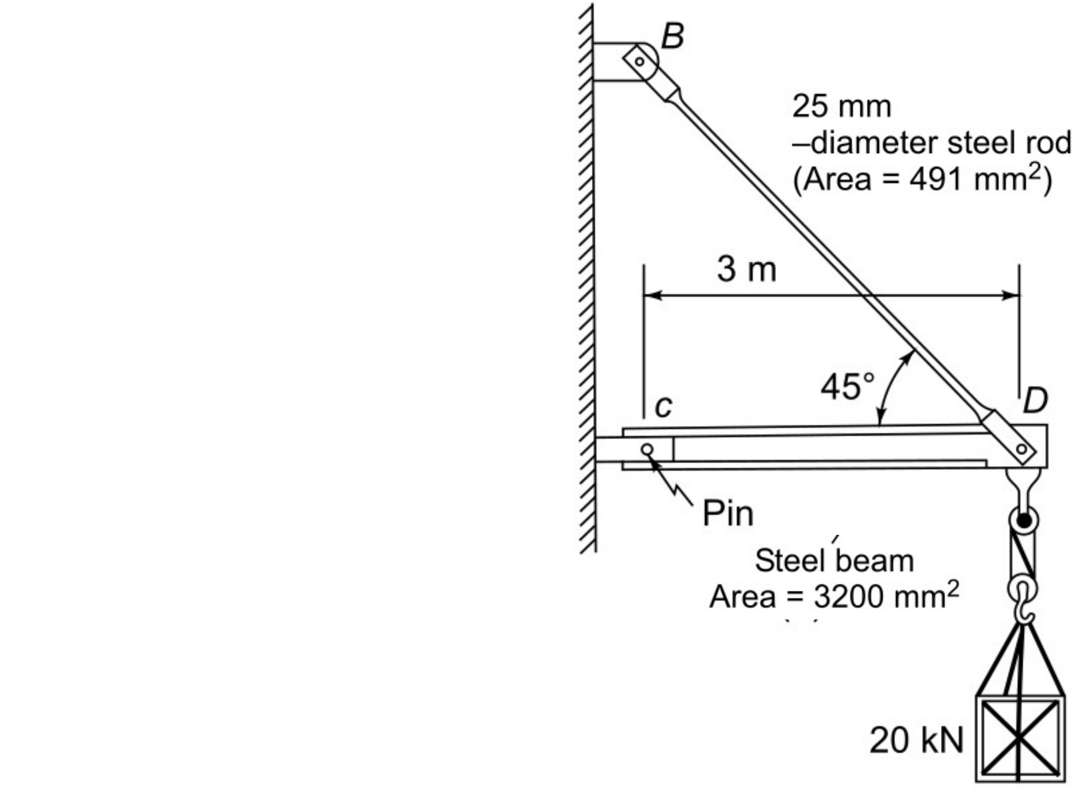
ME231: Solid Mechanics-I

Forces and Moments Transmitted by Slender Members

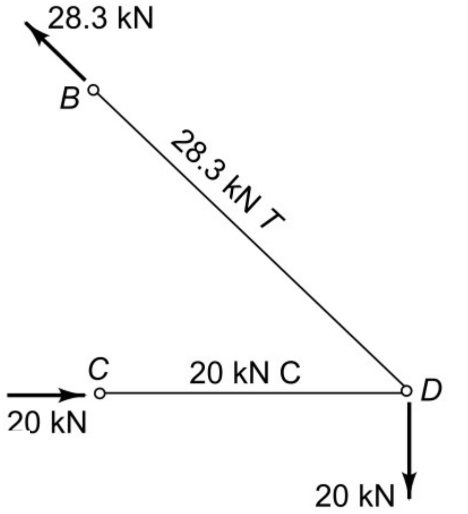
Example 2

A triangular frame supporting a load of 20 kN. Our aim is to estimate the displacement at the point D due to the 20-kN load carried by the chain hoist.

