Q1 (a) we have priors $P(C_1)$ and $P(C_2)$. we also have $p_1 \equiv p(x=0|C_1)$
$p_2 = p(x=0|C_2)$

we know that

$$P(C_i|x) = \frac{P(C_i)\,P(C_i|x)}{P(x)}$$

since $P(x)$ is constant for any one value of x.
and since we classify by comparing values.

$$P(C_i|x) = P(C_i)\,P(C_i|x)$$

$$P(C_i|x) = P(C_1|x=0) + P(C_1|x=1)$$

$$P(C_1|x=0) = P(x=0|C_1)\,P(C_1)$$
$$= p_1 P(C_1).$$

$$P(C_1|x=1) \quad P(x=1|C_1)\,P(C_1)$$
$$= (1-p_1)\,P(C_1)$$ $[P(x=1|C_1) = 1 - P(x=0|C_1)]$

$$P(C_2|x=0) = P(x=0|C_2)\,P(C_2)$$
$$= p_2\,P(C_2)$$

$$P(C_2|x=1) = P(x=1|C_2)\,P(C_2)$$
$$= (1-p_2)\,P(C_2)$$ [similar to above case]

~~P(C₁)~~ For $x=0$.
if $P(C_1|x=0) > P(C_2|x=0)$ then classify x as class 1
otherwise x is classified as class 2

For $x=1$
if $P(C_1|x=1) > P(C_2|x=1)$ then classify x as class 1
otherwise classify as class 2.

Q1(h) $\therefore$ $p_{ij} = P(x_j = 0 | C_i)$

$$P(\underset{\wedge}{\overset{=0}{x_1}}, \underset{\wedge}{\overset{=0}{x_2}} \dots \underset{\wedge}{\overset{=0}{x_D}} | C_i) = P(x_1|C_i) \cdot P(x_2|C_i) \dots P(x_D|C_i)$$

$$= \cancel{p_{ij}}\ p_{i1} \cdot p_{i2} \cdot p_{i3} \dots p_{id}.$$

$$= \prod_{j=1}^{D} p_{ij}$$

$$P(x_1 = 1, x_2 = 1 \dots x_D = 1 | C_i) = \prod_{j=1}^{D} (1 - p_{ij})$$

$P(x_1 \wedge x_2 \dots \wedge x_D.$

~~some~~

To generalize, some x may be 0 and some may 1. so we need to use the Bernoulli distribution function $p_x^{x} (1 - p_x)^{1-x}$

This becomes $\cancel{(1-p_x)^x\ p_{1-x}}$ (

$\cancel{p^x}$ $p_x^{(1-x)} \cdot (1 - p_x)^{x}$

$$p(x_1, x_2 \dots x_D) = \prod_{j=1}^{D} p_{ij}^{(1-x_j)} (1 - p_{ij})^{x_j}$$

$$P(C_i | x_1 x_2 \dots x_D) = P(x_1, x_2 \dots x_D | C_i) P(C_i)$$

$$= \prod_{j=1}^{D} p_{ij}^{(1-x_j)} (1 - p_{ij})^{x_j} \cdot P(C_i)$$

$$P(C_2 | x_1, x_2, \dots x_D) = \prod_{j=1}^{D} p_{ij}^{(1-x_j)} (1 - p_{ij})^{x_j} P(C_2)$$

(c) $p_{11} = 0.6$ $p_{12} = 0.1$ $p_{21} = 0.6$ $p_{22} = 0.9$.

for $P(C_1) = 0.2$ and $P(C_2) = 1-0.2 = 0.8$.

$x = \{0 \ 0\}$.

