MEL7520 **Dr. B. Ravindra**

Railway Active Vehicle Suspension

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Method

In comparison to passive suspensions, the LQR controller designed will allow the active suspension system to dynamically adjust in different operating conditions without compromising rail handling capacity or ride comfort. The active suspension system is operated by an electrohydraulic actuator with a LQR control strategy. Simulink will be used to construct block diagrams and simulate the systems, while MATLAB will be used to build and analyze the state-space model for both passive and active systems.

MATLAB/Simulink

The state-space model of the railway suspension system and the LQR controller for the active suspension system will be developed using MATLAB software, which will also house the model parameters that will be used in conjunction with Simulink. When it comes to the analysis of experimental results, MATLAB is a common program for theoretical calculations.When it comes to analyzing the suspension parameters within the simulation model, the MATLAB environment can come in handy. For comparison and discussion, the results obtained with MATLAB will be used. Simulink is a program that allows you to simulate, analyze, and model complex structures. The program can be used to study the behavior of real-world dynamic systems, and it will be used in this dissertation to study the railway vehicle's quarter-car model suspension system.

MATHEMATICAL MODEL

When designing a control system, the establishment of the mathematical model of the system is needed. Like most engineering control systems, a mathematical model will typically rely on known laws which will then be derived, Newton second law of motion will be the law focus for the quarter car suspension model. The mathematical model's primary purpose is to provide an equation or equations describing how the system behaves.

QUARTER CAR MODEL

A simplified quarter car model, as shown in Figure 1 (a), was used to analyze the parameters related to the suspension system. For the study of the vertical vibration induced by the railway disruption, the quarter car model was chosen because it is the most general and straightforward of the vehicle dynamic vibration models.The railway vehicle shell, also known as the sprung mass, and the railway vehicle bogie, also known as the unsprung mass, make up the mass of the railway vehicle. Suspension springs and dampers attach the sprung and unsprung axles to the track.In contrast to the suspension system's vertical deflections, both transversal and longitudinal deflections are considered negligible. Due to the lack of a control factor in the passive suspension system (see Figure 1), the actuator force will not be considered (b).

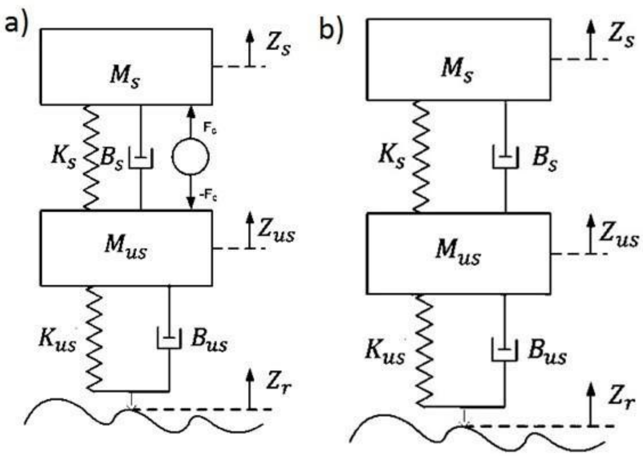


Figure 1: A quarter model for the active suspension system (a) and the passive suspension system (b)

Where:-

M_s= Represents the railway vehicles body mass or sprung mass.

M_{us}= Represents the unsprung mass of the railway vehicle including the bogie, wheel and other associated parts

K_s= Represents the suspension systems spring constant for the sprung mass.

K_{us}= Represents the suspension systems spring constant for the unsprung mass.

B_s= Represents the inherent damping coefficient for the suspension system.

B_{us}=Represents the inherent damping coefficient for the railway vehicles wheel assembly.

F_c= The active suspensions actuator control force.

Z_s= Represents the railway vehicles (sprung mass) body displacement.

Z_{us}= Represents the railway vehicles wheel displacement and the unsprung masses displacement

Z_r= Represents the excitation due to the railway disturbance.

QUARTER RAIL VEHICLE MODEL DESIGN PARAMETERS

Table 1 lists the input parameter values for the railcar model's passive and active suspension systems, which will be used to evaluate and generate performance.

Parameters	Values
M_s	5333 kg
M_{us}	906.5 kg
K_s	430,000 N/m
K_{us}	2,440000 N/m
B_s	20,000 sec/m
B_{us}	40,000 sec/m

Table 1: Quarter rail model design parameters

EQUATION OF MOTION

The equations of motion considered for both the passive and the active suspension systems are derived using Newton's second law of motion.

Equation 1

$$F = ma$$

Equation 2

Transposed for the acceleration, the equation now becomes:

$$a = F/m$$

Equation 3

So, from the figure above, Newton's law of motion, the dynamic equation of the active suspension system and with the forces acting on the sprung mass, the following equation is given:

$$M_s\ddot{Z}_s = B_s\dot{Z}_{us} - B_s\dot{Z}_s - K_s(Z_s - Z_{us}) + F_c$$

Equation 4

Transposed for the sprung mass acceleration, the equation now becomes:

$$\ddot{Z}_s = \frac{B_s\dot{Z}_{us}}{M_s} - \frac{B_s\dot{Z}_s}{M_s} - \frac{K_s(Z_s - Z_{us})}{M_s} + \frac{1}{M_s}F_c$$

Equation 5

The forces acting on the unsprung mass are the following:

$$M_{us}\ddot{Z}_s = -B_s\dot{Z}_{us} - B_{us}\dot{Z}_{us} + B_s\dot{Z}_s + B_{us}\dot{Z}_r - K_s(Z_{us} - Z_s) - K_{us}(Z_{us} - Z_r) - F_c$$

Equation 6

Transposed for the unsprung mass acceleration, the equation now becomes:

$$\ddot{Z}_{us} = -\frac{B_s\dot{Z}_{us}}{M_{us}} - \frac{B_{us}\dot{Z}_{us}}{M_{us}} + \frac{B_s\dot{Z}_s}{M_{us}} + \frac{B_{us}\dot{Z}_r}{M_{us}} - \frac{K_s(Z_{us} - Z_s)}{M_{us}} - \frac{K_{us}(Z_{us} - Z_r)}{M_{us}} - \frac{1}{M_{us}}F_c$$

STATE SPACE REPRESENTATION

The general state-space representation is given by the following:

Equation 7

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x= Represents the state variables vector.