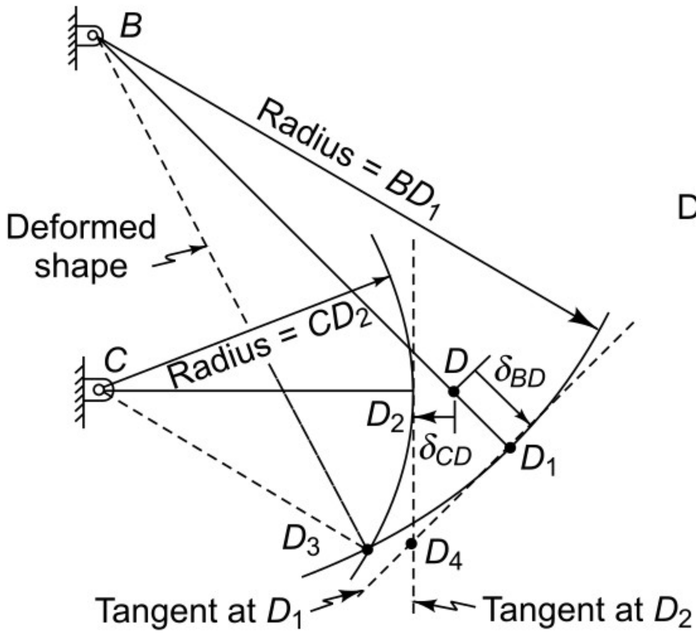


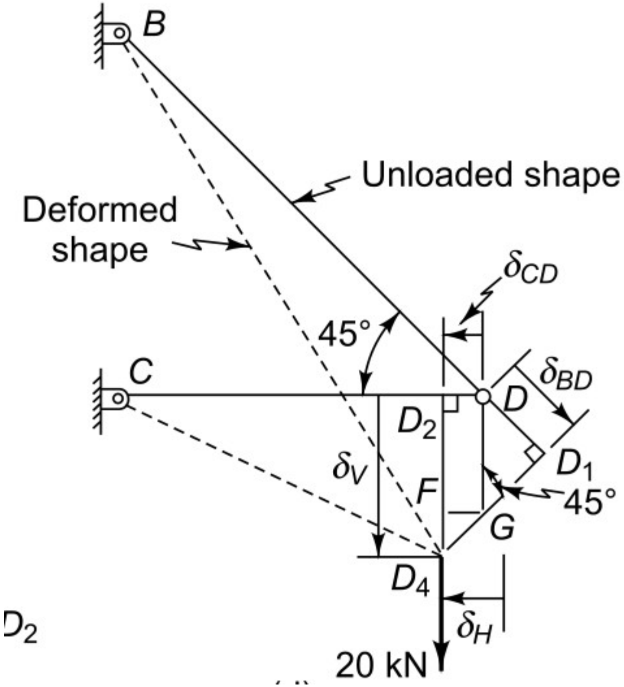
Application of equilibrium conditions:

With the application of equilibrium conditions for the frame as well as for individual links of the frame reaction forces and load in link BD and CD can be calculated (already solved).



Geometric compatibility:





Application of force-displacement relation:

Considering axial load in link BD and CD, axial deflections of links can be calculated as,

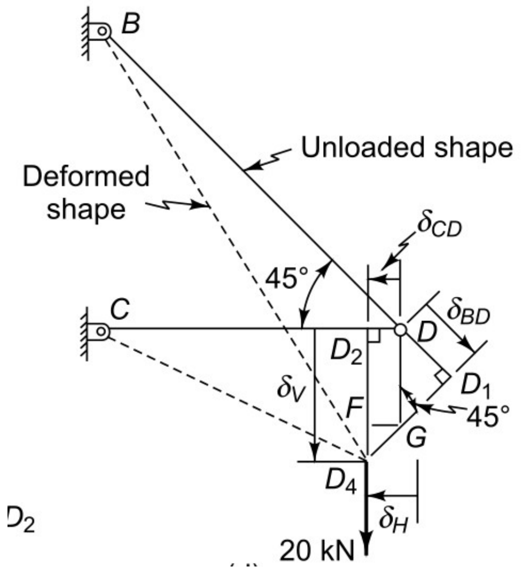
$$\delta_{\mathrm{BD}} = \left(\frac{FL}{AE}\right)_{\mathrm{BD}}, \quad \delta_{\mathrm{CD}} = \left(\frac{FL}{AE}\right)_{\mathrm{CD}}. \qquad \cdots\cdots\cdots\cdots\cdots(2.\mathrm{a})$$

Substituting corresponding given values in (2.a) deflection can be determined.

