B. E. Fourth Semester (CT/IT) / SoE–2014-15 Examination

Course Code : GE 1206/GE 206/GE 510 **Course Name : Discrete Mathematics and Graph Theory**

Time : 3 Hours/4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Assume suitable data wherever necessary.
(4) Illustrate your answers wherever necessary with the help of neat sketches.
(5) Use of Logarithmic tables, non – programmable calculator, is permitted.
(6) For MCQ, first attempt will be considered only.

1. Solve **Q. A or Q. B** :

(A) (A1) If $A_i \subseteq S$, $\forall i = 1, 2, 3, \ldots n \in N$ and $A_i \neq A_j$ for every $i \neq j$, then————————.

(a) $\bigcup_{i=1}^{n} A_i = S$

(b) $P(A) = S$

(c) $\bigcap_{i=1}^{n} A_i = S$

(d) None of these 1

(A2) Explain the converse and inverse of the statements "If a triangle is equilateral, then it is equiangular". 2

(A3) Use principle of mathematical induction to show that :

$$1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \geq 1$$ 4

(B) (B1) Applying propositional logic, which of the following is equivalent to $p \rightarrow q$?

(a) $\sim p \rightarrow q$ (b) $\sim P \vee q$

(c) $\sim P \vee \sim q$ (d) $p \rightarrow q$ 1

(B2) If A and B are any two non–empty sets then show that $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$. 2

(B3) Explain the validity of the following arguments using truth table. "If the initialization is correct and loop terminate then the required output is obtained". "The output has been obtained", therefore, "If the initializaion is correct then loop must terminate". 4

2. Solve **Q. A or Q. B** :

(A) (A1) If $f: R \to R$ defined as $f(x) = -\sin x$, then nature of f is——————————

(a) One–one

(b) Onto

(c) Neither one–one nor onto

(d) One–one and onto. 1

(A2) If A, B and C are any non–empty sets then show that

$A \times (B \cap C) = (A \times B) \cap (A \times C)$ 2

(A3) Let S = {1, 2, 3, 4, 5}. Estimate the relation R_A and R_B generating the partition A = {(1, 2), (3), (4, 5)} and B = {(1, 2, 3), (4, 5)} respectively. Also show that R_A is Equivalence relation. 5

(B) (B1) A relation R $\{(x, y) | 2x + y \leq 5\}$ is defined on the set of positive integers. The relation R is ————————.

(a) Reflexive

(b) Transitive

(c) Symmetric.

(d) None 1

(B2) Let $f: R \to R$ and $g: R \to R$ where R is set of real number and $f(x) = x^2 - 2$, $g(x) = x + 4$ for all $x \in R$. Then estimate $f \circ g$ and $g \circ f$. 2

(B3) Let A = {1, 2, 3}. The relation matrix M_R and M_S are given by :

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, M_S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Then compute $(M_R)^C$, $(M_S)^C$, $M_{R \cup S}$ and $M_{R \cap S}$. 5

3. Solve **Q. A or Q. B** :

(A) (A1) If G = {1, –1, i, –i) is group under ordinary multiplication then 0(–1) is ————————.

(a) 1 (b) 2

(c) 3 (d) 4 1

(A2) Show that set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $\alpha \in R$ form a semi group under matrix multiplication. 2

(A3) Show that the set G = {1, 2, 3, 4, 5, 6} is a finite abelian group under multiplication modulo 7 as composition. 4

(B) (B1) Any Abelian Group not satisfy which of the following property?

(a) Cancellation law.

(b) Reversal law.

(c) Commutative law

(d) Distributive law 1

(B2) Let (Q, *) be an algebraic structure where

$a*b = \frac{a.b}{2}$ $\forall a, b \in Q$. identify the Identity and inverse elements of Q. 2

(B3) Show that the multiplicative group G_1 = {1, i, –1, –i} is isomorphic to the group G_2 = {0, 1, 2, 3} with addition modulo 4 as composition. 4

4. Solve **Q.A or Q. B** :

(A) (A1) A ring R is said to be commutative if ————————.

