Matrix Theory(EE5609) Assignment 11

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Abstract—This document solves a system of given equations

Download all python codes from

https://github.com/anshum0302/EE5609/blob/
master/assignment11/figure.py

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/
master/assignment11/assign11.tex

1 PROBLEM STATEMENT

The system of equations:

$$1.x + 2.x^2 + 3.xy + 0.y = 6$$
$$2.x + 1.x^2 + 3.xy + 1.y = 5$$
$$1.x - 1.x^2 + 0.xy + 1.y = 7$$

1) has solutions in rational numbers
2) has solutions in real numbers
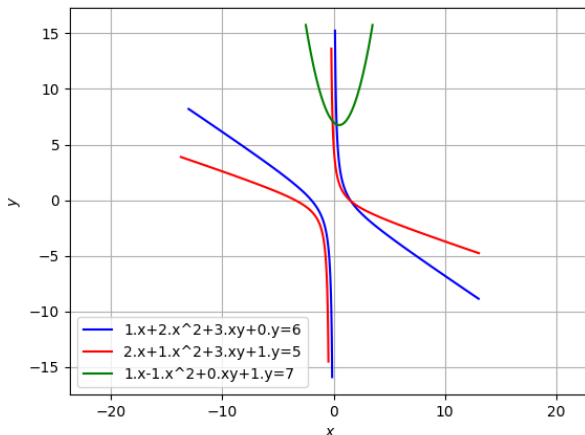3) has solutions in complex numbers
4) has no solution

Solution:

Fig. 1: Graph of Three equations

The general equation of second degree can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$$

For 1st equation we have

$$\mathbf{V} = \begin{pmatrix} 2 & 1.5 \\ 1.5 & 0 \end{pmatrix}$$

$$\implies |\mathbf{V}| = \begin{vmatrix} 2 & 1.5 \\ 1.5 & 0 \end{vmatrix} = -2.25$$

$$\mathbf{u} = \begin{pmatrix} 0.5 & 0 \end{pmatrix}$$

$$f = -6$$

As $|\mathbf{V}| < 0$ this is hyperbola

For 2nd equation we have

$$\mathbf{V} = \begin{pmatrix} 1 & 1.5 \\ 1.5 & 0 \end{pmatrix}$$

$$\implies |\mathbf{V}| = \begin{vmatrix} 1 & 1.5 \\ 1.5 & 0 \end{vmatrix} = -2.25$$

$$\mathbf{u} = \begin{pmatrix} 1 & 0.5 \end{pmatrix}$$

$$f = -5$$

As $|\mathbf{V}| < 0$ this is hyperbola

For 3rd equation we have

$$\mathbf{V} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\implies |\mathbf{V}| = \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$\mathbf{u} = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}$$

$$f = -7$$

As $|\mathbf{V}| = 0$ this is parabola

TABLE 1: Equations in quadratic form

From the figure we can see that all graph of three equations do not meet at the same point.So

the system of equation do not have any solution.

Writing given equations in matrix form	$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 3 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ x^2 \\ xy \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 7 \end{pmatrix}$$
row reduction of the augmented matrix	$$\begin{pmatrix} 1 & 2 & 3 & 0 & 6 \\ 2 & 1 & 3 & 1 & 5 \\ 1 & -1 & 0 & 1 & 7 \end{pmatrix} \xleftrightarrow[R_3=R_3-R_1]{R_2=R_2-2R_1} \begin{pmatrix} 1 & 2 & 3 & 0 & 6 \\ 0 & -3 & -3 & 1 & -7 \\ 0 & -3 & -3 & 1 & 1 \end{pmatrix} \xleftarrow{R_3=R_3-R_2} \begin{pmatrix} 1 & 2 & 3 & 0 & 6 \\ 0 & -3 & -3 & 1 & -7 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix}$$ Third row of above matrix represents equation $$0.x + 0.x^2 + 0.xy + 0.y = 8$$ $$\implies 0 = 8$$ Thus system of equation is inconsistent
Conclusion	Option 4) is correct option 1),2) and 3) are incorrect

TABLE 2: Solving system of given equation