Using the geometry, horizontal as well as vertical deflection of point D is,

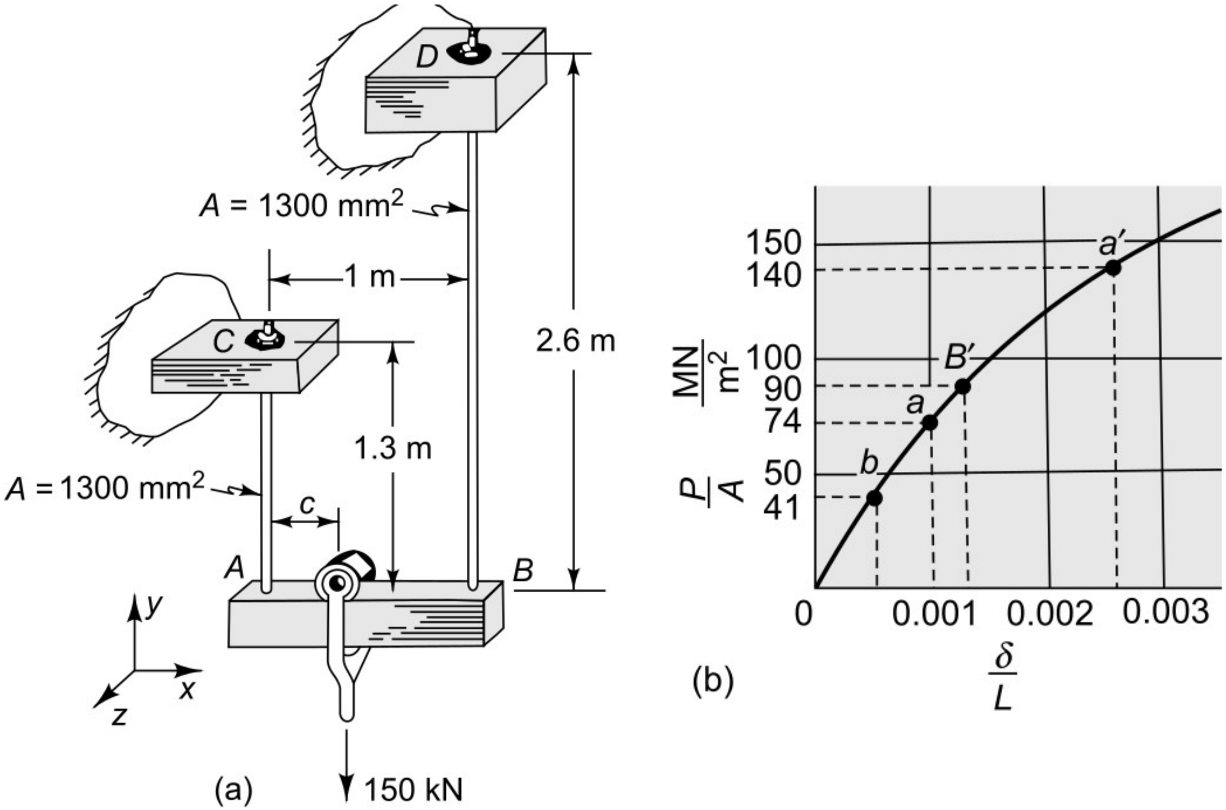
$$\delta_H = \delta_{\mathrm{CD}}$$

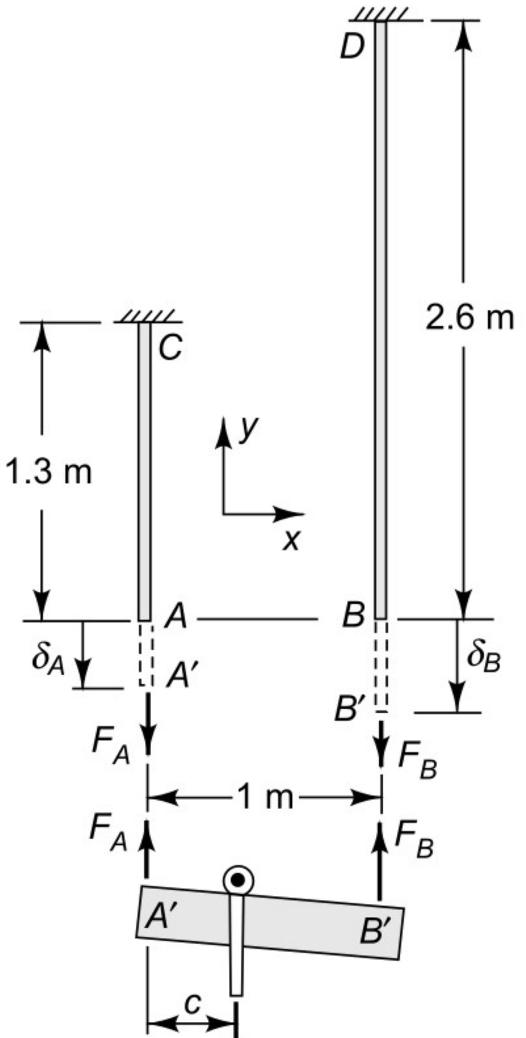
$$\delta_V = \mathrm{D_2F} + \mathrm{FD_4} = \sqrt{2}\delta_{\mathrm{BD}} + \delta_{\mathrm{CD}}$$



Example 3

The stiff horizontal beam AB in is supported by two soft copper rods AC and BD of the same cross-sectional area but of different lengths. The load-deformation diagram for the copper is also shown. A vertical load of 150 kN is to be suspended from a roller which rides on the horizontal beam. We do not want the roller to move after the load is put on, so we wish to find out where to locate the roller so that the beam will still be horizontal in the deflected position.





(c)

As a very first step, isolate the beam AB and draw its FBD.

Consider force equilibrium of beam AB as,

