ME231: Solid Mechanics-I

Fundamental principles of Mechanics

- Mechanics is the science of **forces** and **motions**.
- Applied mechanics is the science of applying the principles of mechanics to systems of practical interest to understand their behavior and design them.
- The general approach towards a problem in applied mechanics involves following steps:

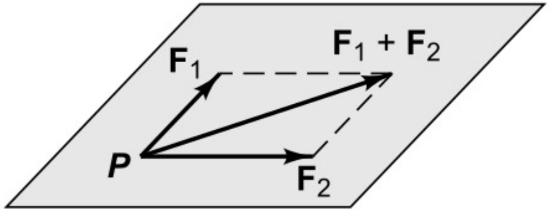
1. Selection of the system of interest.
2. Idealization and simplification of the real situation.
3. Application of principles of mechanics to the idealized model. Deduce the consequences.
4. Compare these predictions with the behavior of the actual system. This usually involves comparison with tests and experiments.
5. If satisfactory agreement is not achieved, the foregoing steps must be reconsidered. Very often progress is made by altering the assumptions regarding characteristics of the system, i.e., by constructing a different idealized model of the system.

Every analysis of a mechanical system involves following three steps:

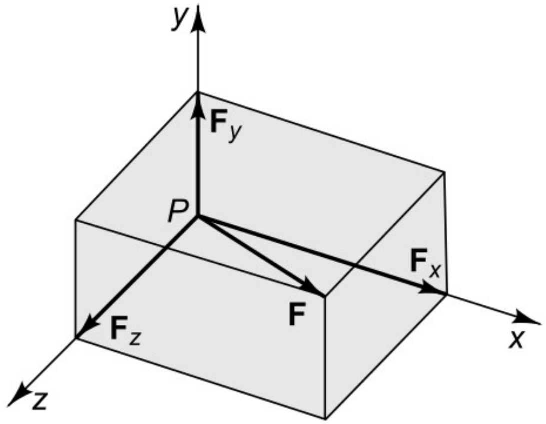
1. Study of forces
2. Study of motion and deformation
3. Application of laws relating the forces to the motion and deformation

Force:

1. Force is a vector interaction which can be characterized by a pair of equal and opposite vectors having the same line of action. When a system is isolated then single vector is also referred as force.

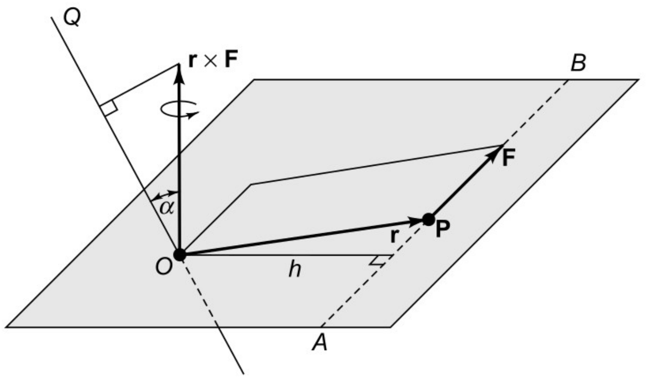


2. When two or more forces act simultaneously, at one point, the effect is the same as if a single force equal to the vector sum of the individual forces were acting.



Moment of a force:

Let, $\boldsymbol{F}$ be a force vector applied at P and let O be a fixed point in space. The moment or torque of $\boldsymbol{F}$ about the point O is defined as the vector cross product $\boldsymbol{r}\times\boldsymbol{F}$, where $\boldsymbol{r}$ is the displacement vector from O to P.

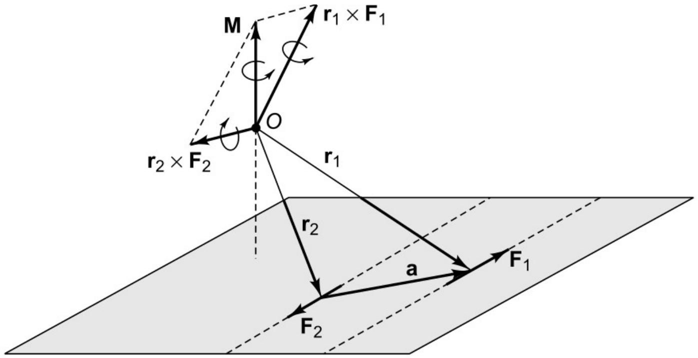


The moment itself is a vector quantity. Its direction is perpendicular to the plane determined by OP and $\boldsymbol{F}$. The sense is fixed by the right-hand rule.

