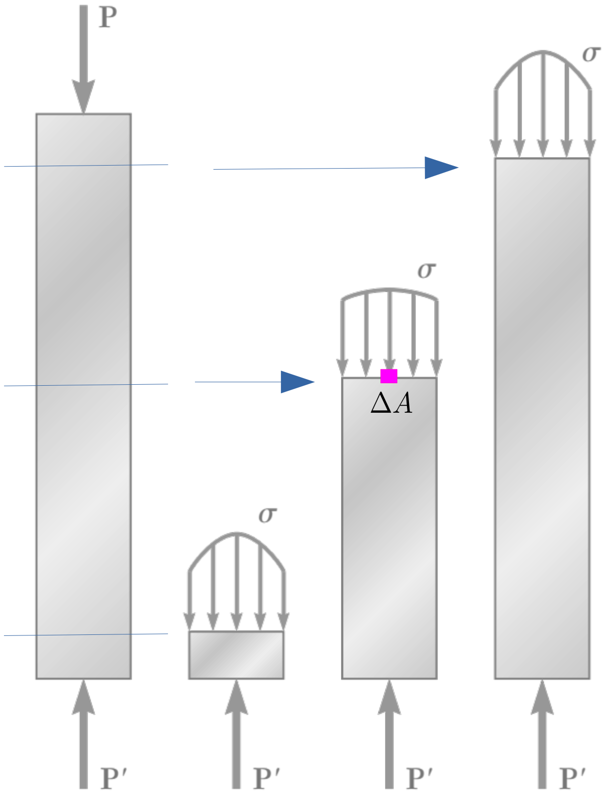
ME231: Solid Mechanics-I

Stress and Strain

Simple state of stress – Axial stress



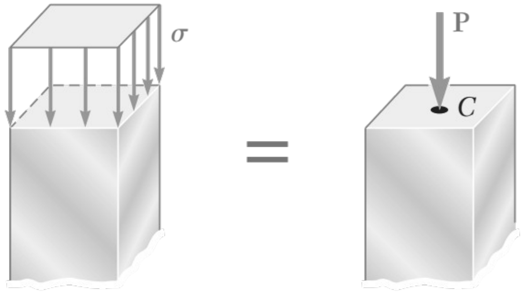
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

