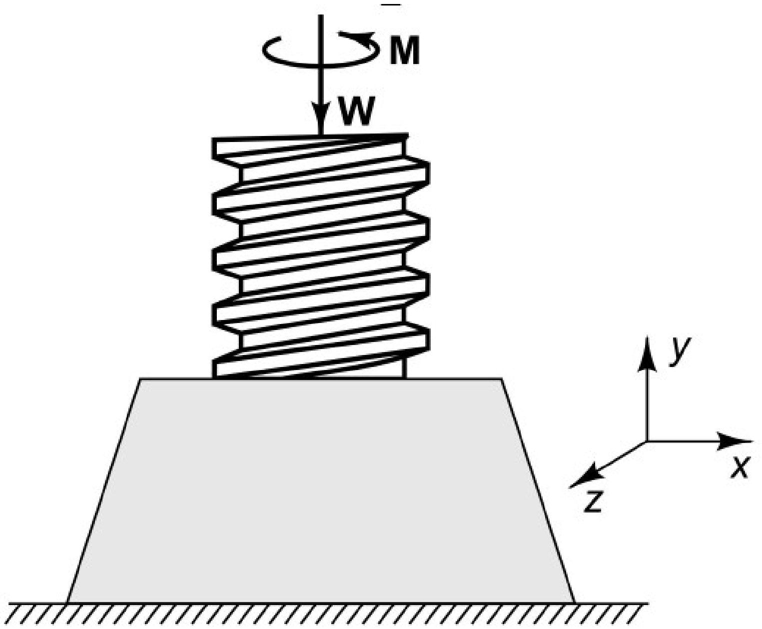
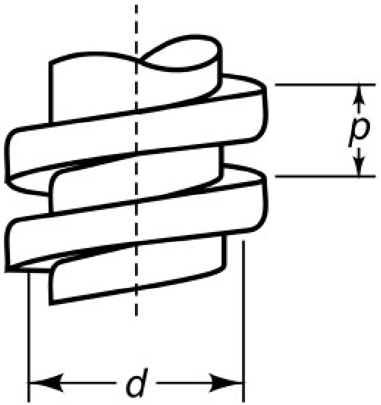
ME231: Solid Mechanics-I

Fundamental principles of Mechanics

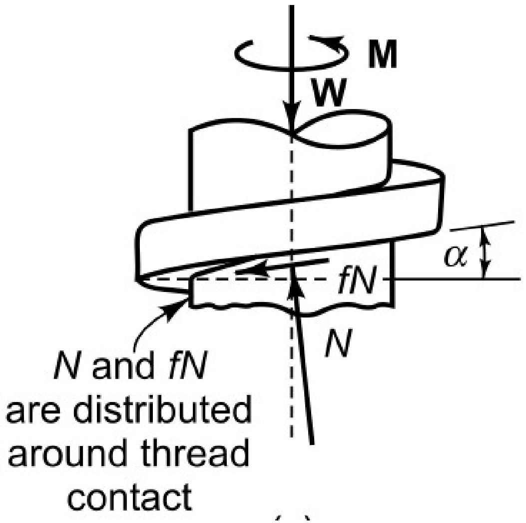
Example 2

A screw jack, which is frequently used to raise or lower weights, is shown. The screw is characterized by a thread pitch p and diameter d. We wish to determine the operating characteristics in the presence of a coefficient of friction f between the screw threads and the jack body. In particular we wish to determine the relationship between the moment necessary to raise and lower the weight W and the frictional and geometrical characteristics of the jack.





Let's isolate the screw. We **idealize** the screw and show the distributed thread loads as acting at one point for convenience of analysis. We see that each portion of the screw must slide up an incline at a helix angle α, where

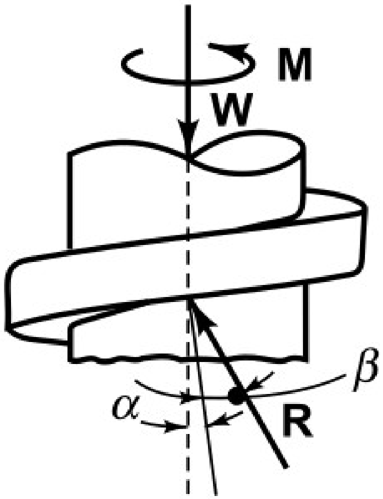


$$\tan\alpha = \frac{p}{\pi d}$$

Above relation can be understood by cutting the thread open and moving over the inclined plane.

