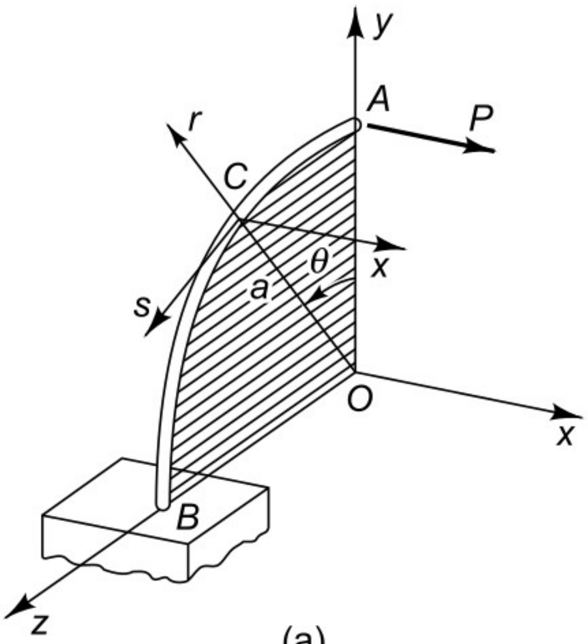
Forces and Moments Transmitted by Slender Members

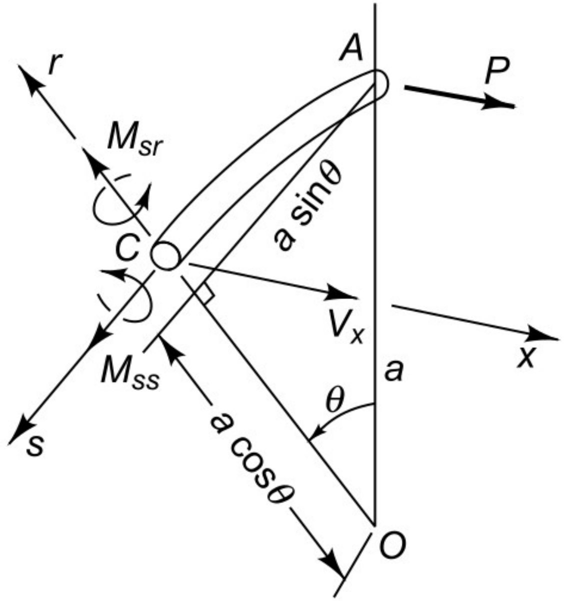
Example 5

A curved slender member AB is anchored at B and bent into a quadrant of a circle of center O and radius a. It is desired to obtain force and moment diagrams for this segment of pipe when a transverse load P is acting as shown.



(a)

It is a three dimensional problem. Basic procedure remain same. We cut a section at any point C. Define the forces and moments at that section and then consider the equilibrium of subsection to determine the forces and moments. As it a curved member, we use a local coordinate system x-r-s along the length of the member. For three-dimensional problems, it will be advisable to use the vector form of equilibrium conditions



FBD of the sub-system along with forces and moments at the section is shown. Applying equilibrium conditions as

$$\sum \boldsymbol{F} = P\boldsymbol{i} + \boldsymbol{V} = 0, \text{ which gives } V_x = -P, V_y = V_z = 0. \qquad \cdots\cdots\cdots\cdots(\text{a.5})$$

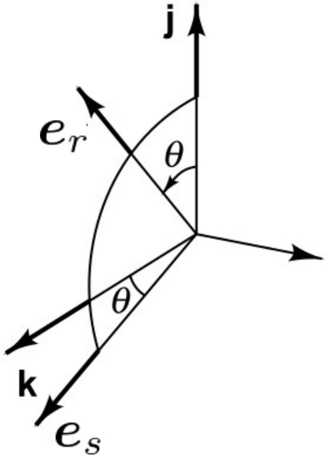
$$\sum \boldsymbol{M}_c = \boldsymbol{r}_{CA} \times P\boldsymbol{i} + M_{sr}\boldsymbol{e}_r + M_{ss}\boldsymbol{e}_s = 0, \qquad \cdots\cdots\cdots\cdots(\text{b.5})$$

where $\boldsymbol{e}_r$ and $\boldsymbol{e}_s$ are unit vectors along r- and s- axis.