$$\int dF = \int_A \sigma dA$$

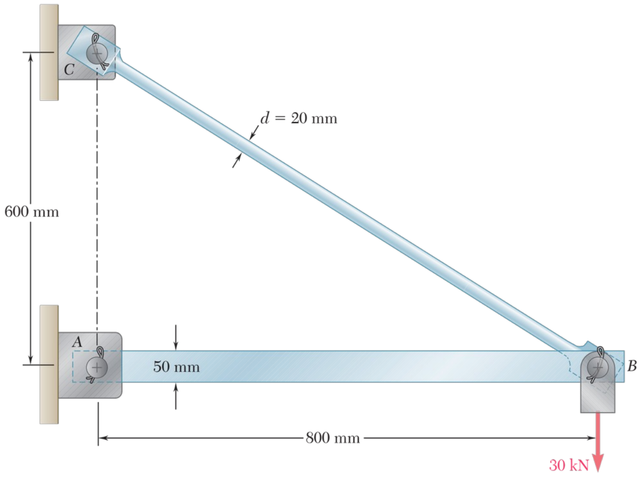
From equilibrium

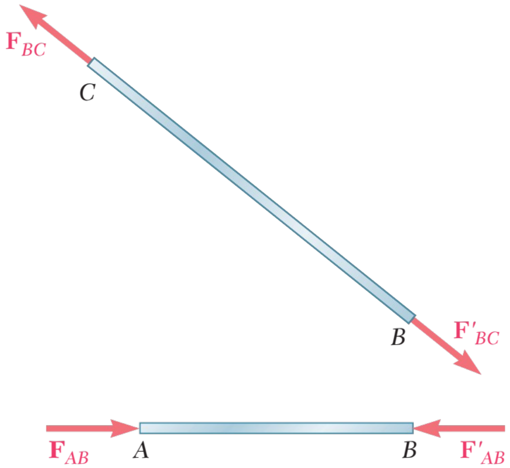
$$P = \int dF = \int_A \sigma dA$$

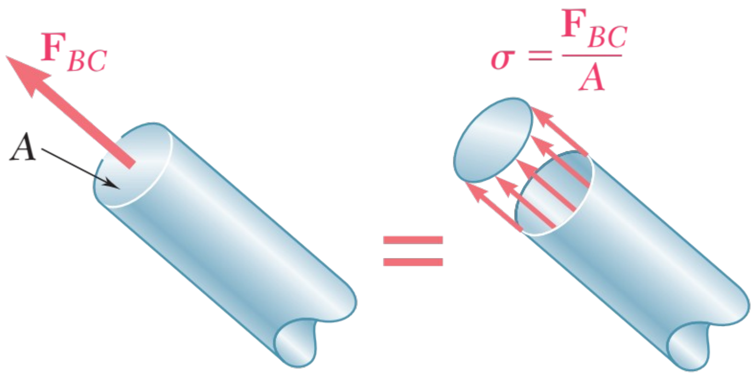
Idealization



Axial stress

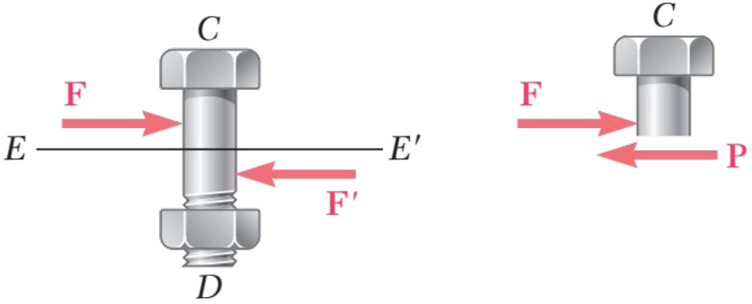


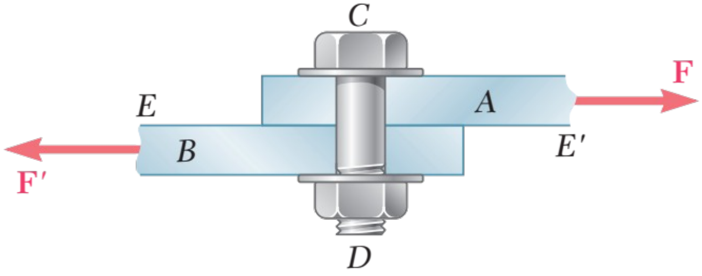




Only normal stress in axial direction is non-zero. All other stress components are zero

