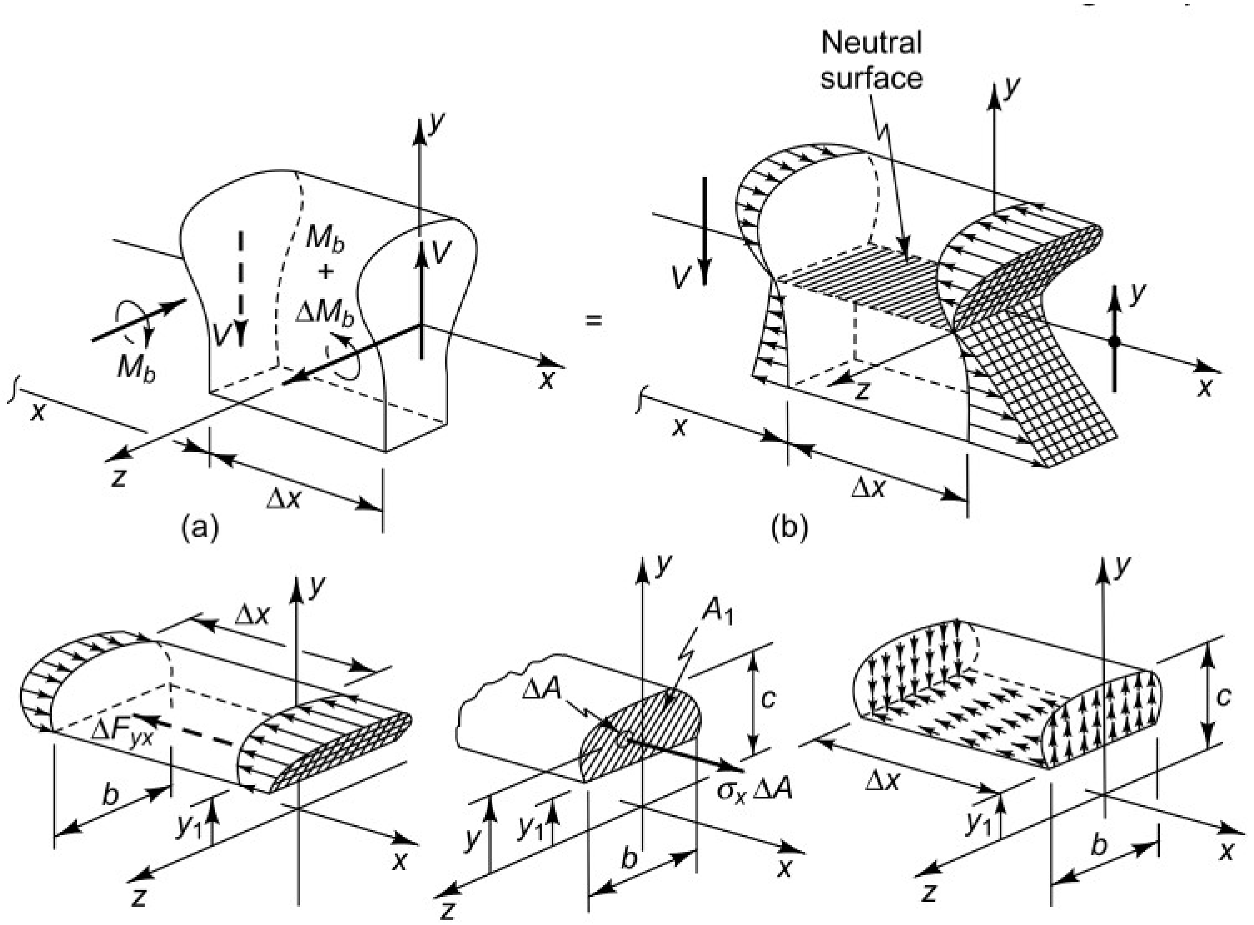
ME231: Solid Mechanics-I

Stresses due to bending

Stresses in symmetrical elastic beams transmitting both Shear force and bending moment



