# CHP- 13

## Information Assymetry & Adverse selection.

## Model layout

- many identical potential firms that hires workers
- Each Firm Produces the same Level of output.
- Identical CRs in tecnology.

# Assumptions:-

- Labour is the only input
- Firms are risk neutral.
- Firms are $\pi$ maximisers
- They act as price takers.
- Price of the goods they produce is $p=1$ (numeraire)
- Let $\theta$ be the no of units of output produced by a worker. And workers differ in their outputs produced.

let

$[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$, denote the set of possible outputs by workers {Basically workers productivity levels}

And

$$0 \leq \underline{\theta} \leq \overline{\theta} < \infty$$

Let, $F(\theta)$ represent the CDF for worker's output or productivity which is defined as

$$P(\theta \leq \theta) = F(\theta)$$

$F(\theta)$ is degenerate implying there is a difference in the productivity levels, ie there are atleast {minimum} 2 types of workers that work in a firm.

Let the total no of workers $= N$

Assumption about workers :-

- worker's maximise the amount that they earn from their work.

- There are 2 options to work from

At Firm

money earned here is $w$

At home.

money earned here is $r(\theta)$

* $r(\theta)$ implies, the earnings through productivity from home.

* Basically it is the money that you could earn if you are at home than work, implying if you have no earnings at home, you will choose to go to work. If you can earn more staying at home, then you won't go to work on decision.

* ie, workers will accept employment iff she recieves a wage $w \geq r(\theta)$

$w$ atleast as good as $r(\theta)$

Q What is the competitive Equilibrium when worker's productivity levels is publicly observable

The problem arises for firms, as they cannot observe a productivity of worker, however suppose, we know the productivity levels of the worker

In that case:-

Worker's Labour is a distinct good

⇒ Distinct wage according to the Labour productivity/output produced is given ie

$W(\theta)$ is the wage given

⇒ At competitive Equilibria:-

$$MR = MC$$

$$P \cdot \theta = W^*(\theta)$$

Since price of Firm = 1

$$\therefore \boxed{w^*(\theta) = \theta}$$

From this, given equillibrium of wage, we can also determine equilibrium of workers accepting employment if

$$\left(\theta : w(\theta) \geq r(\theta)\right) = H$$

$$\parallel$$

ie $\left(\theta : \theta \geq r(\theta)\right)$

This Competive Equillibrium is thus pareto optimal.

Hence pareto optimality is reached, when

$$AS = \int_{\underline{\theta}}^{\overline{\theta}} N\left[I(\theta)\theta + (1 - I(\theta))r(\theta)\right] dF(\theta)$$

where $I(\theta)$ is a binary variable $= 1$ if worker of type $\theta$ works for firm, $I(\theta) = 0$, if else.

$$AS = \int_{\underline{\theta}}^{\bar{\theta}} N\left[I(\theta)\theta + (1 - I(\theta))r(\theta)\right] dF(\theta)$$

Aggregate Surplus is a maximised, the pareto optimality is reached.

## CASE-B

What is Comp-Eq, when worker's productivity is unobs?

Since now, The worker's productivity is unknown,

$$w \neq f(\theta)$$

→ wage rate set by Firms is independent of workers

→ A universal wage rate "w" is set for all workers

→ Labour supply is now $f(w)$

→ workers will hence now accept employment iff $w \geq r(\theta)$

So set of worker who accept employment are

$$H(w) = \{\theta : w \geq r(\theta)\}$$

Now since, we don't know the type of worker's productivity, For competitive equillibrium to hold;

Labour supply = Labour demand

Labour demand is a function of $w$ as well.

Let $Z(w)$ is the lab dd function of Firm. & $Z(w)$ depends on $\mu$, which is the average productivity of workers who are accepting employment.

$$Z(w) = \begin{cases} 0, & \mu < w \\ [0, \infty); & \mu = w \\ \infty, & \mu > w \end{cases}$$

(if productivity is less than what Firm is paying to do the job, the demand then will fall to zero for Labour)

Now equillbirum can only be attained if workers in set $H$ {Labour supply} & Firm's belief of Average productivity of worker is actual Average productivity {$Z(w)$ ie demand = $[0, \infty)$ if $\mu = w$}, then equilibrium of Labour is attained ie

$$\mu = E(\theta \mid \underbrace{\theta \in \Theta^*}_{\text{belief of Firm (about Avg prod)}})$$

