Power, Effective Size, Sample Size, Type I and II Errors

How to determine sample size requirements, how to calculate an effect size, how to assess statistical power

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Learning Objectives

- **Find out how big a sample size is needed for a statistical test to be conducted**
- **Pre-define the minimum size of the effect to be detected (i.e., effect size)**
- **Specify the required probability of detecting the effective size (power)**
- **Specify the significance level (alpha) at which the test will be conducted**

- **Correctly use the vocabulary:**
  - Effect size
  - Power
  - Significance level, alpha
  - Sample size

Sample size affects the ability to detect the real difference between treatments A and B

- **Web testing of a new feature**
  - How long should it run?
  - How many impressions per treatment are needed?
- **Will a hypothesis test reveal a difference between treatments A and B?**
- **The outcome of a hypothesis test (the p-value) depends on:**
  - what the real difference is between two treatments A and B
  - on the luck of the draw: who gets selected for the groups in the experiment
- **The real difference between treatments vs. sample size**
  - the bigger the actual difference between the two treatments, the greater the probability that the experiment will reveal it
  - the smaller the difference between treatments A and B, the more data will be needed to detect it

Power

- **Power**
  - the probability of detecting a specified *effect size* with specified sample characteristics (size and variability)
- **Example**
  - The probability of distinguishing between a 0.33 hitter and a 0.2 hitter in n = 25 at-bats is 0.75
  - The effect size is a difference of 0.13 = 0.33 – 0.2
  - Detection: a hypothesis test will reject the null hypothesis of no difference and concludes that there is a real effect
    - The experiment of sample size $n = 25$ for two hitters, with an effect size of 0.13, has (hypothetical) power of 0.75 or 75%

Moving Parts for Calculating Power or Sample Size

- **Effect size you want to detect**
- **Sample size**
- **Significance level (alpha) at which:**
  - the hypothesis test is conducted or
  - the power is calculated
- **Power**

Specify any three of the moving parts, and the fourth can be calculated.

Motivation

- **A/B Tests**
  - Collecting and processing the data involves some cost
  - Knowing approximately how much data to collect to avoid the situations where the result ends up being inconclusive

Calculating Power: The Mechanics

1. **Start with some hypothetical data that represent your best guess about the data that will result (e.g., e.g., based on prior data)**
   - a hat with 20 ones and 80 zeros to represent a 0.2 hitter or
   - a hat with some observations of "time spent on website"
2. **Create a second sample by adding the desired effect size to the first sample**
   - a second hat with 33 ones and 67 zeros or
   - a second hat with 25 seconds added to each initial "time spent on website"
3. **Draw a bootstrap sample of size n from each box**
4. **Conduct a hypothesis test (permutation- or formula-based) on the two bootstrap samples and record whether the difference between them is statistically significant**
5. **Repeat Steps 3 and 4 many times:**
   - To determine how often the difference was significant
   - This is the estimated power

Sample Size

- **Applications of Power Calculations**
  - To estimate how big a sample needs to be
- **Motivating Example**
  - Experiment: A click-through rates (clicks as a percentage of exposure) for a new add should be tested against an existing ad
  - Policy: a new ad must do better must do better than an existing ad by some percentage (e.g., 10%)
    - otherwise, the existing ad will remain in place
  - This goal, the “effect size” the drives the sample size
- **How many clicks to accumulate in the study?**
  - If you are interested in results that show a huge difference (e.g., 50%), then a relatively small sample might be enough
  - But if even a minor difference would be of interest, then a much larger sample is needed

Summary: Key Ideas and Concepts

- Find out how big a **sample size** is needed for a **statistical test** to be conducted
- Pre-define the minimum size of the effect to be detected (i.e., **effect size**)
- Specify the required probability of detecting the effective size (**power**)
- Specify the significance level (**alpha**) at which the test will be conducted

- The vocabulary:
  - Effect size
  - Power
  - Significance level

Type I and II Errors, Power, Significance Level (α)

