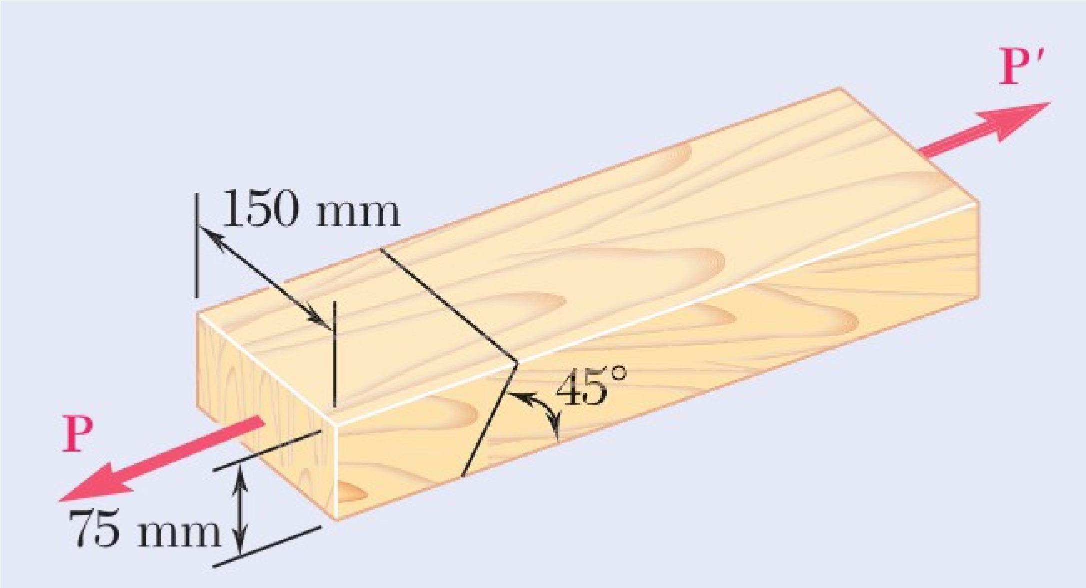
ME231: Solid Mechanics-I

Stress and Strain

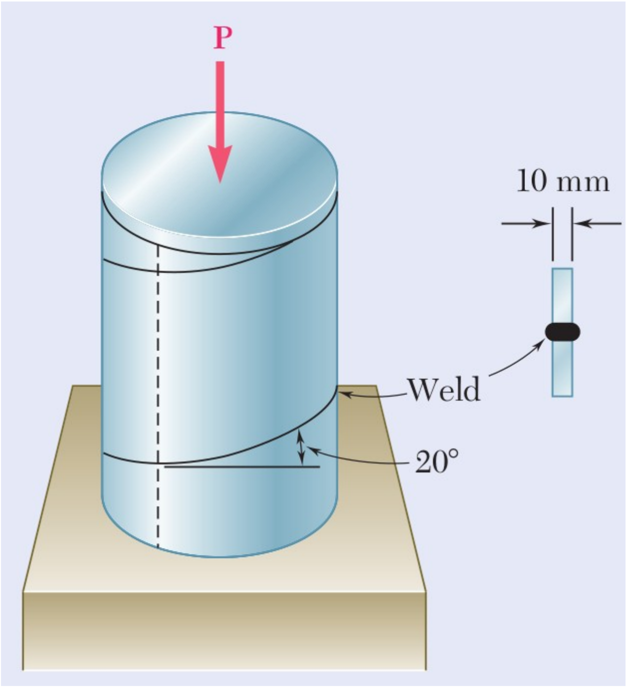
Example 3

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice. If that maximum allowable shearing stress in the glued splice is 620 kPa, determine whether the applied load is within the safe limit or not.



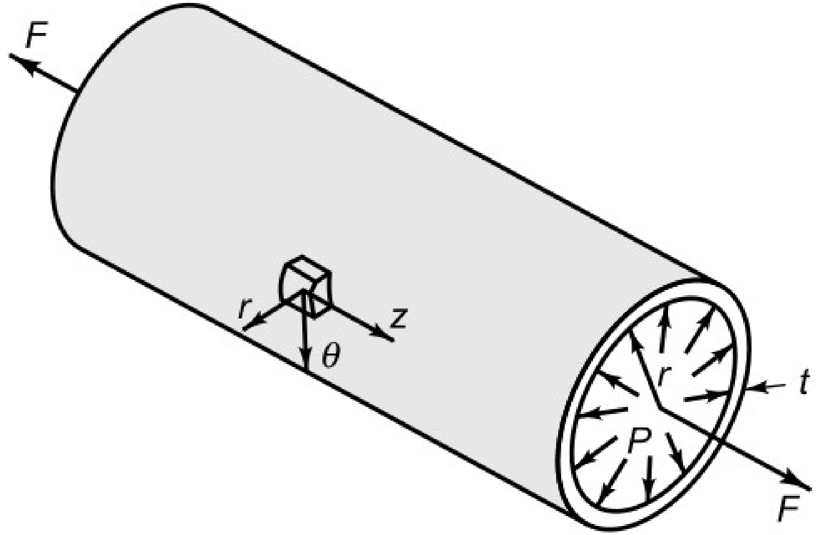
Example 4

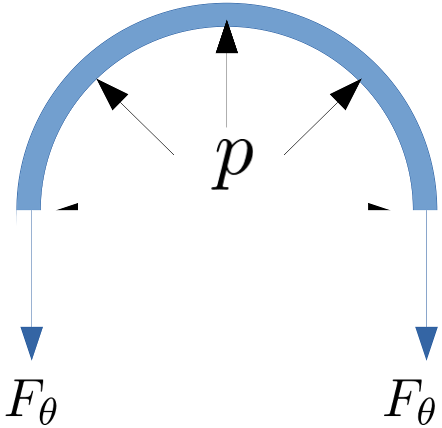
A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force P is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.



