B. E. First Semester (All) / SoE – 2018 – 19 Examination

Course Code : GE 2101 **Course Name : Engineering Mathematics – I**

Time : 3 Hours / 4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Due credit will be given to neatness and adequate dimensions.
(4) Assume suitable data wherever necessary.
(5) Use of Logarithmic tables, non programmable calculator is permitted.

1. (A) Solve any **One** :—

(A1) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$x^2 y_{n+2} + (2n+1)\,x y_{n+1} + (n^2+1) y_n = 0.$ 3

(A2) Given $f(x) = x^3 + 8x^2 + 15x - 24$, apply Taylor's theorem to evaluate

$f\left(\frac{11}{10}\right)$ 3

(B) Solve any **One** :—

(B1) Use Maclaurin's theorem to show that

$$\log(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} \ldots\ldots$$ 3

(B2) If $x = a\cos^4\theta$, $y = a\sin^4\theta$, measure the curvature at $\theta = \frac{\pi}{6}$.

3

(C) Solve any **One** :—

(C1) Prove that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\rho = \frac{a^2b^2}{p^3}$ where p being the perpendicular from the centre upon the tangent at (x, y). 4

(C2) Show that circle of curvature at the origin for the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ is

$$\left(x - \frac{21a}{16}\right)^2 + \left(y - \frac{21a}{16}\right)^2 = \left(\frac{3a}{8\sqrt{2}}\right)^2$$ 4

2. (A) Solve any **One** :—

(A1) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ Find the value of :

$$\frac{\partial(x_1, x_2, x_3)}{\partial(y_1, y_2, y_3)}$$ 3

(A2) If $u = \sin^{-1} x + \sin^{-1} y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ find $\frac{\partial(u, v)}{\partial(x, y)}$, Are u and v functionally related ? If so find the relation. 3

(B) Solve any **One** :—

(B1) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$, show that

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$$ 3

(B2) If $x^x y^y z^z = c$; show that at $x = y = z = \frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$.

3

(C) Solve any **One** :—

(C1) The temperature T at any point (x , y , z) in space is $T = 400\, xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. 4

(C2) If $u = \dfrac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos\left(\dfrac{xy + yz}{x^2 + y^2 + z^2}\right)$, Show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 4 \frac{x^2 y^2 z^2}{x^2 + y^2 + z^2}.$$ 4

3. (A) Solve any **One** :—

(A1) Compute the value of $\Gamma\left(\frac{1}{2} + x\right) \Gamma\left(\frac{1}{2} - x\right) = \dfrac{\pi}{\cos \pi x}$ 3

(A2) Evaluate $\displaystyle\int_0^2 x\,(8 - x^3)^{1/3}\, dx$ 3

(B) Solve any **One** :—

(B1) Trace the curve $3ay^2 = x\,(x - a)^2$. Also find the perimeter of the loop of the curve. 3

(B2) (i) $\displaystyle\int_0^\infty \sqrt{y}e^{-y^2}\, dy \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\, dy = \frac{\pi}{2\sqrt{2}}$ 3

(C) Solve any **One** :—

(C1) Trace the curve $y^2 = x^2\,(1 - x^2)$ and find the area of one of its loop. 4

(C2) Evaluate $\displaystyle\int_0^{\alpha^2} \tan^{-1}\left(\frac{x}{a}\right) dx$

By differentiating under the integral sign. 4

4. (A) Solve any **One** :—

(A1) Change the order of integration and evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$ 3

(A2) Compute the center of gravity of the area bounded by parabola $y^2 = x$, and the line $x + y = 2$. 3

(B) Solve any **One** :—

(B1) Evaluate

$$\int_0^1 \int_0^y xye^{-x^2} \, dydx$$ 3

(B2) Evalulate $\iint e^{2x - 3y}$ dxdy over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$. 3

(C) Solve any **One** :—

(C1) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdxdydz$$. 4

(C2) Find the mass of the tetrahedron bounded by the co – ordinate plane and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, the variable density $\rho = kxyz$. 4

5. (A) Solve any **One** :—

(A1) Given the vector field

$\overline{V} = (x^2 - y^2 + 2xz)\hat{i} + (xz - xy + yz)\hat{j} + (z^2 + x^2)\hat{k}$

Final the curl $\overline{V}$. 2

(A2) Show that

(i) curl (grad Φ) = 0 2

(B) Solve any **One** :—

(B1) A particle moves along the cuve

$\overline{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$, where t is the time. Find the magnitude of tangential components of its acceleration at $t = 2$. 3

(B2) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that

(i) $\text{grad } \vec{r} = \frac{\vec{r}}{r}$

(ii) $\nabla r^n = nr^{n-2}\vec{r}$ 3

(C) Solve any **One** :—

(C1) A vector field is given by

$\overline{A} = (y^2 \cos x. + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$. Show that the field is irrotational and find the scalar potential. 5

(C2) Find the directional derivative of V^2, where

$\overline{V} = xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$ at the point $(2, 0, 3)$ in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(3, 2, 1)$. 5

6. (A) Solve any **One** :—

(A1) If $\overline{F} = 3xy\hat{i} - y^2\hat{j}$ Evaluate $\int_C \overline{F} \cdot \overline{dr}$. Where c is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. 2

(A2) State Stoke's theorem and Divergence theorem. 2

(B) Solve any **One** :—

(B1) Find work done by variable force $\overline{F} = 2y\hat{i} + xy\hat{j}$ on a particle when it is displaced from the origin to the point

$\bar{r} = 4\hat{i} + 2\hat{j}$ along parabola $y^2 = x$. 3

(B2) Evaluate $\oint_C (3x^2 + 2y)\,dx - (x + 3\cos y)\,dy$ by Green's theorem, where C is parallelogram with vertices (0 , 0) , (2 , 0) , (3 , 1) , (1 , 1). 3

(C) Solve any **One** :—

(C1) Apply Stoke's theorem to evaluate $\oint_C [(x + y)\,dx + (2x - z)\,dty + (y + z)\,dz]$ where C is the boundary of the triangle with vertices (2 , 0 , 0) , (0 , 3 , 0) and (0 , 0 , 6). 5

(C2) Use divergence theorem to evaluate $\iint_S \overline{F} \cdot \hat{n}\,ds$, if $\overline{F} = 2xy\hat{i} + yz^2\hat{j} + xy\hat{k}$, where the surface of the sphere is bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$ and $x + 2z = 6$. 5