When several forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, ... , $\boldsymbol{F}_n$, act, their total moment or torque about a fixed point O is defined as the sum

$$\boldsymbol{M} = \boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 + \cdots + \boldsymbol{r}_n \times \boldsymbol{F}_n = \sum_j \boldsymbol{r}_j \times \boldsymbol{F}_j. \qquad \text{............................}(1)$$

Couple:

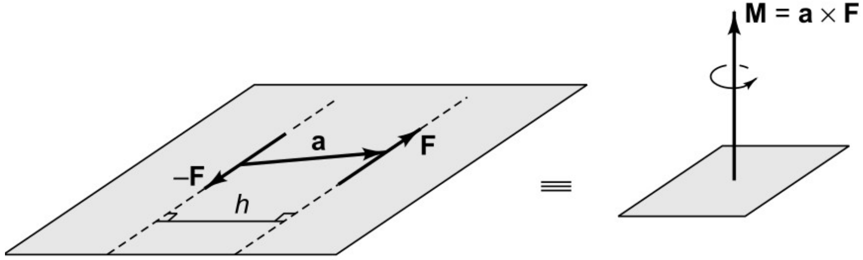


When there are two equal and parallel forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ which have opposite sense, as shown in Figure. Such a configuration of forces is called a ***couple***. It can be shown that the moment of these forces about O will be given as

$$\boldsymbol{M} = \boldsymbol{a} \times \boldsymbol{F}, \qquad \text{.....................}(2)$$

where $\boldsymbol{a}$ is a displacement vector going from an arbitrary point on $\boldsymbol{F}_2$ to an arbitrary point on $\boldsymbol{F}_1$. In fact the moment is **independent of point *O***, which means that the moment remain same about any other point in space. The magnitude of moment is simply defined as $h|\boldsymbol{F}|$, where h is the perpendicular distance between the vectors $\boldsymbol{F}$ and $-\boldsymbol{F}$.

The moment is represented as a vector in the direction of moment and an encircling arrow to avoid any confusion with the force vector.

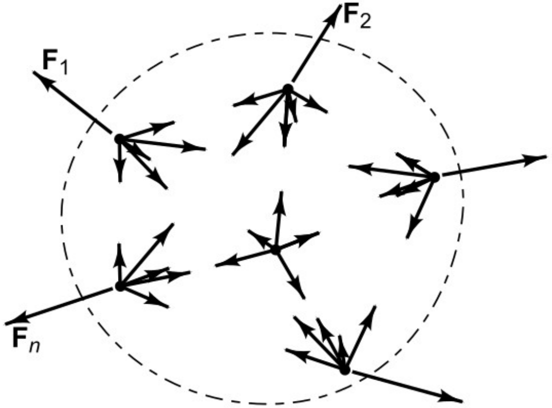


Conditions of equilibrium

According to Newton's law of motion, a particle has no acceleration if the resultant force acting on it is zero and such a particle is said to be in equilibrium. A system which is in equilibrium moves with a constant velocity. We will be considering cases where velocity of the system is zero (i.e. at rest). The study of forces in systems at rest is called ***statics***. If several forces $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, ..., $\boldsymbol{F}_n$ act on a particle, the *necessary and sufficient condition* for the particle to be in ***equilibrium*** is

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots + \boldsymbol{F}_n = \sum_j \boldsymbol{F}_j = 0. \qquad \cdots\cdots\cdots\cdots(3)$$

For an isolated system of particles, it can be shown that for the system to be in equilibrium following are *necessary conditions*:



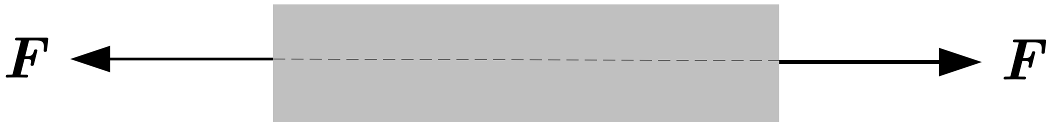
(i) the vector sum of the external forces must be zero i.e.,

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots + \boldsymbol{F}_n = \sum_j \boldsymbol{F}_j = 0. \qquad \cdots\cdots\cdots\cdots(4)$$

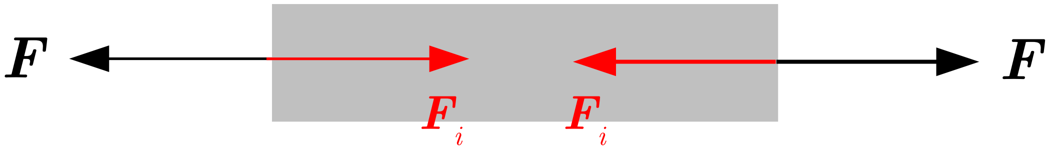
(ii) the total moment of all the external forces about an arbitrary point O must be zero, i.e.,

$$\boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 + \cdots + \boldsymbol{r}_n \times \boldsymbol{F}_n = \sum_j \boldsymbol{r}_j \times \boldsymbol{F}_j = 0. \qquad \cdots\cdots\cdots\cdots(5)$$

Is this system in static equilibrium?



Conditions of equilibrium



The system shown, will be in equilibrium only when the internal forces $\boldsymbol{F}_i = \boldsymbol{F}$.

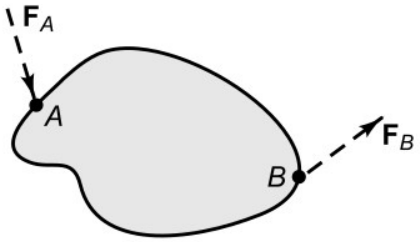
If the system of particles was perfectly rigid so that no pair of particles could separate, the internal forces might automatically adjust themselves so as to provide internal equilibrium whenever the external forces make up an equilibrium set.

Thus, the *necessary and sufficient conditions* for a perfectly rigid body to be in equilibrium are that the set of external forces acting **on the system** should satisfy (4) and (5).

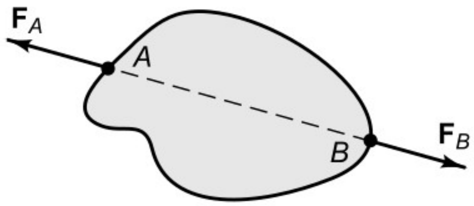
However, a *necessary and sufficient condition* for the **equilibrium of a deformable system** is that the sets of external forces which act **on the system and on every possible subsystem isolated out of the original system** should satisfy both (4) and (5).

Two force member:

Using (4) and (5) it can be very easily shown (exercise for you) that, if a system is in equilibrium under the action of only two external forces applied at A and B. The two forces cannot have random orientation, as shown in figure (a), but must be directed along AB (Fig. b).



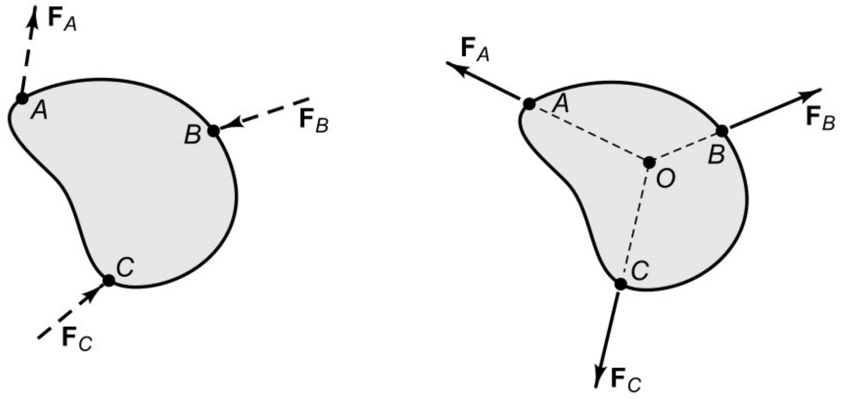
(a)