$\dot{x}$= Represents the time derivative of the state variables vector.

y= Represents the output vector.

u= Represents the input vector.

A= Represents the system matrix.

B= Represents the input matrix.

C= Represents the output matrix.

D= Represents the feedforward matrix.

A state-space model describing the active suspension system will be created using the two equations of motion derived earlier. Let the state variables representing the system be:

$X_1 = Z_s - Z_{us}$

$Z_s - Z_{us}$= Represents the suspension travel.

$X_2 = \dot{Z}_s$

$\dot{Z}_s$= Represents the railway vehicles body velocity.

$X_3 = Z_{us} - Z_r$

$Z_{us} - Z_r$= Represents the wheel deflection.

$X_4 = \dot{Z}_{us}$

$\dot{Z}_{us}$= Represents the wheel vertical velocity.

The state-space model of the active suspension system can be easily obtained and written in the form given in equation 7, which will then be written in matrix form as shown below using the equations of motion contained in equations 3, 4, 5, and 6.

Equation 8

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & -1\\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s}\\ 0 & 0 & 0 & 1\\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_{us}}{M_{us}} & -\frac{B_s+B_{us}}{M_{us}}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \frac{1}{M_s}\\ -1 & 0\\ \frac{B_{us}}{M_{us}} & -\frac{1}{M_{us}}\end{bmatrix}\begin{bmatrix}\dot{Z}_r\\ F_c\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \frac{1}{M_s}\end{bmatrix}\begin{bmatrix}\dot{Z}_r\\ F_c\end{bmatrix}$$

STATE-SPACE CONTROLLABILITY

When it comes to control systems, there are many issues to consider, such as stabilizing chaotic systems with feedback control, and controllability is a crucial aspect to consider when attempting to solve these issues. The system can be regulated if it has a single control input, u, that can shift a system state around in its entire configuration to another state.The controllability matrix P must be maximum rank and equal to the n number of states in a linear time-invariant state system to be controllable.

Equation 9

$$\boldsymbol{P}=\left[\boldsymbol{B}\ \boldsymbol{AB}\ \boldsymbol{A}^2\boldsymbol{B}\cdots\boldsymbol{A}^{n-1}\boldsymbol{B}\right]$$

$$\boldsymbol{rank}(\boldsymbol{P})=\boldsymbol{n}$$

$\boldsymbol{P}$= Represents the controllability matrix.

$\boldsymbol{n}$= Represents the number of state variables.

$\boldsymbol{A}$= represents the coefficient of matrix A.

$\boldsymbol{B}$= represents the coefficient of matrix B.

FULL STATE VARIABLE FEEDBACK CONTROL

Figure 2 shows a full state feedback controller, also known as a pole placement controller, which is the best method for achieving desired pole positions in a closed-loop system because it allows the controller to know all state variables at all times and provide feedback.

The state space matrix is the plant with each state variable being fed back to the control input ,u, through the gain K represented by feedback vector, which can be adjusted to reach the desired closed-loop pole values. So, the system input is given by:

Equation 10

$$u = -Kx$$

Substituting this into equation 7 yields the closed-loop system's state space equation:

Equation 11

$$\dot{x} = Ax - BKx$$

$$\dot{x} = x(A - BK)$$

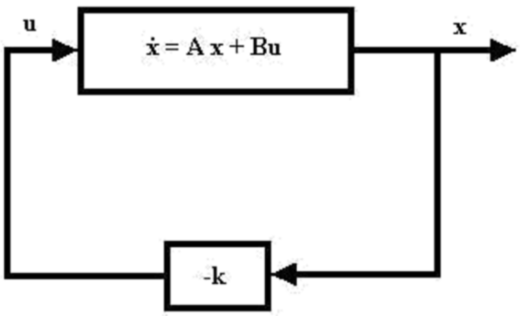


Figure 2: A full-state feedback block diagram

LINEAR QUADRATIC REGULATOR (LQR)

The LQR controller is a common form of state feedback control that provides a systematic way of determining the control gain, K. Since it is one of the more classic control options for linear MIMO time-invariant systems and is easy to design, the LQR approach will be used for the controller design for the active suspension system. One of the benefits of using a LQR controller is that the variables influencing the output index can be weighted based on the individual's desired outcome. The LQR approach will concentrate on improving the rail handling capacity and ride comfort of the quarter rail model.

The primary goal of a LQR controller is to minimize the cost function, J, which is represented by the performance index in equation 12, and to measure the optimal gain, K:

Equation 12

$$J = \frac{1}{2}\int_0^t (x^t Q x + u^t R u)\, dt$$

x^t= Represents the state vector and contains the system state variables.

u^t= Represents the input vector and is the systems control input.

The performance index or quadratic cost function J must be minimised by adjusting both the weighting Q and R matrices, where Q is a diagonal positive definite and R is a positive constant. The desired closed-loop performance is then obtained by tuning the weighting matrices, by penalising bad performance by adjusting the Q matrix or penalising actuator effort by adjusting the R matrix until suitable results regarding the cost function are reached for the plant.