Thus, $\boldsymbol{r}_{CA} = a\boldsymbol{j} + a\boldsymbol{e}_r.$ $\cdots\cdots\cdots\cdots$(c.5)

Vector $\boldsymbol{j}$ can be related to $\boldsymbol{e}_r$ and $\boldsymbol{e}_s$ as,

$$\boldsymbol{j} = \boldsymbol{e}_r \cos\theta - \boldsymbol{e}_s \sin\theta. \qquad \cdots\cdots\cdots\cdots(\text{d.5})$$



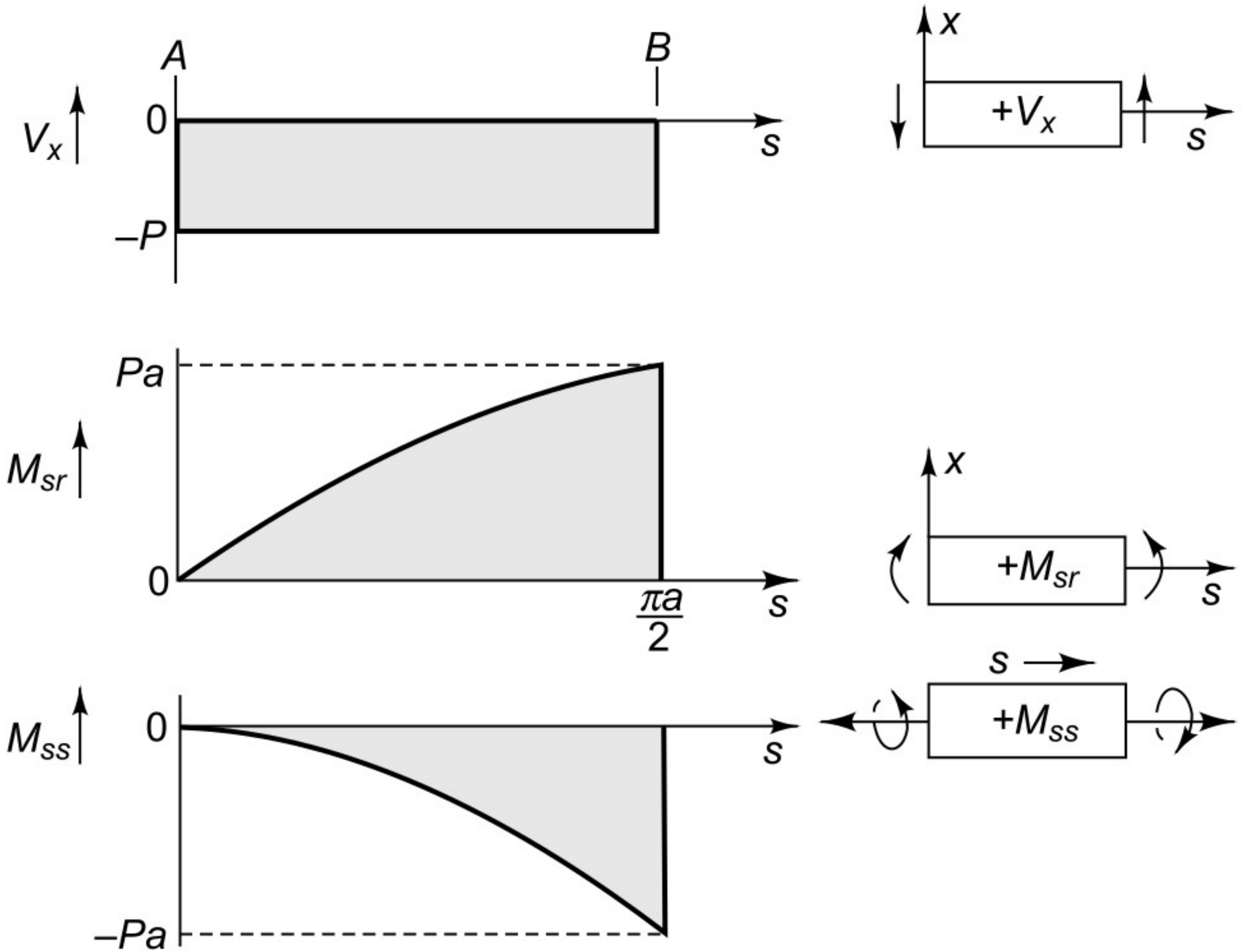
Using (d.5),(c.5) and substituting in (b.5),

