B. E. Third Semester (CE / CT / IT / ME / EL / EE / ET) / SoE – 2018 Examination

Course Code : GE 2201 **Course Name : Engineering Mathematics – III**

Time : 3 Hours / 4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Assume suitable data wherever necessary.
(4) Illustrate your answers wherever necessary with the help of neat sketches.
(5) Use of Logarithmic tables, non programmable calculator is permitted.

1. (A) Solve any **One** :—

(A1) Express $y = 3x^3 + x^2 + x + 1$ in factorial polynomial. 2

(A2) Show that $f(5) = f(4) + \Delta f(3) + \Delta^2 f(2) + \Delta^3 f(2)$. 2

(B) Solve any **One** :—

(B1) Find $\frac{dy}{dx}$ at $x = 1.2$ from the following table

x	1.0	1.5	2.0	2.5
y	27.00	106.75	324.00	783.75

3

(B2) Evaluate $\int_1^{1.4} e^{-x^2}\,dx$ by taking $h = 0.1$ using Simpson's rule. 3

(C) Solve any **One** :—

(C1) Solve a following Difference equation

$y_{n+2} + 5y_{n+1} + 6y_n = 2^n + n.$ 5

(C2) Find out $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ at $t = 1$ sec. for the following data :

t	0.2	0.4	0.6	0.8	1	1.2
θ	0.12	0.49	1.12	2.02	3.2	4.67

5

2. (A) Solve any **One** :—

(A1) Find Laplace Transform of $(t^2 + 1)^2 + e^{-5t} + \cosh 3t$. 2

(A2) Use definition of Laplace Transform and Evaluate

$$\int_0^\infty e^{-2t} \cos t \; dt.$$

2

(B) Solve any **One** :—

(B1) Find Laplace Transform of $\frac{\cos 2t - \cos 3t}{t}$.

3

(B2) Using method of partial fraction evaluate

$$L^{-1}\left\{\frac{(s^2 + 2s + 3)}{(s^2 + 2s + 2)(s^2 + 2s + 5)}\right\}$$

3

(C) Solve any **One** :—

(C1) Using Laplace transform, solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$,

Given $x(0) = 1$, $x(\pi/2) = -1$. 5

(C2) Evaluate $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$ by convolution theorem of Laplace transform. 5

3. (A) Solve any **One** :—

(A1) Compute the values of Z – Transform of $\sin(3n+5)$ and $\cos(n+2)$. 2

(A2) Compute inverse Z – Transform of $\dfrac{z^3}{(z-2)^3}$. 2

(B) Solve any **One** :—

(B1) Using residue method, Evaluate $Z^{-1}\left[\dfrac{16z^3}{(4z-1)(z-1)}\right]$ 3

(B2) Prove that $Z\left(\dfrac{f(n)}{n+k}\right) = Z^{k}\left[\int_{z}^{\infty}\dfrac{F(z)}{z^{k+1}}\,dz\right]$. 3

(C) Solve any **One** :—

(C1) Solve the difference equation by Z – Transform, $y_{n+2} + 3y_{n+1} + 2y_n = u_n$ given that $y_0 = 1$ and $y_n = 0$ for $n < 0$. 5

(C2) Find the inverse Z – Transform of $\dfrac{3z^2+2z+1}{z^2-3z+2}$ using partial fraction method. 5

4. (A) Solve any **One** :—

(A1) Obtain a Fourier series for $f(x) = x^3$. $-1 < x < 1$. 2

(A2) Sketch the graph of a function $f(x) = \begin{cases} \pi + x, & -\pi < x \leqslant 0 \\ \pi - x, & 0 \leqslant x < \pi \end{cases}$ 2

(B) Solve any **One** :—

(B1) Obtain a half range sine series for $f(x) = \pi x - x^2$. $0 \leqslant x \leqslant \pi$. 3

(B2) Obtain a half range cosine series for $f(x) = \sin x$. $0 \leqslant x \leqslant \pi$. 3

(C) Solve any **One** :—

(C1) Find the Fourier series expansion of

$$f(x) = \begin{cases} -\pi \ , & -\pi < x < 0 \\ x \ , & 0 < x < \pi \end{cases}$$

Also show that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots$. 5

(C2) Obtain a Fourier series for $f(x) = \left(\dfrac{\pi - x}{2}\right)^2$, $0 \leqslant x \leqslant \pi$. 5

5. (A) Solve any **One** :—

(A1) Compute the value of complementary function for

$$\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial y^3} = e^{(x + 2y)}.$$ 2

(A2) Compute the value of particular integral for

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = \cos(2x + y).$$ 2

(B) Solve any **One** :—

(B1) Solve $xq = yp + xe^{(x^2 + y^2)}$ where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$. 3

(B2) Solve $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$. 3

(C) Solve any **One** :—

(C1) Solve $\dfrac{\partial^2 z}{\partial x^2} - 3 \dfrac{\partial^2 z}{\partial x \partial y} + 2 \dfrac{\partial^2 z}{\partial y^2} = e^{2x} \sin(x + 3y)$. 5

(C2) Use method of separation of parameter to solve

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad \text{given} \quad u(0, y) = 8e^{-3y}$$

5

6. (A) Solve any **One** :—

(A1) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x, & \text{for} \quad 0 < x < 1 \\ 2 - x, & \text{for} \quad 1 < x < 2 \\ 0, & \text{for} \ x > 2 \end{cases}$$

2

(A2) Find the Fourier sine transform of

$$f(x) = \begin{cases} x, & \text{for} \quad 0 < x < 1 \\ 1, & \text{for} \quad 1 < x < 2 \\ 0, & \text{for} \ x > 2 \end{cases}$$

2

(B) Solve any **One** :—

(B1) Using Parsewals Identity, Show that $\int_0^\infty \frac{dx}{(1+x^2)} = \frac{\pi}{4}$. 3

(B2) Find Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$

3

(C) Solve any **One** :—

(C1) Using the Fourier integral, show that

$$\int_0^\infty \frac{1 + \cos\lambda\pi}{1 - \lambda^2} \cos\lambda x \, \lambda d = \begin{cases} \frac{\pi}{2}\sin x, & 0 \leq x \leq \pi. \\ 0, & x > \pi. \end{cases}$$

5

(C2) Find the Fourier sine trasnform of $e^{-|x|}$ and hence show that

$$\int_0^\infty \frac{x \sin mx}{1 + x^2} \frac{dx}{} = \frac{\pi}{2} e^{-m}, m > 0.$$

5