Based on equation 10, the feedback regulator and solution and to the performance index in is given by:

Equation 13

$$F_c = -Kx$$

Meaning that both the A and B matrices must correspond to the actuator control force in the feedback regulator, giving the matrices shown in equation 14

Equation 14:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_{us}}{M_{us}} & -\frac{B_s + B_{us}}{M_{us}} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M_s} \\ 0 \\ -\frac{1}{M_{us}} \end{bmatrix}$$

LQR CONTROLLER MATLAB IMPLEMENTATION

After determining that both the A and B matrices are controllable, a LQR controller can be built and implemented in MATLAB. The following MATLAB code must be used to represent the system matrix A, the input matrix B, the output matrix C, and the feedforward matrix D:

```
A = [ 0 1 0 -1 ;

    -ks/ms -bs/ms 0 bs/ms;

      0 0 0 1;

    ks/mus bs/mus -kus/mus -(bs+bus)/mus];

B = [0  0 ;

     0 1/ms  ;

    -1  0 ;

    bus/mus -1/mus ];

C = [ 1 0 0 0 ;

    -ks/ms -bs/ms 0 bs/ms ];

D = [0 0; 0 0; 0 0;  0 0; 0 0;
     0 1/ms];
```

Since the open-loop system's controllability has already been established, the quadratic performance index's weighting matrices Q and R can now be tuned for and obtained in MATLAB. The states related to suspension travel and railway vehicle body acceleration were the performance metrics that were of relative significance and needed further attention.The final weighting matrices Q and R are shown below, after trial and error of modifying the nonzero elements in the Q matrix and the input weighting of the R matrix:

Equation 15

$$Q = \begin{bmatrix} 1760 \times 10^6 & 0 & 0 & 0 \\ 0 & 11.6 \times 10^6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = 0.01$$

Using the following MATLAB command, the feedback gain vector K was obtained by entering the Q and R weighting matrices:

```
lqr( A, B(:,2), Q, R )

Q = diag([1760*10^6, 11.6*10^6, 1, 1]);

R = 0.01;

K = lqr( A, B(:,2), Q, R )

K =1.0e+05 *
       1.7075     0.3637     0.7759     0.0052
```

So, the feedback gain vector obtained is:

Equation 16

$$K = [1.7075 \times 10^5 \quad 0.3637 \times 10^5 \quad 0.7759 \times 10^5 \quad 0.0052 \times 10^5]$$

SIMULINK MODEL

The state-space model and the LQR controller can now be used to model both active and passive suspension systems in Simulink. The active and passive suspension models' modeling processes, as well as the functions of the various subsystems, are depicted in the block diagrams below.

When designing the device, the first step was to create a rail disruption that would excite both the passive and active suspensions shown in Figure 3. The first rail disturbance (RD1) is a simple phase input with a 0.06-meter step disturbance height. The step block provides a step input between two definable values at a specific time; for the simulated model, a step time of 0 seconds was selected, along with an initial value of 0.

Figure 3 also shows the second rail disturbance (RD2), which is viewed as a pulse width modulation with square waved impulses at six-second intervals. The chosen amplitude disturbance height was 0.1m, with a pulse width of 50% of the time (three seconds) and a phase delay of 0.1 seconds.The second disruption was selected as the worst-case scenario for the active suspension method, and it is used in the robustness test, which will be discussed in the results and discussion section later in this dissertation.

The next move was to build a first-order filter for the rail disturbances, as well as a derivative block to determine their speeds. The railway disturbance will not actually be a sharp phase disturbance, but rather a smoother edge disturbance with some angle, which is why a filter will be used to smooth out the disturbance for railway vehicles.Since their derivatives will be taken later and the simulation is generating accurate results, a first-order filter in the form of a transfer function was used to smooth out the results generated from both the phase input in RD1 and the pulse width generator in RD2.

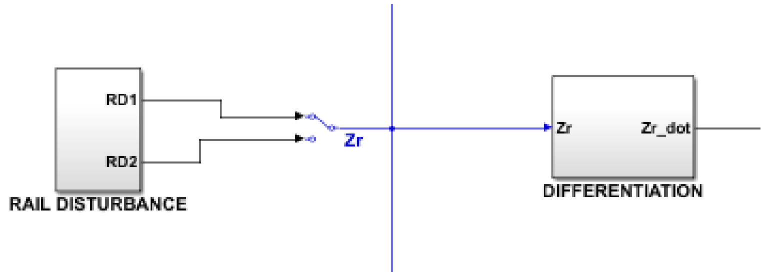


Figure 3: Rail disturbance Simulink model

A state-space block will be used to model the open-loop plant, as shown in Figure 4. The suspension system behavior, as described in equation 7, is implemented using the state-space block.

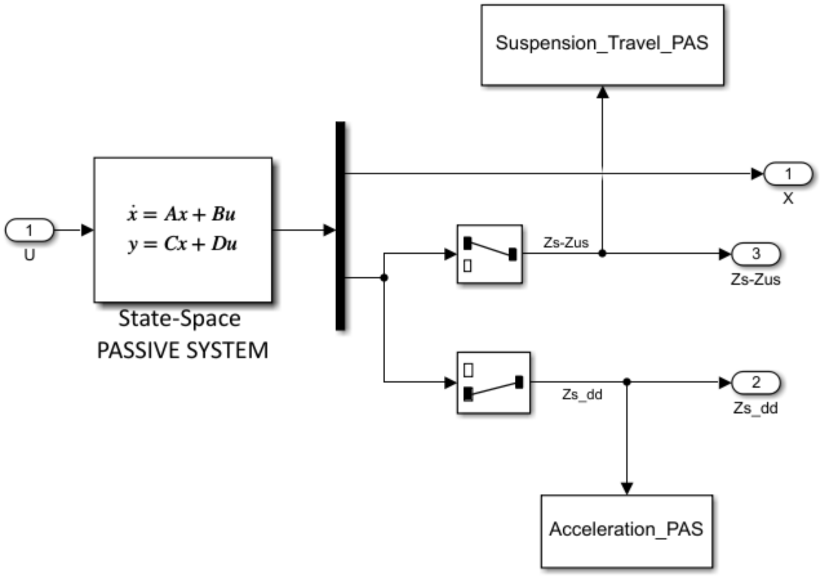


Figure 4: Simulink model of the passive suspension system

Both the active and passive suspension systems' state-space models are divided into two subsystems, as shown in Figure 5.

