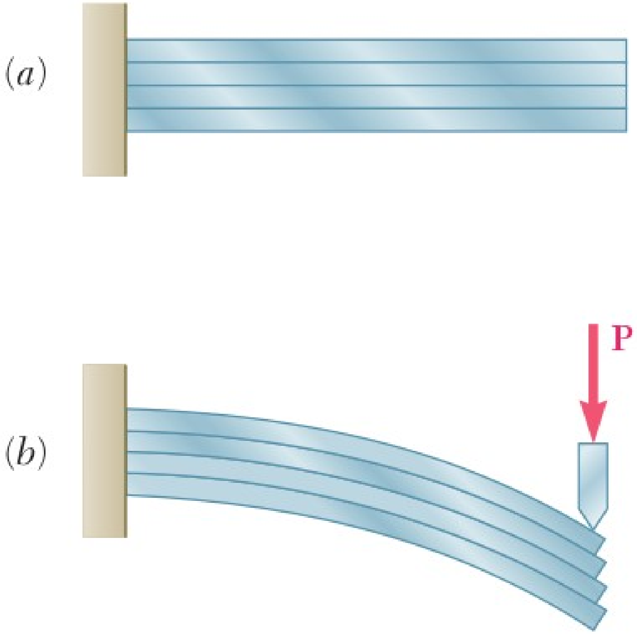
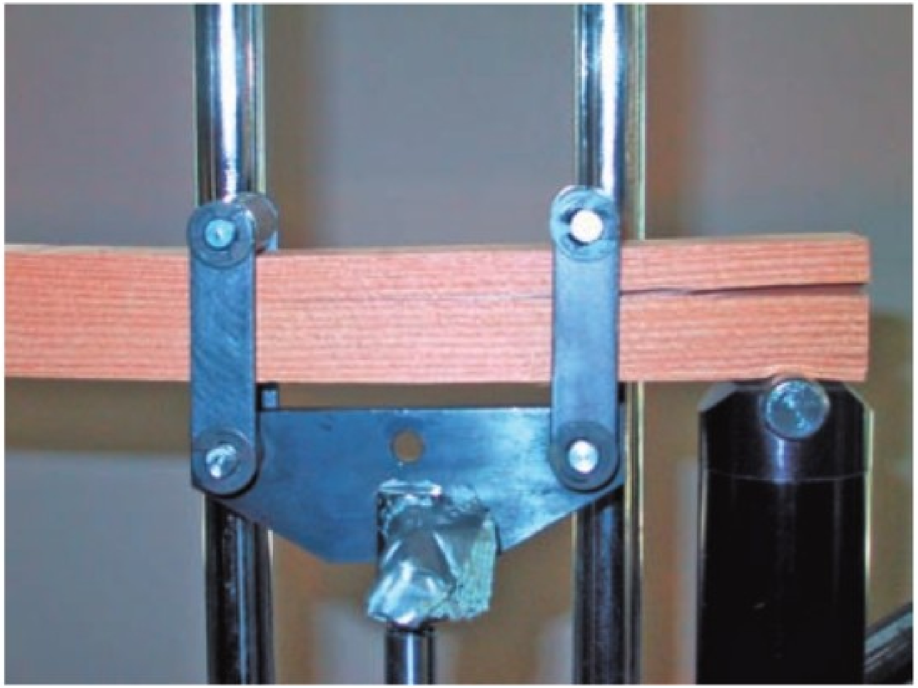
ME231: Solid Mechanics-I

Stresses due to bending

Elastic beams transmitting both shear force and bending moment





Stresses in symmetrical elastic beams transmitting both shear force and bending moment

