Introduction to Machine Learning

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Semi-supervised Learning

- Ground-truth is costly and time consuming
  - Empirical studies show that we need at least (10-30) x dimensions number of samples per class
- Unlabeled samples are plenty and cheap
- Can we combine small number of labeled samples and large number of unlabeled sample to improve learning model?
  - Yes: semi-supervised learning

Self-training

- Key assumption
  - One's own high confidence predictions are correct
- Algorithm
  1. Train f from D_l = (**X**,Y)
  2. Use f to predict on D_{ul} = (**x**, ?)
  3. Add (**x**, f(**x**)) to labeled data D_l
     - Variations: All, most confident, weighted?
  4. Repeat

Self-training

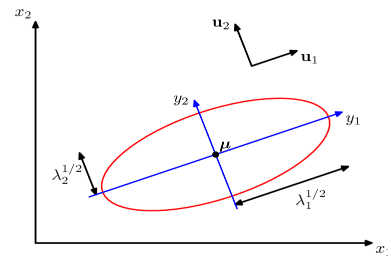
- Advantages
  - Simplest semi-supervised learning
  - Easy to implement (wrapper around existing algorithms)
- Disadvantages
  - Early mistakes may lead to degrade in performance

Probabilistic Approach

- Assume data is generated by a mixture model
  - E.g., Gaussian Mixture Model
- D = Dl + Dul
- How do you estimate parameters? MLE don't work with missing labels
- Solution
  - Expectation Maximization (EM) algorithm

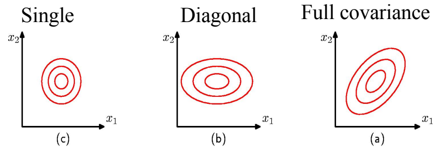
Multivariate Gaussian

- Bivariate covariance matrix representation



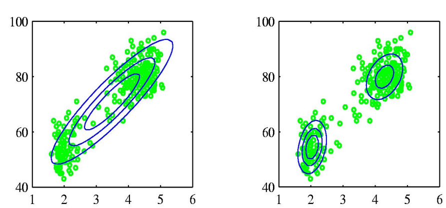
Multivariate Gaussian

- Σ plays key role, but accurate estimation requires large number samples (10-30 x dim; per class)
- However, Σ can be simplified



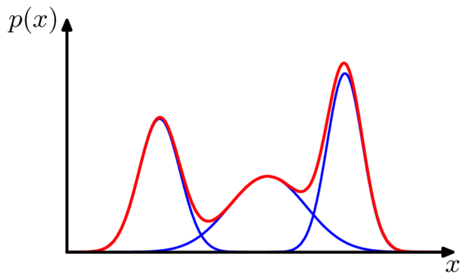
$$\Sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 \end{pmatrix}$$

What If The Data Is Multimodal



- Real world data are rarely unimodal
- Many times we don't know the labels for all components

How to model such data?

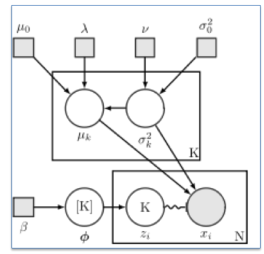


- Mixtures of Gaussians
- Mixture models in general

Mixture Models

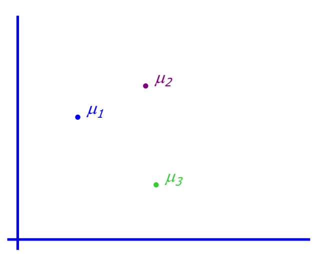
- Mixture model is a probabilistic model for representing the presence of subpopulations within an overall population, without requiring that an observed data set should identify the sub-population to which an individual observation belongs.

- K = # components
- N = # observations
- θ = parameter
- Φ = mixture weight
- z_i = component of observation i
- x_i = i^{th} observation



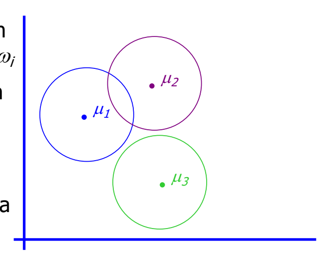
The GMM assumption

- There are k components. The i' th component is called ω_i
- Component ω_i has an associated mean vector μ_i



The GMM assumption

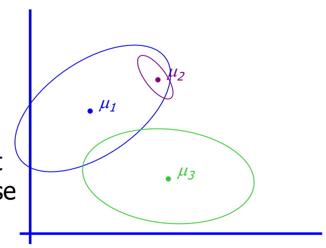
- There are k components. The i' th component is called ω_i
- Component ω_i has an associated mean vector μ_i
- Each component generates data from a Gaussian with mean μ_i and covariance matrix $\sigma^2 \boldsymbol{I}$



