B. E. First Semester (All)/ SoE–2018–19 Examination

Course Code : GE 2101 **Course Name : Engineering Mathematics I**

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Assume suitable data wherever necessary.
(4) Diagrams should be given wherever necessary.
(5) Illustrate your answers wherever necessary with the help of neat sketches.
(6) Use of Logarithmic tables, non – programmable calculator, Drawing instruments, Thermodynamic tables for moist air, is permitted.

1. (A) Solve any **One** :

(A1) Find the n^{th} derivative of : $y = x^2 e^x$. 2

(A2) Find the radius of curvature for the curve $y = 3x^2 - x$ at the point (0, 0). 2

(B) Solve any **One** :

(B1) Given $f(x) = x^3+8x^2+15x-24$, find $f\left(\frac{11}{10}\right)$ by using Taylor's theorem. 3

(B2) Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$, at point $\left(\frac{a}{4}, \frac{a}{4}\right)$ 3

(C) Solve any **One** :

(C1) If $y = [x + \sqrt{1+x^2}]^m$, prove that

$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 5

(C2) Show that circle of curvature at the origin for the curve.

$y = mx + \frac{x^2}{a}$ is $x^2+y^2 = a(1 + m^2)(y - mx)$ 5

2. (A) Solve any **One** :

(A1) If $u = x^2y + y^2z + z^2x$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$ 2

(A2) If $u = \log \left[\frac{x^4-y^4}{x-y}\right]$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ 2

(B) Solve any **One** :

(B1) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x+\tan^{-1}y$ find $\frac{\partial(u, v)}{\partial(x, y)}$ Are u and v functionally related ? If so, find the relation. 3

(B2) If $u = f(x, y)$, where $x = e^r \cos\theta$ and $y = e^r \sin\theta$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right]$$ 3

(C) Solve any **One** :

(C1) If $u = \operatorname{cosec}^{-1}\left[\frac{x^{1/2} + y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$, prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left[\frac{13}{12} + \frac{\tan^2 u}{12}\right]$$ 5

(C2) Divide 24 into three parts such that the continued product of the first, square of second and cube of the third is maximum. 5

3. (A) Solve any **One** :

(A1) Compute the value of $\Gamma 9/2$ 2

(A2) Prove that $\beta(m+1.n) = \frac{m}{m+n}\beta(m,n)$ 2

(B) Solve any **One** :

(B1) Evaluate $\int_0^3 \frac{x^2}{\sqrt{3-x}}\, dx$

(B2) Evaluate $\int_0^1 \frac{x^a-1}{\log x}\,dx$ by differentiating under the integral sign. 3

(C) Solve any **One** :

(C1) Trace the curve $a^2x^2 = y^3(2a - y)$ and show that its area is $\pi \ a^2$. 5

(C2) Show that $\int_0^1 \frac{x^2dx}{\sqrt{1-x^4}} \int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{4}$ 5

4. (A) Solve any **One** :

(A1) Change into polar co–ordinates $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}\,dydx$ 2

(A2) Evaluate $\int_0^1 \int_0^1 xy\ dx\ dy$ 2

(B) Solve any **One** :

(B1) Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dxdy$ 3

(B2) Evaluate $\int_0^{\pi/2} \int_0^{\alpha\sin\theta} \int_0^{(a^2-r^2)/a} r dr d\theta dz$ 3

(C) Solve any **One** :

(C1) Change into polar co–ordinates and evaluate

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dxdy$$ 5

(C2) Find the center of gravity of the area between $y = 6x-x^2$ and $y=x$. 5

5. (A) Solve any **One** :

(A1) Given the vector field

$\bar{V} = (x^2 - y^2 + 2xz)\hat{i} + (xz - xy + yz)\hat{j} + (z^2 + x^2)\hat{k}$ Find the curl $\bar{V}$. 2

(A2) If $\phi = 3x^2y - y^3z^2$ find grad ϕ at the point (1, –2, –1). 2

(B) Solve any **One** :

(B1) A particle moves along the curve

$\bar{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$, where is the time. Find the magnitude of tangential components of its acceleration at t=2. 3

(B2) A particle moves along the curve :

$x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of its velocity and acceleration at $t = 1$ in the direction of :

$\hat{i} - 3\hat{j} + 2\hat{k}$. 3

(C) Solve any **One** :

(C1) Find the constant a, b, c so that

$\bar{A} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational and hence find the function φ such that $\bar{A} = \nabla\varphi$. 5

(C2) Find the value of n for which the vector field $r^n r$ will be solenoidal. Find also whether the vector field $r^n \bar{r}$ is irrotational or not. 5

6. (A) Solve any **One** :

(A1) Evaluate $\bar{F}.\overline{dr}$ at along the curve $\bar{r} = \hat{i}t + \hat{j}t^2 + kt^3$ given $\bar{F} = xy\hat{i} - z^2\hat{j} + xyz\hat{k}$. 2

(A2) Given $\bar{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which is located in first octant then find unit normal vector $\hat{n}$ at any point. 2

(B) Solve any **One** :

(B1) Find work done by the variable force $\overline{F} = 2y\hat{i}+xy\hat{j}$ on a particle when it is displaced from the origin to the point $\overline{r} = 4\hat{i}+2\hat{j}$ along parabola $y^2 = x$. 3

(B2) Use divergence theorem to evaluate $\int\int_s \overline{F}.\hat{n}\ ds$ where $\overline{F} = 4xz\hat{i} - y^2j + yz\hat{k}$ and s is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 3

(C) Solve any **One** :

(C1) Evaluate $\oint_C \overline{F}.\ d\overline{r}$ by Stoke's theorem, where

$\overline{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$ and c is the boundary of the triangle with vertices at (0, 0, 0). (1, 0, 0) and (1, 1, 0). 5

(C2) Use Green's theorem in a plane to evaluate the integral $\oint_C (2x^2-y^2)dx + (x^2+y^2)dy$. Where c is the boundary in xy–plane of the area enclosed by the x–axis and the semi–circle $x^2 + y^2 = 1$ in the upper half xy–plane. 5