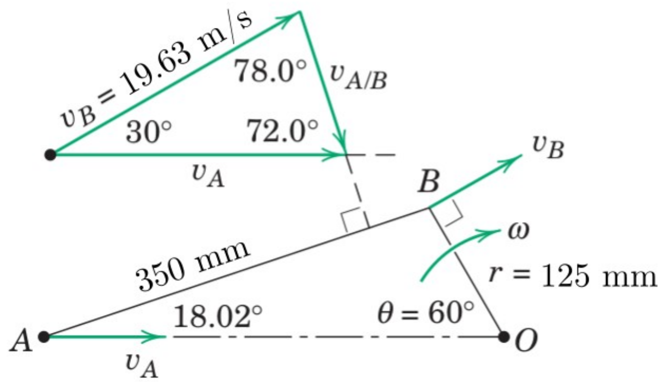
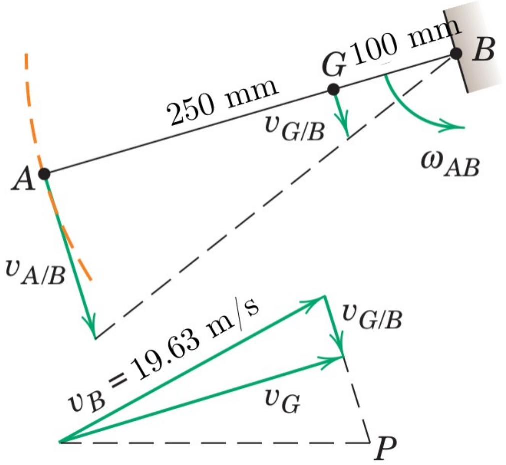
ME232: Dynamics

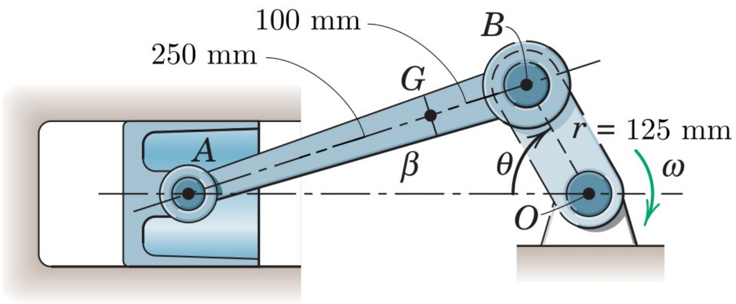
Anshul Faye
afaye@iitbhilai.ac.in
Room # 106

Example 4

The common configuration of a reciprocating engine is that of the slider-crank mechanism shown. If the crank OB has a clockwise rotational speed of 1500 rev/min, determine for the position where $\theta = 60°$ the velocity of the piston A, the velocity of point G on the connecting rod, and the angular velocity of the connecting rod.



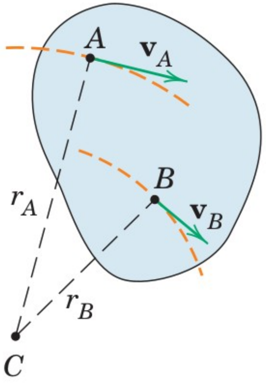




Instantaneous center of zero velocity

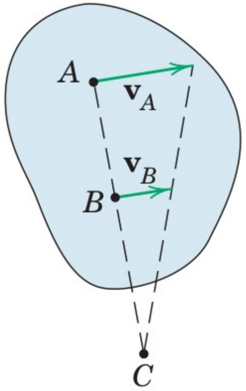
- We determined the velocity of a point on a rigid body in plane motion by adding the relative velocity due to rotation about a convenient reference point to the velocity of the reference point.
- We now solve the problem by choosing a unique reference point which **momentarily has zero velocity.**
- As far as velocities are concerned, the body may be considered to be in pure rotation about an axis, normal to the plane of motion, passing through this point. This axis is called the **instantaneous axis of zero velocity**, and the intersection of this axis with the plane of motion is known as the **instantaneous center of zero velocity**.
- This approach provides a valuable means for visualizing and analyzing velocities in plane motion.

