B. E. Fourth Semester (CT/IT) Examination

Course Code : GE 1206/GE 206 /GE 510

Course Name : Discrete Mathematics and Graph Theory

Time : 3 Hours / 4 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) All questions carry marks as indicated.
(3) For MCQ, only first attempt will be considered.

1. Solve **Q. A or Q. B** :

(A) (A1) The symbolic form of statement "x is an even number iff it is divisible by 2"

(a) $p \longrightarrow q$ (b) $q \longrightarrow p$

(c) $p \leftrightarrow q$ (d) None of these 1

(A2) Write the converse and contrapositive of the following statements. If a triangle is equilateral, then it is equiangular. 2

(A3) Prove the following by applying principle of mathematical induction

$n < 2^n, \quad n \geqslant 0$ 4

(B) (B1) If p and q are the two statements then by De-Morgan's law $\sim (p \wedge q) =$ ________

(a) $\sim p \vee \sim q$ (b) $\sim p \wedge \sim q$

(c) $\sim p \cong \sim q$ (d) None of these 1

(B2) Examine for the tautology, contradiction and contingency ?

$((p \longrightarrow q) \wedge (q \longrightarrow r)) \longrightarrow (p \longrightarrow r)$ 2

(B3) If A, B and C are any non-empty sets then show that

$(A \cup B) \times C = (A \times C) \cup (B \times C)$ 4

2. Solve **Q. A or Q. B** :

(A) (A1) A function f from A to B is said to be Bijective iff is ______

(a) One-to-one (b) Onto

(c) Both one-to-one and onto (d) None of these 1

(A2) Prove that $R = \{(x, y)/x^2 - 4xy + 3y^2 = 0, \forall x, y \in N\}$ is reflexive but not symmetric. 2

(A3) Examine by Using properties of characteristics function

(i) $f_{(A \cup B)}(x) = f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$

(ii) $f_{(A \cap B)}(x) = f_A(x) \cdot f_B(x)$ 5

(B) (B1) A function f from A to B is said to be Onto function if ____

(a) B = Range of f (b) B = Domain of f

(c) B = Co – Domain of f (d) None of these 1

(B2) If $f(x) = x^2$ and $g(x) = 2^x$ then find f 0 g 2

(B3) Let $A = \{0, 1, 2, 3\}$. Find the matrix for $R = \{(x, y)/x + y = 3\}$, $S = \{(x, y)/x + y \leq 4\}$.

Also find $(M_R \cdot M_S)'$ and show that $(M_R \cdot M_S)' \neq (M_R)' \cdot (M_S)'$ 5

3. Solve **Q. A or Q. B** :

(A) (A1) An Algebraic structure (G, *) is called Monoid if it satisfied________

(a) Closure and Associative (b) Associative and Identity

(c) Closure, Associative and Identity (d) None of these 1

(A2) Prove that for any element a in a group G if $a^2 = e$, then G must be an abelian group. 2

(A3) Show that set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, $\alpha \in R$ form a group under matrix multiplication. 4

(B) (B1) An isomorphism of semi group into itself is called semi group________

(a) Homomorphism (b) Endomorphism

(c) Automorphism (d) None of these 1

(B2) Prove that for any element a in a group G if $a^2 = e$, then G must be an abelian group. 2

(B3) Examine whether the set {0, 1, 2, 3, 4} of order 5 is a finite abelian group under multiplication modules as composition. 4

4. Solve **Q. A or Q. B** :

(A) (A1) R be a commutative ring an element $a \neq 0$ in R is called _______ if their exist an element $b \neq 0$ in R such that ab = 0

(a) Identity (b) Zero divisor

(c) Inverse (d) None of these 1

(A2) If R is a ring such that $a^2 = a, \forall a \in R$, then show that

$$a + a = 0, \forall a \in R$$ 2

(A3) Given $A = \left\{\frac{0.8}{3}, \frac{0.9}{4}, \frac{0.6}{5}\right\}$ and $B = \left\{\frac{0.7}{3}, \frac{0.3}{5}\right\}$ be two fuzzy sets.