$\downarrow$ actual productivity

and $$w = E(\theta \mid \theta \in \Theta^*)$$

$$\therefore \boxed{\mu = E(\theta \mid \theta \in \Theta^*) = w}$$

# Assymetric Info & Pareto Inefficiency

Incase of information assymetry like the above case, It will fail to obatain pareto Efficiency

To see this, consider the following:-

1. $r(\theta) = r \ \forall \ \theta$
2. Every worker is equally productive at home.
3. Let $F(r) \in (0,1)$ which is Like the distribution functi of outcome, trying to say there can be atleast 2 types of home earnin

ie $r \geq \theta$ {workers not accepting Firm job}

$r \leq \theta$ { " accepting " " }

Under Pareto optimality, we have

If $\theta \geq r$ ie $w(\theta) \geq r$ : set of worker's join firm

if $\theta < r$ ie $w \leq r$ : don't join.

The problem is, in the competitive Equillbrium we discussed, where $\theta$ is unobserved,

$w(\theta) = w$ { is same for every worker as } $\theta$ is unobs

market equillbrium is thus attained if expectation of Average productivity = Actual Average productivity of Labour, as seen by firm

ie

$$E(\theta : \theta \in \Theta^*) = \mu$$

Given Labour supply of willing workers ie

$$\Theta^* = \{\theta : w \geq r\}$$

$\therefore$ market equilibrium is attained, when

$$\mu = w^*$$

or $E(\theta) = w^*$

However,

(i) if $E(\theta) \geq \gamma$ or $w \geq \gamma$, all workers all employed

(ii) if $E(\theta) < \gamma$ or $w < \gamma$, none accept.

So the problem here is, if there are too many Low productivity workers ie ($\theta > \gamma$), then too many are available for employment all together, an the firm is unwilling to accept these many Labours.

If there is a high fraction of high productivity worker's ie $\gamma > \theta$, the firm unable to see their productivity Level, sets a common $w$ for every $\theta$ level, which discourages high productivity workers to be able to accept employment.

This IS NOT PARETO OPTIMAL

# SIGNALLING

- High Ability workers adopt actions to distinguish themselves From Low ability workers.

$\theta_H$ , $\theta_L$

high effort guy

Low effort guy

& $\theta_H > \theta_L > 0$

Let $\lambda = P(\theta = \theta_H) \in (0,1)$

As we talked, the high' effort individual tries to gain some action to distinguish himself From the low effort person.

Let that action be, Education.

<u>NOTE</u>

Education as an action, is observable. and we <u>assume</u>, Education has nothing to do with worker's productivity

now, to obtain the skill such as educ, you need to pay money. hence naturally it means, there is some cost involved to aquire a Level of eduction ie e$^{\prime\prime}$ by a type $\theta$, which is given by a 2 diff function

$$\underbrace{C}_{\text{cost of educ}}\Big(\overset{e=0}{0},\ \underset{\theta}{\theta}\Big)=0$$

(i) $\overbrace{\dfrac{\partial C(e,\theta)}{\partial e}}^{C_e(e,\theta)} > 0$ {as e↑, c↑}

(ii) $\overbrace{\dfrac{\partial^2 C(e,\theta)}{\partial e^2}}^{C_{ee}(e,\theta)} > 0$ { as e↑, the c↓}

$\overbrace{\dfrac{\partial C}{\partial \theta}}^{C_\theta(e,\theta)} < 0$ {as $\theta=\theta_H$ then c↓}

$\overbrace{\dfrac{\partial^2 C}{\partial \theta \partial e}}^{C_{e,\theta}(e,\theta)} < 0$

→ Cost of higher ability worker's education is Lower: by cost i mean, the actual cost C & Marginal cost M C.

→ It means work needed to put in by high effort individual is Lower than the what effort the low effort guy has to put.

→ Let $U(w, e | \theta)$ be utility of type $\theta$ worker, who give their type, choose educ level $e$ & recieves $w$ as wage.

→ $U(w, e | \theta) = w - C(e, \theta)$

wage from work

cost of educ.

**In equilibrium, high productivity workers choose more educ than the Lower prod workers**

**↳ This is observable to firms {the Level of educ}**

**Hence, this signals for the ability of the worker to the firm.**

# Welfare effects of Signalling.
( Ambiguous )

Firm side

Firms get information about worker's effort through it's educ level etc. This creates effective allocation my workers Labour.

