ME232: Dynamics

Vibration

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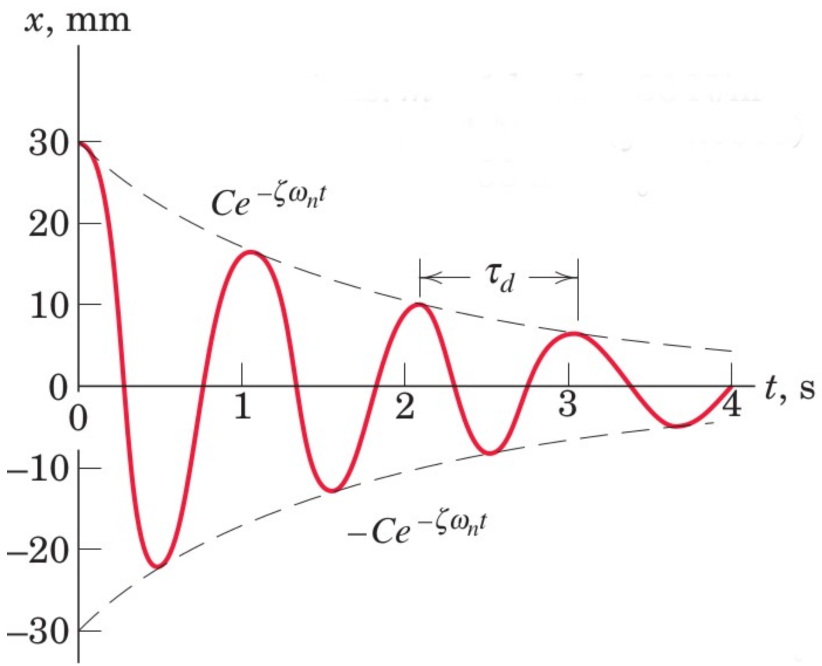
Determination of damping by experiment:

The value of the damping ratio ζ need to be determined experimentally for an underdamped system because the value of the viscous damping coefficient c is not well known.



To determine the damping, we may excite the system by initial conditions and obtain a plot of the displacement x versus time t. We then measure two amplitudes x_1 and x_{N+1}, N full cycle apart and compute their ratio

$$\frac{x_1}{x_{N+1}} = \frac{Ce^{-\zeta\omega_n t_1}}{Ce^{-\zeta\omega_n (t_1+N\tau_d)}} = e^{\zeta\omega_n N\tau_d}. \qquad \text{.............(17)}$$

The logarithmic decrement δ is defined as, $\delta = \frac{1}{N}\ln\left(\frac{x_1}{x_{N+1}}\right) = \zeta\omega_n\tau_d = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$. ......(18)

From this equation, we may solve for ζ and obtain $\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$. ....(19)

Example 1

A body weighing 10 kg is suspended from a spring of constant $k = 2.5$ kN/m. At time $t = 0$, it has a downward velocity of 0.5 m/sec as it passes through the position of static equilibrium. Determine

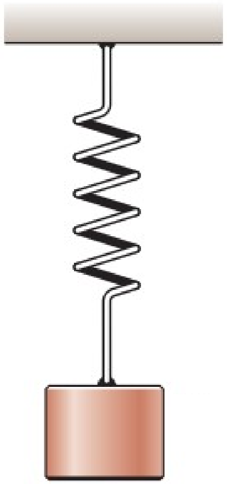
(a) the static spring deflection δ_{st}

(b) the natural frequency of the system in both rad/sec (ω_n) and cycles/sec (f_n)

(c) the system period τ

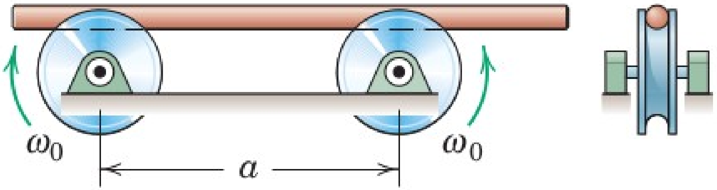
(d) the displacement x as a function of time, where x is measured from the position of static equilibrium

