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## **1 Instructions for preparing laboratory records.**

- 1** All lab. records should be prepared using A-4 size sheets.
2. Submit your lab. record of the previous class for checking on every day of your lab. class.
3. Maintain a thin lab. booklet to maintain a record of observations. These observations should be signed by your lab. instructor on every turn.
4. Each practical should have the following sections;
  - (a) Aim
  - (b) Theory
  - (c) Matlab Code : Note that the code should have sufficient comments which explain the purpose of each section of the code.
  - (d) Results in the form of graphs/tables/etc. All the figures should be labeled properly.
  - (e) Verification of practical results w.r.t. the theoretical background given above.
5. General points to remember:
  - Always begin your MATLAB code the 'close all', 'clear all' and 'clc' commands.
  - Ensure that your variable names and file names are not named as standard MATLAB command names.
  - Kindly be prepared before coming to the laboratory. Revise the required theoretical concepts, and read the lab manual carefully.

## **2 Evaluation**

The internal assessment of the practical course will be done on the basis of

1. Lab Records: 10 marks
2. Mid semester Lab exam, which will be held in the week after the theory mid-sem exams/Viva-voce in the last week of semester: 10 marks
3. Class Attendance: 10 marks

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### 3 Exp.-1: Introduction.

#### AIM

1. Introduction to MATLAB
2. To plot  $\text{sinc}[n]$ .
3. To plot the spectrum of  $\text{sinc}[n]$ .

#### Procedure.

1. Introduction to MATLAB :

(a) Learn the use of various MATLAB commands: *plot* and related commands, *fft* and related commands, *hist* and related commands.

2. To plot  $\text{sinc}[n]$ .
3. To plot the spectrum of  $\text{sinc}[n]$ .

#### MATLAB Commands to be used:

1. *plot* and other command associated with it.
2. *fft,sinc*
3. *fftshift*

Note :

- Take absolute value of vector before plotting, because if a very-2 small complex error is introduced after DFT, IDFT operation, *stem,plot* command will plot real vs. imaginary component of the vector.
- Plot the *DFT* of any vector such that the x-axis is labelled either from  $0-2\pi$  or  $-\pi-\pi$ .

#### 4 Exp.-2: Compute probabilities of given events.

##### AIM

1. Compute probabilities of given events like  $X \leq a$ ,  $b < X \leq c$ ,  $X > d$  (a,b,c and d are constants).
2. Plot the CDF of the random numbers in 'X', 'Y'.

##### Theory:

The probability of an event is the measure of the chance that the event will occur as a result of an experiment. The probability of an event A, symbolized by  $P(A)$ , is a number between 0 and 1, inclusive, that measures the likelihood of occurrence of an event. As per the relative frequency approach to probability,

$$P(A) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} \quad (1)$$

##### Procedure.

1. Generate an  $X_{1 \times N}$ ,  $Y_{1 \times N}$  vectors of random numbers using 'rand', 'randn' commands respectively.
2. Compute the probability of the event  $[b < X \leq c]$ ,  $[b < Y \leq c]$  as given below.

$$\begin{aligned} &\text{Prob.}[b < X \leq c] \\ &\approx \frac{\text{Total count of numbers lying between } [b < X \leq c]}{N} \end{aligned} \quad (2)$$

3. Compare the results for the random variables 'X' and 'Y'
4. Repeat for the events  $X \leq a$ ,  $X > d$  (a,b,c and d are constants).
5. Plot the CDF of the random numbers in 'X', 'Y'.
6. Comment on the results.

##### MATLAB Commands to be used:

1. rand, randn
2. 'for' loop
3. 'if' loop

5 Exp.-3: Verify the PDF of given random numbers, and compute probabilities of given events.

AIM

1. To plot and verify the PDF of Uniform and Gaussian Random Numbers.
2. To verify the properties of PDF of Uniform and Gaussian Random Numbers.
3. To plot and verify the PDF of Speech samples.
4. Compute probabilities of given events like  $X \leq a$ ,  $b < X \leq c$ ,  $X > d$  ( $a, b, c$  and  $d$  are constants).