Consumer side

On consumer's side however, worker's ability to signal the firms, so that they hire them, implies more spending on thir side leading to delcrese in Level of welfare of consumers.

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Let $r(\theta_H) = r(\theta_L) = 0$

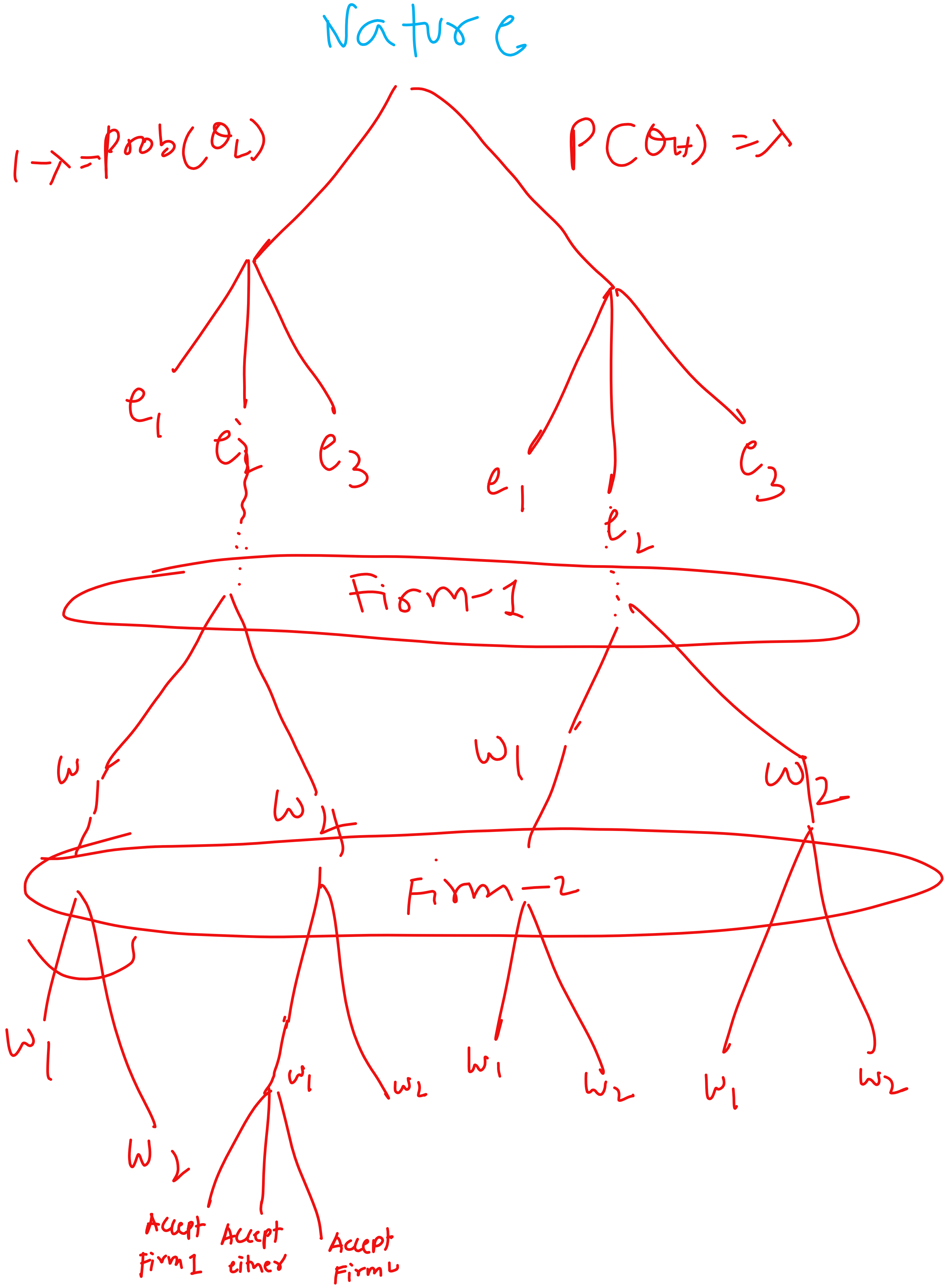
Home earnings

→ Unique Equillibrium in the absence of ability to signal

→ All workers employed by firms at wage, $w^* = E(\theta)$

Pareto Efficienct outcome.