(a) $a \cdot b = b \cdot a, \ \forall a, \ b \in R$

(b) $A + b = b + a, \ \forall a, \ b \in R$

(c) $a \cdot (b + c) = (a \cdot b) + (a \cdot c), \ \forall a, \ b \ c \in R$

(d) None of these. 1

(A2) If R is a ring and $a, b \in R$, then show that $(a + b)^2 = a^2 + ab + ba + b^2 \neq a^2 + 2ab + b^2$. 2

(A3) If X = (x, y, z, u, v, w} and the fuzzy set A and B are defined as

$$A = \left\{\frac{0.1}{x}, \frac{0.3}{y}, \frac{0.4}{z}\right\} \text{ and } B = \left\{\frac{0.4}{u}, \frac{0.5}{v}, \frac{0.6}{w}\right\}$$

Compute $A \cup (A^c \cap B)$ and $A \cap (A \cap B)^c$ 5

(B) (B1) Let $A = \left\{\frac{0.2}{a}, \frac{1}{b}, \frac{0.6}{c}\right\}$ and $B = \left\{\frac{0.4}{a}, \frac{0.5}{b}, \frac{0.3}{c}\right\}$

Then $A \cup B$ = ————————————————

(a) $\left\{\frac{0.4}{a}, \frac{1}{b}, \frac{0.3}{c}\right\}$ (b) $\left\{\frac{0.4}{a}, \frac{0.5}{b}, \frac{0.6}{c}\right\}$

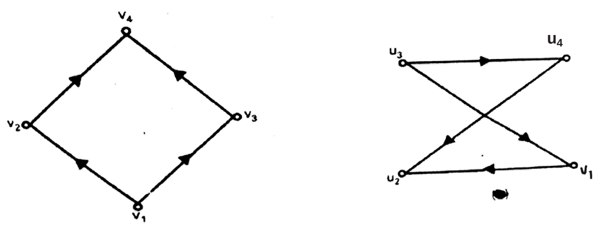
(c) $\left\{\frac{0.4}{a}, \frac{1}{b}, \frac{0.6}{c}\right\}$ (d) $\left\{\frac{0.2}{a}, \frac{0.5}{b}, \frac{0.3}{c}\right\}$ 1

(B2) Draw the lattice $(L^2, \leq 2)$, $(L^3, \leq 3)$, where $L = \{0, 1\}$. 2

(B3) Identify that (R, +6, *6) w.r.t. addition modulo 6 and multiplication modulo 6 as a composition is a commutative ring. Where set R = {0, 1, 2, 3, 4, 5}. 5

5. Solve any **Three** :

(A) Analyze that the following graphs are isomorphic.



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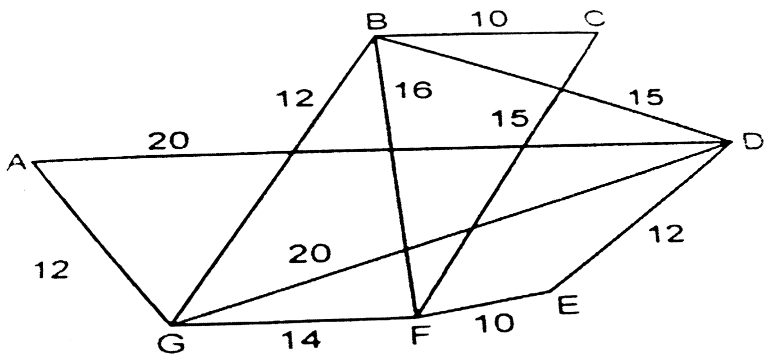
(b) Design a tree For the relation R = {(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)} on a set A = {1, 2, 3, 4, 5, 6, 7}, and also analyse corresponding binary tree. 5

(c) Draw the digraph corresponding to the adjacency matrices.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

show that digraphs of A and B are isomorphic. 5

(D) Using Kruskal's algorithm analyze the minimal spanning tree of following graph and hence compute minimal weight.



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6. Solve any **Three** :

(A) The joint probability function of two discrete random variables X and Y is given by :

$$f(x,y) = \begin{cases} C(x + 2y) &, \quad x = 0,1,2 \quad y = 0, \; 1, \; 2, \; 3 \\ 0 &, \quad \text{otherwise} \end{cases}$$

Determine :

(i) Constant C,

(ii) $P(X \geqslant 1, \; Y \leqslant 3)$ and

(iii) $P(1 \leqslant X \leqslant 2, \; Y \leqslant 2)$ 5

(B) Let X and Y be two continuous random variables having joint density function

$$f(x, y) = \begin{cases} C(x^2 + xy), & 0 \leqslant x \leqslant 1, \quad 0 \leqslant y \leqslant 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Determine :

(i) Constant C (ii) Var. (X) 5

(C) Let X and Y be two random variables with joint probability function:

$$f(x,y) = \begin{cases} \dfrac{2x+y}{10} & , \quad x = 0, 1, 2, \quad y = 0, 1, 2, 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Determine conditional variance of X given Y. 5

(D) Let $f(x, y) = \begin{cases} e^{-(x+y)} & x \geqslant 0, \quad y \geqslant 0 \\ 0, & \text{otherwise} \end{cases}$

be the joint density function of continuous random variables X and Y. Determine :

(i) Marginal density functions of X and Y.

(ii) Conditional density function of X given Y. 5