ME231: Solid Mechanics-I

Stress and Strain

Consider the expressions of σ'_{xx} and τ'_{xy} from (25). Eliminating θ from both the equations as,

$$(\sigma'_{xx} - \sigma_m)^2 + (\tau'_{xy})^2 = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2, \qquad \cdots\cdots\cdots\cdots(27)$$

where,

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

This equation represents the equation of a circle, whose center is at,

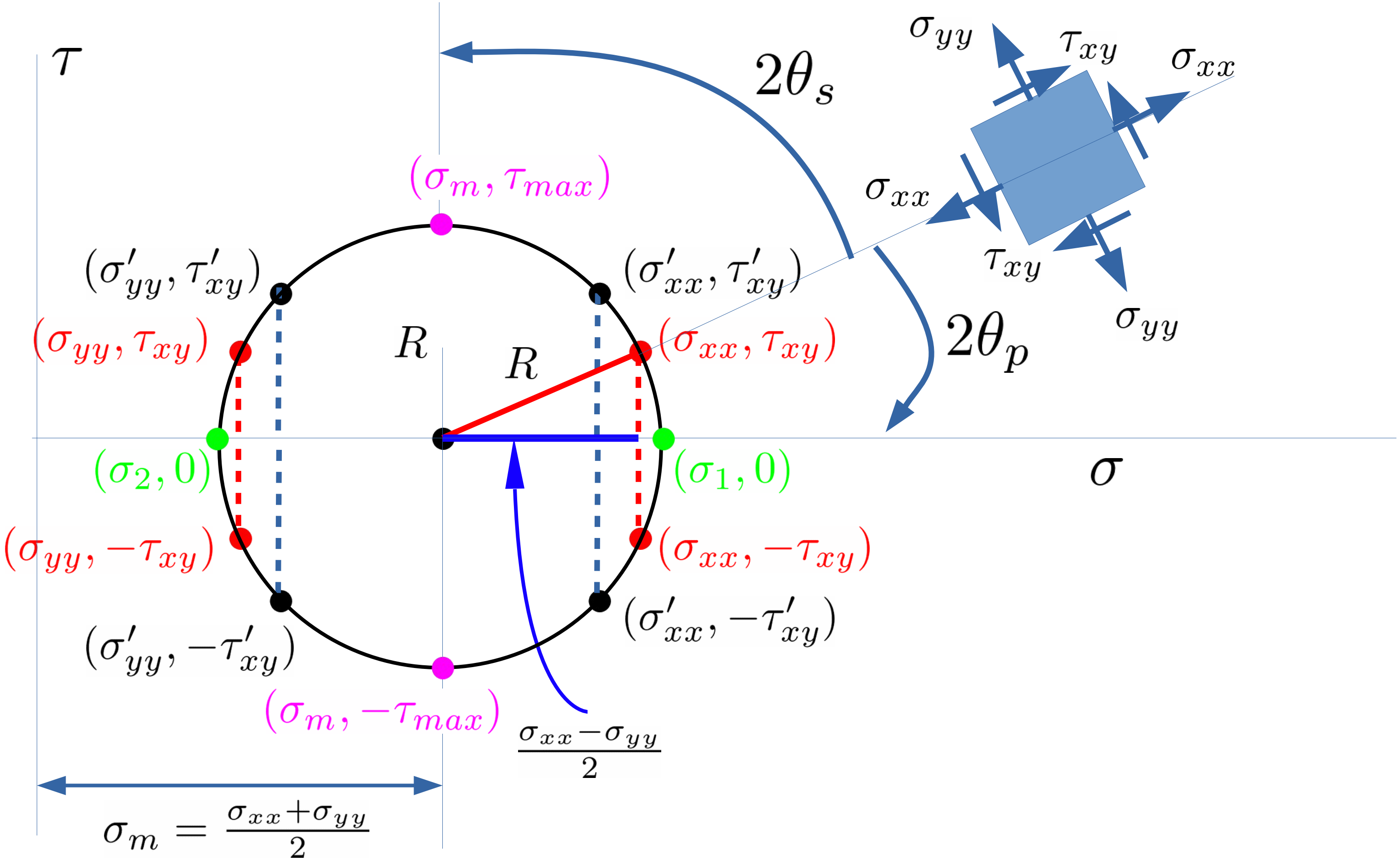
$$(\sigma_m, 0)\,,\ \text{and radius is } R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}. \qquad \cdots\cdots\cdots\cdots(28)$$