→ Signalling leads to potential inefficies.




Two firms simultaneously offer wage offer to worker to hire.

here, probability that worker is of high effort is $P(\theta^H) = \lambda$

for each possible choice 'e' ie level of education, there is $\mu(e) \in [0,1]$

belief of Firm 1 about the ability of $\theta^H$ worker

→ now after determining that the worker is of $\theta^H$, through Firm 1's belief $\mu(e)$ depending on educ level e, the wage w is set

→ This is also seen by Firm 2, ie the belief of Firm 1 & the wage derived, hence belif of firm 2 becomes

$$\mu(e)\, \sigma_1^*(w|e)$$

# Perfect Baysian Equilibrium.

↳ It is a set of strategies and belief function $\mu(e) \in [0,1]$ giving the firm common probability assesment that the worker is of high ability after observing the level of education e, is a PBE if

(i) worker's strategy is optimal given firm's strategy.

(ii) Belief of firm's is derieved from worker's strategy of selecting educ level e. ∴ firm's belief is $\mu(e)$

(iii) The firms wage after following each choice 'e' constitute a Nash eq of the simultaneous-move game for wage offer, in which probability that worker is of high ability is $\mu(e)$

We solve the game through backward induction. We start end. Suppose the Firm after seeing the choice of education level of worker, suppose $e'$, attaches a probability that the worker is of high ability with probability $\mu(e)$. Then Expected productivity of worker here is

$$\boxed{\mu(e)\theta_H + (1-\mu(e))\theta_L}$$

Hence in a simultaneous move game of wage, the Firm's Nash Eq wage equals to

$$\boxed{w^*(e) = w^* = \mu(e)\theta_H + (1-\mu(e))\theta_L}$$

ie $\boxed{\text{productivity of worker} = \text{wage offer}}$

# SEPARATING Equillibrium

Let $e^*(\theta)$ be the worker's equillibrium educ choice, as a function of his/her type.

$w^*(e) \rightarrow$ Firm's equilibrium wage offer as a function of the workers educ level.

$\therefore$ $w^*(e^*(\theta_H)) = \theta_H$ Each worker recieves a wage equal to his/her level of productivity

