ME231: Solid Mechanics-I

Stresses due to bending

$$\sum M_z = -\int_A y\sigma_x dA = \int_A E\frac{y^2}{\rho}dA = \frac{E}{\rho}\int_A y^2 dA = M_b$$

I_{zz}

Second moment of c.s. area

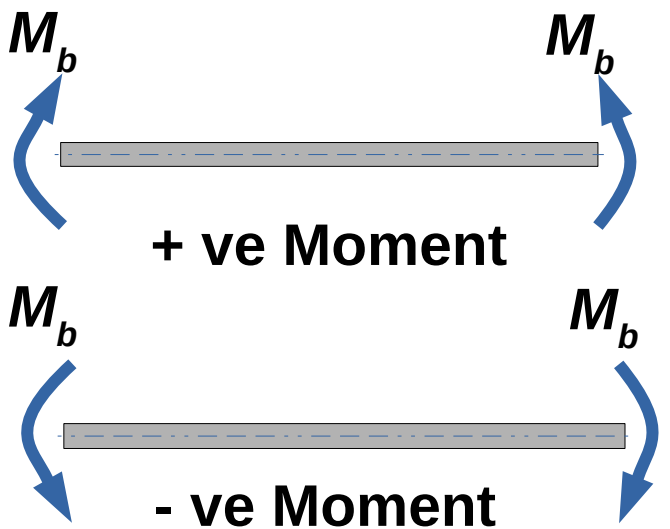
or

Moment of Inertia of the area about the neutral axis

$$M_b = \frac{EI_{zz}}{\rho} \Rightarrow \frac{M_b}{EI_{zz}} = \frac{1}{\rho}.$$

$$\epsilon_x = -\frac{y}{\rho} = -\frac{d\phi}{ds}y \Rightarrow \frac{1}{\rho} = \frac{d\phi}{ds}.$$