Verify De-Morgan's law $\overline{A \cup B} = \bar{A} \cap \bar{B}$ symbols have their usual meaning. 5

(B) (B1) Intersection of two Fuzzy set A and B is a Fuzzy set C =__________

(a) $\max\{\mu_A(x), \mu_B(x)\}$ (b) $\min\{\mu_A(x), \mu_B(x)\}$

(c) $\{\mu_A(x), \mu_B(x)\}$ (d) None of these 1

(B2) If R is a ring and $a, b \in R$, then show that

$$(a + b)^2 = a^2 + ab + ba + b^2 \neq a^2 + 2ab + b^2$$ 2

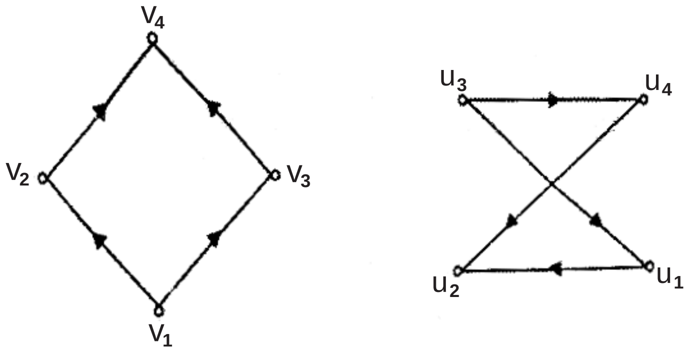
(B3) Show that the set $R = \{a + b\sqrt{2} / a, b \in Q\}$ is a field w. r. t. addition and multiplication. 5

5. Solve any **Three** :

(A) Construct the tree for the following algebraic expression. Also draw its Venn diagram

$$[x + (y + z)] - [a \times (b + c)]$$ 5

(B) Prove that the following graphs are isomorphic



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(C) Draw the digraph of the following adjacency matrices A and B. Are these graphs isomorphic ?

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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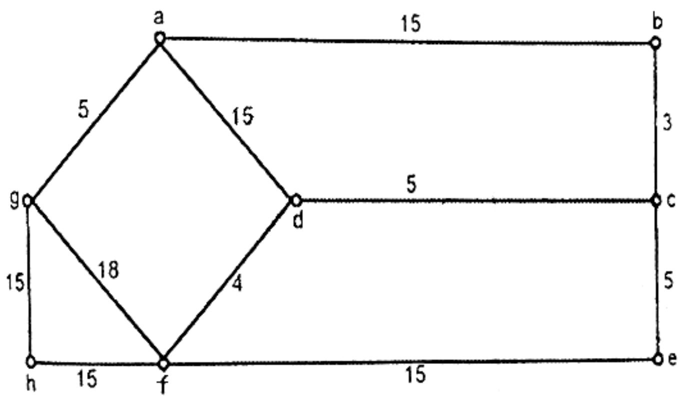
(D) Draw a railway network of minimal cost for the following cities. Also find the minimal cost.



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6. Solve any **Three** :

(A) The joint probability function of two random variable X and Y is given by

$$f(x, y) = \begin{cases} C\,xy, & x = 1, 2, 3,\ y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Find :

(i) The constant C,

(ii) $P(X = 3, Y = 1)$,

(iii) $P(1 \leqslant X \leqslant 2,\ Y \leqslant 3)$,

(iv) $P(X \geqslant 2)$

(v) $P(Y < 2)$ 5

(B) Let X and Y be two continuous random variables having joint density function

$$f(x, y) = \begin{cases} C\,(x^2 + y^2), & 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1 \\ 0, & \text{otherwise} \end{cases}$$

Find :

(i) Constant C,

(ii) $P\left(X < \frac{1}{2},\ Y > \frac{1}{2}\right)$,

(iii) $P\left(\frac{1}{4} < X \frac{3}{4}\right)$,

(iv) $P\left(Y < \frac{1}{2}\right)$ 5

(C) Let X and Y be two random variables with joint probability function

$$f(x, y) = \begin{cases} \dfrac{x + 2y}{27}, & x = 0, 1, 2,\ y = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

Find

(i) $P(X = 0,\ Y = 2)$,

(ii) $P(X \geqslant 1)$

(iii) Marginal probability functions of X and Y. Also determine whether X and Y are independent. 5

(D) Find the conditional density function of (i) X given Y and (ii) Y given X for the distribution

$$f(x, y) = \begin{cases} \dfrac{3(x^2 + y^2)}{2}, & 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1 \\ 0, & \text{otherwise} \end{cases}$$ 5