Matrix Theory(EE5609) Assignment 3

Anshum Agrawal
Roll No- AI20MTECH11006

***Abstract*—This Assignment finds the value of a and b that satisfies the given equation**

Download all python codes from

https://github.com/anshum0302/EE5609/blob/master/assignment3/solu3.py

and latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/master/assignment3/assign3.tex

1 PROBLEM STATEMENT

For the matrix $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$,find the numbers a and b such that $\mathbf{A}^2 + a\mathbf{A} + b\mathbf{I} = 0$.

2 Theory

Cayley-Hamilton Theorem : A matrix satisfies its own characteristic equation.That is if the characteristic equation of an $n \times n$ matrix $\mathbf{A}$ is

$$\lambda^n + a_{n-1}\lambda^{n-1} + ... + a_1\lambda + a_0 = 0, then$$
$$\mathbf{A}^n + a_{n-1}\mathbf{A}^{n-1} + ... + a_1\mathbf{A} + a_0\mathbf{I} = 0$$

3 Solution

For a general square matrix (A), the characteristic equation in variable λ is defined by

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{3.0.1}$$
$$\implies \begin{vmatrix} 3-\lambda & 2 \\ 1 & 1-\lambda \end{vmatrix} = 0 \tag{3.0.2}$$
$$\implies (3-\lambda)(1-\lambda) - 2 = 0 \tag{3.0.3}$$
$$\implies \lambda^2 - 4\lambda + 3 - 2 = 0 \tag{3.0.4}$$
$$\implies \lambda^2 - 4\lambda + 1 = 0 \tag{3.0.5}$$

Now by Cayley-Hamilton Theorem $\mathbf{A}$ satisfies (3.0.5),then replacing λ with $\mathbf{A}$ we get

$$\mathbf{A}^2 - 4\mathbf{A} + \mathbf{I} = 0 \tag{3.0.6}$$

We need to find a and b in

$$\mathbf{A}^2 + a\mathbf{A} + b\mathbf{I} = 0 \tag{3.0.7}$$

Comparing (3.0.6) and (3.0.7) we get a = -4 and b = 1.