$$\frac{1}{\rho} = \frac{d\phi}{ds} = \frac{M_b}{EI_{zz}}$$



Finally from the expressions of stress and strain, we get

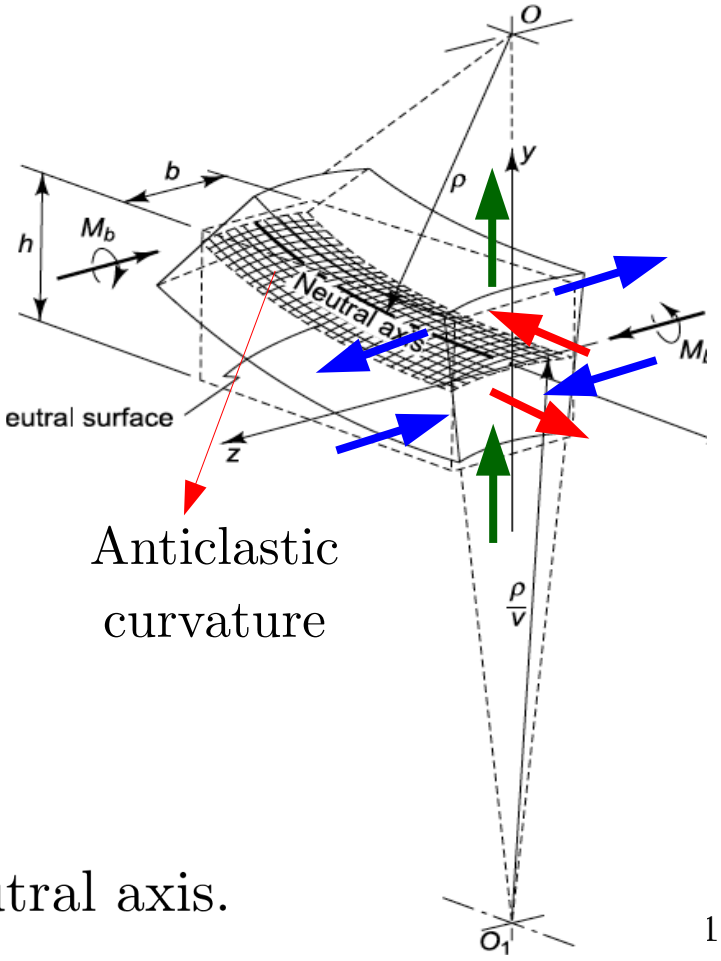
$$\epsilon_x = -\frac{M_b y}{EI_{zz}}$$

$$\sigma_x = -\frac{M_b y}{I_{zz}}$$

$$\epsilon_y = -\nu\epsilon_x$$

$$\epsilon_z = -\nu\epsilon_x$$

$$\gamma_{yz} = 0$$

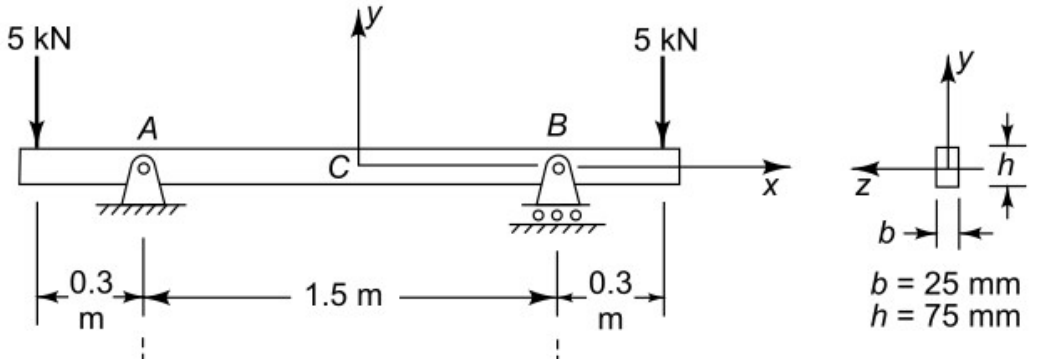


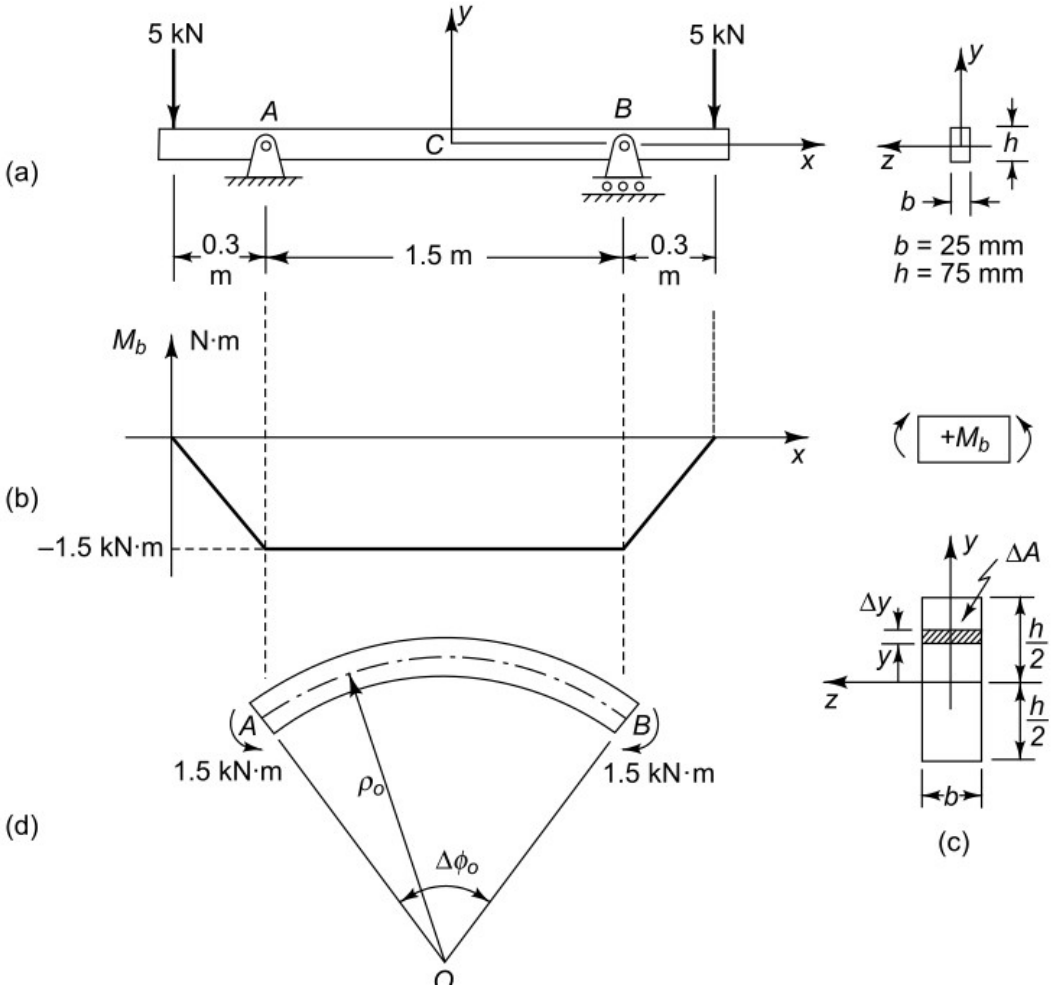
Anticlastic curvature

Section Modulus $S = \frac{I}{c}$
where, c is the maximum distance from the neutral axis.