Assume that the directions of the absolute velocities of any two points A and B on the body are known and are not parallel.

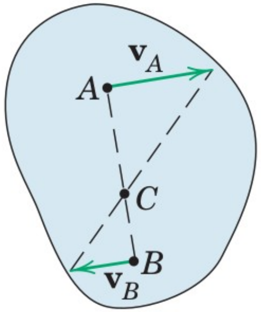


The point about which A has absolute circular motion at the instant considered must lie on the normal to $\boldsymbol{v}_A$ through A. Similarly for B too. The intersection of the two perpendiculars fulfills the requirement for an absolute center of rotation **at the instant considered.**

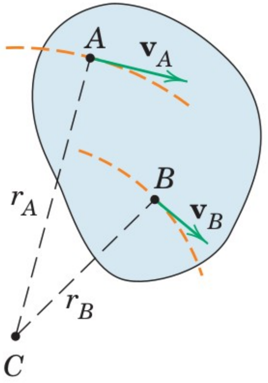
Point C is the instantaneous center of zero velocity and may lie on or off the body. If it lies off the body, it may be visualized as lying on an imaginary extension of the body. **The instantaneous center need not be a fixed point in the body or a fixed point in the plane.**



If we also know the magnitude of the velocity of one of the points, say, $\boldsymbol{v}_A$, we may easily obtain the angular velocity ω of the body and the linear velocity of every point in the body. Thus, the angular velocity of the body is $\omega = v_A/r_A$, which is also the **angular velocity of every line in the body.**

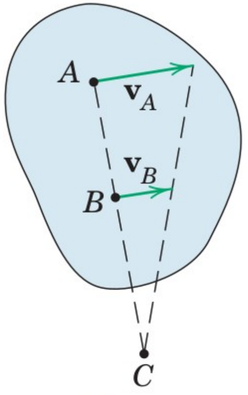


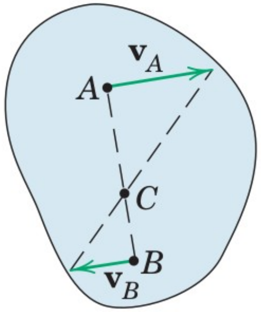
Therefore, the velocity of B is $v_B = r_B\,\omega = (r_B/r_A)v_A$.



Once the instantaneous center is located, the direction of the instantaneous velocity of every point in the body is readily found since it must be perpendicular to the radial line joining the point in question with C.

If the velocities of two points in a body having plane motion are parallel, and the line joining the points is perpendicular to the direction of the velocities, the instantaneous center is located by direct proportion as shown.

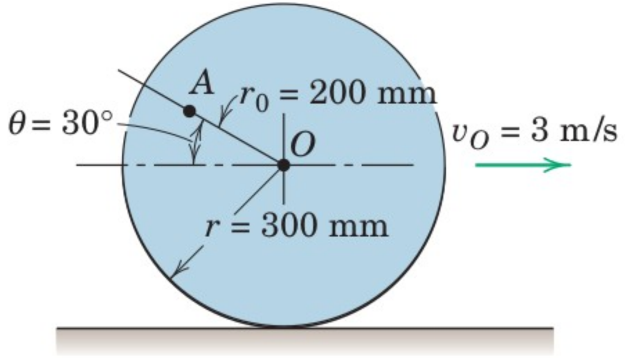




It can be seen that as the parallel velocities become equal in magnitude, the instantaneous center moves farther away from the body and approaches infinity in the limit as the body stops rotating and translates only.

Example 5

The wheel rolls to the right without slipping, with its center O having a velocity $v_O =$ 3 m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.

