Face Recognition with Algorithms in Digital Singal Processing

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Abstract

Recognition of human face is a technology growing explodingly in recent years. This technology relies on algorithms to process and classify digital signals from images or videos. These project helps understanding the ideas and architecture of fundamental algorithms. Eigenface, Fisherface, k-nearest neighbors, support vector machine, and sparse representation classification were implemented on YaleB 32x32 dataset with the optimization of vector selection, whitening, and the size of training data. Dropping top 5 eigenvectors significantly improved the accuracy of eigenface with k-nearest neighbors from 42%to 74%, but not for others. Fisherface shows overall higher accuracies than eigenface, and fisherface with sparse representation classification is the optimum combination with an accuracy at 97%.

1 Introduction

Face recognition starts from the most intuitive way based on the geometric features of a face. This approach is limited by the complicate registration of the marker points, even with state of the art algorithms. Comparatively, another family of algorithms treats images and faces as vectors to classify in a n-domentional space, which is described and introduced in the following paragraphs.

2 Methods

2.1 Eigenfaces

The Eigenfaces algorithm[1] transforms high-dimensional image space to a lower-dimensional representation subspace by Principal Component Analysis (PCA), which identifies the axes with maximum variance.
Let $X = \{x_1, x_2, \ldots, x_n\}$ be a random vector with observations $x_i \in R^d$.
Compute the mean μ

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{1}$$

Compute the the Covariance Matrix S

$$S = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)(x_i - \mu)^T \tag{2}$$

Compute the eigenvalues λ_i and eigenvectors v_i of S

$$Sv_i = \lambda_i v_i, i = 1, 2, \ldots, n \tag{3}$$

Order the eigenvectors descending by their eigenvalue. The k principal components are the eigenvectors corresponding to the k largest eigenvalues.
The k principal components of the observed vector x are then given by:

$$y = W^T(x - \mu) \tag{4}$$

where $W = (v_1, v_2, \ldots, v_k)$. The reconstruction from the PCA basis is given by:

$$x = Wy + \mu \tag{5}$$

The Eigenfaces method then performs face recognition by:
Projecting all training samples into the PCA subspace.
Projecting the query image into the PCA subspace.

Finding the nearest neighbor between the projected training images and the projected query image.

To solve the covariance matrix $S = XX^T$, a transformation is performed to take the eigenvalue decomposition $S = X^T X$ of size NXN instead:

$$X^T X v_i = \lambda_i vi \tag{6}$$

and get the original eigenvectors of $S = XX^T$ with a left multiplication of the data matrix:

$$XX^T(Xv_i) = \lambda_i(Xv_i) \tag{7}$$

The resulting eigenvectors are orthogonal, to get orthonormal eigenvectors they need to be normalized to unit length for further classification[2].

2.2 Fisherface

While the Eigenface using PCA transformation is optimal from a reconstruction standpoint, it doesn't take any class labels into account. Therefore a class-specific projection with a Linear Discriminant Analysis (LDA) was applied to face recognition in [3][4]. The basic idea is to minimize the variance within a class, while maximizing the variance between the classes at the same time. In order to find the combination of features that separates best between classes the LDA maximizes the ratio of between-classes to within-classes scatter. The idea is simple that same classes should cluster tightly together, while different classes are as far away as possible from each other. The algorithm is shown as follow:

Let X be a random vector with samples drawn from c classes:

$$\begin{aligned} X &= \{X_1, X_2, \ldots, X_c\} & (8)\\ X_i &= \{x_1, x_2, \ldots, x_n\} & (9) \end{aligned}$$

The scatter matrices S_B and S_W are calculated as:

$$\begin{aligned} S_B &= \sum_{i=1}^{c} N_i(\mu_i - \mu)(\mu_i - \mu)^T & (10)\\ S_W &= \sum_{i=1}^{c} \sum_{x_j \in X_i} (x_j - \mu_i)(x_j - \mu_i)^T & (11) \end{aligned}$$

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{12}$$

$$\mu_i = \frac{1}{|X_i|} \sum_{x_j \in X_i} x_j \tag{13}$$

Fisher's classic algorithm now looks for a projection W, that maximizes the class separability criterion:

$$W_{opt} = arg\,max_W \frac{|W^T S_B W|}{|W^T S_W W|} \tag{14}$$

Following [3], a solution for this optimization problem is given by solving the General Eigenvalue Problem:

$$\begin{aligned} S_B v_i &= \lambda_i S_w v_i \\ S_W^{-1} S_B v_i &= \lambda_i v_i & (15) \end{aligned}$$