Points $(\sigma'_{xx}, \tau'_{xy})$ and $(\sigma'_{xx}, -\tau'_{xy})$ lie on this circle.

Similarly, considering equations for σ'_{yy} and τ'_{xy} and eliminating θ results in the equation representing the same circle and we see that points $(\sigma'_{yy}, \tau'_{xy})$ and $(\sigma'_{yy}, -\tau'_{xy})$ also lie on the same circle.

It should be observed that points (σ_{xx}, τ_{xy}), $(\sigma_{xx}, -\tau_{xy})$, $(-\sigma_{xx}, \tau_{xy})$, and $(-\sigma_{xx}, -\tau_{xy})$, Also satisfy (27), i.e., all these points also lie on the same circle.

Mohr's circle: Graphical method



Principal stresses and maximum shear stress

- From the equations of transformed stresses, we see that the state of stress at a particular plane depends upon its inclination from x-axis (i.e. θ).
- From the equation of τ'_{xy}, we can find the plane at which shear stress will be zero.

$$\tau'_{xy} = \tau_{xy}\cos 2\theta - \frac{(\sigma_{xx} - \sigma_{yy})}{2}\sin 2\theta = 0,$$

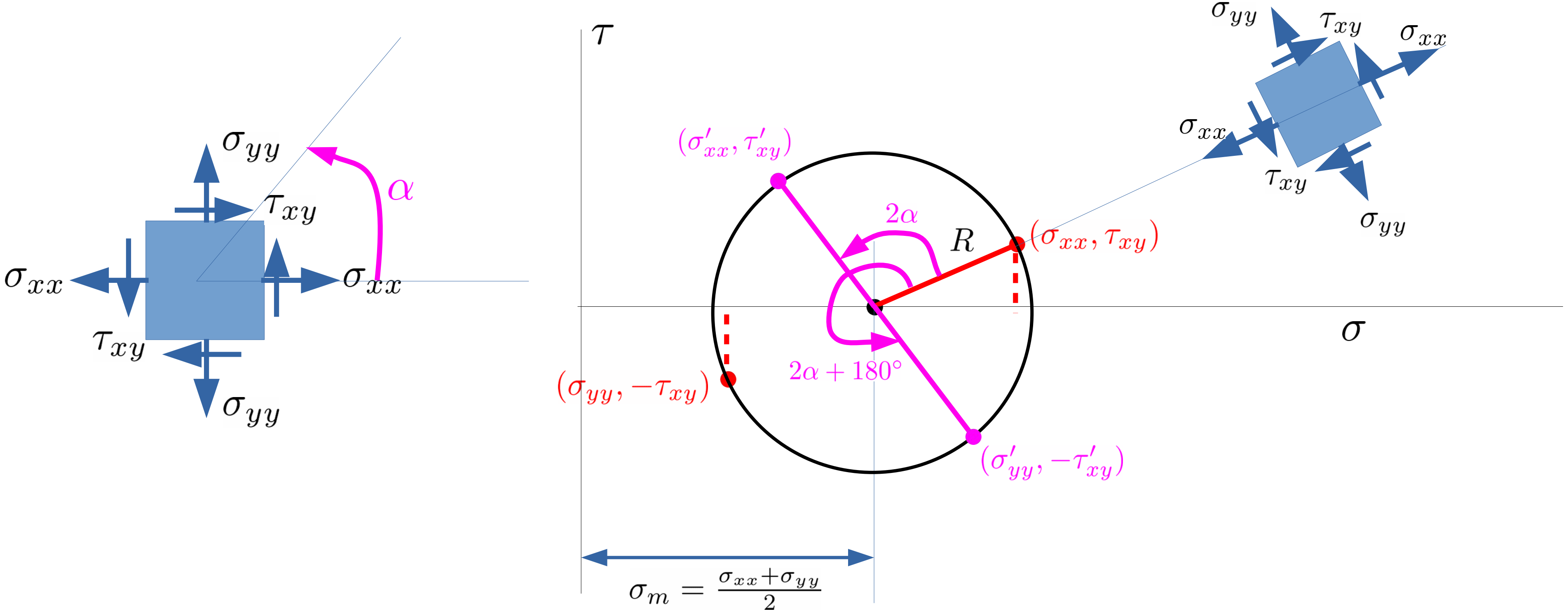
$$\Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