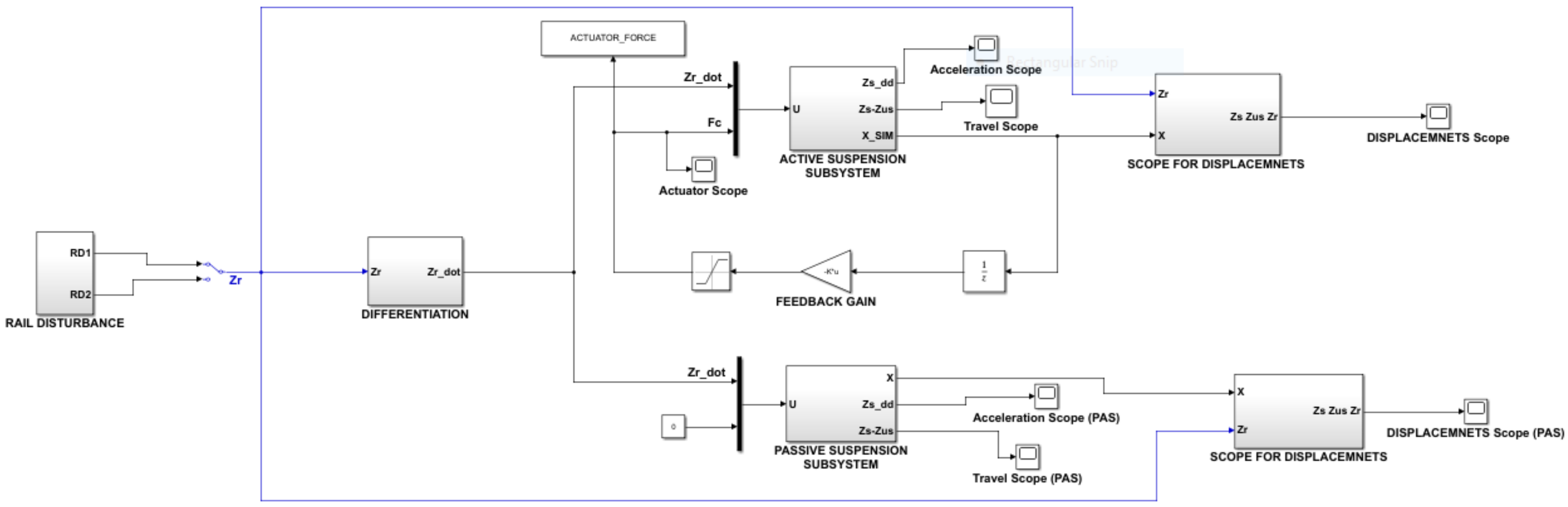


Figure 5: Simulink model of both the active suspension system and the passive suspension system

The active suspension system, on the other hand, has a control function, and all state variables output from the suspension system are fed back to the control input through the gain K, resulting in the actuator force needed to stabilize the system shown in Figure 6.

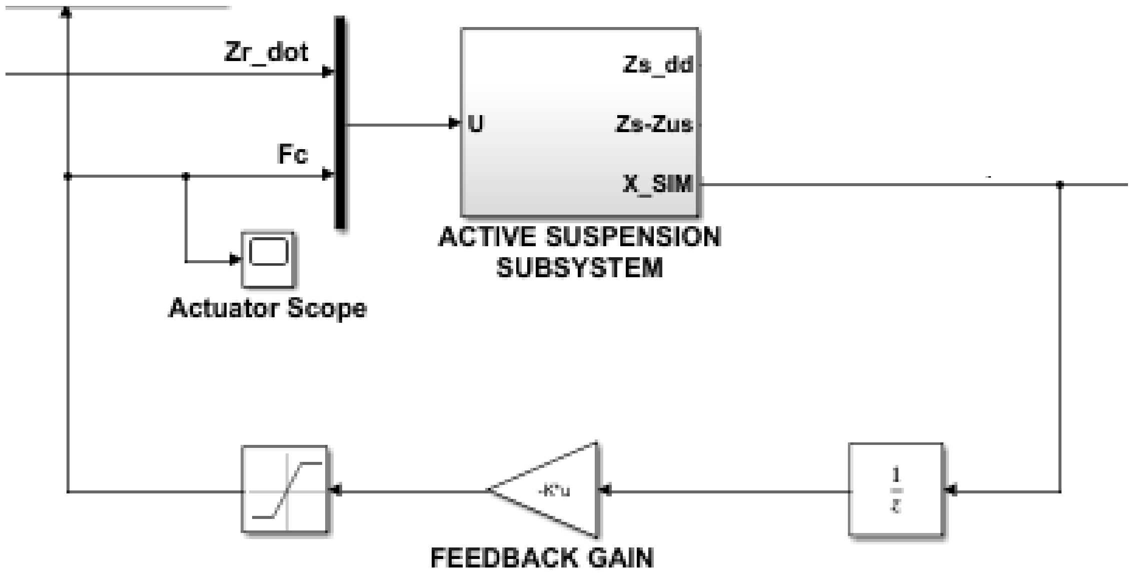


Figure 6:A Simulink model inputting the actuator control force into the active suspension subsystem

Workspace blocks that write signal data into the workspace module in MATLAB are used to input the suspension travel in the sprung mass acceleration results into a workspace. Calculations were carried out in order to obtain the displacements for the sprung and unsprung masses of the railway vehicle for the active suspension system.The suspension travel, wheel deflection, and rail disturbance displacement were added together and measured.