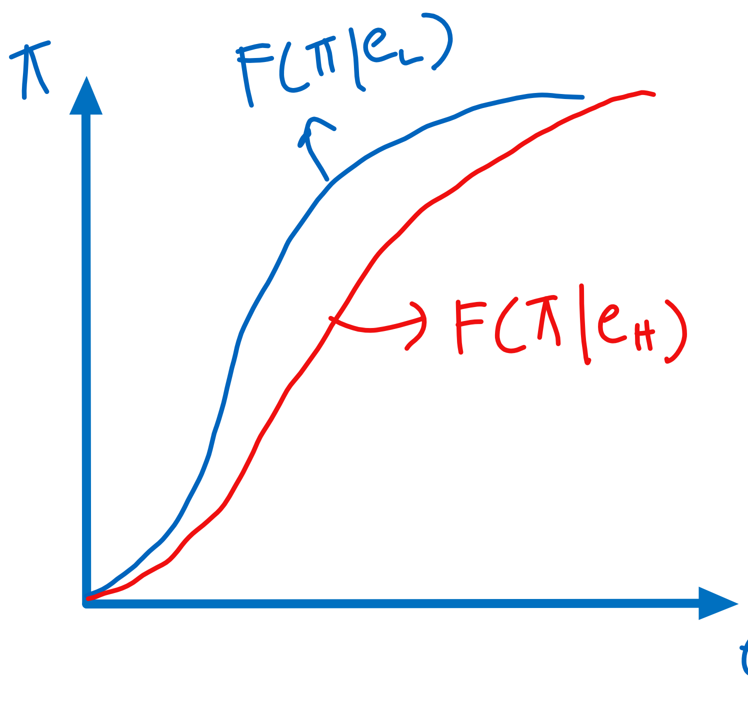
$w^*(e^*(\theta_L)) = \theta_L$

# The principal agent problem.

## Hidden Actions (Moral Hazard)

① owner of project — principal
manager — Agent

- The owner of the plant, the principal wants to hire the manager for a one time project. The manager is the Agent.
- Let $\pi$ denote the profits of the Firm which the owner wants to claim after production, where $\pi \in [\underline{\pi}, \bar{\pi}]$ st $\underline{\pi} < \pi < \bar{\pi} < \infty$
- Let $e$ be the effort Level of the manager, which can be distributed in to 2 types ie high effort. $(e_H)$, Low effort. $(e_L)$ $e_H > e_L$
- Effort of the manager is determined through a possible set of actions — $E \subset \mathbb{R}^m$
- It is assumed, that $e$ partially determines $\pi$ of the firm. where $\pi \in [\underline{\pi}, \bar{\pi}]$
- The stochastic relation between $\pi$ & $e$ is given by a conditional density function $f(\pi|e)$, where $\int_{-\infty}^{\infty} f(\pi|e) = 1$, $f(\pi|e) > 0 \ \forall e \in E$ & $\pi \in [\underline{\pi}, \bar{\pi}] \in \mathbb{R}^m$
- It is considered the manager's have 2 possible effort levels ie
  $e^H$, $e^L$ ↳ low effort
  ↳ high effort
- Distribution $F(\pi|e_H) < F(\pi|e_L)$ ie $F(\pi|e_H)$ FOSD $F(\pi|e_L)$




$\therefore F(\pi|e_H) < F(\pi|e_L)$

FOSD condition implies, the level of Expected profit is more in case of the $e_H$ than $e_L$ st

$$E(\pi) = \int \pi \, dF(\pi|e_H) > \int \pi \, dF(\pi|e_L) \quad \{\text{Expected profit}\}$$

For the Agent (manager), the utility is derieved. And utility is derieved from ??

$w$ (wage) (+ve) (more wage more U↑)

$e$ (effort) (−ve) (more e less U)

Agent's utility function is thus $U(\omega, e)$, and is a Bernoulli's utility function.

st, $U(\omega, e_H) > U(\omega, e_L)$

where; $u(\cdot)$ is a concave utility function. This implies

ⓐ $\frac{\partial U(\omega, e)}{\partial \omega}$ or $U'(\omega, e)$ or $U_\omega(\omega, e) > 0$

ⓑ $\frac{\partial^2 U(\omega, e)}{\partial \omega^2}$ or $U''(\omega, e)$ or $U_{\omega\omega}(\omega, e) < 0$

$\forall \; \omega, e$.

**characteristics of manager**

1) weakly risk averse
2) dislike effort (high effort Levels)
3) Loves more income over Less.

**characteristic of owner**

- risk neutral
- Like $\pi \uparrow$
- Has a diversified Portfolio.

∴ since Bernoulli utility is a Linear Form: $U(\omega, e) = V(\omega) - g(e)$

+ve function of $\omega$
- $V'(\omega) > 0$
- $V''(\omega) \leq 0$

↳ maximising behaviour

−ve function of e.
- $g'(e) < 0$ as $e \uparrow$
- $g''(e) > 0$

↳ minimising behaviour
- $g(e_H) > g(e_L)$.

$$\boxed{\text{OPTIMAL CONTRACT WHEN THE EFFORT IS OBSERVABLE. (Fixed wage contract)}}$$

Let us recall in chp 13, people will go to work iff $w \geq \gamma$, where $\gamma$ though was income from home, it could be thought of as a reservation wage. ie, isko par kiya tab kaam ke bare mein mein sochunga.

Similarly,

When a the principal offers the manager a contract (1 time contract) then the manager can either choose to accept it or reject it

Utility of manager.

$$\left.\begin{array}{ll} \text{if Accept: } U(w,e) = \bar{U}, & \pi > 0 \\ \text{if Reject: } U(w,e) < \bar{U}, & \pi = 0. \end{array}\right\} \pi \text{ of the firm.}$$

where $\bar{U}$ is reservation utility

Intention of owner:- To maximise effort of the manager as it increases $\pi$s. And minimise the compensation or wage paid to the manager. ie $w(\pi)$

↳ wage of manager depends on $\pi$.

