Preference & Choice

We denote the set of alternatives as X

The elements of this X are possible consmp choices

There 2 distinct approaches to modeling individual or decision making

preference behaviour:

(dependent on the primitive character of individual, hence is not observable.)

choice behaviour

(relies on what is best for the individual given a constraint, i.e optimising consmpt. behaviour given a constraint.)

Preference relations →
- Aren't observable
- Depend on rationality assmpt
  - ↳ completeness
  - ↳ transitivity.
- ~~Are more~~ is ~~consistent~~
- treats decision maker's taste as primitive characteristic

Choice relations → Are observable
- Depends on the approach of weak axiom of revealed preference which engages consistency.
- show consistency and parallel to rationality assumption in some way.

→ Treats individual's choice behaviour as primitive

Preference Relations

preference relations are denoted by $\succsim$.

$\succsim$ is a binary relation ~~Rinc~~ (ordered pairs) on the set X.

this allows comparision of the pairs of alternatives.

ie

$x, y \in X$ and $x \succsim y \Rightarrow x$ is atleast as good as y.

two bundles

from $\succsim$, 2 more relation can be derivied :-

1) $\succ$ = strictly prefered

ie $x \succ y \Leftrightarrow x \succsim y$ ~~but $y \not\succsim x$~~ but not $y \succsim x$.

ie x is prefered ~~over~~ to y or x is strictly ~~pre~~ pref to y.

2) $\sim$ ie indifference

if $x, y \in X$

and $x \sim y \Leftrightarrow x$ is indifferent to $y \Leftrightarrow x \succsim y$ and $y \succsim x$.

*: Binary relation of $\succsim$: $\{\succsim \to X \times X ; (x, y) \in X^2$ and $x \succsim y\}$.

~~These~~

Rationality Assumption.

$\boxed{\text{Preference relation is } \underline{\text{rational}}}$ (prove) (Def 1.B.1)

↳ Do it seperately.

(if) as it posses ~~to~~ 2 properties :-

- ~~Completeness :- $x \succsim y$ or $y \succsim x$ or both; a consumer is able to make a decision.~~
- ~~transitivity~~

- Completeness :- $\forall x, y \in X$, ~~&~~ $x \succsim y$ or $y \succsim x$ or both; a consumer is able to make a decision.

- Transitivity :- $\forall x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, ~~then if~~ then $x \succsim z$, despite budget sets. (~~given~~ even when z is not on the budget set as x)

Both completeness & ~~transity~~ transitivity are strong assumptions.

Violation of transitivity :- cyclical preferences.

~~ie if~~ $A \succsim B$, $B \succsim C$, $C \succsim A$.

ie If $A \succsim B$ & $B \succsim C$ & $C \succsim A$

here transitivity ~~can~~ is violation; ~~& fails to be rationale~~ Rationality fails.

Now

Given that $\succsim$ ~~rati~~ is rational is established; several other properties can be identified.

1) $\succ$ is both irreflexive (ie $x \succ x$ never holds) and transitive ie $x \succ y$, $y \succ z$ then $x \succ z$

2) $\sim$ is reflexive ie ($x \sim x$) and transitive ie $x \sim y$, $y \sim z$ then $x \sim z$ and symmetric ie $x \sim y$, then $y \sim x$.

(iii) if ~~$x \succsim$~~ $x > y \succsim z$, then $x > z$.

NOTE,

Rationality of $\succsim$ implies ~~to~~ that both $\succ$ & $\sim$ are transitive.

Individuals preference may fail to ~~st~~ satisfy transitivity in case of cyclical preference.

Utility Functions

Utility function $u(x)$ assigns a numerical value to each element in X, ranking the elements of X according to individual preferences.

NOTE imp (Definition of utility function representing preference relation)

$U: X \to \mathbb{R}$ is a utility function represent preference relation $\succsim$ if $\forall x, y \in X$

$$x \succsim y \Leftrightarrow \cancel{U(x) \succsim} U(x) \geq U(y) \quad (1.B.4)$$

also note utility function representing a preference relation $\succsim$ is not unique

↳ matlab for strictly increasing function

ie $F(U(x))$ is also a ~~utili~~ utility function such that if $\forall$ ~~xc~~ $x, y \in X$; $x \succsim y \Leftrightarrow U(x) \geq U(y)$ then $F: \mathbb{R} \to \mathbb{R} \Rightarrow F(U(x)) > F(U(y))$. (Prove this)

~~Prove of the~~

Properties of utility Function is that ~~a~~, it's ordinal for any STRICTLY INCREASING FUNCTION.

hence preference relation related to ~~utti~~ utility Function is an ordinal property. While the values associated to any bundles in X, are cardinal properties.

Prove (proposition 1.B.2)

A preference relation $\succsim$ can be ~~denoted as~~ represented as a utility function iff $\succsim$ is rational. — doesn't violate completeness, " transitivity.

$\Longleftrightarrow$

Proof To show this, we prove that If ~~U(x)~~ $U(\cdot)$ represents $\succsim$, then $\succsim$ is rational.

Now $U: X \to R$;

1.) if ~~U(x) & U~~ $U(\cdot)$ ~~are~~ is a ~~strictly increasing~~ real valued Function ~~st~~, ~~U: X → R~~ then $U(x) \geq U(y)$, ~~For~~ $\forall$ $x, y \in X$. ~~Then~~ or ~~U(x)~~ $U(y) \geq U(x)$ $\forall$ $x, y \in X$ or Both. But since $U(\cdot)$ ~~reprents~~ represents $\succsim$, $\Rightarrow$ $x \succsim y$ or $y \succsim x$ or both. Hence satisfing completeness.

2.) if $U: X \to R$; $U(\cdot)$ is a real valued function, then if $U(x) \geq U(y)$ & $U(y) \geq U(z)$, $\forall$ $x, y, z \in X$, since $U(\cdot)$ represents $\succsim$, then $x \succsim y$, $y \succsim z$ then $x \succsim z$. $U(x) \succsim U(z)$

So transitivity is satisfied.

Can any rational preference relation ie $\succsim$ some $x \succsim y$ be described as an utility function?

NO. (Why?) (not always)

~~↳ 1.B.4 A. (You yourself has proved it. (You proved it wrong, redo)~~

When X is Finite, we can present rational preference relation as Utility function.

CHOICE RULES (Second Approach to Decision making)

Choice Structure :- It represents the ~~outcome of a~~ ~~Binary representation.~~ choice behaviour given $\mathcal{B}$ represented by $(\mathcal{B}, C(\cdot))$

It consists of 2 ingredients.

① $\mathcal{B}$ is a non empty Family of subsets of X st

for $\forall B \in \mathcal{B}$ { B is an element of $\mathcal{B}$ }.

B set $B \subset X$.

or, simply,

~~set $B \in B \subset X$~~ set $B \in \mathcal{B} \subset X$

$\Rightarrow$ ~~set $B \subset$~~ $\Rightarrow$ set $B \subset X$.

{ infact set B is a subset of $\mathcal{B}$ also and not only an element if written in subset form }

$\mathcal{B}$ are a collection of budget sets, while B is a single budset set inside it.

(11) $C(\cdot)$ is a choice rule ~~or~~ or an outcome of choice behaviour.

~~ie here assigns~~ given a budget set $B \in \mathcal{B}$, st

$$C(B) \subset B, \ \forall B \in \mathcal{B} \subset X.$$

when $C(B)$ contains a single element, ie $C(\{x,y\}) = \{x\}$, then

~~that~~ where $\{x,y\} = B$ ~~and~~, then that element in $C(B)$ is the individual's choice from among other alternatives in B.

However $C(B)$ may ~~coin~~ contain more than one elements as it's outcome, when that happens then we can say that those element of $C(B)$ are alternative(s) in B among other alternative that the decision maker might choose. ie, these are here acceptable or otherwise affordable alts, in B. So in that case of have 2 or more elements as a choice outcome of $C(B)$, tells us that an individual can choose the acceptable alternatives in B if she faces the problem of choosing alternatives in B repeatedly

ex - 1.C.1 (very imp) (VIOLATION OF CONSISTENCY, HENCE WARP)

$X = \{x, y, z\}$ $\nearrow B_1$ $\nearrow B_2$ person chooses x no matter what budget set

$\mathcal{B} = \{\{x,y\}, \{x,y,z\}\}$

$C_1(\cdot)$ is a choice rule st; $C_1(\{x,y\}) = \{x\}$ and $C_1(\{x,y,z\}) = \{x\}$

$C_2(\cdot)$ " st; $C_2(\{x,y\}) = \{x\}$ and $C_2(\{x,y,z\}) = \{x,y\}$

example of 1 or more than 1 choice outcome; ~~given~~ given different budget sets

However, there is an issue is choice rule $C_2(\cdot) = \{x, y\}$

This actually violates the weak axiom of Revealed Preference which states (varian wala)

if X is revealed preferred to Y, then when x is affordable, Y cannot be revealed prefered to X For any budget set comb

~~Weak~~ WARP suggests that individual behaviour displays a certain sense of consistency.

in $C_1(B) = C_1(\{x, y\}) = \{x\}$

$C_1(\{x, y, z\}) = \{x\}$

x was choosen no matter what so individuals' preference on bundles stay consistent.

in $C_2(B) = C_2(\{x, y, z\}) = \{x\}$

$C_2(\{x, y, z\}) = \{x, y\}$

here individual chooses $\{x\}$ in $B_1 = \{x, y\}$ while $\{x, y\}$ in $B_2 = \{x, y, z\}$. This implies that the individual will violate consistency in this manner, hence violating WARP.

ACTUAL DEFINITION OF WARP

<u>I.C.1</u>

if for some $B \in \mathcal{B}$, & $x, y \in B$, we have $x \in C(B)$, then for any $B' \in \mathcal{B}$, & $x, y \in B'$, if $y \in C(B')$, then it must be that $x \in C(B')$

or simplely, if x is choosen, when y is affordable, then there can be no budget set containing both alternatives for which y is chosen and x is not choosen (when x is affordable).

prove WARP: (class mein MB gave).

$\boxed{\text{Denotation of revealed preference } \succsim^*}$

$x \in C(B)$.

if $x \succsim^* y_B \Leftrightarrow$ there is some $B \in \mathcal{B}$, and $x, y \in B$ st ~~$C(B) \ni x$~~.

↳ revealed prefered.

NOTE

revealed preference relation $\succsim^*$ need not be transitive or complete. (How? Prove).

1) why completeness may be not followed ?

Because, under preference theory, one can decide to compare 2 things or bundles for which one they Like better or prefer better or Like them equally. However under choice theory or revealed preference theory, this primitive preference behaviour is not observed. Under RP, we only see the choices that someone makes, given a constraint and not the rationale behind it. If the person under RP theory never had to choose btwn the 2 options he was offered, he would have never choosen one.

2) why transitivity may not be followed?

cyclical preference relation can occur.

NOTE (This can be important to prove 1.C.8)

if x is revealed prefered to y, then we can also say that if $x, y \in B$, $B \in \mathcal{B} \subset X$, and $x \in C(B)$ and $y \notin C(B)$, then x is chosen over y, even when both are feasible or affordable.

The Relationship bt^wn Preference Relation & Choice rules.

Two important relations to be established.

1) ~~t~~ if an ~~d~~ individual has rational preference ordering $\succsim$ does ~~to~~ that generate choice structures $(\mathcal{B}, C(\cdot))$, where $\mathcal{B}$ is the possible budget sets she/he is facing a choice from, satisfy WARP

↳ yes. {ie preference relation ⇒ revealed preference}

2) if individual's ~~was~~ choice behaviour given a budget set $B \in \mathcal{B}$ is captured by a choice structure $(\mathcal{B}, C(\cdot))$ satisfiying WARP, imply or necessarily mean, rational preference relation are consistent with these choices.

↳ may be {ie does $\succsim$ P ⇒ ~~Preferetatio~~ Preference relation}

ie does it mean

$\succsim$ P ⇔ P ??

To answer the first point ie $P \Rightarrow \gamma P$.

- Suppose, an individual ~~chooses~~ wants to maximise it's preference relation $\succsim$, given a budget set or a set of alternatives in X where $B \subset X$.

So given, the individual faces a nonempty subsets budget set of alternatives, in case X is finite.

~~here~~ his preference maximising behaviour would be

$$C^*(B, \succsim) = \{x \in B \subset X : x \succsim y \text{ for } \forall y \in B\}. \text{ \& } C^*(B, \succsim) \neq \phi$$

This holds true when X is a finite set of alternatives, as we can clearly ~~chose or~~ rank the options according to our preference, countably.

~~In case a set where X~~

In case a set where X is infinite st.

$$X = \{x \in R : 0 < x < 1\}$$

here, if preference of an individual is to choose the bundle x between open set (0, 1), of highest rank, then given there are infinite elements between 0 & 1, it becomes impossible to choose the "most preferred" bundle.

ie in $X = \{x \in R : 0 < x < 1\}$, as $x \to 1$ without reaching 1, there can be no single largest value in between which the individual can choose from. So $C^*(B, \succsim) = \phi$.

Now, given a choice structure can be generated from a preference relation $\succsim$ &, we define it as $(\mathcal{B}, C^*(B, \succsim))$, we need to prove it satisfies WARP.

So given $\succsim$ is a rational preference relation, & $(\mathcal{B}, C(B, \succsim))$ is a choice structure ie generated, satisfies WARP. (Prove).

→ Now, given the preference relation $\succsim$, a choice structure $(\mathcal{B}, C(B, \succsim))$ is generated. Then suppose, if $x, y \in B$ and $x \succsim y$, then $x \in C(B)$. Suppose another alternative B', where $x, y, z \in B'$, and $y \succsim z$, then $y \in C(B')$.

But through transitivity, $x \succsim y \Rightarrow x \succsim z \Rightarrow x \in C(B')$ which is our condition to WARP. This means if behaviour is generated by rational pref then it satisfies consistency requirements embodied by WARP

Definition 1.D.1

To RP ⇒ P.

Given the choice structure $(\mathcal{B}, C(\cdot))$ ~~where C(B) satisfies~~ (NOTE, $\succsim$ has not generated that choice structure)

Def 1.D.1

$\succsim$ rationalises $C(\cdot)$ for $B \in \mathcal{B}$, if

$$C(B) = C^*(B, \succsim), \ \forall B \in \mathcal{B}$$

ie if $\succsim$ generates a choice structure $(\mathcal{B}, C(\cdot))$

This can be state mathematically

if $\succsim$ generates a choice structure $(\mathcal{B}, C(\cdot))$, then for $C(B) = C^*(B, \succsim)$, where $C^*(B)$ satisfies WARP, also follows rationality. (RP ⇒ P).

In words,

Rational preference $\succsim$ rationalise $C(B)$, if a choice structure $(\mathcal{B}, C(\cdot, \succsim))$ coincides with $C(\cdot)$ for all budget set $\mathcal{B}$.

In a sense,

preference explain behaviour

- we can conclude by saying that only a choice structure generated from $\succsim$ ~~which satisfies WARP, can~~ which satisfies WARP, can conclude to be rationalised.

(or any choice rule satisfying WARP, can be rationalised)

However, WARP is not a strong assumption to attain that

<u>any</u> choice structure which satisfies WARP, satisfies rationality

eg

Suppose $X = \{x, y, z\}$ and $\mathcal{B} = \{\{x, y\}, \{y, z\}, \{z, x\}\}$

and let c_1, c_2, c_3 be choice rules for $\{x, y\}, \{y, z\}, \{z, x\}$

respectively st:

$$c_1\{x, y\} = \{x\}$$
$$c_2\{y, z\} = \{y\}$$
$$c_3\{z, x\} = \{z\}$$

All 3 of them don't violate WARP. However it's not necessary that under a choice structure that satisfies WARP, may not satisfy rationality, because here

in c_1; $x \succsim y \Rightarrow x \in c_1\{x, y\}$.

in c_2; $y \succsim z \Rightarrow y \in c_2\{y, z\}$

in c_3; $z \succsim x \Rightarrow z \in c_3\{z, x\}$

we observe a cyclical preference order which violates <u>transitivity</u>

ie $x \succsim y$; $y \succsim z$ but $z \succsim x$ (violates transitivity)

Therefore to complete Definition 1.D.1, we add the additional condition

<u>Proposition 1.D.2</u>

if choice structure $(\mathcal{B}, C(B))$: ~~satisfie~~

→ satisfies WARP

→ $\mathcal{B}$ includes all ~~ele~~ subsets of X upto 3 elements.

then there is a rational preference order $\succsim$ which rationalises $C(B)$ ~~rat~~ <u>relative</u> to $\mathcal{B}$, if $C(B) = C^*(B, \succsim)$ for all $B \in \mathcal{B}$. This preference order which does that

Exercise

1.B.1 Prove property (iii) of proposition 1.B.1.

if $\succsim$ is rational, then

(i) $\succ$ is irreflexive and transitive

(ii) $\sim$ is symmetric & "

(iii) $X \succ Y, \succsim Z$, then $X \succ Z$.

prove, $X \succ Y \succsim Z$, then $X \succ Z$

→ if $Y \succ Z$, implies $Y \succsim Z$, then $X \succsim Z$.

if $Z \succsim X$, implies $Z \succsim Y$, then $Y \succsim X$ which contradicts our assumption that $x \succ y$.

$\therefore$ $X \succ Z$ and $Z \not\succsim x$.

1.B.2

$\succ$ is irreflexive and transitive

→ if $X \not\succ X$ never holds. and $X \succ Y, Y \succ Z \therefore X \succ Z$

↳ meaning one cannot prefer a good over it's replica.

$\Rightarrow$ $x \succsim x$ according to completeness axiom. so there is no $x \in X$ st, $x \succ x$

1.B.3

if $f: R \to R$ is a strictly increasing function

$U: X \to \mathbb{R}$ is a utility function representing a preference relation $\succsim$, then $V: X \to R$ defeind as $V(x) = f(U(x))$ is also a utility function representing preference relation.

→ According to definition of utility function

if $x \succsim y$, given $x, y \in X$.

iff

$U(X) \geq U(Y)$, where $U: X \to R$ representing the preference relation $\succsim$

Now, let $F: R \to R$ is a strictly increasing function

then if $x > y$.

then $F(x) \geq F(Y)$

now, since $U(x) \geq U(Y)$

then $F(U(x)) \geq F(U(Y))$

$\Rightarrow$ $V(x) \geq V(Y)$.

$\Leftrightarrow$ $x \succsim y$.

hence f represents $\succsim$.

1.B.4

Given,

iff $U(x) = U(y) \Rightarrow x \sim y$

$U(x) > U(y) \Rightarrow x > y$.

then $U(\cdot)$ represents $\succsim$

$\rightarrow$ if $U(x) = U(y) \Rightarrow x \sim y$ and $U(x) > U(y)$

$\Rightarrow x > y$

$\Rightarrow U(x) \geq U(y) \Rightarrow x \succsim y$.

RE:

1.B.2

(iii) prove

if $x > y \gtrsim z$, then $x \succ z$

Proof

if $x > y \Rightarrow x \gtrsim y$ but not $y \gtrsim x$

and $y \gtrsim z \Rightarrow y > z$ or $y \sim z$

$\Rightarrow x \gtrsim z$ but not $z \gtrsim x$, through transitivity

$\Rightarrow x > z$

Conversely through contradiction

if $z > x$

$\Rightarrow z > y$

then $y > x$ {it's not possible}.

1.B.2

(i) $>$ is both irreflexive ($x > x$ never holds) and is transitive

ie $x > y, y > z \Rightarrow x > z$

Proof

irreflexivity

Through completeness

$x \gtrsim x \Rightarrow x > x$ and $x \sim x$

thus $x > x$ never holds

Transitivity

as $x > y$, $y > z$

then $x > y \gtrsim z$ as $y > z \Rightarrow y \gtrsim z$

thus $x > z$ {From property (iii)}

1.B.3

$f: R \to R$ is a strictly increasing function

$U: X \to R$ is utility function represent $\succsim$, then function $V: X \to R$

defined by $V(x) = f(U(x))$ is a utility function representing $\succsim$

$\to$ if f is strictly increasing function

$\Rightarrow$ $x > y$

$\Leftrightarrow$ $f(x) > f(y)$

now

$U: X \to R$, $U(x)$ represents $\succsim$

$\Rightarrow$ $x \succsim y \Leftrightarrow U(x) \geq U(y)$.

since $U(\cdot)$ is monotonic, ie $U(x) \geq U(y)$

then $F(U(x)) > F(U(y))$

$\Rightarrow$ $x \succsim y \Leftrightarrow U(x) \geq U(y) \Leftrightarrow F(U(x)) \geq F(U(y))$

$\Rightarrow$ $x \succsim y \Leftrightarrow V(x) \geq V(y)$; where

$F(U(\cdot)) = V(\cdot)$

$\forall V: X \to R$

1.B.4

Given;

if $U(x) = U(y) \Rightarrow x \sim y$

if $U(x) > U(y) \Rightarrow x > y$

then $U(\cdot)$ represents $\succsim$ ie $U(x) \geq U(y) \Leftrightarrow x \succsim y$

We know

$\succsim$ is rational

$\Rightarrow$ $x \not\succsim y \Leftrightarrow x > y$ hence $U(x) > U(y)$

$\Rightarrow$ $U(x) > U(y) \Rightarrow x > y \Rightarrow x \succsim y$

Now

1.C.1 B

$(\mathcal{B}, C(\cdot))$

with $\mathcal{B} = \{x, y\}, \{x, y, z\}$

and $C(x, y) = \{x\}$.

show, if $(\mathcal{B}, C(\cdot))$ satisfy WARP, then

$C\{x, y, z\} = \{x\} = \{z\} = \{x, z\}$.

→ NOW

Given $C\{x, y\} = \{x\}$ and $\{\mathcal{B}, C(\cdot)\}$ satisfy WARP

Let $\{x, y\} = B$ and $\{x, y, z\} = B'$

So for WARP to hold, if $C(B) = \{x\}$, where $x, y \in B$, thus

$x \succsim^* y$, then, for any $B' \in \mathcal{B}$, $x, y, z \in B'$ and

$y \succsim^* z$ st. $y \in C(B')$, then x must also $\in C(B')$ as $x \succsim^* y$.

or if $z \succsim^* y$ in B', then since $x \succsim^* y$ and $z \succsim^* y$, then

$x \sim z$, hence $\{x, z\} \in C(B')$ or $\{z\} \in C(B')$

$\therefore C(x, y, z) \in \{x\}, \{z\}, \{x, z\}$

Chapter - 2 Consumer choice.

OVERVIEW

- The fundamental decision unit of microEconomic theory is 'consumer'
- By market Economy; we mean a setting in which good & services that the consumer may aquire are availdble for purchase at known prices.
- consumer is limited by Economic & physical contraints that affect his choice
  - ↳ captured by WALRASIAN BUDGET SET.
  - ↳ captured by consmpt set
- consumer's decision subject to these constraints is captured by the consumer's WALRASIAN DD FUNCTION
  - ↳ in choice Based approach; WALRSIAN DD function is the choice rule of a consumer

Weak axiom of RP ≡ compensated Law of DD.

↳ it's a postulate which says prices & DD quantities move in opposite directions for price changes that leave real weath unchanged.

Commodities (2.B)

The Decision of a consumer in a market is to choose their consumption Level from various goods and services that are available to purchase.

↳ These goods & services together are called commodities.

We denote commodities with $\underset{\sim}{l}$. (used as index).

Total no of commodities = Finite & is denote as L

$\therefore$ ~~$l = \{1, 2, 3 \ldots L\}$~~ $l = 1, 2 \ldots L$

If we capture different commoditie(s) ~~in a~~ ie $l = 1, 2 \ldots L$ as a BUNDLE from which a consumer ~~can~~ chooses ~~[illegible]~~, then it's called a consumption Bundle. Usually a consumer chooses a bundle.