RESULTS AND ANALYSIS

Analyze the effects of the quarter car model's passive suspension system, then examine the reaction of the active suspension system using the state space controller. The LQR controller's effect on the device will also be evaluated to see how important it is. The ride quality and railcar handling will be controlled when discussing the performance of the two suspension systems, with the railcar body acceleration, suspension travel, and wheel deflection, as well as the quarter car model's body displacements, being the parameters of emphasis.

The following criteria will be compared when the two systems are compared:

• Settling Time: This is the time it takes for the machine to oscillate before it reaches a point where it starts to fit the desired value.

• The Rise Time: the time it takes for the device to reach a given percentage within the defined value and how quickly it can react.

• Overshoot: a measure of how much the device initially deviates from the desired answer.

• Steady-State Error: the distance between the desired answer and the final error.

NOTE: The results will be produced using the mathematical quarter railcar model for the suspension system derived in the methodology, as well as the parameters described in the methodology. Within 10 seconds, the quarter railcar model is believed to be traveling at a constant speed along the railway with a rail disturbance height of 0.06m.

When the phase input rail disturbance is met, Figure 7 shows the response of the force produced by the actuator in the active suspension controller. What is noticeable is that the force produced by the actuator is applied in the opposite direction of the sprung mass or railcar body and overshoots; this is due to the sprung mass's tendency to travel upward.The figure shows that the actuator's response is stable and provides an adequate response to the rail disturbance in the form of a 0.06m phase input.

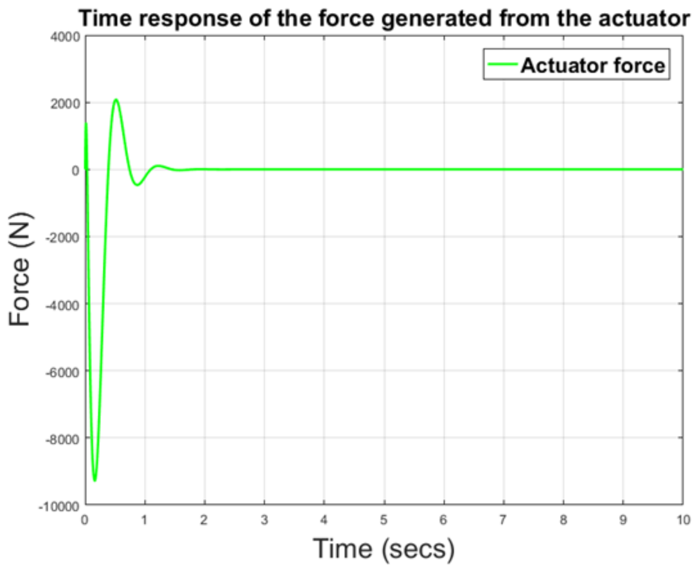


Figure 7: Time response of the force generated from the actuator

Figure 8 shows the impact of inserting a feedback LQR controller on the device in terms of suspension movement as compared to the passive suspension in Figure 35. In comparison to the passive suspensions, a similar overshoot of 0.05 m is present.However, as compared to the passive system, the active suspensions have a significantly shorter settling time of 1.86 seconds, which is a 50% reduction in the response rate for the passive suspensions, resulting in a quicker response, improved ride comfort, and reduced vibrations for the passengers.

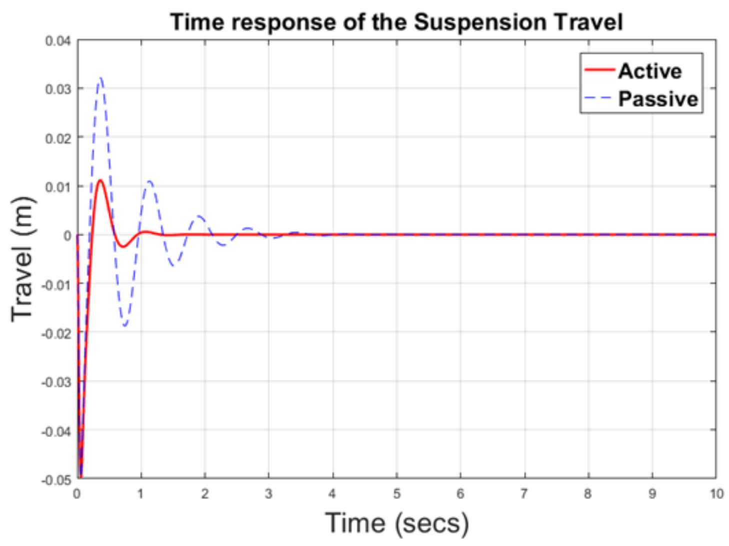


Figure 8: Time response of the suspension travel for the active suspension system

Figure 9 depicts the impact of acceleration on the railcar body after using a LQR controller. As compared to the response of the passive suspension in Figure 7, a similar overshoot of 7.5 m/s2 is present, as shown in Figure 6 for the suspension travel of the active suspension system.The machine, on the other hand, shows a 47 percent increase in ride comfort and road handling with a settling time of 1.75 seconds.

