❖ FORMULATION

For the formulation of **Cost Design and Flight Scheduling** problem, the assumptions and variables that we have taken will be explained in this section.
Let us assume an airline would like to use mathematical programming to schedule its flights to minimize cost (Cost Design). The following map shows the city pairs that act as bases:

Here, we are tackling the problem keeping 3 planes in mind, namely Plane1, Plane2 and Plane3. In the above diagram all the encircled parts depict the various airline bases where the planes will pass through. From here on we will consider **Cost Design** problem as **Problem Part – I** and **Flight Scheduling** problem as **Problem Part – II**.

❖ NOMENCLATURE

x = Total distance covered by Plane1 in km (where x=1 means 100 kms).

y = Total distance covered by Plane2 in km (where y=1 means 100 kms).

z = Total distance covered by Plane3 in km (where z=1 means 100 kms).

a = Number of airports used by Plane1.

b = Number of airports used by Plane2.

c = Number of airports used by Plane3.

❖CONSTRAINTS

For Problem Part – I

▪*Constraint 1*: The sum of the total distance travelled by all three planes should be greater than 7200 km provided plane 1 and plane 3 individually cover double the distance travelled by plane 2.

$$200x + 100y + 200z > 7200$$

▪*Constraint 2*: On analysing every plane on their present conditions it is found that the price spent on each plane’s fuel consumption per km is $102, $204 and $102 for Plane1, Plane2 and Plane3 respectively and the total expenditure allowed on fuel for all planes combined must be less than or equal to $6120.

$$102x + 204y + 102z \leq 6120$$

▪*Constraint 3*: Maintenance cost per km spent on Plane1 is $200, on Plane2 is $100 and on Plane3 is $100. Airlines can spend at most $8000 on maintenance which should be shared among those three planes.

$$200x + 100y + 100z \leq 8000$$

▪*Non-negativity Conditions:* Each Plane should not fly more than 2000 km.

$$20 >= x >= 0$$

$$20 >= y >= 0$$

$$20 >= z >= 0$$

For Problem Part – II

▪*Path Function:* Path function helps to minimize the cost so that we have total no. of airports travelled by each plane. It states that four times the sum of the no. of airports used by planes is equal to sum of the total distances covered by each plane. Hence, Mathematically

$$4(a + b + c) = x + y + z$$

❖COST FUNCTION

For Problem part - I:

Our objective is to minimize total operating cost. The operating cost per km for Plane1, Plane2 and Plane3 are $200, $150 and $400 per km, respectively.

Hence,

$$C_1 = 200x + 150y + 400z$$

For Problem part - II:

Our objective is to minimize the total number of airports used while maximising the total number of distances covered.

Hence,

$$C_2 = (a-1)_2 + (b-1)_2 + c_2$$

▪Conclusion

The Solution of the given problem comes out to be:

$$x = 20$$

$$y = 16$$

$$z = 8$$

Minimum Cost: $960,000

Values of:

$a = 4$

$b = 4$

$c = 3$

So, Plane1 can take 4 paths to travel a total distance of 20,000km. Plane2 can take 4 paths to travel a total distance of 16,000km. Plane3 can take 3 paths to travel a total distance of 8,000km.

So, Plane1 can follow sequence: A1 - A6 - A5 - A1 - A4 - A5 - A1 - A3 Plane2 can follow sequence: A1 - A5 - A4 - A1 - A4 - A5 - A1 Plane3 can follow sequence: A1 - A4 - A1 - A2 - A1