$$P(x_1=0 \ x_2=0 | C_1) = \prod_{j=1}^{D} p_{ij}^{1-x_j} (1-p_{ij})^{x_j}$$

$$= p_{11}^{(1-0)} (1-p_{11})^{0} \cdot p_{12}^{(1-0)} (1-p_{12})^{0}$$

$$= 0.6 \cdot 0.1$$

$$= 0.06.$$

~~$P(x_1=0 \wedge x_2=0 | C_2) = \prod$~~

$$P(x_1=0 \ x_2=0 | C_2) = \prod_{j=1}^{D} p_{ij}^{1-x_j} (1-p_{ij})^{x_j}$$

$$= p_{21}^{(1-0)} \cdot p_{22}^{(1-0)}$$

$$= 0.6 \cdot 0.9$$

$$= 0.54.$$

$$P(C_1 | x_1 x_2) = P(C_1) \times P(x_1=0 \ x_2=0 | C_1)$$

$$= 0.2 \times 0.06$$

$$= 0.018$$

$$P(C_2 | x_1 x_2) = P(C_2) \ P(x_i=0 \ x_2=0 | C_2)$$

$$= 0.8 \times 0.54$$

$$= 0.432$$

since $P(C_2 | x_1 x_2) > P(C_1 | x_1 x_2)$,

~~$P(x_1=0 \ x_2=0$~~ $[0 \ 0]$ belongs to class 2.

$x = \{0 \ 1\}$.

$$P(x_1=0 \ x_2=1 | C_1) = p_{11}^{(1-0)} (1-p_{11})^{0} \cdot p_{12}^{(1-1)} (1-p_{12})^{1}$$

$$= 0.6 \cdot 0.9 = 0.54$$

$$P(x_1=0\ \ x_2=1 \mid C_2) = p_{21}^{1-x_1}(1-p_{21})^{x_1} \cdot p_{22}^{x_2}(1-p_{22})^{1-x_2}$$

$$= \cancel{0.6}\ \cancel{0.4}\ 0.6 \cdot 0.9 = 0.54.$$

$$P(C_1 \mid x_1=0\ \ x_2=1) = P(C_1) \cdot P(x_1=0\ x_2=1 \mid C_1)$$

$$= 0.2 \times 0.54.$$

$$= 0.108$$

$$P(C_2 \mid x_1=0\ \ x_2=1) = P(C_2)\ P(x_1=0\ x_2=1 \mid C_1)$$

$$= 0.8 \times 0.54$$

$$= 0.432$$

Since $P(C_2 \mid x) > P(C_1 \mid x)$ we classify $\{0\ \ 1\}$ as class 2.

$x = [1\ \ 0]$.

$$P(x_1=1\ \ x_2=0 \mid C_1) = p_{11}^{1-x_1}(1-p_{11})^{x_1} \cdot p_{12}^{1-x_2}(1-p_{12})^{x_2}$$

$$= 0.6^{0}(0.4)^{1} \cdot (0.1)^{1}$$

$$= \cancel{0.24}.\ 0.04.$$

$$P(x_1=1\ \ x_2=0 \mid C_2) = p_{21}^{1-x_1} \cdot (1-p_{21})^{x_1} \cdot p_{22}^{1-x_2}\ p_{22}^{x_2}$$

$$= 0.4 \cdot 0.9$$

$$= 0.36.$$

$$P(C_1 \mid x) = \overset{0.04.}{\cancel{0.24}} \times 0.2 = 0.\cancel{048}\ 008$$

$$P(C_2 \mid x) = 0.36 \times 0.8 = \cancel{0.072}.\ 0.288$$

Since $P(C_2 \mid x) > P(C_1 \mid x)$, $x = \{1\ 0\}$ is classified as C_2.

$x = [1 \ 1]$

$$P(x_1 = 1 \ x_2 = 1 | C_1) = p_{11}^{1-x_1} (1 - p_{11})^{x_1} \cdot p_{12}^{1-x_2} (1 - p_{12})^{x_2}$$

$$= (0.4)^1 \cdot (0.9)^1$$

$$= 0.36$$

$$P(x_1 = 1 \ x_2 = 1 | C_2) = p_{21}^{1-x_1} (1 - p_{21})^{x_1} \cdot p_{22}^{1-x_2} (1 - p_{22})^{x_2}$$

$$= 0.4 \times 0.1$$

$$= 0.04.$$

$$P(C_1 | x) = 0.36 \times 0.2 = 0.072$$

$$P(C_2 | x) = 0.04 \times 0.8 = 0.032$$

since $P(C_1|x) > P(C_2|x)$ $x = \{1 \ 1\}$ is classified as C_1.