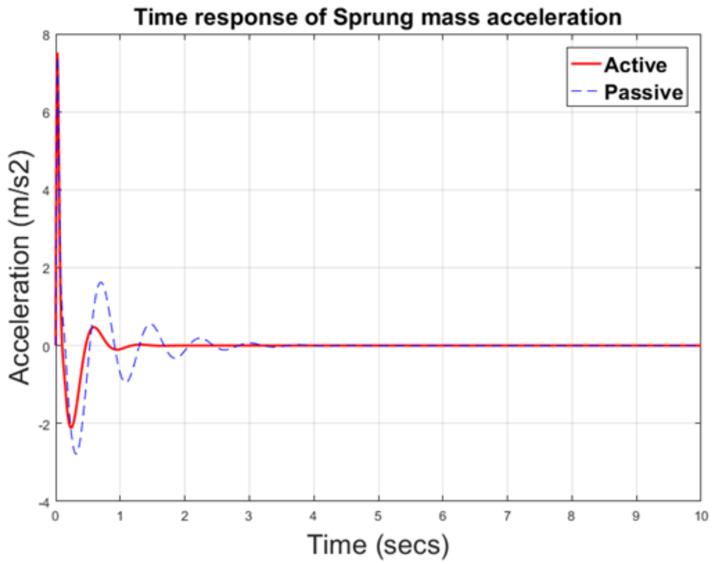


Figure 9: Time response of the sprung mass acceleration for the active suspension system

Figure 10 shows the effects of using the LQR controller to control the active suspension mechanism. The active suspension has a similar overshoot in wheel deflection of 0.0255m as the passive suspension. The settling time, like the previous results for the active suspension system, has improved significantly to 1.74 seconds, a 42 percent increase in the response rate.Figure 10 depicts the consequences of controlling the active suspension mechanism with the LQR controller. The active suspension has a wheel deflection overshoot of 0.0255m, which is similar to the passive suspension. The settling time has increased dramatically to 1.74 seconds, a 42 percent improvement in the response rate, similar to the previous results for the active suspension system.In comparison to the passive suspension, which had a longer settling period, resulting in more significant deflections and poor rail handling, the LQR controller optimized both the amplitude and the settling time, reducing vibrations and avoiding erratic movements of the railcar on the railway.

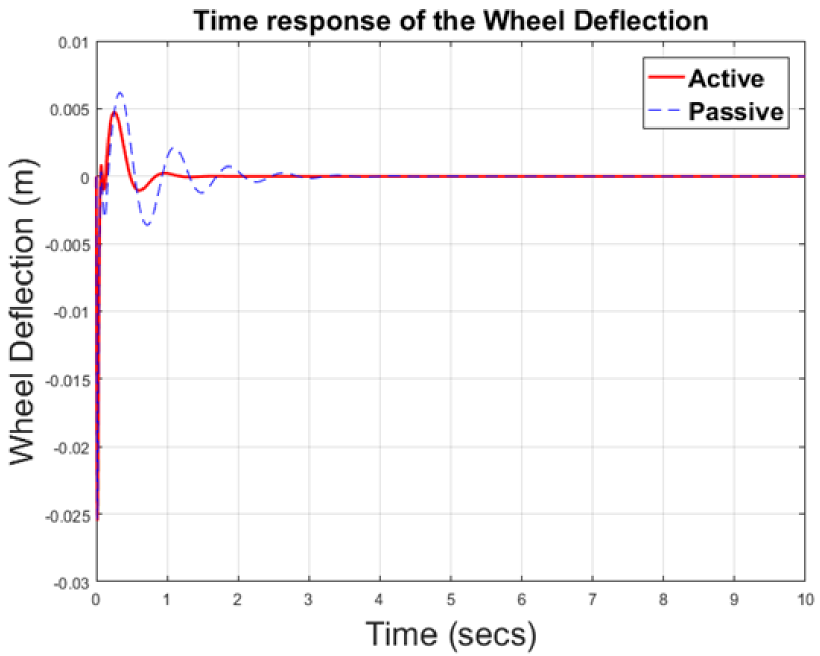


Figure 10: Time response of the wheel deflection for the active suspension system

Figure 11 shows the active suspensions and a contrast of the sprung and unsprung mass body displacements, as well as the displacement of the rail disturbance. The performance response of both the railcar body and the bogie were able to achieve a steady-state close to the same timeframe when the LQR control method was used.As compared to the passive suspension (Figure 12), the overshoot for the sprung mass is marginally lower at 0.0143, and the unsprung mass does not change significantly, but the amplitude and settling period for the unsprung mass have been decreased significantly to 1.85 seconds and 1.42 seconds, respectively, which is a 51 percent and 43 percent increase.In contrast to passive suspension, the active suspension system with LQR control demonstrated a major improvement in the system's ability to minimize the body displacement of railway vehicles, providing reasonable ride comfort for passengers.

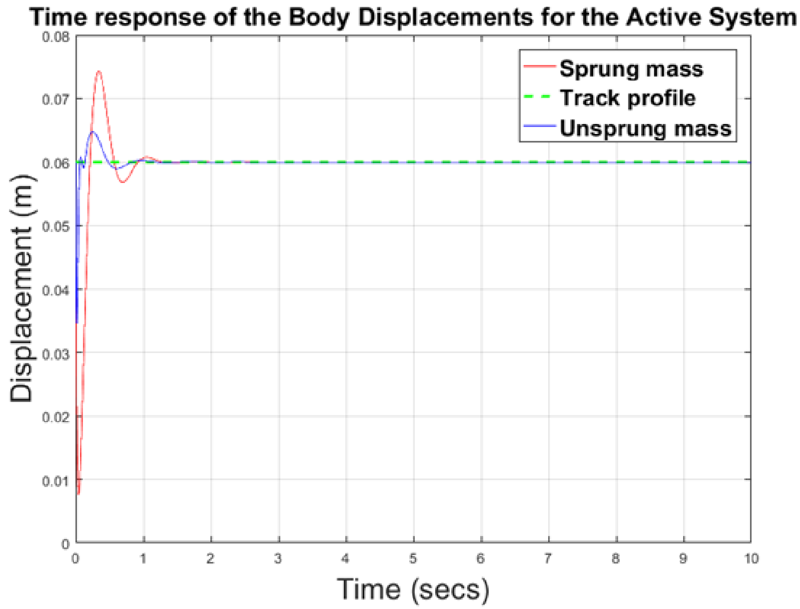


Figure 11: Time response for the body displacements for the active suspension system