The General GMM assumption

Assume that each data point is generated according to the following recipe:

1. Pick a component at random. Choose component i with probability $P(\omega_i)$.
2. Data point ~ $N(\mu_i, \Sigma_i)$



GMM

- GMM (K=M) $p(x|\theta) = \sum_{i=1}^{M} \alpha_i p_i(x|\theta_i)$

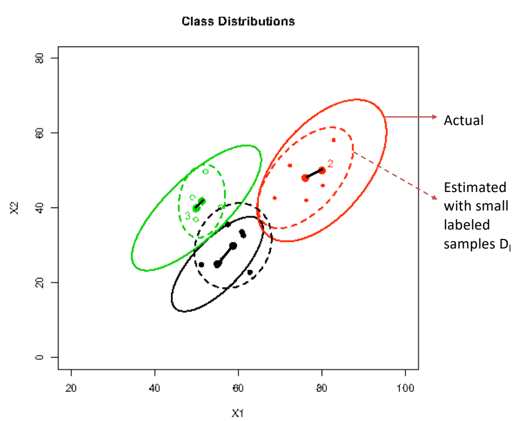
  where $\theta = (\alpha_1, ..., \alpha_M; \theta_1, ...\theta_M)$

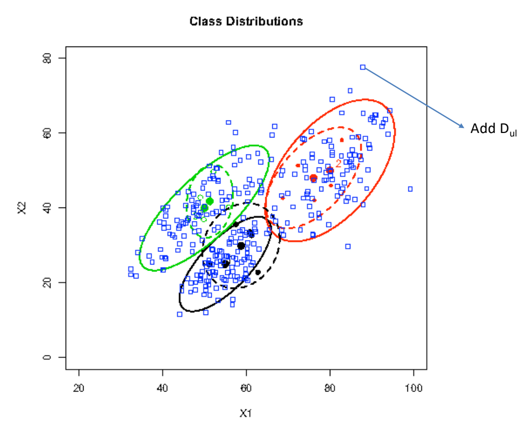
  such that $\sum_1^M \alpha_i = 1, \quad 0 < \alpha_i < 1$ and

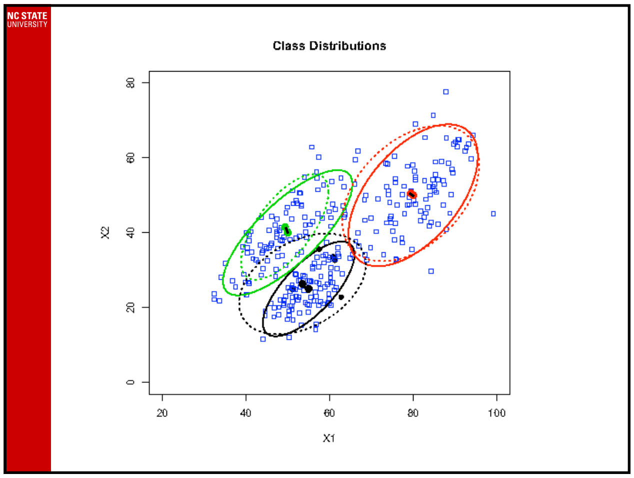
  p_i pdf parameterized by θ_i

- Maximize

$$L(\theta) - L(\theta_i) = \ln \frac{\sum_z p(x|z,\theta)p(z|\theta)}{p(x|\theta_i)}$$