Note that friction opposes motion and therefore, the frictional force will flip depending on the direction of the moment applied in trying to rotate the screw.

Applying equilibrium equation for two different directions of rotation, the appropriate moments necessary to start the movement up or down of the screw.



The resultant $\boldsymbol{R}$ of the normal component $\boldsymbol{N}$ and the frictional component fN acts at an angle β to the normal to the screw thread. Thus, $\boldsymbol{R}$ acts at a $\alpha \pm \beta$ to the vertical, depending on whether the jack is being raised, or is being lowered. If we now sum forces along the y-axis and moments about the y-axis, we have

$$\sum F_y = R\cos(\alpha \pm \beta) - W = 0$$

$$\sum M_y = M - \frac{d}{2} R\sin(\alpha \pm \beta) = 0$$

Thus, $M = \dfrac{Wd}{2}\tan(\alpha \pm \beta)$,

where plus sign appears for the moment necessary to move the screw upward and the minus sign for the moment necessary to lower or unwind the screw.

Note that, when $\beta = \alpha$, the moment M vanishes for equilibrium, and the screw will support the weight W without unwinding. If $\beta > \alpha$, a negative M is required to lower the weight. Thus, a jack that has $\beta \geq \alpha$ is said to be self-locking, a desirable property for a jack to have.

For a system of this type an efficiency η can be defined as the ratio of work input to useful work output (the difference being due to wasted frictional heating).

Here, the work input per revolution is $2\pi M$, while the useful work of raising the weight is pW; thus

$$\eta = \frac{pW}{2\pi M} = \frac{\tan\alpha}{\tan(\alpha+\beta)}.$$

For small value of α and β, efficiency is approximately

$$\eta \approx \frac{\alpha}{\alpha+\beta}.$$

And thus for a self-locking device, $\beta \geq \alpha$, the efficiency cannot surpass 50%.

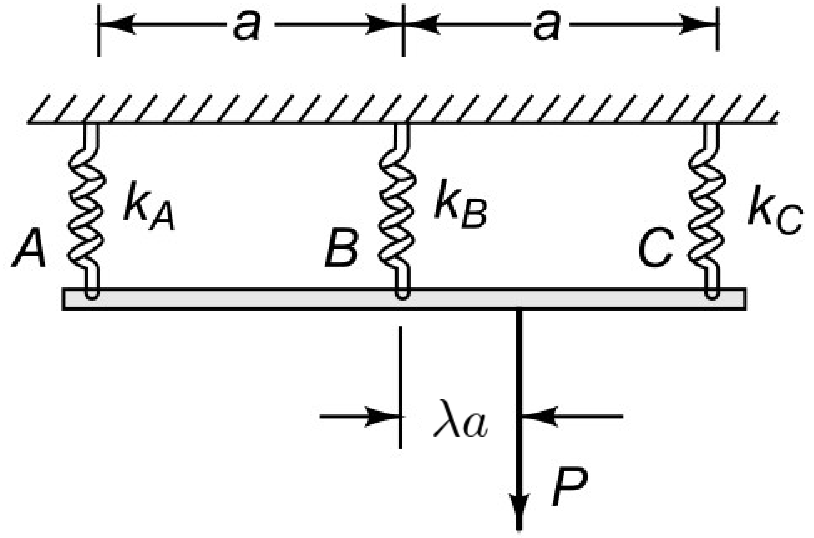
Recall the three steps involved for the analysis based on the principles of mechanics:

1. Study of forces and equilibrium requirements
2. Study of deformation and conditions of geometric fit
3. Applications of force-deformation relations

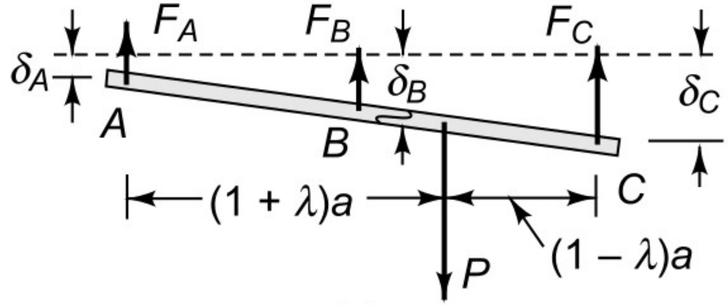
We saw statically determinate problems where step 1 is sufficient to solve the problems. Let us look at some cases now where inclusion of step 2 and 3 become important.

