B. E. Third Semester (CE/CT/IT/ME/EL/EE/ET) / SoE–2018 Examination

Course Code : GE 2201 **Course Name : Engineering Mathematics –III**

Time : 3 Hours / 4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) Use of Logarithmic tables, non – programmable calculator is permitted.

1. (A) Solve any **One** :

(A1) Express $y = x^4 - 12x^3 + 24x^2 - 30x + 9$ in factorial form. 2

(A2) Prove that $\left(\frac{\triangle^2}{E}\right) e^x \left(\frac{Ee^x}{\triangle^2 e^x}\right) = e^x$ 2

(B) Solve any **One** :

(B1) From the following table, obtain $\frac{dy}{dx}$ at the point $x = 1.8$ 3

x	0	0.5	1	1.5	2
y	0.3989	0.3521	0.2420	0.1295	0.0540

(B2) Solve : $y_{n+2} - 2y_{n+1} + 4y_n = 0$. 3

(C) Solve any **One** :

(C1) Evaluate $\int_0^2 \frac{2e^{-x^2}}{\sqrt{\pi}} dx$ by dividing the interval into 10 equal parts by using Simpson's rule. 5

(C2) Solve the difference equation : $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$ 5

2. (A) Solve any **One** :

(A1) Find L.T. of $2t^{3/2} + 5e^{-3t}$ 2

(A2) Find Laplace Transform of $e^{4t}\sin 3t$ 2

(B) Solve any **One** :

(B1) Express $f(t) = \begin{cases} t^2, & 0 < t < 1 \\ 4t, & t > 1 \end{cases}$ in terms of unit step function hence find its Laplace Transform. 3

(B2) Evaluate $L^{-1}\left\{\dfrac{1}{s^3(s+1)}\right\}$ by using convolution theorem. 3

(C) Solve any **One** :

(C1) Find $L^{-1}\dfrac{5s+3}{(s-1)(s^2+2s+5)}$ 5

(C2) Solve $(D^2 + 2D + 5)y = e^{-t}\sin t$, $y(0) = 0$ $y'(0) = 1$ using Laplace transform. 5

3. (A) Solve any **One** :

(A1) Find the Z–Transform of $\dfrac{1}{(n+2)!}$ 2

(A2) Find Z–Transform of na^n 2

(B) Solve any **One** :

(B1) Find inverse Z–Transform of $\dfrac{z-4}{(z-1)(z-2)^2}$ 3

(B2) Find Z–Transform of $a^n \sin n\theta$ 3

(C) Solve any **One** :

(C1) Solve the difference equation by Z–Transform

$$x_{n+2} + 3X_{n+1} + 2X_n = u_n$$

given that $x_0 = 1$ and $x_n = 0$ for $n < 0$, where u_n is an unit step function. 5

(C2) Find inverse Z–Transform of $\dfrac{16z^3}{(4z-1)^2(z-1)}$ 5

4. (A) Solve any **One** :

(A1) Sketch the function $f(x) = \begin{cases} -x^2 & , -\pi < x < 0 \\ x^2 & , \ 0 < x < \pi \end{cases}$ 2

(A2) Find the constant term in the Fourier series for $f(x) = x^2-2$ in the interval, $-2 < x < 2$. 2

(B) Solve any **One** :

(B1) Find the Fourier cosine series for $f(x) = \pi-x$, in the interval, $0 < x < \pi$. 3

(B2) Find the Fourier series expansion for $f(x) = 1-x^2$ in the interval, $-1 < x < 1$. 3

(C) Solve any **One** :

(C1) Find Fourier series for $f(x) = \begin{cases} \pi+x, & -\pi < x \leqslant 0 \\ \pi-x, & 0 \leqslant x < \pi \end{cases}$

Hence show that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots\ldots$ 5

(C2) Expand $f(x) = e^x$ as a Fourier series in the interval $0 < x < 2\pi$. 5

5. (A) Solve any **One** :

(A1) Find complementary function of $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial x \partial y} + 4\dfrac{\partial^2 z}{\partial y^2} = e^{(2x+3y)}$ 2

(A2) Find P.I. for the P.D.E. $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial x \partial y} + 4\dfrac{\partial^2 z}{\partial y^2} = e^{(2x+3y)}$ 2

(B) Solve any **One** :

(B1) Solve $xq = yp+xe^{(x^2+y^2)}$ where $p=\dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$ 3

(B2) Solve $\dfrac{\partial^3 z}{\partial x^3} - 2\dfrac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2y$ 3

(C) Solve any **One** :

(C1) Use method of separation of parameter to solve :

$4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial t} = 3u$ given $u(0, t) = 3e^{-t} - e^{-5t}$ 5

(C2) Solve $\dfrac{\partial^2 z}{\partial x^2} - 3\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = e^{2x}\sin(x+3y)$ 5

6. (A) Solve any **One** :

(A1) Find the Fourier sine transform of e^{-ax} 2

Express $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. 2

(B) Solve any **One** :

(B1) Using Parsevel's identity, show that $\displaystyle\int_0^{\infty} \frac{x^2 dx}{(x^2+1)^2} = \frac{\pi}{4}$ 3

(B2) Find the Fourier sine transform of $e^{-|x|}$ and hence show that

$\displaystyle\int_0^{\infty} \frac{x\sin mx\,dx}{1+x^2} = \frac{\pi}{2}e^{-m}, \quad m > 0.$ 3

(C) Solve any **One** :

(C1) Find the Fourier transform $f(x) = \begin{cases} 1 - x^2, & \text{for } |x| \leq 1 \\ 0, & \text{for } |x| > 1 \end{cases}$

and hence find $\displaystyle\int_0^{\infty} \left(\frac{\sin x - x\cos x}{x^3}\right) \cos\left(\frac{x}{2}\right) dx$ 5

(C2) Solve for f(x),the integral equation $\displaystyle\int_0^{\infty} f(x)\sin\lambda x\,dx = \begin{cases} 1, & 0 \leq \lambda \leq 1 \\ 2, & 1 \leq \lambda \leq 2 \\ 0, & \lambda \geq 2 \end{cases}$

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