For $P(C_1) = 0.6$ $P(C_2) = 0.4$

$x = [0 \quad 0]$

$$P(x | C_1) = 0.06.$$

$$P(x | C_2) = 0.54.$$

$$P(C_1 | x) = 0.06 \times 0.6 = 0.036$$

$$P(C_2 | x) = 0.54 \times 0.4 = 0.216$$

$x = [1 \quad 0]$

$P(x|C_1) = 0.04$

$P(x|C_2) = 0.36$

$P(C_1|x) = 0.04 \times 0.6 = 0.024$

$P(C_2|x) = 0.36 \times 0.4 = 0.144.$

Since $P(C_2|x) > P(C_1|x)$ we classify $x = [1 \quad 0]$ as C_2

$x = [1 \quad 1]$

$P(x|C_1) = 0.36$

$P(x|C_2) = 0.04$

$P(C_1|x) = 0.36 \times 0.6 = 0.216$

$P(C_2|x) = 0.04 \times 0.4 = 0.016$

Since $P(C_1|x) > P(C_2|x)$ we classify $x = [1 \quad 1]$ as C_1

Priors $P(C_1) = 0.8 \qquad P(C_2) = 0.2$

$x \; [0 \quad 0]$

$P(x|C_1) = 0.06$

$P(x|C_2) = 0.54.$

$P(C_1|x) = 0.06 \times 0.8 = 0.048$

$P(C_2|x) = 0.54 \times 0.2 = 0.108$

Since $P(C_2|x) > P(C_1|x)$ we classify $x = [0 \quad 0]$ as C_2

$x = [0 \quad 1]$

$P(x|C_1) = 0.54$

$P(x|C_2) = 0.54$

$P(C_1|x) = 0.54 \times 0.8 = 0.432$

$P(C_2|x) = 0.54 \times 0.2 = 0.108$

Since $P(C_1|x) > P(C_2|x)$ we classify $x = [0 \quad 1]$ as C_1.

$x = [1 \quad 0]$

$P(x|c_1) = 0.04$

$P(x|c_2) = 0.36.$

$P(c_1|x) = 0.04 \times 0.8 = \cancel{0.32}\ 0.032$

$P(c_2|x) = 0.36 \times 0.2 = 0.072$

since $P(c_2|x) > P(c_1|x)$ we classify $x = (1\ 0)$ as c_2.

$x = [1 \quad 1]$

$P(x|c_1) = 0.36$

$P(x|c_2) = 0.04$

$P(c_1|x) = 0.36 \times 0.8 = 0.288$

$P(c_2|x) = 0.04 \times 0.2 = 0.008$

since $P(c_2|x) > P(c_2|x)$ we classify $x = [1\ 1]$ as c_1.

summary of classifications

x_1	x_2	$P(c_1) = 0.2$	$P(c_1) = 0.6$	$P(c_1) = 0.8$
0	0	c_2	c_2	c_2
0	1	c_2	c_1	c_1
1	0	c_2	c_2	c_2
1	1	c_1	c_1	c_1

Q1d

```
    'No. Sigma   value'


ans =

    '1. -5       23.595506
      '


ans =

    '1. -4       20.224719
      '


ans =

    '1. -3       22.471910
      '


ans =

    '1. -2       21.348315
      '


ans =

    '1. -1       23.595506
      '


ans =

    '1. 0        28.089888
      '


ans =

    '1. 1        28.089888
      '
```

```
ans =

    '1. 2        32.584270
     '


ans =

    '1. 3        32.584270
     '


ans =

    '1. 4        32.584270
     '


ans =

    '1. 5        31.460674
     '


ans =

    'minimum sigma is -4'


ans =

    'The error rate for the test data with best sigma is 14.606742 percent '
```

