B. E. First Semester (All) / SoE – 2018-19 Examination

Course Code : GE 2102 **Course Name : Engineering Mathematics – II**

Time : 3 Hours/4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.

(2) All questions carry marks as indicated.

(3) Assume suitable data wherever necessary.

(4) Illustrate your answers wherever necessary with the help of neat sketches.

(5) Use of Logarithmic tables, non – programmable calculator is permitted.

1. (A) Solve any **One** :—

(A1) Compute the value of Particular Integral from D.E.

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{2x}$$ 2

(A2) Compute the value of C.F. for the following D.E.

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = \sin^2 x$$ 2

(B) Solve any **One** :—

(B1) Evaluate the Solution of $[(1 + \log xy)]\,dx + \left[1 + \frac{x}{y}\right] dy = 0.$ 3

(B2) Solve the differential equation $y \log y \frac{dy}{dx} + x - \log y = 0.$ 3

(C) Solve any **One** :—

(C1) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sin hx + \sin x.$ 5

(C2) Use Method of Variation of parameter and solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. 5

2. (A) Solve any **One** :—

(A1) Convert following D. E. Into Differential equation with constant Coefficient form

$$(x + 3)^2 \frac{d^2y}{dx^2} - 4(x + 3)\frac{dy}{dx} + 6y = \log(x + 3)$$ 2

(A2) Develop a differential equation for an electrical circuit connecting Resistance of R ohm, inductance of L Henry and capacitance of C Farade in series with emf of E volt. 2

(B) Solve any **One** :—

(B1) When a switch closed in a circuit containing a battery E, resistance R and inductance L, the current i builds up at the rate given by

$L\frac{di}{dt} + Ri = E$. Find current i as function of t. 3

(B2) Compute the value of C.F. for the following D.E.

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3 \sin(\log x).$$ 3

(C) Solve any **One** :—

(C1) Solve $(3x + 2)^2 \frac{d^2y}{dx^2} + 5(3x + 2)\frac{dy}{dx} - 3y = x^2 + x + 1$. 5

(C2) Solve $\frac{dx}{dt} + 3x - 2y = 1$, $\frac{dy}{dt} - 2x + 3y = e^t$ given that $x(0) = y(0) = 0$. 5

3. (A) Solve any **One** :—

(A1) Express $\sqrt{3} - i$ in polar form. 2

(A2) Compute the general values of $\log(1 + i) + \log(1 - i)$. 2

(B) Solve any **One** :—

(B1) If $\tan (A + iB) = x + iy$ then

prove that (i) $\tan 2A = \dfrac{2x}{1 - x^2 - y^2}$, (ii) $\tanh 2B = \dfrac{2y}{1 + x^2 + y^2}$ 3

(B2) If $i^{\alpha+\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$. 3

(C) Solve any **One** :—

(C1) Prove that $\sinh^{-1} z = \log (z + \sqrt{z^2 + 1})$. 5

(C2) If $\cosh (x + iy) = u + iv$ then show that

$\dfrac{u^2}{\cosh^2 x} + \dfrac{v^2}{\sinh^2 x} = 1$ and $\dfrac{u^2}{\cos^2 x} - \dfrac{v^2}{\sin^2 x} = 1$ 5

4. (A) Solve any **One** :—

(A1) Test whether following functions are analytic or not

$f(z) = \log z$ 2

(A2) Evaluate $\oint_C \left(\dfrac{3z^2 + 7z + 1}{z + 1} \right) dz$ where $|z| = 1{\cdot}5$. 2

(B) Solve any **One** :—

(B1) Expand the following functions by Laurent's Series —

$f(z) = \dfrac{z}{(z-1)\ (2-z)}$ for the region $|z - 1| > 2$. 3

(B2) Use method of Milnes Thomson and find $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is an analytical function,

Where $u(x, y) = e^{-2xy} \sin (x^2 - y^2)$. 3

(C) Solve any **One** :—

(C1) Use Method of Residue and evaluate $\oint_C \dfrac{12z - 7}{(z-1)^2\ (2z+3)} dz$,

Where (i) $|z| \leqslant 2$ and (ii) $|z + i| \leqslant 3/2$. 5

(C2) Evaluate $\int_{0}^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$, by contour Integration. 5

5. (A) Solve any **One** :—

(A1) Find the rank of the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ 2

(A2) Compute the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ 2

(B) Solve any **One** :—

(B1) Test the consistency of the following Equation :

$$5x + 3y + 7z = 4, \quad 3x + 26y + 2z = 9, \quad 7x + 2y + 10z = 5.$$

And if so, find the solution. 3

(B2) Verify the Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

3

(C) Solve any **One** :—

(C1) Using Sylvester's theorem Verify $\log_e e^A = A$, where $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

5

(C2) Find the Eigen value and Eigen vectors and modal matrix for the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 5

6. (A) Solve any **One** :—

(A1) Compute the normal equations for the curve $y = ax^b$. 2

(A2) Find the ranks of X and Y from the following data :

x	24	13	27	12	31	42	13	29	17	11
y	24	25	21	25	22	19	24	20	25	26

2

(B) Solve any **One** :—

(B1) Use Method of Least square and Fit a curve $y = ax + \frac{b}{x}$ to the following data :

x	1	2	3	4	5	6	7	8
y	5•43	6•28	8•23	10•32	12•63	14•86	17•27	19•51

3

(B2) Obtain the rank correlation coefficient for the following data :

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

3

(C) Solve any **One** :—

(C1) Two lines of regression are given by $8x - 10y + 66 = 0$ and $40x - 18y = 214$.

If $\sigma_x^2 = 9$, find

(i) Mean values of x and y,

(ii) The coefficient of correlation between x and y and

(iii) Variance of y. 5

(C2) Find the equation of regression lines and the coefficient of correlation for the following data :

x	3	5	6	8	9	11
y	2	3	4	6	5	8

5