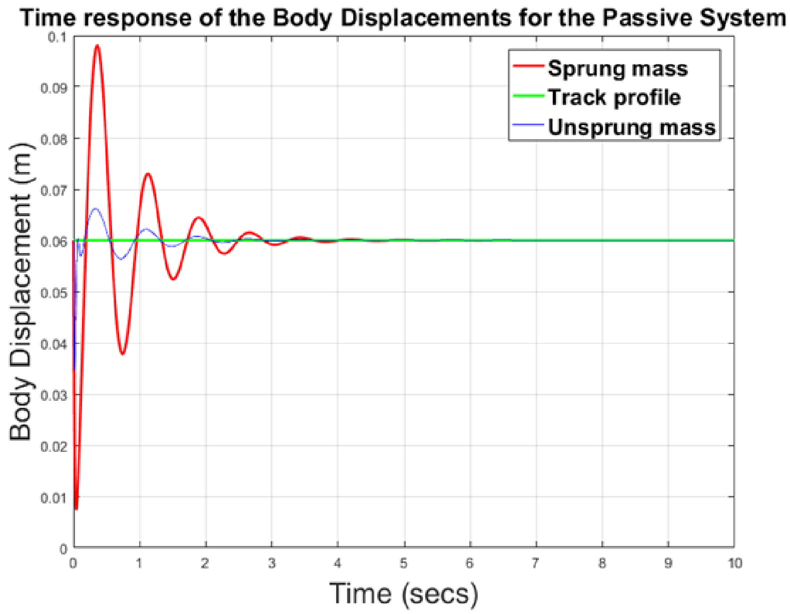


Figure 12: Time response of the body displacements for the passive suspension system

ROBUSTNESS ASSESSMENT

A robustness test is performed after the LQR controller for the active suspension system has been designed and tested. The primary explanation for this test is that when using a LQR control for a system, the stability margins are not guaranteed to remain in place if the system's parameters are changed.A robustness evaluation is required to ensure that a change in the parameters does not result in a significant reduction in the performance of the active suspension system.

The test also revealed that, depending on the modified parameters, the device could have some flexibility.

Table 2 below lists the parameters of the changed values. The parameter variations are likely to be on the more extreme side, and might not be conceivable in reality, but this test is intended to test the system's robustness and for the worst-case scenario.As a worst-case scenario, for this robustness measure, a square signal amplitude reflecting a railway disturbance height of 0.1m with a time of six seconds was used to model the rail disturbance.

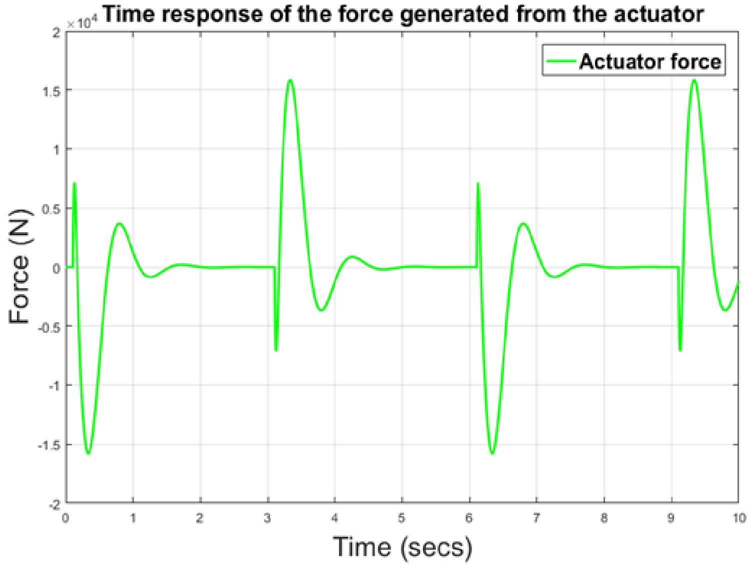
Modified Quarter rail vehicle design parameters	
Parameters	Values
M_s	8532.8 kg
M_{us}	906.5 kg
K_s	361,200 N/m
K_{us}	2,440000 N/m
B_s	14,600 sec/m
B_{us}	40,000 sec/m

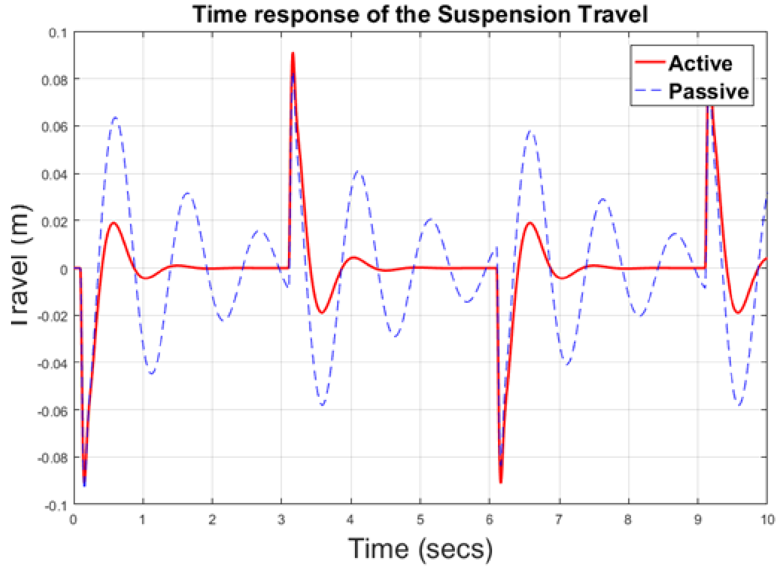
Table 2: Modified Quarter rail vehicle design parameters