Q2a

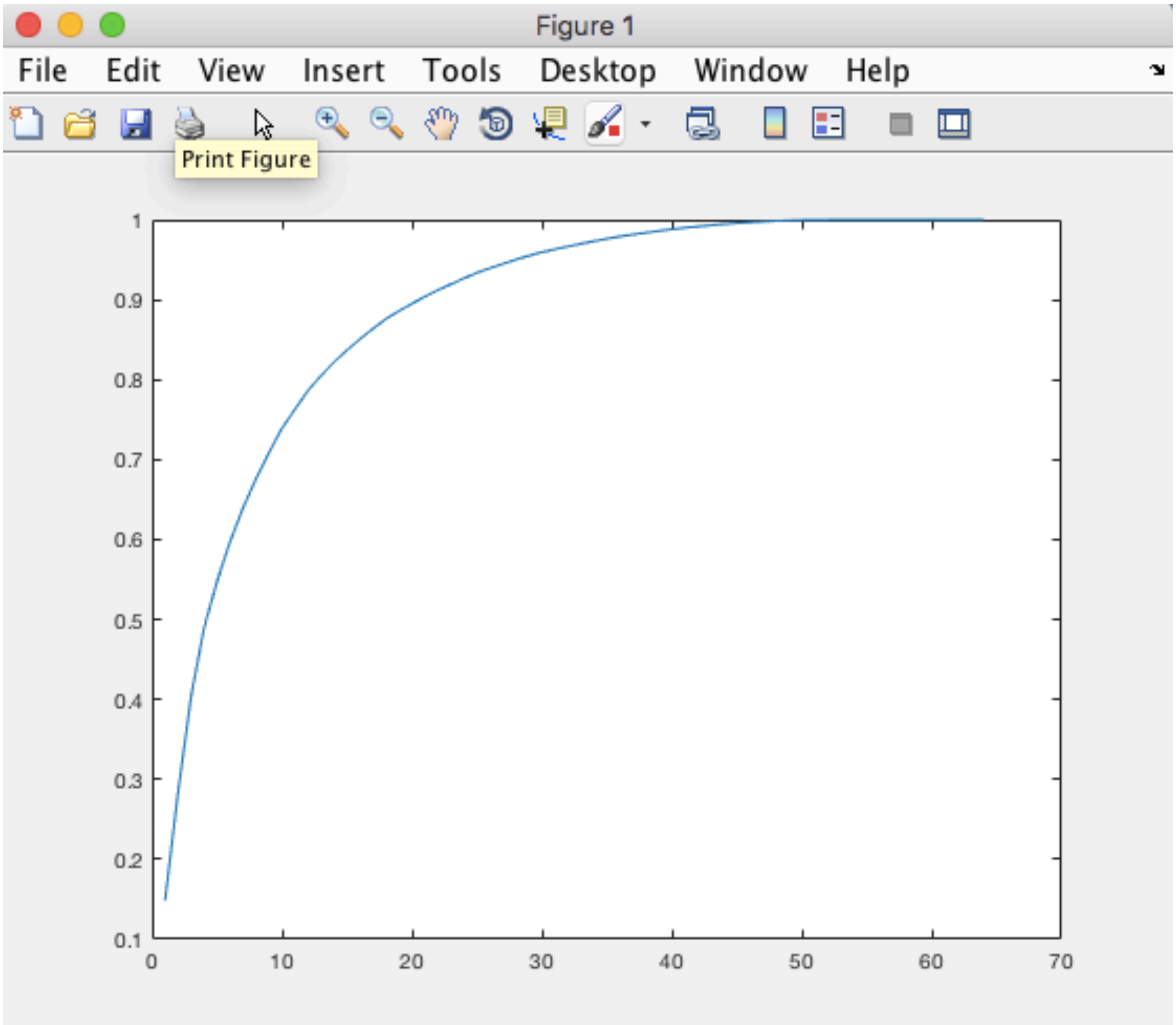
```
>> Q2a

ans =

    'Error rate for k=1 is 5.39 percent'


ans =

    'Error rate for k=3 is 4.04 percent'


ans =

    'Error rate for k=5 is 4.38 percent'


ans =

    'Error rate for k=7 is 5.39 percent'
```

.

