Editors' Suggestion | Featured in Physics

Fragmentation: Principles versus Mechanisms

Emmanuel Villermaux*
Aix Marseille Université, CNRS, Centrale Marseille, IRPHE UMR 7342, 13384 Marseille, France and Institut Universitaire de France, 75005 Paris, France

(Received 10 June 2025; accepted 23 September 2025; published 26 November 2025)

When it comes to understanding how a cohesive object breaks up, there are two types of temptations: either seek detailed mechanisms (capillary instabilities for liquids, cracks, propagation in brittle solids...), or rely on a general principle to infer the multiplicity of the fragments' sizes. Here we show that an original conservation law coupled with a maximal randomness principle provides new, unifying predictions. We explain when this principle is likely to apply, and why the fragment's size distribution is a power law $p(d) \sim d^{-\beta}$, in that case, with exponent $\beta = D + 1 - \{\pi^{D/2}/[2^D(D/2)!]\}$, a function of the dimensionality of the breaking object D. Examples including crushed and grinned brittle materials like solid bars, plates, and shells; or cubes and spheroids; but also liquid drops and bubbles; exploding liquid shells; plastic debris in the ocean; and remnants from the cavemen industry are considered. The discussion is supplemented by an original experiment.

DOI: 10.1103/r7xz-5d9c

Despite being of a trivial common experience, the origin of the multiplicity of fragment sizes produced by a glass accidentally dropped on a floor, and its many variants, remains a fundamentally unanswered question. It concerns a vast landscape of applications all involving the breakup of cohesive objects, ranging from crushed [1] or impacted solid bars [2–5], plates [6,7], rings and shells [8–13], or spheroids [14–18], liquid fragments from impacted drops [19], and exploding liquid shells [20], to the formation of bubbles under breaking waves [21–23] and even the remnants from the cavemen industry [24]. In all cases the fragments are broadly distributed in size according to a decaying power law with an exponent β, a notable function of the aspect ratio, or dimensionality D of the initial object [25,26].

We invoke here a "maximal randomness" principle (the analog of Boltzmann's *Stosszahlansatz* in the kinetic theory of gases [27]) to describe the distribution of the fragment sizes which, when applied to the appropriate function and coupled with an original conservation law provides new, unifying results.

We call $n\,\mathrm{d}d$ the number of fragments in the size range $\{d, d + \mathrm{d}d\}$ formed from an object of size R dismantled under maximal randomness conditions, subjected to some constraints. Because there are many microscopic factors influencing the formation of a given fragment, in an environment which is itself varying concomitantly with the breakup process [8,28], a deterministic description of n is out of reach (see however, Refs. [29,30] in a simple geometry). But precisely because of that microscopic random nature, the size partition n in a given experiment is likely, among all those that are possible, to be the one with maximal probability. In systems where there are many degrees of freedom and very few microscopic constraints at play like in gases [31], or turbulence [32,33], the statistics is readily obtained by computing the number of occurrences of a given microstate, which amounts to maximizing an entropy [34]. With $n!$ arrangements per size class ($\ln n! \approx n \ln n - n$), our principle thus amounts to maximizing

$$S = \int \{n \ln n - n + \text{constraints}\}\mathrm{d}d. \tag{1}$$

We need now to make the constraints explicit. A conservation law for fragmenting objects, first discovered in [12] is key to the present discussion. Let a D-dimensional object of volume R^D dismantle into a collection of fragments with sizes $d(\mathbf{x}, t)$ at position $\mathbf{x}$ and time t during the process. When it is over, fragment sizes d are independent of t, and are distributed according to n. We may define a density in the breaking object extensive to the local fragment volume as $\psi(\mathbf{x}, t) \sim d(\mathbf{x}, t)^D/R^D$, a globally conserved field moving at velocity $\mathbf{u}(\mathbf{x}, t)$ as the fragments separate (see the Supplemental Material, [35], Sec. I). This density obeys the conservation equation $\partial_t\psi + \nabla(\psi\mathbf{u}) = 0$, which also writes $\partial_t \ln\psi + \mathbf{u}\nabla\ln\psi = -\nabla\mathbf{u}$, singling out the field divergence $\nabla\mathbf{u}$. That divergence, of order $\dot{R}/R$ is usually small since cracks in a macroscopic solid material propagate at the velocity of sound $c \gg \dot{R}$ and involve minute microscopic displacements so that fragments are formed before they have separated appreciably. Until breakup is completed at $t = \mathcal{O}(R/c)$, the net expansion of the object is at most $\dot{R}/R \times R/c = \mathcal{O}(\dot{R}/c) \ll 1$, and we have