Decision

Actual	Reject H0	Fail to Reject H0
H0 True	Type I Error	correct
H0 False	correct	Type II Error

Power = 1 – Probability (Type II Error)

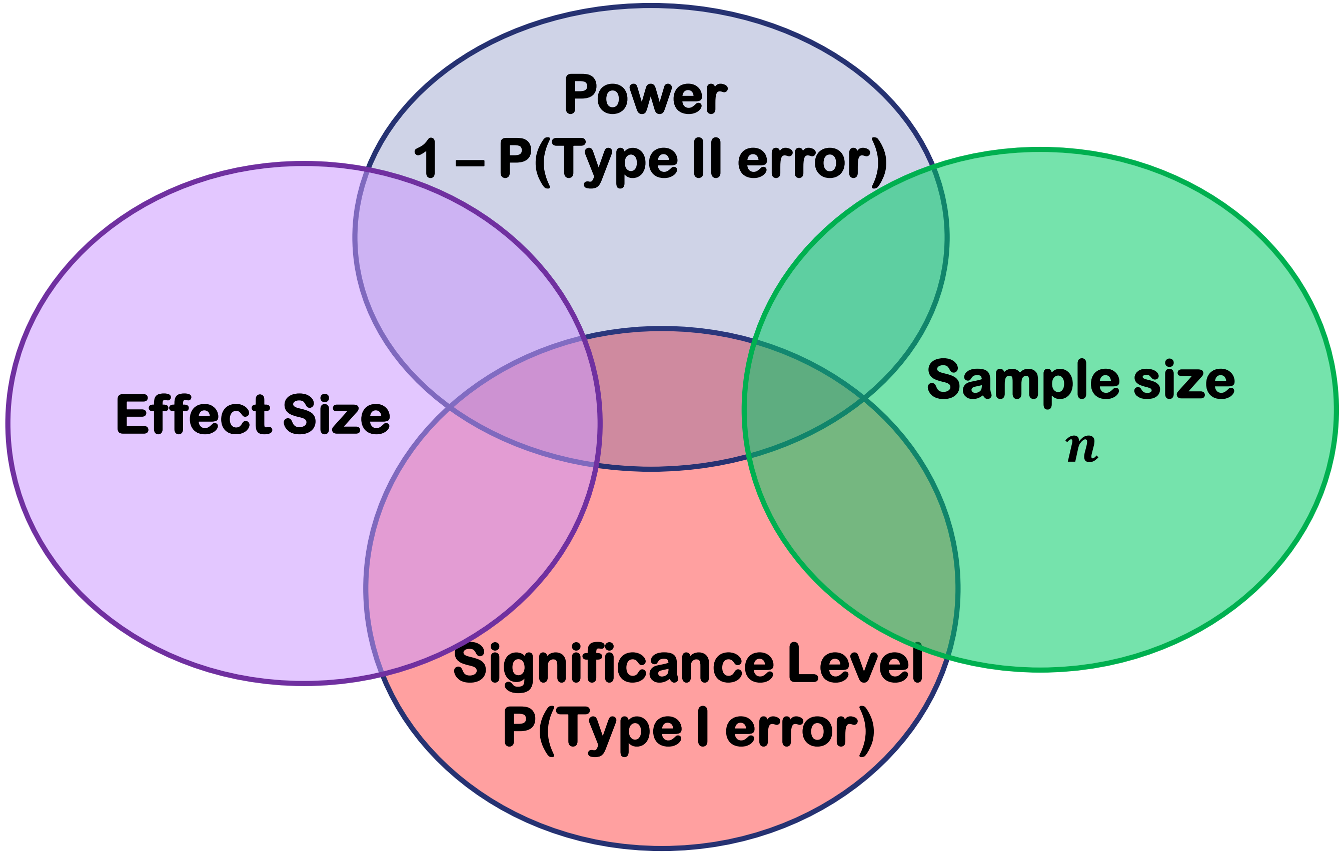
α = Probability (Type I Error)