$$\sum \boldsymbol{M}_c = \left[a\left(\boldsymbol{e}_r \cos\theta - \boldsymbol{e}_s \sin\theta\right) + a\boldsymbol{e}_r\right] \times P\boldsymbol{i} + M_{sr}\boldsymbol{e}_r + M_{ss}\boldsymbol{e}_s = 0,$$

Simplifying and collecting components,

$$-Pa(1+\cos\theta) + M_{ss} = 0, \quad -Pa\sin\theta + M_{sr} = 0.$$

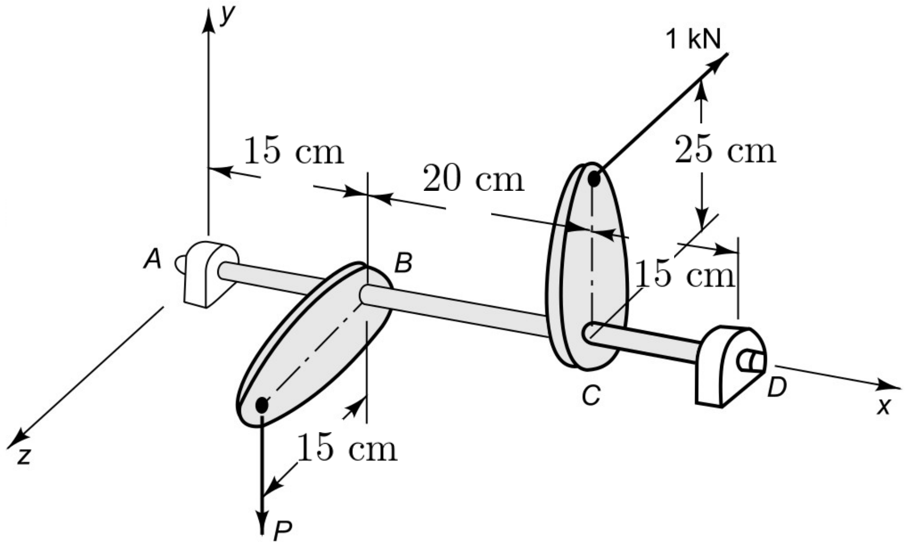
Thus, $M_{ss} = Pa(1+\cos\theta), \quad M_{sr} = Pa\sin\theta.$