(b)

Three force member:

If a system is in equilibrium under the action of only three external forces applied at A, B, and C, the three forces cannot have random orientation, as shown in figure (a). It can be shown (exercise for you) that to satisfy equilibrium conditions (5), **they must all lie in the plane** ABC if the total moment about each of the points A, B, and C is to vanish. Further more, **they must all intersect at a common point** $\boldsymbol{O}$, otherwise the total moment about the intersection of any two of the lines of action could not vanish.



(a) (b)

Think about the limiting case, when point O moves off at great distance from A, B, and C.

Engineering applications

Many practical engineering problems involve structures or machines in equilibrium. Certain forces, usually loads, are specified, and it is necessary to determine the reactions which come into play to balance the loads. The general method of analysis involves following steps:

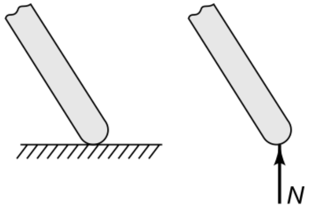
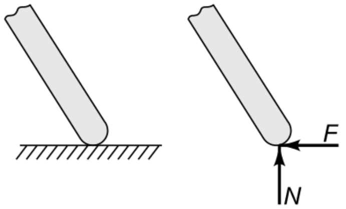
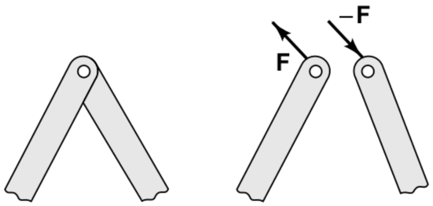
1. Selection of system
2. Idealization of system characteristics

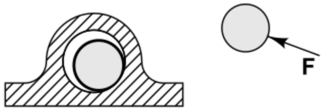
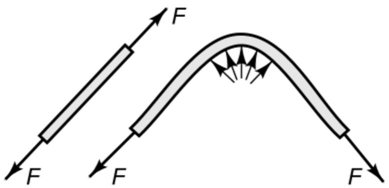
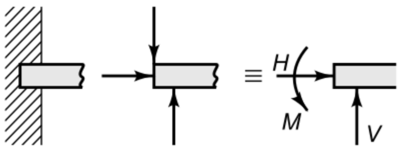
These are followed by an analysis based on the principles of mechanics, which includes the following steps:

1. Study of forces and equilibrium requirements
2. Study of deformation and conditions of geometric fit
3. Applications of force-deformation relations

In some systems it is possible to determine all the forces involved by using only the equilibrium requirements without regard to the deformations (i.e., step 2). Such systems are called **statically determinate**.

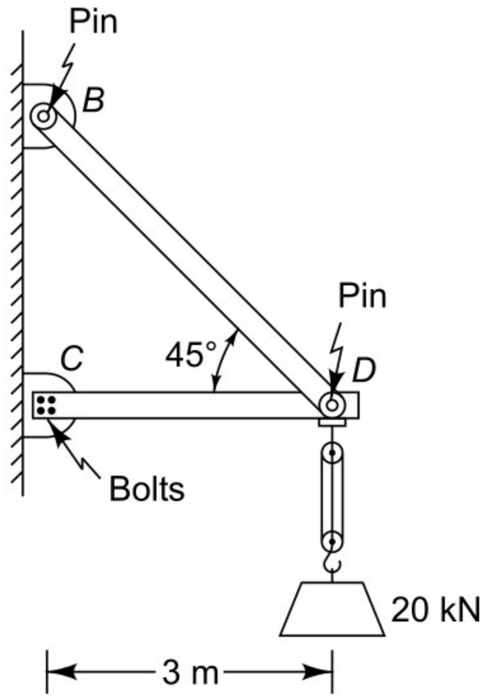
A very important and the first step towards analyzing a system drawing a **free body diagram (FBD).** A FBD is the sketch of the isolated system (or subsystem) and all the external forces acting on it. Depending upon the complexity of the system, a single or many isolation may be required for complete analysis. Following table shows the force-transmitting properties of several mechanical elements.