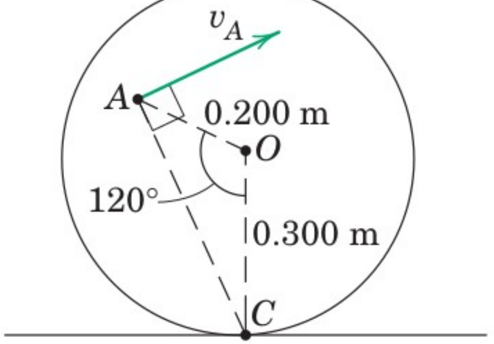


The point on the rim of the wheel in contact with the ground has no velocity if the wheel is not slipping; it is, therefore, the instantaneous center C of zero velocity. The angular velocity of the wheel becomes

$\omega_O = v_O / OC = 3/0.3 = 10$ rad/s

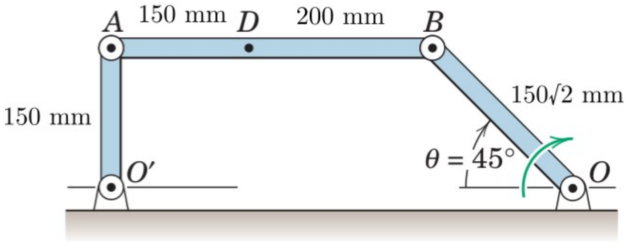
The distance $AC = 0.436$ m

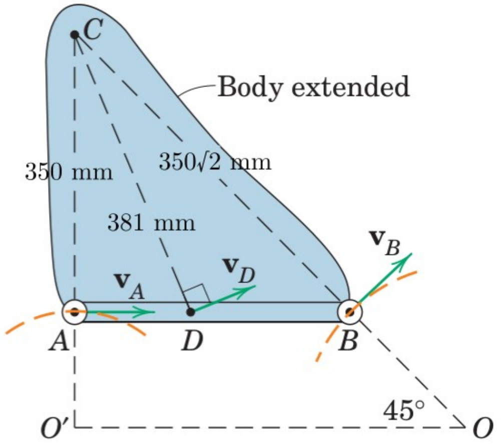
The velocity of A is $v_A = \omega \cdot AC = 4.35$ m/s, which is perpendicular to AC.



Example 6

Arm OB of the linkage has a clockwise angular velocity of 10 rad/sec in the position shown where $\theta = 45°$. Determine the velocity of A, the velocity of D, and the angular velocity of link AB for the position shown.





The directions of the velocities of A and B are tangent to their circular paths about the fixed centers O' and O.

The intersection of the two perpendiculars to the velocities from A and B locates the instantaneous center C for the link AB. The distances AC, BC, and DC can be computed or scaled from the drawing.

The angular velocity of BC, considered a line on the body extended, is equal to the angular velocity of AC, DC, and AB and is

$$\omega_{BC} = v_B / BC = OB \cdot \omega_{OB} / BC = 4.29 \text{ rad/s (CCW)}$$

Thus velocity of A and D are,

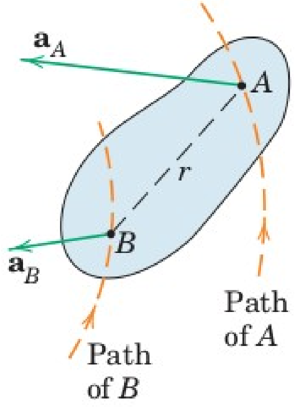
$$v_A = 4.29 \times AC = 1.5 \text{ m/s}, \; v_D = 4.29 \times CD = 1.632 \text{ m/s}$$

Relative acceleration

Consider the equation $\boldsymbol{v}_A = \boldsymbol{v}_B + \boldsymbol{v}_{A/B}$, which describes the relative velocities of two points A and B in plane motion in terms of nonrotating reference axes. By differentiating the equation with respect to time, we obtain the relative-acceleration as

$$\boldsymbol{a}_A = \boldsymbol{a}_B + \boldsymbol{a}_{A/B} . \qquad \text{..................(7)}$$

If point A and B belong to the same rigid body then during the plane motion, the observer moving with B perceives A to have circular motion about B.

