Matrix Theory(EE5609) Assignment 9

Anshum Agrawal
Roll No- AI20MTECH11006

***Abstract*—Given AB = -BA, this documents finds value of trace(A) and trace(B).**

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/master/assignment9a/assign9.tex

1 Problem Statement

Let $\mathbf{A}$ and $\mathbf{B}$ be real invertible matrices such that

$$\mathbf{AB} = -\mathbf{BA}. \tag{1.0.1}$$

Then

1) trace$\mathbf{A}$ = trace($\mathbf{B}$) = 0
2) trace$\mathbf{A}$ = trace($\mathbf{B}$) = 1
3) trace$\mathbf{A}$ = 0, trace($\mathbf{B}$) = 1
4) trace($\mathbf{A}$) = 1, trace($\mathbf{B}$) = 0

Solution:

Definition	Matrix $\mathbf{A}$ is said to be similar to matrix $\mathbf{B}$ if there exists matrix $\mathbf{P}$ such that $\mathbf{A} = \mathbf{PBP}^{-1}$
Properties	Similar matrices have same eigenvalues Sum of eigenvalue of a matrix equals its trace

TABLE 1: Similar matrices and Properties

trace($\mathbf{A}$) = 0 trace($\mathbf{B}$) = 0	From (1.0.1) we have $\mathbf{AB}$=-$\mathbf{BA}$ or $\mathbf{A}$ = $\mathbf{B}$(-$\mathbf{A}$)$\mathbf{B}^{-1}$. So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.From properties of similar matrices their eigenvalues are same.Also sum of eigenvalue = trace of matrix.Therefore trace($\mathbf{A}$) = trace(-$\mathbf{A}$) or trace($\mathbf{A}$) = -trace($\mathbf{A}$) or trace($\mathbf{A}$) = 0. Similarly we can write $\mathbf{B}$ = $\mathbf{A}$(-$\mathbf{B}$)$\mathbf{A}^{-1}$.So, matrix $\mathbf{B}$ and (-$\mathbf{B}$) are similar.Therefore similar to trace($\mathbf{A}$), trace($\mathbf{B}$) = 0. So this statement is true
trace($\mathbf{A}$) = 1 trace($\mathbf{B}$) = 1	From (1.0.1) we have $\mathbf{AB}$=-$\mathbf{BA}$ or $\mathbf{A}$ = $\mathbf{B}$(-$\mathbf{A}$)$\mathbf{B}^{-1}$. So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.From properties of similar matrices their eigenvalues are same.Also sum of eigenvalue = trace of matrix.Therefore trace($\mathbf{A}$) = trace(-$\mathbf{A}$) or trace($\mathbf{A}$) = -trace($\mathbf{A}$) or trace($\mathbf{A}$) = 0. As trace($\mathbf{A}$) = 0 this statement is false
trace($\mathbf{A}$) = 0 trace($\mathbf{B}$) = 1	From (1.0.1) we have $\mathbf{AB}$=-$\mathbf{BA}$ or $\mathbf{B}$ = $\mathbf{A}$(-$\mathbf{B}$)$\mathbf{A}^{-1}$. So, matrix $\mathbf{B}$ and (-$\mathbf{B}$) are similar.From properties of similar matrices their eigenvalues are same.Also sum of eigenvalue = trace of matrix.Therefore trace($\mathbf{B}$) = trace(-$\mathbf{B}$) or trace($\mathbf{B}$) = -trace($\mathbf{B}$) or trace($\mathbf{B}$) = 0. As trace($\mathbf{B}$) = 0 this statement is false
trace($\mathbf{A}$) = 1 trace($\mathbf{B}$) = 0	From (1.0.1) we have $\mathbf{AB}$=-$\mathbf{BA}$ or $\mathbf{A}$ = $\mathbf{B}$(-$\mathbf{A}$)$\mathbf{B}^{-1}$. So, matrix $\mathbf{A}$ and (-$\mathbf{A}$) are similar.From properties of similar matrices their eigenvalues are same.Also sum of eigenvalue = trace of matrix.Therefore trace($\mathbf{A}$) = trace(-$\mathbf{A}$) or trace($\mathbf{A}$) = -trace($\mathbf{A}$) or trace($\mathbf{A}$) = 0. As trace($\mathbf{A}$) = 0 this statement is false

TABLE 2: Calculation of trace