Simple state of stress – Shear stress



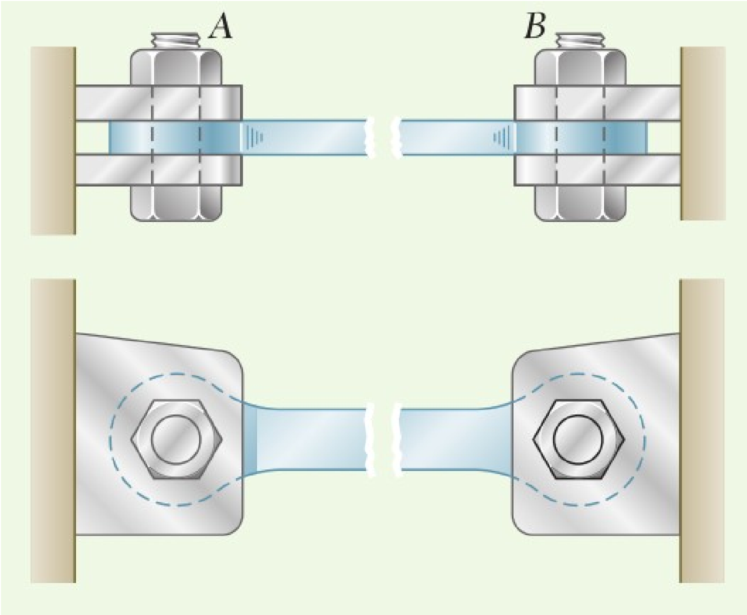


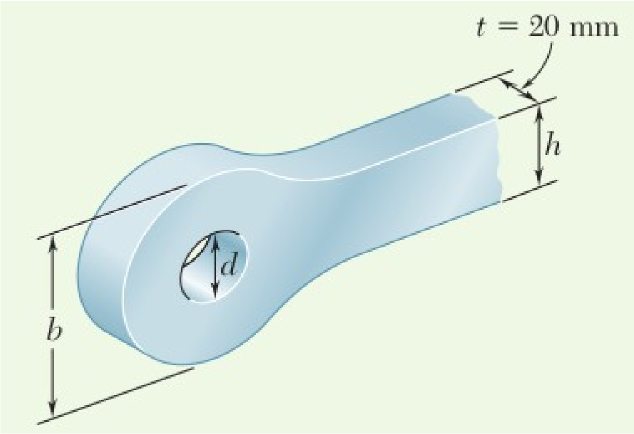
Average shear stress

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{F}{A}$$

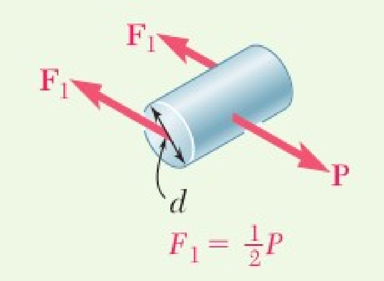
Example 1

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. The maximum allowable stresses are $\sigma = 175$ MPa, $\tau = 100$ MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, and (c) the dimension h of the bar.





Diameter of the bolt:



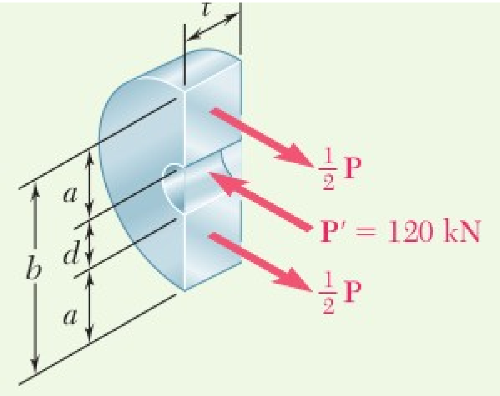
Shear stress at any of the cross section of the bolt is

$$\tau = F_1/A = P/(2A)$$

$$\tau = P/(2A) = 4P/(2\pi d^2) \leq \tau_{\text{allowable}}$$

Minimum required value of d can be obtained.

Dimension b of the bar:



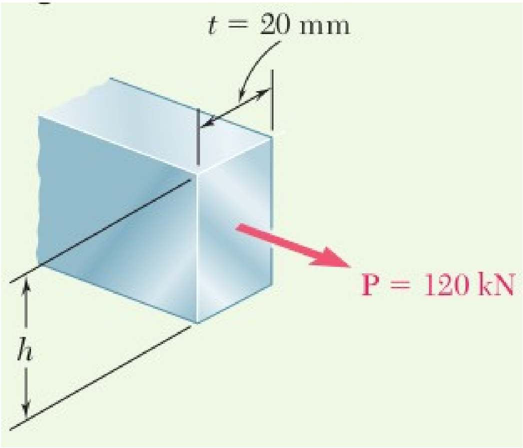
Normal stress at the cross section is

$$\sigma = \frac{P/2}{A} = \frac{P/2}{at}$$

$$\sigma = \frac{P}{2at} \leq \sigma_{\text{allowable}}$$

Thus for a given value of t, minimum required value of a can be obtained. Then $b = 2a + d$.