$$\sum F = F_A + F_B - 150 = 0, \quad \cdots\cdots\cdots\cdots\cdots(3.a)$$

$$\sum M_A = F_B \cdot 1 - 150 \cdot c = 0, \quad \cdots\cdots\cdots\cdots\cdots(3.b)$$

Now, geometric requirement for the beam to be in horizontal position is

$$\delta_A = \delta_B. \quad \cdots\cdots\cdots\cdots\cdots(3.c)$$

(3.c) can also be written as,

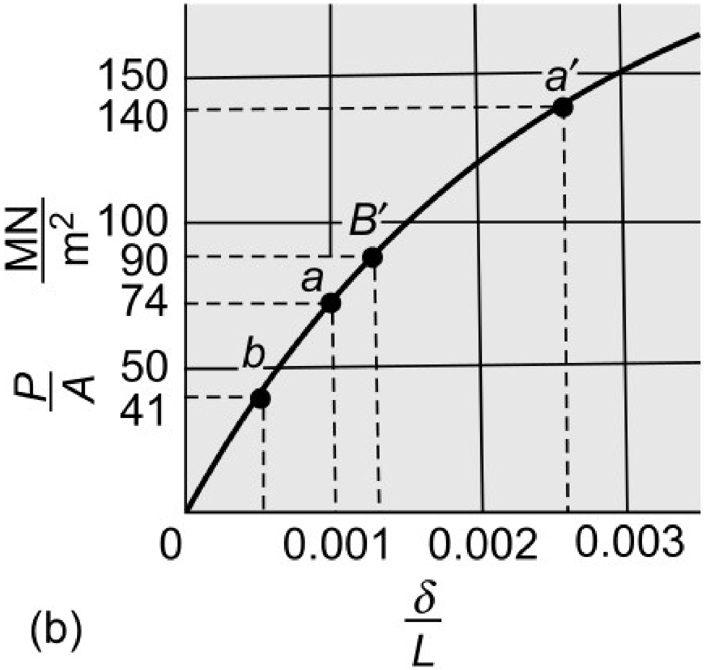
$$\frac{\delta_A}{L_A} = \frac{\delta_B}{L_A} = \frac{\delta_B}{L_B}\frac{L_B}{L_A} = 2\frac{\delta_B}{L_B}. \quad \cdots\cdots\cdots\cdots\cdots(3.d)$$

We can also write (3.a) as,

$$\frac{F_A}{A_A} + \frac{F_B}{A_A} = \frac{150}{A_A} \quad \text{or} \quad \frac{F_A}{A_A} + \frac{F_B}{A_B} = 115\ \mathrm{MN/m^2} \quad \cdots\cdots\cdots\cdots\cdots(3.e)$$

$$(\because A_A = A_B)$$

As we do not have an analytical expression for relation between load and deflection, solution for this problem will not be easy. An iterative approach will be required for the solution.



