Active Suspension Control

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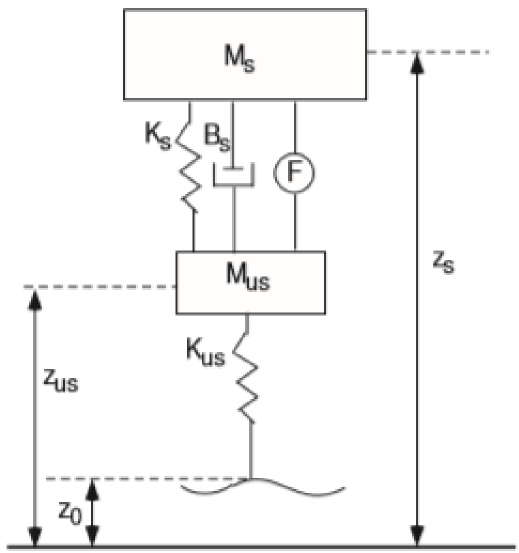
Summary

The project focuses on minimizing rider discomfort while penalizing tire deflection and suspension stroke for an Active Suspension system. In this project, we also analyze the stability and disturbance attenuation of the model by implementing Full State Feedback and Observer with State Feedback.

Introduction

A Quarter car model is used to implement the Active Suspension design in an automotive suspension system with sprung mass, M_s as vehicle and unsprung mass, M_{us} as tire and wheels combined. The tire has a stiffness, K_s and a passive suspension connecting the vehicle and tire has a damping, B_s. An Active suspension is also present in parallel with the Passive suspension that exerts a force F on the 2 masses.



States of Quarter Car Suspension Model are

x_1: *Tire Deflection*, x_2: *Unsprung mass velocity*, x_3: *Suspension Stroke*, x_4: *Sprung mass velocity*

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} z_{us} - z_0 \\ \dot{z_{us}} \\ z_s - z_{us} \\ \dot{z}_s \end{bmatrix}$$

Control Input is the force applied by the actuator normalized by the value of the sprung mass and is represented as

$$u = F/M_s$$

Changes in road elevation results in disturbance

$$d = \dot{z_0}$$

The system is controlled by taking sensor reading for the suspension stroke and the overall output is

$$y_m = x_3 + n$$

where n represents Noise from the sensor readings.

State Space Description

The State Space description for the Quarte Car suspension with Active suspension is

$$\dot{x} = Ax + Bu + Ed$$
$$y_m = Cx + n$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_{us}}{M_{us}} & -\frac{B_s}{M_{us}} & \frac{K_{us}}{M_{us}} & \frac{B_s}{M_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{B_s}{M_s} & -\frac{K_s}{M_s} & -\frac{B_s}{M_s} \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ \frac{M_s}{M_{us}} \\ 0 \\ -1 \end{bmatrix}$$
$$C = [0\;0\;1\;0], \;\; E = [-1\;0\;0\;0]^T$$

Control and Observer Design

1. Cost Function

Human perception of ride quality is sensitive to acceleration, and thus we include a term proportional to $\dot{x}_4^2$ in the cost index

$$J = \int_0^\infty q_1 x_1^2 + q_3 x_3^2 + \gamma \dot{x}_4^2 + r u^2 \, dt$$

The above cost function was reduced to a general form of

$$J = \int_0^\infty x^T Q x + u^T R u + 2x^T S u \, dt,$$

where

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \gamma \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (B_s/M_s)^2 & (-B_s K_s/M_s^2) & -(B_s/M_s)^2 \\ 0 & (-B_s K_s/M_s^2) & (K_s/M_s)^2 & (B_s K_s/M_s^2) \\ 0 & -(B_s/M_s)^2 & (B_s K_s/M_s^2) & (B_s/M_s)^2 \end{bmatrix}$$
$$S^T = \gamma \begin{bmatrix} 0 & -B_s/M_s & K_s/M_s & B_s/M_s \end{bmatrix}, \quad R = \gamma + r.$$

Parameters used

$$M_{us} = 30\,kg, \;\; M_s = 250\,kg, \;\; K_{us} = 15000\,N/m, \quad B_s = 1000 \frac{N}{m-s}$$

2. State Feedback

State feedback gain were computed by solving Ricatti equation for specific weights.

$$K = [7.5407 \quad -0.8734 \quad -10.7107 \quad -7.9666]$$

With Full State feedback, the closed loop system is represented as

$$\dot{x} = (A - BK)x + Ed$$

The closed loop State Feedback response from disturbance, d (due to road elevations) to rider discomfort, $\dot{x}_4$

$$\dot{x}_4 = (A - BK)(4,:)x$$

3. Observer

Similarly, observer gain were computed by solving Ricatti equation with noise and disturbances being zero mean white gaussian noise processes with specific covariance matrices.