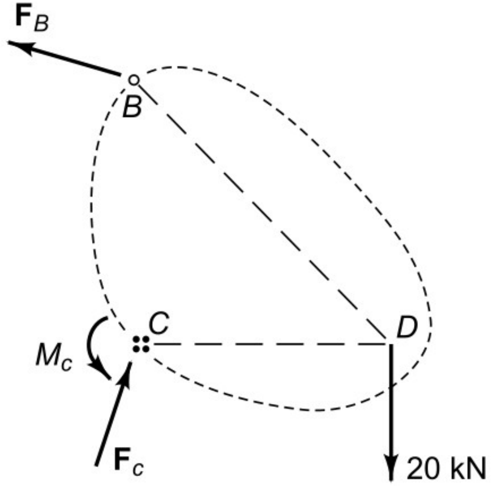
	(a) A *frictionless* surface can exert only a normal contact force N.
	(b) When there is *friction* the surface can exert a tangential force F as well as a normal force N. The force F assumes *any* value necessary to prevent motion up to a maximum value $F_s = f_s N$, where f_s, is the *coefficient of friction.*
	(c) A *frictionless pinned joint* transmits a force **F** which passes through the pin. No torque about the pin is transmitted.

	(d) A *frictionless bearing* exerts a force **F** on the shaft, which passes through the center of the shaft. No torque about the shaft is transmitted.
	(e) A weightless *flexible string* or *cable* transmits force along its length. Each element is subjected to equal and opposite tensile forces F along the string. Compressive forces cannot be sustained. If the string passes over a frictionless peg or pulley, the direction of the force in the string is altered but its magnitude remains constant.
	(f) An ideal *clamped* support provides complete restraint against longitudinal or transverse motion and against rotation. It can supply force reactions H and V and a moment reaction M.

Example 1

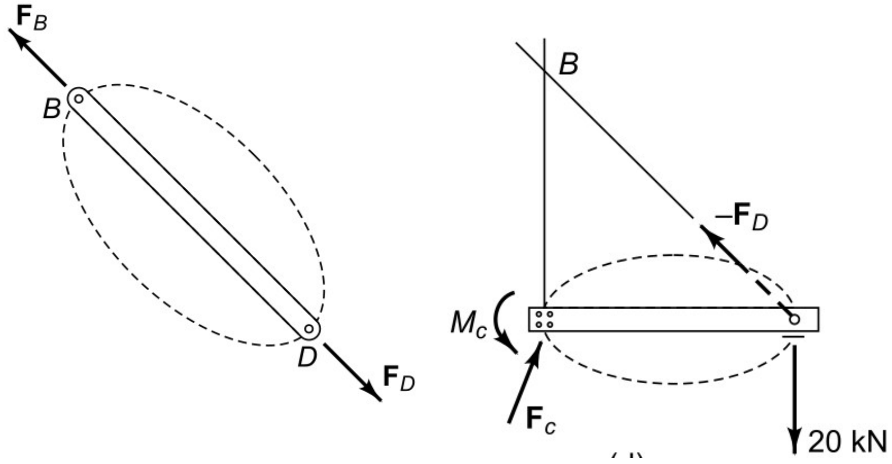
The simple triangular frame shown in figure is used to support a small chain hoist. The chain hoist is supporting its rated capacity of 20 kN. The rod BD is pinned at its ends. The member CD is pinned at D and secured with four bolts at C. Predict the forces acting on the wall at B and C?





As a very first step isolate the frame from wall support B and C, and apply appropriate reactions. Here, we idealize the links to be massless.

It can be observed that there are 5 scaler unknowns and only 3 scalar equations are available to solve them. Since, we cannot obtain a complete solution at this stage, we further isolate subsystems (i.e., links) as shown.



