Matrix Theory(EE5609) Assignment 10

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***Abstract*—Given matrix T find matrix T[C] which represents matrix T with respect to basis C.**

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/master/assignment10/assign10.tex

1 Problem Statement

Let $\mathbf{C} = \left(\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}\right)$ be a basis for $\mathbb{R}^2$ and $\mathbf{T}:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-2y\end{pmatrix}$. If $\mathbf{T}[\mathbf{C}]$ represents the matrix of $\mathbf{T}$ with respect to basis $\mathbf{C}$ then which of the following is true

1) $\mathbf{T}[\mathbf{C}] = \begin{pmatrix}-3 & -2\\3 & 1\end{pmatrix}$
2) $\mathbf{T}[\mathbf{C}] = \begin{pmatrix}3 & -2\\-3 & 1\end{pmatrix}$
3) $\mathbf{T}[\mathbf{C}] = \begin{pmatrix}-3 & -1\\3 & 2\end{pmatrix}$
4) $\mathbf{T}[\mathbf{C}] = \begin{pmatrix}3 & -1\\-3 & 2\end{pmatrix}$

Solution:

Linear Transformation and change of Basis	If matrix **A** represents Linear Transformation with respect to standard ordered basis and matrix **B** represents same transformation with respect to basis **V**,Then $$\mathbf{B} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}$$

TABLE 1: Linear Transformation and change of basis

In above question $\mathbf{A} = \mathbf{T}, \mathbf{B} = \mathbf{T}[\mathbf{C}], \mathbf{V} = \mathbf{C}$.

Evaluate **T**	For linear transformation **T** we have $$\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-2y\end{pmatrix}$$ $$\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1 & 1\\1 & -2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$ $$\implies \mathbf{T} = \begin{pmatrix}1 & 1\\1 & -2\end{pmatrix}$$
Evaluate inverse of basis **C**	To find inverse of matrix **C** we row reduce augmented matrix **CI** $$\begin{pmatrix}1 & 2 & 1 & 0\\2 & 1 & 0 & 1\end{pmatrix} \xleftrightarrow[R_2=-\frac{1}{3}R_2]{R_2=R_2-2R_1} \begin{pmatrix}1 & 2 & 1 & 0\\0 & 1 & \frac{2}{3} & -\frac{1}{3}\end{pmatrix}$$ $$\xleftrightarrow{R_1=R_1-2R_2} \begin{pmatrix}1 & 0 & -\frac{1}{3} & \frac{2}{3}\\0 & 1 & \frac{2}{3} & -\frac{1}{3}\end{pmatrix}$$ $$\therefore \mathbf{C}^{-1} = \begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\\frac{2}{3} & -\frac{1}{3}\end{pmatrix}$$
Evaluate **TC**	$$\mathbf{TC} = \begin{pmatrix}1 & 1\\1 & -2\end{pmatrix}\begin{pmatrix}1 & 2\\2 & 1\end{pmatrix}$$ $$= \begin{pmatrix}3 & 3\\-3 & 0\end{pmatrix}$$
Evaluate $\mathbf{T}[\mathbf{C}]=\mathbf{C}^{-1}\mathbf{TC}$	$$\mathbf{T}[\mathbf{C}] = \mathbf{C}^{-1}\mathbf{TC}$$ $$= \begin{pmatrix}-\frac{1}{3} & \frac{2}{3}\\\frac{2}{3} & -\frac{1}{3}\end{pmatrix}\begin{pmatrix}3 & 3\\-3 & 0\end{pmatrix}$$ $$\implies \mathbf{T}[\mathbf{C}] = \begin{pmatrix}-3 & -1\\3 & 2\end{pmatrix}$$
Conclusion	Option 3) is correct.Options 1),2) and 4) are incorrect

TABLE 2: Calculation of $\mathbf{T}[\mathbf{C}]$