- **Type I Error**
  - Probability of False Positives
  - Fail to find an effect that IS there
- **Type II Error**
  - The probability of False Negatives
  - Find an effect that is NOT there

- **Power** is defined as one minus the probability of making a Type II error
  - The probability of finding an effect that *IS* there
- **Significance level** (α) is the probability of making a Type I error
  - The probability of finding an effect that is *NOT* there

Type I & II Errors, Power, Significance Level, Effect Size

The moving parts



given any three,
the fourth can be determined

Power Calculations in R with pwr Package

Function	Samples	Power Calculations for	Compare:Data Type
pwr.2p.test()	2	Two proportions (equal n)	Proportions
pwr.2p2n.test()	2	Two proportions (un-equal n)	Proportions
pwr.p.test()	1	Proportion (one sample)	Proportions
pwr.t.test()	1, 2, paired	t-tests	Continuous, means
pwr.t2n.test()	2	t-test (un-equal n)	Continuous, means
pwr.chisq.test()	2	Relationship between 2	Categorical variables
pwr.anova.test()	Multiple	Balanced one-way ANOVA	Continuous, means
pwr.r.test()	2	Correlation between two continuous variables	Continuous, correlation coefficients
pwr.f2.test()	2	General linear model	Impact of a set of predictors on an outcome or impact of one set of predictors above and beyond a second set of predictors (or covariates)

R: Power Analysis with pwr.t.test(): t-tests of means
(one sample, two samples and paired samples)

```
pwr.t.test(n = NULL, d = NULL, sig.level = 0.05, power = NULL,
    type = c("two.sample", "one.sample", "paired"),
    alternative = c("two.sided", "less", "greater"))
```

• n	Number of observations (per sample)
• d	Effect size
• sig.level	Significance level (Type I error probability)
• power	Power of test (1 minus Type II error probability)
• type	Type of t test : one- two- or paired-samples
• alternative	a character string specifying the alternative hypothesis, must be one of "two.sided" (default), "greater" or "less"

Python: Power Analysis: t-tests of means
(one sample, two samples and paired samples)

```
from statsmodels.stats import power
```

```
Signature: power.TTestIndPower.solve_power(self, effect_size=None, nobs1=None, alpha=None,
power=None, ratio=1.0, alternative='two-sided')
Docstring:
solve for any one parameter of the power of a two sample t-test
```

Parameters
effect_size : float: standardized effect size, difference between the two means divided by the standard deviation. `effect_size` has to be positive.
nobs1 : int or float: number of observations of sample 1. The number of observations of sample two is ratio times the size of sample 1, i.e. ``nobs2 = nobs1 * ratio``
alpha : float in interval (0,1) significance level, e.g. 0.05, is the probability of a type I error, that is wrong rejections if the Null Hypothesis is true.
power : float in interval (0,1) power of the test, e.g. 0.8, is one minus the probability of a type II error. Power is the probability that the test correctly rejects the Null Hypothesis if the Alternative Hypothesis is true.
ratio : float: ratio of the number of observations in sample 2 relative to sample 1. see description of nobs1. The default for ratio is 1; to solve for ratio given the other arguments it has to be explicitly set to None.
alternative : string, 'two-sided' (default), 'larger', 'smaller‘ extra argument to choose whether the power is calculated for a two-sided (default) or one sided test. The one-sided test can be either 'larger', 'smaller'.

Returns
value : float: The value of. The value solves the power equation given the remaining parameters.

Example: t-test of means: two.sample

Case Study: Cell phone usage and driving reaction time

Null Hypothesis: Ho: $\mu_1 - \mu_0 = 0$ ⟵ **two sample**

The mean response time (μ_1) for drivers using a cell phone is the same as the mean response time (μ_0) for drivers that are cell phone free.