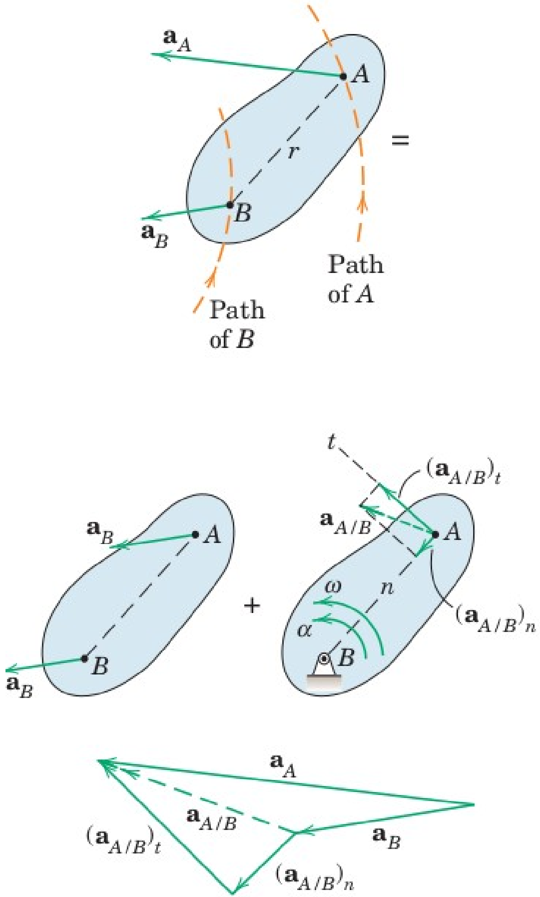


Because the relative motion is circular, it follows that the relative-acceleration term will have both a normal component directed from A toward B due to the change of direction of $\boldsymbol{v}_{A/B}$ and a tangential component perpendicular to AB due to the change in magnitude of $\boldsymbol{v}_{A/B}$.

Thus we may write,

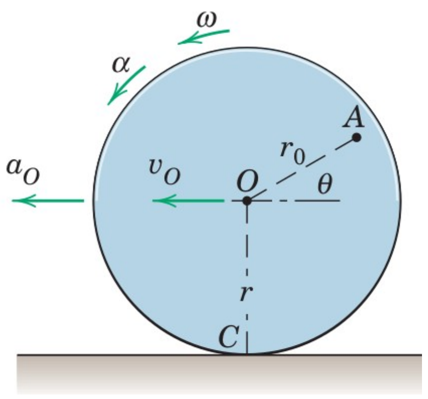
$$\boldsymbol{a}_A = \boldsymbol{a}_B + (\boldsymbol{a}_{A/B})_n + (\boldsymbol{a}_{A/B})_t \,, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(8)$$

where $(\boldsymbol{a}_{A/B})_n = \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$, and $(\boldsymbol{a}_{A/B})_t = \boldsymbol{\alpha}\times\boldsymbol{r}$.



Example 7

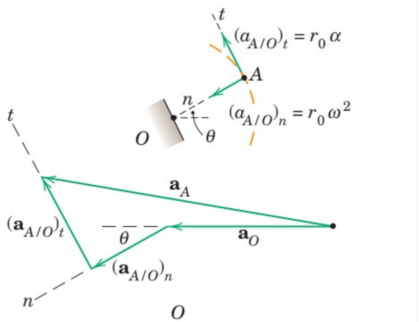
The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\boldsymbol{v}_O$ and an acceleration $\boldsymbol{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.



Angular velocity $\omega = v_O/r$, angular acceleration $\alpha = a_O/r$.

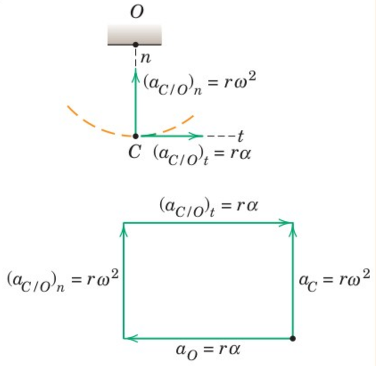
The acceleration of A, $\boldsymbol{a}_A = \boldsymbol{a}_O + \boldsymbol{a}_{A/O} = \boldsymbol{a}_O + (\boldsymbol{a}_{A/O})_n + (\boldsymbol{a}_{A/O})_t$.

where, magnitudes $(a_{A/O})_n = r_0\omega^2 = r_0(v_O/r)^2$, $(a_{A/O})_t = r_0\alpha = r_0(a_O/r)$.