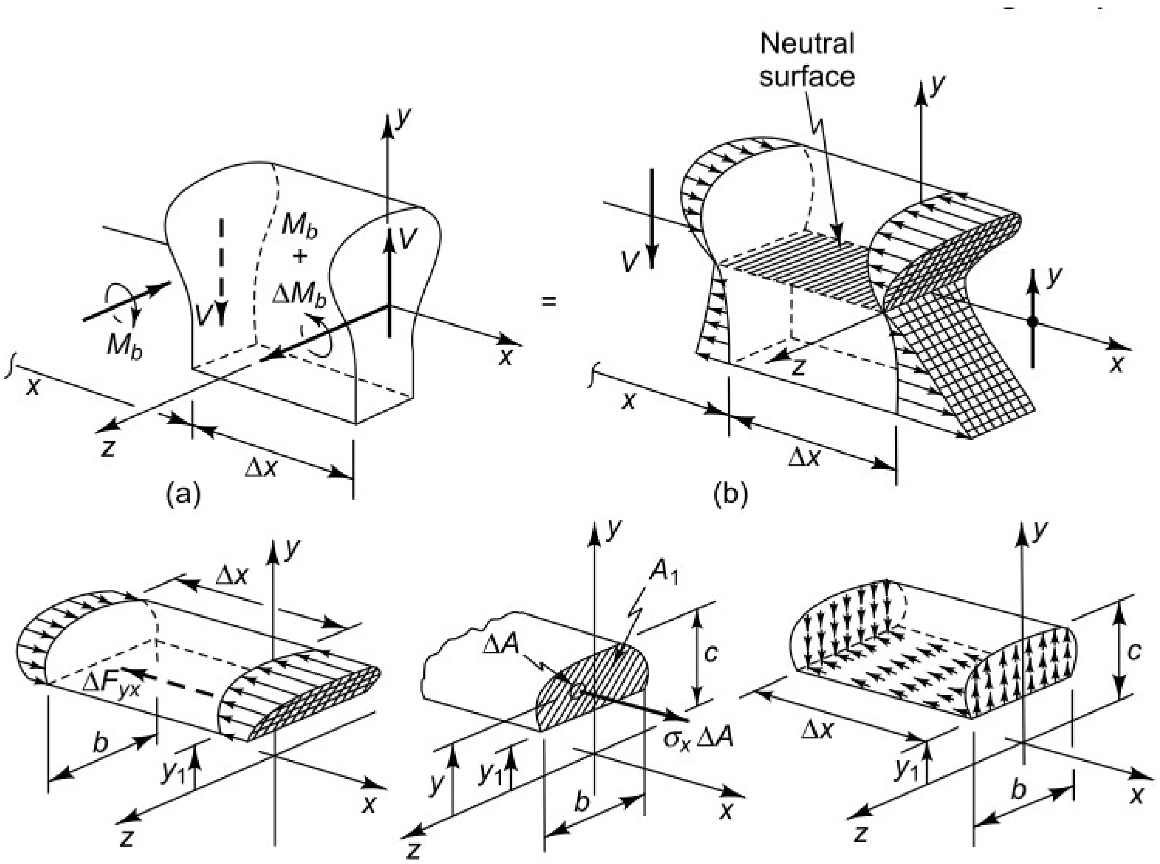
- Pure bending is a relatively less common type of loading for a beam.
- Generally a shear force is present with the bending moment.
- In such cases bending moment does not remain constant along the beam and it varies, hence if is more difficult to obtain an exact solution for such problem. Exact solutions are available for certain cases from theory of elasticity.
- To developing a solution with engineering judgment, we assume that the bending stress distribution is valid even when the bending moment varies along the beam, i.e., when a shear force is present.
- Experimental evidence and comparison with available exact solutions show that the estimates of the stress solutions obtained with the aforementioned assumption are satisfactory for most engineering purpose.

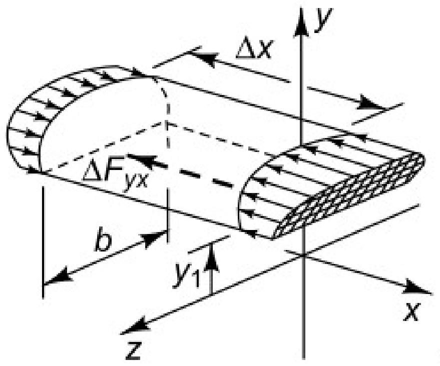
We consider an element where no transverse load is acting, so that transverse shear force is independent of x.

