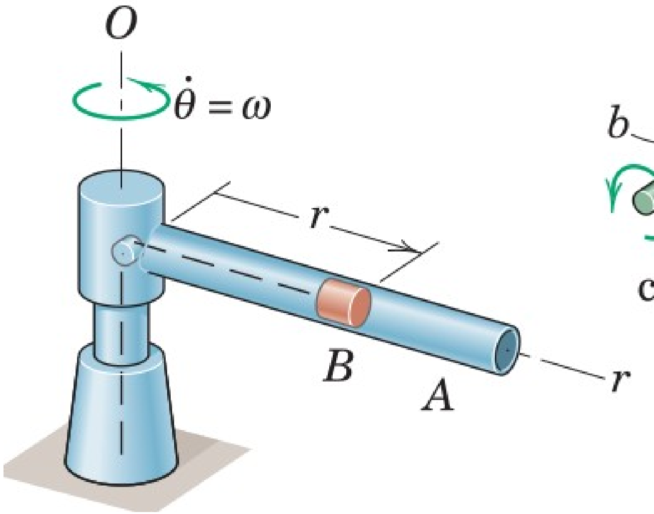
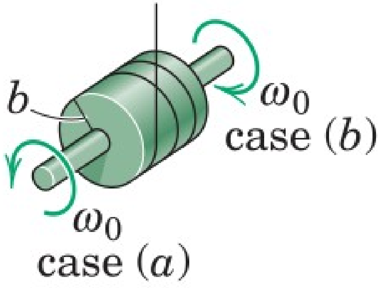
ME232: Dynamics

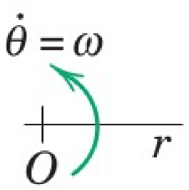
Anshul Faye
afaye@iitbhilai.ac.in
Room # 106

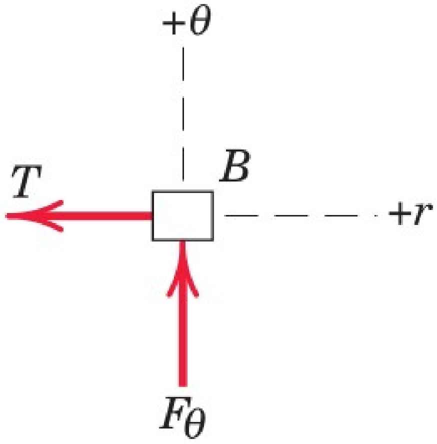
Example 2

Tube A rotates about the vertical O-axis with a constant angular rate ω and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b. Determine the tension T in the cord and the horizontal component F of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.



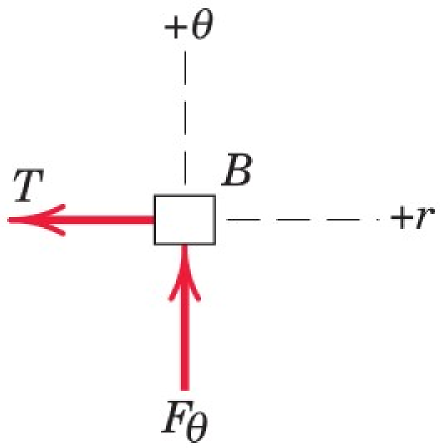






We use the polar-coordinate form of the equations of motion.

The free-body diagram of B is shown in the horizontal plane. The equations of motion are

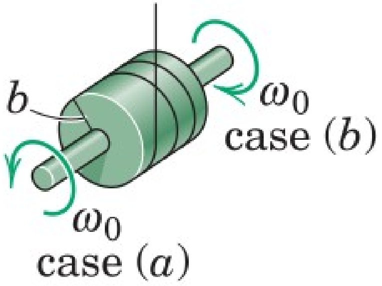


$$\sum F_r = ma_r \Rightarrow -T = m(\ddot{r} - r\dot{\theta}^2) \Rightarrow T = -m(\ddot{r} - r\omega^2),$$

$$\sum F_\theta = ma_\theta \Rightarrow F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \Rightarrow F_\theta = 2m\dot{r}\omega.$$