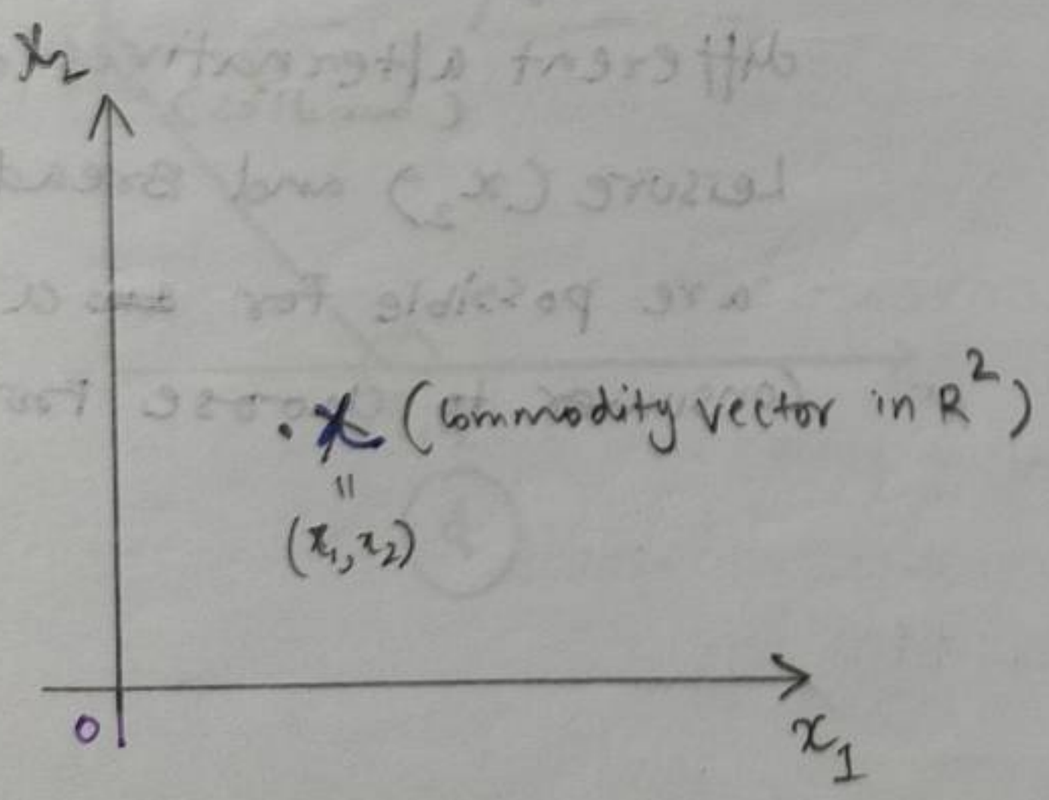
we denote as consumption Bundle ~~as a vector~~ ~~called~~ ~~consump~~ or commodity vector or ~~commodi~~ consumption vector.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_L \end{bmatrix}$$; where $x_1, x_2 \ldots x_L$ are commodities with index $l = 1, 2, 3 \ldots L$

where $x \in R^L$ and is a point in R^L space; R^L = commdity space

ie if $x \in R^2 \Rightarrow R^2 (R \times R)$ {Two commodity space}

$$\therefore x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



where $(x_1, x_2) \in R^2$

R^L_+ = commodity is space of L commodities & is non-negative space

In $x = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix}$; x_L stands for the Lth commodity consume

Consumption Set (2.c) { A physical constraint }

A consumption set acts as a physical contraint that limit's consumer's choice among consmp bundle.

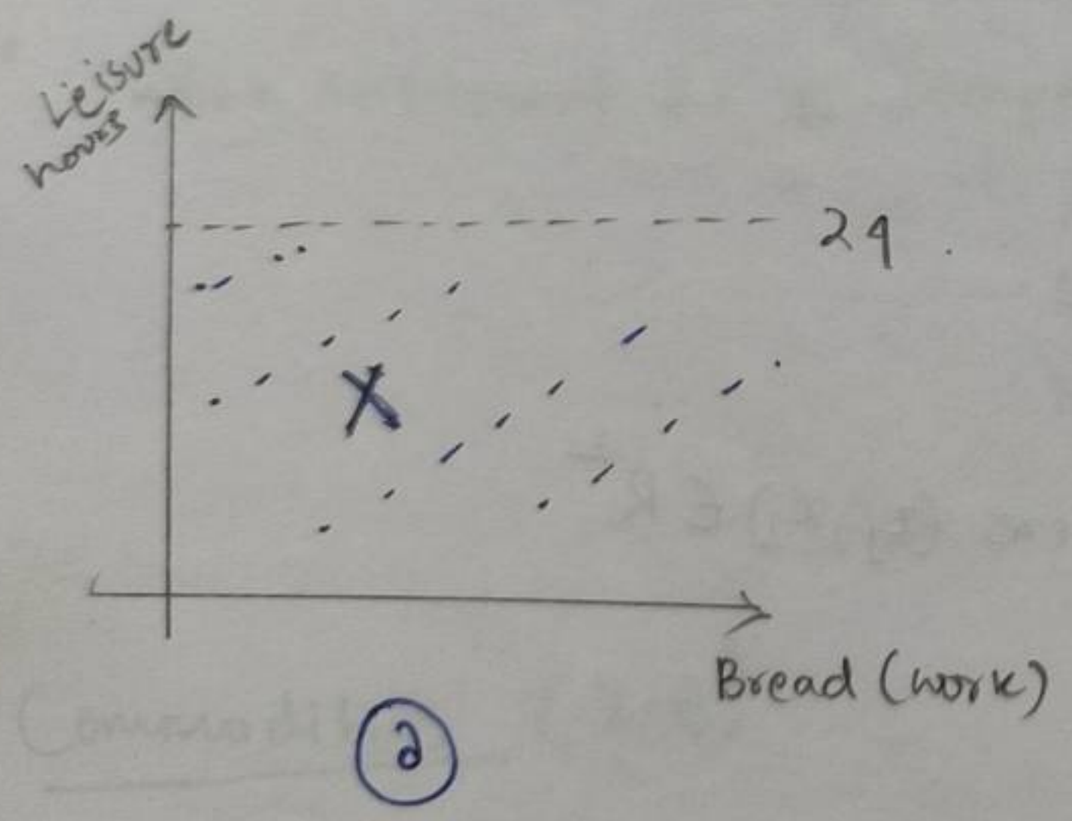
A consumption bundle is a part of consumption set; which itself is a ~~ele~~ subset of R^L (commodity space)

ie $x \in X \subset R^L$ { can include integers, natural nos etc }

X is also called the set of alternatives (Chp-1)

x is a bundle inside X; which the consumer chooses.

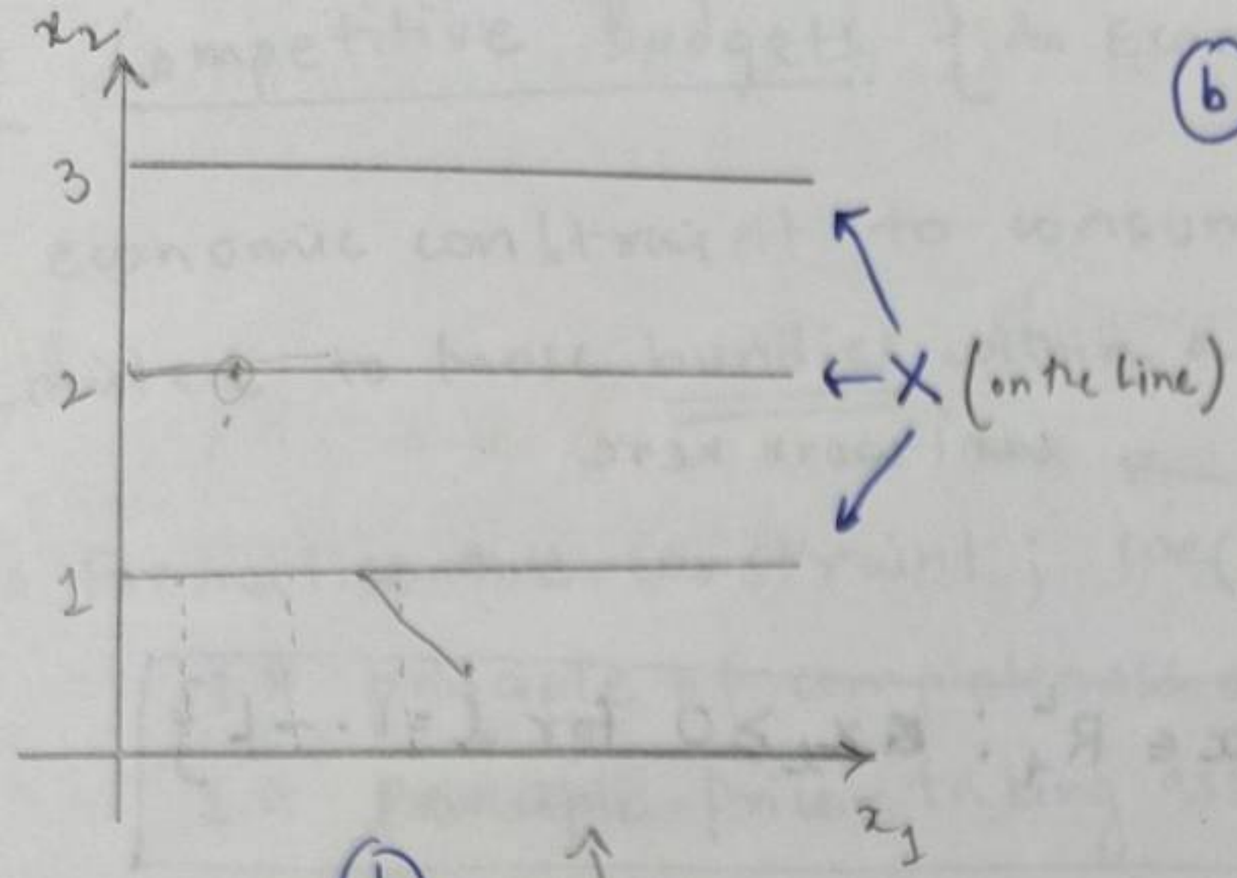
Suppose $L=2$ { two commodity space ; R^2 } { Examples of consp se



(a)

A consmp set

(a) → is a figure of ~~of~~ consumption set; in which different alternatives (bundles) of Leisure (x_2) and Bread (x_1) are possible for ~~an~~ a consumer to choose from.



(b)

A consmp set with integer amount

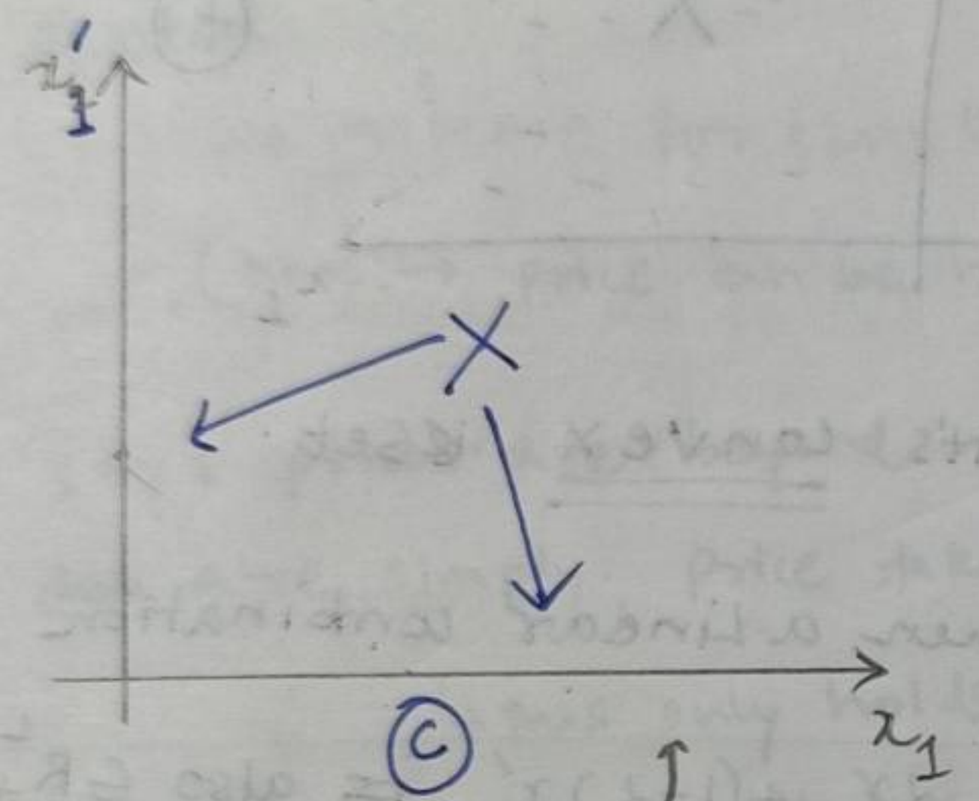
(b) → is a Figure where x_2 & x_1 are 2 commodities; and x_1 is available in continuity while x_2 is avaible in discrete amounts.

∴ commodity bundles are in different Levels :-

$(3, x_1)$ or $(2, x_1)$ or $(1, x_1)$

$\forall\ x_1 \in R$ but $x_2 \in Z$

~~So since X ⊂ R, where X ⊂ R×Z~~



(c)

A consmp set, only one good is consumed.

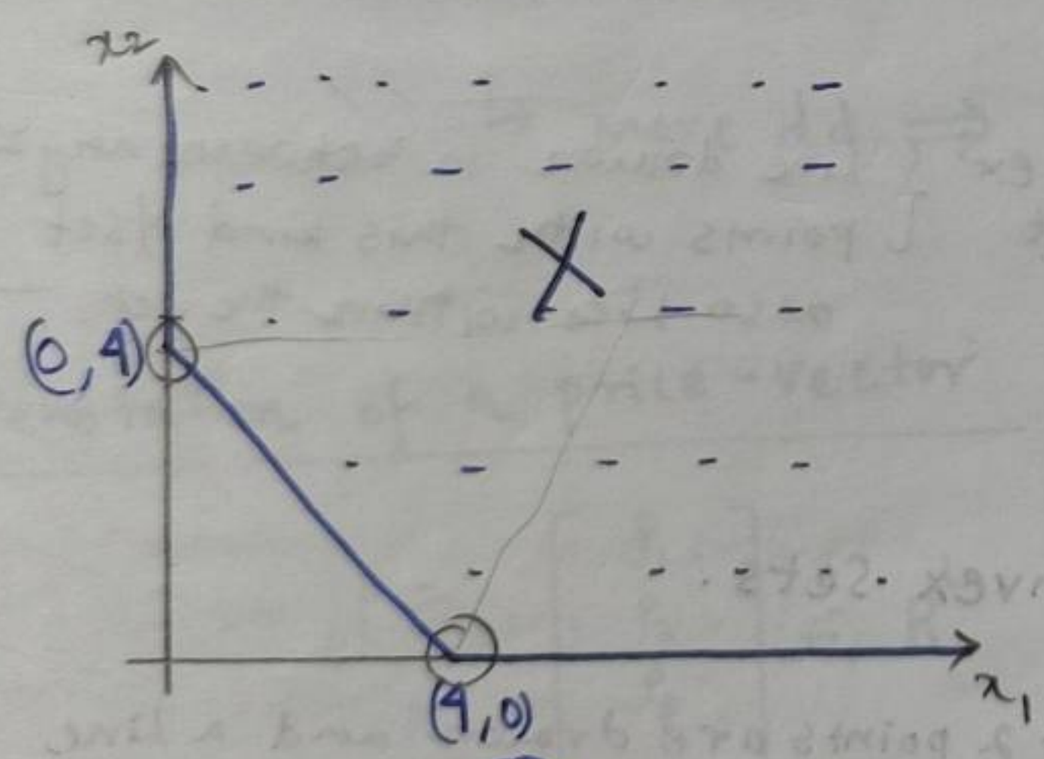
(c) → $x_1 \in R$ and let's say x_1 and x_1' are the same commodities at different Location. So it is impossible for a consumer to consume. (at the same time)

∴ consmp set is $X = x_1$ or

$X = x_1'$ or

$X = 0$

but $X \neq \{x_1, x_1'\}$

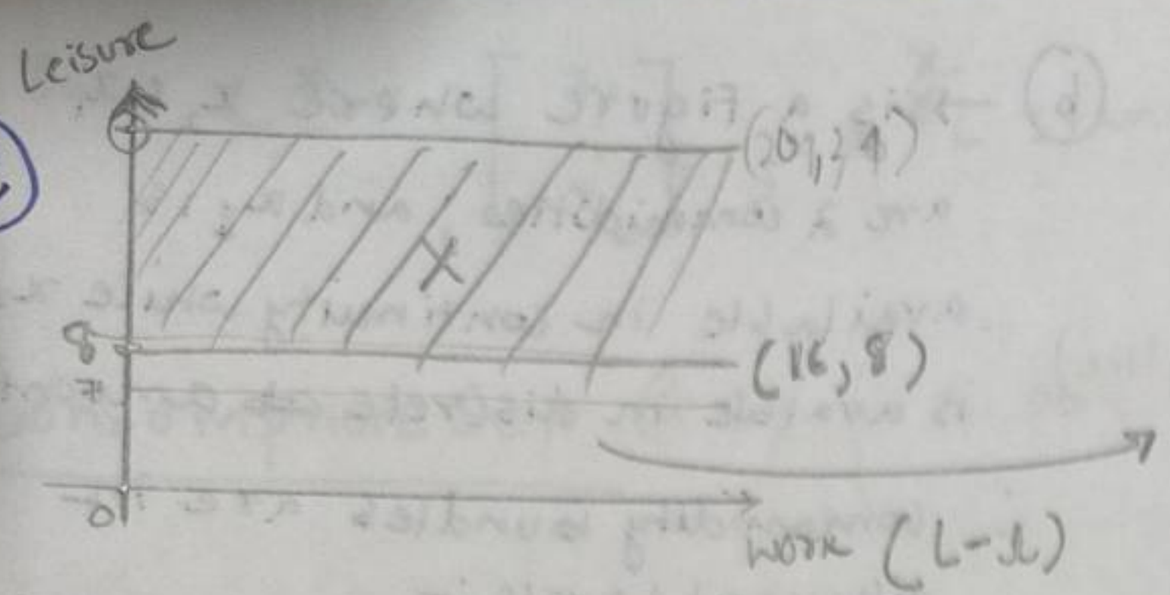


(d)

(d) → It is a consmp set where; ~~beyond~~ below (4,0) to (0,4) as ~~a~~ ~~Comptione~~ Consmption doesn't happen. ie X reflects a consmp^n set requiring minimum of either (0,4) (kink1) or (4,0) (kink2)

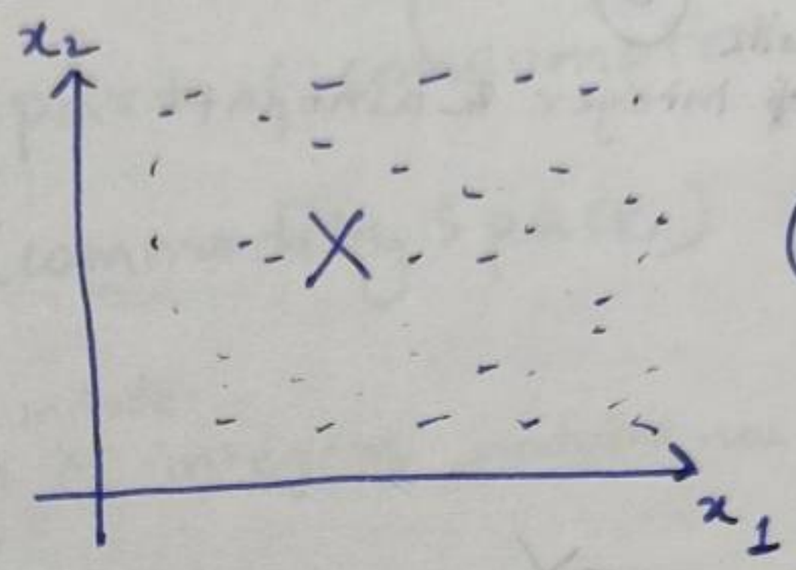
Ex-I A Law requires that no-one works more than 16 hrs. Draw the consmp set

(e)



$$\boxed{\text{Consumption set: } X = R^L_+ = \{x \in R^L_+ : x_l \geq 0 \text{ for } l = 1 \ldots L\}}$$

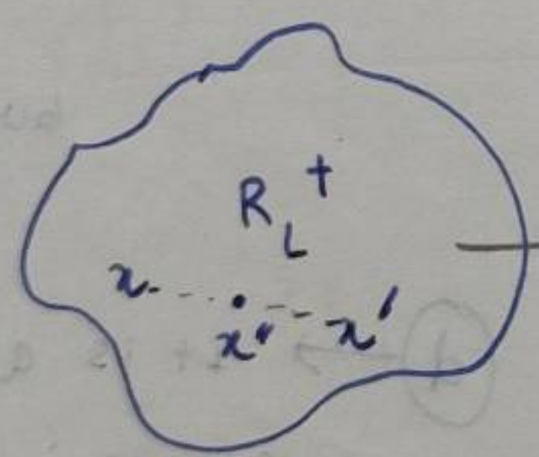
set of non-negative commodity bundles. if $L=2$ →



(f)

one special feature of $X = R^L_+$ is; it's <u>convex</u> set.

ie if x and x' $\in R^L_+$; then a Linear combination of x and x' denoted as $x'' = \alpha x + (1-\alpha)x'$, is also $\in R^L_+$



→ convex set { Line drawn in between any 2 points with this kind of set also Lies within the set }

Figure (a), (d), (e) and (f) are all convex sets.

figure (b), (c) are not because if 2 points are drawn and a Line connecting them is drawn, the Line Lies out side the defined set.

2.D Competitive Budgets {An Economic constraint}

An economic constraint to consumption ⇒ his consumption is Limited to those bundles within a consmpt set which he can afford

To formalise the constraint; we take into account 2 assmptions

1* principle of completeness or universality of markets

2* ~~Principle~~ price-taking assumption

1* → says L commodities are traded in $ prices that are publically Quoted.

we assume for simplicity; $p >> 0 \ \forall l = 1, 2 \ldots L$

(note:→ price can be negative; in that case it's subsidy)

2* → says prices are beyond influence of consumers. ie consumers are simply; price takers.

↳ this only holds if consumer's demand for a particular commodity is representing a small fraction of the total demand for a commodity.

↳ more dd ⇒ more bargaining power

Denotation of a price-vector

$$p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_L \end{bmatrix} \in R^L \quad \text{or } R^L_+ \text{ (given our assmp).}$$

Conditions of affordability

→ prices in market $p = (p_1 \ldots p_L)$

→ wealth or income of consumer W in (dollars)

thus, a consmp bundle $x \in R^L_+$ is only affordable if it satisfies

$p.x = p_1 x_1 + p_2 x_2 \cdots p_L x_L \leq W$

ie a consumption bundle x is only affordable iff it costs doesn't exceed the consumer's wealth.

here in

$$B_{p,w} = \underbrace{p \cdot x}_{\text{vector multiplication}} = p_1 x_1 + p_2 x_2 \cdots p_L x_L \leq w \qquad x \in R^L_+$$

Price of x_1 commodity

x_1 is a commodity; not a bundle here

This is also known as walrasian budget set or ~~competive~~ competitive budget set.

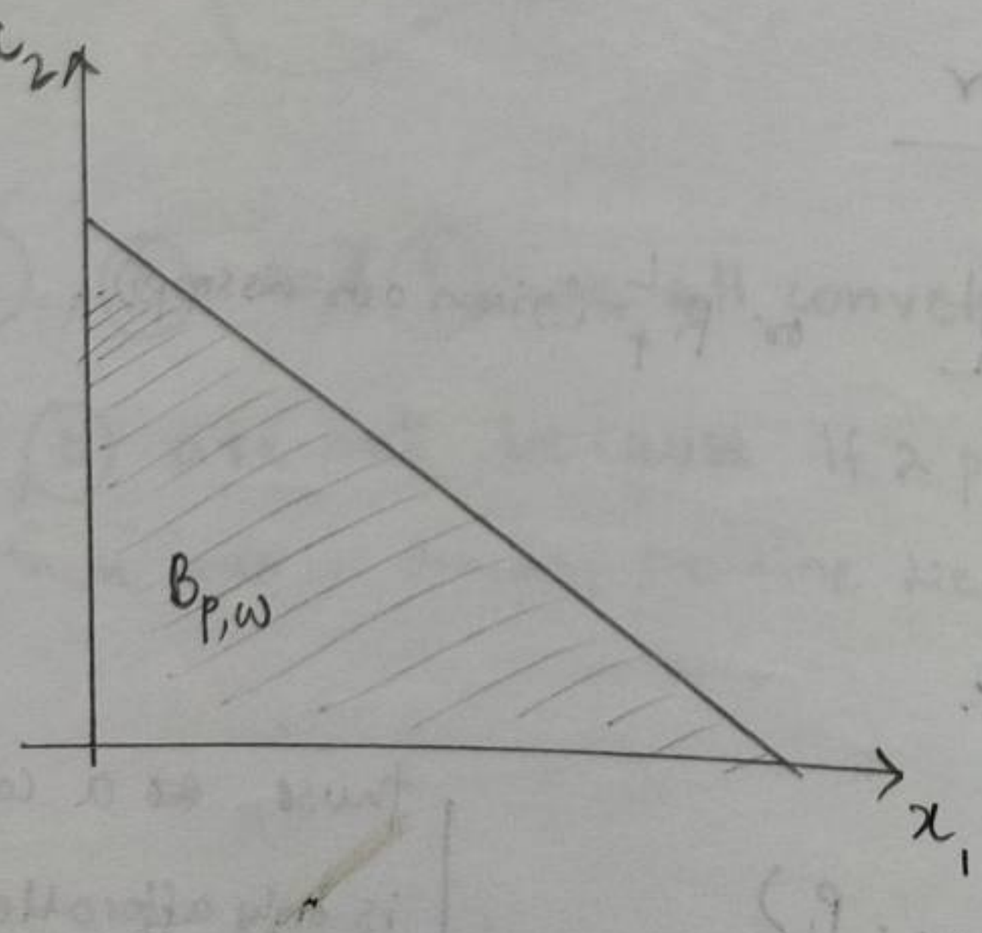
<u>set representation of walrasian budget set</u>

$$\{x \in R^L_+ ; p \cdot x \leq w\} = B_{p,w}$$

<u>Def-2.D.1</u> : A walrasian budget set is a set of <u>all</u> <u>Feasible</u> <u>consmp</u> bundles for a consumer who faces market price vector p and wealth w. ie $B_{p,w} = \{x \in R^L_+ : p \cdot x \leq w\}$

A CONSUMER'S PROBLEM:- Given a budget set ; p & w; chose an x bundle from $B_{p,w}$

<u>Depiction of Budget set</u>



we assume;

① consumer has a nondegerate choice problem ie; the commodities he has to choose from are distinct; meaning there is a clear trade off between them.