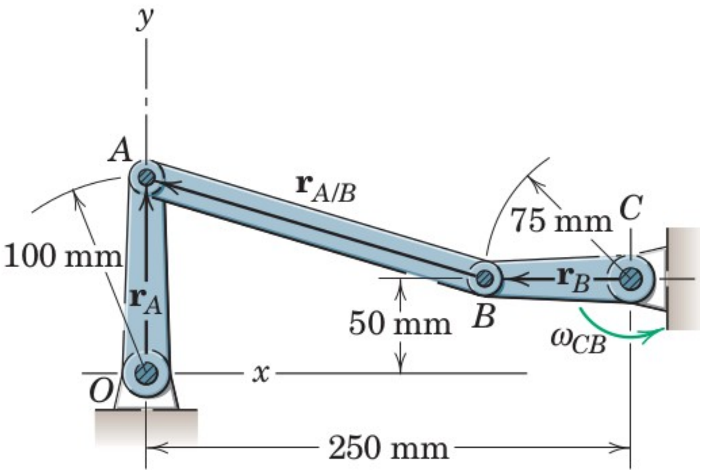
The acceleration of C, $\boldsymbol{a}_C = \boldsymbol{a}_O + \boldsymbol{a}_{C/O} = \boldsymbol{a}_O + (\boldsymbol{a}_{C/O})_n + (\boldsymbol{a}_{C/O})_t$.

where, magnitudes $(a_{C/O})_n = r\omega^2$, $(a_{C/O})_t = r\alpha$.



Example 8

Crank CB has a constant counterclockwise angular velocity of 2 rad/s in the position shown during a short interval of its motion. Determine the angular acceleration of links AB and OA for this position. Solve by using vector algebra.



We already solved for ω_{AB} = -6/7 rad/s, ω_{OA} = -3/7 rad/s.

The acceleration of A, $\boldsymbol{a}_A = \boldsymbol{a}_B + (\boldsymbol{a}_{A/B})_n + (\boldsymbol{a}_{A/B})_t$.

$$\boldsymbol{a}_A = \boldsymbol{\alpha}_{OA} \times \boldsymbol{r}_A + \boldsymbol{\omega}_{OA} \times (\boldsymbol{\omega}_{OA} \times \boldsymbol{r}_A)$$

$$= \alpha_{OA}\, \boldsymbol{k} \times 100\, \boldsymbol{j} + (-3/7\, \boldsymbol{k}) \times (-3/7\, \boldsymbol{k} \times 100\, \boldsymbol{j})$$

$$= -100\, \alpha_{OA} \boldsymbol{i} \;\; -100(3/7)^2 \boldsymbol{j} \text{ mm/s}^2$$

$$\boldsymbol{a}_B = \boldsymbol{\alpha}_{CB} \times \boldsymbol{r}_B + \boldsymbol{\omega}_{CB} \times (\boldsymbol{\omega}_{CB} \times \boldsymbol{r}_B)$$

$$= \mathbf{0} + (2\boldsymbol{k}) \times (2\boldsymbol{k} \times -75\, \boldsymbol{i}) = 300\, \boldsymbol{i} \text{ mm/s}^2$$

The acceleration of A, $\boldsymbol{a}_A = \boldsymbol{a}_B + (\boldsymbol{a}_{A/B})_n + (\boldsymbol{a}_{A/B})_t$.

$(\boldsymbol{a}_{A/B})_n = \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \boldsymbol{r}_{AB}) = [-6/7\ \boldsymbol{k}] \times [-6/7\ \boldsymbol{k} \times (-175\ \boldsymbol{i} + 50\ \boldsymbol{j})]$

$\quad = (6/7)^2\ (175\ \boldsymbol{i} - 50\ \boldsymbol{j})$ mm/s^2

$(\boldsymbol{a}_{A/B})_t = \boldsymbol{\alpha}_{AB} \times \boldsymbol{r}_{AB} = \alpha_{AB} \boldsymbol{k} \times (-175\ \boldsymbol{i} + 50\ \boldsymbol{j}) = -\ 50\ \alpha_{AB} \boldsymbol{k}\ -175\ \alpha_{AB} \boldsymbol{j}$ mm/s^2 .

$a_{AB} = -0.1050$ rad/s^2, $a_{OA} = -4.34$ rad/s^2.