Verified C Implementation of Dijksta's Algorithm

Anshuman Mohan, Aquinas Hobor

APLAS Student Research Competition
December 3, 2019

Saluting the Mothership

Certifying Graph-Manipulating C Programs via Localizations within Data Structures

SHENGYI WANG, National University of Singapore, Singapore
QINXIANG CAO, Shanghai Jiao Tong University, China
ANSHUMAN MOHAN, National University of Singapore, Singapore
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VST + CompCert + 25000 LOC library

Powerful enough to verify executable code
against realistic specifications
expressed with mathematical graphs

[Wang *et. al.*, PACMPL OOPSLA 2019]

This Work

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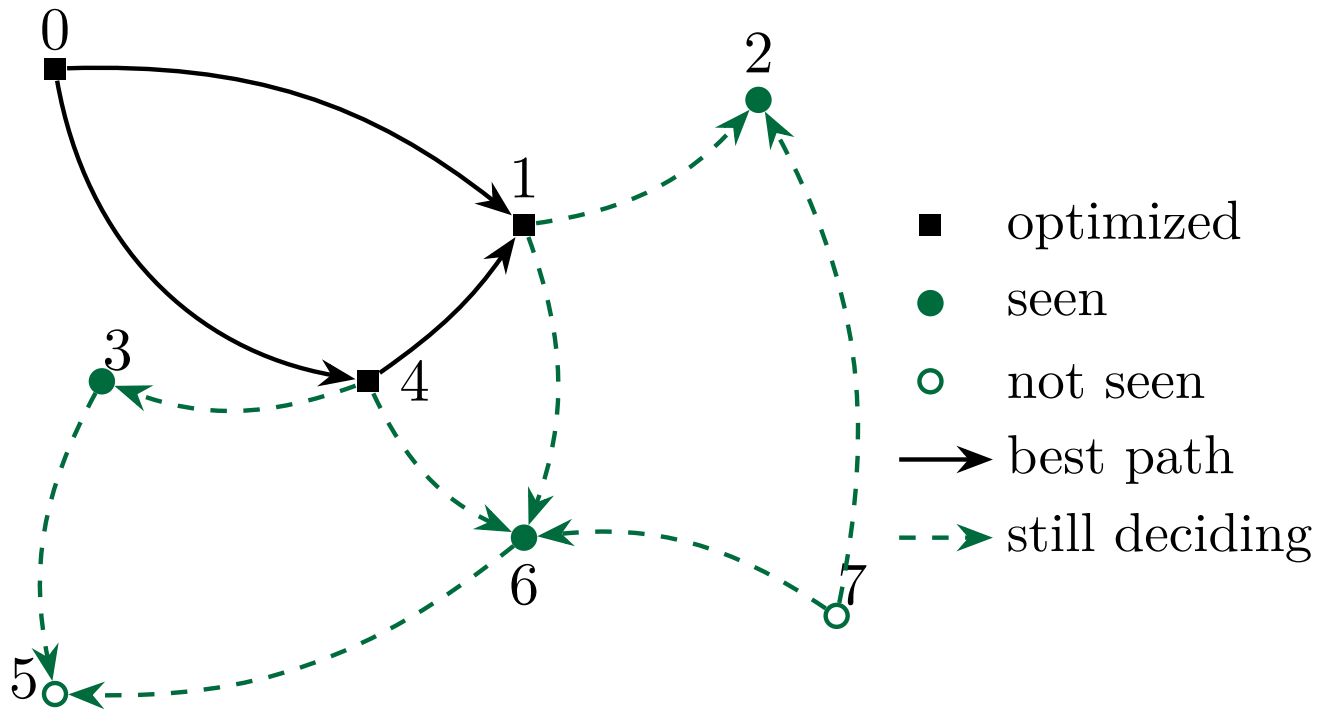
Verify Dijkstra's one-to-all shortest path algorithm

Challenges

Using CompCert C
- executable and realistic
- real-world complications

Aiming for full functional correctness
- and not just safety

Refresher



Instantiating DijkGraph

A PreGraph is a hextuple (`VType`, `EType`, `vvalid`, `evalid`, `src`, `dst`)

$$\textbf{Dijk\_PG}(\gamma) \stackrel{\text{def}}{=} \texttt{VType := Z}$$
$$\texttt{EType := VType * VType}$$
$$\texttt{src := fst}$$
$$\texttt{dst := snd}$$
$$\forall v.\ \texttt{vvalid}(\gamma, v) \Leftrightarrow 0 \leqslant v < \texttt{SIZE}$$
$$\forall s, d.\ \texttt{evalid}(\gamma, (s, d)) \Leftrightarrow \texttt{vvalid}(\gamma, s) \wedge \texttt{vvalid}(\gamma, d)$$