Dimension h of the bar:



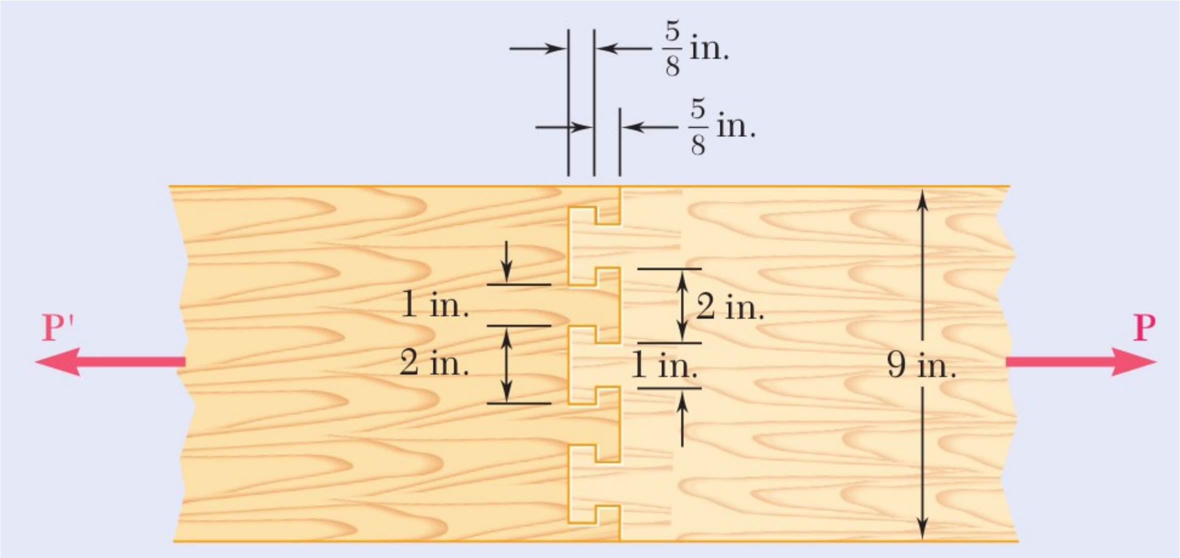
Normal stress at the cross section is

$$\sigma = \frac{P}{A} = \frac{P}{th} \leq \sigma_{\text{allowable}},$$

which gives us the minimum value of h required.