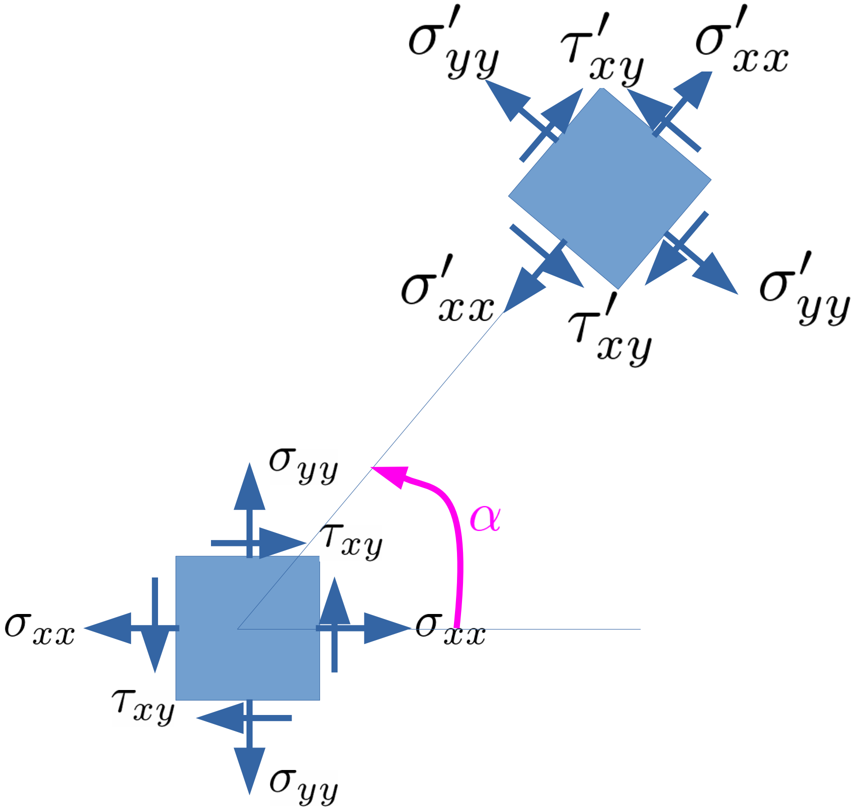
- Planes at which **shear stresses are zero** are called **principal planes** and the normal stresses at these planes are called **principal stresses**.
- It can also be seen that maximum value of shear stress is given as,

$$\tau'_{\max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2},$$ at angle θ satisfying $\tan 2\theta_s = -\frac{\sigma_{xx} - \sigma_{yy}}{2\tau_{xy}}$.