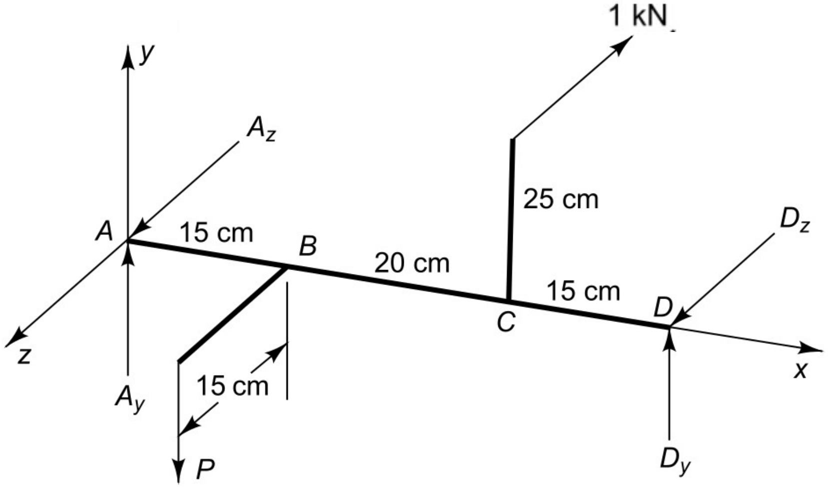


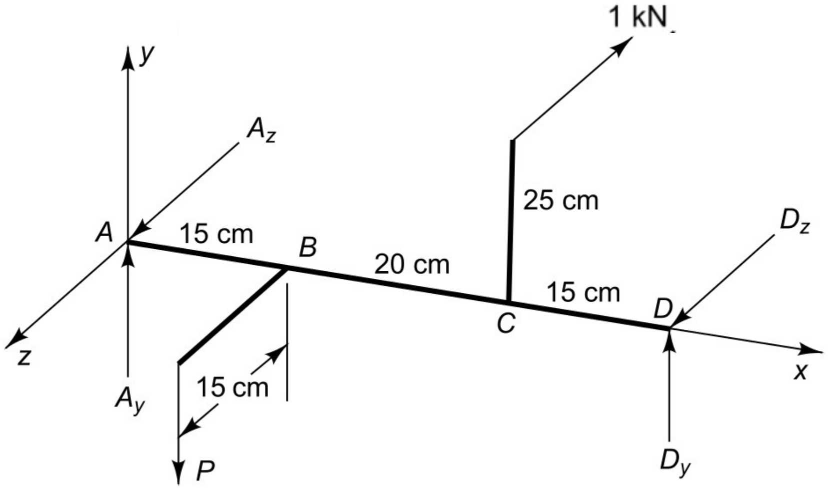
A
B
V_x
0
s
–P
x
$+V_x$
s
Pa
M_{sr}
0
$\frac{\pi a}{2}$
s
x
$+M_{sr}$
s
s
M_{ss}
0
s
$+M_{ss}$
–Pa

Example 6

Consider the offset bell-crank mechanism. A shaft supported in journal bearings at A and D is loaded as shown and has offset links attached at B and C. The problem is to obtain diagrams showing the variation of shear force, bending moment, and twisting moment in the shaft AD.







Applying equilibrium conditions,

$$\sum F_y = A_y + D_y - P = 0, \qquad \cdots\cdots\cdots\cdots\text{(a.6)}$$

$$\sum F_z = A_z + D_z - 1 = 0, \qquad \cdots\cdots\cdots\cdots\text{(b.6)}$$

$$\sum M_A = -0.15\,(\boldsymbol{k}+\boldsymbol{i})\times(-P\boldsymbol{j}) + (-0.25\boldsymbol{j} - 0.35\boldsymbol{i})\times(-1\boldsymbol{k}) - 0.5\boldsymbol{i}\times(D_z\boldsymbol{k} + D_y\boldsymbol{j}) = 0, \qquad \cdots\cdots\cdots\cdots\text{(c.6)}$$

Now, by simplifying and collecting components, we get,

D_z = 0.7 kN (from c.6)

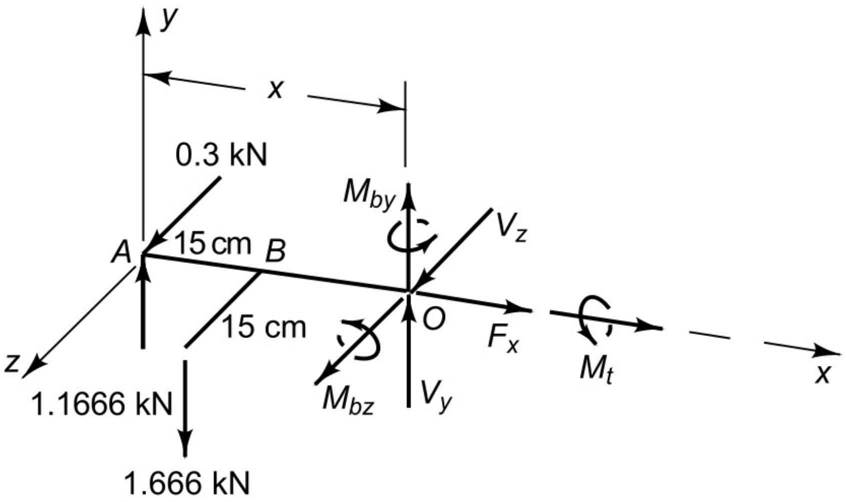
A_z = 0.3 kN (from b.6)

P = 1.666 kN (from c.6)

D_y = -0.5 kN (from c.6)

A_y = 1.666 kN (from a.6)

After determining all support reactions and load P required for equilibrium, now let us consider the equilibrium of sub-sections.