Theory Consider the Probability density function given in the figure below.

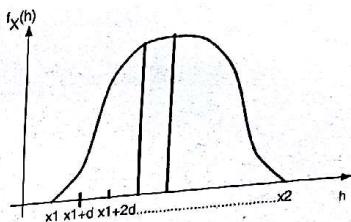


Figure 1: Probability Density Function.

$$\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d] = \int_{x_1+nd}^{x_1+(n+1)d} f_X(h) dh \quad \text{for } n = 1, 2, \dots, (x_2 - x_1)/d \quad (3)$$

For small values 'h', the above equation can be approximated as

$$\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d] \simeq d \cdot f_X(x_1 + nd) \quad (4)$$

Thus, from Eqn.4

$$f_X(x_1 + nd) \simeq \frac{\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d]}{d} \quad (5)$$

In order to plot the probability density function for a set of numbers, we should compute the value of  $f_X(x_1 + nd)$  for  $n = 0, 1, 2, \dots, (x_2 - x_1)/d$ , and plot it with respect to the h-axis vector  $h = [x_1, x_1 + d, x_1 + 2d, \dots, (x_2 - d)]$ .

Numerically,  $\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d]$  can be computed as

$$\frac{\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d]}{\frac{\text{Total count of numbers lying between } [x_1 + nd < X \leq x_1 + (n+1)d]}{\text{Total count of numbers generated}}} \quad (6)$$

Thus, from Eqn.6 and Eqn.5

$$\begin{aligned} & f_X(x_1 + nd) \\ & \approx \frac{\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d]}{d} \\ & \approx \frac{1}{d} \cdot \frac{\text{Total count of numbers lying between } [x_1 + nd < X \leq x_1 + (n+1)d]}{\text{Total count of numbers generated}} \end{aligned} \quad (7)$$

#### Procedure.

1. (a) Generate an  $X_{1 \times N}$  vector of Gaussian/Uniform random numbers.
- (b) Compute  $x_1 = \text{MIN}(X)$  and  $x_2 = \text{MAX}(X)$ .
- (c) Define  $d = (x_2 - x_1)/M$ .
- (d) Generate  $f_X = [f_X(x_1) f_X(x_1 + d) \dots f_X(x_1 + (M-1)d)]_{1 \times M}$  using the *HIST* command i.e.,  
Compute  $\text{Prob.}[x_1 + nd < X \leq x_1 + (n+1)d]$  as given in Eqn.7.
- (e) Generate h-axis vector  $h = [x_1, x_1 + d, x_1 + 2d, \dots, (x_2 - d)]$
- (f) Plot  $f_X$  versus  $h$ .
- (g) Label the above plot appropriately.
- (h) Superimpose the theoretical *pdf* over the one obtained through simulation to verify the results.

2. To verify the properties of the *pdf* we have to compute the integral

$$\int_{-\infty}^{\infty} f_X(\alpha) d\alpha, \quad (8)$$

and see that it equals 'ONE'. This integral can practically be approximated as

$$\int_{-\infty}^{\infty} f_X(\alpha) d\alpha \approx d * \text{sum}(f_X). \quad (9)$$

3. Repeat the above by using speech samples to generate the random vector  $X_{1 \times N}$

4. (a) Compute the probability of the event  $[b < X \leq c]$  for Gaussian/Uniform random variables as given below.

$$\text{Prob.}[b < X \leq c] \approx \frac{\text{Total count of numbers lying between } [b < X \leq c]}{N} \quad (10)$$

- (b) Compute the above theoretically.  
(c) Compare theoretical results with the practical result obtained in a. above.  
(d) Repeat for the events  $X \leq a$ ,  $X > d$  (a,b,c and d are constants).

