Last Time:

- LQG
- Kalman Filter

Today:

- Rocket Soft Landing

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* The Rocket Soft-Landing Problem

  - Go from from some initial state x_0 to some final position r_f with $z_f = 0$ and $v_f = 0$

  - Minimize some combination of fuel consumption and landing position error $\| r_f - r_g \|$

  - Respect thrust limits + safety constraints

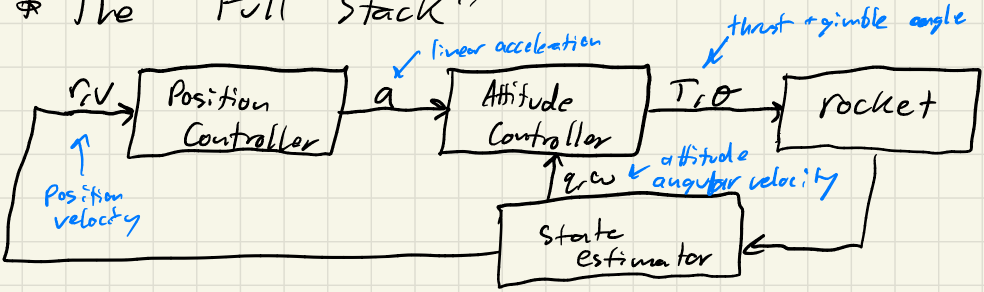
* Examples:

  - NASA Curiosity "Sky Crane" (2012)

  - SpaceX Falcon 9, Starship

  - NASA Perseverance w/TRN (2021)

* The "Full Stack"



- State Estimation:

  SpaceX: GPS + IMU with good filtering, ~1 m position accuracy, < 1 cm/sec velocity, ~1° attitude. Enables precision landing.

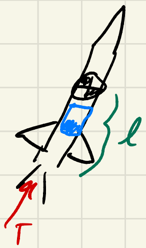
  Mars: No GPS. IMU + Radar Altimeter + Vision ~30 meter accuracy. Avoid Boulders.

- Decoupled Control Loop:

  High-level Position Controller: Uses a point-mass model. Reasons about safety, thrust, and fuel. Generates acceleration commands. Runs at ~ Hz

  Low-level Attitude Controller: Reasons about attitude, flexible modes, fluid slosh, generates thrust + gimble commands to track desired acceleration. Runs at ~10 Hz

* Rocket Dynamics



- Rigid Body

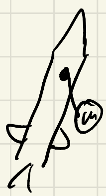
Position Control (slow)
$$\begin{cases} \dot{v} = -g + \dfrac{T}{m} & \leftarrow \text{point mass} \\ \dot{m} = -\alpha T & \leftarrow \text{fuel burn} \end{cases}$$

attitude controller (fast)
$$J\dot{\omega} + \omega \times J\omega = \underbrace{l \times T}_{\text{torque}} \quad \leftarrow \text{attitude}$$

Inertia

- Fuel can be 80% + of initial vehicle mass. Have to account for this

- Fluid slosh: Highly nonlinear, hard to model



← pendulum with l_p and m_p fit to data

- Flexible modes: Rockets are built to be very light ⇒ not stiff ⇒ low-frequency bending modes. First mode ~Hz. Dealt with by adding notch filters to the attitude controller at the bending frequencies to avoid excitinging it.

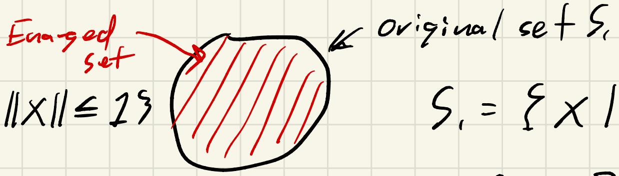
- Aerodynamic Forces: Mostly ignore. Velocity constraints in position controller to make sure these stay small.

- Lots of model uncertainty. Linear robust control ideas are used in the attitude controller.

* Backround: Convex Relaxation

- Sometimes we have a non-convex constraint that can be expressed as the boundary of a larger convex set:

$S_2 = \{x \mid \|x\| \leq 1\}$



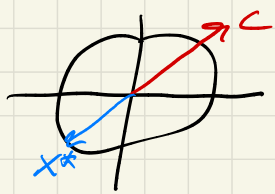
$$S_1 = \{x \mid \|x\| = 1\}$$

$$S_1 = \partial S_2$$

- Replacing the original constraint with larger convex one is called "convex relaxation"

- Sometimes if the cost is "nice" we can still get the answer to the original problem by solving the relaxed version:

$$\min \ c^T x$$
$$\text{s.t.} \ \|x\| = 1$$
$$\Downarrow$$
$$\|x\| \leq 1$$

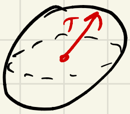


- When this happens we call it a "tight relaxation"

* Convex Relaxation of Thrust Constraint

- Maximum thrust constraint

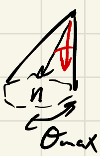
$$T \in \mathbb{R}^3, \quad \|T\| \leq T_{max}$$



(convex)

- Thrust angle constraint

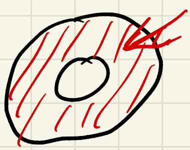
$$n^T \frac{T}{\|T\|} \leq \cos(\theta_{max})$$



(convex)

- Rocket engine also has minimum thrust constraint:

$$T_{min} \leq \|T\| \leq T_{max}$$



Not Convex!

- Let's add a new "slack variable" $\Gamma \in \mathbb{R}$ that equals the thrust magnitude:

1) $\|T\| = \Gamma$ $\}$ Boundary of a convex set (sphere)

2) $T_{min} \leq \Gamma \leq T_{max}$

3) $n^T T \leq \Gamma \cos(\theta_{max})$ $\}$ Convex

- Now we can convexify the constraint by relaxing 1)

1*) $\|T\| \leq \Gamma$

2) $T_{min} \leq \Gamma \leq T_{max}$

3) $n^T T \leq \Gamma \cos(\theta_{max})$

- The paper proves that this relaxation is tight using Pontryagin's minimum principle