B. E. First Semester (All)/SOE18-19_Rev_FY-201 Examination

Course Code : GE 101 **Course Name : Engineering Mathematics**

Time : 2 Hours] [Max. Marks : 40

Instructions to Candidates :—

(1) Attempt any **Four** questions out of **Six**.
(2) All questions carry **Ten** marks.
(3) Assume suitable data wherever necessary.
(4) All questions carry marks as indicated.
(5) Illustrate your answers wherever necessary with the help of neat sketches.
(6) Use of non – programmable calculator are permitted.

1. Solve Either A–Part or B–Part :

(A) (A1) If $y = \sin^{-1}x$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$ 4(CO1)

(A2) Find the radius of curvature for the curve $y = xe^{-x}$ at the point at maximum value of y. 6(CO1)

OR

(B) (B1) Use Machaurin's theorem to show that

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ \ldots.$$ 4(CO1)

(B2) If $y = e^{m\cos^{-1}x}$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (m^2 + n^2)y_n = 0$$ Hence find $(y_n)_0$. 6(CO1)

2. Solve Either A–Part Or B–Part :

(A) (A1) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$ 4(CO2)

(A2) An open rectangular tank is to have a capacity 256 cu. Meter. Find the dimension of the tank so that its surface area is minimum. 6(CO2)

OR

(B) (B1) If $u = \frac{yz}{x}$ $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ 4(CO2)

(B2) If $x + y = 2e\theta \cos$ and $x - y = 2ie\theta \sin\varphi$, then show that

$$\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 u}{\partial\varphi^2} = 4xy \frac{\partial^2 u}{\partial x\partial y}$$ 6(CO2)

3. Solve Either A–Part OR B–Part :

(B) (A1) Prove that $\int_0^\infty \frac{xc}{cx} dx = \frac{\sqrt{c+1}}{(\log c)^{\overline{c+1}}}$ 4(CO3)

(A2) Use differentiate under the integral sign and evaluate

$$\int_0^1 \frac{x^a - x^b}{\log x} dx \quad \text{where } a, b > 0$$ 6(CO3)

OR

(B) (B1) Prove that

$$\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$$ 4(CO3)

(B2) Trace the curve $y^2(a + x) = (a - x)^3$. Find the area between the Curve and its asymptote. 6(CO3)

4. Solve Either A–Part Or B–Part :

(A) (A1) Evaluate $\iint_R \frac{xy}{\sqrt{1-y^2}}$ dxdy, where R is the region of circle $x^2 + y^2 = 1$ in first quadrant. 4(CO3)

(A2) Find the smaller area bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x = 3y = 6$ 6(CO3)

OR

(B) (B1) Evaluate $\int_0^\infty \int_0^x x e^{-x^2/y}\, dy\, dx$ by changing order of integration. 4(CO3)

(B2) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\ dz\ dy\ dx$ 6(CO3)

5. Solve Either A–Part Or B–Part :

(A) (A1) Find the directional derivative of the function $\varphi = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (–1, 2, 1) 5(CO4)

(A2) Show that $\bar{F} = (2xy + z^3)\hat{i} + (x^2)\hat{j} + (3xz^2)\hat{k}$ is a conservative vecor field and find a function such that $\bar{F} = \nabla\Phi$. 5(CO4)

OR

(B) (B1) A particle moves along the curve

$\bar{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}.$

where t is the time. Find the magnitude of tangential components of its acceleration at $t = 2$. 5(CO4)

(B2) Find the constant a and b so that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2 - bz^3 = 4$ may cut orthogonally at point (1, –1, 2) 5(CO4)

6. Solve Either A–Part Or B–Part :

(A) (A1) Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the rectangle bounded by $y = 0$, $x = a$, $y = b$ and $x = 0$ in XY-plane.

5(CO5)

(A2) Evaluate $\iint_S \bar{F}.\ \bar{F}.\hat{n}\ ds$ over the surface of a sphere $x^2 + y^2 + z^2 = 1$ in the positive octant, where $\bar{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ 5(CO4)

OR

(B) (B1) Apply stokes theorem to evaluate

$\oint_C [(x + y)dx + (2x - z)dy + (y + z)dz]$

where c is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) 5(CO4)

(B2) Evaluate $\oint_C [(3x^2 + 2y)dx - (x + 3\cos y)dy]$ by Green's theorem, where C is parallelogram with vertices (0, 0), (2, 0), (3, 1), (1, 1) 5(CO4)