(b)

First assume a value of δ_A/L_A, and find the value of δ_B/L_B using (3.d).

Find the corresponding values of F_A/A_A and F_B/A_B from the curve.

Substitute these values in (3.e) and check if these values satisfy the equation. If yes, F_A and F_B are known.

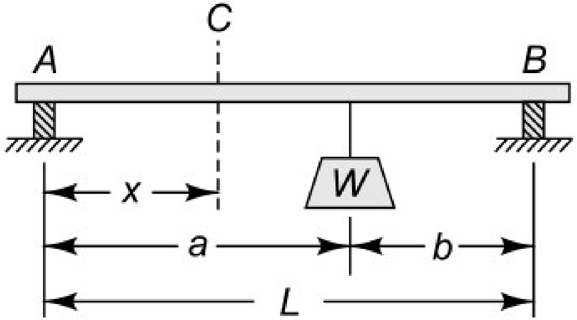
If not, then assume a different value of δ_A/L_A, and repeat the previous steps till (3.e) is satisfied.

Once F_B is known, (3.b) can be used to determine the value of c.

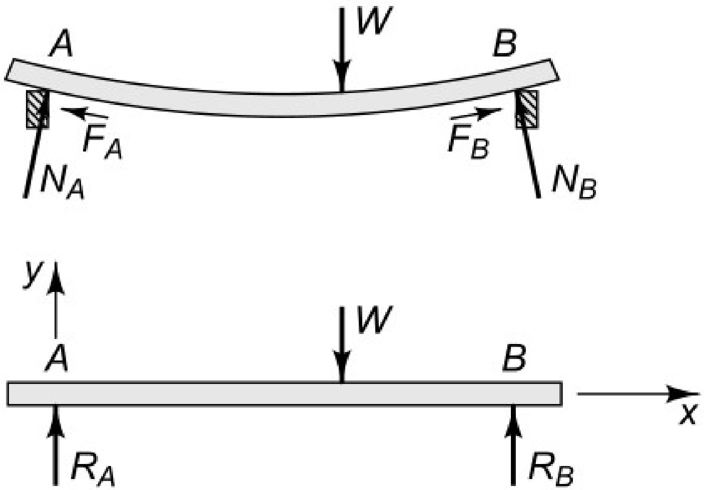
Bending and Torsion Loads

Example 4

Let us consider a beam supporting a weight near the center and resting on two other beams, as shown in Figure. It is desired to find the distribution of forces and moments along the beam.



Idealization



Find the reaction forces by applying the equilibrium equations.

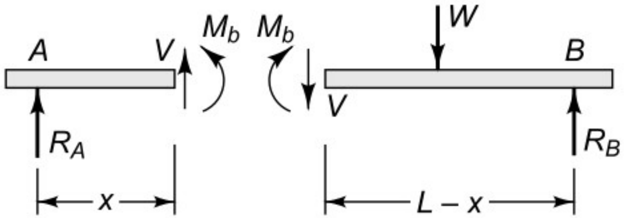
$$\sum F_y = R_A + R_B - W = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(4.a)}$$

$$\sum M_A = R_B \cdot L - W \cdot a = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(4.b)}$$

Using (4.a) and (4.b), R_A and R_B can be determined as,

$$R_A = Wb/L, \quad R_B = Wa/L. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{(4.c)}$$

Now all the external forces are known. Let us find the distribution of internal forces and moments along the length of the beam. To do that we consider a section at any point along the beam and apply equilibrium equation to the sub-systems.