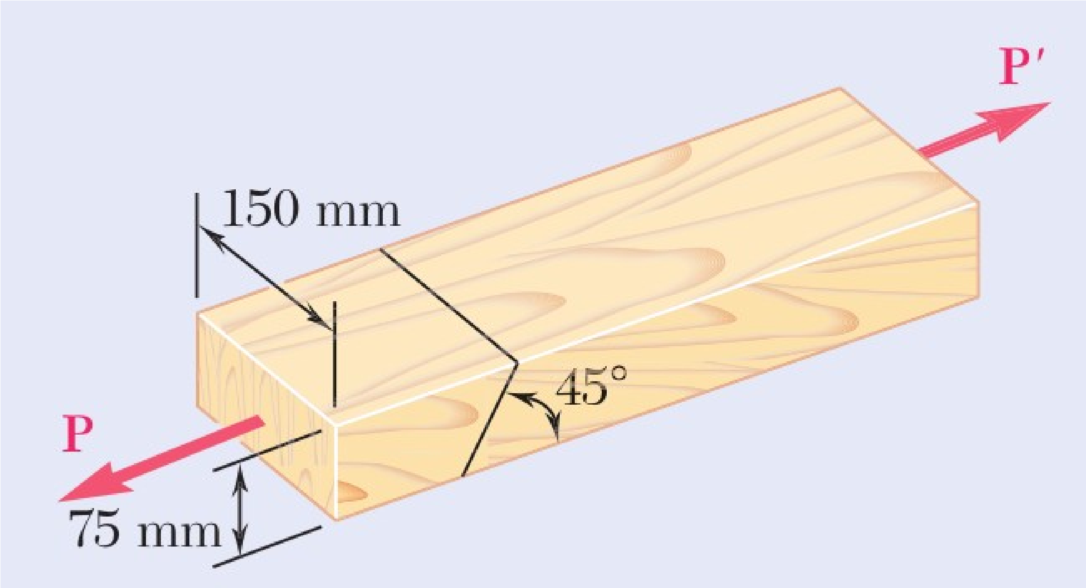
Example 2

Two wooden planks, each 12 in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load that will cause the joint to fail.



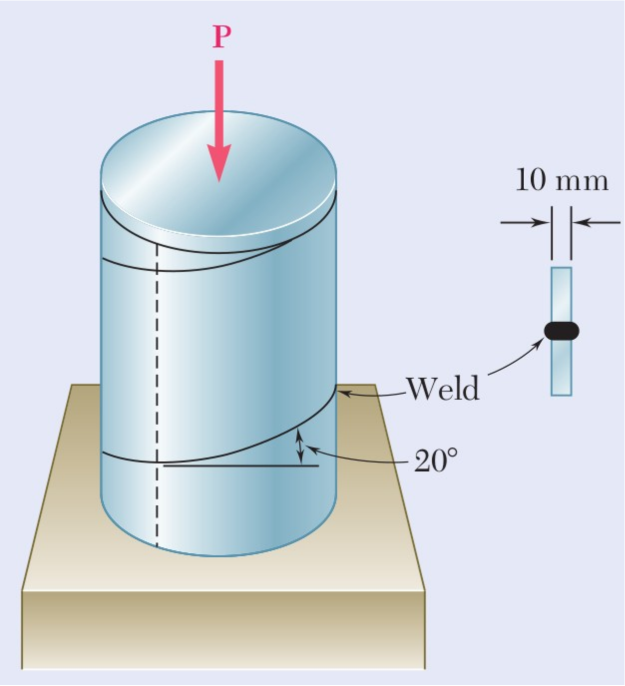
Example 3

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11$ kN, determine the normal and shearing stresses in the glued splice. If that maximum allowable shearing stress in the glued splice is 620 kPa, determine whether the applied load is within the safe limit or not.