Instantiating DijkGraph

A LabeledGraph is a quadruple (PreGraph, `VL`, `EL`, `GL`)

$$\textbf{Dijk\_LG}(\gamma) \stackrel{\text{def}}{=} \text{Dijk\_PG as shown}$$

```
VL := list EL
EL := Z
GL := unit
```

Instantiating DijkGraph

A GeneralGraph adds arbitrary soundness conditions

$$\textbf{DijkGraph}(\gamma) \stackrel{\text{def}}{=} \text{Dijk\_LG as shown, and}$$
$$\mathit{FiniteGraph}(\gamma) \wedge$$
$$\forall i, j.\ \texttt{vvalid}(\gamma, i) \wedge \texttt{vvalid}(\gamma, j) \Rightarrow$$
$$i = j \Rightarrow \texttt{elabel}(\gamma, (i, j)) = 0 \wedge$$
$$i \neq j \Rightarrow 0 \leqslant \texttt{elabel}(\gamma, (i, j)) \leqslant \lfloor \texttt{MAX/SIZE} \rfloor$$

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Representing DijkGraph in Memory

$$\mathsf{list\_rep}(\gamma, i) \stackrel{\text{def}}{=} \texttt{data\_at array graph2mat}(\gamma)[i]\ \texttt{list\_addr}(\gamma, i)$$

$$\mathsf{graph\_rep}(\gamma) \stackrel{\text{def}}{=} \mathop{\Large *}_{\texttt{vvalid}(\gamma, v)} v \mapsto \mathsf{list\_rep}(\gamma, v)$$

Code and Specification

```
#define IFTY INT_MAX - INT_MAX/SIZE

void dijkstra (int graph[SIZE][SIZE], int src,
                          int *dist, int *prev) {
```

$\{$ DijkGraph$(\gamma)\}$

Code and Specification

```
#define IFTY INT_MAX - INT_MAX/SIZE

void dijkstra (int graph[SIZE][SIZE], int src,
                              int *dist, int *prev) {
```

$\{$ DijkGraph(γ) $\}$

```
 int pq[SIZE];
 int i, j, u, cost;
 for (i = 0; i < SIZE; i++)
 {  dist[i] = INF; prev[i] = INF; pq[i] = INF;  }
 dist[src] = 0; pq[src] = 0; prev[src] = src;
```

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```

$\{$ DijkGraph$(\gamma) \wedge \mathit{dijk_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq})\}$

```
 // big while loop
```

Code and Specification

$$\{\ \mathsf{DijkGraph}(\gamma) \wedge \mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq})\}$$

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```
while (!pq_emp(pq)) {
 u = popMin(pq);
 for (i = 0; i < SIZE; i++) {
  cost = graph[u][i];
   if (cost < INF) {
    if (dist[i] > dist[u] + cost) {
     dist[i] = dist[u] + cost; prev[i] = u; pq[i] = dist[i];
}}}}
```

Code and Specification

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 }}}}
```

$\left\{\begin{array}{l} \mathsf{DijkGraph}(\gamma) \wedge \forall\, \mathit{dst} \in \mathit{priq}.\ \mathit{priq}[\mathit{dst}] = \texttt{INF} \wedge \\ \mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq}) \end{array}\right\}$

```
return;
}
```