*Contact author: emmanuel.villermaux@univ-amu.fr

$$\int \mathrm{d}\mathbf{x}^D\{\partial_t \ln \psi + \mathbf{u}\cdot\nabla \ln \psi\} = -\int \mathrm{d}\mathbf{x}^D \nabla\cdot\mathbf{u} \approx 0, \quad (2)$$

meaning that $\ln \psi$ (or $\ln d$) summed over the whole volume of the breaking object is conserved during the internal rearrangements leading to its fragmentation. Conversely, among the many routes branching cracks may take [28], those permitted by this conservation law must leave $\int\{n \ln d\}\,\mathrm{d}d$ unchanged.

We first consider the consequences of this constraint only. With β a Lagrange multiplier, we thus maximize S in Eq. (1) as

$$\delta S = \delta\int\{n \ln n - n + \beta n \ln d\}\mathrm{d}d = \delta n\int\{\ln n + \beta \ln d\}\mathrm{d}d = 0 \quad \text{whatever } \delta\mathrm{n}, \quad (3)$$

implying that $n \sim d^{-\beta}$, a power law reflecting the empirical observations reported above. We choose a rigid representation involving the initial size of the object R,

$$n = \frac{1}{R}\left(\frac{d}{R}\right)^{-\beta}, \quad \text{for } d < R, \quad (4)$$

to formulate the conservation of mass, more precisely the conservation of volume, accounting for the empirical observation that fragments tend to isotropy [18,36,37]. The volume of a sphere of diameter d is $(d/2)^D\pi^{D/2}/(D/2)!$ in dimension D so that

$$R^D = \int_0^R \frac{\pi^{D/2}}{2^D\left(\frac{D}{2}\right)!} d^D\, n\,\mathrm{d}d, \quad (5)$$

which solves for β in a closed form:

$$\beta = D + 1 - \frac{\pi^{D/2}}{2^D\left(\frac{D}{2}\right)!}. \quad (6)$$

The formula above, which solely comes from the kinematic constraint in Eq. (2) agrees well, in trend and order of magnitude ($\beta \gtrsim D$) with the measured ones for solid bars (1D), plates and shells (2D), or cubes, spheroids, but also for liquid drops and bubbles (3D), as seen in Fig. 1. Some original data in this figure are given in mass, and/or cumulated distributions (see the Supplemental Material, [35], Sec. II, and [2,3]).

We have $\beta \approx 1.3$ for 1D brittle glass bars [4], or spaghetti [2,5] (the original data of C. C. Lienau are plotted in the Supplemental Material, [35], Sec. III) either dropped on a rigid floor, or impacted longitudinally by a heavy mass [5]. The value $\beta \approx 2.4$ is a good average for dropped 2D brittle plates [6] and exploded ceramic tubes [10], as well as for

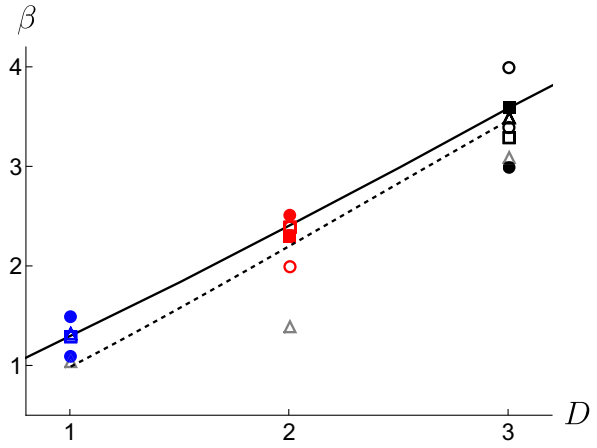

FIG. 1. Exponents β from fragment size partition $n \sim d^{-\beta}$ measured by several authors in the fragmentation of both solid and fluid objects, of dimension D: blue disk [4], blue circle [2], blue triangle [3], gray triangle [25], red disk [6], red circle [7], red square [10], red filled square [24], black disk [14], black circle [18], black square [22], black filled square [19], black triangle: present Letter. The dashed line is Eq. (6), and the continuous line results from mass conservation in Eq. (5), using Eq. (8).

plastic fragments collected in the ocean [7]. Most of those come from the degradation of thin packagings (bottles, envelopes, etc.) and therefore pertain to the 2D paradigm ([35], Sec. IV). Flakes from stones used as hammers by chimpanzee (and possibly early hominins) are produced via a 2D expansion process at the tool impact, and fall into this 2D category ([35], Sec. V, and [24]). Rocks progressively breaking in dense granular flow under a sustained shear in a gap first display a 2D signature, and finally a 3D exponent when interparticle stresses (grinding) have become isotropic [38].