② we choose $L=2$. Hence we draw a budget LINE { For $L>2$; it is called budget hyperplane which satisfies

$$\{x \in R^2_+ : p \cdot x = w\}$$

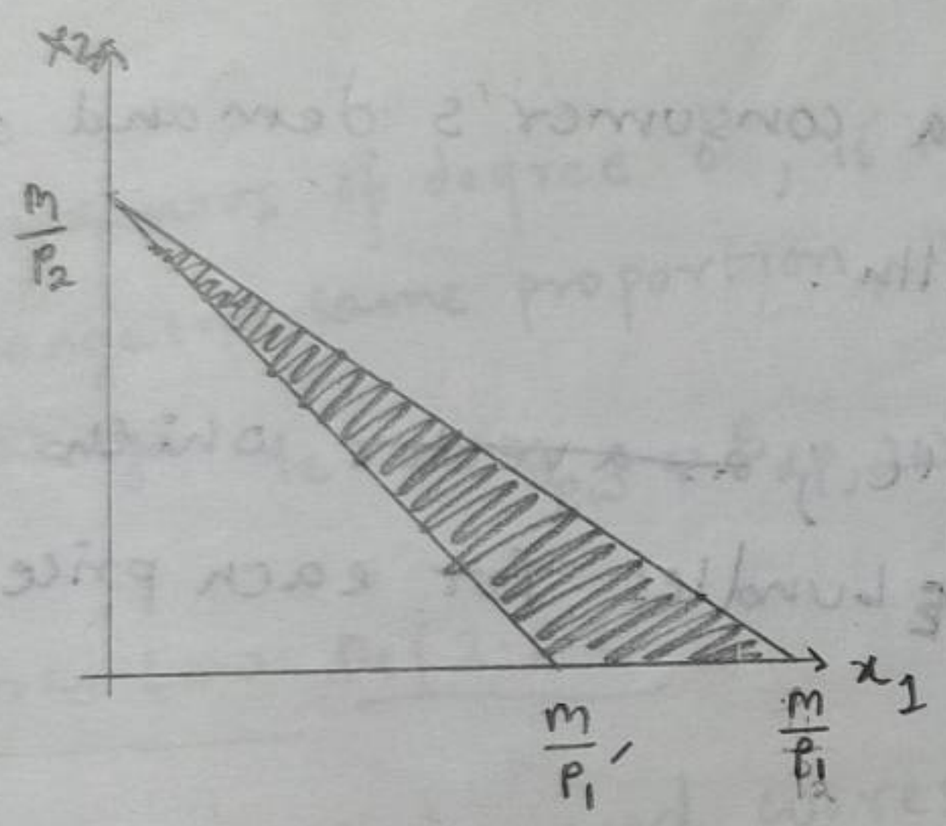
The budget line $\{x \in R^2_+ : p \cdot x = w\}$ represents the upper boundary of the budget set which is $B_{p,w} \rightarrow \{x \in R^2_+ : p \cdot x \leq w\}$.

eq representation of budget hyperplane $\boxed{x \in R^L_+ : px \leq w}$.

now; if for the following figure has the ~~axis~~ axes $y = \frac{m}{p_2}$ and $x = \frac{m}{p_1}$

then the slope of the budget line is $\frac{p_1}{p_2} \left(\approx \frac{y}{x}\right)$.

SLOPE & INTERCEPT TRANSFORMATION FOR BUDGET LINE

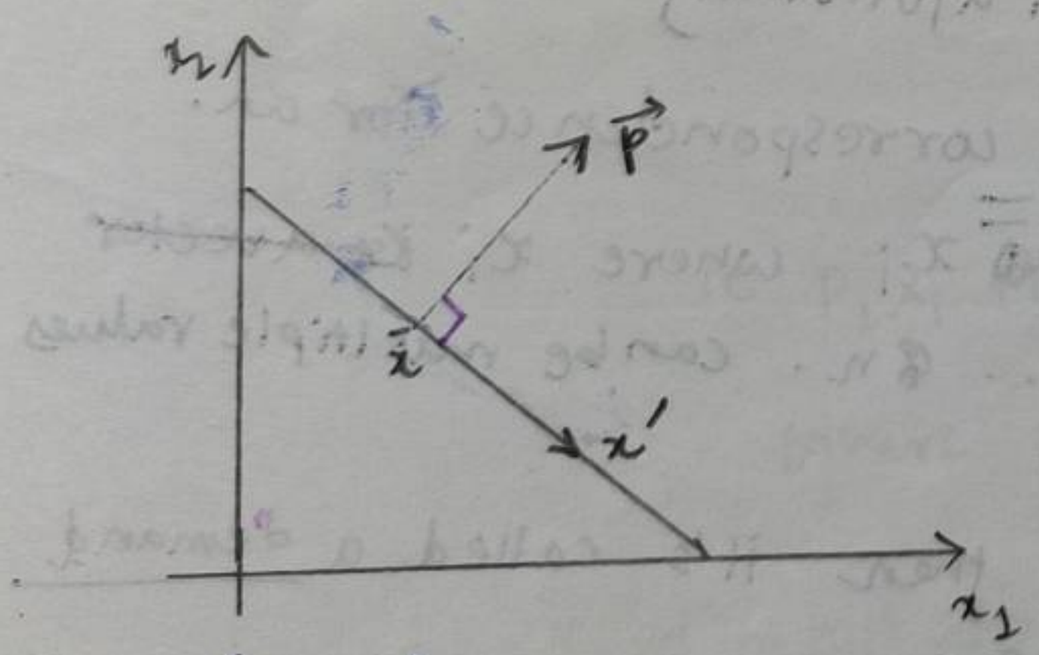


Suppose $p_1 \rightarrow p_1'$ such that

$p_1' > p_1$

The price p_1 as it increases; the budget set shrinks as the $p_1 \rightarrow p_1'$

On the other hand; a geometric representation between price and commodity bundle can also be shown ie the relation $p \cdot x = w$.



for $L = 2$.

$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$ $x' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$

$\therefore \Delta x = \begin{bmatrix} \bar{x}_1 - x_1' \\ \bar{x}_2 - x_2' \end{bmatrix}$

here; $\vec{p}$ is the price vector from point $\bar{x}$

$\vec{x}'$ is the commodity bundle starting from $\bar{x}$. Since $\vec{p}$ is perpendicular to the plane w; $\vec{p}$ and $\vec{x}'$ are both orthogonal to each other at point $\bar{x}$ with angle $90°$ st; $\vec{p} \cdot \vec{x} = p \cdot x \cos\theta$

where $\theta = 90°$

$\Rightarrow p \cdot x \cos 90 = 0$

we denote $\vec{x}$ as $\bar{x} - x' = \Delta x$

$\therefore p \cdot \Delta \vec{x} = 0$.

However; $p \cdot \bar{x} = p \cdot x' = w$

were $\bar{x}$ and x' are point vectors.

Convexity of $B_{p,\omega}$

The budget set $B_{p,\omega} = \{x \in R^L_+ : p \cdot x \leq \omega\}$ is convex set if any two points x and x' inside the budget set if joined, then $x'' = \alpha x' + (1-\alpha) x$ ~~lies~~ also lies inside the budget set st ~~$p \cdot x \leq \omega$ & $p \cdot x'$ then~~ $p \cdot x \leq \omega$, $p \cdot x' \leq \omega$ then $p x''$

$\Rightarrow p(\alpha x' + (1-\alpha) x) = \alpha p x' + (1-\alpha) p x \leq \omega$. $\therefore x'' \in B_{p,\omega}$.

Convexity of $B_{p,\omega}$ depends on convexity of R^L_+ {which is also convex}

(2.E) Demand Functions & Comparative Statics.

Demand Correspondence associates to a consumer's demand of bundles or bundle given price-wealth.

$x(p,\omega)$ is a demand correspondence; ~~as a vector~~ which assigns a set ~~ton~~ of chosen consumption bundles for each price-wealth pair (p,ω).

This correspondence is multivalued in real life $\Rightarrow$ there are multiple consumption vectors assigned to price-wealth pair (p,ω). {basically it's not a function}

in this can demand correspondence for an individual ie $x(p,\omega) = x_i$ where x_i ~~is a vector~~ ~~of multi~~ $\forall i = 1, 2 \ldots n$. can be multiple values

However, iff $x(p,\omega)$ is single valued then it's called a demand function given price-wealth pair.

correspondence $\neq$ function

many to one

~~one-one~~ unique value of the y

2 main assumptions :- (For Walrasian demand correspondence $x(p,\omega)$)

* Homogenity of degree zero {dd function are homog of degree zero}.
* Satisfies WALRAS LAW {consumer consumes his entire wealth}

↳ $\boxed{p_1 x_1 + p_2 x_2 = w}$

Homogenity Def - 2.E.1

$X(p,w)$ is a demand correspondence (or demand function) of p and w.

then $X(\lambda p, \lambda w) = \lambda^0 X(p,w)$

$= X(p,w)$ for p, w and $\lambda > 0$.

Homogenous of degree 0 ; is implying that both prices and wealth change in same proportion, then the consumption choices doesn't change. ie $B_{p,w} = B_{\alpha p, \alpha w}$ where $\alpha = \lambda > 0$

Walras Law Def 2.E.2

if $x(p,w)$ is demand correspondence (or dd function), then it satisfies WALRAS LAW.

↳ ie if $p >> 0$ and $w > 0$, we have $p \cdot x = w$ for $x \in x(p,w)$

vector

ie $p_1 x_1 + p_2 x_2 = w$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

ie a consumer fully expends his entire wealth for desirable goods that he wants to spend on. However A consumer's budget might also be intertemporal ; ie he saves today and consume tomorrow. WALRAS LAW also applies here ; ~~incase~~ where a generalised WALRAS LAW applies for consumer over his Life time.

Exercise 2.E.1 Check homogenity & WALRAS LAW

if $L=3$ ie $\cancel{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$ $x(P,\omega) = \begin{bmatrix} x_1(P,\omega) \\ x_2(P,\omega) \\ x_3(P,\omega) \end{bmatrix}$

where $x_1(P,\omega) = \dfrac{\omega P_2}{P_1(P_1+P_2+P_3)}$

$$x_2(P,\omega) = \frac{\omega P_3}{P_2(P_1+P_2+P_3}$$

$$x_3(P,\omega) = \frac{\beta \omega P_1}{P_3(P_1+P_2+P_3)}$$

Checking for homogenity for $x_1(P,\omega)$ & $x_3(P,\omega)$ for $B=1$

$$x_1(\lambda P,\lambda \omega) = \frac{\cancel{\lambda^2} \omega P_2}{\cancel{\lambda^2} P_1(P_1+P_2+P_3)} = x_1(P,\omega)$$

$$x_3(\lambda P,\lambda \omega) = \frac{\cancel{\lambda^2} \beta \omega P_1}{\cancel{\lambda^2} P_3(P_1+P_2+P_3)} = X_3(P,\omega)$$

$P_1x_1+P_2x_2+P_3x_3 = W$

$$\frac{WP_2}{(P_1+P_2+P_3)} + \frac{WP_3}{(P_1+P_2+P_3)} + \frac{\beta WP_1}{(P_1+P_3+P_2)}$$

$\Rightarrow$ if $B=1$

$$W\frac{(P_1+P_1+P_2)}{(P_1+P_2+P_3)} = W \quad \{\text{WALRAS LAW HOLD for } B=1\}$$

$\boxed{\text{For } B=(0,1) \text{ ; WALRAS LAW WON'T HOLD}}$

~~An implication of $x(P,\omega)$ being homogenous~~

we assume that $x_l(P,\omega)$ is now a function {single valued} rather than a correspondence {in generality}. Hence we can write the $x(P,\omega)$ vector in terms of commodity specifice demand function

$$x(P,\omega) = \begin{bmatrix} x_1(P,\omega) \\ x_2(P,\omega) \\ \vdots \\ x_L(P,\omega) \end{bmatrix}$$

where $x_l(P,\omega)$ are commodities $\forall\, l = 1, 2 \ldots L$

we also assume, $x(P,\omega)$ to be continous and differentiable. (then it's a function)

Comparative Statics

underlying changes in $x(P,\omega)$ due to economic parameters P, ω is called comparative statics.

WEALTH EFFECTS & ENGEL CURVE (note, W is not a vector. It's a single variable)

fix p in $x(\bar{P},\omega)$ and study the effect of ω in $x(\bar{P},\omega)$; ~~this~~ ~~given~~ if $x(\bar{P},\omega)$ is just a function of ω is called a consumer's Engel curve or Engle function.

A function is has it's image.

ie $F(x) = y$ → image.

∴ For Engel function; it's image is called wealth Expansion. ~~{curve se~~

ie ~~E(x)~~ $E(x(\bar{P},\omega)) = y$

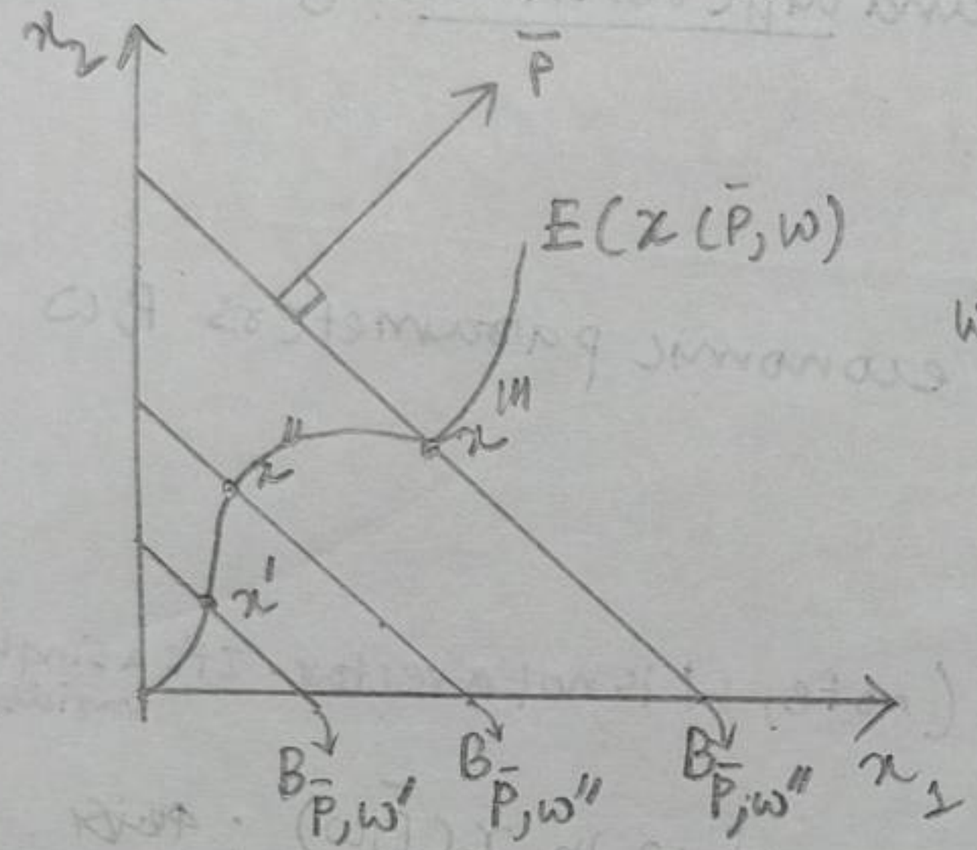
or $E_p = \{x(\bar{P},\omega) : \omega > 0\}$ in R^L_+

Wealth Effects:- $\dfrac{\partial x_L(P,\omega)}{\partial \omega}$ at any (P,ω) ~~is for L^{th} good $x_L \in x$~~ for L^{th} good $x_L(P,\omega) \in x(P,\omega)$ is called income Effect or wealth effect.

NOTE

- $x(P,\omega)$ = dd function ; if $x(P,\omega)$ is single valued
- $x(\bar{P},\omega)$ = Engel function or $E(x(P,\omega))$; where P is fixed.
- $E(x(P,\omega)) = y$

  y image is called wealth (Income) Expansion path.
- $\frac{\partial x_l(P,\omega)}{\partial \omega}$ = income effect $\forall\ x_l(P,\omega) \in x(P,\omega)$



(normal good Engel curve)

where $x', x'', x''' \in x(\bar{P},\omega)$

- $\omega' > \omega'' > \omega'''$
- $B_{\bar{P},\omega'''} > B_{\bar{P},\omega''} > B_{\bar{P},\omega'}$
- P is fixed ie $P' = P'' = P''' = \bar{P}$ (Price vector)
- $E(x(P,\omega))$ is Engle function and the curve is called wealth expansion path.

$x_1, x_2 \in X$

Properties of income effect:-

- if l commodity is normal; $\frac{\partial x_l(P,\omega)}{\partial \omega} \geq 0$ ie it's non decreasing in wealth
- if l comodity is inferior ; $\frac{\partial x_l(P,\omega)}{\partial \omega} \leq 0$ ie it's decreasing in wealth.

goods are usually considered normal unless stated otherwise. The large aggregated goods ~~like~~ or the essential goods' demand increase as wealth increases (normal demand behaviour) but for commodities like particular type of shoes or disaggregated commodity; we substitute amongst them from lower quality commodities to higher quality commodities, making these lower quality commodities as wealth increases, an inferior good.

MATRIX REPRESENTATION OF WEALTH EFFECTS

Wealth or income Effect :- $\frac{\partial x_L(P,\omega)}{\partial \omega}$ {For a commodity}

in matrix; $x(P,\omega) \rightarrow \begin{bmatrix} x_1(P,\omega) \\ \vdots \\ x_L(P,\omega) \end{bmatrix}$

$$\Rightarrow \frac{\partial x(P,\omega)}{\partial \omega} = \begin{bmatrix} \partial x_1(P,\omega)/\partial\omega \\ \partial x_2(P,\omega)/\partial\omega \\ \vdots \\ \partial x_L(P,\omega)/\partial\omega \end{bmatrix}$$

PRICE EFFECTS AND OFFER CURVE (P is a vector. It has $P_1, P_2, P_3 \ldots P_L$)

Effect on commodity with ω as fixed, if price changes.

Let $L=2$ {Two commodity space}, ω is fixed and P_1 is also fixed.

we observe the study of x_2 good due to changes in it's $\boxed{\text{own price.}}$

(i) own price Effect

$$\frac{\partial x_K(P,\omega)}{\partial P_K}.$$

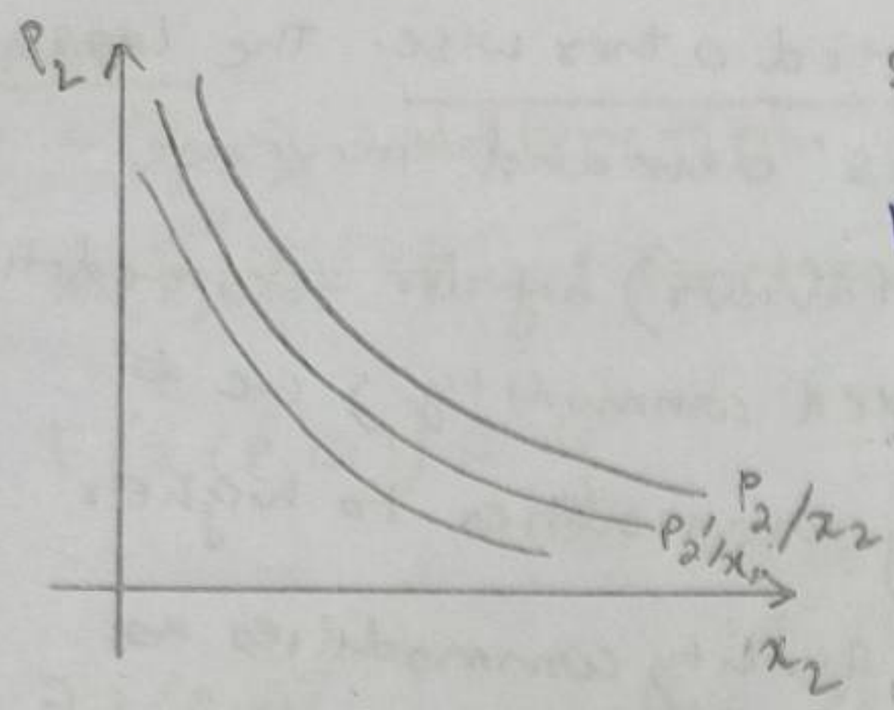
ie change in K^{th} good due to price of Kth good

(ii) Cross price Effect.

$$\frac{\partial x_L(P,\omega)}{\partial P_K}$$

ie change in L^{th} good due to price of Kth good.

