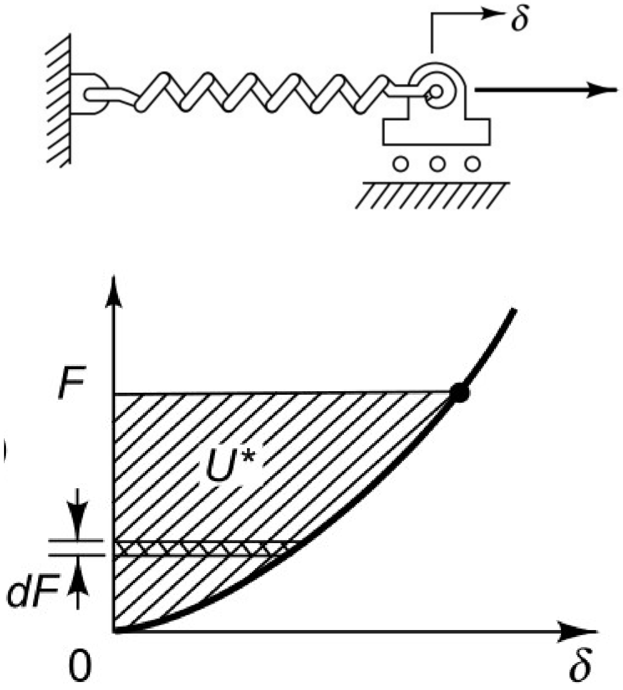
Consider an elastic spring (not necessarily linear) fixed at one end and subjected to a load F at the other end. If at a deflection of δ spring is in equilibrium, then the potential energy stored in the spring is given as,

$$U = \int_0^{\delta} F d\delta.$$

Note that this energy is same as the area under the load-deflection curve. The magnitude of the energy that can be stored by a given spring or member is an important consideration in mechanical design. Components which are subjected to impact loads are often selected based on the their capacity to absorb energy.

When the point of application of a variable force $\boldsymbol{F}$ undergoes a displacement $\boldsymbol{s}$, the *complementary work* can be defined as

$$W^* = \int_s \boldsymbol{s} \cdot d\boldsymbol{F}.$$



Corresponding to the the gradual loading of a nonlinear spring, the *complementary energy* U^* associated with a force F is defined as,

$$U^* = \int_0^F \delta dF.$$

It should be observed that for a liner elastic system $U = U^*$.

In later part of the course we will see an important application of strain energy or complementary energy to determine the deflection of linear-elastic systems.

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Forces and Moments Transmitted by Slender Members

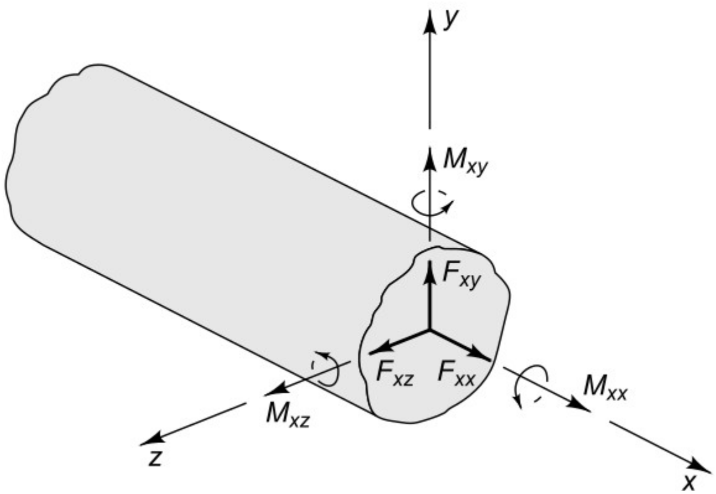
A large number of engineering structures has **slender members** as load carrying-members.

A slender member can be defined as a member whose **length is relatively higher** (at least five time or more) than the other two (cross-sectional) dimensions.

Beams, columns, shafts, rods, stringers, struts, and links can be classified as slender members. A hoop or coil is also considered as slender structure as they are form by a long thin rod.

In general, a slender member can have axial, torsion or bending loads.

If slender member is in equilibrium, all the sub-systems isolated by assuming a hypothetical cuts/sections at the point of interest will also be in equilibrium. Considering equilibrium of these sub-systems, forces and moments at each cross-section will be determined.