Dijkstra_Correct

$$
\begin{aligned}
&\mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{prev}, \mathit{dist}, \mathit{priq}) \stackrel{\text{def}}{=} \\
&\quad \forall \mathit{dst}.\ \mathit{dst} \in \mathit{popped}(\mathit{priq}) \ \Rightarrow \\
&\qquad\qquad \exists \mathit{path}.\ \mathit{path\_correct}(\gamma, \mathit{prev}, \mathit{dist}, \mathit{path}) \wedge \\
&\qquad\qquad \mathit{path\_glob\_optimal}(\gamma, \mathit{dist}, \mathit{path}) \wedge \\
&\qquad\qquad \mathit{path\_entirely\_in\_popped}(\gamma, \mathit{prev}, \mathit{path}) \wedge \\
&\qquad \mathit{priq}[\mathit{dst}] < \texttt{INF} \ \Rightarrow \\
&\qquad\qquad \texttt{let}\ m\ \texttt{:=}\ \mathit{prev}[\mathit{dst}]\ \texttt{in}\ m \in \mathit{popped}(\mathit{priq}) \wedge \\
&\qquad\qquad \forall m' \in \mathit{popped}(\mathit{priq}).\ \mathit{cost}(\mathit{path2m}+ :: (m, \mathit{dst})) \leqslant \\
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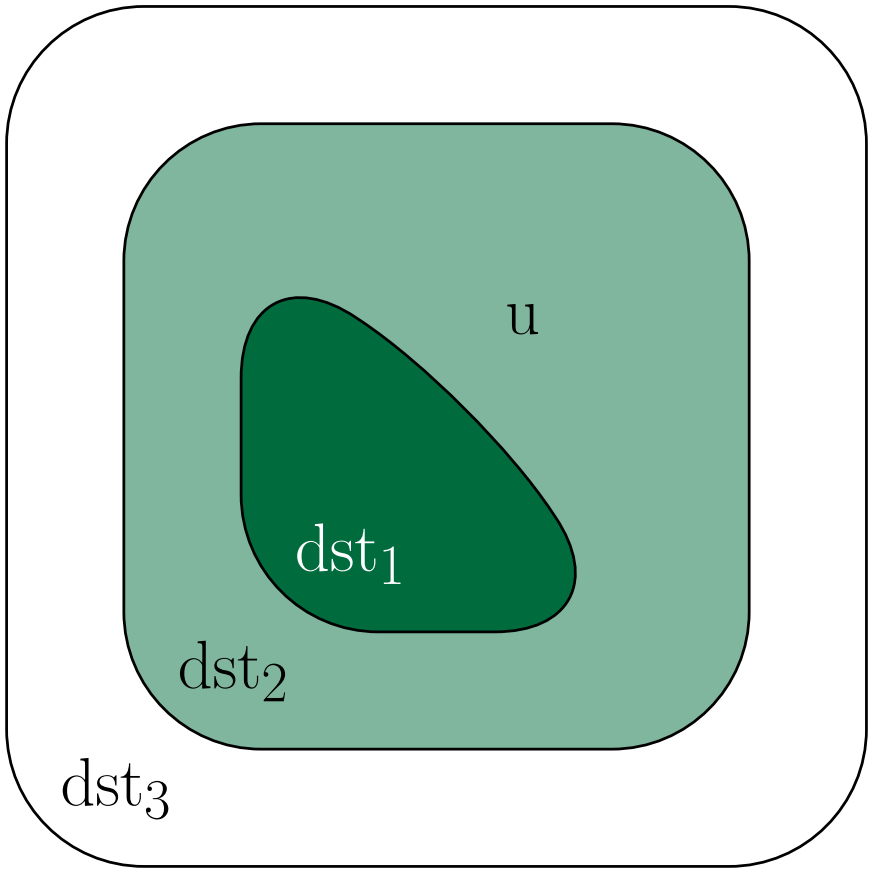
Key Transformation: Growing the Subgraph