$$L = [107.32 \quad -12487 \quad 154.08 \quad -616.32]^T$$

4. Observer with State Feedback

With Observer based State feedback, the closed loop system is represented as

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = A_{cl} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} d$$

where

$$A_{cl} = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix}$$

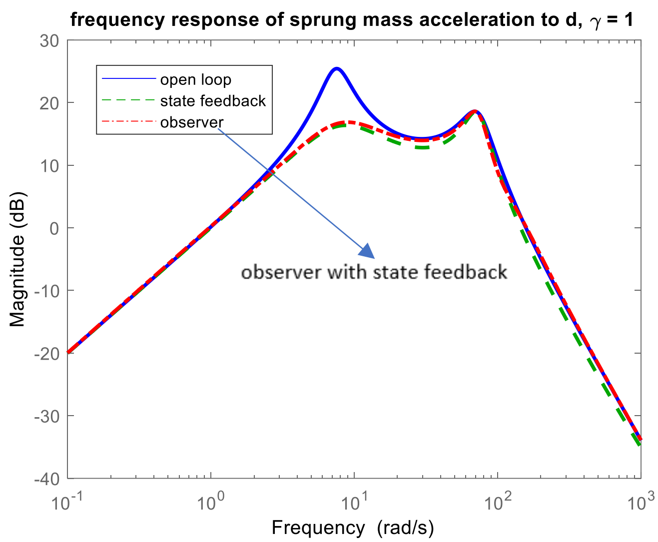
The closed loop response of Observer with State feedback from disturbance, d (due to road elevations) to rider discomfort, $\dot{x}_4$

$$\dot{x}_4 = A_{cl}(4,:) \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Frequence Response and Stability Analysis

1. Frequency Response

A comparison of Bode plots for response of rider discomfort to the disturbance is made between Open loop, state feedback and observer with state feedback systems.



As we penalize rider discomfort more (by increasing γ) in the cost function, disturbance attenuation is achieved and we see that the response of rider discomfort reduces at all frequencies, except near 70 rad/s. This is because $P_{\dot{x}_{4u}}$ has a zero near 70 rad/s that is not shared with $P_{\dot{x}_{4d}}$, where

$P_{\dot{x}_{4u}}$ represents response of rider discomfort from control input

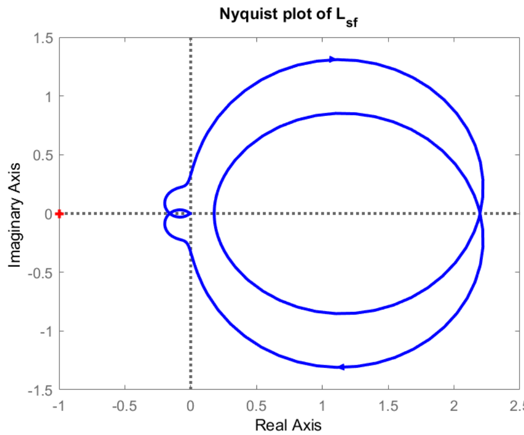
$P_{\dot{x}_{4u}}$ represents response of rider discomfort from disturbance

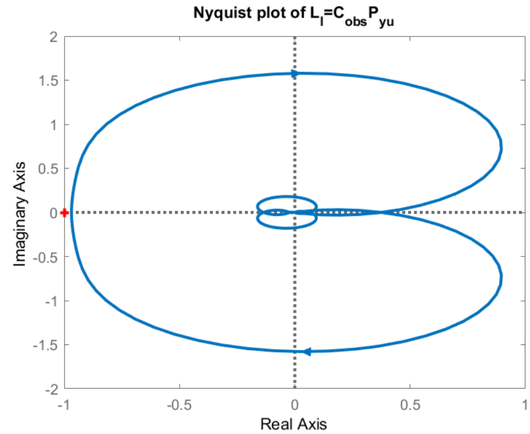
$$\dot{X}_4(j\omega) = P_{\dot{x}_{4u}}(j\omega)U(j\omega) + P_{\dot{x}_{4d}}(j\omega)D(j\omega)$$

Hence, $P_{\dot{x}_{4u}}(j\omega)$ contains a zero near 70 rad/s and thus it is not possible to alter the transfer function from d to $\dot{x}_4$ at any frequency for which $P_{\dot{x}_{4u}}(j\omega) = 0$, independently of how the control signal is generated.

2. Stability

The Stability radius with Observer based State feedback reduces to 0.032 as shown below. The figure below represents Nyquist plot for State feedback (left) and Observer with State feedback (right) with critical point -1 marked as red (and the Stability radius being the smallest distance between critical point and the Nyquist plot)





The Bode plot for Sensitivity function is also plotted below with State feedback in left and Observer with State feedback in right