For case (a), $\dot{r} = +b\omega_0, \ddot{r} = 0,$ thus,

$$T = mr\omega^2, \qquad F_\theta = 2mb\omega_0\omega.$$

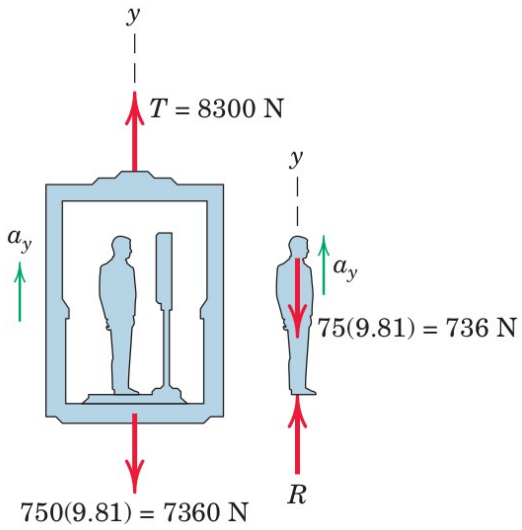


For case (b), $\dot{r} = -b\omega_0, \ddot{r} = 0,$ thus,

$$T = mr\omega^2, \qquad F_\theta = -2mb\omega_0\omega.$$

Example 3

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.



From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$$\sum F_y = ma_y \Rightarrow 8300 - 7360 = 750a_y \Rightarrow a_y = 1.257 \text{ m/s}^2.$$

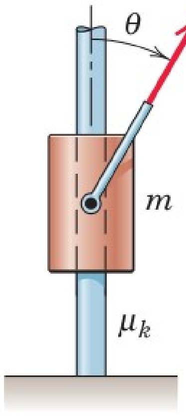
The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R act on the man together with its weight, hence, equation of motion is,

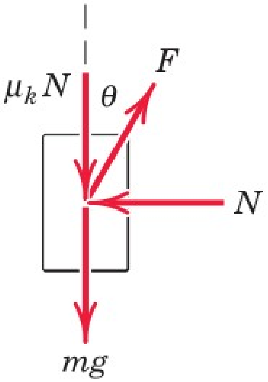
$$\sum F_y = ma_y \Rightarrow R - 736 = 75 \times 1.257 \Rightarrow R = 830 \text{ N}.$$

Velocity after 3 second can be calculated using the expression derived for constant acceleration.

Example 4

The collar of mass m slides up the vertical shaft under the action of a force F of constant magnitude but variable direction. If $= kt$ where k is a constant and if the collar starts from rest with $= 0$, determine the magnitude F of the force which will result in the collar coming to rest as reaches $\pi/2$. The coefficient of kinetic friction between the collar and shaft is μ_k .





$$\sum F_x = ma_x$$

$$F\sin\theta - N = 0 \Rightarrow N = F\sin\theta.$$

$$\sum F_y = ma_y$$

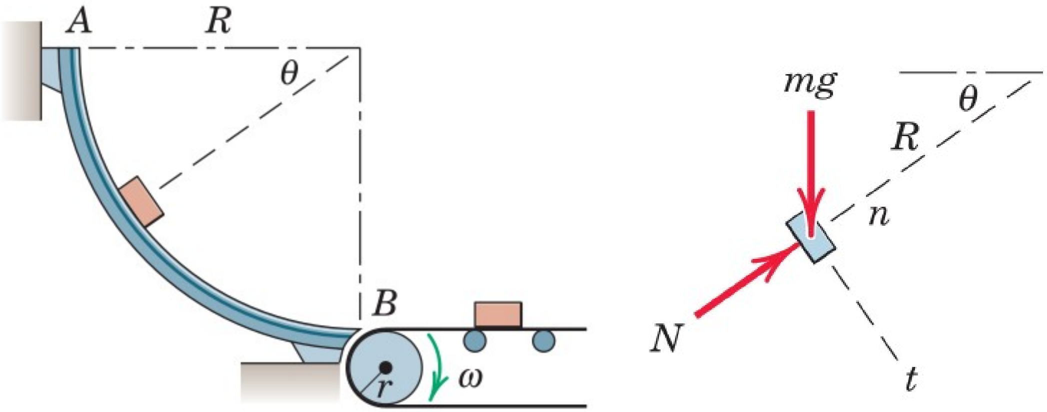
$$F\cos\theta - \mu_k N - mg = mdv/dt,$$