↳ This the cost for owner (principal)

Optimal contract eq$^n$s set by principal →
- → Profit eq$^n$ of Firm (maximise)
- → wage eq$^n$ of manager (minimise)

① profit eq$^n$. {principal's problem}

$$\max_{e \in \{e_H, e_L\}, w(\pi)} \int_{\underline{\pi}}^{\bar{\pi}} (\underset{\text{Revenue}}{\pi} - \underset{\text{cost}}{w(\pi)}) \underbrace{f(\pi|e)\, d\pi}_{\text{distribution function of } \pi} = \int \pi f(\pi|e)\, d\pi - \underbrace{w(\pi) f(\pi|e)\, d\pi}_{\text{wage eq}^n}$$

↳ wage eq$^n$ → ②, which we minimise, find an answer, then put it back into eq$^n$ ① to derive optimal $\pi$.

Sub to:
$$\int_{\underline{\pi}}^{\bar{\pi}} \underbrace{\underset{\text{utility from wage}}{v(w(\pi))} f(\pi|e)\, d\pi - \underset{\text{utility \{dis utility\} from effort.}}{g(e)}}_{U(w(\pi)) \geq \bar{U} \text{ \{written previously\}}} \geq \bar{U}$$
→ constraint is basically manager's utility max problem.

this is called as participation constraint

② **wage eq$^n$** $(w(\pi))$ {principal's problem} {cost}

$$\min \int w(\pi)\, f(\pi|e)\, d\pi$$

$$\text{sub to. } \int v(w(\pi)) \cdot f(\pi|e)\, d\pi - g(e) \geq \bar{U} \rightarrow$$ constraint is basically manager's utility max problem.

Participation constraint.

Solution to wage eq$^n$. $(w(\pi))$

FOC (taking derivative wrt wage $w$ )

$$L = \int w(\pi) f(\pi|e)\, d\pi + \lambda \left[ \int \bar{U} - v(w(\pi)) \cdot f(\pi|e) d\pi \right]$$

$$\frac{\partial L}{\partial w} = f(\pi|e) d\pi + \lambda \left[ \int 0 - v'(w(\pi)) \cdot f(\pi|e) d\pi \right]$$

removing $d\pi$ from eq$^n$ as it can be ignored since we take derivative wrt $w$ only.

$$\Rightarrow \quad f(\pi|e) - \lambda v'(w(\pi)) \cdot f(\pi|e) = 0.$$

$$\Rightarrow \quad \text{or} \quad \cancel{f(\pi|e)} = \lambda v'(w(\pi))\, \cancel{f(\pi|e)}$$

$$\Rightarrow \quad \boxed{\frac{1}{v'(w(\pi))} = \lambda}$$ multiplier

The FOC tells that $\frac{\partial U(w(\pi))}{\partial \pi} = \text{const}$

$\Rightarrow \; w^*(\pi) = \text{const}.$

We know, manager is moderate risk averse. Thus suppose if manager is risk averse, then, $v'(w(\pi))$ is going to strictly decrease in $w$ : meaning $v''(w) < 0$

ie $\frac{\partial v'(w)}{\partial w} < 0$

now, since $w^*(\pi)$ is a constant derived from FOC of wage eq$^n$, it means: given that the contract dictates on manager's effort levels, & that there is no possibility of providing incentives for the manager to work {since effort is observable}, the owner fully insures the risk averse manager with a const wage according effort level such that he is atleast as good as his reservation utility (Basically derived optimal for manager.)

$$\text{ie } U(w^*(\pi)) \geq \bar{U}$$

$$\text{where } U(w^*) = v(w^*) - g(e)$$

$$\text{then } w^*(\pi) = \text{const}$$

Increasing compensation!

If $g(e_H) > g(e_L) \Rightarrow U_{e_H}(w) \geq \bar{U}_{e_H}$ & $U_{e_L}(w) \geq \bar{U}_{e_L}$ $\therefore \bar{U}_{e_H} > \bar{U}_{e_L}$ {ie the reservation utility of the higher effort is greater than lower effort}

This Implies, $w_{e_H}(\pi) > w_{e_L}(\pi)$

For Risk neutral manager

$v(w) = w$, ie at any wage, the manager will be willing to work.

↳ here fixed wage is one of many other schemes.

Any compensation function $w(\pi)$ that gives manager an expected wage payment $= \bar{U} + g(e)$ is also optimal.

??