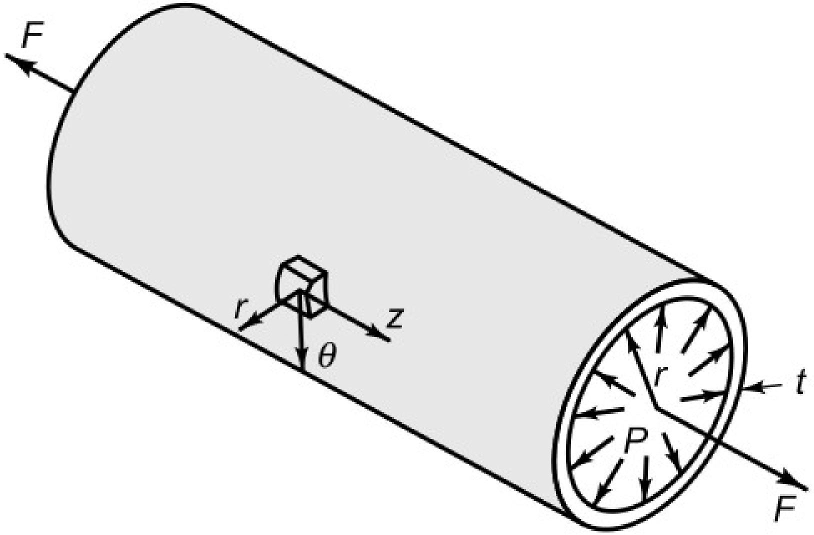
Example 4

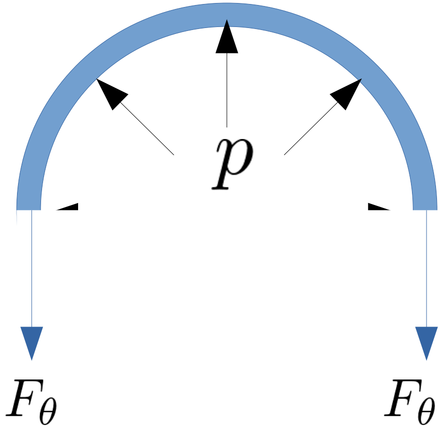
A steel pipe of 400-mm outer diameter is fabricated from 10-mm-thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 300-kN axial force P is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.



Example 5

Consider a thin-walled cylinder of internal radius r and thickness t. If the cylinder is subjected to an internal pressure p and an axial force F, show that the r, θ, z directions are the principal stress directions. Show also that if the wall is so thin that $t/r << 1$, then determine the stresses in the pipe wall.





Detach a part of cylinder and its content by a *rz*-plane and considering force equilibrium in -direction as,

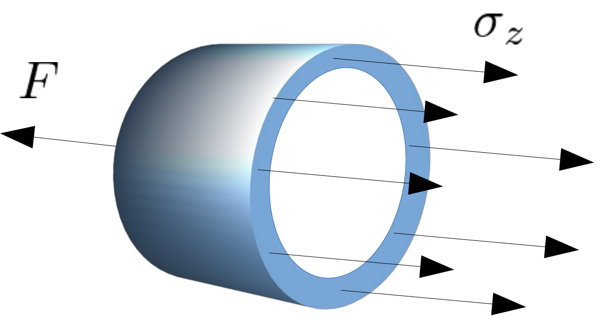
$$\sum F_\theta = 2F_\theta - pA = 0,$$

where A is the projected area at *rz*-plane. Thus,

$$2F_\theta - pA = 2\sigma_\theta \cdot l \cdot t - p \cdot 2r \cdot l = 0$$

or $\sigma_\theta = pr/t$. **(called as hoop stress)**

To determine the stresses in longitudinal direction detach a part of cylinder and its content by a plane parallel to *r*-plane and considering force equilibrium in *z*-direction as,



$$\sum F_z = \sigma_z \cdot 2\pi r \cdot t - F = 0,$$

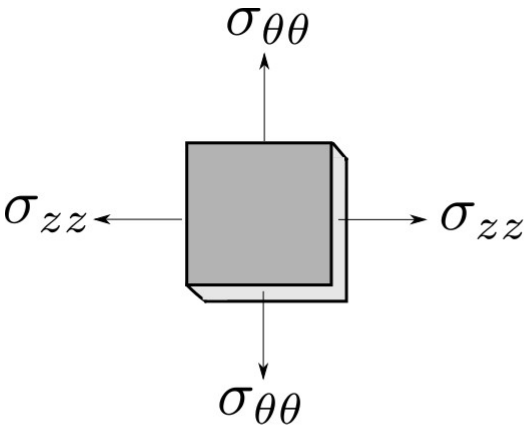
or $\sigma_z = F/(2\pi r t)$.

For a cylinder with end caps, axial force F is caused due to pressure on the end caps, where $F = p{\cdot}\pi r^2$. Thus for cylinder with end caps,

$$\sigma_z = pr/(2t). \quad \textbf{(called as longitudinal stress)}$$

The accuracy of derived result depends on the vessel being **thin-walled**, ($r >> t$). At the surfaces of the vessel wall, a radial stress σ_r must be present to balance the pressure there. But the inner-surface radial stress is equal to p, while the circumferential (or hoop) and longitudinal stresses are p times the ratio (r/t) and $(r/2t)$, respectively. When this ratio is large, the **radial stresses can be neglected** in comparison with the hoop or longitudinal stresses.

Thus, the state of stress an element of the pressure vessel is as follows,



which is a case of plane stress.