MATLAB Commands to be used:

1. rand, randn
2. wavread
3. max, min, hist

## 6 Exp.-4: Mean and Variance.

### AIM

1. Compute the mean and variance of given Random Variables.
2. Verify the properties of the Expectation operator.
3. Generate Uniform and Gaussian Random numbers with desired mean and variance.

### Theory

1. Consider a random variable  $X$  whose probability density function is given by  $f_X(a)$ . The mean is  $\mu_X$  and variance is  $\sigma_X^2$  are computed as

$$\begin{aligned}\mathbb{E}[X] = \mu_X &= \int_{-\infty}^{\infty} af_X(a)da \\ \mathbb{E}[(X - \mu_X)^2] = \sigma_X^2 &= \int_{-\infty}^{\infty} a^2 f_X(a)da\end{aligned}\quad (11)$$

2. The operator  $\mathbb{E}$  in the above equation is known as the *EXPECTATION* operator. It is known that the Expectation operator is a linear operator i.e.

$$\mathbb{E}[aX + bY + c] = a\mathbb{E}[X] + b\mathbb{E}[Y] + c \quad (12)$$

3. Consider a random variable  $X : U(p, q)$ , then

$$\begin{aligned}\mathbb{E}[X] = \mu_X &= \int_{-\infty}^{\infty} af_X(a)da = \frac{p+q}{2} \\ \mathbb{E}[(X - \mu_X)^2] = \sigma_X^2 &= \int_{-\infty}^{\infty} a^2 f_X(a)da = \frac{(q-p)^2}{12}\end{aligned}\quad (13)$$

Let  $Y = aX + b$ , then

$$\begin{aligned}\mathbb{E}[Y] = a\mu_X + b &= a\frac{p+q}{2} + b \\ \mathbb{E}[(Y - \mu_Y)^2] = \sigma_Y^2 &= a^2 \frac{(q-p)^2}{12}\end{aligned}\quad (14)$$

Similar results can be given for the gaussian Random Variable.

Procedure.

1. (a) Generate an  $X_{1 \times N}$  vector of Gaussian/Uniform random numbers using *rand*/*randn* command.

(b) Compute the mean and variance as given below

$$\begin{aligned}\mu'_X &= \frac{1}{N} \sum_{i=1}^N X(i) \\ \sigma'^2_X &= \frac{1}{N} \sum_{i=1}^N (X(i) - \mu'_X)^2\end{aligned}\quad (15)$$

(c) Compute the mean and variance using in-built MATLAB commands

$$\begin{aligned}\mu''_X &= \text{mean}(X) \\ \sigma''^2_X &= \text{Var}(X)\end{aligned}\quad (16)$$

(d) Compute the theoretical values of mean and variance.

(e) Compare the results of (b), (c), and (d).

2. (a) Generate a  $X_{1 \times N}$  vector of Gaussian/Uniform random numbers using *rand*/*randn* command.

(b) Generate a  $Y_{1 \times N}$  vector as given by

$$Y = aX + b \quad (17)$$

(c) Compute the mean of  $Y$  using in-built MATLAB commands.

(d) Compute the theoretical value of mean of  $Y$  using Eqn.12

(e) Compare the results of (c) and (d).

3. To generate Uniform/Gaussian random variables 'Y' with mean  $\mu_Y$  and variance  $\sigma_Y^2$ .

(a) Generate a  $X_{1 \times N}$  vector of Gaussian/Uniform random numbers using *rand*/*randn* command.

(b) The *rand* command generates numbers uniformly distributed between [0,1], thus the mean is  $\frac{1}{2}$  and variance is  $\frac{1}{12}$ . The *randn* command generates Gaussian distributed numbers with mean 'zero' and variance 'one'.