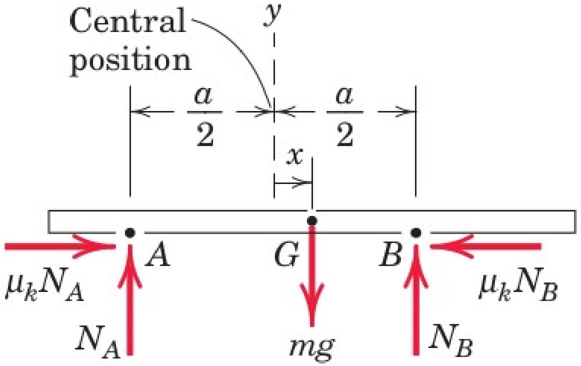
(e) the maximum velocity v_{max} attained by the mass.



Example 2

The two fixed counter rotating pulleys are driven at the same angular speed ω_0 . A round bar is placed off center on the pulleys as shown. Determine the natural frequency of the resulting bar motion. The coefficient of kinetic friction between the bar and pulleys is μ_k .





The governing equations are

$$\sum F_x = m\ddot{x} \qquad \mu_k N_A - \mu_k N_B = m\ddot{x},$$

$$\sum F_y = 0 \qquad N_A + N_B - mg = 0,$$

$$\sum M_A = 0 \qquad aN_B - (a/2 + x)mg = 0,$$

Eliminating N_A and N_B from the first equation yields

$$\ddot{x} + \frac{2\mu_k g}{a}x = 0.$$

Thus, natural frequency is,

$$\omega_n = \sqrt{\frac{2\mu_k g}{a}}.$$

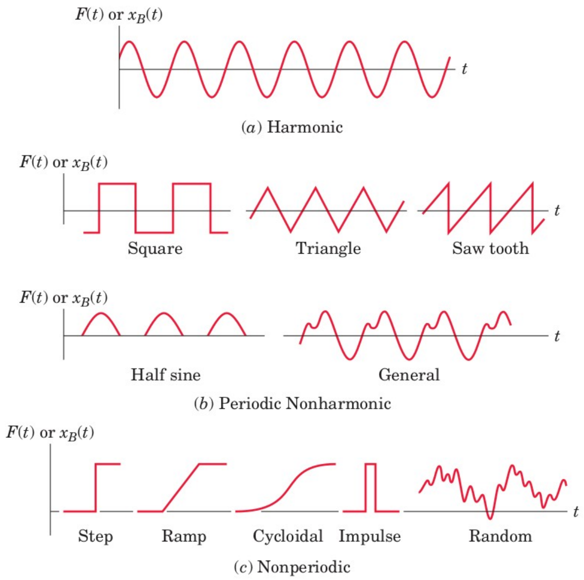
Forced vibration

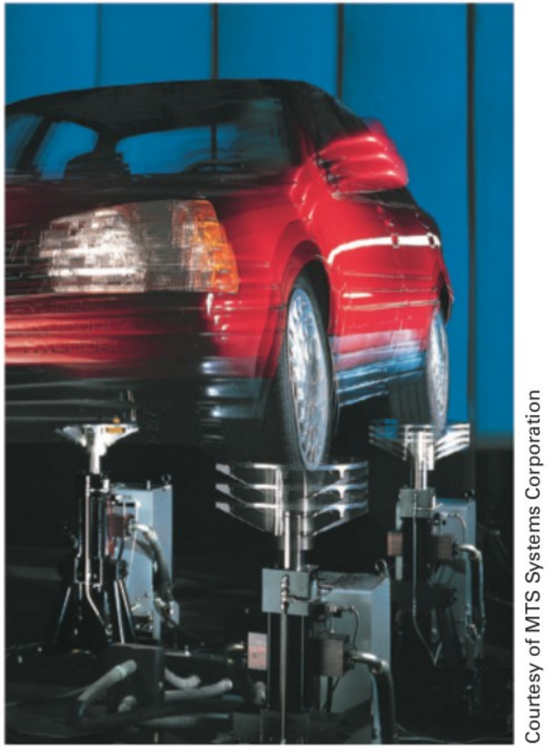
Although there are many significant applications of free vibrations, the most important class of vibration problems is that where the motion is continuously excited by a disturbing force.