Example 3

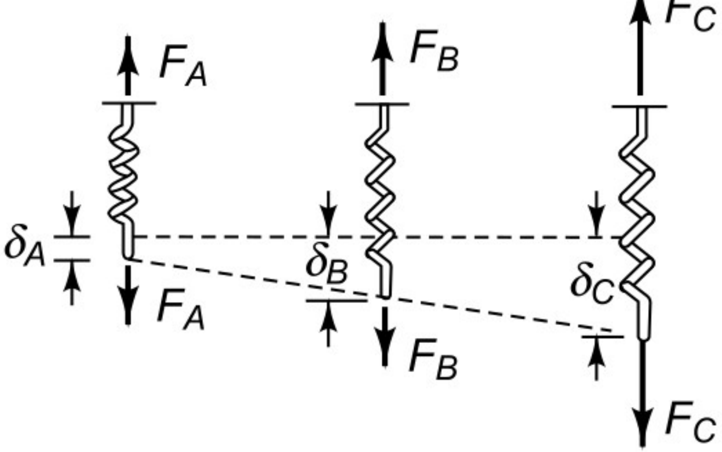
A light rigid bar ABC is supported by three springs, as shown in figure. Before the load P is applied, the bar is horizontal. The distance from the center spring to the point of application of P is λa, where λ is a dimensionless parameter which can vary between $\lambda = -1$ and $\lambda = 1$. The problem is to determine the deflections in the three springs as functions of the load position parameter λ. We shall obtain a general solution for arbitrary values of the spring constants and display the results for the particular set $k_A = (1/2)k$, $k_B = k$ and $k_C = (3/2)k$.



Idealization: The bar is relatively rigid and massless.



(c)



FBD of the bar as well as springs are shown.

Application of equilibrium conditions:
For bar:

$$\sum F = F_A + F_B + F_C - P = 0, \qquad \cdots\cdots\cdots\text{(a)}$$

$$\sum M_A = F_B \cdot a + F_C \cdot 2a - P \cdot (1+\lambda)a = 0, \qquad \cdots\cdots\cdots\text{(b)}$$

It can be observed that the problem is statically indeterminate.

Study of deformation conditions and geometric fit:
If springs has to be connected with the bar, they have to follow specific deflections, as shown in the figure. Relation between the deflections can be written as,

$$2\delta_B = \delta_A + \delta_C. \qquad \cdots\cdots\cdots\text{(c)}$$

Application of force-displacement relations:
Force in each spring can be related to corresponding deflection of spring as,

$$\delta_A = \frac{F_A}{k_A}, \quad \delta_B = \frac{F_B}{k_B}, \quad \text{and} \quad \delta_C = \frac{F_C}{k_C}. \qquad \text{...............(d)}$$

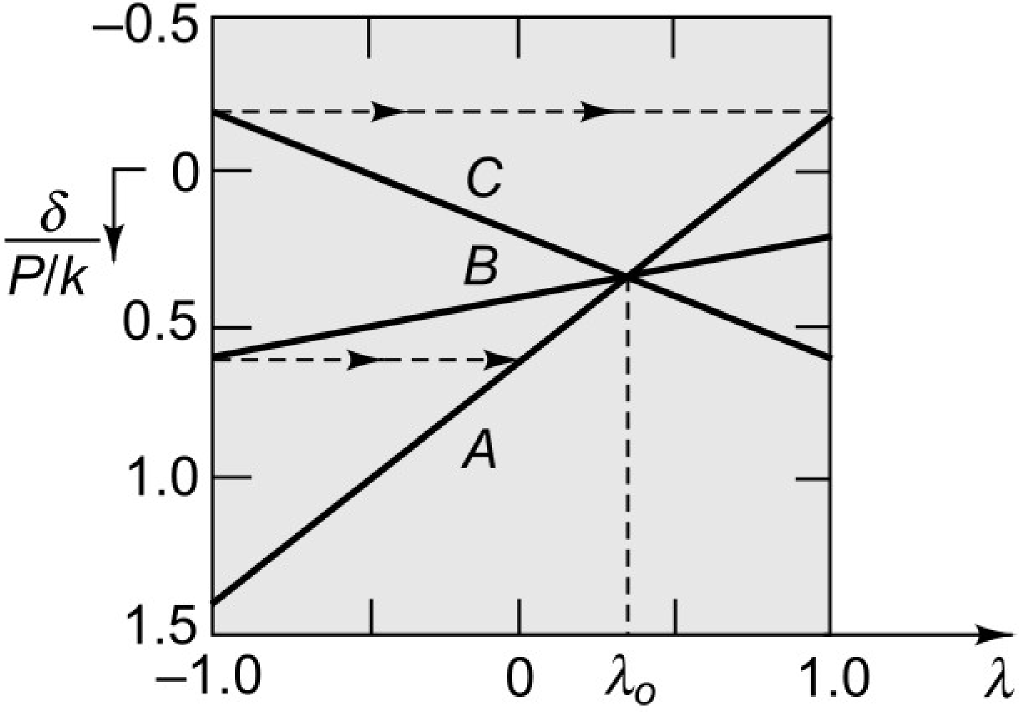
Equations (a)-(d) are six independent equations in six unknowns (three forces and three deflections). These equations can be solved very easily for all six unknowns.
Deflections are found to be,

