

CS335: Introductory Lab Assignment

Hands-on with Python

August 7, 2019

Definitions

Null Space: The null space of a matrix A , referred to as $N(A)$, is the space of all solutions to the equation $Ax = 0$. The null space of an $m \times n$ matrix A is a subspace of \mathbb{R}^n .

Basis of a Space is a set of vectors v_1, v_2, \dots, v_n with two properties, viz.,

1. The vectors v_1, v_2, \dots, v_n are independent and
2. These vectors span the space.

Eigenvector: Vector $x \in \mathbb{R}^n$ is called an eigenvector of an $n \times n$ matrix A , iff $Ax = \lambda x, \exists \lambda \in \mathbb{R}$. The scalar λ is called an eigenvalue of A , corresponding to the eigenvector x .

Problem 1

Complete the code for computing the rref of a matrix in `Q1.py`. Check if your code works for the matrices given in the code. See the attached python code. (Do not use any library that directly computes inverse, rref, eigen values or any other function)

1. Write code for computing the the basis and dimension of column space of matrix. Print the basis and dimension for each matrix listed in the code. Verify if you get the same dimension for column space and row space of the matrix.
2. Add the code to find the basis and dimension of the nullspace of a matrix. Print the basis and dimension of the nullspace of each of the matrices named A, B, \dots, Q

Problem 2

Consider following matrices

$$A = \begin{bmatrix} 0.1 & 0.25 & 0.3 & 0.35 \\ 0.2 & 0.25 & 0.3 & 0.05 \\ 0.3 & 0.25 & 0.4 & 0.15 \\ 0.4 & 0.25 & 0 & 0.45 \end{bmatrix} \quad B = \begin{bmatrix} 0.35 & 0.3 & 0.35 \\ 0.2 & 0.4 & 0.4 \\ 0.45 & 0.3 & 0.25 \end{bmatrix}$$

1. Create a python program to calculate eigen values for the matrix.
2. Find all the eigen values of matrix A and B.
3. What is the range of eigen values for both the matrices ?
4. Are these matrices singular? You might need to create `python` script for calculating determinant

Problem 3

We have a real dataset *college_univs.data*, which records statistics concerning universities and colleges. Each column records a certain kind of property of each school. Right now, you may find it's totally mysterious what the meaning of each number. However, we are only interested in the correlation between data in different columns.

1. In this experiment, you are going to determine which columns are positively correlated, negatively correlated, or uncorrelated. To achieve this, first, you need to choose several columns, then compute their covariance and correlation coefficients. You should pick at least 3 pairs of columns for this experiment.
2. Generate scatter plots for the correlated and uncorrelated variables using `matplotlib` library.

Problem 4

1. Plot the probability mass function and the cumulative distribution function of a binomial distribution for a few different values of the parameter p . How does their shapes changes as the function of p ?
2. Plot the probability mass function and the cumulative distribution function of a geometric distribution for a few different values of the parameter p . How does their shapes changes as the function of p ?

Problem 5.

Sameer has invented a new game. The problem is that he has no idea what the expected payout is, so he doesn't know what to charge for it! The game works like this: the player starts with a score of 0. The computer generates a random real number between 0 and 1, inclusive. The number is added to the player's score and the player gets Rs. 10. If the player's score is still below 1, this is repeated- the computer picks a random real number between 0 and 1, adds it to the player's score, and the player gets Rs. 10. The game ends when the player's score is 1 or greater. For example, suppose the computer outputs the numbers 0.2153, 0.561148, 0.392327. In this case, the player wins Rs. 30, and a fourth number is not generated; the game stops because the total score is 1 or larger.

1. Write a simulation that plays this game. Run it many times (at least 10,000) and take the estimate of the expected payoff of the game by averaging your results. What is this number? Describe your findings and include some sort of plot.
2. What if, instead, the real number returned by the computer is squared before it's added to the player's score? Repeat part (a) with this change. Now what is the expected payoff? Describe your findings and include some sort of plot.

Submission Instructions

All the codes must be submitted in python as .py file. You are required to use **numpy** library unless specified.

Create separate file for each question in the following format. Q1.py, Q2.py and so on. Follow the following directory structure.

```
lab0_<roll_number>
├── Q1.py
├── Q2.py
├── Q3.py
├── Q4.py
├── Q5.py
└── solutions.txt
```

Create a zip file of your submissions and name your zip file as lab0_<roll_number>.zip

Zip file can be created using following command on Linux terminal :

```
zip -r lab0_<roll_number>.zip lab0_<roll_number>
```

Create a separate file **solutions.txt** for answering questions that require written explanations.