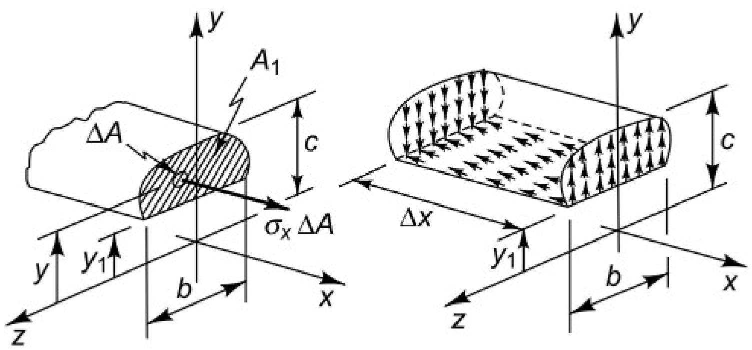
Bending moment varies along the length.

Now consider the equilibrium of an isolated element, which is obtained by cutting the beam through a plane at $y=y_1$.

Due to unbalnace of bending stresses, a force ΔF_{yx} is expected on the bottom surface of the element (as shown).







Considering equilibrium in x-direction

$$\sum F_x = \left[\int_{A_1} \sigma_x dA\right]_{x+\Delta x} - \Delta F_{yx} - \left[\int_{A_1} \sigma_x dA\right]_x = 0$$

Using formula for bending stress

$$\Delta F_{yx} = -\int_{A_1} \frac{(M_b + \Delta M_b)y}{I_z} dA + \int_{A_1} \frac{(M_b)y}{I_z} dA_x$$

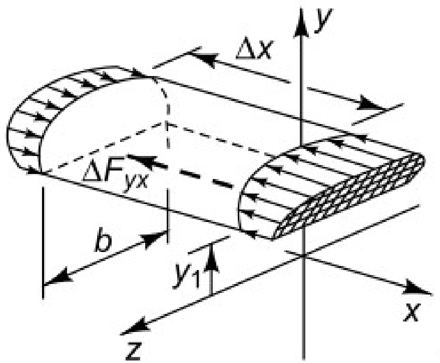
$$\Delta F_{yx} = -\frac{\Delta M_b}{I_z} \int_{A_1} y dA$$

Dividing both sides by Δx and taking the limit,

$$\frac{dF_{yx}}{dx} = -\frac{dM_b}{dx} \frac{1}{I_z} \int_{A_1} y dA = \frac{V}{I_z} \int_{A_1} y dA$$

............(12)

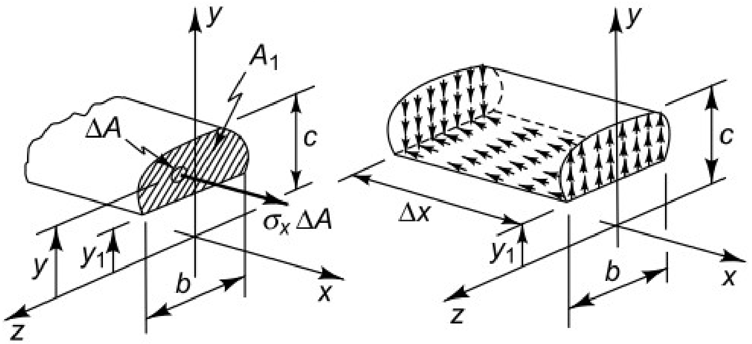
We write equation (12) as



$$q_{yx} = \frac{VQ}{I_z}, \qquad \cdots\cdots\cdots\cdots(13)$$

where,

$$q_{yx} = \frac{dF_{yx}}{dx}, \quad \text{and} \quad Q = \int_{A_1} y dA. \qquad \cdots\cdots\cdots\cdots(14)$$

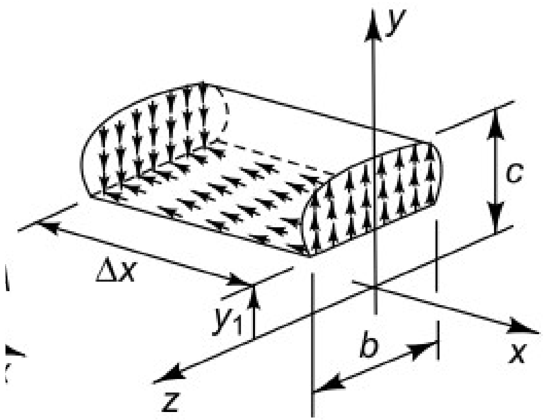


Here, q_{yx} is the total longitudinal shear force transmitted across the plane defined by $y=y_1$ per unit length along the beam, is called shear flow, and Q is the first moment of shaded area A_1.