$$\delta_A = P\frac{2k_C - \lambda(k_B + 2k_C)}{k_A k_B + 4k_A k_C + k_B k_C}, \qquad \delta_B = P\frac{k_A + k_C + \lambda(k_A - k_C)}{k_A k_B + 4k_A k_C + k_B k_C}, \quad \text{and}$$

$$\delta_C = P\frac{2k_A + \lambda(k_B + 2k_A)}{k_A k_B + 4k_A k_C + k_B k_C}. \qquad \text{...............(e)}$$

Note that deflections of all springs are proportional to P and linear function of λ.

Let us try to understand more about the deformation. We plot all deflections as function of λ for the given stiffness values.



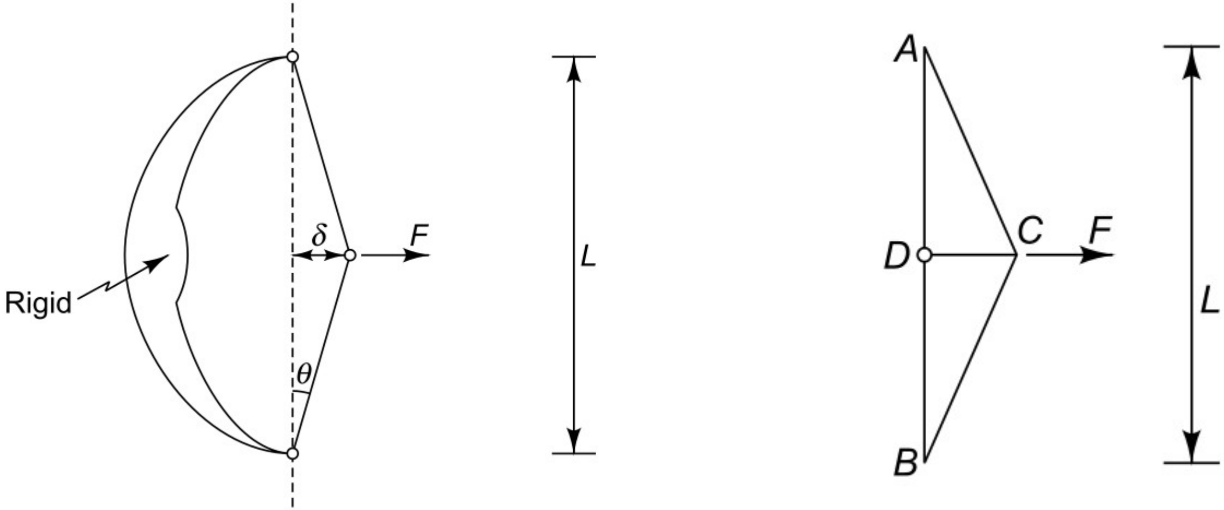
λ_0 can be determined as,

$$\lambda_0 = \frac{k_C - k_A}{k_A + k_B + k_C},$$

and deflection of all points correction to λ_0 is

$$\delta(\lambda_0) = \frac{P}{k_A + k_B + k_C}.$$

Example 4



For the bow shown in figure, plot Force vs displacement relation. The string is under initial tension. The string follows a linear load-deflection relation.