Steps to solve problems using Mohr's circle

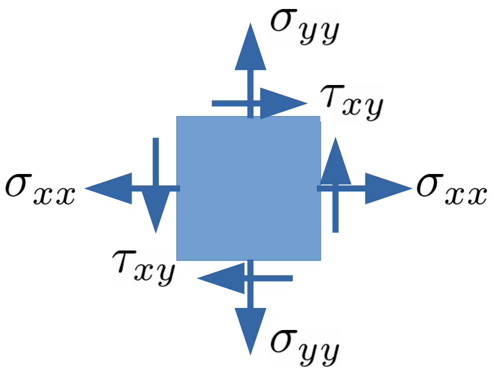
For the given state of stress, find the stresses for an element which is inclined at an angle α from x-axis.

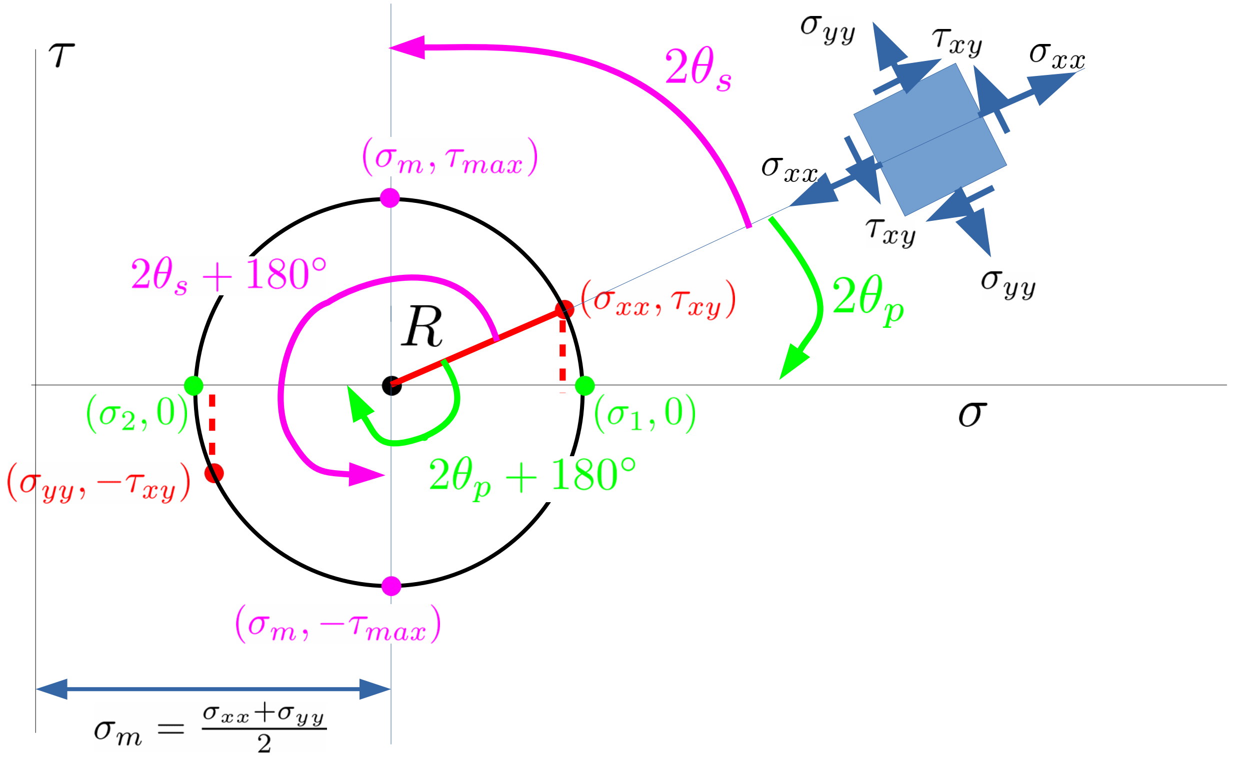


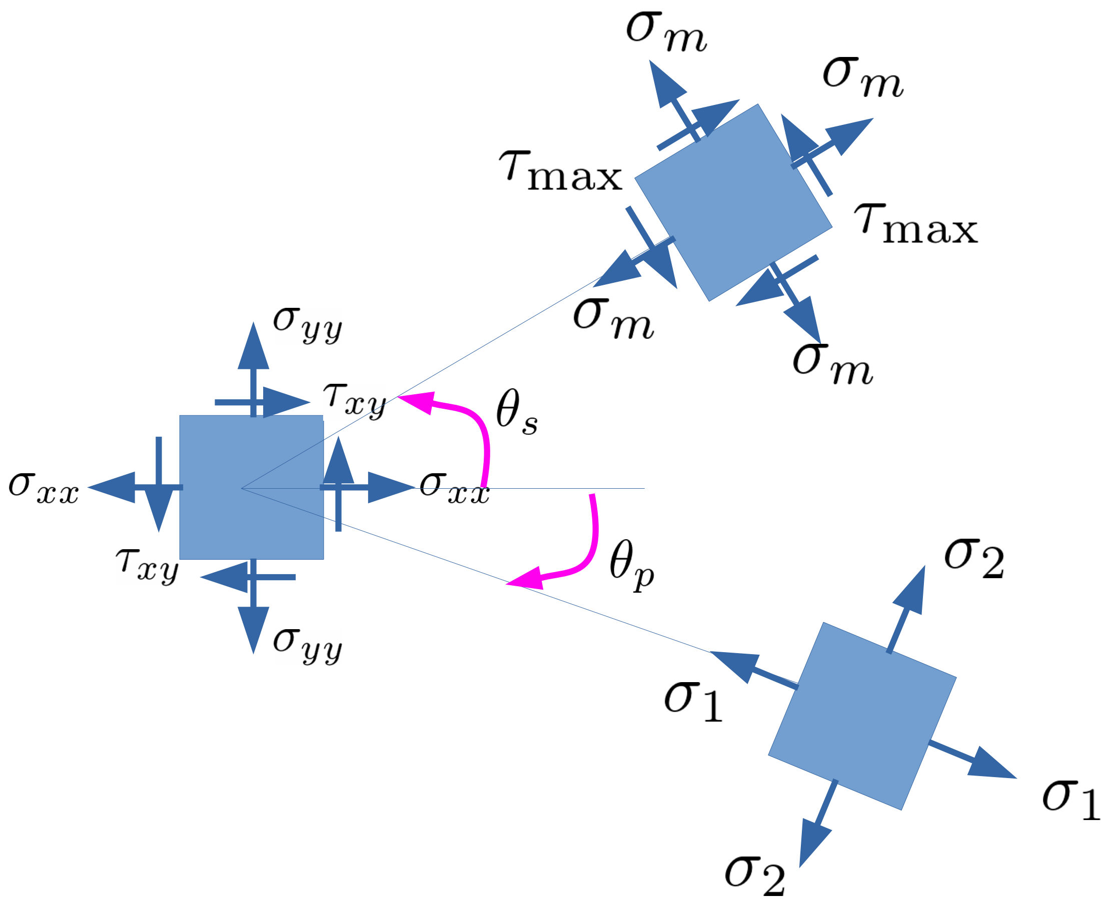


σ'_{yy}
τ'_{xy}
σ'_{xx}
σ'_{xx}
τ'_{xy}
σ'_{yy}
σ_{yy}
τ_{xy}
α
σ_{xx}
σ_{xx}
τ_{xy}
σ_{yy}