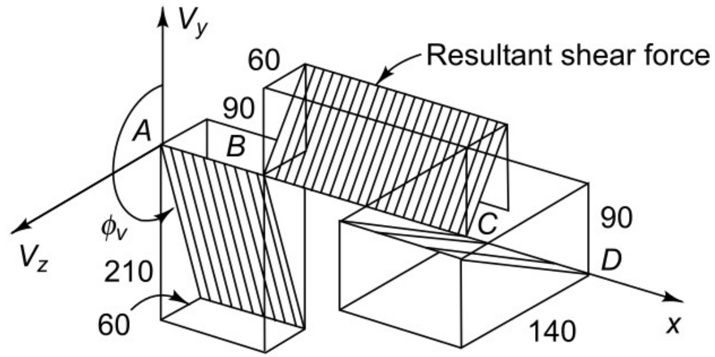


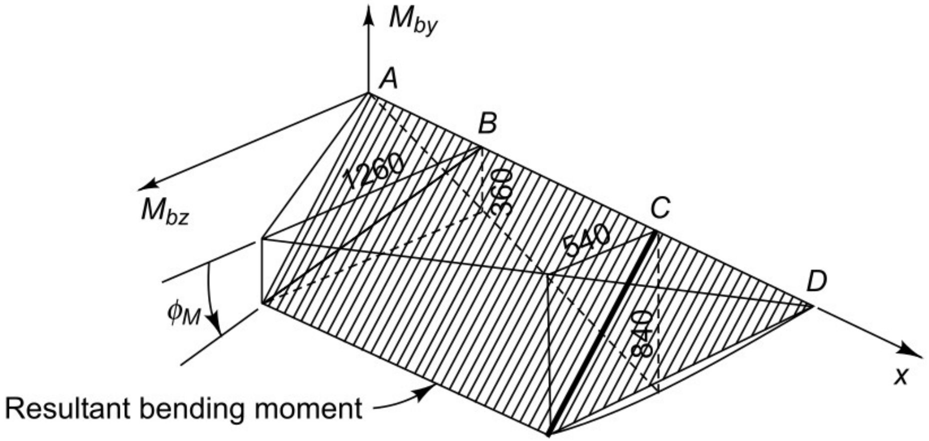
Take a section at a distance $0.35 < x < 0.5$ m. Determine the expressions defining the distribution of all section forces (F_x, V_y, V_z) and moments (M_t, M_{by}, M_{bz}) by applying equilibrium equations. These expressions will be valid for $0.35 < x < 0.5$ m.

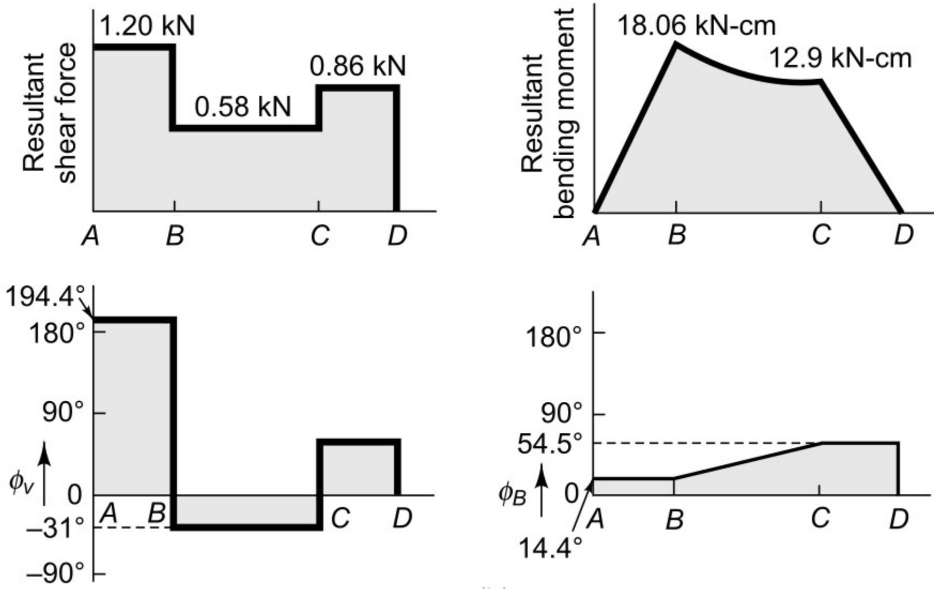
Repeat similar exercise by considering a section between 0 and 0.35 m length.