So now; given we want to study the own price Effect; we draw the consumer demand function curve assuming the good as an <u>ordinary</u> good



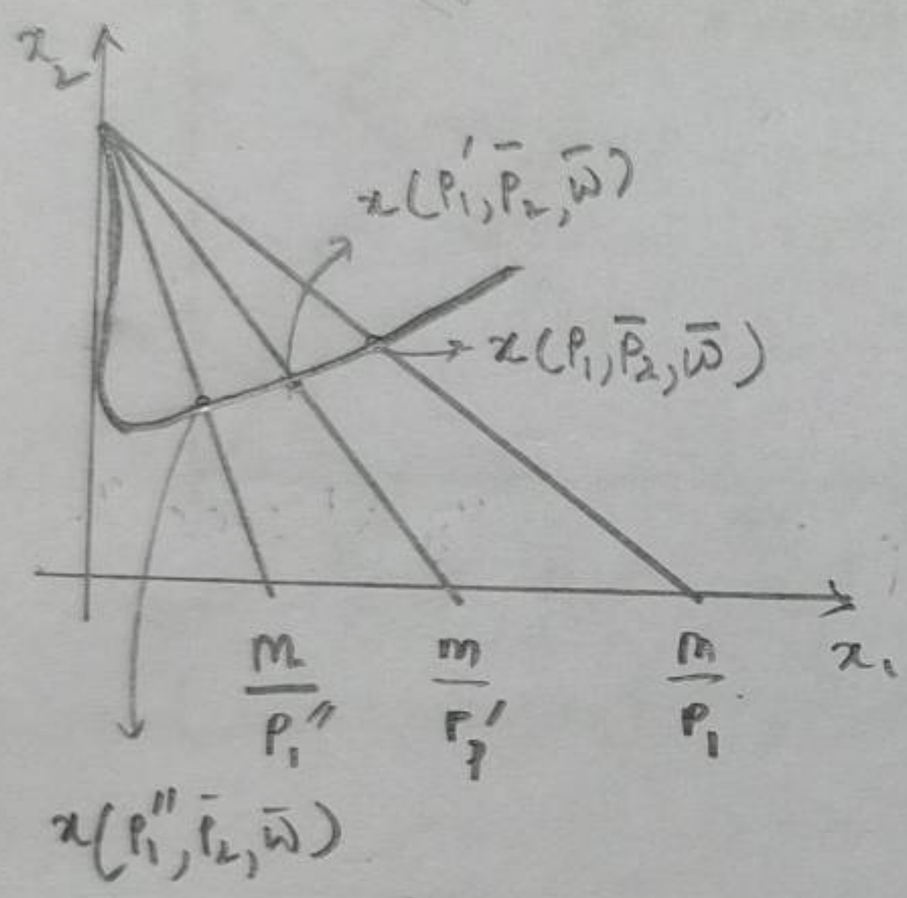
slope of dd $= \dfrac{dP}{dx}$

price Effect $= \dfrac{dx}{\partial P}$

$\boxed{\text{in case of ordinary good } \dfrac{\partial x_k}{\partial P_k} < 0}$

~~connecting the locus of all points on the DEMAND CURVE~~ gives us the price offer curve

REPRESENTATION OF PRICE EFFECT IN R^2 SPACE

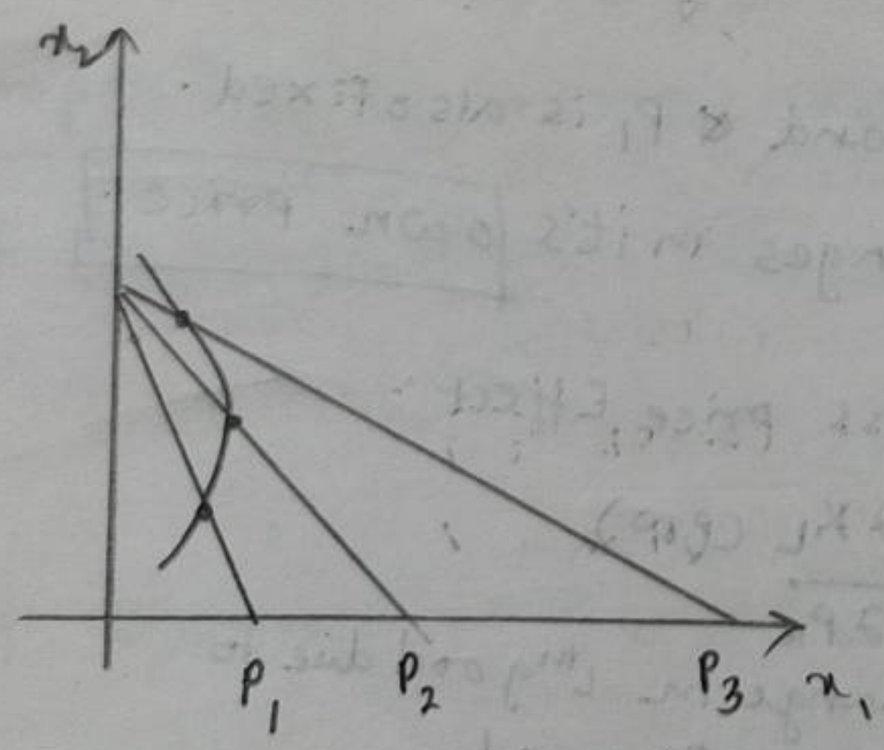


Locus of all points on the budget lines, due to change in prices; gives us the POC or price offer curve.

$P_1'' > P_1' > P_1$

$x_1'' < x_1' < x_1$

GIFFEN GOODS and POC



suppose x_1 is giffen good.

then $\dfrac{\partial x_1}{\partial P_1} > 0$ ie as $P\uparrow, x\uparrow$

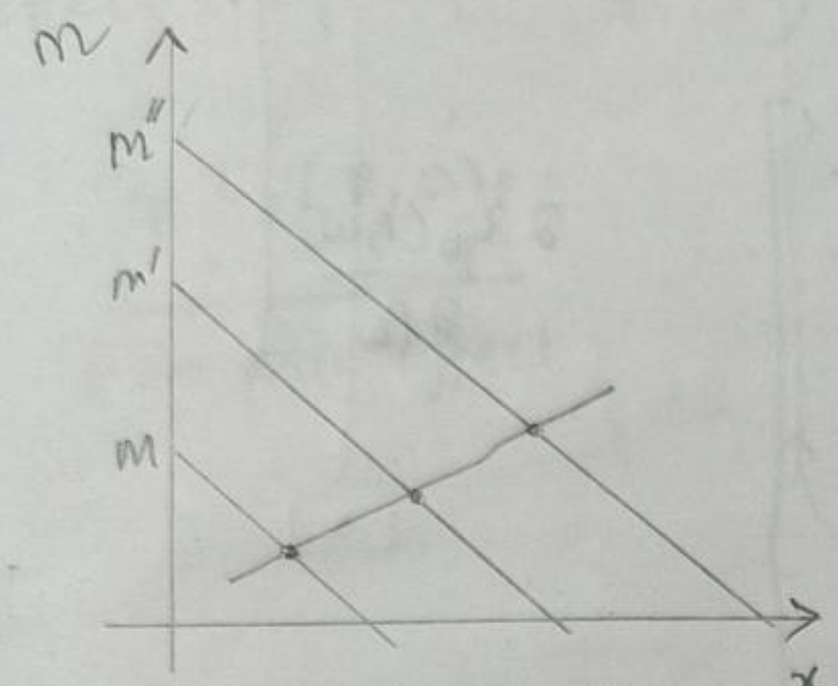
or $P\downarrow, x\downarrow$.

as $P_3 < P_2 < P_1$

$x_3 < x_2 < x_1$

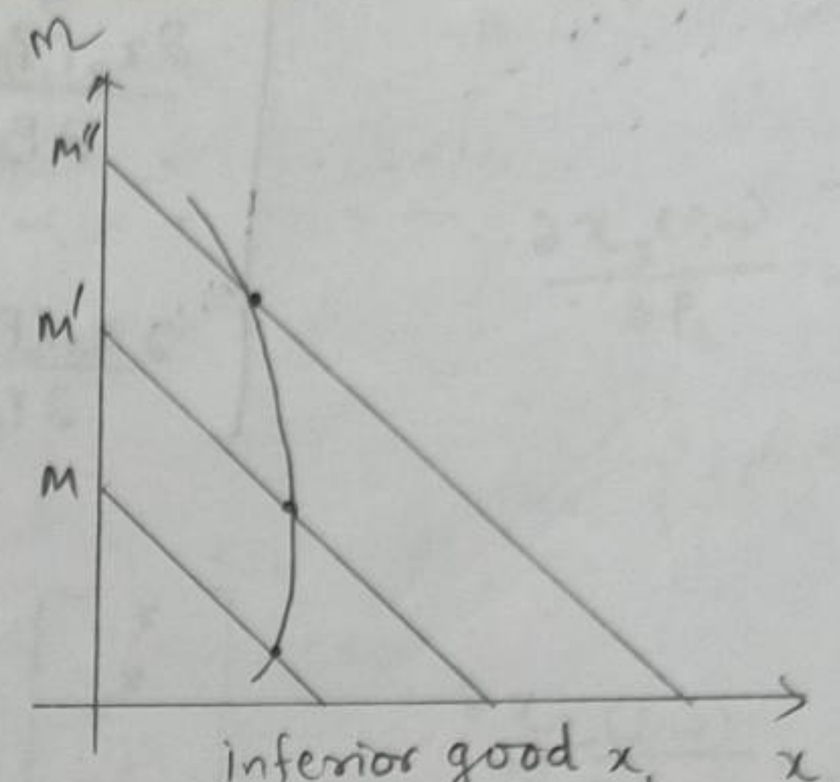
ENGEL CURVE REPRESENTATION & PRICE OFFER CURVE

Normal goods & inferior goods. (m, x) space



Engel curve for x_1 is normal

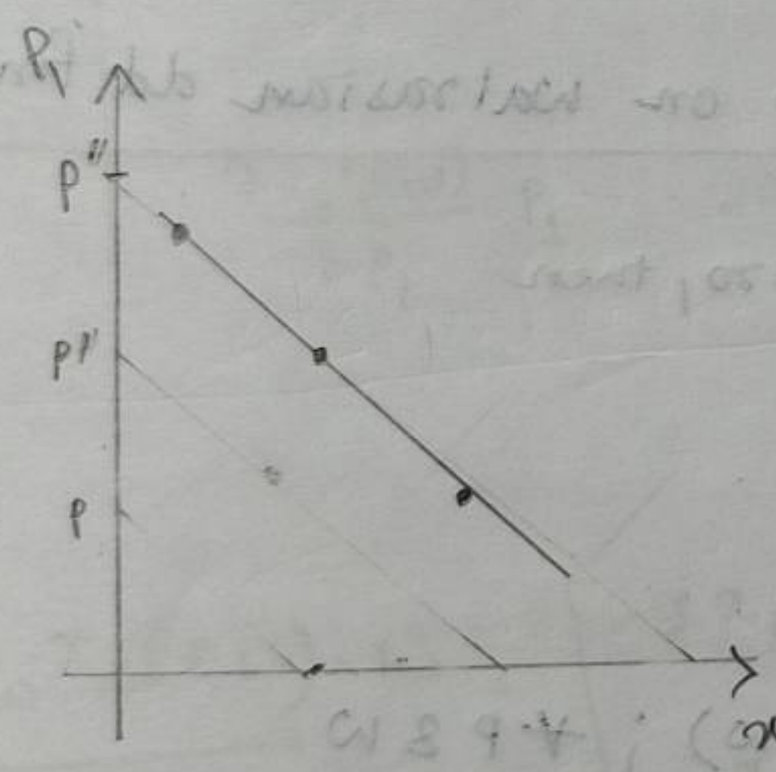
ie $\frac{\partial x_1}{\partial m} > 0$



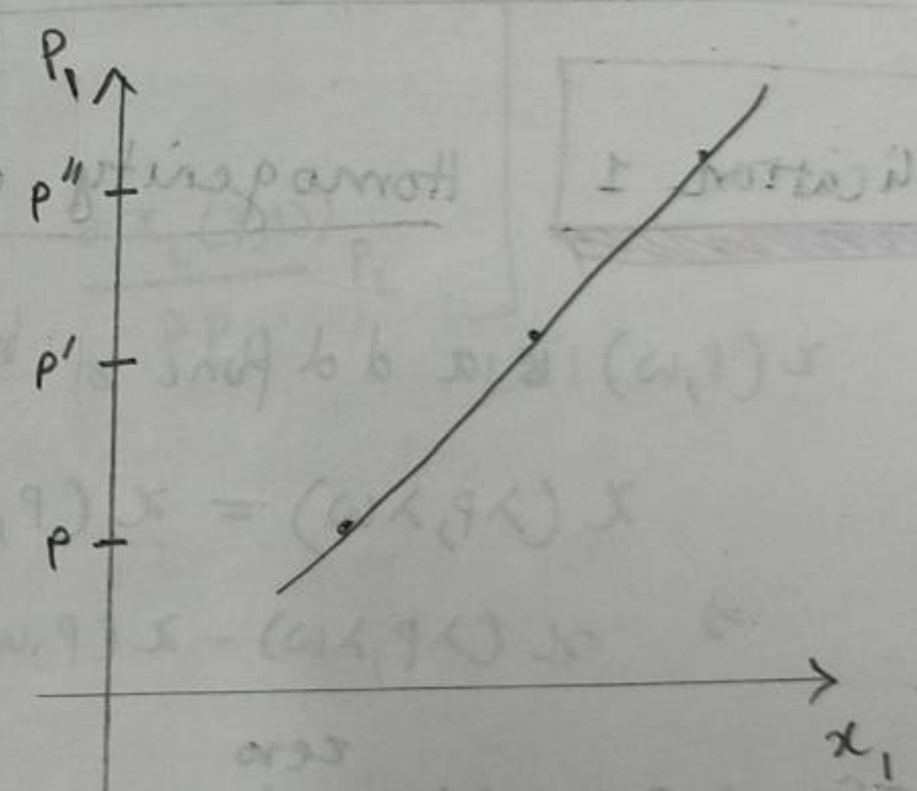
inferior good x_1

ie $\frac{\partial x_1}{\partial m} < 0$

Ordinary goods & Giffen good (P, x) space {basically dd space}



Price offer curve for ordinary good



Price offer curve of giffen (violation of g Law of dd)

Price Effect Matrix representation

$$D_p x(p,w) = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} & \frac{\partial x_1(p,w)}{\partial p_2} & \cdots & \frac{\partial x_1(p,w)}{\partial p_L} \\ \frac{\partial x_2(p,w)}{\partial p_2} & & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial x_n(p,w)}{\partial p_1} & \cdots & \cdots & \frac{\partial x_L(p,w)}{\partial p_L} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_L \end{bmatrix}_{n\times 1} \begin{bmatrix} p_1, p_2 \cdots p_L \end{bmatrix}_{n\times n}$$

Implications of our 2 Assumptions of demand functions:-

ie ✓ Homogenity of degree zero.

✓ Walras Law.

on Wealth and Price Effect.

Implication 1 Homogenity of degree zero on walrasian dd func

if $x(p,w)$ is a dd func of homogenity zero, then

$$x(\lambda p, \lambda w) = x(p,w) \quad \forall \lambda \in R$$

$$\Rightarrow \quad x(\lambda p, \lambda w) - x(p,w) = 0$$

Proposition 2.E.1 Given zero homogenity of $x(p,w)$; $\forall p$ & w

$$\sum_{k=1}^{L} \frac{\partial x_l(p,w)}{\partial p_k} \cdot p_k + \frac{\partial x_l(p,w)}{\partial w} w = 0 \quad \text{for } l = 1 \cdots L$$

result of Euler's formula

under Matrix notation

$$D_p x(p,w)\, p + D_w x(p,w)\, w = 0.$$

Prove $D_p x_p(p,w)p + D_w x(p,w)w = 0$.

we know From price Effect matrix ~~no~~ representation & Income effect representation,

$$\underbrace{D_p x_p(p,w)p}_{\text{from price Effect}} = \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} p_1 & \cdots & \frac{\partial x_1(p,w)}{\partial p_L} p_L \\ \vdots & \frac{\partial x_2(p,w)}{\partial p_2} p_2 & \vdots \\ \frac{\partial x_L(p,w)}{\partial p_1} p_1 & \cdots & \frac{\partial x_L(p,w)}{\partial p_L} p_L \end{bmatrix}_{L\times L} \begin{bmatrix} p_1 \\ \vdots \\ p_L \end{bmatrix}_{L\times 1}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial x_1(p,w)}{\partial p_1} p_1 + \frac{\partial x_1(p,w)}{\partial p_2} p_2 + \cdots \frac{\partial x_1(p,w)}{\partial p_L} \cdot p_L \\ \vdots \\ \frac{\partial x_L(p,w)}{\partial p_1} p_1 \cdots \frac{\partial x_L(p,w)}{\partial p_L} p_L \end{bmatrix}_{L\times 1}$$

$$\underbrace{D_w x_w(p,w)\cdot w}_{\text{income Effect}} = \begin{bmatrix} \partial x_1(p,w)/\partial w \cdot w \\ \partial x_2(p,w)/\partial w \cdot w \\ \vdots \\ \partial x_L(p,w)/\partial w \cdot w \end{bmatrix}_{L\times 1}$$

then

$$\begin{bmatrix} \frac{\partial x_1(p,\omega)}{\partial p_1}\cdot p_1 + \cdots + \frac{\partial x_1(p,\omega)}{\partial p_L}\cdot p_L \\ \vdots \\ \frac{\partial x_L(p,\omega)}{\partial p_1} p_1 + \cdots + \frac{\partial x_L(p,\omega) p_L}{\partial p_L} \end{bmatrix}_{L\times 1} + \begin{bmatrix} \partial x_1/\partial \omega \cdot \omega \\ \vdots \\ \partial x_L/\partial \omega \cdot \omega \end{bmatrix}_{L\times 1}$$

$$= \begin{bmatrix} \frac{\partial x_1(p,\omega)}{\partial p_1} p_1 + \cdots \quad \partial x_1/\partial \omega \cdot \omega \\ \vdots \\ \frac{\partial x_L(p,\omega)}{\partial p_1}\cdot p_1 \quad \partial x_L/\partial \omega \cdot \omega \end{bmatrix} = 0$$

Now

How did we transform

$$x(\lambda p, \lambda \omega) = x(p,\omega)$$

$$\Rightarrow x(\lambda p, \lambda \omega) - x(p,\omega) = 0$$

into $\sum_{k=1}^{L} \frac{\partial x_l(p,\omega)}{\partial p_k}\cdot p_k + \frac{\partial x_l(p,\omega}{\partial \omega} \quad \forall\, l = 1 \cdots L$?

or $D_p\, x(p,\omega)\, p + D_\omega\, x(p,\omega)\cdot \omega = 0$??

EULER'S FORMULA

given a homogenous function such that

$$f(tx_1 \cdots tx_n) = t^{\gamma} f(x_1 \cdots x_n)$$

is homogenous of degree γ

then the partial derivative function is homogenous of degree $r-1$

ie $f(tx_1, \dots tx_n) = t^r f(x_1 \dots x_n)$

$\Rightarrow$ ~~$\frac{\partial f(tx_1 \dots tx_n)}{\partial x}$~~ doing partial derivative

$$\frac{\partial f(tx_1 \dots tx_n)}{\partial tx_1} \cdot \frac{\partial tx_1}{\partial t} \dots + \frac{\partial f(tx_1 \dots tx_n)}{\partial tx_n} \cdot \frac{\partial tx_n}{\partial t}$$

$$= t^r \left\{ \frac{\partial f(\cdot)}{\partial x_1} + \frac{\partial f(\cdot)}{\partial x_2} \dots \frac{\partial f(\cdot)}{\partial x_n} \right\}$$

$$\Rightarrow t \left\{ \cancel{\frac{\partial f}{\partial tx_1}} \cdot f_1' + f_2' \dots f_n' \right\} = rt^{r-1} f(\cdot)$$

$$\Rightarrow \boxed{\{f_1' + f_2' \dots f_n'\} = rt^{r-1} \times} f(\cdot)$$

From this important result;

if $x(\lambda p, \lambda \omega) = x(p, \omega)$

$\Rightarrow$ $x(\lambda p, \lambda \omega) - x(p, \omega) = 0$, then doing partial diff wrt λ

$$\Rightarrow \frac{\partial x(\lambda p, \lambda \omega)}{\partial \lambda p} \cdot \frac{\partial \lambda p}{\partial \lambda} + \frac{\partial x(\cdot)}{\partial \lambda \omega} \cdot \frac{\partial \lambda \omega}{\partial \lambda} - \frac{\partial x(p, \omega)}{\partial \lambda} \cdot - \frac{\partial x(\cdot)}{\partial \lambda} = 0$$

$$\Rightarrow \boxed{D_p x(p, \omega)\, p + D_\omega x(p, \omega) \cdot \omega = 0.}$$

from homogenity of degree zero Assmp of dd function.

Given $D_p x(p, \omega) p + D_\omega x(p, \omega) \omega = 0$; we can also attain elasticities of the dd function.

For $l = L$

$$\sum_{k=1}^{L} \frac{x_l(P,\omega)}{\partial P_k} \cdot P_k + \frac{x_l(P,\omega)}{\partial \omega} \cdot \omega = 0$$

Elasticities can as be defined from the above equation

For individual commodity

$$\frac{\partial x_1(P,\omega)}{\partial P_1} \cdot P_1 + \frac{\partial x_1(P,\omega)}{\partial \omega} \cdot \omega = 0$$

multiplying $\frac{1}{x_1}$ on both sides.

$$\Rightarrow \frac{\partial x_1(P,\omega)}{\partial P_1} \cdot \frac{P_1}{x_1} + \frac{\partial x_1(P,\omega)}{\partial \omega} \cdot \frac{\omega}{x_1} = 0$$

$$\Rightarrow \varepsilon_P + \varepsilon_\omega = 0$$

For x_l commodities $\forall l = 1, 2 \ldots L$

$$\Rightarrow \boxed{\sum_{k=1}^{L} \varepsilon_{P_k} + \varepsilon_\omega \quad \forall l = 1, 2 \ldots L}$$

So from $\uparrow$, we conclude implication-1's result saying for a dd function of homogenity zero, equal percentage change in price & wealth leads to no change in demand.

Implication-2 → A demand function satisfies WALRAS LAW ie

$$p \cdot x(p,w) = w$$

prop-2.E.2 (cournot aggregation)

~~then~~ $$\sum_{k=1}^{L} \frac{\partial x_l(p,w)}{\partial p_k} p_k + x_k(p,w) = 0$$ [1st part of implication with respect to p.]

Suppose $L = 1$

$p_1 x_1 = w$ {diff wrt p_1}

$$\frac{\partial x_1}{\partial p_1} p_1 + x_1 = 0 \quad \text{or} \quad x_1 \left\{ \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1} + 1 \right\} = 0.$$

$\Rightarrow x_1 (e_{x,p} + 1) = 0.$

$\Rightarrow e_{x,p} = -1$ (unit elasticity)

In matrix

$$D_p x(p,w)\, p + \boxed{x^T(p,w)} \rightarrow ??$$

↳ After diff. $\left\{ \frac{d\, 2AX}{\partial X} = 2A^T \right\}$

Prop. 2.E.3 (Engel aggregation)

$$p \cdot x(p,w) = w$$

$$\Rightarrow \sum p_l \frac{\partial x(w,p)}{\partial w} = 1$$

suppose $L = 1$

$p_1 x_1 = w$

$$\frac{\partial x_1}{\partial w} p_1 = 1 \quad = \cancel{\frac{\partial x_1}{\partial w} \cdot \frac{p_1}{w}}$$

In matrix

$$p\, D_w x(p,w) = 1$$

What does ~~pp~~ prop 2.E.2 & 2.E.3 talk about??

Cournot aggregation:- given elasticity of price for demand, ~~as~~ if unit elasticity is achieved, then there is no change in expenditure ie if $p\uparrow$, consumer will balance out it's expenditure on x_1 by buying less of it. ie $\frac{\partial E}{\partial p} = 0$ {where E = expenditure}

Cournot aggregation shows us the relationship be[illegible] elasticity of demand and ~~dem~~ consumer Expenditure

Engel aggregation :- The ~~is~~ sum of all income elasticities on different good should add up to 1. Intuition is if income increas then demand for goods in a way that it adds up to the change in wealth.

eg if 20% like in income
↳ you increas 10% food consmp
you i " 10% Alcohol consmp

but total consomption now should be equal to what your wealth is; not more or less than that & as it violates WALRAS LAW

Ex-2.E.2. ??

WEAK AXIOM OF REVEALED PREFRENCE & LAW OF DEMAND.

We will study the implications of ~~law of demand~~ WARP on the ~~law of demand~~ consumer demand.

The WARP is a consistency ~~axiom~~ axiom, ~~which satisf~~ for choice based approach in decision theory. Since Walrasian dd function ~~x~~ is a single valued correspondence, satisfies Walras Law ~~and~~, is homogenous of degree zero, and is the choice rule of a consum given a budget, then It also fits, that WARP does apply to $x(p,w)$ as well.

NEW DEFINITION OF WARP (2.F.1)

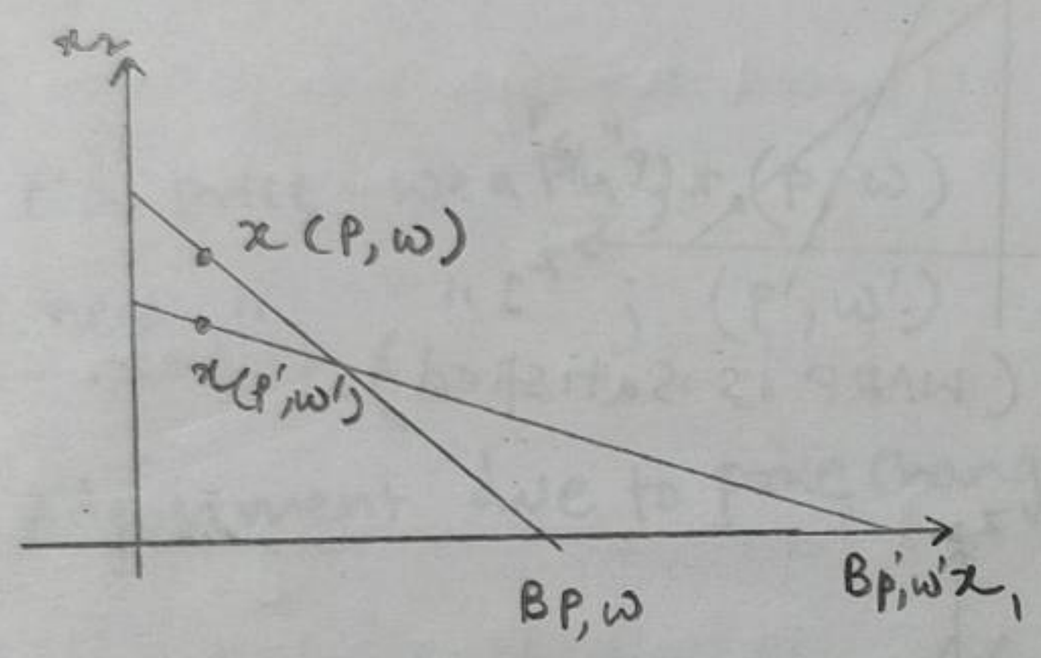
The WALRASIAN DD function satisfies WARP if the following property holds given the price-wealth arguments (p,ω) ; (p',ω') and $x(p',\omega') \neq x(p,\omega)$

ie if $p \cdot x(p',\omega') \leq \omega$ then for (p,ω), $p' x(p,\omega) > \omega'$

ie if new bundle is avaible at old price-wealth argument; then it should not happen that the old bundle is available at new price-wealth argument ie old bundle is not affordable at new price-wealth argument.