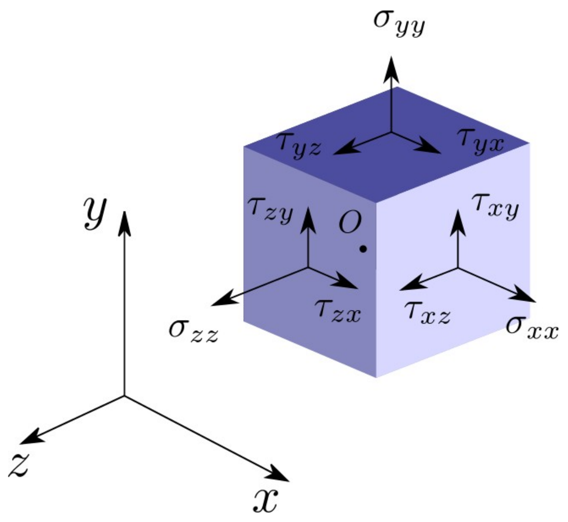
For a given state of stress, find the principal stresses and maximum shear stresses and inclination of principal planes and planes of maximum shear stresses.



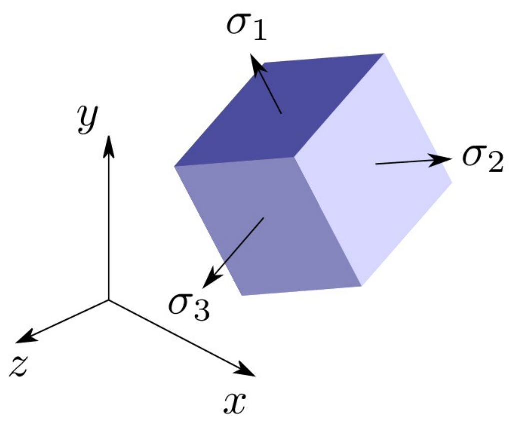


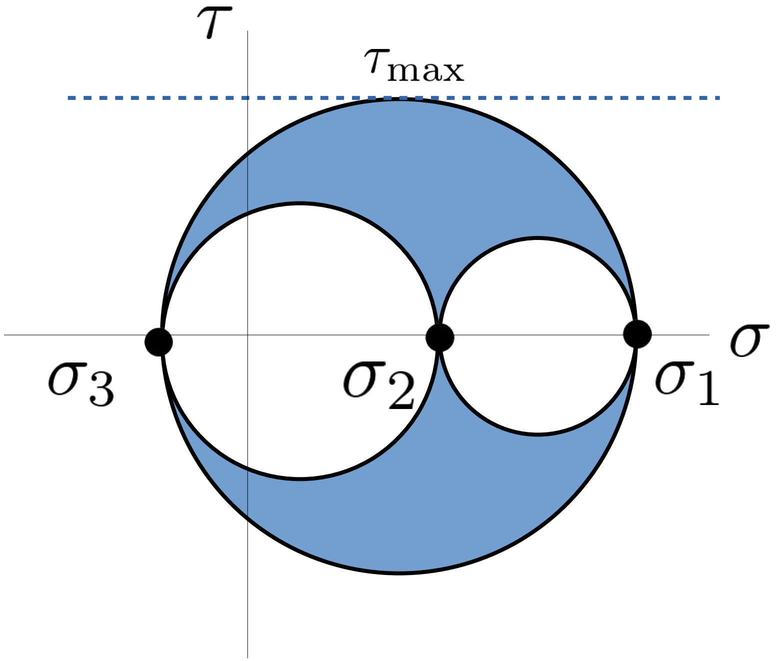

σ_m
σ_m
$\tau_{\max}$
$\tau_{\max}$
σ_m
σ_m
σ_{yy}
τ_{xy}
θ_s
σ_{xx}
σ_{xx}
θ_p
τ_{xy}
σ_{yy}
σ_2
σ_1
σ_1
σ_2

For a general 3D state of stress



For a general three dimensional state of stress there exist three mutually perpendicular plane at which shear stresses are zero. These are called **principal planes** and normal to these planes are called **principal axes**. Normal stresses on these planes are called **principal stresses**.





$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Generally principal stresses are ordered as,

$\sigma_1 > \sigma_2 > \sigma_3$, then

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$$

Examples

Draw the Mohr's circle for following cases.