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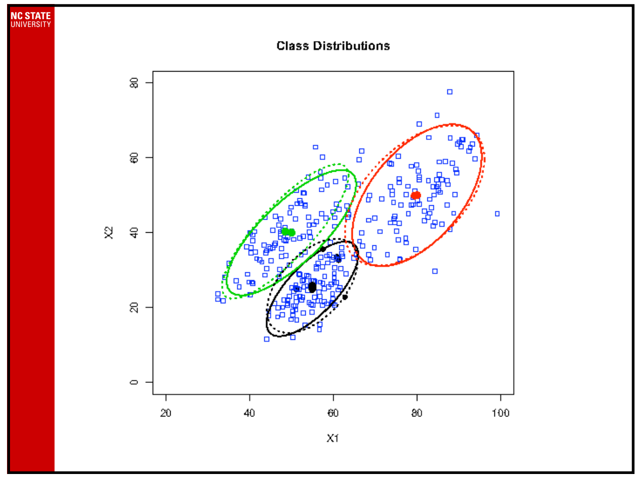
Class Distributions
X1
X2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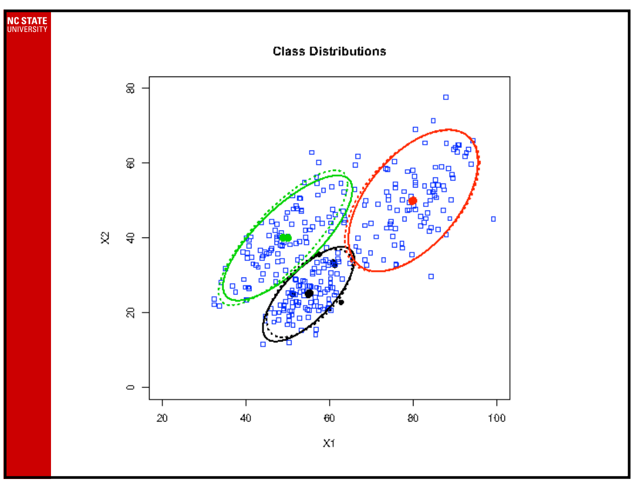
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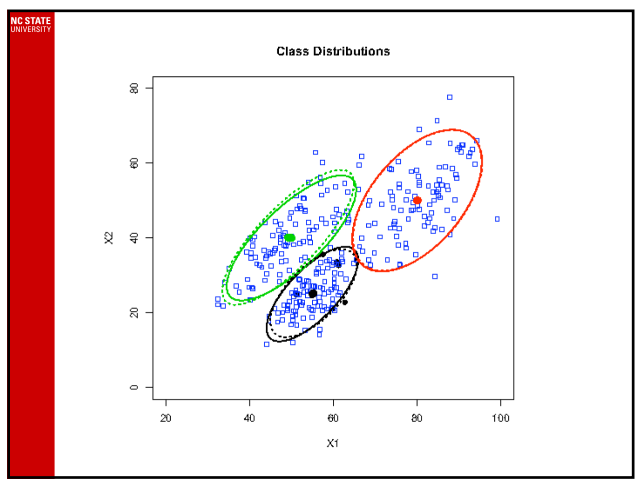
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Class Distributions
X1
X2

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Semi-supervised Learning

EM to estimate GMM parameters

- E-Step

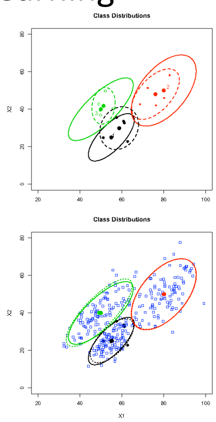
$$e_{ij} = \frac{\left|\hat{\Sigma}_j^k\right|^{-1/2} \exp\left\{-\frac{1}{2}\left(x_i - \hat{\mu}_j^k\right)^T \hat{\Sigma}_j^{-1,k}\left(x_i - \hat{\mu}_j^k\right)\right\}}{\sum_{l=1}^{M}\left|\hat{\Sigma}_l^k\right|^{-1/2} \exp\left\{-\frac{1}{2}\left(x_i - \hat{\mu}_l^k\right)^T \hat{\Sigma}_l^{-1,k}\left(x_i - \hat{\mu}_l^k\right)\right\}}$$

- M-Step

$$\alpha_j = \frac{\sum_{i=1}^{N} e_{ij}}{N}, \qquad \hat{\mu}_j^{k+1} = \frac{\sum_{i=1}^{N} e_{ij} x_i}{\sum_{i=1}^{N} e_{ij}},$$

$$\text{and} \quad \hat{\Sigma}_j^{k+1} = \frac{\sum_{i=1}^{N} e_{ij}\left(x_i - \hat{\mu}_j^{k+1}\right)\left(x_i - \hat{\mu}_j^{k+1}\right)^T}{\sum_{i=1}^{N} e_{ij}}$$

i^{th} data vector, j^{th} class



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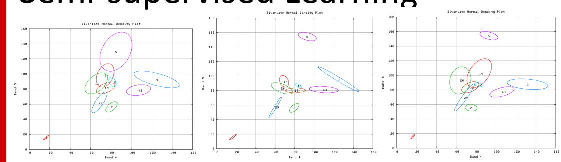
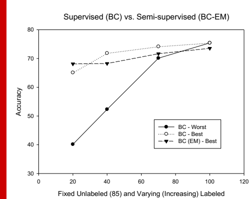
Semi-supervised Learning



10 Classes, 100 Training Samples (10-30) x No of dimensions / class

Small Subset of 20 Training Samples

20 labeled + 80 unlabeled samples



Ranga Raju Vatsavai, Shashi Shekhar, Thomas E. Burk: A Semi-Supervised Learning Method for Remote Sensing Data Mining. ICTAI 2005: 207-211