Shear stress at $y{=}y_1$ is,

$$\tau_{yx} = \frac{VQ}{bI_{zz}} = \tau_{xy},$$

where,

$$Q = \int_{A_1} y dA.$$

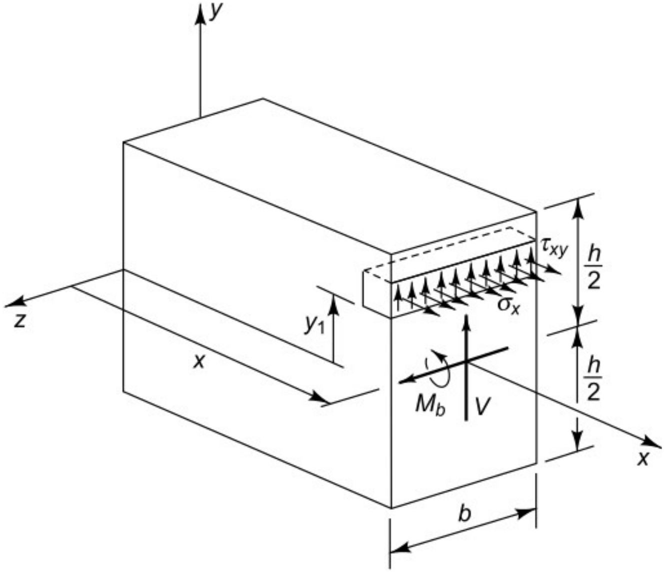
Shear flow due to bending,

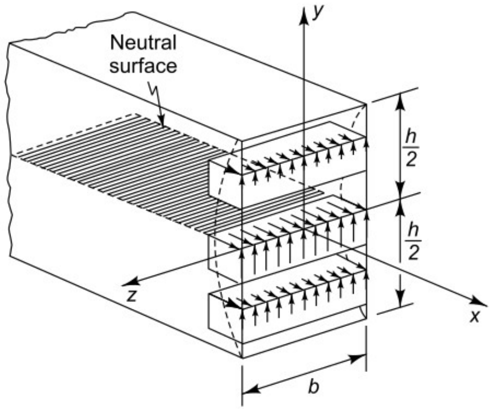
$$q_{yx} = b\tau_{yx} = \frac{VQ}{I_{zz}}.$$

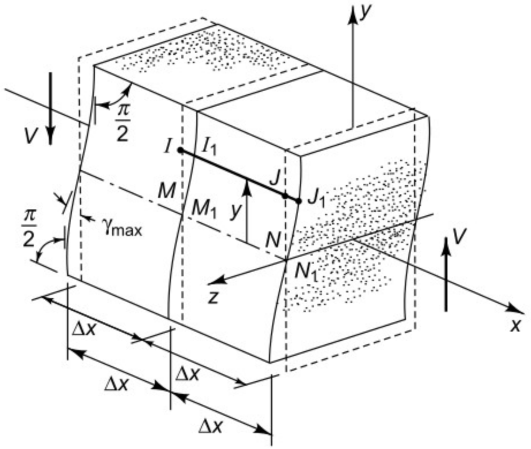
Shear stress distribution in rectangular beams

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$$







$$\tau_{xy} = \frac{V}{2I_{zz}}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right]$$

or

$$\tau_{xy} = \frac{3V}{2A}\left[1 - \left(\frac{y_1}{h/2}\right)^2\right]$$

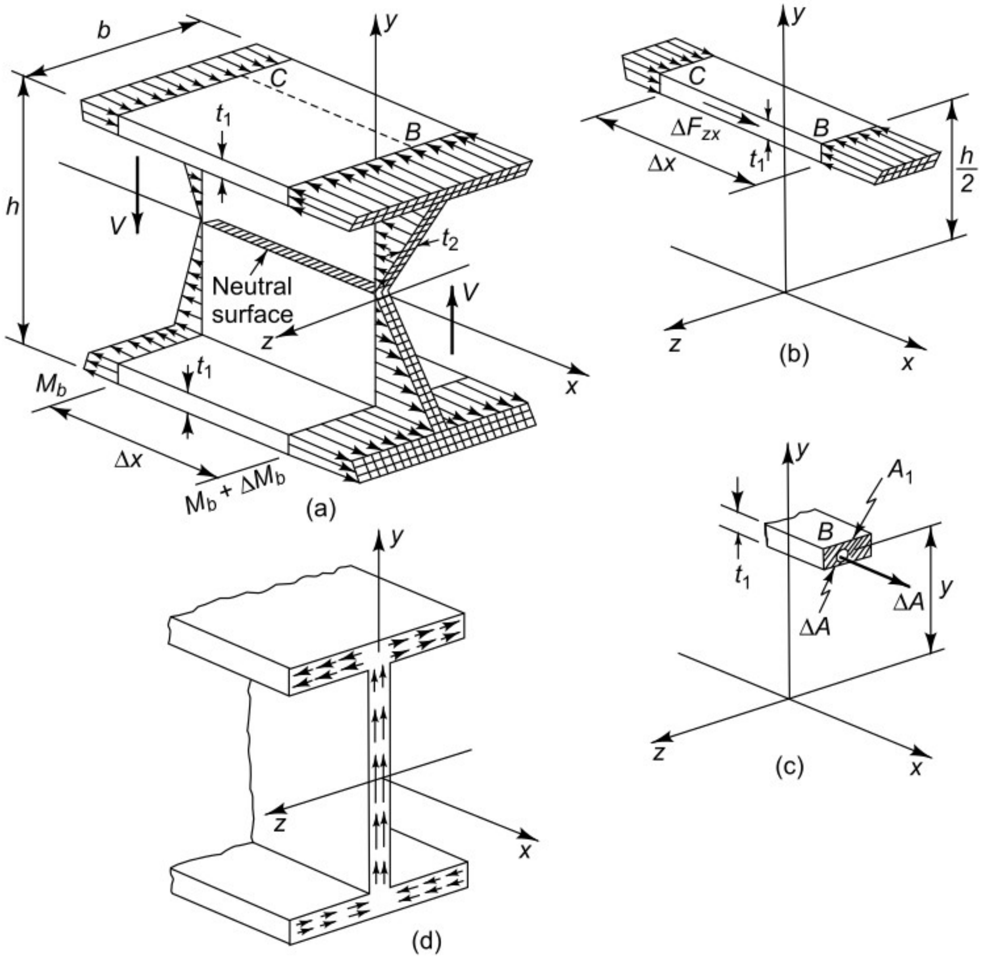
$$\boxed{(\tau_{xy})_{\text{avg}} = \frac{3V}{2A}}$$