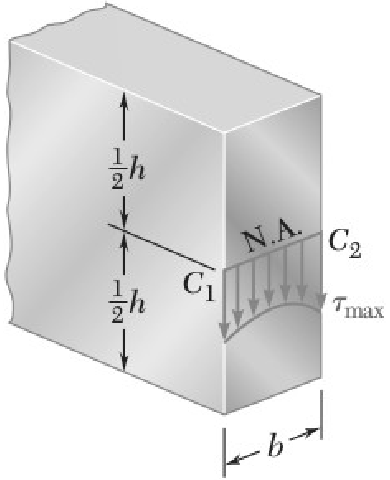
If it is assumed that the shear stress τ_{yx} is uniform across the beam, then the shear stress at $y=y_1$ is

$$\tau_{yx} = \frac{q_{yx}}{b} = \frac{VQ}{bI_z}. \qquad \cdots\cdots\cdots\cdots(15)$$

From the moment equilibrium, uniform shear stress at x-faces can be obtained as,



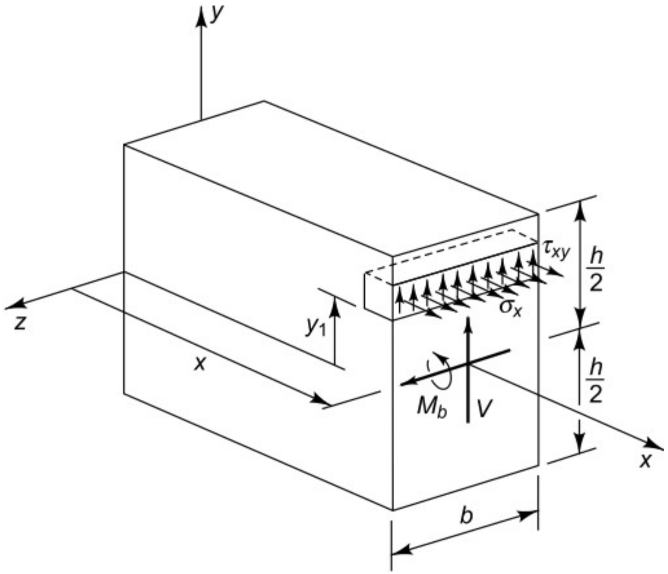
$$\tau_{yx} = \tau_{xy} = \frac{VQ}{bI_z}. \qquad \cdots\cdots\cdots\cdots(16)$$

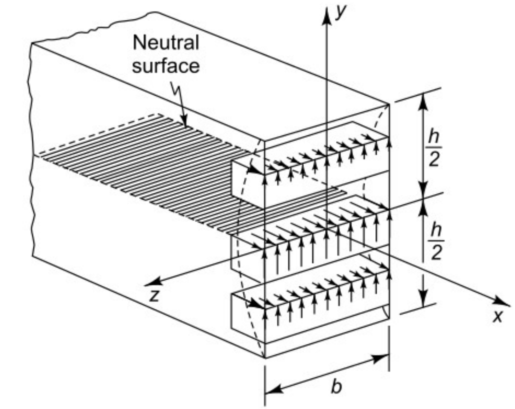


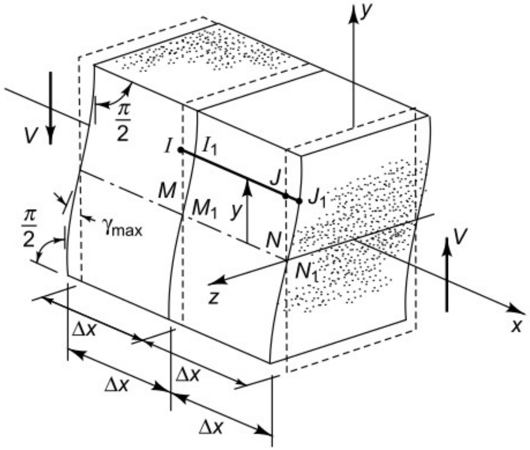
For a narrow beam, the assumption of constant shear stress across the beam is justified.