$U(w,e) \geq \bar{U}$ For some $w^*(\pi)$

or $U(w,e) = \bar{U}$

$\Rightarrow v(w) - g(e) = \bar{U}$

$\Rightarrow v(w) = \bar{U} + g(e)$

$\Rightarrow$ if $v(w) = w$

$\therefore w^* = \bar{U} + g(e)$

So the optimal amount of $\pi$ that can be derived from the principal's profit eq$^n$, give optimal e is observable, optimal wage ie fixed wage is derived is:

$$\max_{e \in (e_L, e_H)} \int \pi f(\pi|e)\, d\pi - \underbrace{\text{cost}}_{\text{wage eq}^n}$$

$$= \int \pi f(\pi|e)\, d\pi - w(\pi) f(\pi|e)\, d\pi)$$

$$= \int \pi f(\pi|e)\, d\pi - \underbrace{\frac{1}{v'(w(\pi))}}_{\text{optimal from minimisation of wage eq}^n}$$

$$\Rightarrow \int \pi f(\pi|e)\, d\pi - v^{-1}(w(\pi))$$

since $w = g(e) + \bar{U}$

$$\Rightarrow \boxed{\int \pi f(\pi|e)\, d\pi - v^{-1}(g(e) + \bar{U})}$$

↳ optimal $\pi$ max eq$^n$.

Thus an optimal contract is the one, where principal's $\pi$ eq$^n$ is maximised ie $\int \pi f(\pi|e) d\pi - v^{-1}(g(e)+\bar{U})$ given, $w$ = fixed wage due to observable levels of effort.

imp

# OPTIMAL CONTRACT WHEN EFFORT IS UNOBSERVABLE.

When Effort Levels are unobserved, there can be 2 cases of optimal contract:-

## (a) When Manager is risk neutral

In case of a risk neutral manager, the optimal contract generates the Effort Level & Expected utilities same as the observed case.

in case of Risk neutral, we know previously that

$$V(w) = w.$$

Thus optimal Effort Level which solves when effort were observable:

$$\int \pi f(\pi|e)d\pi - w(\pi)f(\pi|e)d\pi = \int \pi f(\pi|e)d\pi - w = \int \pi f(\pi|e)d\pi - g(e) - \bar{u}$$ { $\pi$ max$^n$ of Firm owner

Now consider a situation where e isn't observable.

Now, Let

$w = \pi - \alpha$ { ie we Flip the concept of profit. earlier $\pi$ was supposed to be the income Left after cost was subtracted from the total revenue, for the principal }

→ This means, if principal sold away his business to the manager, the manager's wealth is equal to income of principal minus the sales price.

If, the manager chooses to accept this contract, he will maximise his expected utility with e. $\left\langle \begin{matrix} e_H \\ e_L \end{matrix} \right.$ } depending on what contract demands.

as, acceptance of contract depends on whether or not reservation utility is satisfied or not.

Utility of manager: $U(w(\pi)) = V(w) - g(e)$

since $w = V(w)$

$$\Rightarrow \int u(w(\pi)) f(\pi|e) d\pi = \int \{V(w) - g(e)\} f(\pi|e) d\pi$$

$$\Rightarrow \int w(\pi) f(\pi|e) d\pi - \underbrace{g(e)}_{\text{const}}$$

since $w = \pi - \alpha.$

$$\Rightarrow \int \pi f(\pi|e) d\pi - \underbrace{\alpha}_{\text{const}} - \underbrace{g(e)}_{\text{const}}$$

$$\therefore \boxed{\int U(w(\pi)) f(\pi|e) d\pi = \int w(\pi) f(\pi|e) d\pi - g(e) = \int \pi f(\pi|e) d\pi - \alpha - g(e)}$$

The eqn in Last page is maximised, is comparable to $\Pi$ eqn of observable case (Last page)

manager will accept this contract, as long as $U(w,e) \geq \bar{U}$

since $\bar{U}(w,e) = \int \pi f(\pi|e)\, d\pi - \alpha - g(e) \geq \bar{U}$ holds.

let $\alpha^*$ be the level at which

$$\int w(\pi) f(\pi|e)\, d\pi - g(e) = \int \pi f(\pi|e)\, d\pi - \alpha - g(e) \quad \{\text{for worker}\}$$

$\therefore$ given $e^*$, $\alpha^*$, principal's payoff becomes

$$\alpha^* = \int \pi f(\pi|e)\, d\pi - g(e) - \bar{U}$$

then with compensation scheme $w = \pi - \alpha^*$, the owner & manager get the same compensation as ~~wen~~ when efforts was affordable.

When manager is risk Averse