The rank of S_W is at most $(N - c)$, with N samples and c classes. Similar to Eigenface, the reduction in dimention was solved [3] by performing a PCA on the data and projecting the samples into the $(N - c)$-dimensional space.
The optimization problem can be rewritten as:

$$\begin{aligned} W_{pca} &= arg\,max_W |W^T S_T W| & (16)\\ W_{fld} &= arg\,max_W \frac{|W^T W_{pca}^T S_B W_{pca} W|}{|W^T W_{pca}^T S_W W_{pca} W|} & (17) \end{aligned}$$

The transformation matrix W, that projects a sample into the $(c-1)$-dimensional space is then given by:

$$W = W_{fld}^T W_{pca}^T \tag{18}$$

2.3 Support Vector Machine

Support Vector Machines (SVM) were first introduced by Vapnik and Chervonenkis[5]. The core idea is to find the optimal hyperplane to seperate different classes. While there are theoretically infinite hyperplanes to seperate the dataset, an optimum hyperplane is chosen so that the distance to the nearest datapoint of both classes is maximized. The points spanning the hyperplane are the *Support Vectors*[6]. Given a Set of Datapoints $\mathcal{D}$:

$$\mathcal{D} = \{(x_i, y_i) | x_i \in R^p, y_i \in \{-1, 1\}\}_{i=1}^n$$

where x_i is a point in p-dimensional vector, and y_i is the corresponding class label
We search for $\omega \in R^n$ and bias b, forming the Hyperplane H:

$$\omega^T x + b = 0$$

that seperates both classes so that:

$$\omega^T x + b = 1, \text{if } y = 1$$
$$\omega^T x + b = -1, \text{if } y = -1$$

The optimization problem that needs to be solved is:

$$\min \frac{1}{2}\omega^T \omega$$

$$\omega^T x + b \geq 1, y = 1$$
$$\omega^T x + b \leq 1, y = -1$$

The kernel trick is used for classifying non-linear datasets. It works by transforming data points into a higher dimensional feature space with a *kernel function*, where the dataset can be seperated again. Commonly used kernel functions are *RBF kernels*:

$$k(x, x') = exp(\frac{\|x - x'\|^2}{\sigma^2})$$

or *polynomial kernels*:

$$k(x, x') = (x \cdot x')^d$$

.

2.4 Sparse Representation Classification

Sparse Representation-based Classification(SRC) is an algorithm represent query images with all the samples in the training set, proposed by Wright[7]. This is equal to solve the linear system of equation y = Ax, where y are query images and A is the matrix columns of training samples. Since linear system of equations is usually underdetermined, the target of SRC is to find sparsest solution by L1-minimization instead of unique solution.

$$\arg\min_{\infty} = \arg\min \|w\|_1 \tag{19}$$

For classification task, the image was reconstructed corresponding to each class of train samples to compute the residual of it between the real query image. Its intuitive idea is to reconstruct with the smallest residual, which is most likely to be the true class of query image. The process can be expressed as:

$$\min_i r_i(y) = \|y - A\gamma_i(\hat{x})\|_2 \tag{20}$$

where γ_i is a function that selects the coefficients within the ith class.

3 Experiment and Discussion

The experiment is performed on a dataset from Yale database which includes around 64 near frontal images under different illuminations of each of 38 individual. In this study, certain number of images from each individual p = (10, 20, 30, 40, 50) was used for training and the rest for testing. Multiple python libraries were used in the experiment including LDA, kNN and SVM from sklearn and SRC from skmultilearn.