$$\Rightarrow F\cos kt - \mu_k F\sin kt - mg = mdv/dt,$$

Find the expression of force as a function of time and velocity by integrating above the equation. Substituting $= \pi/2k$ and v=0 in that expression will give the required force.

Example 5

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.



The normal force N depends on the n-component of the acceleration which, in turn, depends on the velocity.
The velocity will be cumulative according to the tangential acceleration a_t. To find a_t we use,

$$\sum F_t = ma_t,$$

$$mg\cos\theta = ma_t \Rightarrow a_t = g\cos\theta. \quad \cdots\cdots\cdots\text{(a)}$$

$$\sum F_n = ma_n,$$

$$N - mg\sin\theta = ma_n,$$

$$\Rightarrow N = mv^2/R + mg\sin\theta. \quad \cdots\cdots\cdots\text{(b)}$$

To represent N as a function of , we first need to express v in terms of using (a) as,

$$a_t = \dot{v} = vdv/ds = g\cos\theta,$$

$$\Rightarrow vdv = g\cos\theta ds = g\cos\theta(Rd\theta),$$

integrating the above relation as,

$$\Rightarrow \int_0^v vdv = \int_0^\theta gR\cos\theta d\theta, \qquad \Rightarrow v^2 = 2gR\sin\theta, \qquad \cdots\cdots\cdots\text{(c)}$$

substituting (c) in (b), we get,

$$\Rightarrow N = 3mg\sin\theta.$$

To find the required rotational velocity,

$$v|_{\theta=\pi/2} = \omega r = \sqrt{2gR}, \text{ hence, } \omega = \sqrt{2gR}/r.$$

Work and Energy

We have already seen application of Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$ to various problems of particle motion to establish the instantaneous relationship between the net force acting on a particle and the resulting acceleration of the particle.

There are two general classes of problems in which the cumulative effects of unbalanced forces acting on a particle are of interest to us.

(1) integration of the forces with respect to the displacement of the particle, and

(2) integration of the forces with respect to the time they are applied.

Integration with respect to displacement $\rightarrow$ the equations of work and energy

Integration with respect to time $\rightarrow$ the equations of impulse and momentum.

Work

A force $\boldsymbol{F}$ is acting on a particle at A. The position vector of A is $\boldsymbol{r}$, and $d\boldsymbol{r}$ is the differential displacement from A to A'.

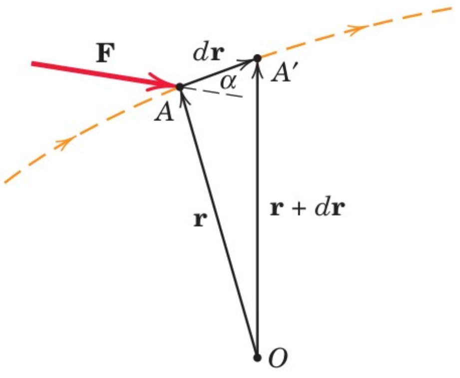
The work done by the force $\boldsymbol{F}$ during the displacement $d\boldsymbol{r}$ is

$$dU = \boldsymbol{F} \cdot d\boldsymbol{r} = F\ ds\ \cos a, \qquad \cdots\cdots\cdots(9)$$