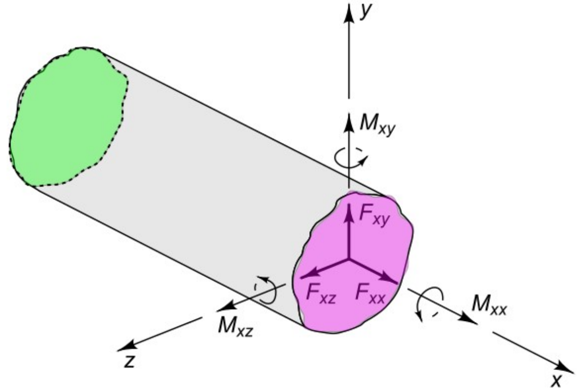
F_{xx} – Axial force in x-direction (also denoted as F_x),

F_{xy}, F_{xz} – Shear forces at x-plane (also denoted as V_y, V_z),

M_{xx} – Twisting moment (also denoted as M_t or T),

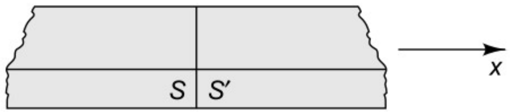
M_{xy}, M_{xz} – Bending moment (also denoted as M_{by}, M_{bz}).

Sign conventions



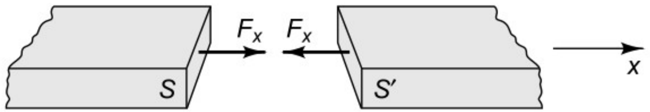
The cross-sectional face will be called

- **positive** when the outward normal points in the positive direction (magenta colored face)
- **negative** when the outward normal points in the negative direction (green colored face)

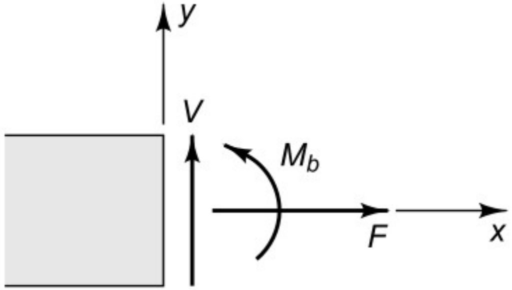


The force or moment component are positive when

- acting on a positive face in a positive direction
- acting on a negative face in a negative direction

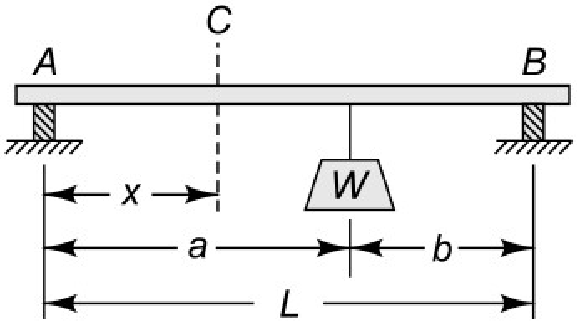


For a two-dimensional case positive components of force and moments are shown

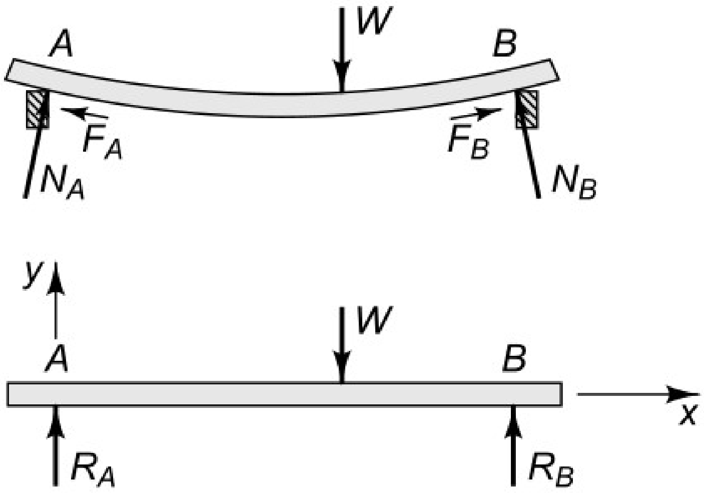


Example 1

Let us consider a beam supporting a weight near the center and resting on two other beams, as shown in Figure. It is desired to find the distribution of forces and moments along the beam.



Idealization



Find the reaction forces by applying the equilibrium equations.

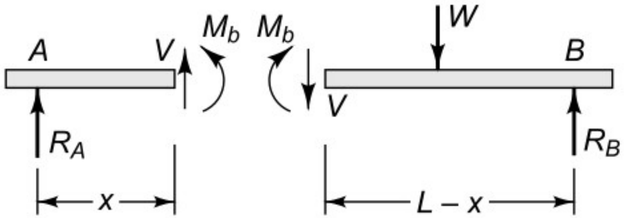
$$\sum F_y = R_A + R_B - W = 0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(1.a)}$$

$$\sum M_A = R_B \cdot L - W \cdot a = 0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(1.b)}$$

Using (1.a) and (1.b), R_A and R_B can be determined as,

$$R_A = Wb/L, \quad R_B = Wa/L. \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(1.c)}$$

Now all the external forces are known. Let us find the distribution of internal forces and moments along the length of the beam. To do that we consider a section at any point along the beam and apply equilibrium equation to the sub-systems.