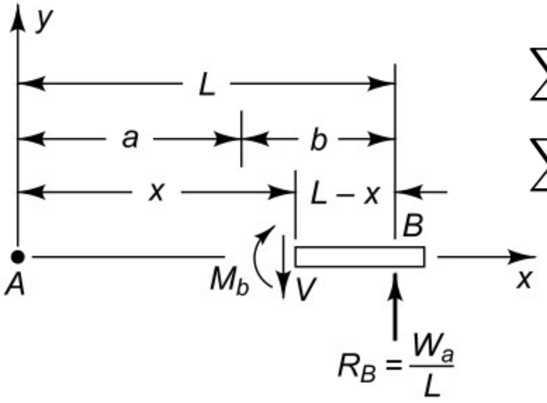
Considering the equilibrium of left part,

$$\sum F_y = R_A + V = 0 \Rightarrow V = -R_A = -Wb/L \quad \cdots\cdots\cdots\text{(4.d)}$$

$$\sum M_A = V \cdot x + M_b = 0 \Rightarrow M_b = -Vx = Wbx/L$$

Note that above equations are application at any point between point A and the point of application of W, i.e., $0 \leq x \leq a$.

To get the distribution for rest of the beam, section is taken between the point of application of W and point B and consider the the equilibrium of the subsection.

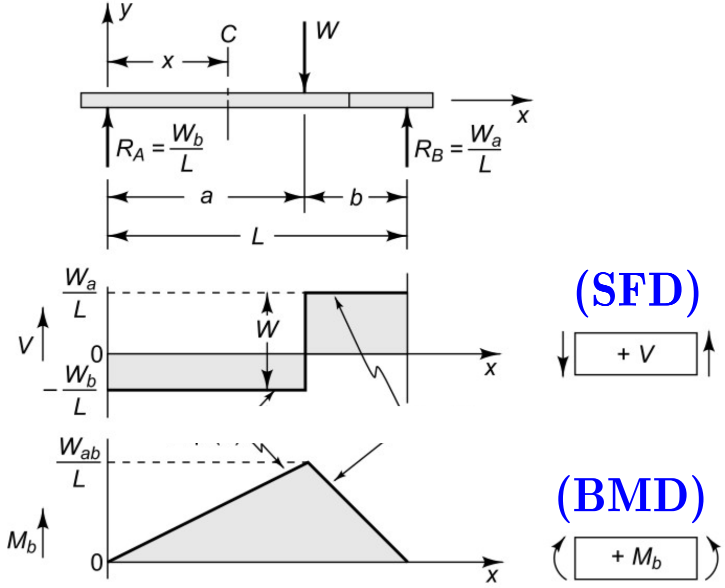


$$\sum F_y = R_B - V = 0 \Rightarrow V = R_A = Wa/L$$

$$\sum M_A = V \cdot (L-x) - M_b = 0 \Rightarrow M_b = V(L-x) = Wa(L-x)/L \quad \text{(4.e)}$$

Equations (4.e) are application for i.e., $a \leq x \leq b$.

Distribution of forces and moments (Equations 4.d and 4.e) can be graphically shown using **Shear Force Diagram (SFD)** and **Bending Moment Diagram (BMD)**.

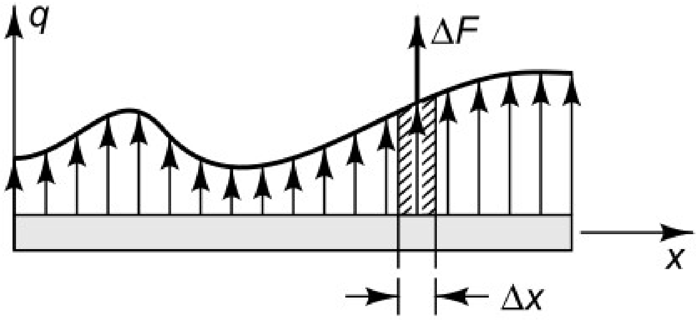


Distributed loads

In Example 4, we idealized the forces working between the slender member and supports or loading mechanism as **point forces (concentrated at a single point)**.

Another idealization which is generally used is the concept of **continuously distributed** loading.