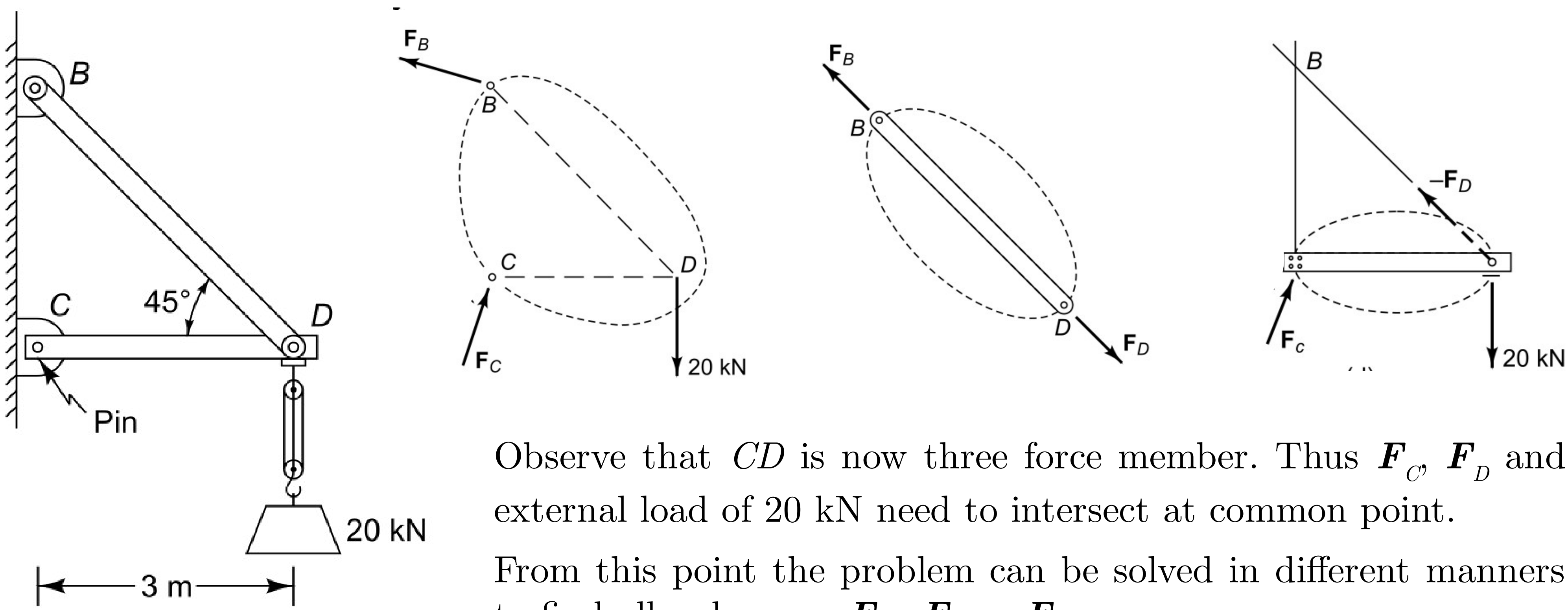
Observe that link BD is a two force member. Hence, both $\boldsymbol{F}_B$ and $\boldsymbol{F}_D$ will act along BD, have same magnitude and opposite directions. Thus,

$\boldsymbol{F}_B = -\boldsymbol{F}_D.$

Consider link CD now. Direction of FD is known, however there are still four scalar unknowns with three equilibrium equations. Thus, we can not solve this problem with only equilibrium equations.

This is, in fact, the **statically indeterminate** case.

Now, suppose the joint C to be a pin joint. This avoids the present of moment at point C and we can see that the problem now convert to a statically determinate problem.



Observe that CD is now three force member. Thus $\boldsymbol{F}_C$, $\boldsymbol{F}_D$ and external load of 20 kN need to intersect at common point.

From this point the problem can be solved in different manners to find all unknowns $\boldsymbol{F}_C$, $\boldsymbol{F}_D$ or $\boldsymbol{F}_B$.