Example 5

Consider a thin-walled cylinder of internal radius r and thickness t. If the cylinder is subjected to an internal pressure p and an axial force F, show that the r, θ, z directions are the principal stress directions. Show also that if the wall is so thin that $t/r << 1$, then determine the stresses in the pipe wall.





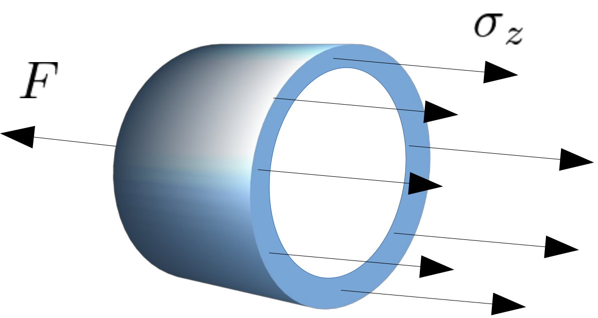
Detach a part of cylinder and its content by a *rz*-plane and considering force equilibrium in -direction as,

$$\sum F_\theta = 2F_\theta - pA = 0,$$

where A is the projected area at *rz*-plane. Thus,

$$2F_\theta - pA = 2\sigma_\theta \cdot l \cdot t - p \cdot 2r \cdot l = 0$$

$$\text{or } \sigma_\theta = pr/t. \qquad \textbf{(called as hoop stress)}$$



To determine the stresses in longitudinal direction detach a part of cylinder and its content by a plane parallel to *r*-plane and considering force equilibrium in *z*-direction as,

$$\sum F_z = \sigma_z \cdot 2\pi r \cdot t - F = 0,$$

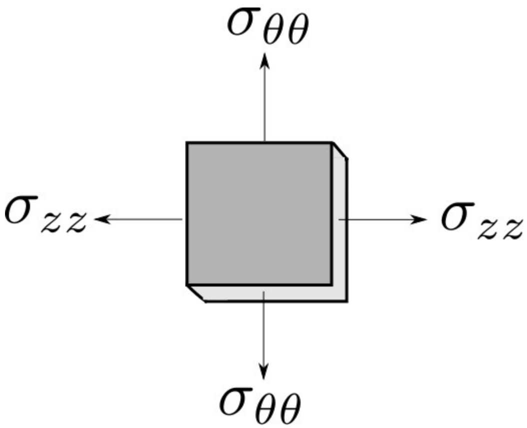
$$\text{or } \sigma_z = F/(2\pi r t).$$

For a cylinder with end caps, axial force F is caused due to pressure on the end caps, where $F = p{\cdot}\pi r^2$. Thus for cylinder with end caps,

$$\sigma_z = pr/(2t). \quad \textbf{(called as longitudinal stress)}$$

The accuracy of derived result depends on the vessel being **thin-walled**, ($r >> t$). At the surfaces of the vessel wall, a radial stress σ_r must be present to balance the pressure there. But the inner-surface radial stress is equal to p, while the circumferential (or hoop) and longitudinal stresses are p times the ratio (r/t) and $(r/2t)$, respectively. When this ratio is large, the **radial stresses can be neglected** in comparison with the hoop or longitudinal stresses.

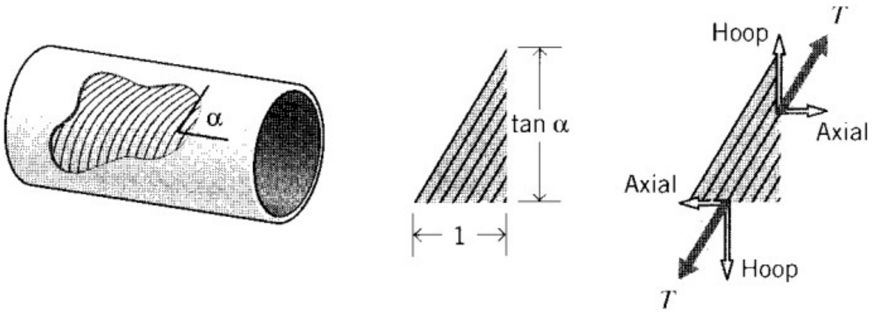
Thus, the state of stress an element of the pressure vessel is as follows,



which is a case of plane stress.

Example 6

Consider a cylindrical pressure vessel to be constructed by filament winding, in which fibers are laid down at a prescribed helical angle α. Taking a free body of unit axial dimension along which n fibers transmitting tension T are present, the circumferential distance cut by these same n fibers is then tan α. To balance the hoop and axial stresses, the fiber tensions must satisfy the relations



$$nT\sin\alpha = \frac{pr}{t}\cdot 1\cdot t$$

$$nT\cos\alpha = \frac{pr}{2t}\cdot\tan\alpha\cdot t$$

Dividing the first of these expressions by the second and rearranging, we have

$$\tan^2\alpha = 2 \quad \text{or} \quad \alpha = 54.7^\circ.$$

This is the angle for filament wound vessels, at which the fibers are inclined just enough toward the circumferential direction to make the vessel twice as strong circumferentially as it is axially.

Firefighting hoses are also braided at this same angle, since otherwise the nozzle would jump forward or backward when the valve is opened and the fibers try to align themselves along the correct direction.