The force may be externally applied or may be generated within the system by such means as unbalanced rotating parts.

Forced vibrations may also be excited by the motion of the system foundation.

Various forms of excitation forcing functions $F = F(t)$ and foundation displacements $x_B = x_B(t)$ are shown in figure.





An automobile undergoing vibration testing of its suspension system.

Harmonic excitation:

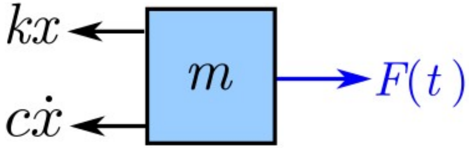
As a very first step we start with the harmonic force which occurs frequently in engineering.

The understanding of the analysis associated with harmonic forces is also a necessary step in the study of more complex forms.

Consider a system where the body is subjected to the external harmonic force $F = F_0 \sin \omega t$, in which F_0 is the force amplitude and ω is the driving frequency (in radians/second).

Note that ω_n, which is a property of the system, is different from ω, which is a property of the force applied to the system.

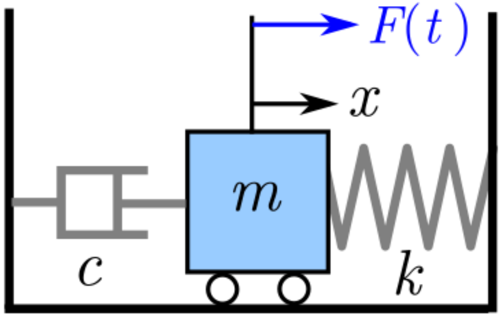
From the free-body diagram, we apply Newton's second law to obtain



$$-kx - c\dot{x} + F_0 \sin \omega t = m\ddot{x} \qquad \text{or} \qquad m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t.$$

Alternate form of the above equation is,

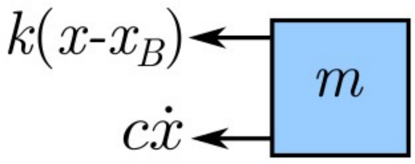
$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t. \qquad \cdots\cdots\cdots\cdots(20)$$



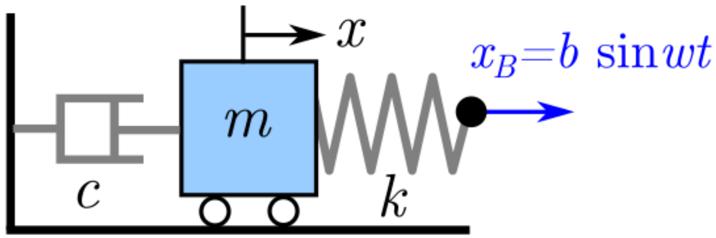
Base excitation:

In many cases, the excitation of the mass is due to the movement of the base or foundation to which the mass is connected by springs or other compliant mountings; e.g., seismographs, vehicle suspensions, and structures shaken by earthquakes.

Harmonic movement of the base is equivalent to the direct application of a harmonic force.



Consider the system where the spring is attached to a movable base. x is the displacement of mass from the equilibrium position when the base were in its neutral position.



The base, is assumed to have a harmonic movement $x_B = b\ \sin\omega t$. The spring deflection is the difference between the inertial displacements of the mass and the base.

From the free-body diagram, Newton's second law gives

$$-k(x - x_b) - c\dot{x} = m\ddot{x} \quad \text{or} \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{kb}{m}\sin\omega t. \qquad \cdots\cdots\cdots\cdots(21)$$

It can be observed (21) is similar to (20).