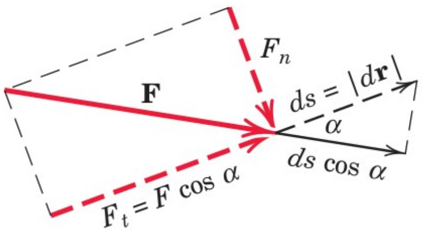
where a is the angle between $\boldsymbol{F}$ and $d\boldsymbol{r}$, and ds is the magnitude of $d\boldsymbol{r}$.

Equation (9) may be interpreted as the displacement multiplied by the force component $F_t = F \cos a$ in the direction of the displacement.

Alternatively, the work dU may be interpreted as the force multiplied by the displacement component $ds \cos a$ in the direction of the force.



(a)



(b)

With this definition of work, it should be noted that the component $F_n = F \sin a$ normal to the displacement does no work. Thus, the work dU may be written as

$$dU = F_t \, ds. \qquad \cdots\cdots\cdots\cdots(10)$$

Work is positive if the working component F_t is in the direction of the displacement and negative if it is in the opposite direction.

Forces which do work are termed **active forces**.

Constraint forces which do no work are termed **reactive forces**.

The SI units of work are those of force (N) times displacement (m) or N·m. This unit is given the special name **joule** (J), which is defined as the work done by a force of 1 N acting through a distance of 1 m in the direction of the force.

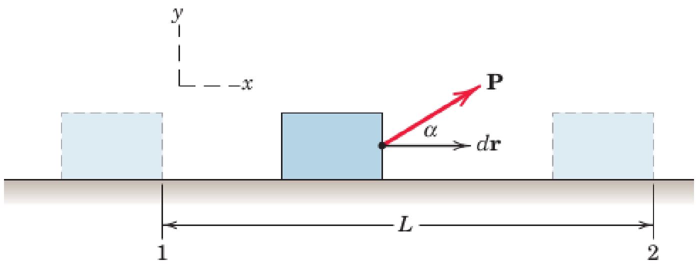
Calculation of work

During a finite movement of the point of application of a force, the force does an amount of work equal to

$$U = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_1^2 (F_x dx + F_y dy + F_z dz) \quad \text{or} \quad U = \int_1^2 F_t \, ds. \qquad \cdots\cdots\cdots(11)$$

1) Work associated with a constant external force:

Consider the constant force $\boldsymbol{P}$ applied to the body as it moves from position 1 to position 2. With the force $\boldsymbol{P}$ and the differential displacement $d\boldsymbol{r}$ written as vectors, the work done on the body by the force is

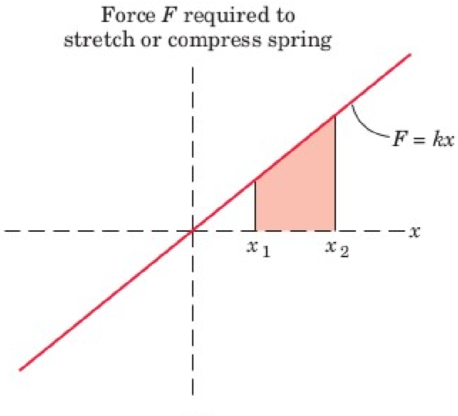


$$U_{1-2} = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_1^2 (P \cos\alpha \, \boldsymbol{i} + P \sin\alpha \, \boldsymbol{j}) dx \, \boldsymbol{i},$$

$$U_{1-2} = \int_1^2 P \cos\alpha \, dx = P\cos\alpha(x_2 - x_1) = PL\cos\alpha. \qquad \cdots\cdots\cdots(12)$$

2) Work associated with a spring force:

Consider the linear spring of stiffness k where the force required to stretch or compress the spring is proportional to the deformation x.



