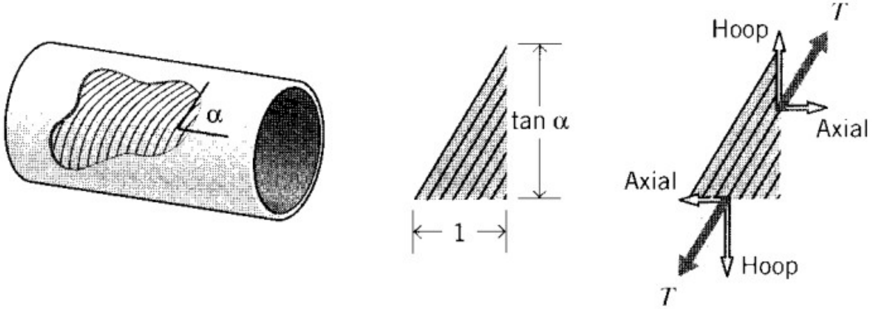
ME231: Solid Mechanics-I

Stress and Strain

Example 6

Consider a cylindrical pressure vessel to be constructed by filament winding, in which fibers are laid down at a prescribed helical angle α. Taking a free body of unit axial dimension along which n fibers transmitting tension T are present, the circumferential distance cut by these same n fibers is then tan α. To balance the hoop and axial stresses, the fiber tensions must satisfy the relations



$$nT\sin\alpha = \frac{pr}{t}\cdot 1\cdot t$$

$$nT\cos\alpha = \frac{pr}{2t}\cdot \tan\alpha\cdot t$$

Dividing the first of these expressions by the second and rearranging, we have

$$\tan^2\alpha = 2 \quad \text{or} \quad \alpha = 54.7^\circ.$$

This is the angle for filament wound vessels, at which the fibers are inclined just enough toward the circumferential direction to make the vessel twice as strong circumferentially as it is axially.

Firefighting hoses are also braided at this same angle, since otherwise the nozzle would jump forward or backward when the valve is opened and the fibers try to align themselves along the correct direction.

Example 7

For a slender beam, the equilibrium equations were derived as,

$$\frac{dV}{dx} + q = 0 \quad \text{and} \quad \frac{dM}{dX} + V = 0.$$

We will prove these equations again using general equilibrium equations derived in this chapter, i.e.,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \cdots\cdots\cdots\cdots(7\text{a})$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0. \qquad \cdots\cdots\cdots\cdots(7\text{b})$$

Integrate (7b) in the thickness direction (i.e., y-direction) as,

$$\int_{y=-h/2}^{y=h/2} \left(\frac{\partial \tau_{xy}}{\partial x} dy + \frac{\partial \sigma_{yy}}{\partial y} dy \right) = 0, \quad \Rightarrow \frac{\partial}{\partial x} \int_{y=-h/2}^{y=h/2} \tau_{xy} dy + \int_{y=-h/2}^{y=h/2} d\sigma_{yy} = 0$$

$$\Rightarrow \frac{\partial V}{\partial x} + \sigma_{yy}\big|_{(y=h/2)} - \sigma_{yy}\big|_{(y=-h/2)} = 0, \quad \text{where,} \quad V = \int_{y=-h/2}^{y=h/2} \tau_{xy} dy, \quad q = \sigma_{yy}\big|_{(y=h/2)} \text{ and } \sigma_{yy}\big|_{(y=h/2)=0}\,.$$

Thus, $$\frac{\partial V}{\partial x} + q = 0.$$

Now, multiply (7a) with y and integrate in the thickness direction (i.e., y-direction) as,

$$\int_{y=-h/2}^{y=h/2} \left(\frac{\partial \sigma_{xx}}{\partial x} y dy + \frac{\partial \tau_{xy}}{\partial y} y dy \right) = 0, \Rightarrow \frac{\partial}{\partial x} \int_{y=-h/2}^{y=h/2} \sigma_{xx} y dy + \int_{y=-h/2}^{y=h/2} \left(\frac{\partial \tau_{xy}}{\partial y} y \right) dy = 0,$$

$$\Rightarrow -\frac{\partial M}{\partial x} + \int_{y=-h/2}^{y=h/2} \left(\frac{\partial \tau_{xy}}{\partial y} y \right) dy = 0, \quad \text{where} \quad -M = \int_{y=-h/2}^{y=h/2} \sigma_{xx} y dy,$$

