

Q1 (a) we have priors $P(C_1)$ and $P(C_2)$. we also have $p_1 \equiv P(x=0|C_1)$
 $p_2 \equiv P(x=0|C_2)$

we know that

$$P(C_i|x) = \frac{P(C_i) P(C_i|x)}{P(x)}$$

Since $P(x)$ is constant for any one value of x .
 and since we classify by comparing values.

$$P(C_i|x) = P(C_i) P(C_i|x)$$

$$P(C_i|x) = P(C_i|x=0) + P(C_i|x=1)$$

$$P(C_1|x=0) = P(x=0|C_1) P(C_1) \\ = p_1 P(C_1)$$

$$P(C_1|x=1) = P(x=1|C_1) P(C_1) \\ = (1 - p_1) P(C_1)$$

$$[P(x=1|C_1) = 1 - P(x=0|C_1)]$$

$$P(C_2|x=0) = P(x=0|C_2) P(C_2) \\ = p_2 P(C_2)$$

$$P(C_2|x=1) = P(x=1|C_2) P(C_2) \\ = (1 - p_2) P(C_2)$$

[similar to above case]

For $x=0$.

if $P(C_1|x=0) > P(C_2|x=0)$ then classify x as class 1
 otherwise x is classified as class 2

For $x=1$

if $P(C_1|x=1) > P(C_2|x=1)$ then classify x as class 1
 otherwise classify as class 2.

Q11h)

$$p_{ij} = P(x_j = 0 | c_i)$$

$$P(\bar{x}_1^0, \bar{x}_2^0 \dots \bar{x}_D^0 | c_i) = P(x_1 | c_i) \cdot P(x_2 | c_i) \dots P(x_D | c_i)$$

$$= p_{i1} \cdot p_{i2} \cdot p_{i3} \dots p_{iD}$$

$$= \prod_{j=1}^D p_{ij}$$

$$P(x_1=1, x_2=1 \dots x_D=1 | c_i) = \prod_{j=1}^D (1 - p_{ij})$$

$$P(x_1, x_2, \dots, x_D)$$

where

To generalize, some x may be 0 and some may 1.

so we need to use the Bernoulli distribution

$$\text{function } p^x (1-p)^{1-x}$$

This becomes

$$\frac{(1-p)^x p^{1-x}}{p^x (1-p)^{1-x}} = \frac{(1-p)^x}{p^x (1-p)^{1-x}}$$

$$p(x_1, x_2, \dots, x_D) = \prod_{j=1}^D p_{ij}^{(1-x_j)} (1-p_{ij})^{x_j}$$

$$P(c_1 | x_1, x_2, \dots, x_D) = P(x_1, x_2, \dots, x_D | c_1) P(c_1)$$

$$= \prod_{j=1}^D p_{ij}^{(1-x_j)} (1-p_{ij})^{x_j} \cdot P(c_1)$$

$$P(c_2 | x_1, x_2, \dots, x_D) = \prod_{j=1}^D p_{ij}^{(1-x_j)} (1-p_{ij})^{x_j} P(c_2)$$

$$(c) \quad p_{11} = 0.6 \quad p_{12} = 0.1 \quad p_{21} = 0.6 \quad p_{22} = 0.9.$$

$$\text{For } P(C_1) = 0.2 \quad \text{and } P(C_2) = 1 - 0.2 = 0.8.$$

$$x = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

$$\begin{aligned} P(x_1=0, x_2=0 | C_1) &= \prod_{j=1}^D p_{ij}^{1-x_j} (1-p_{ij})^{x_j} \\ &= p_{11}^{(1-0)} (1-p_{11})^0 \cdot p_{12}^{(1-0)} (1-p_{12})^0 \\ &= 0.6 \cdot 0.1 \\ &= 0.06. \end{aligned}$$

$$\begin{aligned} P(x_1=0, x_2=0 | C_2) &= \prod_{j=1}^D p_{ij}^{1-x_j} (1-p_{ij})^{x_j} \\ &= p_{21}^{(1-0)} (1-p_{21})^0 \cdot p_{22}^{(1-0)} (1-p_{22})^0 \\ &= 0.6 \cdot 0.9 \\ &= 0.54. \end{aligned}$$

$$\begin{aligned} P(C_1 | x_1, x_2) &= P(C_1) \times P(x_1=0, x_2=0 | C_1) \\ &= 0.2 \times 0.06 \\ &= 0.012 \end{aligned}$$

$$\begin{aligned} P(C_2 | x_1, x_2) &= P(C_2) \times P(x_1=0, x_2=0 | C_2) \\ &= 0.8 \times 0.54 \\ &= 0.432 \end{aligned}$$

Since $P(C_2 | x_1, x_2) > P(C_1 | x_1, x_2)$,

~~$P(x_1=0, x_2=0)$~~ $[0 \ 0]$ belongs to class 2.

$$x = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

$$\begin{aligned} P(x_1=0, x_2=0 | C_1) &= p_{11}^{(1-0)} (1-p_{11})^0 \cdot p_{12}^{(1-0)} (1-p_{12})^0 \\ &= 0.6 \cdot 0.9 = 0.54 \end{aligned}$$

$$P(x_1=0, x_2=1|C_2) = p_{21}^{1-x_1} (1-p_{21})^{x_1} \cdot p_{22}^{1-x_2} (1-p_{22})^{x_2}$$

$$= 0.6 \cdot 0.4 \cdot 0.6 \cdot 0.9 = 0.54.$$

$$P(C_1|x_1=0, x_2=1) = P(C_1) \cdot P(x_1=0, x_2=1|C_1)$$

$$= 0.2 \times 0.54.$$

$$= 0.108$$

$$P(x_1=0, x_2=1|C_2) = P(C_2) \cdot P(x_1=0, x_2=1|C_2)$$

$$= 0.8 \times 0.54$$

$$= 0.432$$

Since $P(C_2|x) > P(C_1|x)$ we classify $\{0, 1\}$ as class 2.