The force in the string AC can be calculated as,

$$F_{AC} = F/\left(2\sin\theta\right) \qquad \cdots\cdots\cdots\cdots\cdots(l)$$

Elongation of the string AC,

$$\delta_{\mathrm{AC}} = AC - AB/2 = L/(2\cos\theta) - L/2 \qquad \cdots\cdots\cdots\cdots\cdots(m)$$

Using load-deflection relationship for string as,

$$F_{\mathrm{AC}} = k\delta_{\mathrm{AC}} \qquad \cdots\cdots\cdots\cdots\cdots(n)$$

Using (l)-(n), we get

$$\frac{F}{2\sin\theta} = \frac{kL}{2}\frac{1-\cos\theta}{\cos\theta} \Rightarrow F = 2k\frac{L\tan\theta}{2}(1-\cos\theta)$$

$$\Rightarrow F = 2k\delta_{\mathrm{CD}}(1-\cos\theta) = 4k\delta_{\mathrm{CD}}\sin^2(\theta/2) = k\delta_{\mathrm{CD}}\theta^2 = 4\delta_{\mathrm{CD}}^3 k/L^2.$$

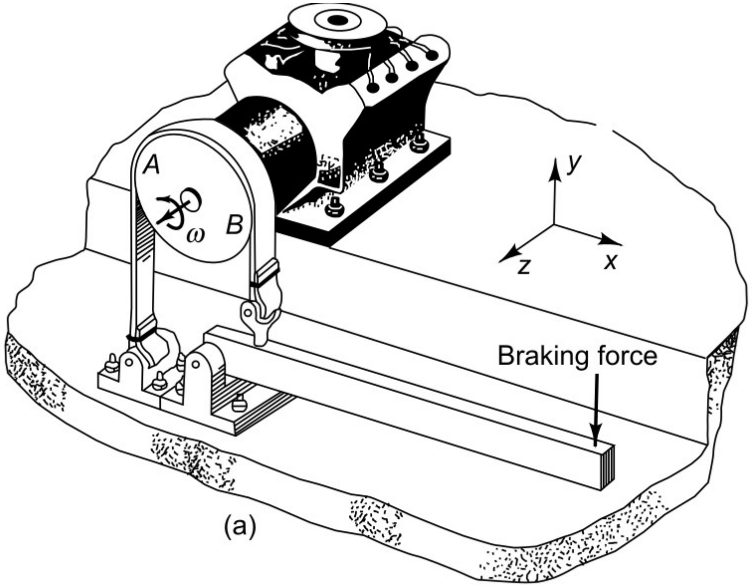
$$\text{(for small } \theta,\ \tan\theta \approx \theta = 2\delta_{\mathrm{CD}}/L)$$

$$\Rightarrow F = \frac{4k\delta_{\mathrm{CD}}^3}{L^2}.$$

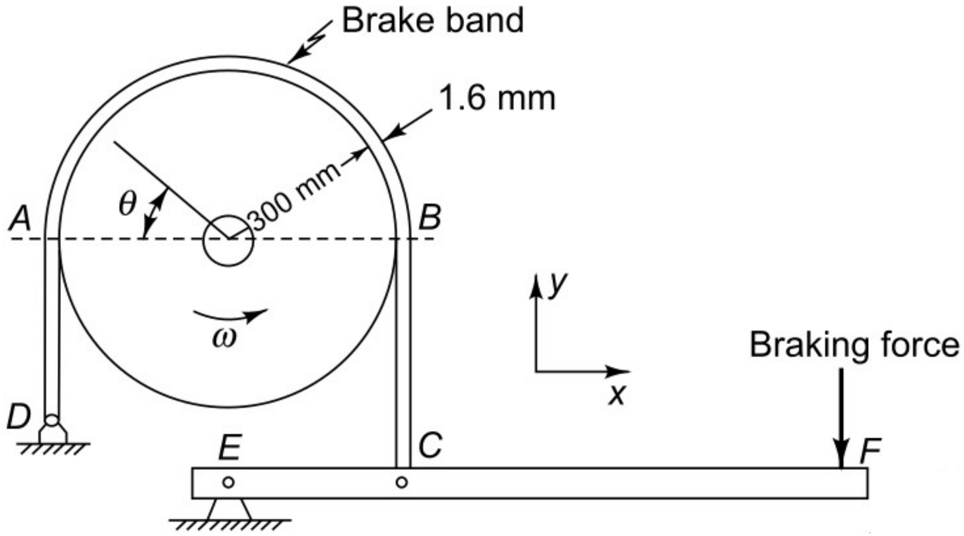
We thus get a non-linear response. Notice importantly that it is possible to obtain a non-linear response from linear springs as shown.