Example 1

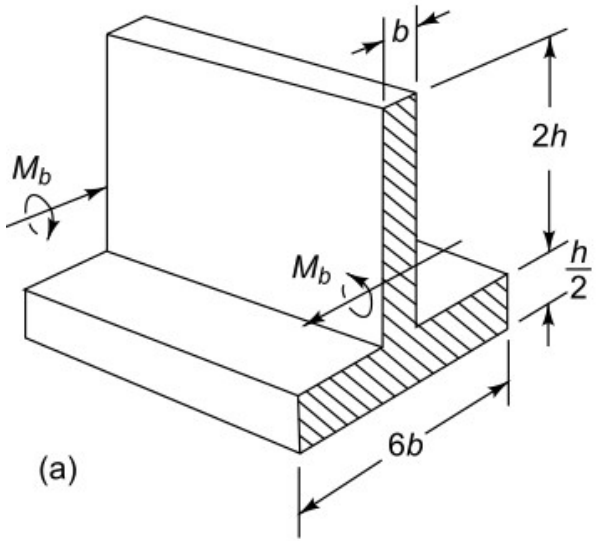
A steel beam 25 mm wide and 75 mm deep is pinned to supports at points A and B, as shown in figure, where the support B is on rollers and free to move horizontally. When the ends of the beam are loaded with 5-kN loads, we wish to find the maximum bending stress at the mid-span of the beam and also the angle $\Delta\phi_O$ subtended by the cross sections at A and B in the deformed beam.