As for fragmentation in liquids, we must mention plunging breaking waves which entrain air pockets below the ocean surface, breaking up in the transient turbulent agitation of the water. The size distribution of the resulting bubbles has been interpreted along several different lines, notably one invoking a Kolmogorov type of turbulence below the wave [21,23], leading to $\beta = 10/3$, a value not incompatible with recent data [22]. The 3D present paradigm suggests $\beta \approx 3.5$ which is not incompatible either (see the Supplemental Material, [35], Sec. VI), but has an essentially different origin, assuming randomness, not necessarily turbulence. Likewise, a good fit with $\beta \approx 3.5$ is obtained for the fragments of a liquid drop entrainment into a pool of immiscible liquid [19] as it impacts the surface of the pool at a large Weber number ([35], Sec. VII).

A standard sugar cube (made of compacted dried saccharose crystals [39]) shattered into pieces when crushed on a rigid floor by a heavy mass falling by its own weight offers an ideal illustration of a 3D fragmentation protocol ([35], Sec. VIII). As Fig. 2 shows, fragments issuing from a single cube are more numerous and smaller

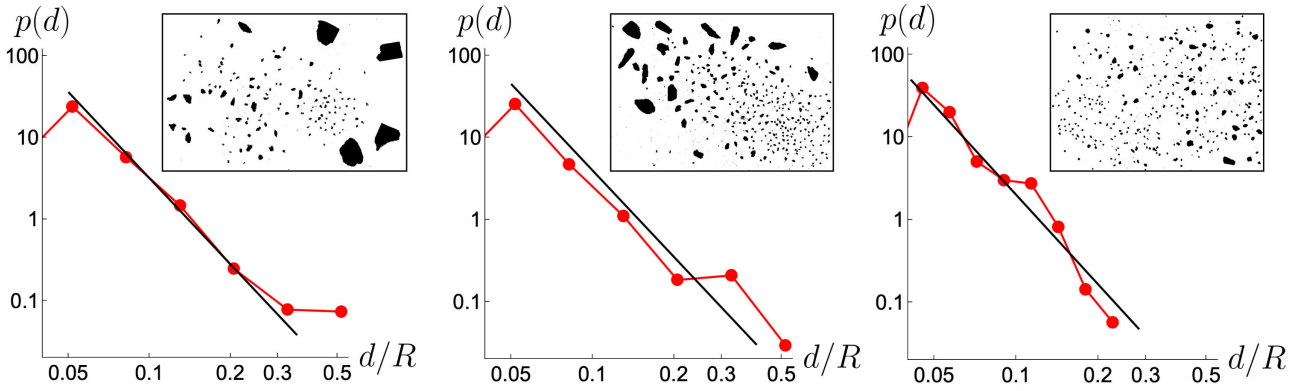

FIG. 2. Fragment size distribution $p(d)$ from the shattering of a single sugar cube of size $R = 2.5$ cm lying on a rigid floor, crushed by a heavy mass ($m = 0.4$ kg), falling by its own weight from a height h. The image width in the inserts is 15 cm. Left, $h = 10$ cm; center, $h = 50$ cm; Right, $h = 150$ cm. In each case, all fragments larger than $0.04R = 1$ mm are collected, and deposited on a dark support for analysis. The fragment sizes are measured from their projected area A (the black spots in the inserts) as $d = \sqrt{4A/\pi}$. Whatever h, we have $p(d) \sim d^{-\beta}$ with $\beta = 3.5$ (black lines).