$$\Rightarrow -\frac{\partial M}{\partial x} + y\tau_{xy}\Big|_{y=-h/2}^{y=h/2} - \int\limits_{y=-h/2}^{y=h/2} \tau_{xy} dy = 0, \quad \text{where,} \quad V = \int\limits_{y=-h/2}^{y=h/2} \tau_{xy} dy, \text{ and } \; y\tau_{xy}\big|_{y=-h/2} = y\tau_{xy}\big|_{y=h/2} = 0.$$

Thus, $$\frac{\partial M}{\partial x} + V = 0.$$

Stress, Strain and Temperature relationship

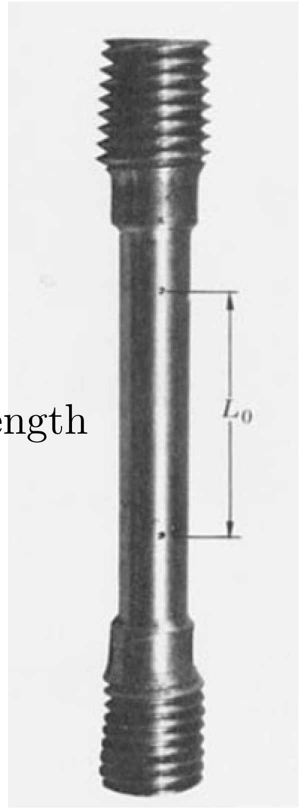
Introduction

- In the previous chapter, we derived three equations (for 3D) of equilibrium for the six components of stress.
- In addition to that there are three components of displacements in the six equations relating strain to displacement.
- Thus, to determine the distributions of stress and strain in a body more equations are required.
- The distribution of stress and strain will depend on the material behavior of the body.
- In this chapter we shall discuss the relations between stress and strain (constitutive law).
- Different materials follow different relationship between stress and strain, and development of constitutive model for materials is an active field of research.