(If exact stress variation is assumed)

$$\boxed{(\tau_{xy})_{\text{avg}} = \frac{V}{A}}$$

(If constant stress variation is assumed)

Shear stress distribution in I-beams



Shear flow,

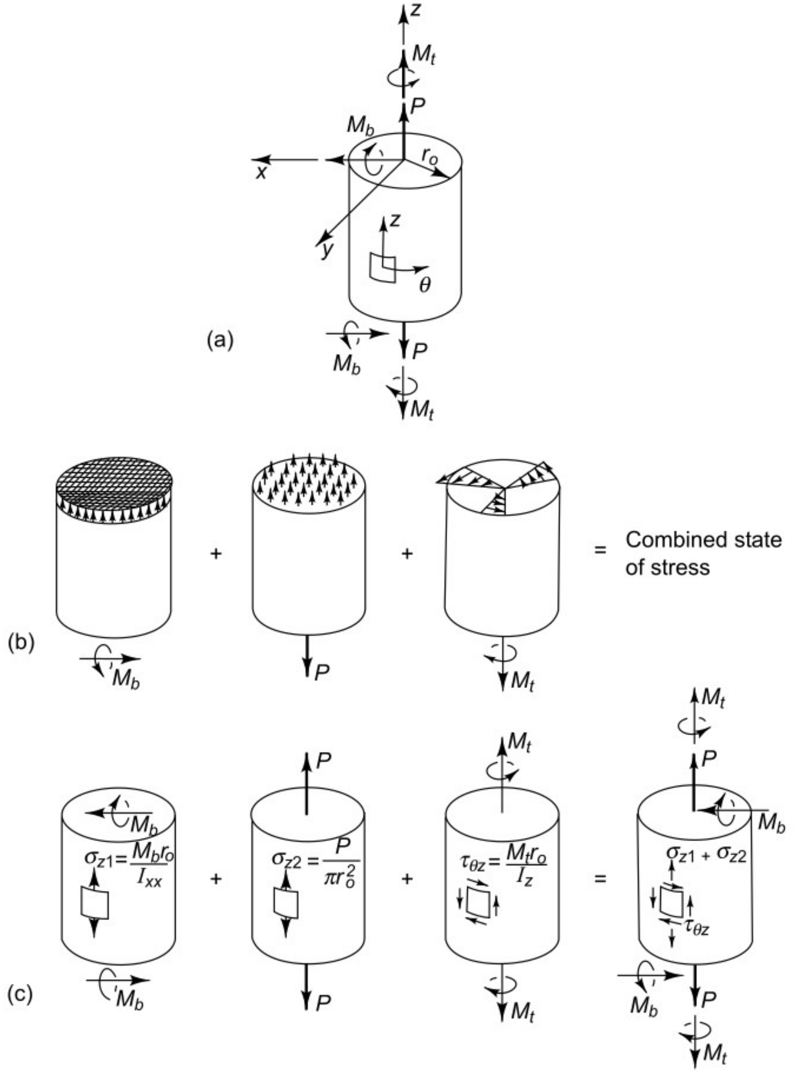
$$q_{zx} = t_1 \tau_{zx} = -\frac{VQ}{I_{zz}}.$$

