B. E. Third Semester (CE / CT / IT / ME / EL / EE / ET) / BECV – 18-19 – Rev – SoE – CV-201 Examination

Course Code : GE 2201 **Course Name : Engineering Mathematics III**

Time : 2 Hours] [Max. Marks : 40

Instructions to Candidates :—

(1) Attempt any **Four** questions out of **Six**.
(2) All questions carry **Ten** marks.
(3) Assume suitable data wherever necessary.
(4) Use of Logarithmic tables, non programmable calculator is permitted.

1. Solve Either **A – Part** or **B – Part** :— (CO1)

(A) (A1) Express the given function in factorial notation

$$f(x) = x^4 + 5x^3 - 3x^2 + 7x + 5$$ 3

(A2) Prove that $\triangle \log f(x) = \log \left[1 + \frac{\triangle f(x)}{f(x)}\right]$ 3

(A3) Solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$. 4

OR

(B) (B1) Find the function whose first order forward difference is e^x. 3

(B2) Find $\frac{dy}{dx}$ at $x = 1.2$ from the following table :

x	1.0	1.5	2.0	2.5
y	27.00	106.75	324.00	783.75

3

(B3) Evaluate $\int_1^{1.4} e^{-x^2}\, dx$ by taking $h = 0.1$ using Simpson's rule. 4

2. Solve Either **A – Part** or **B – Part** :— (CO2)

(A) (A1) Find Laplace Transform of $f(t) = \frac{\sin t}{t}$ 4

(A2) Find $L^{-1}\left\{\log\left(\frac{S+a}{S+b}\right)\right\}$ 3

(A3) If $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \sin t + \cos 2t$, given $y(0) = 1$, $y^1(0) = 0$ then find Laplace Transform of $y(t)$. 3

OR

(B) (B1) Find Laplace Transform of $f(t) = (t^2 + 1)^3 + e^{-5t}$ 4

(B2) Evaluate $L^{-1}\left\{\frac{1}{S^3(s+1)}\right\}$ by using convolution theorem. 3

(B3) If $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$, given $y(0) = 1$, $y^1(0) = -1$ then find Laplace Transform of $y(t)$. 3

3. Solve Either **A – Part** or **B – Part** :— (CO2)

(A) (A1) Find $Z\left\{\frac{1}{n!}\right\}$ by using definition of Z – Transform. 3

(A2) Find inverse Z – Transform of $F(z) = \frac{z^2 + 2}{(z-1)(z-4)}$

. Using Residue Method. 3

(A3) Solve the difference equation by Z – Transform
$y_{n+2} - 2\cos\alpha\, y_{n+1} + y_n = 0$, given $y_0 = 0$, $y_1 = 1$. 4

OR

(B) (B1) Find Z – Transform of $e^{in\theta}$. Hence find $Z\{\sin(3n)\}$. 3

(B2) Find inverse Z – Transform of $F(z) = \dfrac{z}{(z+2)(z-1)(z+1)}$

Using Partial fraction Method. 3

(B3) Solve the difference equation by Z – Transform $y_{n+2} + 4y_{n+1} + 3y_n = 2^n$, given $y_0 = 0$, $y_1 = 1$. 4

4. Solve Either **A – Part** or **B – Part** :— (CO3)

(A) (A1) Obtain the Fourier series expansion for $f(x) = x^3, -\pi < x < \pi$. 4

(A2) If half range cosine series of $f(x) = (2x - 1), 0 < x < 1$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(n\pi x)$$

then calculate Value of a_n. 3

(A3) If Fourier series of $f(x) = x - x^2, -1 < x < 1$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(n\pi x) + \sum_{n=1}^{n=\infty} b_n \sin(n\pi x)$$

then calculate Value of b_n. 3

OR

(B) (B1) Find the Fourier series for $f(x) = \sin 2x, \; -\pi < x < \pi$ 4

(B2) If Fourier series of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$, is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(nx) + \sum_{n=1}^{n=\infty} b_n \sin(nx)$$

then calculate Value of b_n. 3

(B3) If Fourier series Of $f(x) = \left(\dfrac{\pi - x}{2}\right)^2$, $0 < x < 2\pi$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{n=\infty} a_n \cos(nx) + \sum_{n=1}^{n=\infty} b_n \sin(nx)$$

then calculate Value of a_n. 3

5. Solve Either **A – Part** or **B – Part** :— (CO4)

(A) (A1) Solve $xq = yp + xe^{(x^2 + y^2)}$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ 3

(A2) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0$ 3

(A3) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} = \cos x \cos 2y$ 4

OR

(B) (B1) Solve $\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = 5z + \tan(y - 3z)$ 3

(B2) Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{(x + 2y)}$ 3

(B3) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$ 4

6. Solve Either **A – Part** or **B – Part** :— (CO2)

(A) (A1) Express $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as Fourier integral. 3

(A2) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$

where $a > 0$. 3

(A3) Using Parseval's identity, show that $\int_0^x \frac{dx}{(x^2 + 1)^2} = \frac{\pi}{4}$ 4

OR

(B) (B1) Find the Fourier Sine integral of $f(x) = \begin{cases} 1, & 0 \leqslant x \leqslant \pi \\ 0, & x > \pi \end{cases}$ 3

(B2) Find the Fourier transform of $f(x) = \begin{cases} x, & \text{for } |x| \leqslant a \\ 0, & \text{for } |x| > a \end{cases}$ 3

(B3) Find $f(x)$ from the integral equation

$$\int_0^x f(x) \cos \alpha x \, dx = \begin{cases} 1 - \alpha, & 0 \leqslant \alpha \leqslant 1 \\ 0, & \alpha > 1 \end{cases}$$

Hence show that $\int_0^\infty \frac{\sin^2 t}{t^2} \, dt = \frac{\pi}{2}$ 4