Figure shows a distributed loading on the beam. Such forces might result from fluid or gas pressures, or from magnetic or gravitational attractions. If the total force on a length Δx be denoted by ΔF; then the intensity of loading q is defined as the limit as



$$q(x) = \lim_{\Delta x \to 0} \frac{\Delta q}{\Delta x}.$$

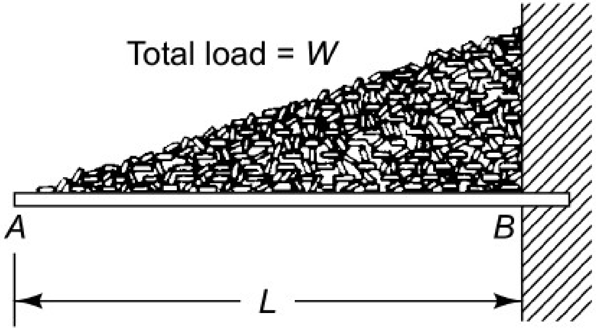
Most commonly used distributed loading in engineering applications are **uniformly distributed loading** (i.e., q is constant) and **linearly varying distribution** (i.e. $q(x)=A+Bx$)

Example 5

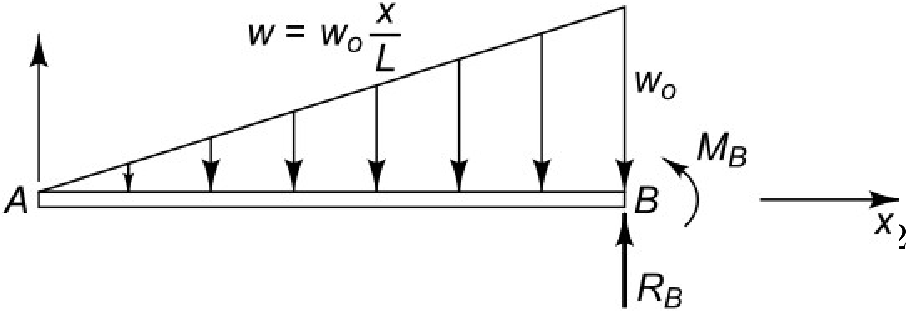
Consider a cantilever beam built in at the right end. Bricks having a total weight W have been piled up in triangular fashion. It is desired to obtain shear-force and bending-moment diagrams.

Total load W can be idealized as a linearly distributed load on the beam.
i.e., $w(x) = A+Bx$. Constants A and B can be determined as,
(i) considering that at x=0, q=0, thus A=0.
(ii) $\int_0^L w(x)dx = \int_0^L Bxdx = W \Rightarrow B = 2W/L^2$.

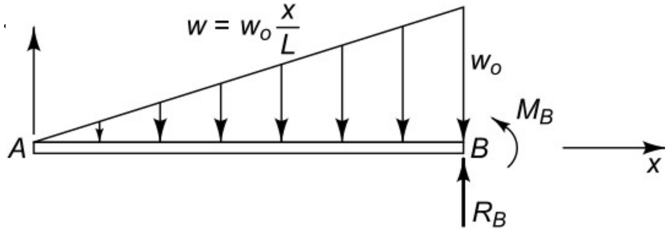
Thus, $w(x) = 2Wx/L^2 = w_0 x/L$, where w_0 is the load at x=L, i.e., $w(L) = w_0 = 2W/L$.



Idealization

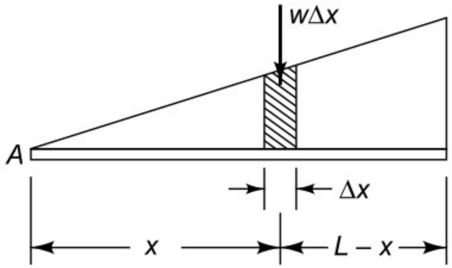


Find the support reactions by applying the equilibrium equations.



$$\sum F_y = R_B - \int_0^L w(x)dx = 0$$

$$\Rightarrow R_B = \int_0^L w_0 x/L dx = w_0 L/2. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5.a)$$



$$\sum M_b = M_B + \int_0^L w(x)(L-x)dx = 0$$

$$\Rightarrow M_B = WL/3. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5.b)$$

Now, to get the distribution of shear force and bending moment along the length of the beam, we take a section at a distance x and consider the equilibrium of sub-system as,

$$\sum F_y = V - \int_0^x w(\xi)d\xi = 0$$

$$\Rightarrow V = \frac{w_0 x^2}{2L}. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5.c)$$

$$\sum M = M_b + \int_0^x w(\xi)(L-\xi)d\xi = 0$$

$$\Rightarrow M_b = -\frac{w_0 x^3}{6L}. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5.d)$$