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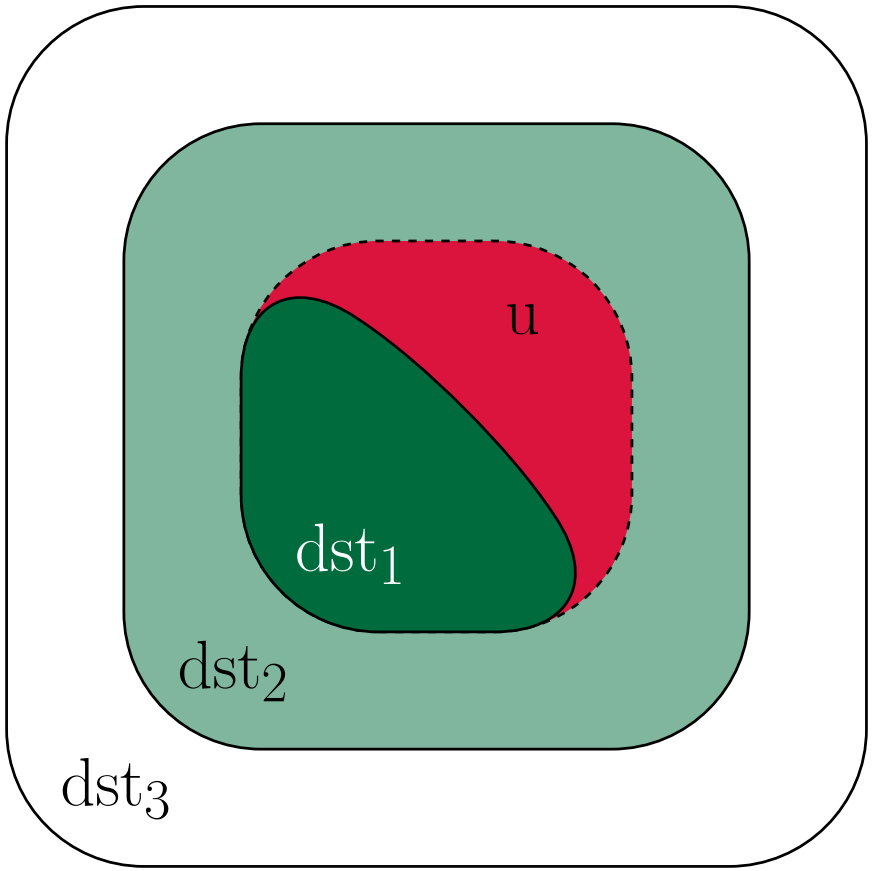

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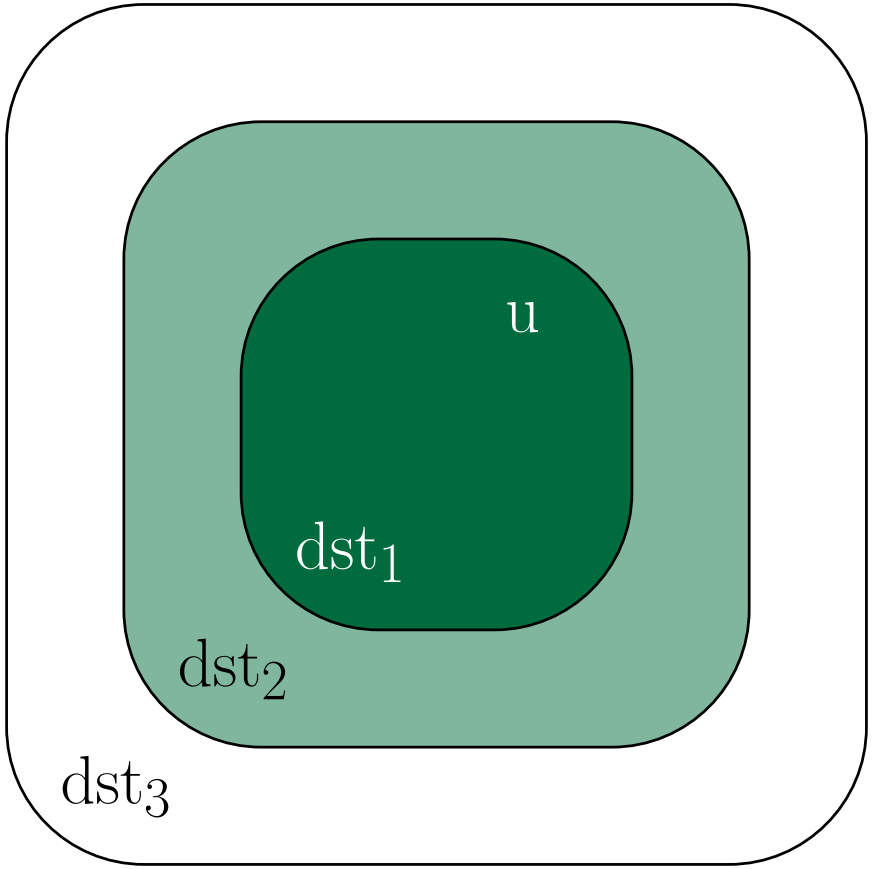

Overflow Strikes Again

The longest optimal path has `SIZE-1` links

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so say we set `elabel`'s upper bound to $\lfloor$`MAX/(SIZE-1)`$\rfloor$