for a larger impact velocity, but their size distribution is always well fitted by a power law with $\beta \approx 3.5$ (in any case better than a Log normal fit; see the Supplemental Material, [35], Secs. VIII and IX). The interest of this experiment lies not so much in the fact that it confirms known things (Fig. 1), but because it invites one to consider another principle, the one of energy conservation: the energy of the impactor is finite. The application of this principle is delicate in fragmentation since we know that equating the available mechanical energy with the amount of created surface energy may lead to notorious mistakes, of which we have several examples with liquids (see Sec. 11 in Ref. [40]). The reason is that there are many forms of energy (kinetic, sound, light, heat, etc.), and many sources of energy dissipation in fragmentation, friction (by viscosity, or plastic deformations) being a notable one. Nothing guaranties in general that the energy input will be exclusively converted into surface creation, although most of it actually does so in brittle materials. In this restricted limit, the famous and helpful [29,41] closure proposed by Griffith [42] is legitimate. A certitude however is that surface creation energy cannot be an infinite sink, a fact which constrains both the range of admissible fragment sizes d, and their distribution n. Let σ be the fracture energy per unit surface area (surface tension in liquids). The net energy consumed in surface formation, proportional to

$$\sigma \int_{d_{\min}}^{R} d^{D-1} n \mathrm{d}d \sim d_{\min}^{D-\beta} \tag{7}$$

must remain finite, implying, since $\beta > D$, that $d_{\min} \neq 0$. The minimal fragment size cannot be zero (within the continuum limit); this constitutes a prediction relevant to a long standing question [40]. As a consequence, the net number of fragments is finite, their size distribution $p(d) = n/\int_{d_{\min}}^{R} n \,\mathrm{d}d$ exists, and $\int_{d_{\min}}^{R} d\, n \,\mathrm{d}d$ is finite as well. Therefore, maximizing S in Eq. (1) with this additional constraint provides ([35], Sec. X)

$$n = \frac{1}{R}\left(\frac{d}{R}\right)^{-\beta} e^{-\frac{d}{R}}, \tag{8}$$

exhibiting the exponential falloff characteristic of empirical distributions [2,9,10,14,25]. Table I gives the values of β using Eqs. (5) and (8).

In the experiments of Fig. 2, the impactor energy (mass m and release height h) is $w = mgh$ with $g = 9.81\ \mathrm{m\,s^{-2}}$. Sugar cubes are fairly stiff and brittle objects (from the data in [39], we estimate the Young modulus to be $E \approx 4 \times 10^8$ Pa, the critical stress $\tau_c \approx 2300$ kPa, and the fracture energy $\sigma = \frac{3}{4}\tau_c^2 R/E \approx 200\ \mathrm{J\,m^{-2}}$). Assuming the brittle limit to be valid, we equate w with the net surface energy creation $\sigma \int_{d_{\min}}^{R} \pi d^2 n \,\mathrm{d}d$ and find

$$\frac{d_{\min}}{R} \approx \left(\frac{\beta - 3}{\pi}\frac{w}{\sigma R^2}\right)^{-\frac{1}{\beta-3}}, \tag{9}$$

suggesting that for $\beta = 3.5$, $R \approx 2$ cm and $h = 0.5$ m, $d_{\min}/R \approx 0.06$, an order of magnitude compatible with the one found in Fig. 2. The ratio $w/\sigma R^2$ in Eq. (9) would be, in liquids, called the *Weber number* We (in [19], $m \sim \rho R^3$ with ρ the liquid density, $gh = \frac{1}{2}v^2$, and $\mathrm{We} = \rho v^2 R/\sigma$). For instance, a droplet of size R in translation at velocity v in a gas stream with density ρ_a

TABLE I. The exponent β of the fragment number partition in Eq. (8), computed from Eq. (5); see also Fig. 1.

Dimension	1D	2D	3D
β	1.3	2.4	3.5

destabilizes and fragments when the Weber number $\rho_a v^2 R/\sigma \gtrsim 6$, although we know that this process involves few relatively simple, well-defined mechanisms (drop flattening into a pancake, then bag-formation and rim-ligament breakup [43]).

But precisely, are general principles alone enough to describe fragmentation in all cases, independently of any specific mechanism? Certainly not, as the example above suggests, and there are several others: The breakup of a smooth liquid thread produces droplets with uniform size proportional to the jet diameter via a deterministic capillary instability [40]; even violently expanded liquid shells bursting by the random nucleation of holes followed by ligament breakup are too regular to give rise to a power law distribution as in Eq. (8), but are instead well described by a bell-shaped distribution representative of a noisy capillary instability [20]. In that situation, the microscopic rearrangements within the ligaments constrain deterministically the variance of the fragments' diameters to be proportional to their mean squared, i.e., $(\langle d^2\rangle - \langle d\rangle^2)/\langle d\rangle^2 = 1/\nu$, a feature found similarly in the radial expansion of a necklace of magnetic beads studied in [12]. The fragments are then gamma distributed with order ν:

$$n \sim \frac{1}{\langle d\rangle}\left(\frac{d}{\langle d\rangle}\right)^{\nu-1} e^{-\nu\frac{d}{\langle d\rangle}}, \tag{10}$$