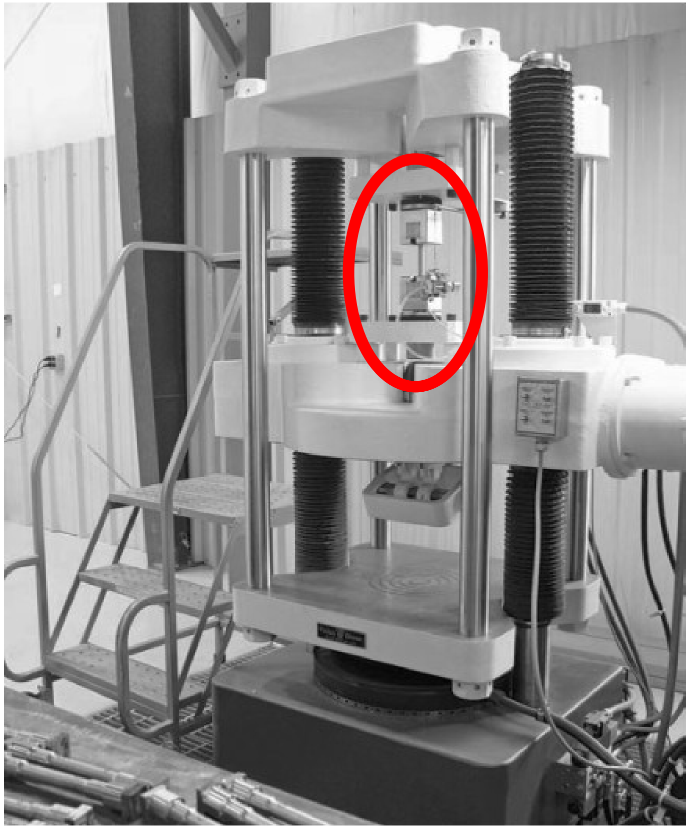
Tensile test

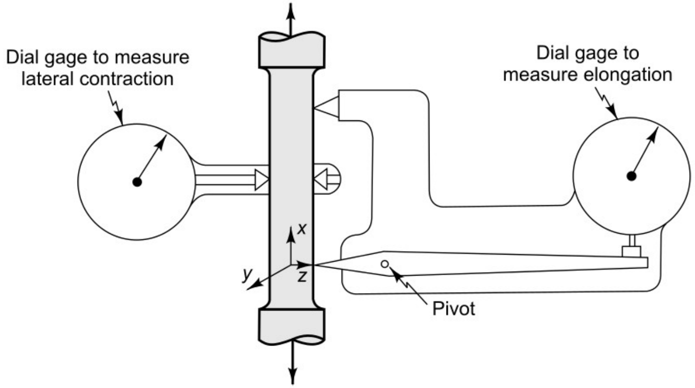
Tensile test specimen

Gage length



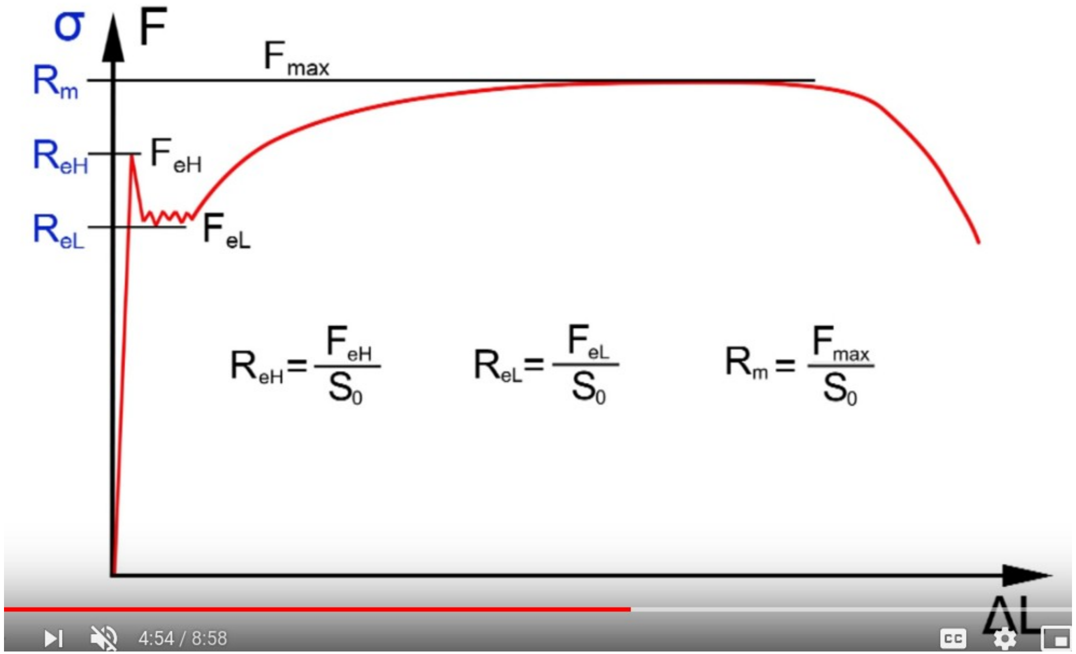
Universal Testing Machine (UTM)