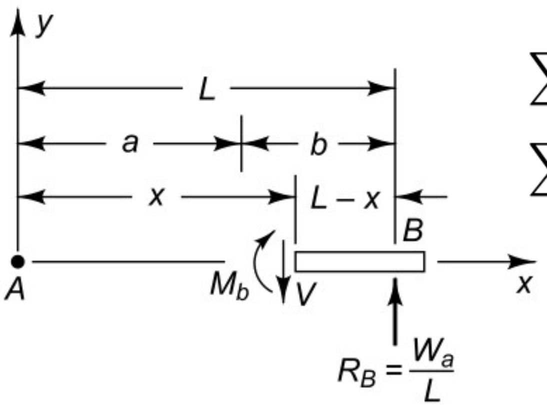
Considering the equilibrium of left part,

$$\sum F_y = R_A + V = 0 \Rightarrow V = -R_A = -Wb/L \qquad \cdots\cdots\text{(1.d)}$$

$$\sum M_A = V \cdot x + M_b = 0 \Rightarrow M_b = -Vx = Wbx/L$$

Note that above equations are application at any point between point A and the point of application of W, i.e., $0 \leq x \leq a$.

To get the distribution for rest of the beam, section is taken between the point of application of W and point B and consider the the equilibrium of the subsection.

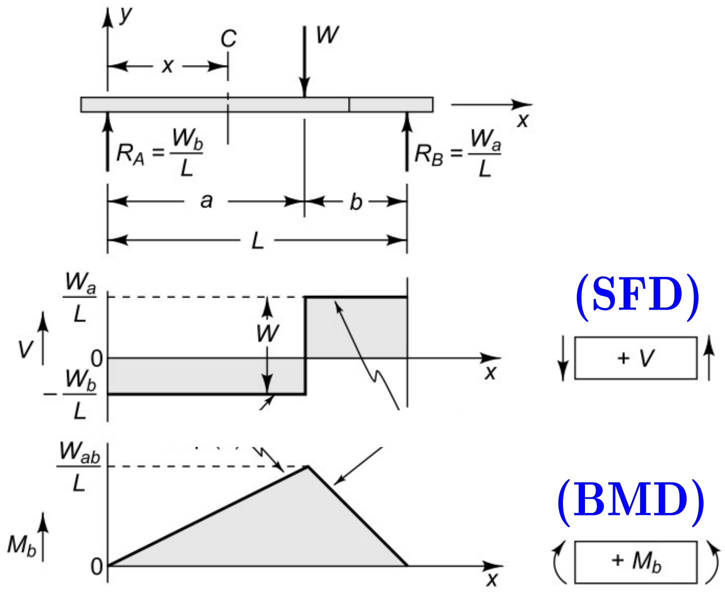


$$\sum F_y = R_B - V = 0 \Rightarrow V = R_A = Wa/L$$

$$\sum M_A = V \cdot (L - x) - M_b = 0 \Rightarrow M_b = V(L - x) = Wa(L - x)/L \qquad \text{(1.e)}$$

Equations (1.e) are application for i.e., $a \leq x \leq b$.

Distribution of forces and moments (Equations 1.d and 1.e) can be graphically shown using **Shear Force Diagram (SFD)** and **Bending Moment Diagram (BMD)**.

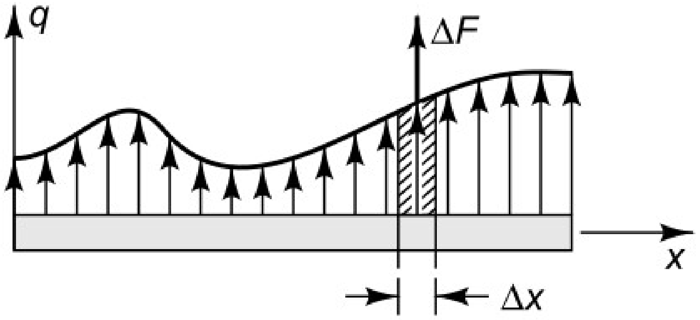


Distributed loads

In Example 1, we idealized the forces working between the slender member and supports or loading mechanism as **point forces (concentrated at a single point)**.

Another idealization which is generally used is the concept of **continuously distributed** loading.

Figure shows a distributed loading on the beam. Such forces might result from fluid or gas pressures, or from magnetic or gravitational attractions. If the total force on a length Δx be denoted by ΔF; then the intensity of loading q is defined as the limit as



$$q(x) = \lim_{\Delta x \to 0} \frac{\Delta q}{\Delta x}.$$

Most commonly used distributed loading in engineering applications are **uniformly distributed loading** (i.e., q is constant) and **linearly varying distribution** (i.e. $q(x)=A+Bx$)