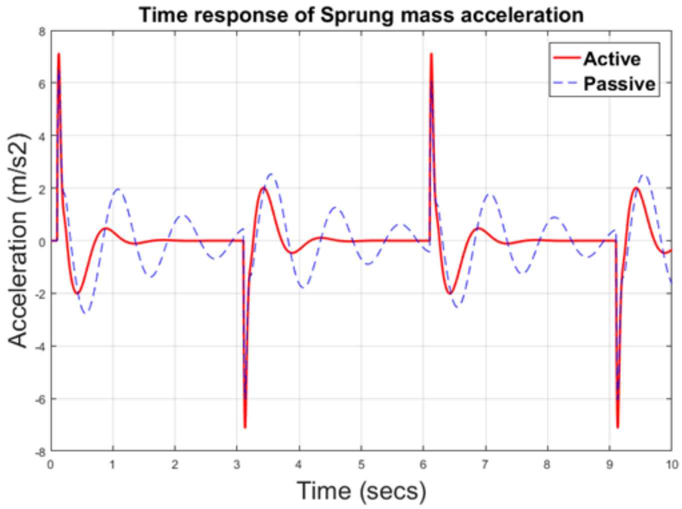
The feedback gain vector, K, obtained for this robustness assessment is:

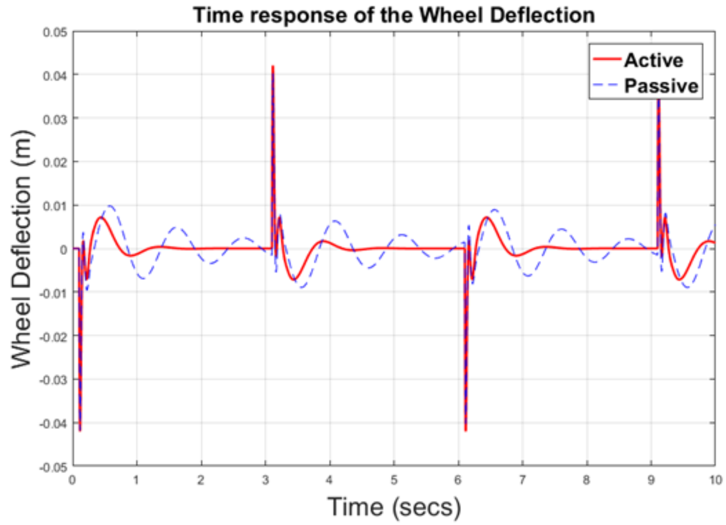
$$K = [1.9239 \times 10^5 \quad 0.5007 \times 10^5 \quad 1.2249 \times 10^5 \quad 0.0019 \times 10^5]$$

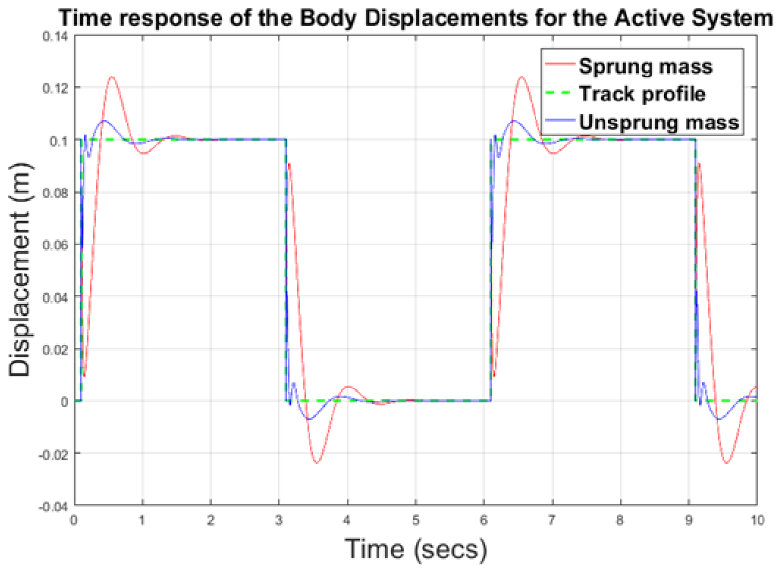
The results of the robustness test are shown below:

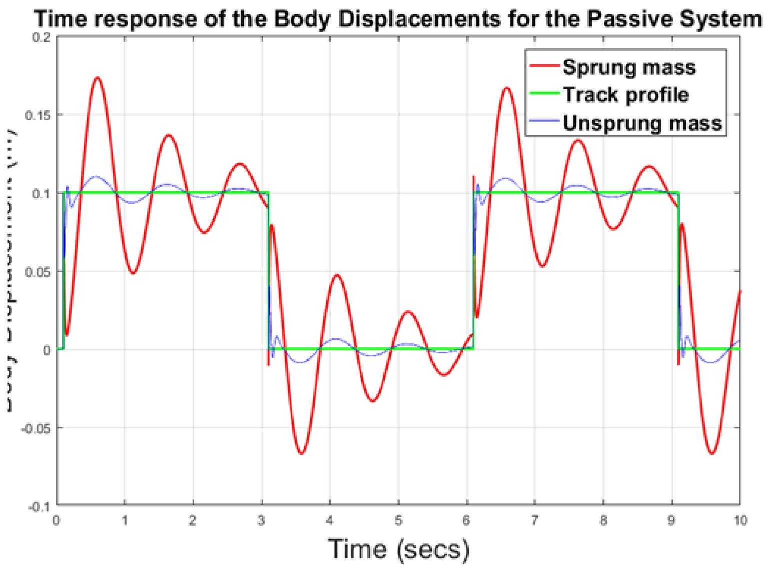












CONCLUSION

The test revealed little improvement in ride comfort, with the only notable difference being in the force results for the actuator, which revealed that 70 percent increases in force are needed to operate the device and achieve similar results as the LQR controller with the initial parameters.Furthermore, it demonstrates that, with the changed parameters selected, the active suspension system, with the LQR control strategy in place, demonstrates its robustness and can adjust relatively well to variations of parameters chosen for robustness testing.

NOTE:-Please install all the required Tool Boxes.The most important Tool Box I had to install was **Control System Toolbox**