$$x = [1, 0].$$

$$P(x_1=1, x_2=0|C_1) = p_{11}^{1-x_1} (1-p_{11})^{x_1} \cdot p_{12}^{1-x_2} (1-p_{12})^{x_2}$$

$$= 0.6 (0.4)^1 \cdot (0.8)^1$$

$$= 0.24 \cdot 0.8 = 0.192$$

$$P(x_1=1, x_2=0|C_2) = p_{21}^{1-x_1} (1-p_{21})^{x_1} \cdot p_{22}^{1-x_2} (1-p_{22})^{x_2}$$

$$= 0.4 \cdot 0.9$$

$$= 0.36$$

$$P(C_1|x) = 0.192 \times 0.2 = 0.0384$$

$$P(C_2|x) = 0.36 \times 0.8 = 0.288$$

Since $P(C_2|x) > P(C_1|x)$, $x = \{1, 0\}$ is classified as C_2 .

$$x = [1 \ 1]$$

$$\begin{aligned} P(x_1=1, x_2=1|C_1) &= p_{11}^{1-x_1} (1-p_{11})^{x_1} \cdot p_{12}^{1-x_2} (1-p_{12})^{x_2} \\ &= (0.4)^1 \cdot (0.9)^1 \\ &= 0.36 \end{aligned}$$

$$\begin{aligned} P(x_1=1, x_2=1|C_2) &= p_{21}^{1-x_1} (1-p_{21})^{x_1} \cdot p_{22}^{1-x_2} (1-p_{22})^{x_2} \\ &= 0.4 \times 0.1 \\ &= 0.04 \end{aligned}$$

$$P(C_1|x) = 0.36 \times 0.2 = 0.072$$

$$P(C_2|x) = 0.04 \times 0.8 = 0.032$$

Since $P(C_1|x) > P(C_2|x)$ $x = [1 \ 1]^T$ is classified as C_1 .

$$\text{For } P(C_1) = 0.6 \quad P(C_2) = 0.4$$

$$x = [0 \ 0]$$

$$P(x_1=0, x_2=0|C_1) = 0.06$$

$$P(x_1=0, x_2=0|C_2) = 0.54$$

$$P(C_1|x) = 0.06 \times 0.6 = 0.036$$

$$P(C_2|x) = 0.54 \times 0.4 = 0.216$$

$$x = [1 \ 0]$$

$$P(x|c_1) = 0.04$$

$$P(x|c_2) = 0.36$$

$$P(c_1|x) = 0.04 \times 0.6 = 0.024$$

$$P(c_2|x) = 0.36 \times 0.4 = 0.144$$

Since $P(c_2|x) > P(c_1|x)$ we classify $x = [1 \ 0]$ as c_2

$$x = [1 \ 1]$$

$$P(x|c_1) = 0.36$$

$$P(x|c_2) = 0.04$$

$$P(c_1|x) = 0.36 \times 0.2 = 0.072$$

$$P(c_2|x) = 0.04 \times 0.2 = 0.008$$

Since $P(c_1|x) > P(c_2|x)$ we classify $x = [1 \ 1]$ as c_1

$$\text{Prior } P(c_1) = 0.8$$

$$P(c_2) = 0.2$$

$$x = [0 \ 0]$$

$$P(x|c_1) = 0.66$$

$$P(x|c_2) = 0.54$$

$$P(c_1|x) = 0.66 \times 0.8 = 0.528$$

$$P(c_2|x) = 0.54 \times 0.2 = 0.108$$

Since $P(c_1|x) > P(c_2|x)$ we classify $x = [0 \ 0]$ as c_1

$$x = [0 \ 1]$$

$$P(x|c_1) = 0.54$$

$$P(x|c_2) = 0.54$$

$$P(c_1|x) = 0.54 \times 0.8 = 0.432$$

$$P(c_2|x) = 0.54 \times 0.2 = 0.108$$

Since $P(c_1|x) > P(c_2|x)$ we classify $x = [0 \ 1]$ as c_1 .

$$x = [1 \ 0]$$

$$P(x|c_1) = 0.04$$

$$P(x|c_2) = 0.36$$

$$P(c_1|x) = 0.04 \times 0.8 = \cancel{0.032} \quad 0.032$$

$$P(c_2|x) = 0.36 \times 0.2 = 0.072$$

since $P(c_2|x) > P(c_1|x)$ we classify $x = [1 \ 0]$ as c_2 .

$$x = [1 \ 1]$$

$$P(x|c_1) = 0.36$$

$$P(x|c_2) = 0.04$$

$$P(c_1|x) = 0.36 \times 0.8 = 0.288$$

$$P(c_2|x) = 0.04 \times 0.2 = 0.008$$

$$x = [1 \ 1]$$

since $P(c_1|x) > P(c_2|x)$ we classify x as c_1 .

Summary of classifications

x_1	x_2	$P(c_1) = 0.2$	$P(c_1) = 0.6$	$P(c_2) = 0.8$
0	0	c_2	c_2	c_2
0	1	c_2	c_1	c_1
1	0	c_2	c_2	c_2
1	1	c_1	c_1	c_1