a distribution also interpretable in terms of maximal entropy, but now with β no more given by the space dimension D as in Eq. (8), but by $\beta = 1-\nu < 0$ (since $\nu \gtrsim 4$, typically [12,20]). Incidentally, the minimum fragment size $d_{\min}$ can actually be zero in that case ($\sigma \int_0^R d^{D-1} n\, \mathrm{d}d$ is finite for $\nu > 1-D$), the small sizes being however themselves in vanishingly small proportion ($\sim d^{\nu-1}$ as $d \to 0$). Randomness (which sets the form of the distribution), and microscopic mechanisms (setting the distribution parameters $\langle d\rangle$ and ν) cooperate in the overall statistics n.

Similarly in solids, patterns and fragments are well understood from deterministic mechanisms in some simple geometries both in quasistatic [29,44], or dynamic fracture [5,30,45]. However, even in situations where randomness prevails, microscopic interactions between the fragments may influence the distribution shape: the observations in [18] which would normally fall into the category $\beta \approx 3.5$ if the material were brittle, display a markedly different exponent $\beta_\star \approx 1.6$ with plastic materials, where cracks have now the possibility to "heal," hindering fragment separation. The relaxation time $\tau \sim \eta/E$ in a viscoelastic material (where η is a shear viscosity, see Ch. 36 in Ref. [46]) defines a persistence length $d_\star \approx c\tau$ below which cracks heal, preventing the formation of smaller fragments (see Sec. 3.2 in Ref. [47]). One may qualitatively anticipate that only a fraction $1-e^{-d/d_\star}$ of bonds linking fragments of size d have actually succeeded in separating, thus depleting n from its small sizes, and conversely favoring larger ones, hence a shallower d dependence. The fragments interact with their neighbors along $D-1$ degrees of freedom (i.e., their surface in 3D), and we thus anticipate that

$$n \sim d^{-\beta}(1-e^{-d/d_\star})^{D-1} \underset{d\to 0}{\longrightarrow} d^{-\beta+D-1}. \tag{11}$$

The reasoning above suggests that $\beta_\star \approx 3.5\text{–}2 \approx 1.5$, qualitatively consistent with the findings in [18].

To conclude, we have shown that for brittle materials or fragmentation in turbulent flows, a kinematic constraint applied to a maximal randomness principle infers both the power law shape of the fragment size distribution and the value of its dimensionality-dependent exponent. Considering several counter examples, we have also delimitated the domain of validity of this description which assumes "molecular chaos" in the sense of Boltzmann, and holds in the absence of microscopic constraints between the fragments, although these can be accommodated—subject to the appropriate mechanism—to the description in some cases. The present contribution also intends to make more precise the area where maximum entropy approaches can be useful, or not.

Acknowledgments—I thank the Ecole Doctorale 353 of Aix-Marseille University for offering me to teach a series of lectures on fragmentation, which were the opportunity to refine the thinking on the experiments presented here, made during the summer of 2005.

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

[1] J. M. Coulson, J. F. Richardson, J. R. Backhurst, and J. H. Harker, *Chemical Engineering*, 5th ed. (Butterworth-Heinemann, Oxford, 2002), Vol. 2.
[2] C. C. Lienau, J. Franklin Inst. **221**, 485 (1936).
[3] T. Ishii and M. Matsushita, J. Phys. Soc. Jpn. **61**, 3474 (1992).
[4] E. S. C. Ching, S. L. Lui, and K. Q. Xia, Physica (Amsterdam) **287A**, 83 (2000).
[5] J. R. Gladden, N. Z. Handzy, A. Belmonte, and E. Villermaux, Phys. Rev. Lett. **94**, 035503 (2005).
[6] T. Kadono, Phys. Rev. Lett. **78**, 1444 (1997).
[7] M. Poulain-Zaros, N. Pujara, G. Verhille, and M. J. Mercier, C.R. Phys. **25**, 1 (2024).
[8] N. F. Mott, Proc. R. Soc. A **189**, 300 (1947).
[9] F. Wittel, F. Kun, H. J. Hermann, and B. H. Kröplin, Phys. Rev. Lett. **93**, 035504 (2004).
[10] H. Katsugari, S. Ihara, and H. Honjo, Phys. Rev. Lett. **95**, 095503 (2005).
[11] D. E. Grady, *Fragmentation of Rings and Shells* (Springer-Verlag, Berlin, 2006).
[12] A. Vledouts, N. Vandenberghe, and E. Villermaux, Proc. R. Soc. A **471**, 20150678 (2015).