Example 5

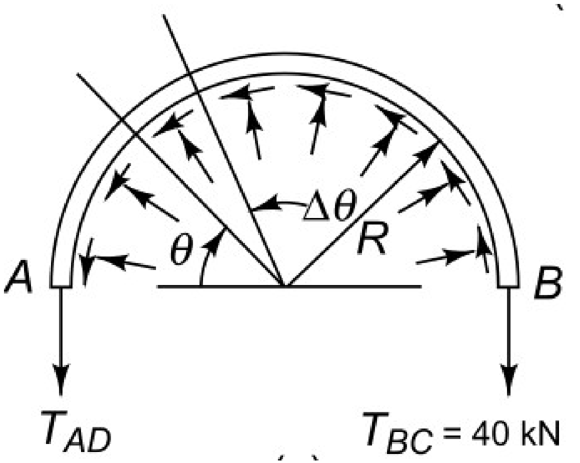
In a test on an engine, a braking force is supplied through a lever arm EF to a steel brake band $CBAD$ which is in contact with half the circumference of a 600-mm-diameter flywheel. The brake band is 1.6 mm thick and 50 mm wide and is lined with a relatively soft material which has a kinetic coefficient of friction $f = 0.4$ with respect to the rotating flywheel. The operator wishes to predict how much elongation there will be in the section AB of the brake band when the braking force is such that there is a tension of 40 kN in the section BC of the band.



(a)



As a very first step let us draw the FBD of section AB of the strap.

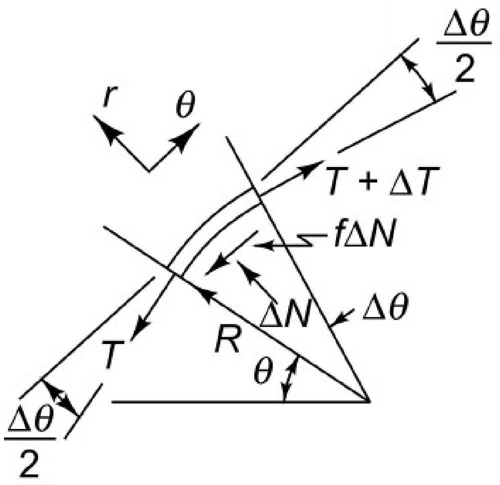


Distributed Normal force and the the friction force between the drum and the strap is shown.

Now consider a small portion of the strap. FBD of the section is shown. Writing equilibrium equation for the section,

$$\sum F_r = \Delta N - T\sin(\Delta\theta/2) - (T + \Delta T)\sin(\Delta\theta/2) = 0,$$

$$\Rightarrow \Delta N - 2T\sin(\Delta\theta/2) - \Delta T\sin(\Delta\theta/2) = 0, \qquad \cdots\cdots(\text{o})$$



$$\sum F_\theta = (T + \Delta T)\cos(\Delta\theta/2) - T\cos(\Delta\theta/2) - f\Delta N = 0,$$

$$\Rightarrow \Delta T\cos(\Delta\theta/2) - f\Delta N = 0, \qquad \cdots\cdots(\text{p})$$

For small angle $\Delta\theta$, we can write (o) as,

$$\Delta N = (2T + \Delta T)\Delta\theta/2,$$

Neglecting ΔT compared to T, $\Delta N = T\Delta\theta$.

Also (p) become, $\Delta T = f\Delta N = fT\Delta\theta$, or $\dfrac{\Delta T}{\Delta\theta} = fT$.

For $\Delta\theta \to 0$,

$$\lim_{\Delta\theta\to 0} \frac{\Delta T}{\Delta\theta} = \frac{dT}{d\theta} = fT.$$

Integrating above relation and applying the boundary condition $T = T_{AD}$ at $\theta = 0$,we get,

$$T = T_{AD}e^{f\theta},$$

T_{AD} can be obtained by considering that $T = T_{BC} = 40$ kN at $\theta = \pi$.

Finally, $$T = 11.38e^{f\theta}. \quad \cdots\cdots\cdots\cdots\text{(p)}$$

Note that tension in the strap varies exponentially with angle θ. For fully wrapped strap (i.e., θ=2π) tension at one end will be even higher.

To determine the elongation in section AB, we first calculate the elongation in small element as,

$$\Delta\delta = \frac{T}{K} = \frac{T}{AE/R\Delta\theta}. \quad \cdots\cdots\cdots\cdots\text{(q)}$$

Total elongation in section AB can be obtained by integration of (q) after substituting (p) as,

$$\delta_{AB} = \int_A^B d\delta = \int_{\theta=0}^{\pi} \frac{TR}{AE} d\theta.$$