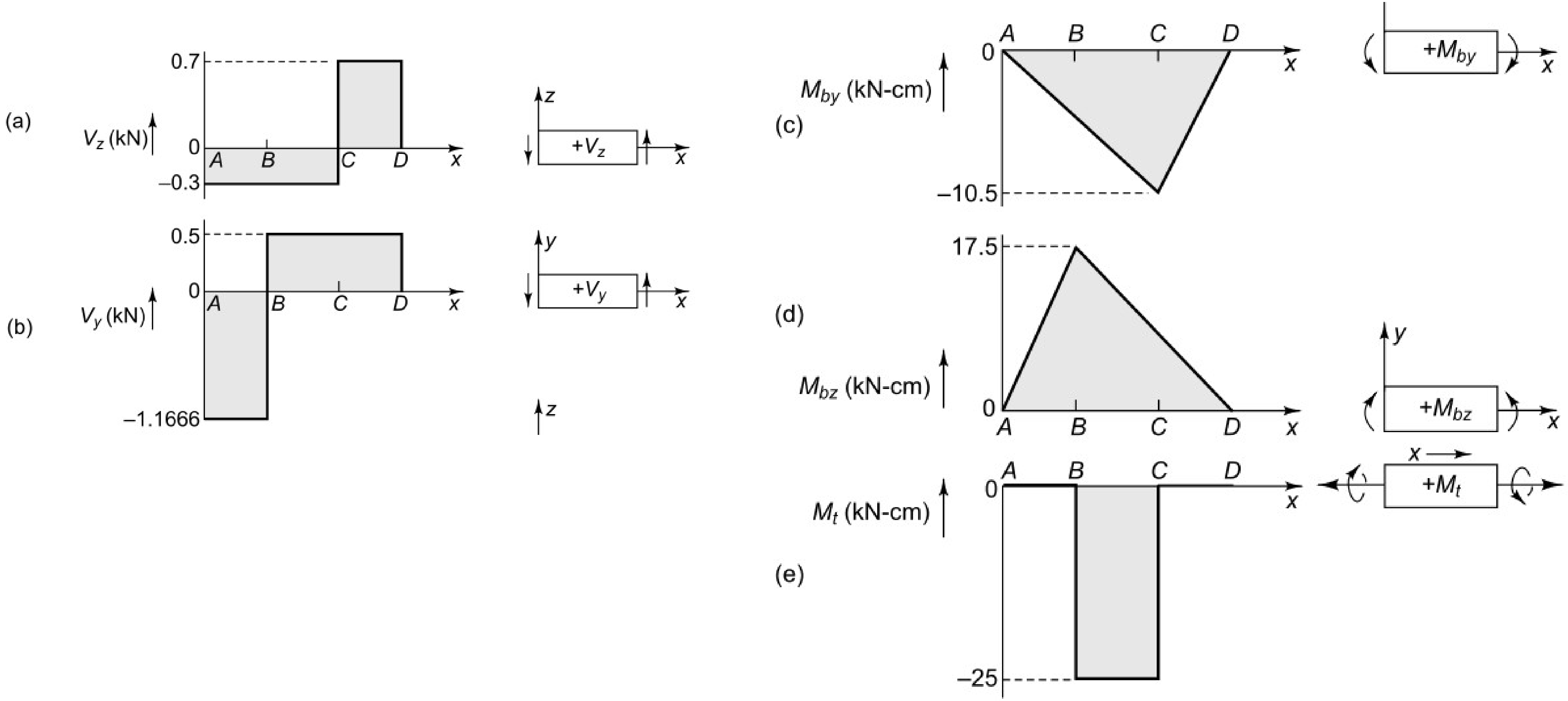
After completion of above exercise we will have all information to draw the distribution of all forces and moments along the length of the shaft.