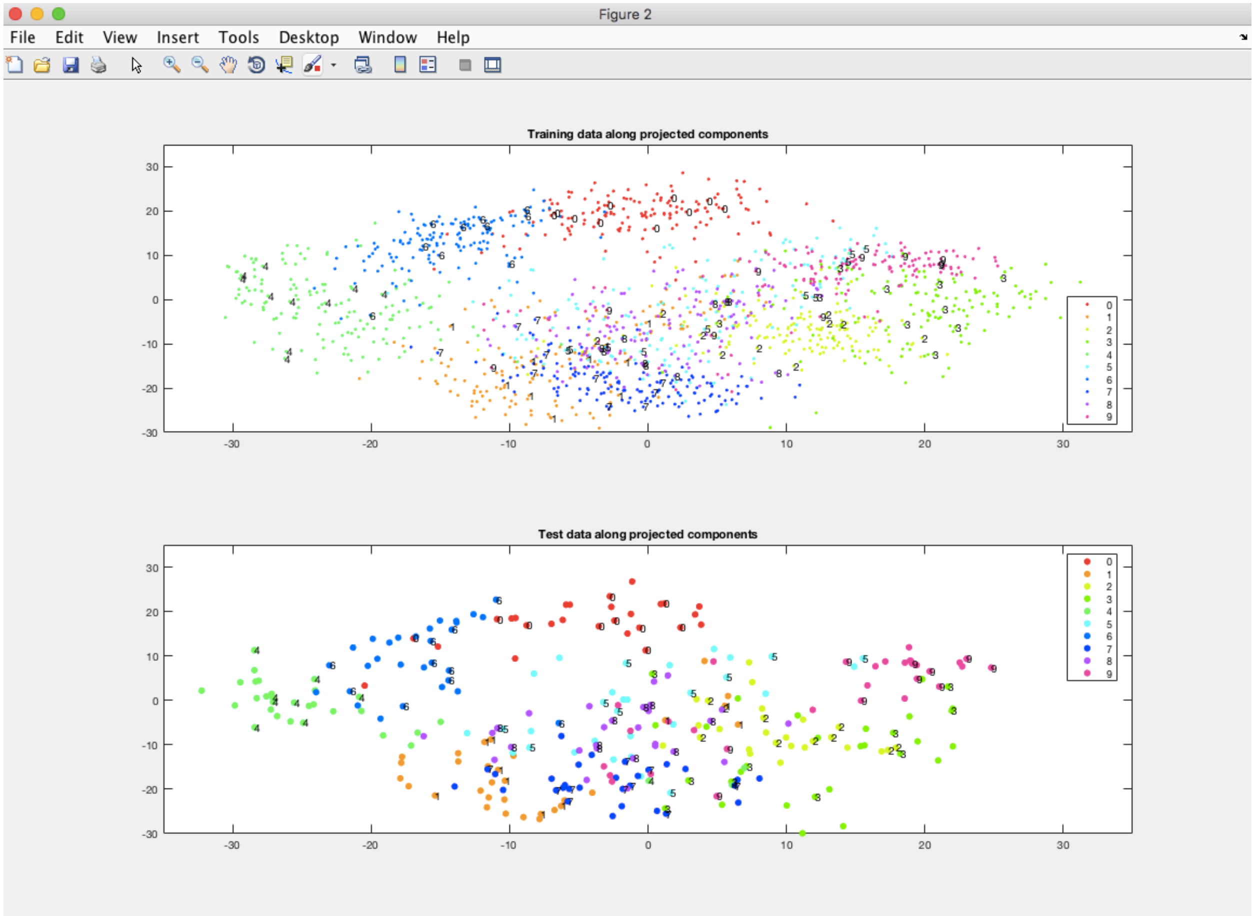
Q2b

```
>> Q2b

ans =

    'Error rate for k=1 with projected_data is 4.71 percent'


ans =

    'Error rate for k=3 with projected_data is 4.71 percent'


ans =

    'Error rate for k=5 with projected_data is 5.39 percent'


ans =

    'Error rate for k=7 with projected_data is 5.39 percent'
```

The plot of variance is given below, the principal number of components that explain 90% of varance is 21.



Q2c. The plot of the data on the first 2 prinicipal components is given below.



The plotted data of both test and train data is given.

2d

```
ans =

    'Error rate for L=2 and K=1 using LDA is 44.78 percent'


ans =

    'Error rate for L=2 and K=3 using LDA is 41.41 percent'


ans =

    'Error rate for L=2 and K=5 using LDA is 40.74 percent'


ans =

    'Error rate for L=4 and K=1 using LDA is 19.19 percent'


ans =

    'Error rate for L=4 and K=3 using LDA is 18.52 percent'


ans =

    'Error rate for L=4 and K=5 using LDA is 15.82 percent'


ans =

    'Error rate for L=9 and K=1 using LDA is 9.76 percent'


ans =

    'Error rate for L=9 and K=3 using LDA is 9.43 percent'


ans =

    'Error rate for L=9 and K=5 using LDA is 9.43 percent'
```

