Matrix Theory(EE5609) Assignment 9

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***Abstract*—Given AB = -BA, this documents finds value of trace(A) and trace(B).**

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/master/assignment9a/assign9.tex

1 PROBLEM STATEMENT

Let $\mathbf{A}$ and $\mathbf{B}$ be real invertible matrices such that

$$\mathbf{AB} = -\mathbf{BA}. \tag{1.0.1}$$

Then

1) trace$\mathbf{A}$ = trace($\mathbf{B}$) = 0
2) trace$\mathbf{A}$ = trace($\mathbf{B}$) = 1
3) trace$\mathbf{A}$ = 0, trace($\mathbf{B}$) = 1
4) trace($\mathbf{A}$) = 1, trace($\mathbf{B}$) = 0

Solution:

Definition	Matrix $\mathbf{A}$ is said to be similar to matrix $\mathbf{B}$ if there exists matrix $\mathbf{P}$ such that $\mathbf{A} = \mathbf{PBP}^{-1}$
Properties	Similar matrices have same eigenvalues Sum of eigenvalue of a matrix equals its trace From above two properties we can conclude that similar matrices have same trace

TABLE 1: Similar matrices and Properties

trace($\mathbf{A}$) = 0 trace($\mathbf{B}$) = 0	From (1.0.1) we have $\mathbf{AB} = -\mathbf{BA}$ $\implies \mathbf{A} = \mathbf{B}(-\mathbf{A})\mathbf{B}^{-1}$ So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.$\therefore$ $trace(\mathbf{A}) = trace(-\mathbf{A})$ $\implies trace(\mathbf{A}) = 0$ Similarly From (1.0.1) we have $\mathbf{AB} = -\mathbf{BA}$ $\implies \mathbf{B} = \mathbf{A}^{-1}(-\mathbf{B})\mathbf{A}$ So, matrix $\mathbf{B}$ and (-$\mathbf{B}$) are similar.$\therefore$ $trace(\mathbf{B}) = trace(-\mathbf{B})$ $\implies trace(\mathbf{B}) = 0$ So this statement is true
trace($\mathbf{A}$) = 1 trace($\mathbf{B}$) = 1	From (1.0.1) we have $\mathbf{AB} = -\mathbf{BA}$ $\implies \mathbf{A} = \mathbf{B}(-\mathbf{A})\mathbf{B}^{-1}$ So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.$\therefore$ $trace(\mathbf{A}) = trace(-\mathbf{A})$ $\implies trace(\mathbf{A}) = 0.$ As trace($\mathbf{A}$) = 0 this statement is false
trace($\mathbf{A}$) = 0 trace($\mathbf{B}$) = 1	From (1.0.1) we have $\mathbf{AB} = -\mathbf{BA}$ $\implies \mathbf{B} = \mathbf{A}^{-1}(-\mathbf{B})\mathbf{A}$ So, matrix $\mathbf{B}$ and (-$\mathbf{B}$) are similar.$\therefore$ $trace(\mathbf{B}) = trace(-\mathbf{B})$ $\implies trace(\mathbf{B}) = 0.$ As trace($\mathbf{B}$) = 0 this statement is false
trace($\mathbf{A}$) = 1 trace($\mathbf{B}$) = 0	From (1.0.1) we have $\mathbf{AB} = -\mathbf{BA}$ $\implies \mathbf{A} = \mathbf{B}(-\mathbf{A})\mathbf{B}^{-1}$ So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.$\therefore$ $trace(\mathbf{A}) = trace(-\mathbf{A})$ $\implies trace(\mathbf{A}) = 0.$ As trace($\mathbf{A}$) = 0 this statement is false

TABLE 2: Calculation of trace