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Matrix Theory(EE5609) Assignment 4

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Abstract—This Assignment proves the property of a using (2.0.2) we get given triangle

Download latex-tikz codes from

https://github.com/anshum0302/EE5609/blob/ master/assignment4/assign4.tex

1 PROBLEM STATEMENT

 $\triangle ABC$ is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle BCD$ is a right angle.

2 Solution

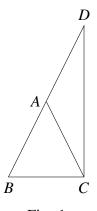


Fig. 1

We are given that AB = AC. So

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.1}$$

Also BA is produced to D such that AB = AD. Therefore we have

$$\mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{A} \tag{2.0.2}$$

Taking dot product of vectors $(\mathbf{B} - \mathbf{C})$ and $(\mathbf{D} - \mathbf{C})$ we get

$$(\mathbf{B} - \mathbf{C})^{T}(\mathbf{D} - \mathbf{C})$$

$$= (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{C})^{T}(\mathbf{D} - \mathbf{A} + \mathbf{A} - \mathbf{C})$$

$$= ((\mathbf{B} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}))^{T}((\mathbf{D} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}))$$

$$(\mathbf{B} - \mathbf{C})^{T}(\mathbf{D} - \mathbf{C})$$
= $((\mathbf{B} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}))^{T}((\mathbf{D} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}))$
= $((\mathbf{B} - \mathbf{A}) + (\mathbf{A} - \mathbf{C}))^{T}((\mathbf{A} - \mathbf{B}) + (\mathbf{A} - \mathbf{C}))$
= $(-(\mathbf{A} - \mathbf{B}) + (\mathbf{A} - \mathbf{C}))^{T}((\mathbf{A} - \mathbf{B}) + (\mathbf{A} - \mathbf{C}))$
= $-\|\mathbf{A} - \mathbf{B}\|^{2} - (\mathbf{A} - \mathbf{B})^{T}(\mathbf{A} - \mathbf{C})$
+ $(\mathbf{A} - \mathbf{C})^{T}(\mathbf{A} - \mathbf{B}) + \|\mathbf{A} - \mathbf{C}\|^{2}$

 $(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C})$ and $(\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B})$ are both dot product of vectors (A - B) and (A - C), therefore

$$(\mathbf{B} - \mathbf{C})^{T}(\mathbf{D} - \mathbf{C})$$

$$= -\|\mathbf{A} - \mathbf{B}\|^{2} - (\mathbf{A} - \mathbf{B})^{T}(\mathbf{A} - \mathbf{C})$$

$$+ (\mathbf{A} - \mathbf{C})^{T}(\mathbf{A} - \mathbf{B}) + \|\mathbf{A} - \mathbf{C}\|^{2}$$

$$= -\|\mathbf{A} - \mathbf{B}\|^{2} + \|\mathbf{A} - \mathbf{C}\|^{2}$$

using (2.0.1) we get

$$(\mathbf{B} - \mathbf{C})^{T}(\mathbf{D} - \mathbf{C}) = -\|\mathbf{A} - \mathbf{B}\|^{2} + \|\mathbf{A} - \mathbf{C}\|^{2} = 0$$
(2.0.3)

since $(\mathbf{B} - \mathbf{C})^T (\mathbf{D} - \mathbf{C}) = 0$, therefore BC \perp CD and $\angle BCD$ is a right angle.