Graphically (satisfying WARP)

L=2



$x(p,\omega)$ = old bundle

$x(p',\omega')$ = new "

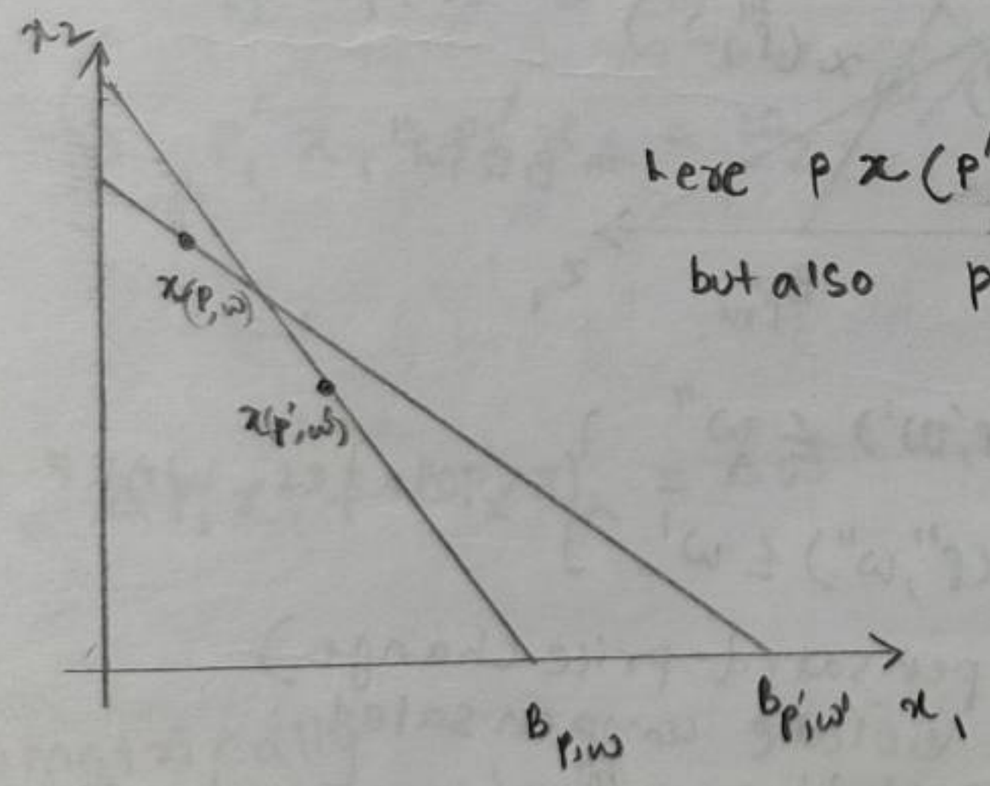
(p,ω) = old price wealth

(p',ω') = new price wealth.

→ to satisfy ~~we~~ WARP; we cannot have $p' x(p,\omega) \leq \omega'$ and $p\, x(p',\omega') \leq \omega$ at the same time

(Violating warp)

L=2



here $p\, x(p',\omega') \leq \omega$
but also $p' x(p,\omega) \leq \omega'$ } violates WARP

This diagram however violates WARP; hence that

(According to book)

Under consumer demand setting the idea of WARP can be put as follows:-

if $p \cdot x(p',\omega') \leq \omega$ and $\underline{x(p',\omega') \neq x(p,\omega)}$, then we know that when consumer faces a price wealth situation (p,ω); the consumer chooses $x(p,\omega)$ bundle or vector even though $x(p',\omega')$ is affordable. ie we interpret it as $x(p,\omega) \succsim^* x(p',\omega')$.

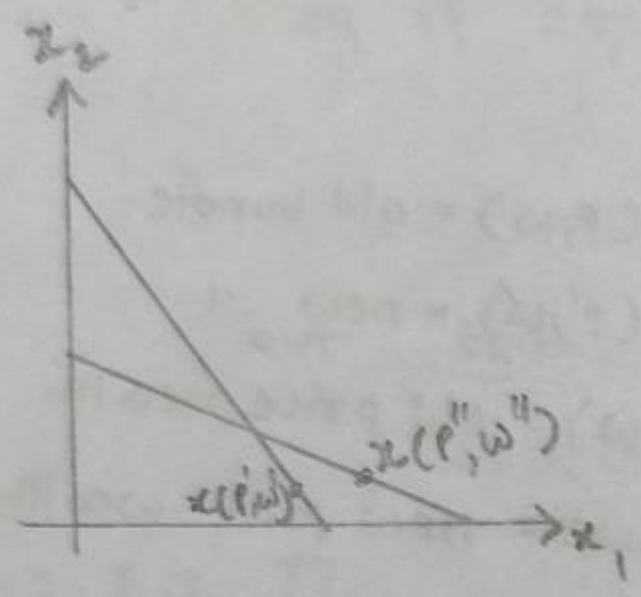
So whenever $x(p',\omega')$ and $x(p,\omega)$ are both affordable under (p,ω), he will choose

$x(p,w)$ over $x(p',w')$ showing consistency. This implies for price wealth argument (p',w') which is the budget composition for $x(p',w')$, $x(p,w)$ must not be affordable under that budget. ie $p'x(p,w) > w'$

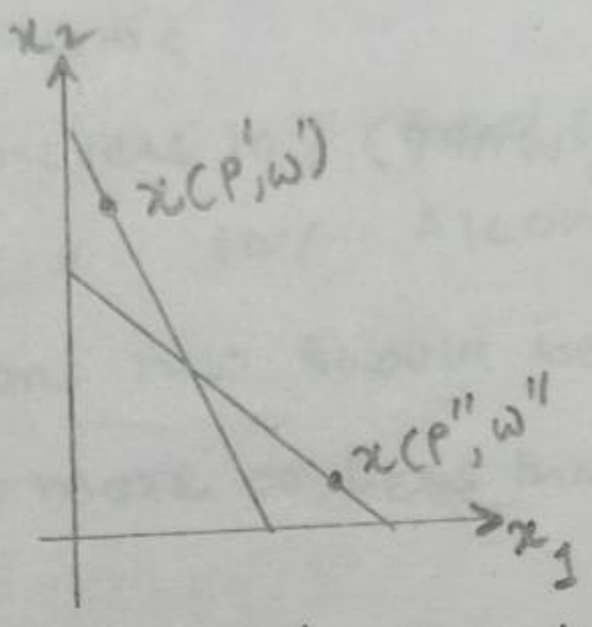
WARP is also a restriction on consumer demand behaviour.

Implications of WARP

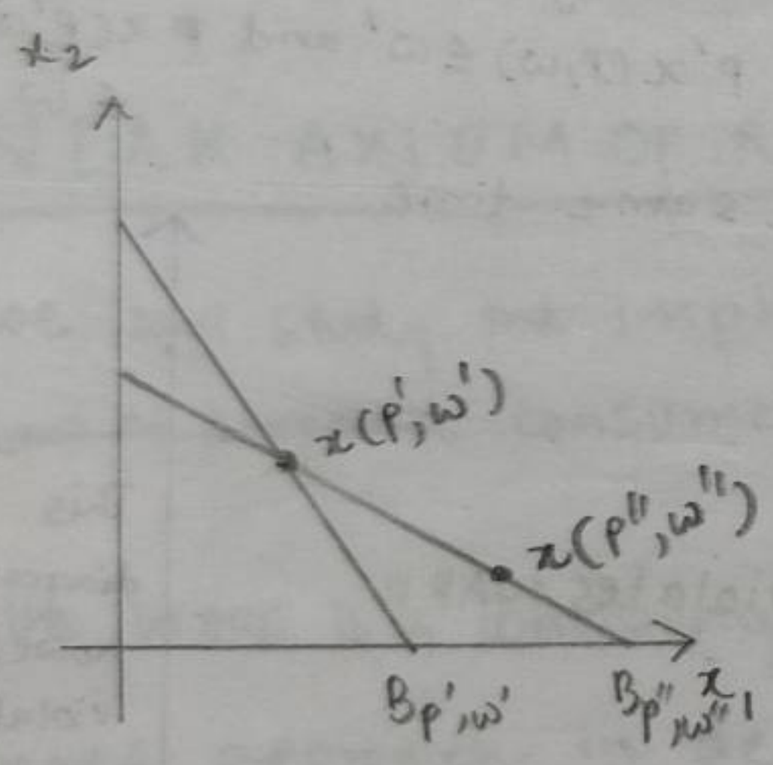
Few examples of WARP satisfying & violating



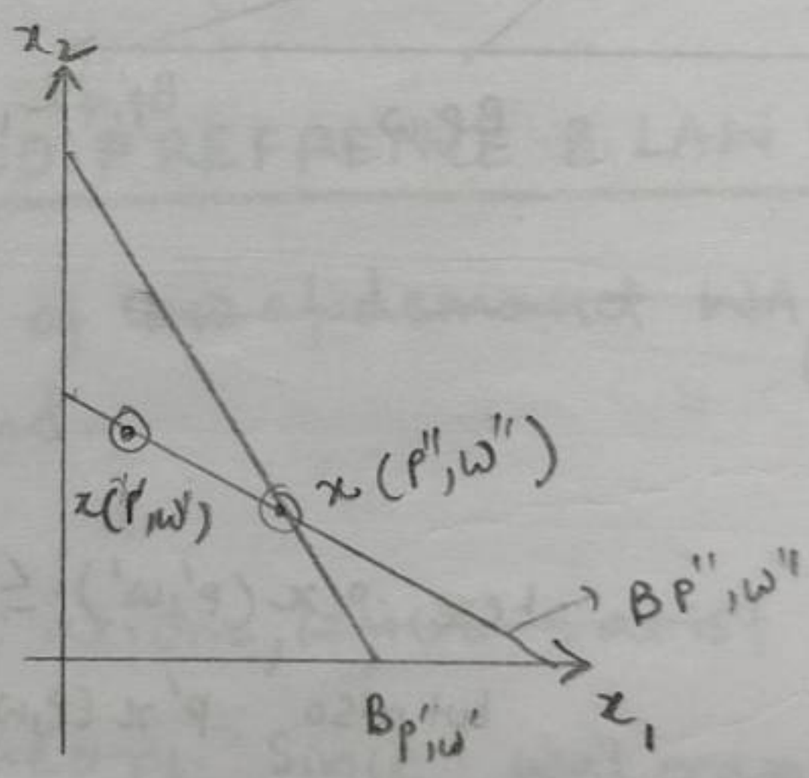
(satisfy WARP)
(no compensated price change)



(WARP is satisfied)



$p'x(p'',w'') \geq w'$
$p''x(p',w') \leq w''$
} WARP holds



$p''x(p',w') \leq w''$
$p'x(p'',w'') \leq w'$
} violates WARP

(compensated price change but violate compensated Law of dd as $p'' < p'$, but x is decreasing)

Implication of WARP :-

Price Effect = Substitution Effect + Income Effect
{relative change in prices of commodity} {relative change in consumer's income or real wealth.}

To associate WARP into these Effects, we only need to take into account the relative change in price of commodities. (Substitution Effect)

The map to understand the implication of WARP; is to understand what compensated Law of demand means. For compensated demand we need to understand what compensated prices or ~~stut~~ (Slutsky) compensated price changes. (don't assume

↳ change in prices is accompanied by a change in the consumer's wealth that makes his initial consumption bundle ~~just~~ exactly affordable at new prices. ie

if (p, w) are price wealth pair for demand $x(p, w)$

then a change in $(p, w) \longrightarrow (p', w')$ in a way that $x(p, w)$ is still affordable is called compensated price change.

$\therefore$ at old price-wealth; (p, w)

at new " " ; (p', w')

Wealth Adjustment due to price changes; $\Delta w = \Delta p \cdot x$

eg if $p_1 \longrightarrow p_1'$ and $p_2 \longrightarrow p_2'$ st $p_1' > p_1$ and $p_2' > p_2$

$\therefore$ $p_1 x_1 + p_2 x_2 = w_1$

$p_1' x_1 + p_2' x_2 = w_2$

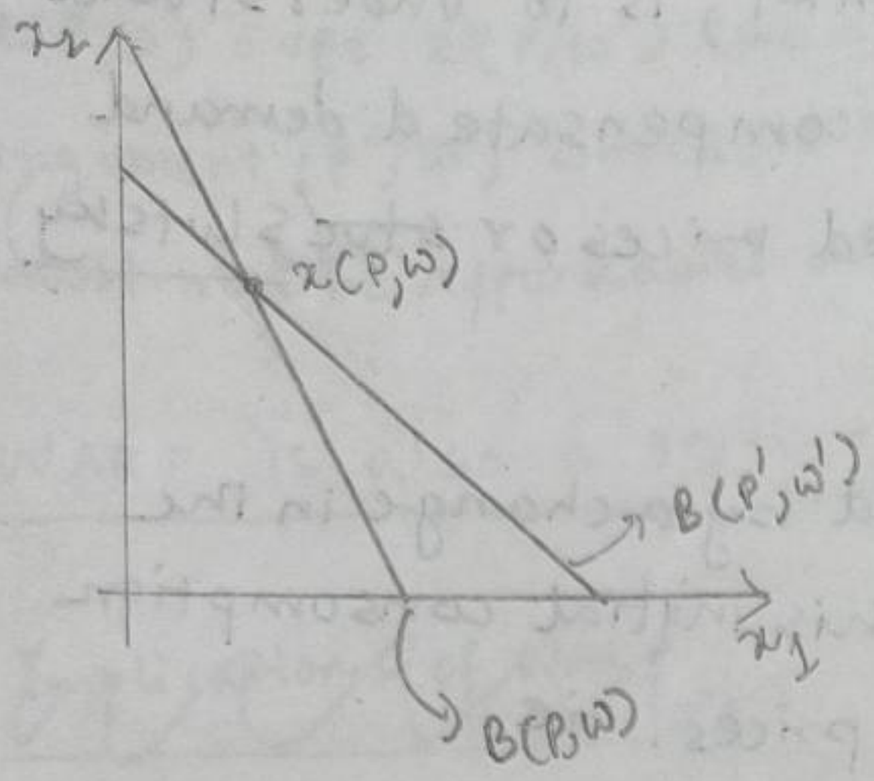
where x_1 & x_2 are commodities and (x_1, x_2) are same bundles with different Budget ~~set~~ Line $\{L=2$; hyperplane if $L>2\}$

$\therefore$ $\Delta p_1 x_1 + \Delta p_2 x_2 = \Delta w$

$\Rightarrow$

Geometrically

Budget Line $B_{p',w'}$ from $\{p', w'\}$ passes through the same x vector where $B_{p,w}$ also passes through.



So under compensated price change

$$p' x(p,w) \leq w' \ \& \ p\, x(p,w) \leq w$$

This kind of wealth adjustment due to change in prices is called slutsky wealth compensa

ie $\Delta w = \Delta p\, x$.

Now we try to understand what implication happens due to this price chang

objective

① WARP $\overset{A}{\Rightarrow}$ CP $\}$ → proof - 1

② CP $\overset{A}{\Rightarrow}$ WARP. → proof 2

reflected through proposition 2·F·1

ie CP $\Leftrightarrow$ WARP.

2·F·1 { proof 1 $\Leftrightarrow$ proof 2 }

2·F·1 → A walrasian demand function is homogenous of degree zero; & satisfies WALRASLAW then it satisfies WARP $\boxed{\text{iff}}$ the following property holds.

Property → given a $\boxed{\text{compensated price change}}$ from $p \to p'$ implying a wealth adjustment $w \to w'$; we have from old price-wealth pair to new price wealth pair (p', w') = $(p', p'x(p,w))$; then

{ $px(p,w) = w \to p'x(p,w) = w'$ } { compensated price chan

$$A = \boxed{(p' - p)\left[x(p',w') - x(p,w)\right] \leq 0} \rightarrow \text{compensated law of demand}$$

$= 0$ if $x(p',w') = x(p,w)$

< 0 if $x(p',w) \neq x(p,w)$.

proof (1) WARP implies inequality, with strict inequality if $x(p,\omega) \neq x(p',\omega)$.

To prove $A = (p'-p)\left[x(p',\omega') - x(p,\omega)\right] \leq 0$ given $\underbrace{(p,\omega) \to (p',\omega)}$

ie there is a compensated price change.

Case-1 if $x(p',\omega') = x(p,\omega)$ {the price change is about the same bundle}

~~then~~ LHS

$$(p'-p)\left[x(p',\omega') - x(p',\omega')\right] = (p'-p)(0) = 0$$

Case-2 if $x(p',\omega') \neq x(p,\omega)$ {a different bundles are consumed} {Through WARP}

LHS

$$(p'-p)\left[x(p',\omega') - x(p,\omega)\right] = \underbrace{p'\left[x(p',\omega') - x(p,\omega)\right]}_{\text{part (1)}} - \underbrace{p\left[x(p',\omega') - x(p,\omega)\right]}_{\text{part (2)}}$$

~~Now~~ NOW

From WALRAS LAW; $px(p,\omega) = \omega$ and $p'x(p',\omega') = \omega'$

(Part 1) $\rightarrow$ $p'x(p',\omega') - p'x(p,\omega) = \omega' - p'x(p,\omega)$

from our assumption of compensated price change for bundle $x(p,\omega)$

we had gotten; $(p,\omega) \to (p',\omega')$

then $px(p,\omega) = \omega$ and $p'x(p,\omega) = \omega'$

$\therefore$ $p'x(p',\omega') - p'x(p,\omega) = \omega' - \omega' = 0$

$\therefore$ part 1 $= 0$.

(Part 2) $\rightarrow$ $p\left[x(p',\omega') - x(p,\omega)\right]$

$\Rightarrow$ $px(p',\omega') - \underbrace{px(p,\omega)}_{\omega \text{ (walras Law)}}$

$\Rightarrow$ $\underbrace{px(p',\omega')}_{??} - \omega$ —— (2)

~~Now~~ $px(p',\omega')$ is a budget. Now according to our assumption of

of compensated price change; we have $p' \cdot x(p,w) \leq w$ is not only affordable under (p,w) but also p', w'. Then $x(p',w')$ be another bundle which affordable under (p',w'), <u>will not be</u> affordable under (p,w) {As it would otherwise violate WARP}

hence

$$\text{if } p' x(p,w) \leq w$$
$$\text{then } p\, x(p',w') > w$$

$\therefore$ (part 2) $\rightarrow$ $p\, x(p',w') - w$

where $\underset{\downarrow}{>} w$.

$$\therefore \quad p\, x(p',w') - w > 0$$

~~Hence we proved: WARP $\Rightarrow$ inequality equation.~~

$\therefore$ combining results of (part 1) and (part 2); then

$$A = \underset{\overset{\parallel}{0}}{\text{(part 1)}} - \underset{>0}{\text{(part 2)}}$$

$$= 0 - [p\, x(p',w') - w]$$

$$A \leq 0$$

$\boxed{\text{Hence we proved WARP} \Rightarrow A \leq 0}$ $\boxed{\text{Proof - 1}}$

CP $\Rightarrow$ WARP

(ii) <u>Proof (2)</u> $\rightarrow$ for all compensated prices changes with <u>strict inequality</u> if $x(p',w') \neq x(p,w)$ <u>implies</u> WARP holds.

or simply we have ~~to prove~~ a fact that ~~First that~~ WARP holds if for any 2 price-wealth pairs (p,w) and (p',w'), we have $p' x(p,w) > w'$ and $p\, x(p',w') =$ and $x(p',w') \neq x(p,w)$ {Proof 2.2}

What we are actually trying to prove is if inequality A hold by all compensated price changes then WARP is implied to hold

proof 2.2

The goal of the proof is to show that if WARP is violated, it means there must be a compensated price change - for ~~wch~~ which weak ~~anix~~ an xiom is be violated. (CP → WARP)

Strategy → Assume WARP is being violated; so for ~~a ⊗ compen st~~ so this means for any 2 price - wealth pair (p', w') and (p'', w'') :-

$\rightarrow p'' x(p', w') \leq w''$
$\rightarrow p'' x(p'', w'') \leq w'$ } hold, hence violates WARP.

~~Now, this violation of WARP will be through compensated price change if one of the inequalities become a equality.~~

~~ie if orginally $x(p', w')$ was affordable at (p', w') then if there is a compensated price change $(p', w') \rightarrow p''$ and $p'' x(p', w') = w''$ holds. then~~

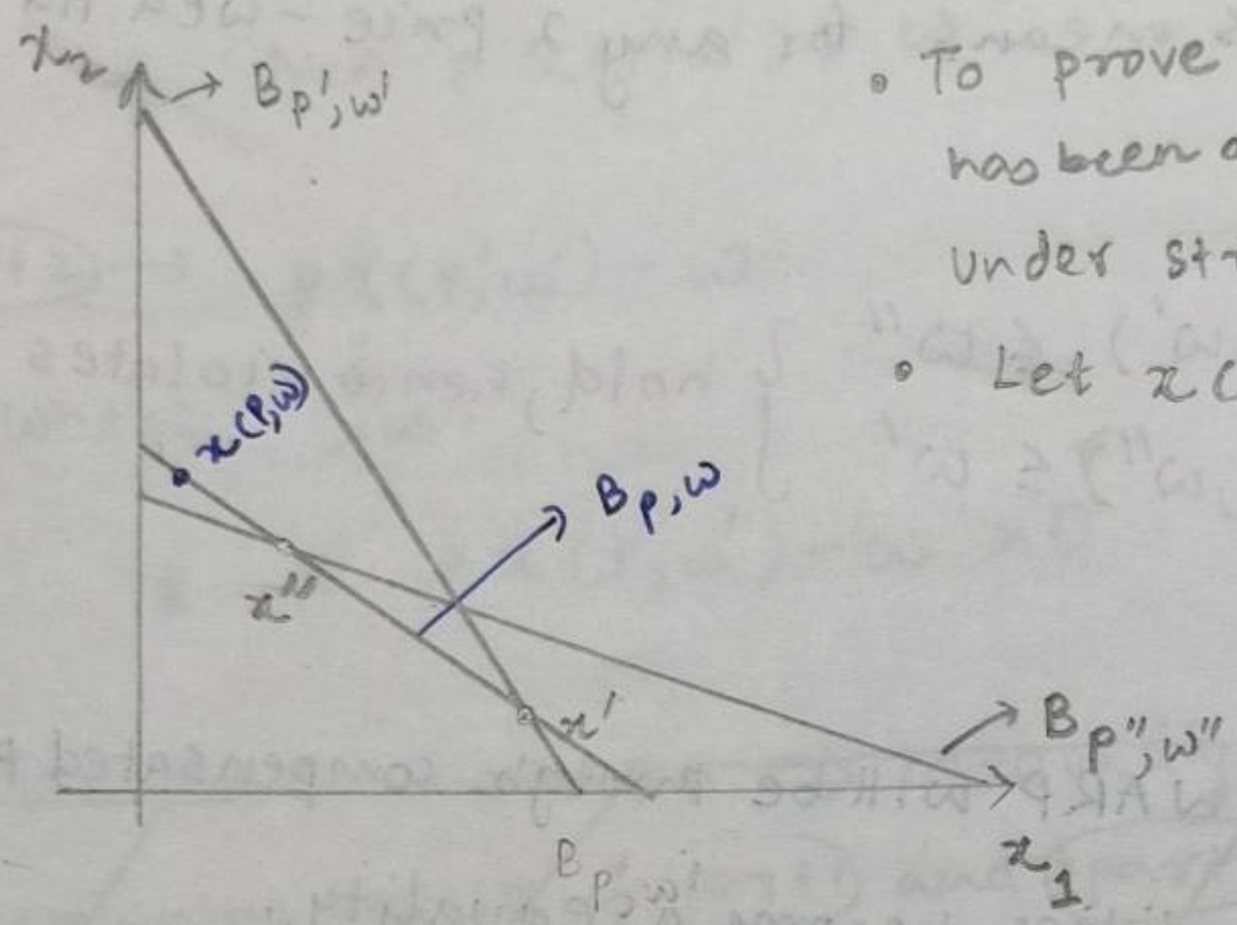
Now

a compensated price change is shown when; suppose $p' x(p', w') = w'$ is the orignial budget line; now if $p' \rightarrow p''$ ~~then~~ and $w' \rightarrow w''$; and $p'' x(p', w') = w''$; then it's a compensated price change. It implies under WARP since $p'' x(p', w') \leq w''$ and $p' x(p'', w'') \leq w'$ cannot hold simultaneously; ~~but~~ as ~~it's~~ due a CP now $x(p', w')$ is affordable at (p'', w''). So if either of these 2 inequalities become equality, WARP is violated ~~gives to violate~~. ~~And this is contradictory to our violation of WARP~~. As now; it is possible that $p'' x(p', w') = w''$ and also $p' x(p'', w'') \leq w'$

So there must be a compensated price change for WARP being violated

Now suppose we assume WARP is being violated with <u>strict inequality</u>

ie $p' x(p'', w'') < w'$ and $p'' x(p', w') < w''$



- To prove WARP violates because there has been a compensated price change under strict inequality.
- Let $x(p'', w'') = x''$ and $x(p', w') = x'$

<u>Step-1</u> :- we construct a new price-wealth combination which the convex-averages or weighted averages of (p', w') and (p'', w''). We consider this new budget line from p, w as a compensated price change

$\therefore$ $p = \alpha p' + (1-\alpha) p''$

$w =$ ~~$\alpha w' + (1-\alpha) w''$ or $(\alpha p' + (1-\alpha) p'')$ assuming~~

$= (\alpha p' + (1-\alpha) p'') x(p', w')$: assuming $(\alpha p + (1-\alpha) p'') x(p', w') = (\alpha p + (1-\alpha) p'') \cdot x(p', w')$

$\therefore$ new price wealth combination is (p, w) or $(\alpha p' + (1-\alpha) p'', \alpha w' + (1-\alpha) w'')$

now; given a new price wealth combination, we must have a new demand function $x(p, w)$. And which itself is a weighted or convex-combination of bundles ~~$x(p', w')$~~ at (p', w') and (p'', w'')

from the figure

$$\cancel{w = \alpha w' + (1-\alpha) w''}$$

$$\cancel{\Rightarrow w > \alpha p' x(p', \alpha p' x(p', w') + (1-\alpha) p''}$$

now

$$w = \alpha p' x(p', w') + (1-\alpha) p'' x(p', w') < \alpha w' + (1-\alpha) w''$$

$$\Rightarrow w < \alpha w' + (1-\alpha) w''$$

$$\Rightarrow p \cdot x(p, w) < \alpha w' + (1-\alpha) w''$$

since $p = \alpha p' + (1-\alpha) p''$

$$\Rightarrow \alpha p' x(p, w) + (1-\alpha) p'' x(p, w) < \alpha w' + (1-\alpha) w''$$

$$\Rightarrow p' x(p, w) < w' \Rightarrow x(p, w) \neq x(p', w') \quad \text{— (1)}$$

or $p'' x(p, w) < w'' \Rightarrow x(p, w) \neq x(p'', w'')$

For eqn (1) if it holds.