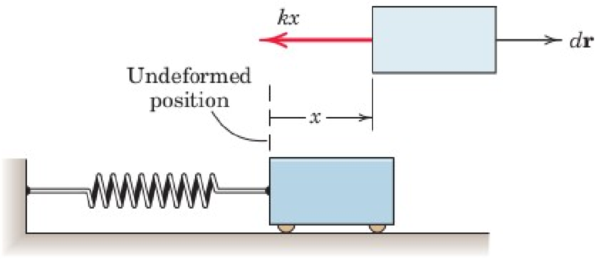
(a)

The force exerted by the spring on the body is $\boldsymbol{F} = -kx\ \boldsymbol{i}$.

The work done on the body by the spring force as the body undergoes an arbitrary displacement from an initial position x_1 to a final position x_2 is

$$U_{1-2} = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_1^2 -kx\boldsymbol{i} \cdot dx\,\boldsymbol{i} = \int_{x_1}^{x_2} -kx dx,$$

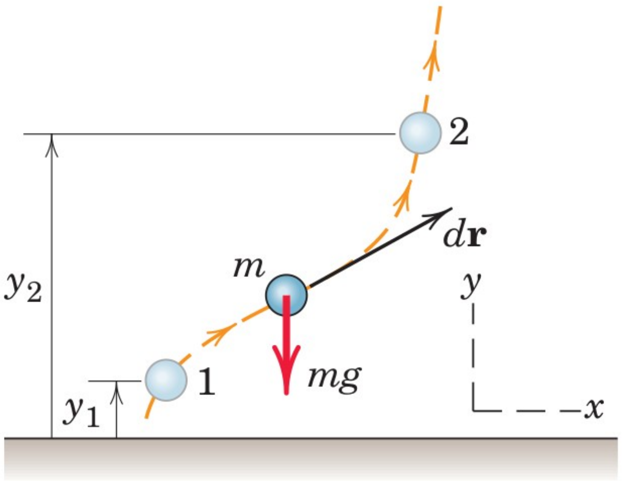
$$U_{1-2} = \frac{1}{2}k(x_1^2 - x_2^2). \qquad \cdots\cdots\cdots\cdots(13)$$



(b)

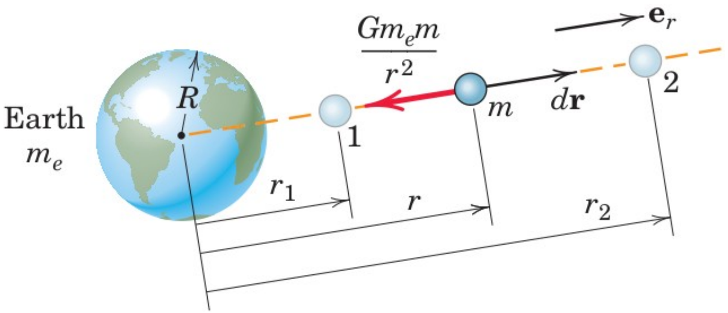
3a) Work associated with weight: ($\boldsymbol{g}$ = constant)

For the sufficiently low variation in altitude (so that the acceleration of gravity $\boldsymbol{g}$ may be considered constant), the work done by the weight $m\boldsymbol{g}$ of the body as the body is displaced from an arbitrary altitude y_1 to a final altitude y_2 is



$$U_{1-2} = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_1^2 -mg\boldsymbol{j} \cdot (dx\,\boldsymbol{i} + dy\,\boldsymbol{j}) = \int_{y_1}^{y_2} -mgdy = -mg(y_2 - y_1). \quad \cdots\cdots\cdots(14)$$

3b) Work associated with weight: ($g \neq$ constant)



