Matrix Theory(EE5609) Assignment 4

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***Abstract*—This Assignment proves the property of a given triangle**

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/master/assignment4/assign4.tex

1 Problem Statement

$\triangle ABC$ is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle BCD$ is a right angle.

2 Solution

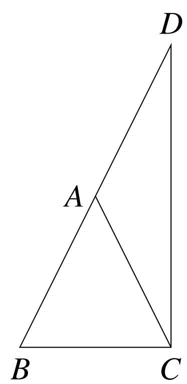

Fig. 1

We are given that AB = AC. So

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.1}$$

Also BA is produced to D such that AB = AD. Therefore we have

$$\mathbf{A}-\mathbf{B} = \mathbf{D}-\mathbf{A} \tag{2.0.2}$$

Taking dot product of vectors **CB** and **CD** we get

$$\begin{aligned}\mathbf{CB}.\mathbf{CD} &= (\mathbf{B}-\mathbf{C})^T(\mathbf{D}-\mathbf{C})\\ &= (\mathbf{B}-\mathbf{A}+\mathbf{A}-\mathbf{C})^T(\mathbf{D}-\mathbf{A}+\mathbf{A}-\mathbf{C})\\ &= ((\mathbf{B}-\mathbf{A})+(\mathbf{A}-\mathbf{C}))^T((\mathbf{D}-\mathbf{A})+(\mathbf{A}-\mathbf{C}))\end{aligned}$$

using (2.0.2) we get

$$\begin{aligned}\mathbf{CB}.\mathbf{CD} &= ((\mathbf{B}-\mathbf{A})+(\mathbf{A}-\mathbf{C}))^T((\mathbf{D}-\mathbf{A})+(\mathbf{A}-\mathbf{C}))\\ &= ((\mathbf{B}-\mathbf{A})+(\mathbf{A}-\mathbf{C}))^T((\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{C}))\\ &= (-(\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{C}))^T((\mathbf{A}-\mathbf{B})+(\mathbf{A}-\mathbf{C}))\\ &= -\|\mathbf{A}-\mathbf{B}\|^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})\\ &\quad +(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})+\|\mathbf{A}-\mathbf{C}\|^2\end{aligned}$$

now $(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})$ and $(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})$ are both dot product of vectors $(\mathbf{A}-\mathbf{B})$ and $(\mathbf{A}-\mathbf{C})$ and so

$$\begin{aligned}\mathbf{CB}.\mathbf{CD} &= -\|\mathbf{A}-\mathbf{B}\|^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})\\ &\quad +(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})+\|\mathbf{A}-\mathbf{C}\|^2\\ &= -\|\mathbf{A}-\mathbf{B}\|^2+\|\mathbf{A}-\mathbf{C}\|^2\end{aligned}$$

using (2.0.1) we get

$$\mathbf{CB}.\mathbf{CD} = -\|\mathbf{A}-\mathbf{B}\|^2+\|\mathbf{A}-\mathbf{C}\|^2 = 0 \tag{2.0.3}$$

since $\mathbf{CB}.\mathbf{CD} = 0$,therefore BC $\perp$ CD and $\angle BCD$ is a right angle.