~~$p' x(p', w')$~~ then as w passes through x', then

~~$p x' = w$ and~~ $\underbrace{p x(p', w') = w}_{\text{compensated price change.}}$ and $p' x(p, w) < w'$ (hold simultaneously)

thus it violates WARP.

From $A = (p' - p)\left[x(p', w') - x(p, w)\right] \leq 0$

can be written as

$$= \Delta p \, \Delta x^c \leq 0$$

where Δx^c reminds of the compensated change in x. This inequality is called the Compensated Law of Demand: which states that

If the price of commodity goes up, you demand less of it. That is why
And this is ~~the implied~~ derieved from WARP ~~of~~. That is why
~~Way~~ why we were proving CP $\Rightarrow$ WARP and WARP $\Rightarrow$ CP

$\Rightarrow \therefore$ CP $\Leftrightarrow$ WARP.

Simplest case to see the effect of compensated Demand due to compensated price and Application of WARP is to see for a single commodity suppose x_1

we know; $\Delta P \, \Delta x \leq 0$.

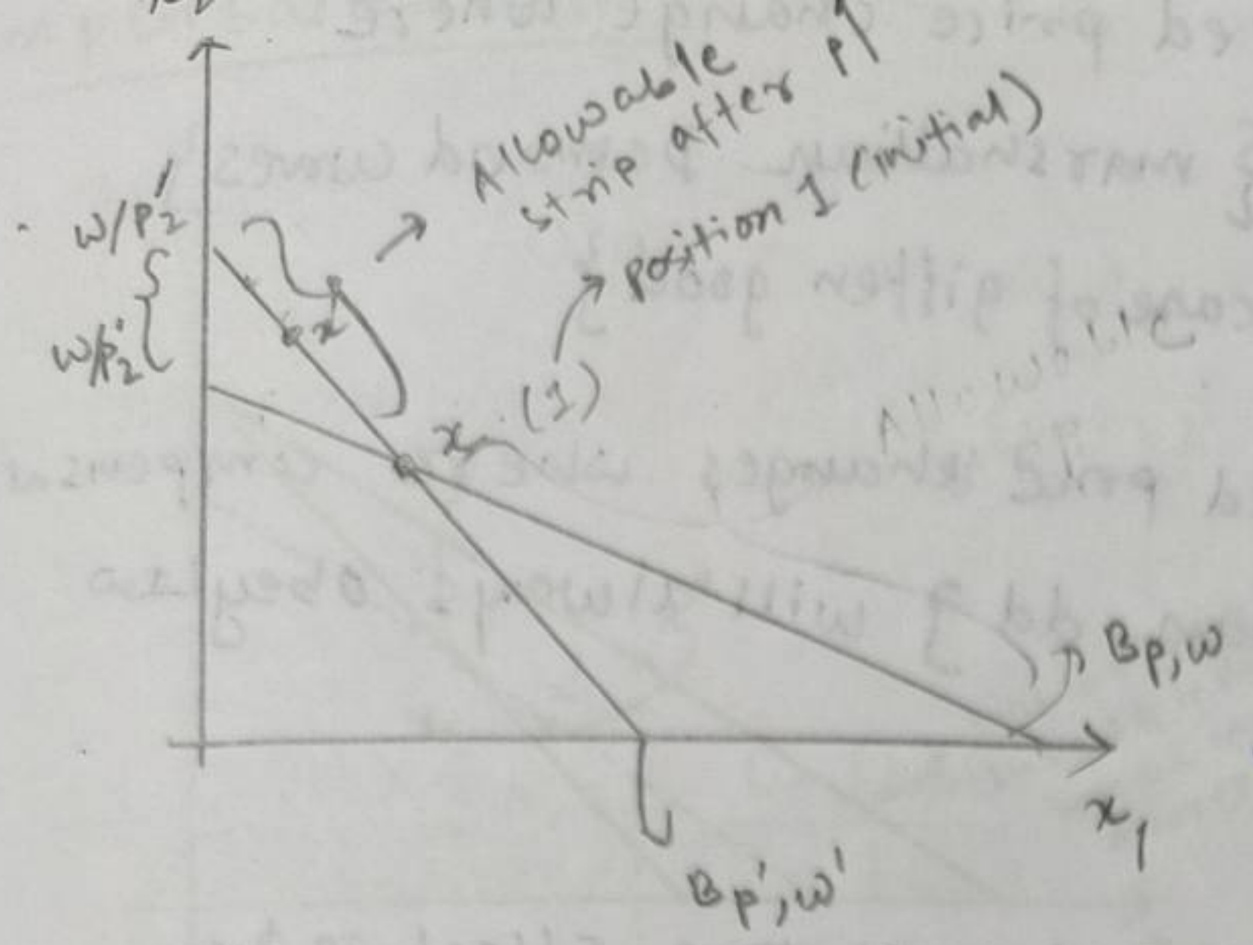
So if $L = 1$. and x_1 is the commodity

then $x = \begin{bmatrix} x_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $P = \begin{bmatrix} P_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore \quad \Delta P \cdot \Delta x = \Delta P_1 \Delta x_1$.

So according to proposition 2.F.1; if there is a compensated price change; WARP is equivalent to compensated law of demand which states that price and demand move imposite direction.

Then if ~~$\Delta P_1 \uparrow$; then $\Delta x \downarrow$~~ if $P_1 \uparrow$ to P_1' ~~and~~ ie $(P_1' - P_1)$ or $\Delta P_1 > 0$ then $x_1 \rightarrow x_1'$ ie $(x_1' - x_1) < 0$ or $\Delta x < 0$



$B_{p,w} \to B_{p',w'}$

for $(p,w) \to (p',w')$

st $\underline{p' > p}$

→ figure ~~2.5~~ 2.F.4

since $p \to p'$; according CLD $(p'-p)(x(p',w') - x(p,w)) \leq 0$

then $x \to x'$; "

and this satisfy WARP (How?)

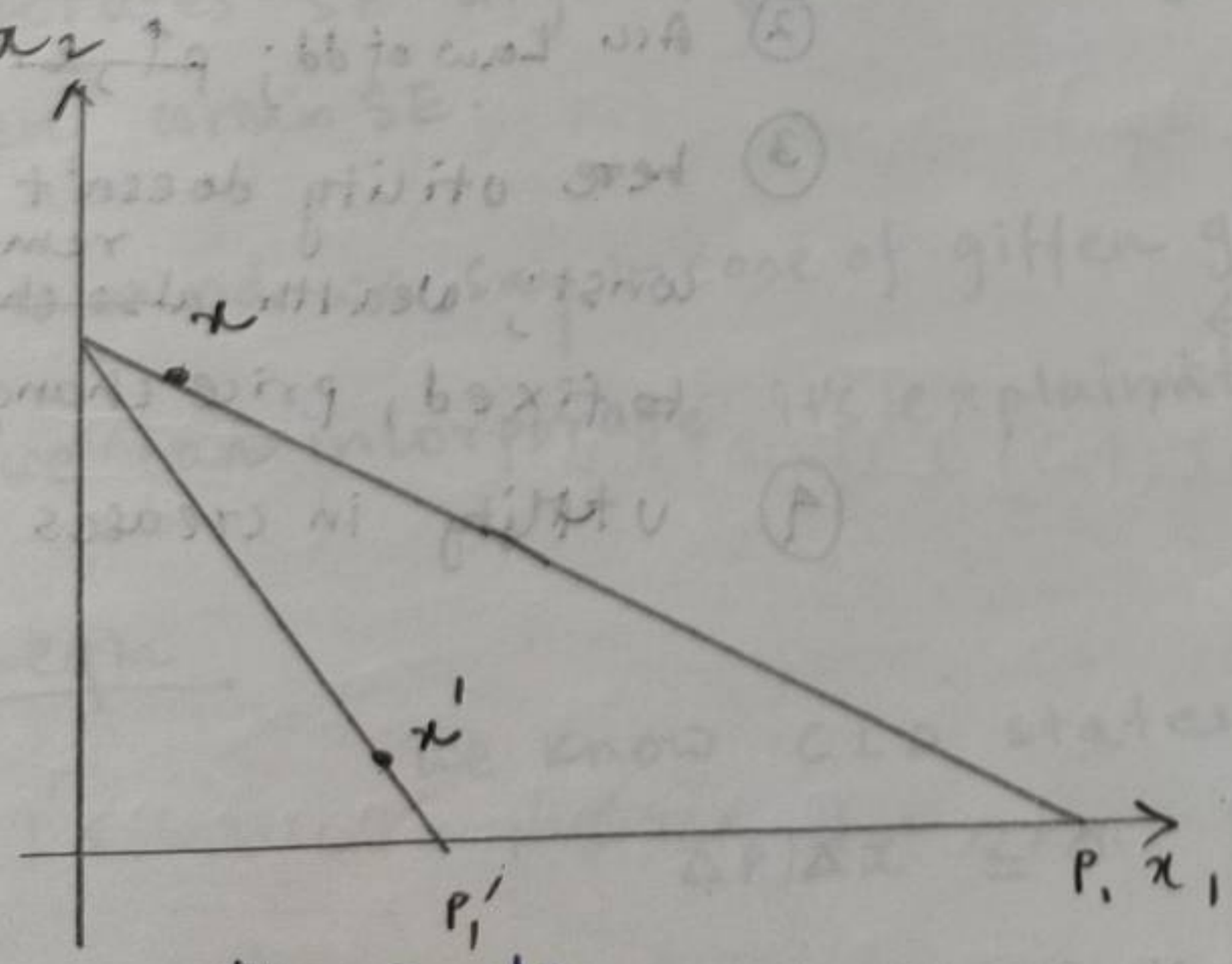
$px = w$ {according WL}

and $p'x = w'$ is affordable at New price-wealth.

then $px' > w$

So WARP holds. {Inshort we can say WARP imposes restriction on where we can place the bundle or comodity in case of ~~CD~~ CLD with comp price change.

Eg of Non-compensated DD Law ~~with WARP~~. (Figure 2.F.5)



giffen goods.

Suppose there is no comp^n price change; only price of p_1 increases. Then according to Law of dd.

$\Delta p \Delta x \leq 0$

but this might not follow as if $p\uparrow$, $x(p,w)\uparrow$ in case of giffen goods. So WARP doesn't put any restrictions here such that Law of dd is not violated.

hence; in case of uncompensated price change

uncompensated Demand occurs { marshallian Demand curves }

can violates <u>Law of demand</u>. { case of giffen good }

However in case of compensated price changes where compensated demand curves occurs { hicksian dd } will always obey law of demand.

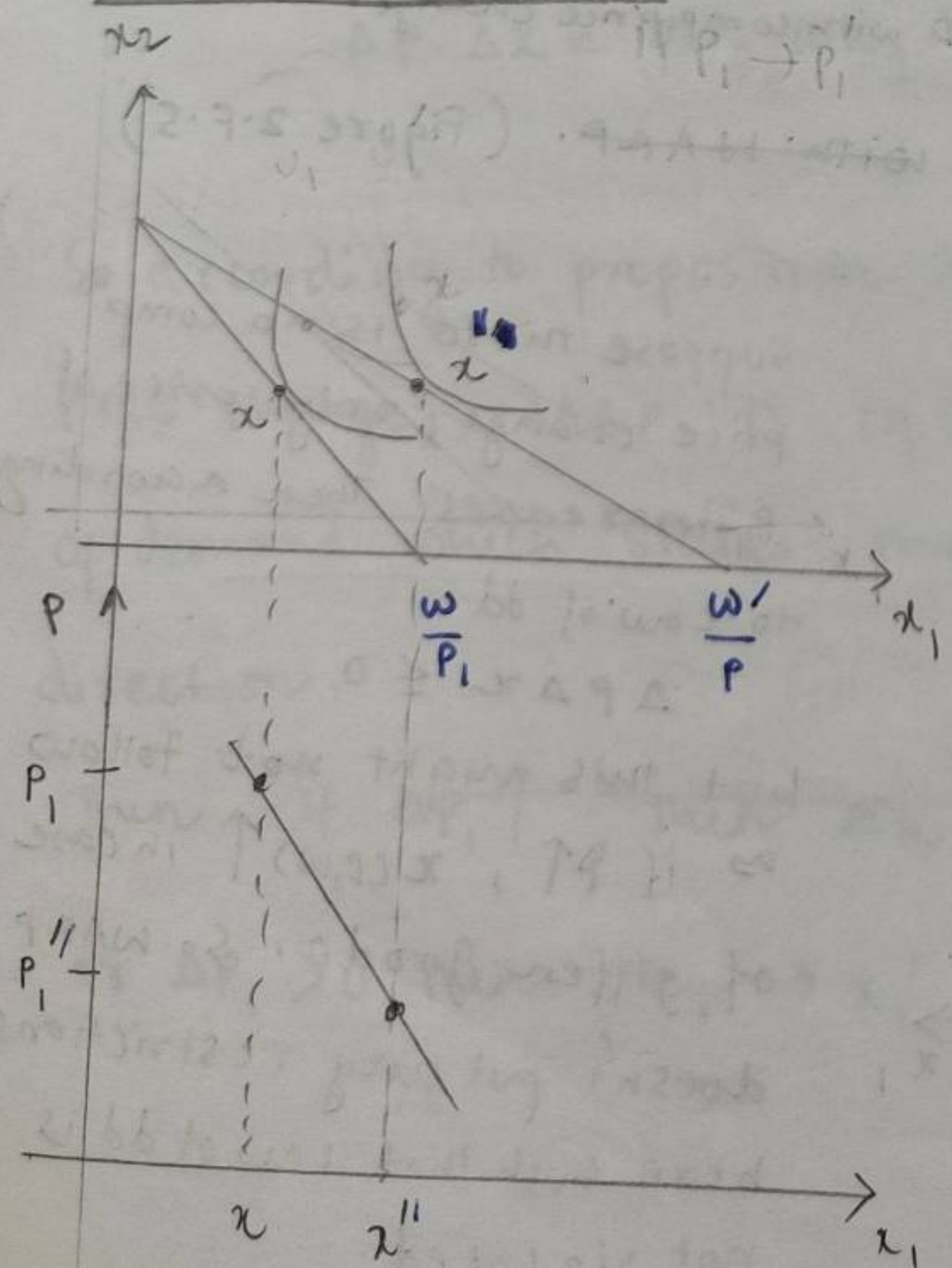
→ compensated dd Law only has substitution Effect ie the

- $\frac{\partial x}{\partial p}$; hence is steeper

→ while uncomp dd (marshalian DD.) incorporates income effect; hence is flatter.

Diagrams to support visualisation of compensated and uncomp demand curves.

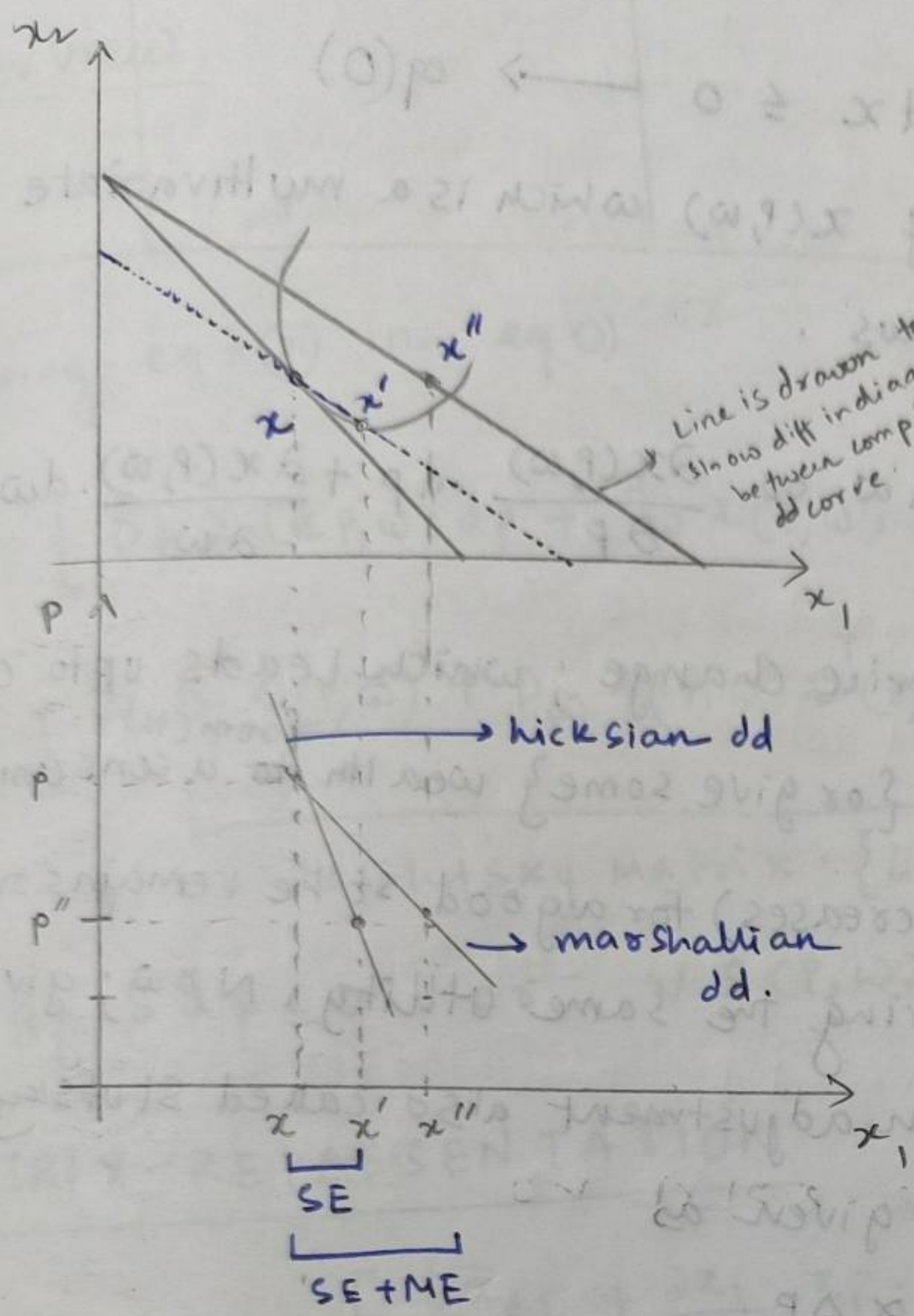
<u>Uncompensated DD</u>



Suppose :-

1. $p_1 \rightarrow p_1''$ st $p_1'' < p_1$
2. Acc Law of dd; ~~p↑, x~~ p↓, x↑
3. here utility doesn't remain const; wealth ~~also changes~~ remains fixed, price changes.
4. utility increases

compensated dd



(I) now suppose, $P_1 \longrightarrow P_1'$ st $P_1' < P_1$

(II) But no, without actually increasesing nominal income, we take away that relative increase in income due to fall in price st utility remains constant as the person is just ~~at~~ able to afford the original bundle.

(III) Now of; if $P\downarrow$ according to Law of dd, $x\uparrow$

(IV) Let $x \longrightarrow x'$ and $x' > x$ as $P \longrightarrow P'$ and $P' < P$

SUBSTITUTION MATRIX

Another implication of compensated Law of demand is that ~~is~~ it incorporates SE only. or just Let's say, it coorporate SE with ME ~~in~~ hidden within SE.

keep in mind we say incase of giffen good; it violates Law of dd.

but we can incorporate it's explaination within CLD.

To begin

we know CLD states

$$\Delta P \Delta x \leq 0.$$

Since we assumed that $x(P,w)$ is demand function which satisfy homogenity of degree zero and WALRAS LAW; A direct implication

To this is; it is differentiable {and continuous}

we can reinstate CLD as

$$dp \cdot dx \leq 0 \longrightarrow eq(0)$$

as dx is total differential of $x(p,w)$ which is a multivariate function of price-wealth; thus.

$$dx = x_p \cdot dp + x_w \, dw \text{ or } \frac{\partial x(p,w)}{\partial p} \cdot dp + \frac{\partial x(p,w)}{\partial w} \cdot dw \quad (I)$$

Now under compensated price change; which leads upto CLD we found that we take away {or give some} wealth ~~to~~ from (to) a consumer if price ~~incre~~ decreases (increases) for a good st he remains on the same original bundle, having the same utility. Now; given that assumption, the wealth adjustment also called Slutsky wealth compensation was given as

$$\Delta w = x \, \Delta p.$$

in differential term;

$$dw = x \, dp \quad \text{— (II)}$$

replacing (II) in eq^n (I)

$$dx = x_p \, dp + x_w \cdot (x \, dp)$$

in Matrix notation

$$dx = D_p x(p,w) \, dp + D_w x(p,w) \, x^T(p,w) \, dp. \qquad \left\{ \frac{d \, x^T A x}{dx} = 2A^T x \right\}$$

now

dx is a $L \times 1$ matrix

and $D_p x(p,w) = L \times L$

$D_w x(p,w) = L \times 1$

$dp = L \times 1$

$\therefore dw = 1 \times 1$

$$\therefore dx = \underbrace{D_p x(p,w) \, dp}_{L \times 1} + \underbrace{D_w x(p,w)}_{L \times 1} \underbrace{x^T(p,w)}_{1 \times L} \underbrace{dp}_{L \times 1}$$

$= L \times 1$ matrix. — (III)

putting eqn (III) in eq(0)

$$dp\{ D_p x(p,\omega)\, dp + D_\omega x(p,\omega)\, x^T(p,\omega)\, dp \} \le 0$$

$$dp\{ \underbrace{D_p x(p,\omega) + D_\omega x(p,\omega) x^T(p,\omega)}_{\text{The Slutsky Matrix. } \{L \times L \text{ order}\}} \} dp \le 0$$

$$dp\, S\, dp \le 0 \quad \text{or} \quad v\, S(p,\omega)\, v \text{ (in matrix) where } v = dp$$

MATRIX REPRESENTATION

$$S(p,\omega) = \begin{bmatrix} \frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial \omega} \cdot x_1 & \frac{\partial x_1}{\partial p_2} + \frac{\partial x_1}{\partial \omega} x_2 & \cdots & \frac{\partial x_1}{\partial p_L} + \frac{\partial x_1}{\partial \omega} \cdot x_1 \\ \vdots & & & \vdots \\ \frac{\partial x_L}{\partial p_1} + \frac{\partial x_L}{\partial \omega} x_1 & \cdots & & \frac{\partial x_L}{\partial p_L} + \frac{\partial x_L}{\partial \omega} \cdot x_L \end{bmatrix}_{L \times L}$$

Diagonal Elements show SE with own price.