If large changes in altitude occur, then the weight (gravitational force) is no longer constant. We must therefore use the gravitational law and express the weight as a variable force of magnitude

$$F = G\frac{m_e m}{r^2}. \qquad \cdots\cdots\cdots(15)$$

Now, the work can be expressed as,

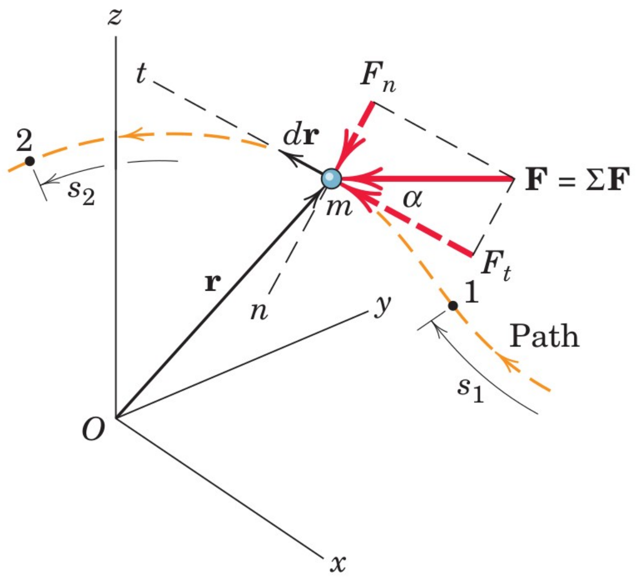
$$U_{1-2} = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_1^2 -\frac{Gm_e m}{r^2}\mathbf{e}_r \cdot dr\, \mathbf{e}_r = -Gm_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= Gm_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = mgR^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \qquad \cdots\cdots\cdots(16)$$

where the equivalence $Gm_e = gR^2$ is used.

Work and curvilinear motion:

Let us consider the work done on a particle of mass m, moving along a curved path under the action of the force $\boldsymbol{F}$. The position of m is specified by $\boldsymbol{r}$, and its displacement along its path during the time dt is represented by the change $d\boldsymbol{r}$ in its position vector. The work done by $\boldsymbol{F}$ during a finite movement of the particle from point 1 to point 2 is



$$U = \int_1^2 \boldsymbol{F} \cdot d\boldsymbol{r} = \int_{s_1}^{s_2} F_t \, ds.$$

where the limits specify the initial and final end points of the motion. Substituting Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$, the expression for the work of all forces becomes

$$U = \int_1^2 m\boldsymbol{a} \cdot d\boldsymbol{r} = \int_{s_1}^{s_2} ma_t \, ds = \int_{s_1}^{s_2} m\dot{v} \, ds = \int_{v_1}^{v_2} mv \, dv = \frac{1}{2}m(v_2^2 - v_1^2).$$

.........(17)

Work and kinetic energy:

The **kinetic energy** ***T*** of the particle is defined as, $T = \frac{1}{2}mv^2.$ ....(18)

It is the total work which must be done on the particle to bring it from a state of rest to a velocity v. Kinetic energy T is a scalar quantity with the units of N·m or joules (J) in SI units. Kinetic energy is always positive, regardless of the direction of the velocity.

Accordingly, (17) can now be written as, $U_{1\text{-}2} = T_2 - T_1 = \Delta T.$ ....(19)

which is **the work-energy equation** for a particle.

The equation states that the total work done by all forces acting on a particle as it moves from point 1 to point 2 equals the corresponding change in kinetic energy of the particle. Although T is always positive, the change ΔT may be be positive, negative, or zero.

Alternatively, the work-energy relation may be expressed as, $T_1 + U_{1\text{-}2} = T_2$ ....(19a)

A major advantage of the method of work and energy is that it **avoids the necessity of computing the acceleration and leads directly to the velocity changes** as functions of the forces which do work.

Further, the work-energy equation **involves only those forces which do work** and thus give rise to changes in the magnitude of the velocities.