Q2e

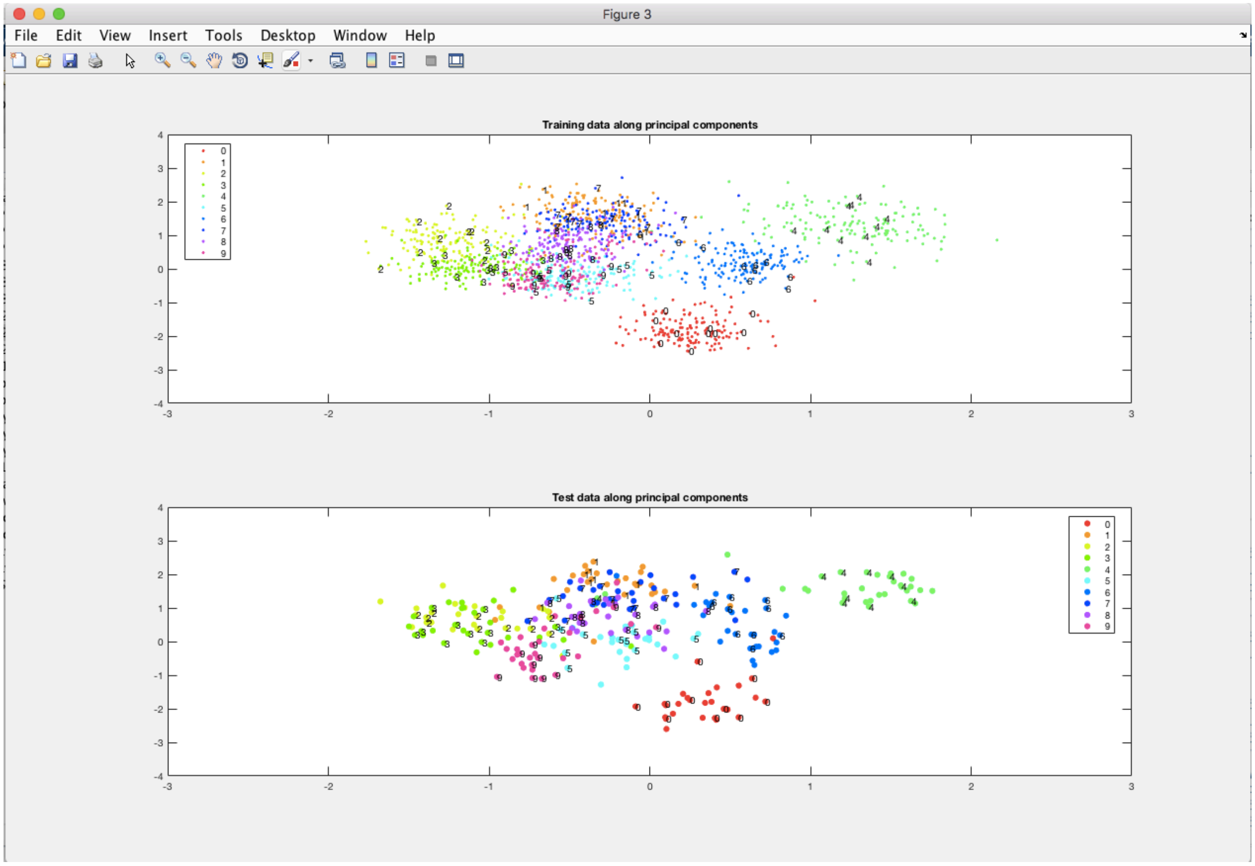
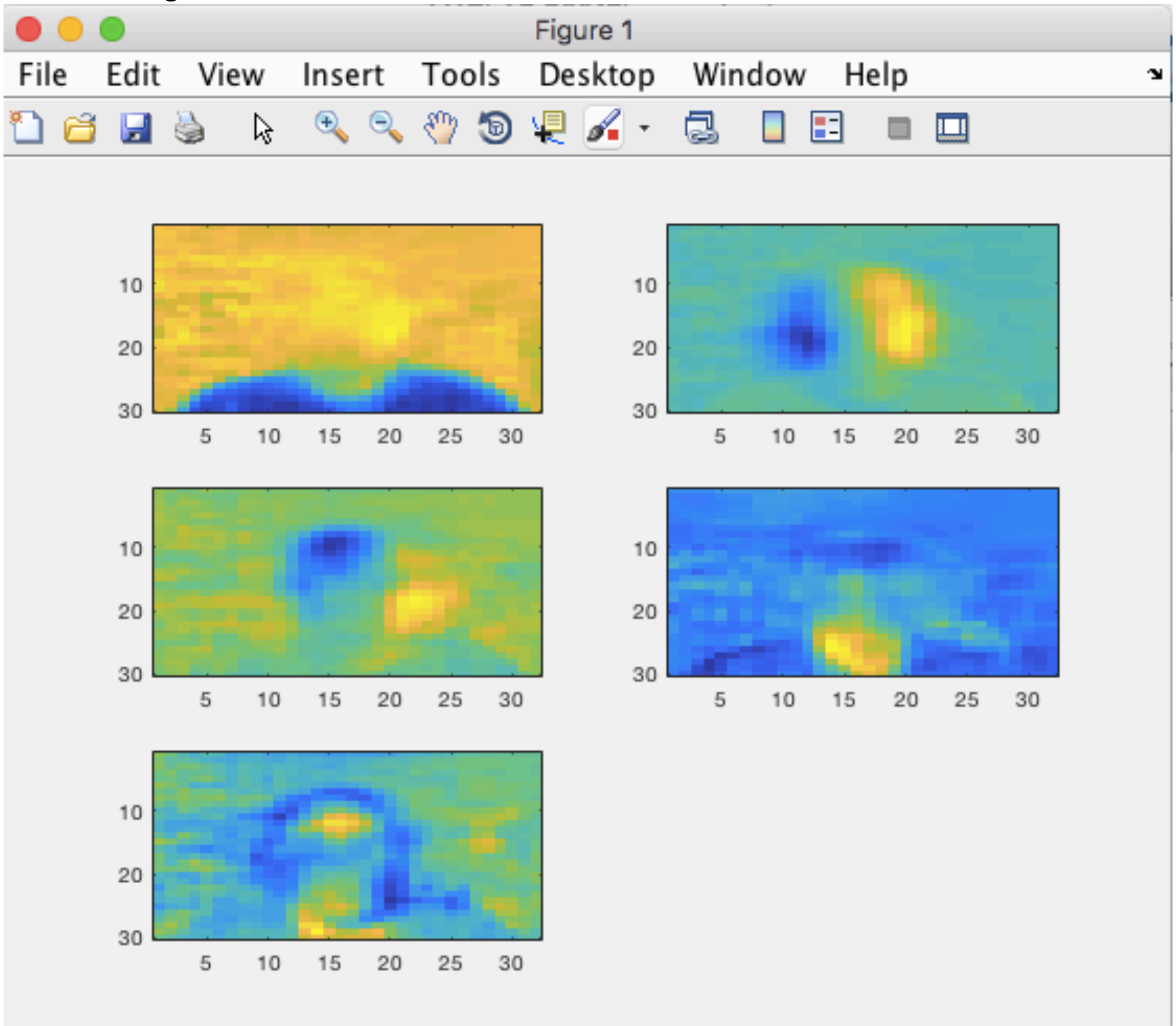


Figure 3
File Edit View Insert Tools Desktop Window Help
Training data along principal components
Test data along principal components

Q3a

The first 5 eigenfaces are:



Q3b. The k-values that explains 90% of variance and the error rates of using these principal components with knn is given below. The knn error peaks at k=1.

```
>> Q3b

ans =

    'The k value that explains 90 pct of the variance is 41'


ans =

    'Error rate for k=1 is 11.29'


ans =

    'Error rate for k=3 is 23.39'


ans =

    'Error rate for k=5 is 41.13'


ans =

    'Error rate for k=7 is 43.55'
```

Q3c. The first five faces from reconstruction are as follows. As we can see, when k=10, the images are highly pizelated, and pictures get clearer as we increase k to 50 and 100. This can be explained by the fact that when we use lesser k values and reconstruct, we use the mean of the observed features to estimate and the lesser features lead to poorer estimates.