[13] A. Vledouts, N. Vandenberghe, and E. Villermaux, Proc. R. Soc. A **472**, 20150679 (2016).
[14] J. J. Gilvarry and B. H. Bergstrom, J. Appl. Phys. **32**, 400 (1961).
[15] A. Meiborn and I. Balslev, Phys. Rev. Lett. **76**, 2492 (1996).
[16] S. Z. Wu, K. T. Chau, and T. X. Yu, Powder Technol. **143–144**, 41 (2004).
[17] K. Schönert, Powder Technol. **143–144**, 2 (2004).
[18] G. Timar, J. Blömer, F. Kun, and H. J. Herrmann, Phys. Rev. Lett. **104**, 095502 (2010).
[19] H. Lhuissier, C. Sun, A. Prosperetti, and D. Lohse, Phys. Rev. Lett. **110**, 264503 (2013).
[20] A. Vledouts, J. Quinard, N. Vandenberghe, and E. Villermaux, J. Fluid Mech. **788**, 246 (2016).
[21] C. Garrett, M. Li, and D. Farmer, J. Phys. Oceanogr. **30**, 2163 (2000).
[22] C. E. Blenkinsopp and J. R. Chaplin, IEEE J. Oceanic Eng. **35**, 388 (2010).
[23] E. Villermaux, X. Wang, and L. Deike, PNAS Nexus **1**, 1 (2022).
[24] J. Mercader, M. Panger, and C. Boesch, Science **296**, 1452 (2002).
[25] L. Oddershede, P. Dimon, and J. Bohr, Phys. Rev. Lett. **71**, 3107 (1993).
[26] J. A. Aström, F. Ouchterlony, R. P. Linna, and J. Timonen, Phys. Rev. Lett. **92**, 245506 (2004).
[27] C. Cercignani, *Ludwig Boltzmann, The Man Who Trusted Atoms* (Oxford University Press, New York, 1998).
[28] E. Sharon, S. P. Gross, and J. Fineberg, Phys. Rev. Lett. **74**, 5096 (1995).
[29] N. Vandenberghe and E. Villermaux, Soft Matter **9**, 8162 (2013).
[30] S. Moulinet and M. Adda-Bedia, Phys. Rev. Lett. **115**, 184301 (2015).
[31] M. Born, *Atomic Physics* (Dover, New York, 1969).
[32] B. Castaing, J. Phys. II France **6**, 105 (1996).
[33] H. Tao, T. A. Zaki, and C. Meneveau, J. Fluid Mech. **973**, R6 (2023).
[34] R. Englman, J. Phys. Condens. Matter **3**, 1019 (1991).
[35] See Supplemental Material at http://link.aps.org/supplemental/10.1103/r7xz-5d9c for an explicit presentation of the status of the density $\psi(\mathbf{x}, t)$, details about notations and change of variables in the definition of distributions, the original data together with the fits proposed in the main text in a range of different situations, details about the experimental setup and procedures, a discussion about the status of the Lognormal distribution and the maximal entropy calculation with average size constrained.
[36] F. Kun, F. K. Wittel, H. J. Herrmann, B. H. Kroplin, and K. J. Maloy, Phys. Rev. Lett. **96**, 025504 (2006).
[37] G. Domokos, D. J. Jerolmack, F. Kun, and J. Torok, Proc. Natl. Acad. Sci. U.S.A. **117**, 18178 (2020).
[38] Y. Li, W. Hu, R. Huang, C.-S. Chang, and M. McSaveney, JGR Earth Surf. **129**, e2024JF007844 (2024).
[39] M. Tapias, E. E. Alonso, and J. Gili, Soils Found. **56**, 805 (2016).
[40] E. Villermaux, J. Fluid Mech. **898**, P1 (2020).
[41] B. Bourdin, G. Francfort, and J. Marigo, J. Elast. **91**, 5 (2008).
[42] A. A. Griffith, Phil. Trans. R. Soc. A **221**, 163 (1921).
[43] E. Villermaux and B. Bossa, Nat. Phys. **5**, 697 (2009).
[44] B. Roman, Int. J. Fract. **182**, 209 (2013).
[45] B. Audoly and S. Neukirch, Phys. Rev. Lett. **95**, 095505 (2005).
[46] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, Course of theoretical physics (Pergamon Press, Oxford, 1986).
[47] E. Villermaux, Int. J. Fract. **206**, 171 (2017).