(c) Generate a  $Y_{1 \times N}$  vector as given by

$$Y = aX + b \quad (18)$$

(d) Referring to Eqn.14 we get

i. for the case of uniformly distributed numbers

$$\begin{aligned} E[Y] = \mu_Y &= \frac{a}{2} + b \\ E[(Y - \mu_Y)^2] = \sigma_Y^2 &= \frac{(a)^2}{12} \end{aligned} \quad (19)$$

'a', 'b' can be computed from the above equation

ii. for the case of gaussian distributed numbers

$$\begin{aligned} E[Y] = \mu_Y &= b \\ E[(Y - \mu_Y)^2] = \sigma_Y^2 &= (a)^2 \end{aligned} \quad (20)$$

'a', 'b' can be computed from the above equation

(e) Compute the mean and variance of 'Y' using MATLAB commands.

(f) Compare the result obtained above with the actual values of  $\mu_Y, \sigma_Y^2$  (as chosen by the student).

#### MATLAB Commands to be used:

1. rand, randn

2. mean, var

## 7 Exp.-5: Transformation of Random Variables and Central Limit Theorem.

### AIM

1. To plot the pdf of the sum of two Random Variables, and verify the result.
2. To verify that the sum of 2 Gaussian Random Variables is also Gaussian.
3. To verify the Central Limit Theorem

### Theory

1. It is known that the pdf of the sum of two independent random variables is the convolution of the individual pdfs. Let  $X, Y$  be two independent random variables with pdf  $f_X(a)$  and  $f_Y(a)$  respectively. We define a random variable  $Z$  as

$$Z = X + Y \quad (21)$$

then

$$f_Z(a) = f_X(a) * f_Y(a) \quad (22)$$

2. It is known that the pdf of the sum of two independent Gaussian random variables is also Gaussian. Let  $X, Y$  be two independent Gaussian random variables with mean  $\mu_X, \mu_Y$  and variance  $\sigma_X^2, \sigma_Y^2$  respectively. We define a random variable  $Z$  as

$$Z = pX + qY \quad (23)$$

then

$$Z : \text{GAUSSIAN}(p\mu_X + q\mu_Y, p^2\sigma_X^2 + q^2\sigma_Y^2) \quad (24)$$

3. Let  $X_i$  for  $i = 1, 2, \dots, N$  be  $N$  zero mean, independent identically distributed random variables. The Central Limit Theorem states that the random variable  $Y$ , defined as

$$Y = \lim_{N \rightarrow \infty} \left[ \frac{1}{\sqrt{N}} \sum_{i=1}^N X_i \right] \quad (25)$$

tends to have a Gaussian density function.

Procedure.

1. (a) Generate  $X_{1 \times M}, Y_{1 \times M}$  vectors of Uniform random numbers using *rand* command.
  - (b) Generate  $Z_{1 \times M} = X_{1 \times M} + Y_{1 \times M}$
  - (c) Plot the pdf of  $Z_{1 \times M}$ .
  - (d) Verify that the pdf obtained above is the convolution of two rectangular pdfs (of Uniform Random Variables).
2. (a) Generate  $X_{1 \times M}, Y_{1 \times M}$  vectors of Gaussian random numbers using *randn* command.
  - (b) Generate an  $Z_{1 \times M} = pX_{1 \times M} + qY_{1 \times M}$
  - (c) Plot the pdf of  $Z_{1 \times M}$ .
  - (d) Verify that the pdf obtained above is Gaussian with mean  $(p\mu_X + q\mu_Y)$  and variance  $(p^2\sigma_X^2 + q^2\sigma_Y^2)$ .
3. (a) Generate  $X_{j:1 \times M}, j = 1, 2, \dots, N$  vectors of zero mean Uniform random numbers using *rand* command.
  - (b) Generate an  $Z_{1 \times M} = \left[ \frac{1}{\sqrt{N}} \sum_{i=1}^N X_i \right]$
  - (c) Plot the pdf of  $Z_{1 \times M}$ .
  - (d) Verify that the pdf obtained above is Gaussian with zero mean.
  - (e) Compute the variance and verify the result theoretically.

MATLAB Commands to be used:

1. *rand*, *randn*
2. *mean*, *var*