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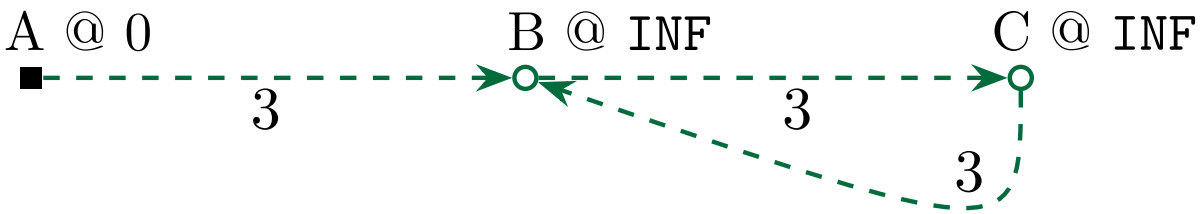
`MAX` $= 7$, `SIZE` $= 3$, so $0 \leqslant$ `elabel`$(\gamma, e) \leqslant 3$.



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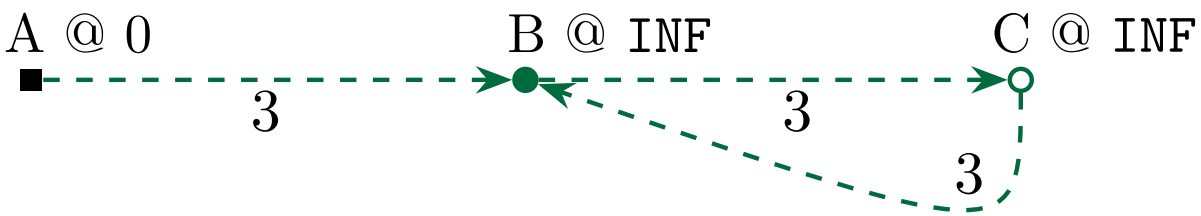

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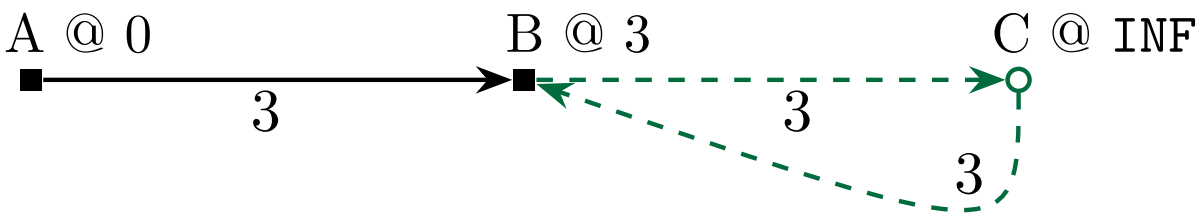

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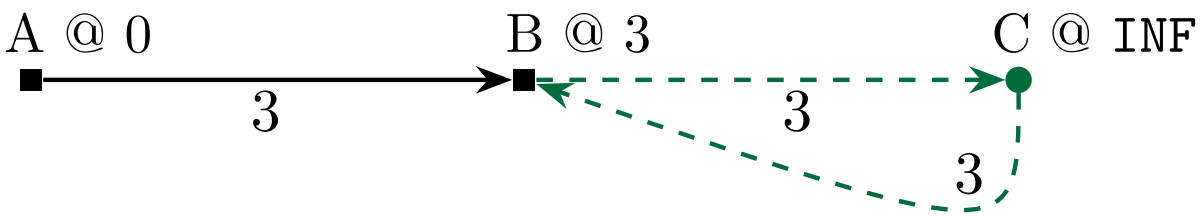

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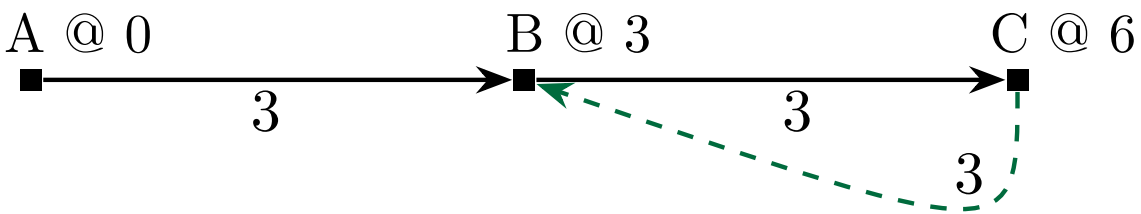