Consider now a system of two particles joined together by a rigid connection. The forces in the connection are equal and opposite, and their points of application necessarily have identical displacement components in the direction of the forces. Therefore, the net work done by these internal forces is zero during any movement of the system. Thus, (19) is applicable to the entire system, where $U_{1\text{-}2}$ is the total or net work done on the system by forces external to it and ΔT is the change, $T_2 - T_1$, in the total kinetic energy of the system. The total kinetic energy is the sum of the kinetic energies of both elements of the system. Thus, another advantage of the work-energy method is that it **enables us to analyze a system of particles joined in the manner described without dismembering the system**.

Power

- The capacity of a machine is measured by the time rate at which it can do work or deliver energy.
- The total work or energy output is not a measure of this capacity since a motor, no matter how small, can deliver a large amount of energy if given sufficient time.
- On the other hand, a large and powerful machine is required to deliver a large amount of energy in a short period of time.
- Thus, the capacity of a machine is rated by its power, which is defined as the time rate of doing work.
- Accordingly, the power P developed by a force $\boldsymbol{F}$ which does an amount of work U is $P = dU/dt = \boldsymbol{F}\cdot d\boldsymbol{r}/dt$. Because $d\boldsymbol{r}/\mathrm{dt}$ is the velocity $\boldsymbol{v}$ of the point of application of the force, we have

$$P = \boldsymbol{F}\cdot\boldsymbol{v}\ . \qquad \cdots\cdots\cdots(20)$$

- Power is a scalar quantity, and in SI it has the units of N·m/s = J/s. The special unit for power is the watt (W), which equals one joule per second (J/s).
- In U.S. customary units, the unit for mechanical power is the horsepower (hp). These units and their numerical equivalences are

$$1 \text{ W} = 1 \text{ J/s}$$

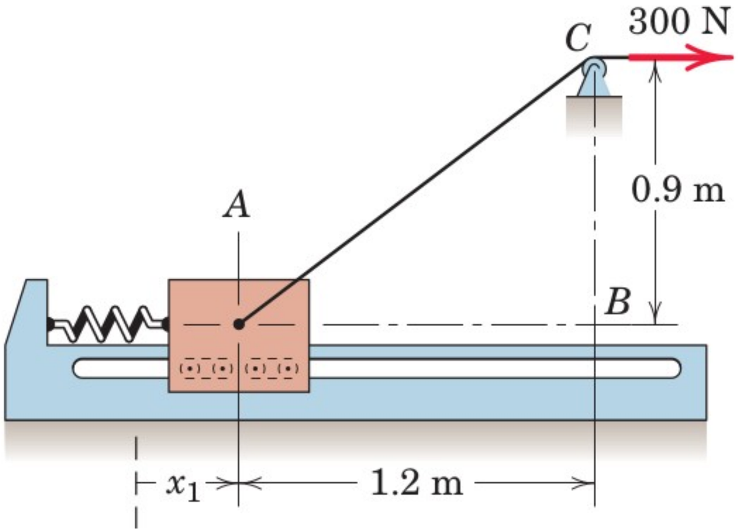
$$1 \text{ hp} = 746 \text{ W} = 0.746 \text{ kW}$$

- The ratio of the work done by a machine to the work done on the machine during the same time interval is called the mechanical efficiency e_m of the machine. Efficiency is always less than unity since every device operates with some loss of energy and since energy cannot be created within the machine. The mechanical efficiency at any instant of time may be expressed in terms of mechanical power P by

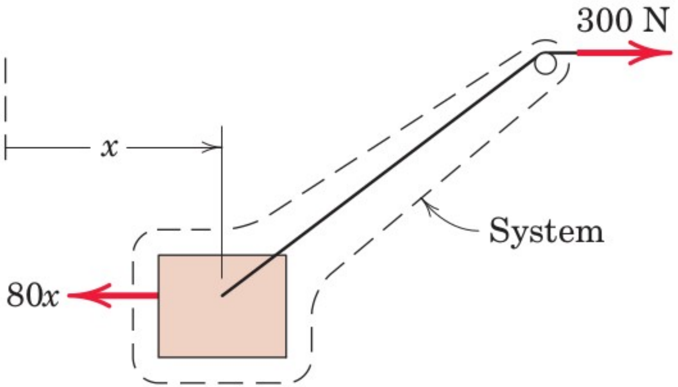
$$e_m = P_{\text{output}}/P_{\text{input}}. \qquad \cdots\cdots\cdots(21)$$