Now, if we recall negative semidefiniteness;

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$; given a symmetric matrix

if $a \leq 0$ and $c \leq 0$ and $\underbrace{ac - b^2}_{\text{Determinant}} \geq 0$; then the matrix is negative semidefinite. (used in optimisation problems for matrices to show convexity & concavity

Q: $A \leq 0$ or $A = NSD$.

NSD condition :- $x^T A x \leq 0$ ie $v S v \leq 0$

Now, since under compensated price change, a person's ~~utf~~ utility remains fixed while price changes as he is able to afford his original ~~utf~~ bundle.

~~Now suppose if $p\downarrow$, $x\uparrow$ demand for x for the~~

Now suppose price P_l of lth good $\downarrow$; then as per Law of demand $x_l \uparrow$; thus $\frac{\partial x_l(p,w)}{\partial p_l} < 0$

thus if $\frac{\partial x_l}{\partial P_l} + \frac{\partial x_l}{\partial w} \cdot x_l = 0$; then diagonals are either $= 0$ or

~~hence~~ $\boxed{S_{ll}(p,w) \text{ is negative } \cancel{\text{semidefinite}} \text{ or } 0.}$ or quasi negative semidefinite as $S_{lk}(p,w)$ is symmetric

S_{lk} is symmetric for $L=2$ & for $L>2$; it may may not hold not be symmetric given homogenity, WALRAS LAW & WARP as assumptions. So additional assumptions are required.

So concluding the chapter :-

1. The consistency requirement of WARP is equivalent to CLD given ~~under~~ compensated price change
2. CLD $\Rightarrow$ negative semidefiniteness of Slutsky matrix
3. S isn't symmetric for $L>2$.

Curious case of Giffen Good & how it doesn't violate CLD.

For Giffen good, if suppose x_l is giffen in nature then according to normal Law of dd ; $\frac{\partial x_l}{\partial P_l} > 0$.

However, since CLD doesn't get violated due to presence of giffen good and $S_{ll}(P,w) \leq 0$ still ie Substitution ~~matrix~~ matrix S remains negative semidefinite because;

$S_{l,l}$ ie Substitution ~~matrix~~ of own price good.

$$\underset{(>0)}{\frac{\partial x_l}{\partial P_l}} \cdot P_l + \underset{(<0)}{\frac{\partial x}{\partial w}} \cdot x_l \leq 0$$

ie if $\frac{\partial x_l}{\partial P_l} > 0$ then $\frac{\partial x_l}{\partial w} < 0$ ie good x_l must alsobe inferior if it's giffen such that the diagonal elements of own price becomes zero or become negative, income effects overweighs substitution effect.

Chp-3 Classical Demand Theory

Overview

We study preference based approach to consumers demand.

3B We study Basic properties of consumer preference relation

- convexity
- monotonicity. (or Local non satiation)

3C Takes into account the consumer's utility function and takes about it properties and why it exists.

→ not all preference relations are representable by a utility function

→ preferences must be continous ~~to~~ (sufficient condition) to satisfy as utility function.

3D focuses on utility maximisation st a Walrasian budget const given consumer is a price taker

3E Focuses on consumer's Expenditure problem.

↳ compensated demand correspondence

or

Hicksian demand "

3.B Preference Relations: Basic properties.

Analysis of consumer behaviour is studied through assesing consumer's preference pattern over a period of time.

Preference relation is shown as $\succsim$ {at least as good as}

we assume $\succsim$ as rational — transitivity, completeness. {Def. 3.B.1}

Two main assumptions of rational preference — monotonicity, convexity

Desirability assumptions (monotonicity)

~~Less~~ more is preferred to Less. Quantity of a 'good' good assuming Non satiation

Def: 3.B.2 A $\succsim$ is

→ **Strict Monotonicity ~~of~~ or Strong monotonicity: or Strong monotone**

if ~~$y \geq x$~~ $y \geq x$ and $y \neq x \Rightarrow y \succsim x$

ie $y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{matrix} \to x_1 \\ \to x_2 \end{matrix}$

even though commodity x_2 is insame amount to x_2 in vector y; consumer shows a STRONG choice of bundle y over x. This is strong monotonicity.

→ **Monotonicity:** (allows the possibility of indifference in bundles)

if ~~$y \gg x$~~ thus if $y \gg x$ {bundle y is extremly Large than x}

then $y > x$ for $x, y \in X$ (otherwise if $y \geq x$ then $y \succsim x$ or $y \sim x$)

Assumption of monotonicity hold iff ~~goods~~ commodity are 'good' not 'bads'

However, we can change ~~the~~ or redefine consmp activity to make a bad into good.

According to Def 3.B.2 $\succsim$ is <u>monotone</u>; we may have indifference when there is an increase in the amount of some but not all commodities

However if $\succsim$ is strict monotone; if y is larger than ~~some~~ x by some amount, and no less for any other, y is strictly prefered to x.

<u>Def - 3.B.3</u>

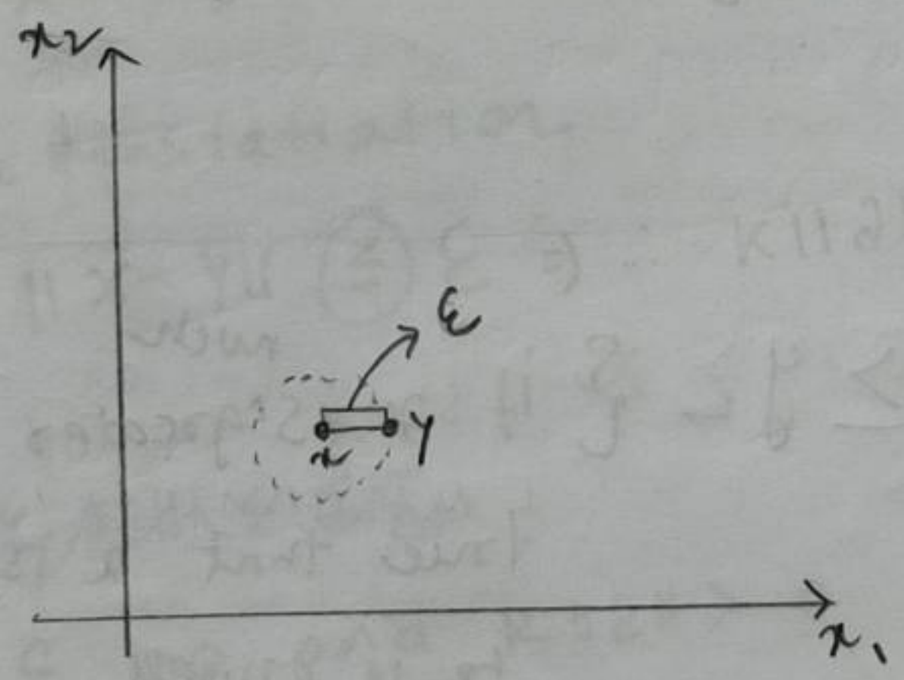
The preference Relations $\succsim$ on X is locally non satiated if for every $x \in X$ and $\varepsilon > 0$, there is $y \in X$ st $\|y - x\| \leq \varepsilon$ and $y \succ x$

→ Locale non satiation is a weak assmp than monotonicity.

if monotonicity holds ⇒ Local non satiation holds

but the reverse is not true.

The assumption of local ~~sat~~ non~~sation~~ satiation says that there is always a feasible bundle nearby that a person can have than his current bundle.



hence it entails that suppose that there is a very very small increment ε in the x bundle st $y \succ x$ where y is a new bundle st

$$y = x + \varepsilon$$

then as a consumer will prefer y over x ie $y \succ x$.

~~This distan~~ This is mathematically represented as $\forall x \in X$,

$$\forall \varepsilon < 0 \; ; \; \exists y \in \underset{\hookrightarrow \subset R^L}{X} \text{ st } \underbrace{\|y - x\|}_{\text{distance between } x \text{ and } y \text{ bundle}} \leq \varepsilon \Rightarrow y \succ x \; \hookrightarrow \subset R^L$$

(the increment)

Monotonicity & Local Non Satiation also makes sure that IC's are thin & not thick.

NOTE

When $X = R^L_+$ then there are no situation in which commodi are bads {all goods in R^L_+ are goods; thus $x = 0$ if bads are considered

Ex-3.B-1

② If $\succsim$ is strongly monotone, then it's monotone

Strongly monotone $\Leftrightarrow$

if $x \geq y$ and $x \neq y \Rightarrow x \succ y \;\; \forall x \in X \subset R^L_+$

Now suppose

$x \gg y \Rightarrow x \geq y$ {if x is much greater than y then it is true that x is equal or greater to y as well

From this condition, if $x \gg y \Rightarrow x \geq y$

then since $\succsim$ is strongly monotone

such that

$x \geq y$ and $x \neq y$

we can write

$\cancel{y \Rightarrow}$ $x >> y \Rightarrow x \geq y \Rightarrow x > y$

or $x >> y \Rightarrow x > y$ {which is the monotone definition}.

(b) If $\succsim$ is monotone, then it's locally nonsatiated.

→ Monotone means

if $x >> y \Rightarrow x > y \quad \forall x, y \in X, \; X \subseteq R^L_+$

given monotone conditions

Let $x = y + \underbrace{ka}$

↳ vector a multiplied to scalar k, where a is a unit vector & $k > 0$

Now since $x >> y \Rightarrow x > y$

We can write $\|x - y\| = \|y + ka - y\| = \|ka\| =$
$= k\|a\|$

~~To prove~~ Local nonstatiation

$\|x - y\| \leq \varepsilon \Rightarrow \therefore k\|a\| \leq \varepsilon$ {this is true

So we pose

$k\|a\| \leq \varepsilon$

$\Rightarrow$ since $\varepsilon > 0$ and $k\|a\| > 0$

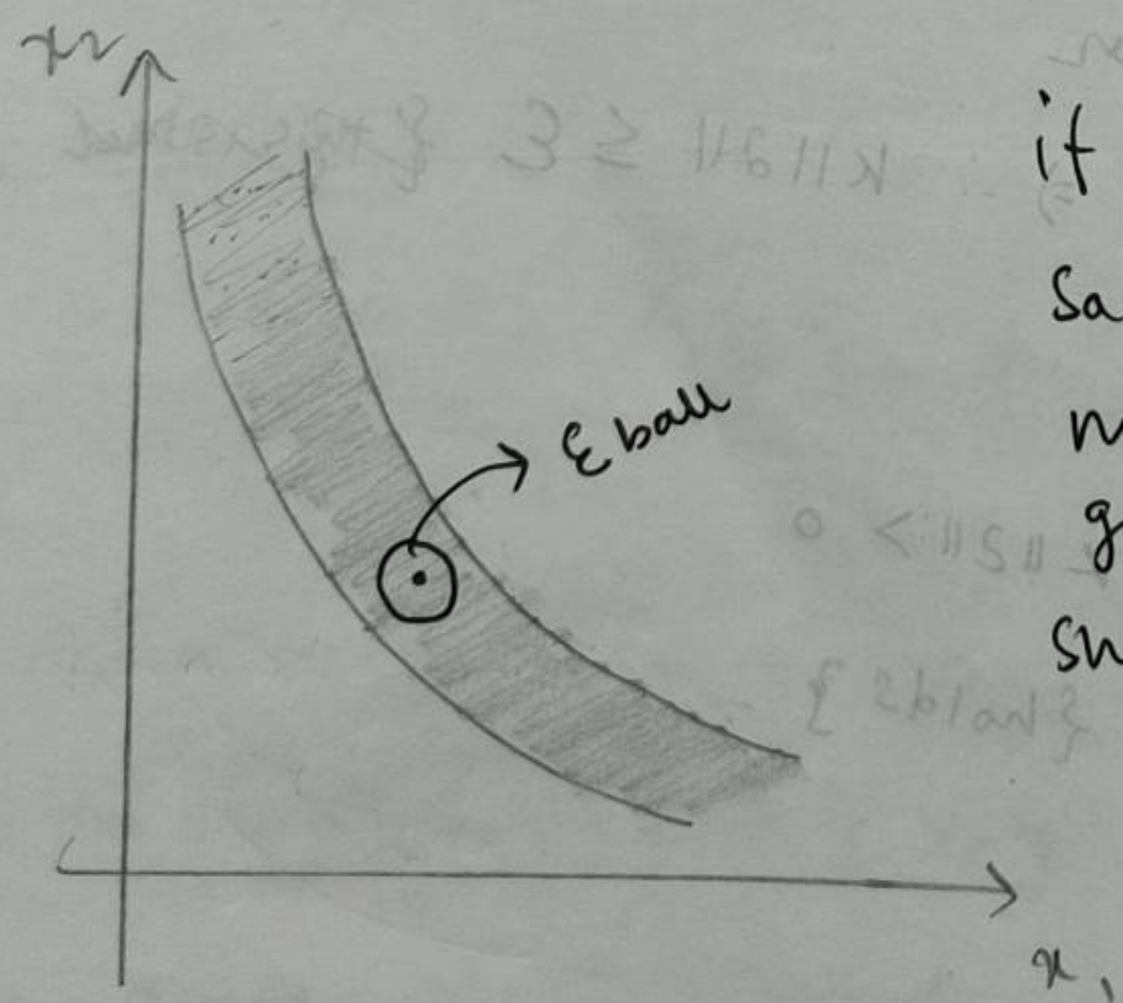
$\Rightarrow$ $\|x - y\| = k\|a\| \leq \varepsilon$ {holds}.

∴ To sum up

$$\boxed{\begin{array}{l}\text{Strong monotonicity} \Rightarrow \text{monotonicity} \\ \text{and monotonicity} \Rightarrow \text{Local non satiation.}\end{array}}$$

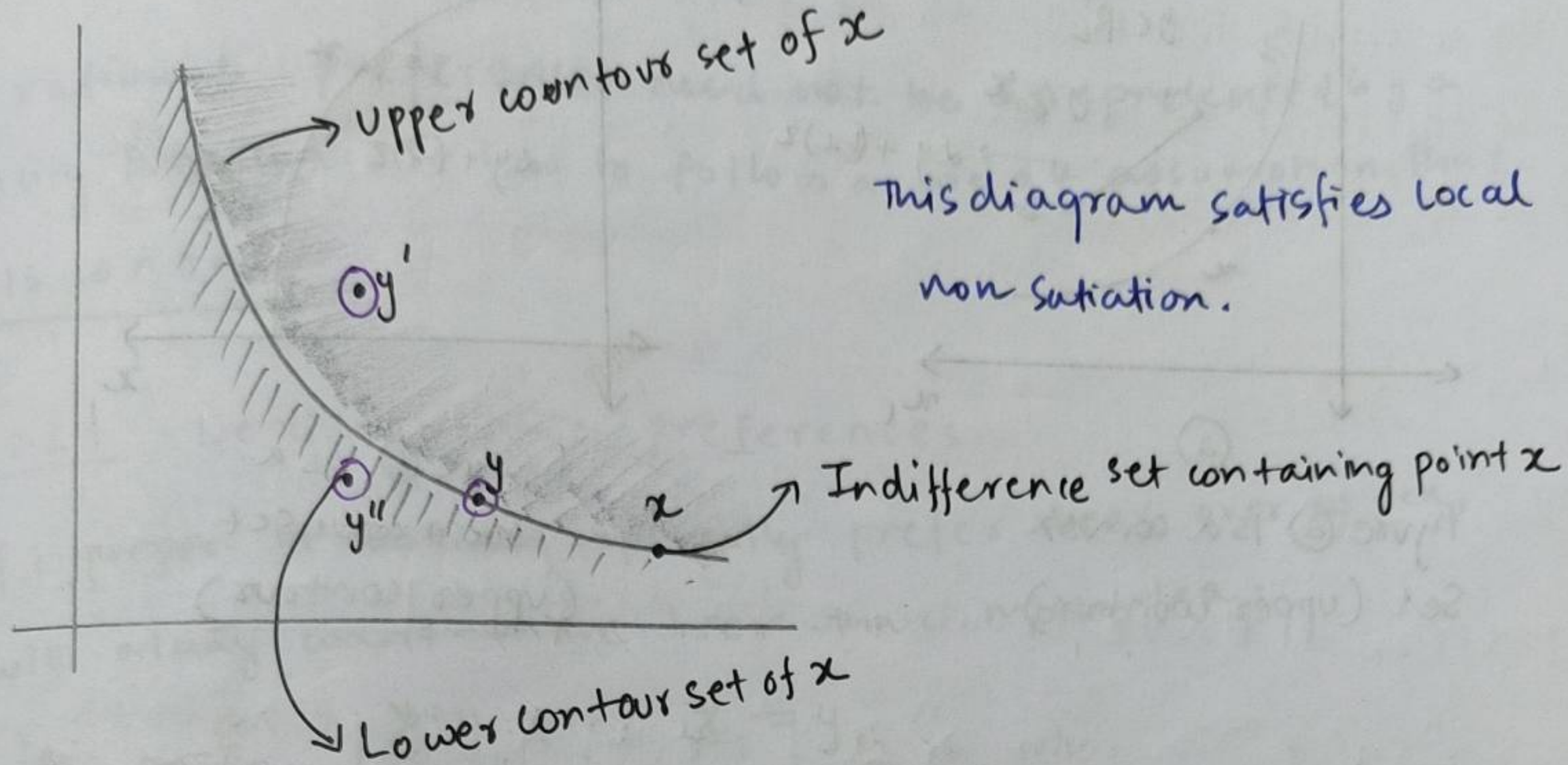
<u>Local non satiation</u> is a weaker imposition than monotonicity; as it allows ~~to~~ some goods to be non monotone or bads. While monotone needs $y \gg x$ or vice versa, Local non satiation allows bundles to be Less than ~~each~~ eachother and ~~strict~~ still have a strict preference. Monotone needs more of atleast some good while Local non satiation ~~the~~ says that around any bundle, there will exist prefered bundle no matter <u>you need more or Less of any good.</u>

<u>Implications of monotonicity (hence non satiation) is that ICs are not thick, they are thin.</u>



if IC's are thick, they cannot satisfy local non satiation or monotonicity (if one good or both goods are more) as higher bundle should give more utility.

components of the IC or sets related to consumption bundle



This diagram satisfies local non satiation.

Indifference set: $A = \{x, y \in X : x \sim y\}$

Upper contour set: $B = \{x, y \in X : y' \succsim x\}$

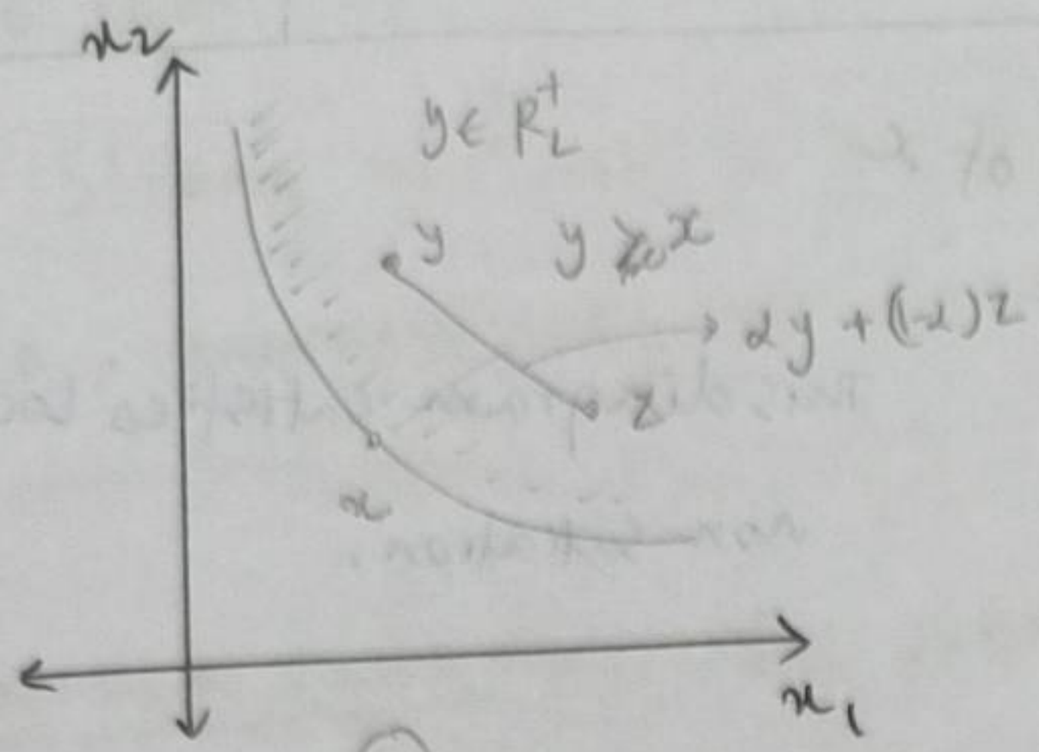
Lower counter set: $C = \{x, y \in X : x \succsim y''\}$

Convexity Assumption

A $\succsim$ order is convex. It implies the trade offs that consumer is willing to make among different goods.

Definition 3.B.4 (Convexity)

A preference relation $\succsim$ on X is convex if for every $x \in X$, the upper contour set $\{x, y \in X : y \succsim x\}$ is convex ie if $y \succsim x$ and $z \succsim x$ then $\alpha y + (1-\alpha) z \succsim x$ for any $\alpha \in (0,1)$



(a)

Figure (a) is a convex set (upper countor)

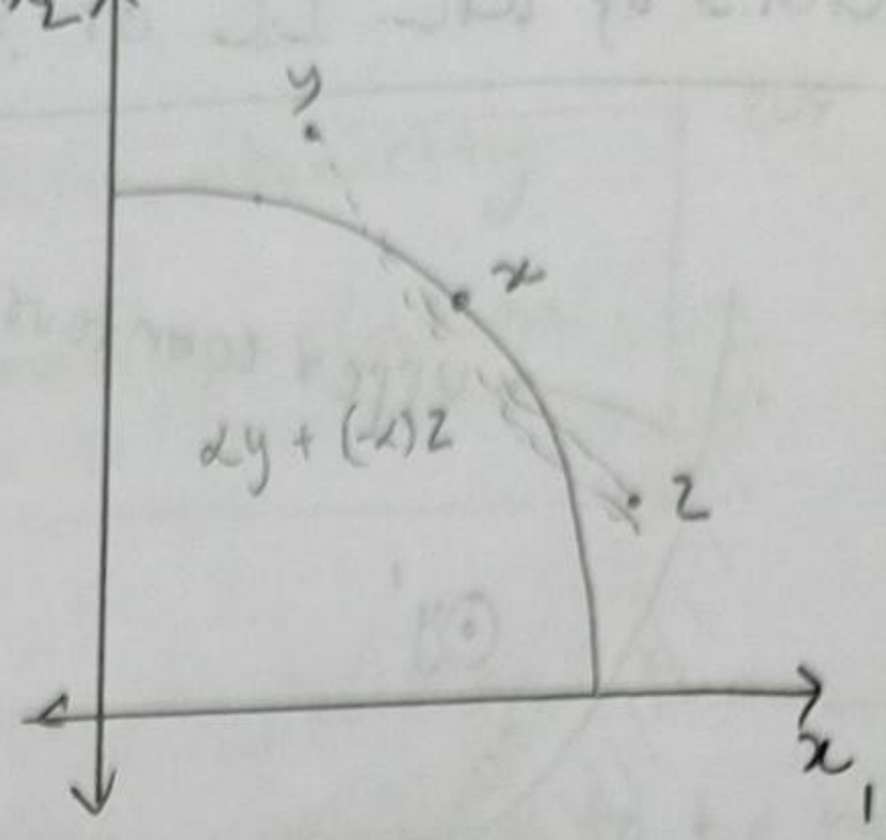


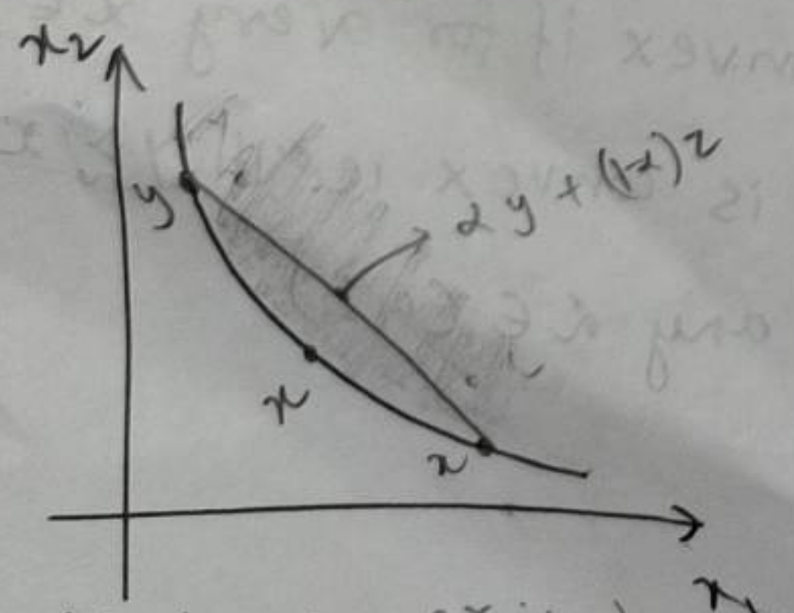
figure (b) is a non convex set (upper contour)

Convexity is ~~the~~ a strong and centeral assumption in economics; where it is important in defining DMRS of IC's. It says that to compensate for increasing amount of a good x, A good y in that same commodity space has to be losing it's unit's inside a consumption set X.