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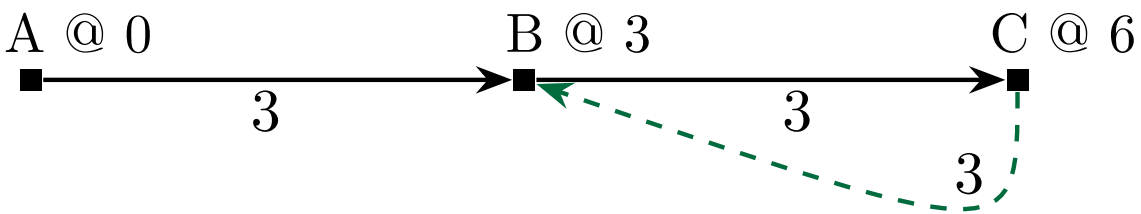

if $3 > 9$ then relax C $\rightsquigarrow$ B

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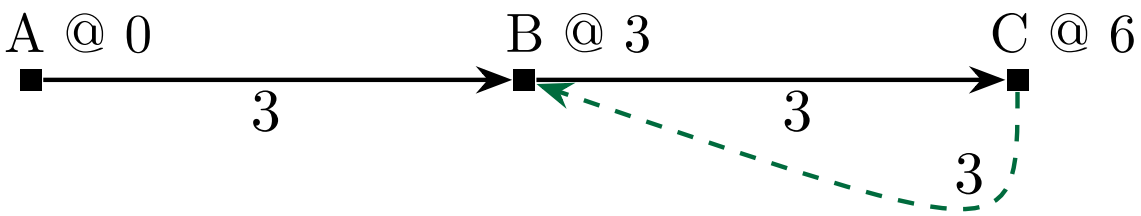

if $3 > 1$ then relax C $\rightsquigarrow$ B

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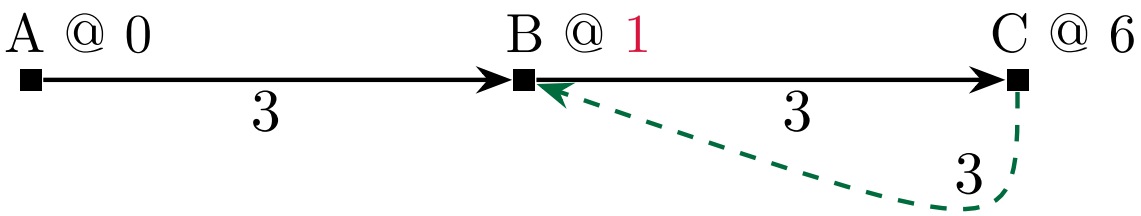

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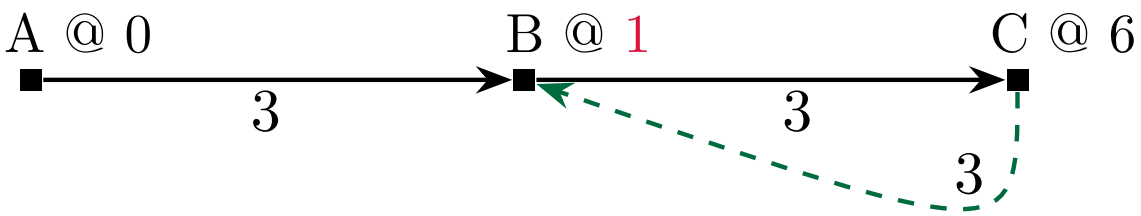

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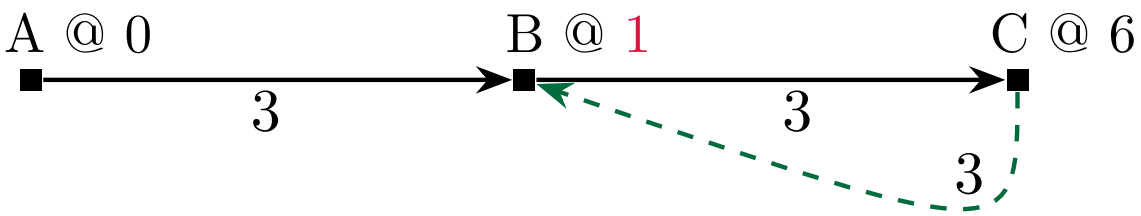

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Max path cost is then $\lfloor$`MAX/SIZE`$\rfloor$ `* (SIZE-1) = MAX -` $\lfloor$`MAX/SIZE`$\rfloor$

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There are other ways to fix this!

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- Intuition supports `INF = MAX`

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- No reason to do any of the above

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- No reason to do any of the above... until today