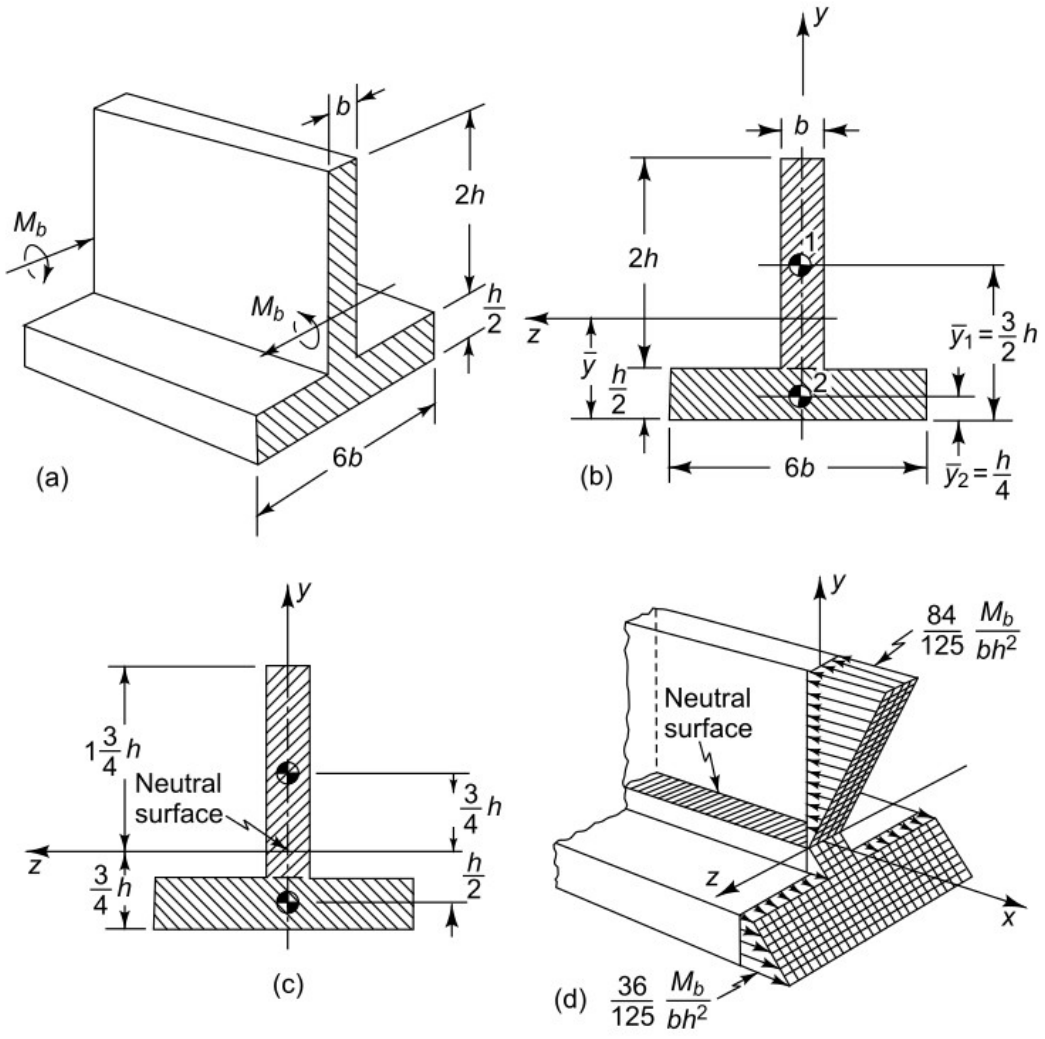


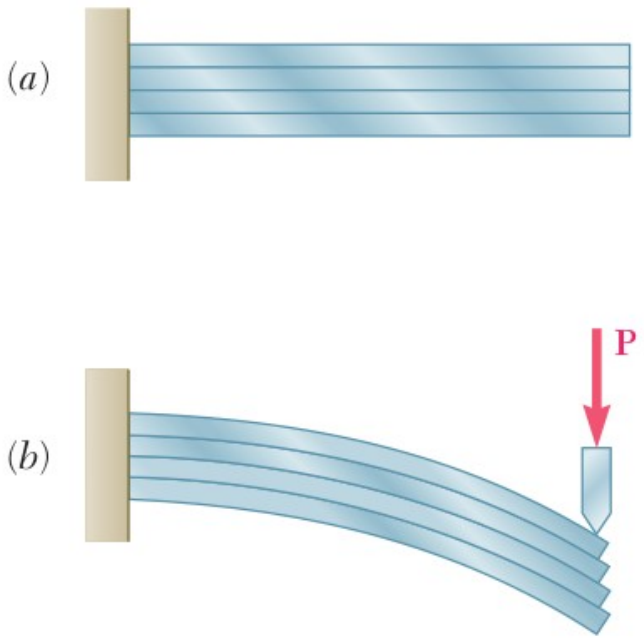

5 kN
5 kN
y
A
B
C
x
(a)
0.3 m
1.5 m
0.3 m
y
z
h
b
b = 25 mm
h = 75 mm
M_b N·m
x
+M_b
(b)
–1.5 kN·m
y
ΔA
Δy
y
z
h/2
h/2
b
(c)
A
B
1.5 kN·m
1.5 kN·m
ρ_o
$\Delta\phi_o$
O
(d)

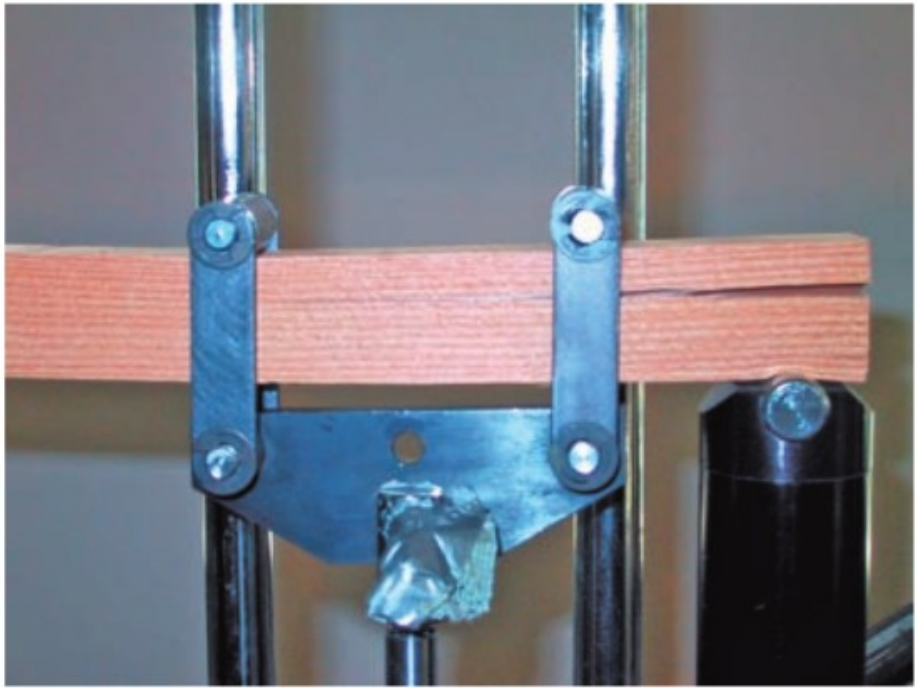
Example 2

We wish to find the maximum tensile and compressive bending stresses in the symmetrical T beam of under the action of a constant bending moment M_b.




b
2h
M_b
M_b
$\frac{h}{2}$
6b
(a)
y
b
2h
1
z
$\bar{y}$
$\frac{h}{2}$
2
$\bar{y}_1 = \frac{3}{2}h$
6b
$\bar{y}_2 = \frac{h}{4}$
(b)
y
$1\frac{3}{4}h$
Neutral surface
$\frac{3}{4}h$
z
$\frac{3}{4}h$
$\frac{h}{2}$
(c)
y
$\frac{84}{125}\frac{M_b}{bh^2}$
Neutral surface
z
x
(d)
$\frac{36}{125}\frac{M_b}{bh^2}$


(a)
(b)
P



Stresses in symmetrical elastic beams transmitting both Shear force and bending moment