Example 6

The 50 kg block at A is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300 N force in the cable. The block is released from rest at A, with the spring to which it is attached extended an initial amount $x_1 = 0.233$ m. The spring has a stiffness $k = 80$ N/m. Calculate the velocity v of the block as it reaches position B.



We assume that the stiffness of the spring is small enough to allow the block to reach position B.
The active-force diagram for the system composed of both block and cable is shown for a general position.



As the block moves from $x_1 = 0.233$ m to $x_2 = 0.233 + 1.2 = 1.433$ m, the work done by the spring force acting on the block is

$$U_{1-2} = \frac{1}{2}k(x_1^2 - x_2^2) = \frac{1}{2}80 \times (0.233^2 - 1.433^2) = -80\,\text{J}.$$

The work done on the system by the constant 300-N force in the cable is the force times the net horizontal movement of the cable over pulley C, which is $\sqrt{1.2^2 + 0.9^2} - 0.9 = 0.6$ m. Thus, the work done is $300 \times 0.6 = 180$ J.

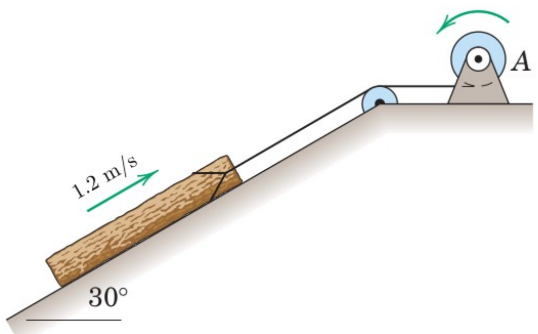
Now apply the work-energy equation to the system as

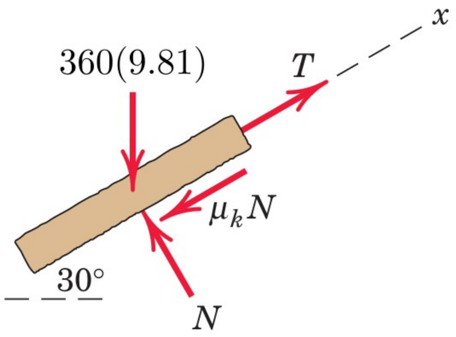
$T_1 + U_{1\text{-}2} = T_2$.

$0 - 80 + 180 = 1/2 \times 50 \times v^2$, which gives $v = 2$ m/s.

Example 7

The power winch A hoists the 360 kg log up the 30° incline at a constant speed of 1.2 m/sec. If the power output of the winch is 4 kW, compute the coefficient of kinetic friction μ_k between the log and the incline. If the power is suddenly increased to 6 kW, what is the corresponding instantaneous acceleration a of the log?





From the free-body diagram of the log,

$$\sum F_x = T - 3531.6\cos 30°\mu_k - 3531.6\sin 30° = 0$$

$$\Rightarrow T = 3058.6\mu_k + 1765.8 \qquad \cdots\cdots\cdots(a)$$

Power out of the winch gives the tension in the cable,

$P = Tv$, which gives, $T = 4000/1.2 = 3330$ N

Now, μ_k can be calculated from (a) as $\mu_k = 0.513$.

When the power is increased, the tension momentarily becomes,

T $= 6000/1.2 = 5000$ N,

Then the corresponding acceleration is given by

$\Sigma F_x = ma_x$, $\quad 5000 - 3058.6\times 0.513 + 1765.8 = 360a$

Hence, $a = 4.63$ m/s^2