SFDs and BMDs drawn in the previous slide can be combined to draw resultant shear force and bending moment and their directions as follows.



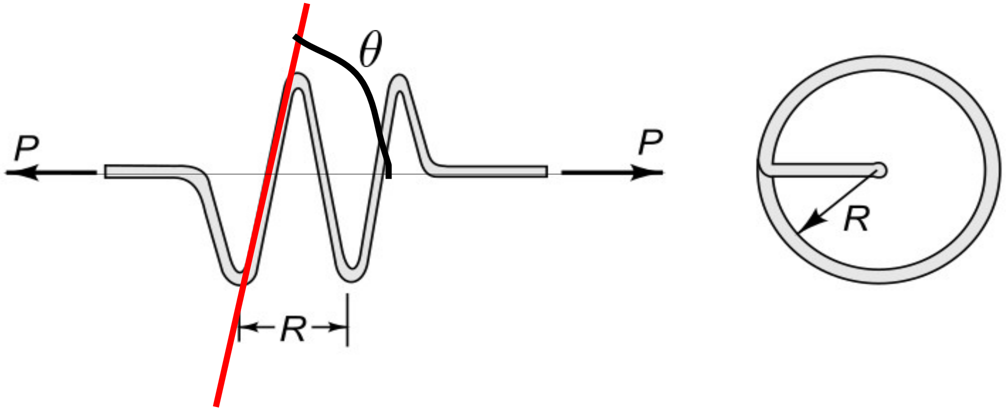





(a)
V_z (kN)
0.7
0
–0.3
A B C D x
$+V_z$
z
x
(b)
V_y (kN)
0.5
0
–1.1666
A B C D x
$+V_y$
y
x
z
(c)
M_{by} (kN-cm)
0
–10.5
A B C D x
$+M_{by}$
x
(d)
M_{bz} (kN-cm)
17.5
0
A B C D x
$+M_{bz}$
y
x
(e)
M_t (kN-cm)
0
–25
A B C D x
$+M_t$
x

Example 7

When the load P is applied, the coil spring shown has a pitch equal to the coil radius. Find the maximum bending and twisting moments in the coil.

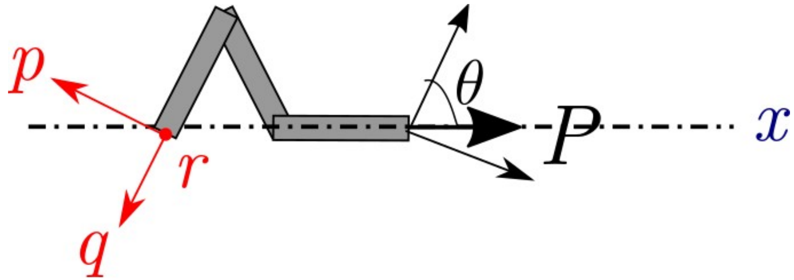


$$\text{Pitch} = R = 2\pi R\ \text{cot}$$

$$\cot\theta = 1/(2\pi)$$

$$\theta = 80.9^\circ$$

Resolve the load P in p and q direction, and apply equilibrium conditions to find the moments.



$$\sum M_r = 0.$$

$$\sum M_p = (M_b)_{\max} - (P\cos\theta)R = 0 \Rightarrow (M_b)_{\max} = PR\cos\theta.$$

$$\sum M_q = (M_t)_{\max} - (P\sin\theta)R = 0 \Rightarrow (M_t)_{\max} = PR\sin\theta.$$