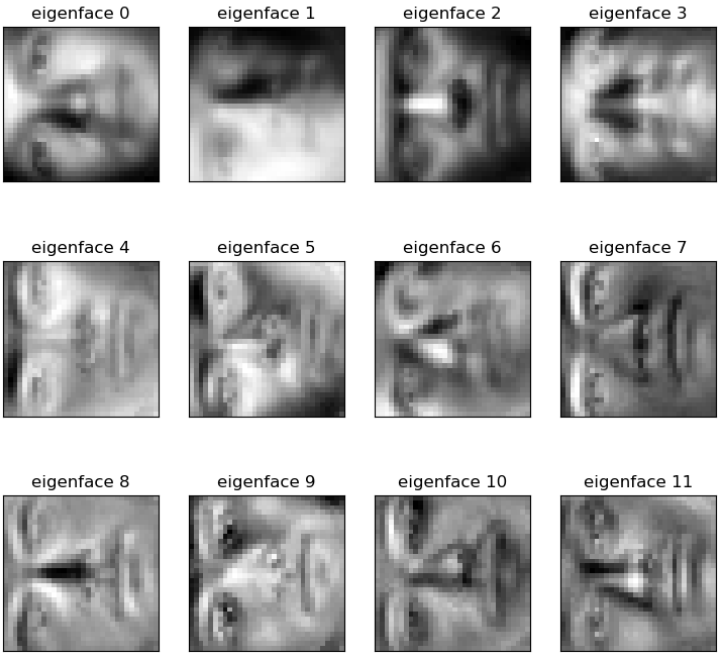

Figure 1: Eigenfaces obtained from YaleB dataset

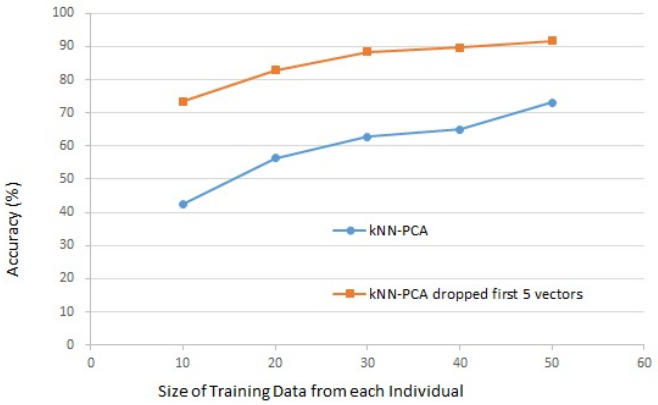

Figure 2: Effect of dropping top eigenvectors

Figure 1 shows eigenfaces obtained from YaleB dataset. Top eigenfaces is related to the illuminations, and may affect the accuracy of face recognition. The accuracy of eigenfaces with and without dropping top eigenfaces were tested and shown in Figure 2. The accuracy after dropping increases by $\sim$ 20% with kNN as the classifier. This result supports the assumption of illumination. The accuracy increases with the size of training from 73% to 92%.

Anothor key paramter of eigenface is the number of vectors. The number of 100 and 150 were tested and shown in Figure 3. Higher accuracy (90% vs 85% at p=50) is observed at number of vectors is 150, as an effect of classification at higher dimension. This number of 150 is used in the following study.

PCA and LDA were implemented with classifiers of kNN, SVM, and SRC in Figure 4.Unlike PCA, dropping top eigenfaces (eigenvectors) does not affect LDA. Whitening was required in SVM and applied

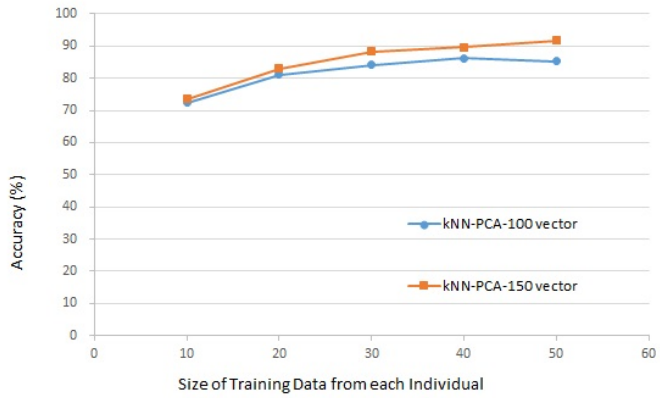

Figure 3: Effect of number of eigenvectors

in all tests in Figure 4. LDA shows overall higher accuracy than PCA with each classifier, due to consider images grouply instead of individually as a class. SVM gives higher accuray than kNN, and SRC gives higher accuracy than SMV generally for both PCA and LDA. The highest accuracy was obtained at LDA with SRC.

Figure 4: PCA and LDA test using kNN, SVM, and SRC as classifier

4 Conclusions

This study implemented Eigenface (PCA) and fisherface (LDA) with classifiers of kNN, SVM, and SRC on YaleB dataset. The accuracy was improved by dropping top eigenvectors of PCA with kNN and increasing size of training data. In generall, LDA shows higher accuracy than PCA, and SRC shows higher accuracy than kNN and SVM. The highest accuracy at 97% was obtained using LDA and SRC.

References

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