b/h	0.25	0.5	1	2	4	6	10	20	50
τ_{max}/τ_{ave}	1.008	1.033	1.126	1.396	1.988	2.582	3.770	6.740	15.65
τ_{min}/τ_{ave}	0.996	0.983	0.940	0.856	0.805	0.800	0.800	0.800	0.800

Shear stress distribution in rectangular beams







$$\tau_{xy} = \frac{V}{2I_{zz}}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right]$$

or

$$\tau_{xy} = \frac{3V}{2A}\left[1 - \left(\frac{y_1}{h/2}\right)^2\right]$$

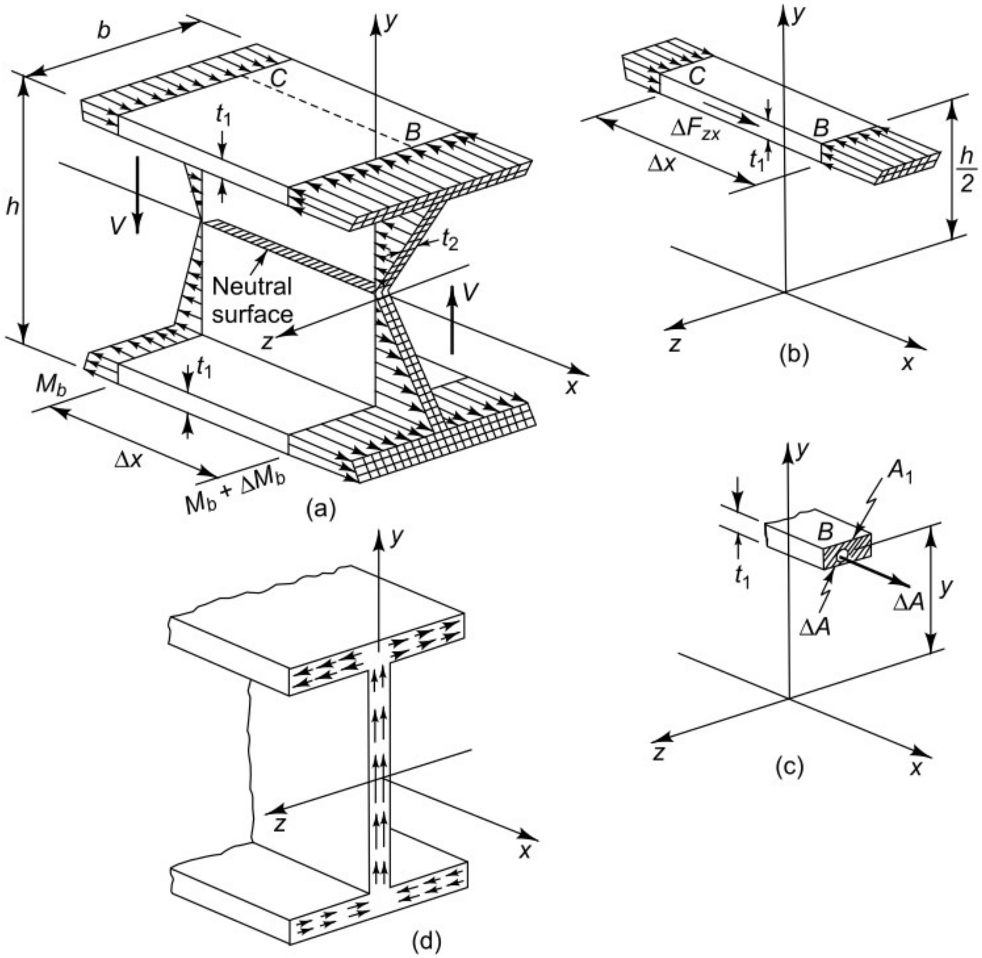
$$\boxed{(\tau_{xy})_{\text{avg}} = \frac{3V}{2A}}$$