Alternative Hypothesis: H1: $\mu_1 \neq \mu_0$ ⟵ **two sided**

Empirical Data (past experience):

- 1-second difference in reaction time is considered an important difference
- reaction time has a standard deviation of 1.25 seconds

Effect size (d) = $\frac{\mu_1 - \mu_0}{\sigma} = \frac{1}{1.25} = 0.8$ or larger

↑

standardized mean difference

R Example: t-test of means: Equal Participants

Case Study: Cell phone usage and driving reaction time

```r
# File: power_analysis.R

#install.packages("pwr")
library (pwr)

# Cell phone usage and driving reaction time
# d = 0.8 or larger: standarded mean difference
pwr.t.test(d=0.8, sig.level=0.05, power=0.9,
           type="two.sample",
           alternative="two.sided")
```

Output

```
     Two-sample t test power calculation

              n = 33.82555
              d = 0.8
      sig.level = 0.05
          power = 0.9
    alternative = two.sided

NOTE: n is number in *each* group
```

sample size = 34 participants in each group in order to detect an effect size of 0.8 with 90% certainty (power) and no more than a 5% chance (alpha) of erroneously concluding that a difference exists when, in fact, it does not

Python: t-test of means: Equal Participants

Case Study: Cell phone usage and driving reaction time

```python
from statsmodels.stats import power
```

```python
power.TTestIndPower().solve_power(0.8, power=0.9, ratio=1,
                                  alpha=0.05,
                                  alternative='two-sided')
```

```
33.825543836843956
```

Output

```
     Two-sample t test power calculation

              n = 33.82555
              d = 0.8
      sig.level = 0.05
          power = 0.9
    alternative = two.sided

NOTE: n is number in *each* group
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sample size = 34 participants in each group in order to detect an **effect size** of 0.8 with 90% certainty (**power**) and no more than a 5% chance (**alpha**) of erroneously concluding that a difference exists when, in fact, it does not

R Example: t-test of means: Un-equal Participants

Case Study: Cell phone usage and driving reaction time

- Constraints:
  - Assume that you want to detect 0.5 standard deviation difference in population means
  - You limit the chances of falsey declaring population means to be different to 1 out of 100
  - You can only afford to include 40 participants in the study, with equal number in each group
- Question:
  - What is the probability to detect a difference between the population means that is that large, given the above constraints?

```
pwr.t.test(n = 20, d=0.5, sig.level=0.01,
           type="two.sample",
           alternative="two.sided")
```

Output

```
     Two-sample t test power calculation

              n = 20
              d = 0.5
      sig.level = 0.01
          power = 0.1439551
    alternative = two.sided

NOTE: n is number in *each* group
```

you have less than 14% chance of declaring a difference of 0.625 seconds or less (d = 0.5 = 0.625 / 1.25). Conversely, there is 86% chance that you will miss the effect that you are looking for.

Python Example: t-test of means: Un-equal Participants

Case Study: Cell phone usage and driving reaction time

- Constraints:
  - Assume that you want to detect 0.5 standard deviation difference in population means
  - You limit the chances of falsey declaring population means to be different to 1 out of 100
  - You can only afford to include 40 participants in the study, with equal number in each group
- Question:
  - What is the probability to detect a difference between the population means that is that large, given the above constraints?

```
power.TTestIndPower().solve_power(0.5, nobs1=20, ratio=1,
                                  alpha=0.01,
                                  alternative='two-sided')
```

```
0.1439550828602357
```

you have less than 14% chance of declaring a difference of 0.625 seconds or less (d = 0.5 = 0.625 / 1.25). Conversely, there is 86% chance that you will miss the effect that you are looking for.

Output

```
     Two-sample t test power calculation

              n = 20
              d = 0.5
      sig.level = 0.01
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NOTE: n is number in *each* group
```

<u>Goal</u>: Maximize the power of statistical tests, while maintaining an acceptable α with small sample size

- **Under more direct control:**
  - **sample size**
  - **significance level**
- **Indirect control**
  - **power**
  - **effect size**

Maximize chances of finding a real effect and minimize the chances of finding an effect that isn't there, while keeping the study costs reasonable.

- **Relax significance level**
  - **make it easier to reject H0**
  - **power increases**
- **Increase sample size**
  - **power increases**