Shear stress at point B,

$$\tau_{zx} = -\frac{VQ}{t_1 I_{zz}} = \tau_{xz}.$$

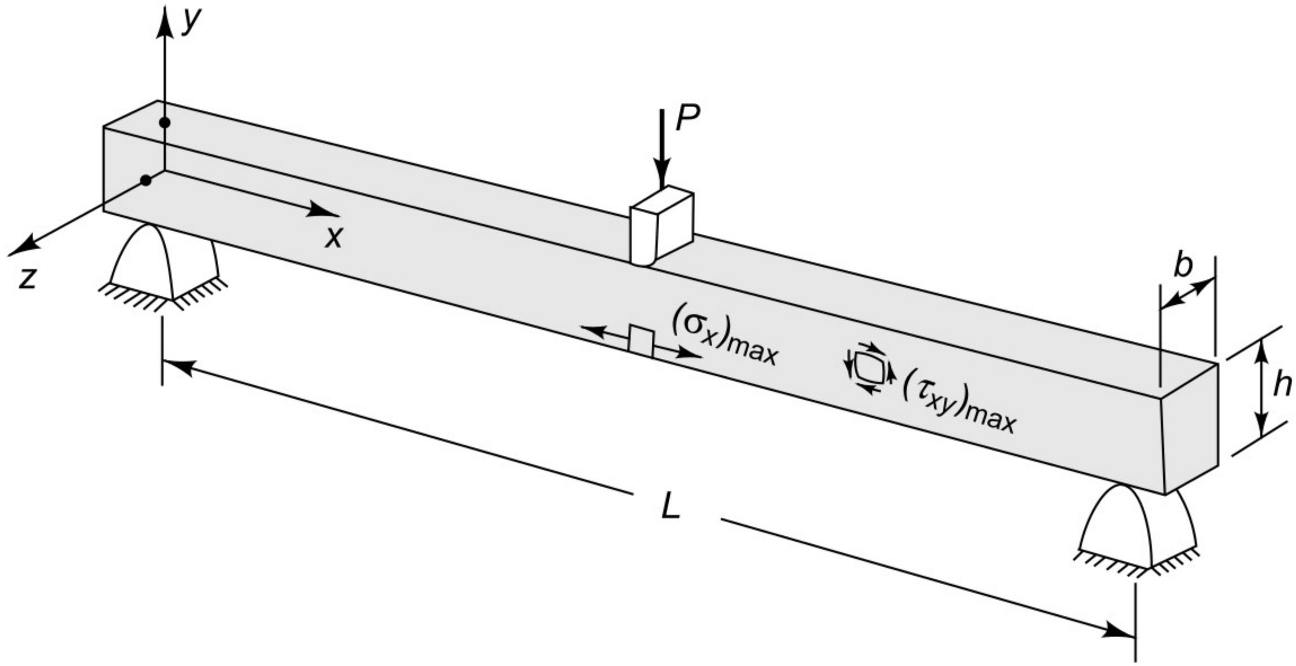
Example 3: Stress analysis in bending-Combined stresses

In figure an elastic circular shaft is shown transmitting simultaneously a bending moment M_b , an axial tensile force P, and a twisting moment M_t . We wish to study the state of combined stress.



Example 4

A rectangular beam is carried on simple supports and subjected to a central load. We wish to find the ratio of the maximum shear stress (τ_{xy}) max to the maximum bending stress $(\sigma_x)_{\max}$.



Strain energy in bending

Pure bending

$$U = \frac{1}{2}\iiint \sigma_x \epsilon_x dx\, dy\, dz = \iiint \frac{\sigma_x^2}{2E} dx\, dy\, dz$$

$$U = \iiint \frac{1}{2E}\left(\frac{M_b y}{I_{zz}}\right)^2 dx\, dy\, dz = \int_L \frac{M_b^2}{2E I_{zz}{}^2} dx \iint_A y^2 dy\, dz$$

$$U = \int_L \frac{M_b^2}{2EI_{zz}} dx$$

Bending with transverse loads

$$U = \frac{1}{2}\iiint (\sigma_x \epsilon_x + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz}) dx\, dy\, dz$$

$$= \iiint \frac{\sigma_x^2}{2E} dx\, dy\, dz + \iiint \frac{\tau_{xy}{}^2 + \tau_{xz}{}^2}{2G} dx\, dy\, dz$$