B. E. First Semester (All) / SoE–2018-19 Examination

Course Code : GE 2102 **Course Name : Engineering Mathematics–II**

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Assume suitable data wherever necessary.
(4) Illustrate your answers wherever necessary with the help of neat sketches.
(5) Use of Logarithmic tables, non – programmable calculator, Thermodynamic tables for moist air, is permitted.

1. (A) Solve any **One** :

(A1) Calculate the Complimentary function $\frac{d^2y}{dx^2} + a^2y = \cos ax$ **2**

(A2) Calculate the particular integral of $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$ 2

(B) Solve any **One** :

(B1) Solve $[2x - y + 1]dx = [x - 2y + 1]dy$ 3

(B2) Solve $x\left[\frac{dy}{dx} + y\right] = 1 - y$ 3

(C) Solve any **One** :

(C1) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} + \sin x$ 5

(C2) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^{-x}\sec^3 x$$ 5

2. (A) Solve any **One** :

(A1) Convert the differential equation into linear differential equation with constant coefficient $(x+3)^2 \frac{d^2y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = \log(x+3)$ 2

(A2) Calculate the Complimentary function of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$ 2

(B) Solve any **One** :

(B1) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = \cos(\log x)$ 3

(B2) Solve $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} + 2xy = 10(x^2+1)$ 3

(C) Solve any **One** :

(C1) Solve $(3x+2)\frac{d^2y}{dx^2} + 5(3x+2)\frac{dy}{dx} - 3y = x^2 + x + 1$ 5

(C2) Solve $t\frac{dx}{dt} + y = 0$, $t\frac{dy}{dt} + x = 0$ 5

3. (A) Solve any **One** :

(A1) Separate the function $\cosh(x+iy)$ into real and imaginary parts. 2

(A2) Find the smallest positive integer n for which $\left[\frac{1+i}{1-i}\right]^n = 1$. 2

(B) Solve any **One** :

(B1) If $\tan(A+iB) = x + iy$, prove that

(i) $\tan 2A = \frac{2x}{1-x^2-y^2}$

(ii) $\tanh 2B = \frac{2y}{1+x^2+y^2}$ 3

(B2) Express $1+\sin\alpha + i\cos\alpha$ in the modulus and amplitude form. 3

(C) Solve any **One** :

(C1) If $i^{\alpha+i\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$ 5

(C2) If $\tan(\theta+i\phi) = \cos\alpha + i\sin\alpha$,

show that $2\theta = n\pi + \frac{\pi}{2}$, $e^{2\phi} = \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$ 5

4. (A) Solve any **One** :

(A1) Show that $u = 2x - x^3 + 3xy^2$ is harmonic. 2

(A2) Test whether the function $f(z) = z^2$ is analytic or not. 2

(B) Solve any **One** :

(B1) Expand the following functions by Laurent's Series

$f(z) = \frac{z}{(z-1)(2-z)}$ valid for $1 < |z| < 2$ 3

(B2) If f(z) is analytic function with constant modulus show that f(z) is constant. 3

(C) Solve any **One** :

(C1) Use cauchy residue method, evaluate $\int \frac{12z-7}{(z-1)^2(2z+3)}\,dz$ where c is circle $|z+i| = \sqrt{3}$ 5

(C2) Evaluate the following Integral by contour Integration

$\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta}\,d\theta$ 5

5. (A) Solve any **One** :

(A1) Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & 3 \end{bmatrix}$. 2

(A2) Find eigen values of matrix $A = \begin{bmatrix} 2 & 9 \\ 1 & 2 \end{bmatrix}$. 2

(B) Solve any **One** :

(B1) Test the consistency and Solve $2x + 6y + 11 = 0$, $6x + 20y - 6z + 3 = 0$ and $6y - 18z + 1 = 0$. 3

(B2) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 101$ as a linear polynomial of A. 3

(C) Solve any **One** :

(C1) Find the Eigen value and Eigen vectors and modal matrix for the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 5

(C2) Use Sylvester's theorem to show that $e^A = e^x \begin{bmatrix} \cos hx & \sin hx \\ \sin hx & \cos hx \end{bmatrix}$ where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ 5

6. (A) Solve any **One** :

(A1) Calculate rank correlation coefficient for the following data :

x	2	4	5	6	8	11
y	18	12	10	8	7	5

2

(A2) Find the normal equations of $y = a + bx + cx^2$ 2

(B) Solve any One :

(B1) Fit a curve $y = ax^2+b$ for the following data :

x	12	16	20	22	24	26	30
y	6.44	7.5	6.9	10.76	10.76	11.76	14.00

3

(B2) Find the coefficient of correlation and equation of lines of regression for the following data :

x	6	2	10	4	8
y	9	11	5	8	7

3

(C) Solve any **One** :

(C1) Obtain the rank correlation coefficient for the following data:

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

5

(C2) Two lines of regression are given by $8x-10y+66=0$ and $40x-18y = 214$ if $\sigma_x^2 = 9$, find :

(i) Mean values of x and y.

(ii) The coefficient of correlation between x and y.

(iii) The standard deviation of y.

(iv) Variance of y. 5