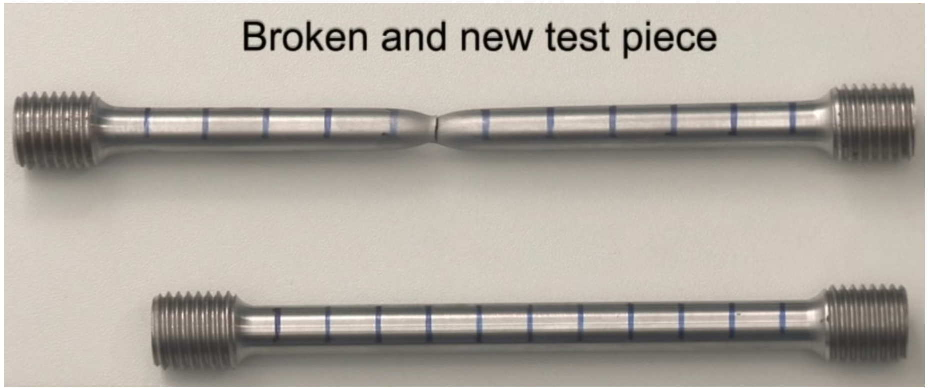




Tensile test

https://www.youtube.com/watch?v=D8U4G5kcpcM

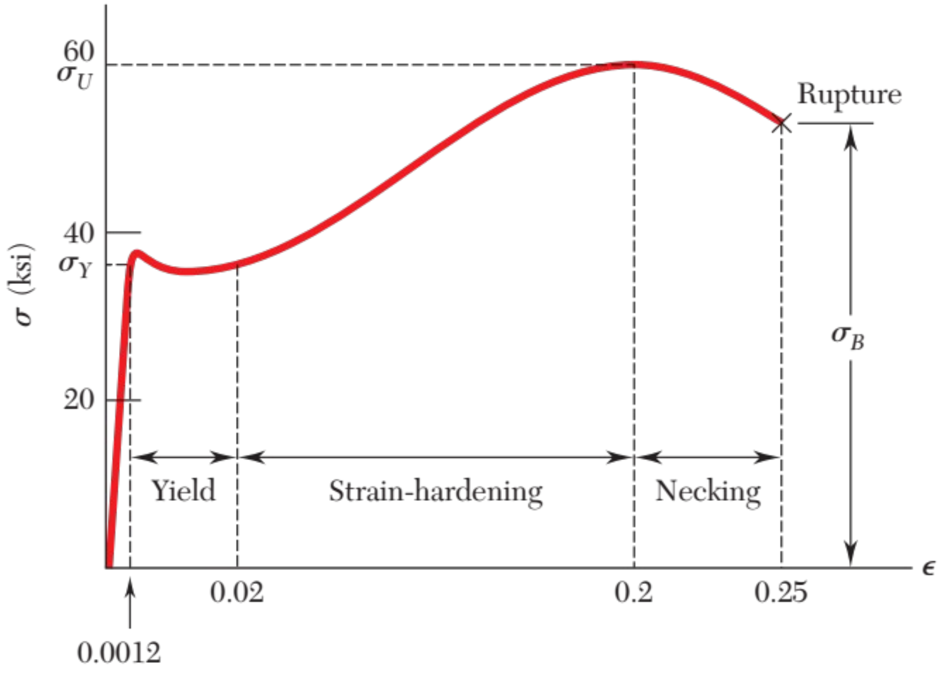




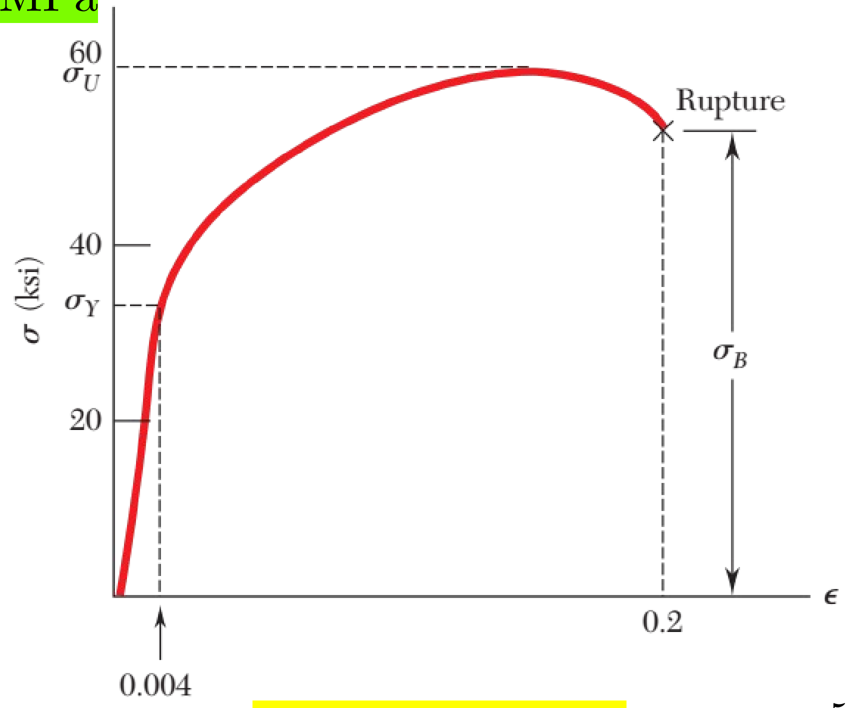
Stress-strain curve

$$\text{Engineering Stress } \sigma = \frac{P}{A_0}, \text{ Engineering Strain } \epsilon = \frac{\delta}{L_0}.$$