(If exact stress variation is assumed)

$$\boxed{(\tau_{xy})_{\text{avg}} = \frac{V}{A}}$$

(If constant stress variation is assumed)

Shear stress distribution in I-beams



Shear flow,

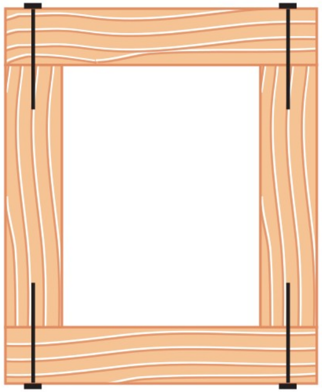
$$q_{zx} = t_1 \tau_{zx} = -\frac{VQ}{I_{zz}}.$$

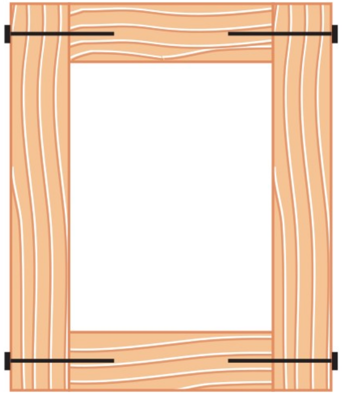
Shear stress at point B,

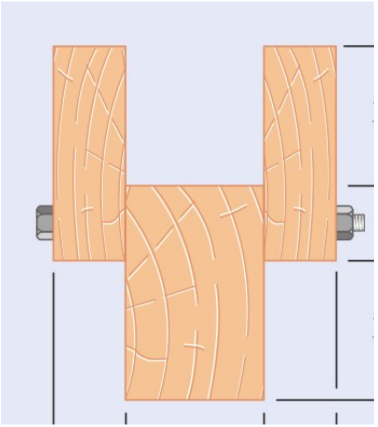
$$\tau_{zx} = -\frac{VQ}{t_1 I_{zz}} = \tau_{xz}.$$

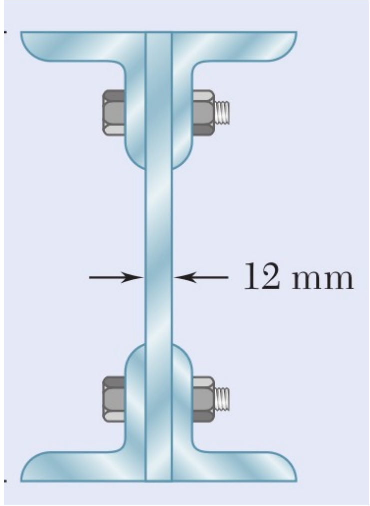
.............(17)

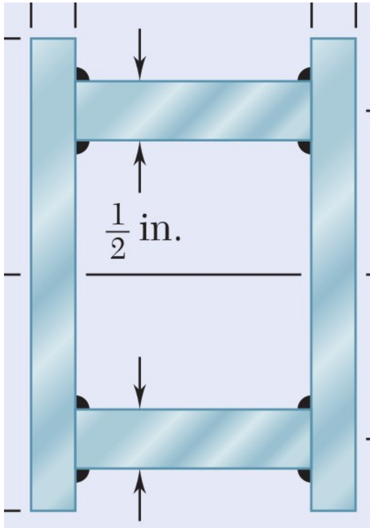
Built-up beams

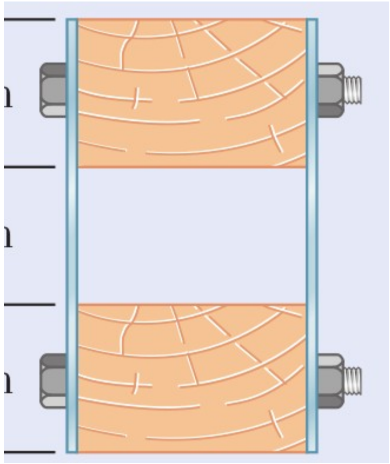












Example 3: Stress analysis in bending-Combined stresses

In figure an elastic circular shaft is shown transmitting simultaneously a bending moment M_b , an axial tensile force P, and a twisting moment M_t . We wish to study the state of combined stress.