1 ksi = 6.89476 MPa

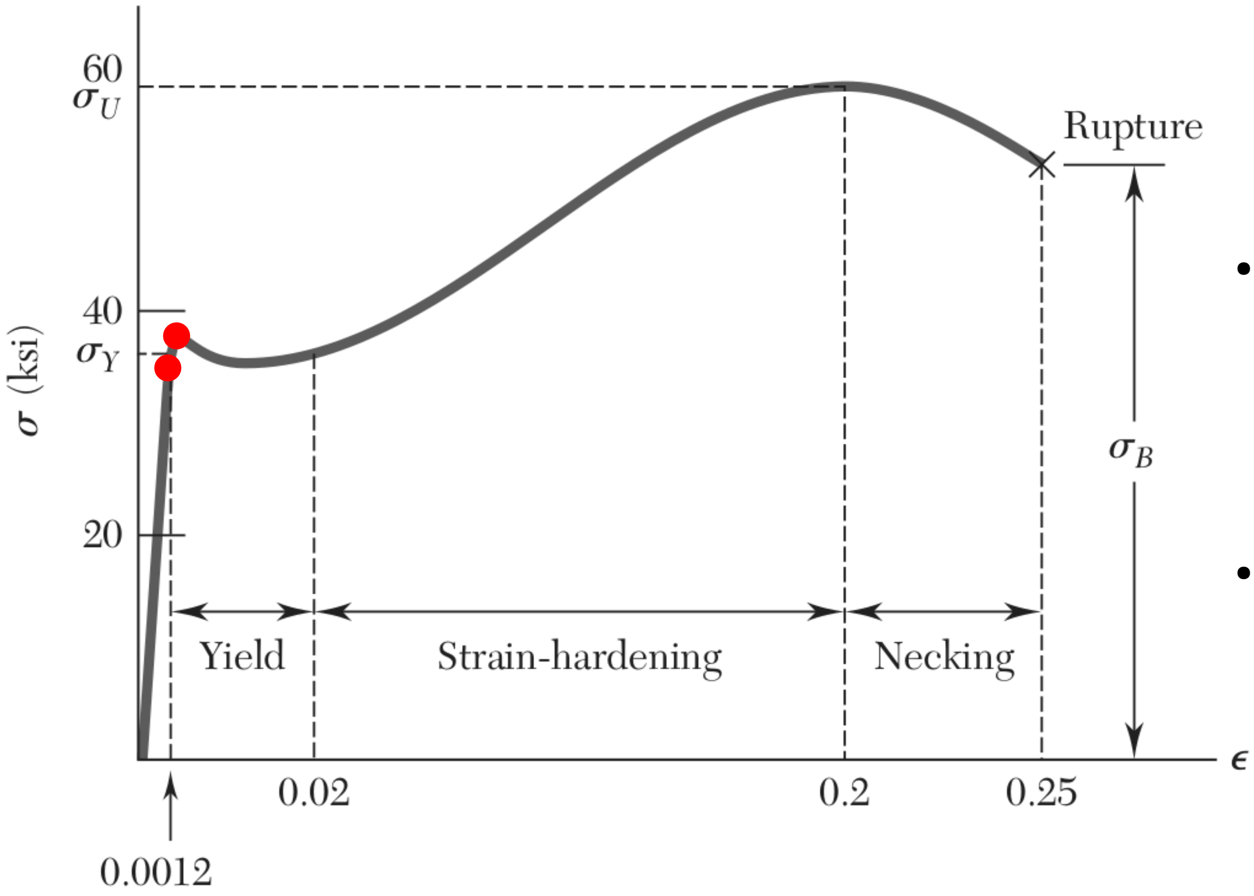


Low carbon steel

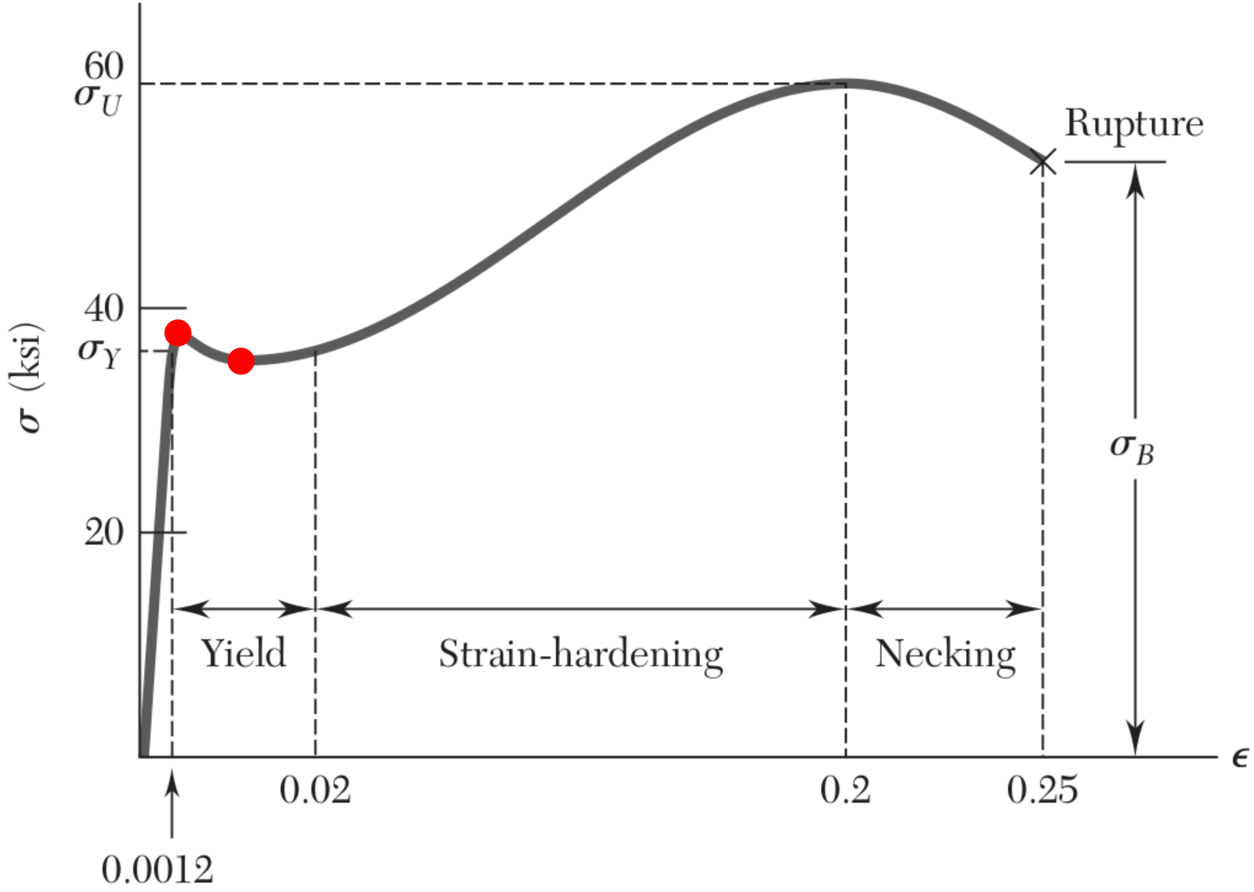


Aluminum alloy

Features of stress-strain curve



- The very first region is where stress is linearly proportional to the strain. The proportional limit is defined as the maximum stress upto which this proportionality exists.
- The elastic limit is defined as the maximum stress up to which material behaves elastically, i.e., there is no permanent strain on release of stress.
- However, neither the proportional nor the elastic limits can be determined precisely. They deal with the limiting cases of zero deviation from linearity and of no permanent set.



- Since plastic deformations of the order of the elastic strains are often unimportant, instead of reporting the elastic limit it has become standard practice to report a quantity called the yield strength, which is the stress required to produce a certain arbitrary plastic deformation.
- For many of the common steels the plastic deformation begins abruptly, resulting in an increase of strain with no increase, or perhaps even a decrease, in stress.

- For such materials, the stress at which plastic deformation first begins is called the upper yield point; subsequent plastic deformation may occur at a lower stress, called the lower yield point.