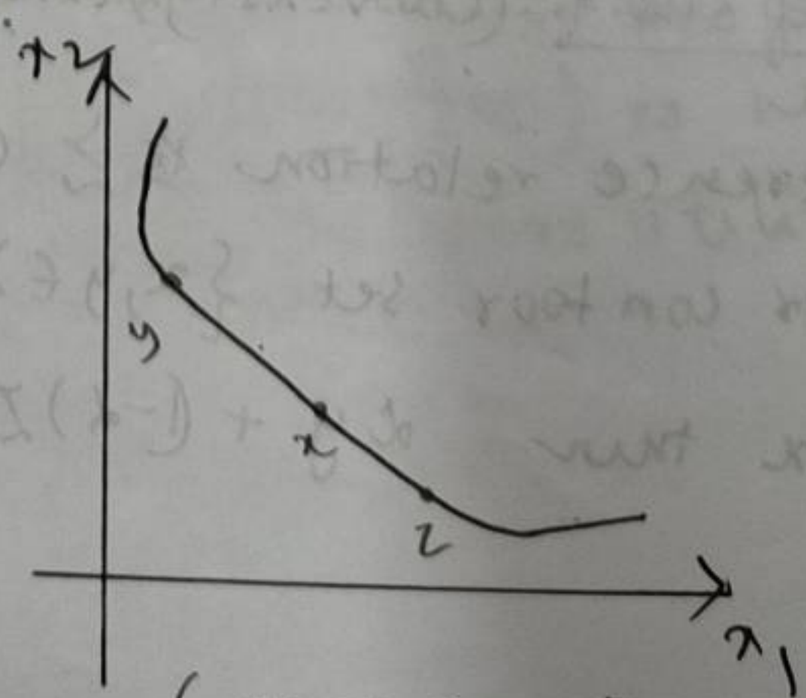
STRICT CONVEXITY

Definition 3.B.5:- The preference relation $\succsim$ on X is strictly convex if for every x, we have $y \succsim x$, $z \succsim x$ and $y \neq z \Rightarrow$

$\alpha y + (1-\alpha) z \succ x$ For all $0 \alpha \in (0,1)$



(Strict convexity)



(weak convexity)

3.C Preference & Utility

A rational preference need not be represented by a utility function. It has to follow a weak assumption that it is continous.

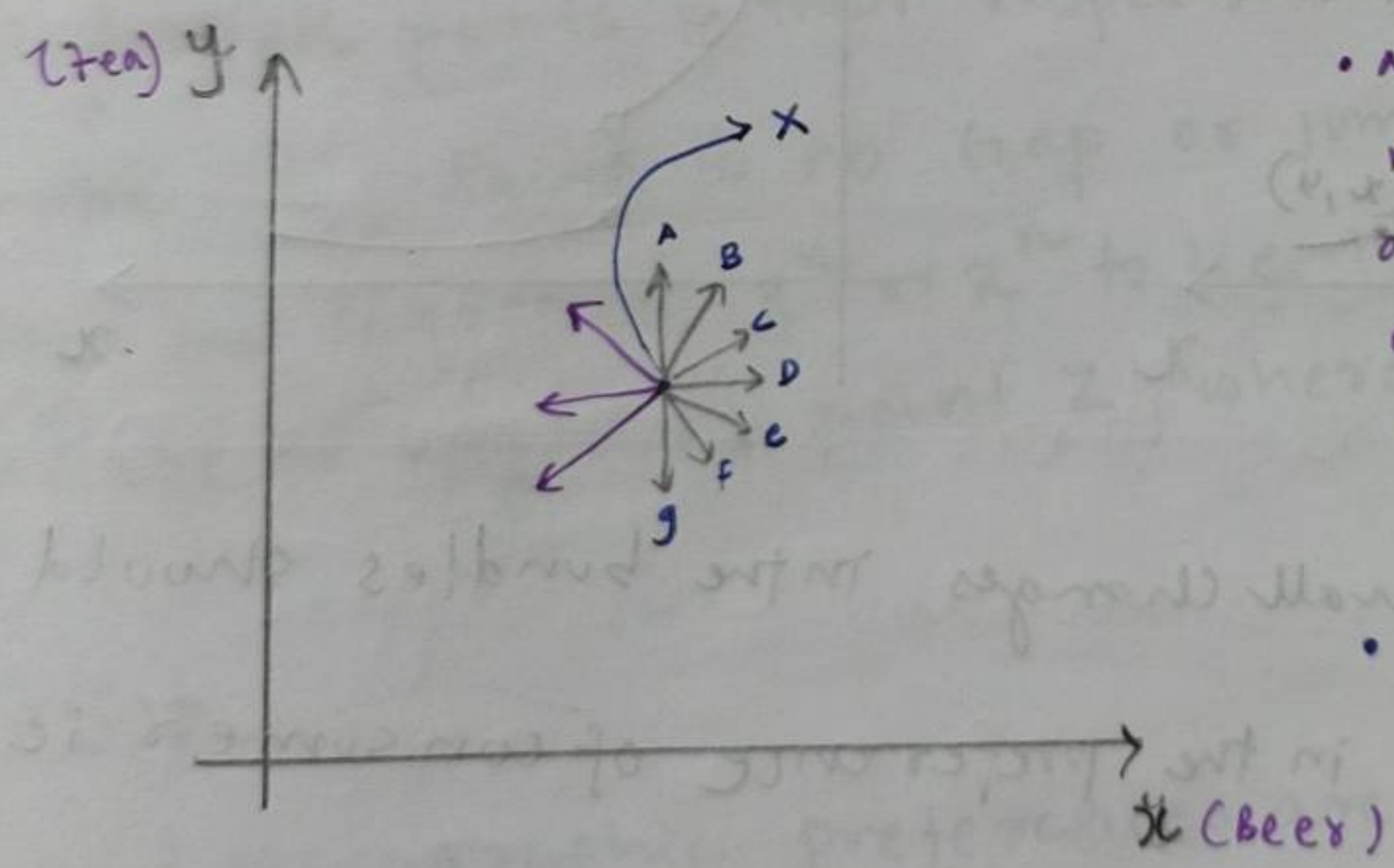
Ex. 3.C.1 Lexicographic preferences.

If i ~~prefer~~ ~~be~~ Lexicographicaly prefer beer over ~~to~~ tea, i will alway want more beer than more tea.

ie $x_1 > y_1$ or $x_1 = y_1$

~~then~~ and $x_2 \geq y_2$.

ie If beer is greater in amount than tea or is equal to tea (no less than).



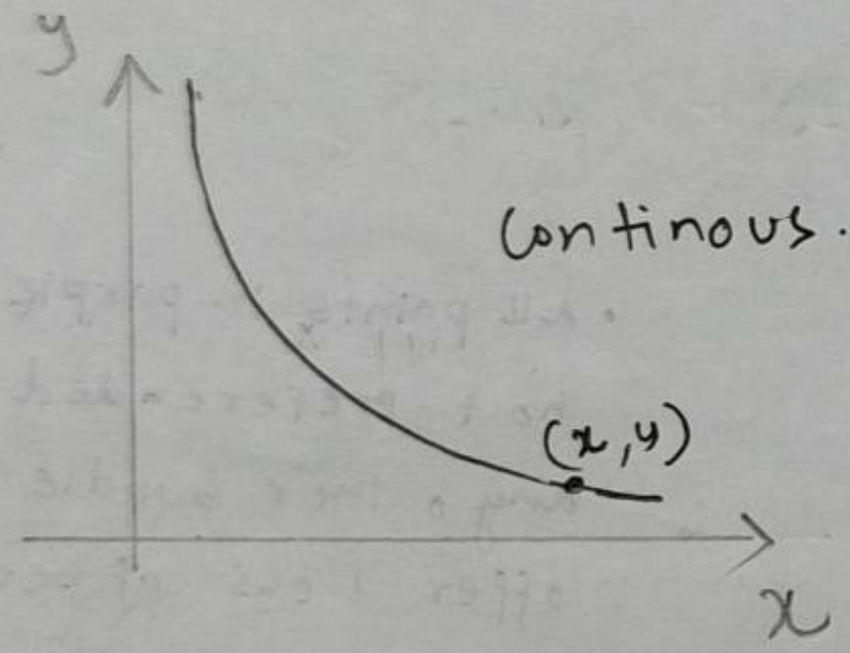
- All points in purple are not preferred over any other bundle as they offer less of beer and more or equal amount of tea.
- point A, B, C, D, E, F, g are points or bundles that are prefered over X. These bundles are lexicographically prefered over to X.

In case of lexicographic preferences, ~~as~~ they cannot be repre
by a utility function as ~~all~~ there is no possibility of 2 distin
~~indifference between bundles~~. bundles being indifferent.
~~Ie~~ ie Indifference sets are <u>singletons</u> (only 1 bundle in each I

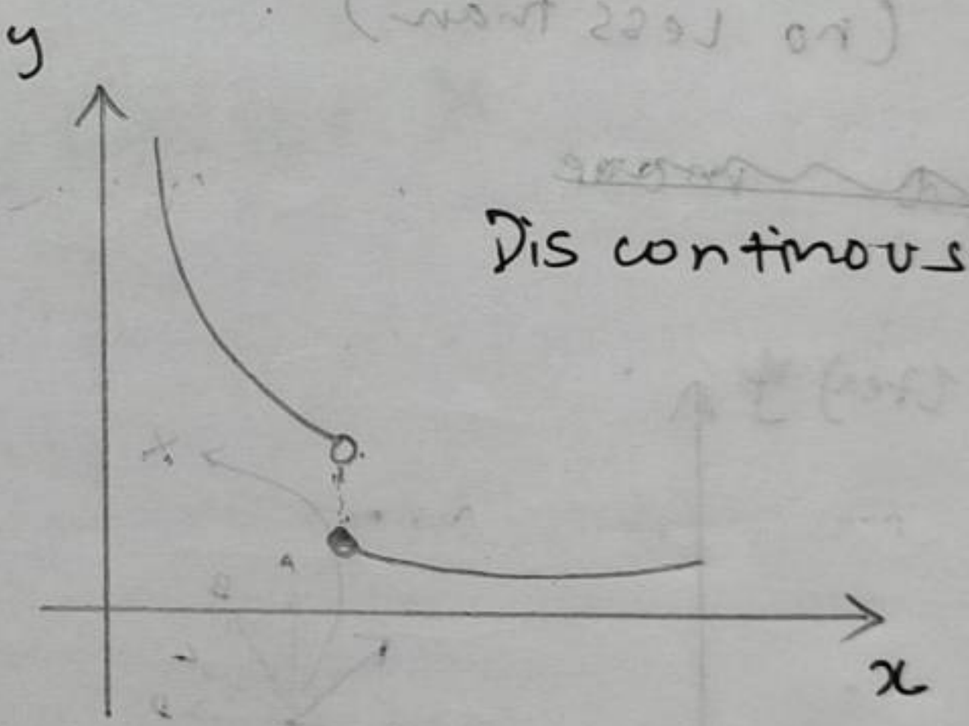
Def. 3.C.1

The preference relation $\succsim$ on X is continous if it is preserved under Limits. ie for any <u>sequence</u> of pairs $\{x^n, y^n\}_{n=1}^{\infty}$ with $x^n \succsim y^n \ \forall n$, $X = \lim_{n \to \infty} x^n$ and $Y = \lim_{n \to \infty} y^n$, <u>we have $X \succsim Y$</u> $\forall n \to \infty$

→



Continous.

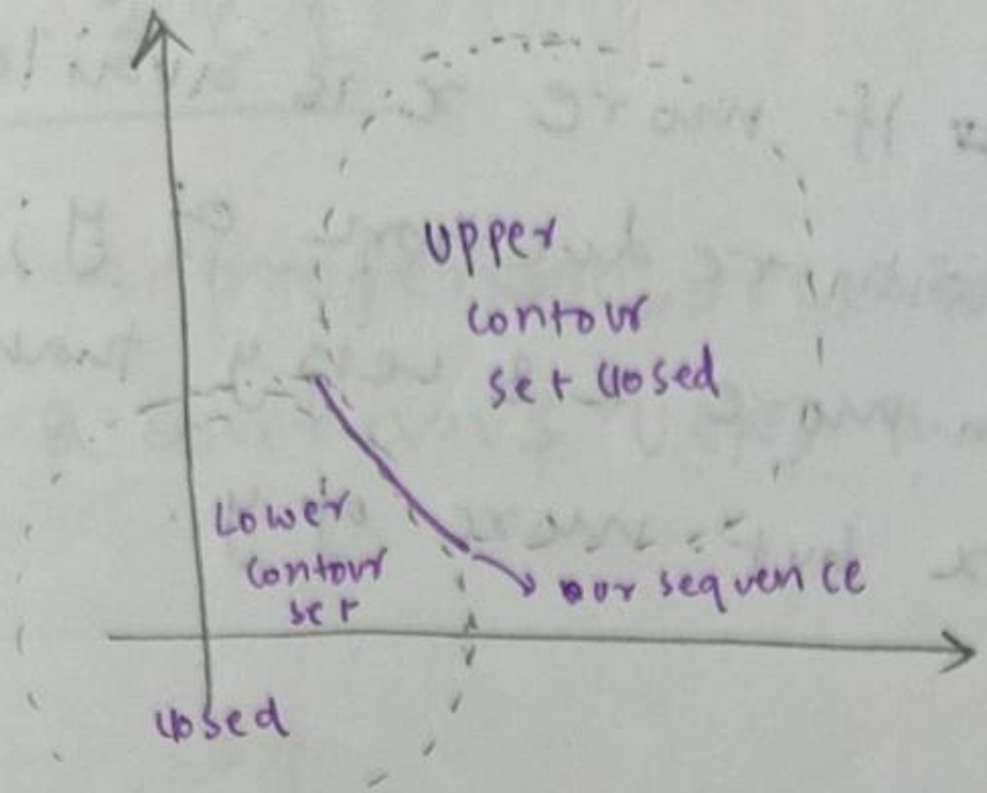


Dis continous

→ It says that small changes in the bundles should not lead to jumps in the preference of consumer ie if ~~at~~ $x^n \succsim y^n$ ~~over~~ where x^n & y^n are 2 sequence; then it should be that at the Limit also; $x \succsim y$ also an $n \to \infty$

not $y \succsim x$ {this will happen in case of lexicographic pref}

↳ reversal of preference.



An equivalent way to state <u>continuity</u> is to say that upper and Lower countour sets are closed.

How?

Upper contour set: $U(y) : \{ y \in X : y \succsim x \}$ → upper contour; no tutility; should contain all bundles where $y \succsim x$

Lower contour set: $L(x) : \{ x \in X : x \succsim y \}$ → should have all bundles where $x \succsim y$

<u>Neighbour hood concept:-</u>

$N_x^{\epsilon} : \{ y \in X : d_2(x,y) < \epsilon \}$ → if $L(x)$ is closed set

$N_y^{\epsilon} : \{ x \in X : d_2(x,y) < \epsilon \}$ → if $U(y)$ is closed set.

Then if we have $y \succsim x$ & $x \succsim y$; where $U(x)$ & $L(x)$ includes all points in their respective set, then For all points on the IC; there is no Gap or jump For any point. From the sequence y^n or x^n to Lie in $U(x)$ or $L(x)$ and converge to any other point z; where $x \not\succsim y$ or $y \not\succsim x$

<u>Ex - 3.C.1</u>

~~Lexi~~ Lexicographic preferences are not continous. <u>prove</u>

Suppose $(x_1, y_1) = A$

$(x_2, y_2) = B$

and person Lexicographically prefers $x \succsim y$

then in that case; ~~er~~ if more x is available for consumer to consume with reduction of y; the consumer will chose bundle with more x & less y than the one will no change in x but more of y.

→

$$x^n = \left(\frac{1}{n}, 0\right)$$

$$y^n = (0, 1).$$

if $x \succsim y$

~~as~~ then as $n \to \infty$; $x^n \to 0$. and at $n \to \infty$

$y \to 0$.

$\therefore$ Limiting value of x, y ~~to~~ converge to $(0,0)$

~~as~~ <u>at</u> x $(0,0)$

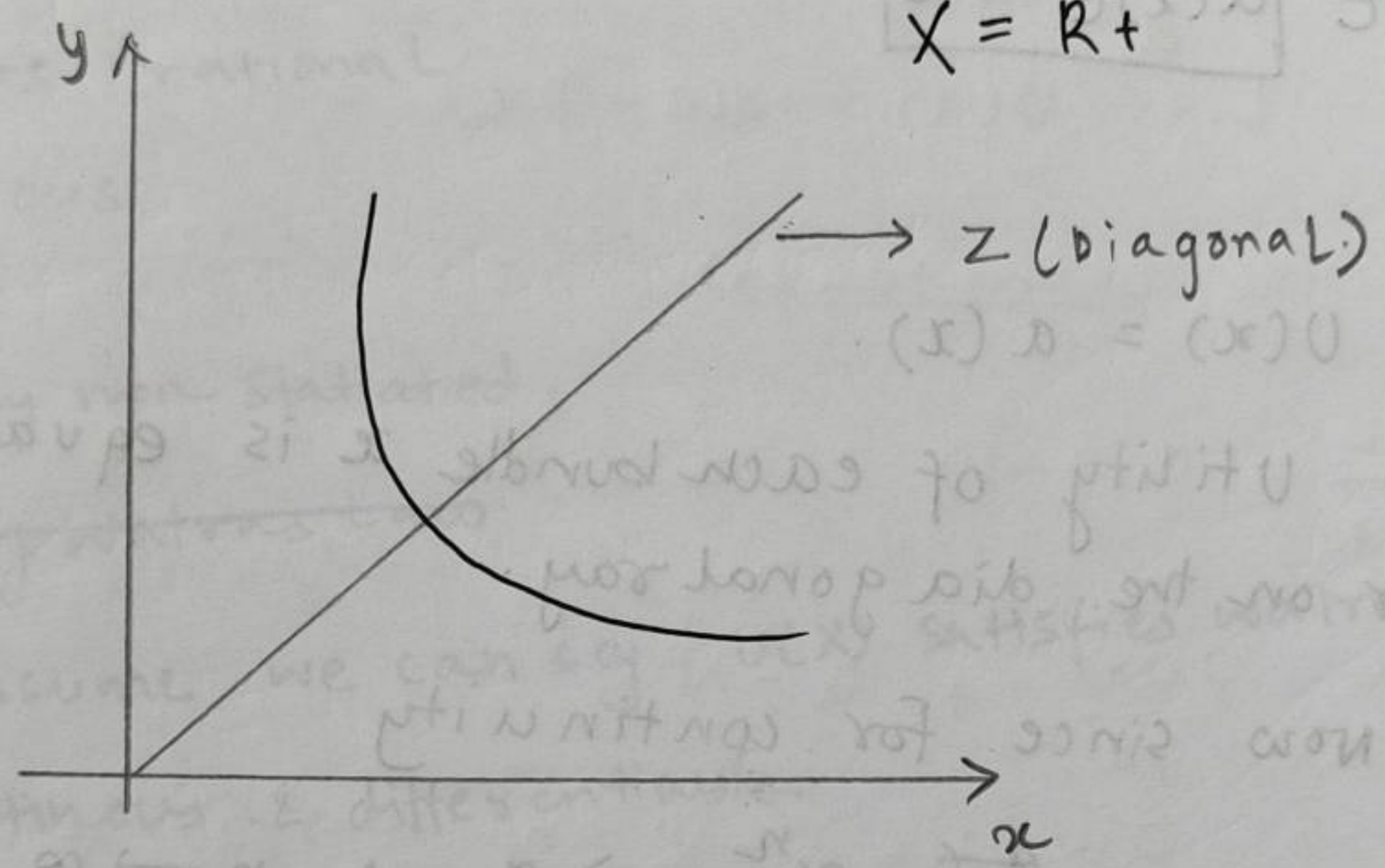
$x \sim y$ } ~~which to vi~~

ie $x \succsim y$ or $y \succsim x$; there is a change or reversal in preference order which should not happen under continuity $\therefore$ Hence Lexicographic preferences are not continous.

Proposition. 3.C.1

If a rational preference relation $\succsim$ on X is continous then, $\exists$ a continous $U(x)$ that represents. $\succsim$.

$X = R^L_+$



Proof:-

construct a diagonal Z in the non-negative orthant

ie $X = R^L_+$.

Let $a(x)$ be a scalar st $a(x) \geq 0$ & $a(x)e$ is on the diagonal ray. Here e is the vector of all ones. $(1, 1, 1, 1 \ldots)$

This mean $a(x)e$ have same components in them.

~~Now, we know $\succsim$ is rational & continous on X.~~

ie since $a(x)e$ lie on Z; $Z = \{ae : a \geq 0\}$.

ie each point of this ray is $(a, a, a, a, \ldots)$

Now

due to $x \geq 0$ ie monotonicity of commodities.

if $x \succsim y$

then mapping this onto the diagonal ray

we get $a(x) \geq a(y)$; which preserves order.

$\Rightarrow$ ~~ae~~ $\boxed{a(x)e \sim x}$.

<u>Now</u>

Let $U(x) = a(x)$.

then Utility of each bundle x is equal to a unique scalar on the diagonal ray.

~~Since~~ Now since for continuity

if $x^n \succsim y^n$ ~~at~~ $x^n \longrightarrow x$ as $n \to \infty$ st $x \succsim y$

$y^n \longrightarrow y$ as $n \to \infty$

and since $a(x)e \sim x$

$\Rightarrow$ $a(x^n)e \longrightarrow a(x)e$ as $n \to \infty$

$a(y^n)e \longrightarrow a(y)e$ as $n \to \infty$

~~Since~~ then since $U(x) = a(x)e$ and $x^n \succsim y^n$

$\Rightarrow$ $U(x^n) \geq U(y^n)$.

$\Rightarrow$ $U(x) \geq U(y)$ $\Leftrightarrow$ $x \succsim y$

$\therefore$ $U(x)$ is continous and represents $\succsim$

The Utility Maximisation problem 3.D

The problem of utility maximisation needs a few assumption about preferences

- preferences are well behaved: ~~ie rational~~ ~~ie rational~~.
  - ↳ ~~continous~~ rational
  - ↳ continous
  - ↳ convex
  - ↳ Locally non Statiated.
  - ~~↳ satisfy Walras Law~~.

Given these assume we can say $U(X)$ satisfies walras Law and is continous & differentiable.

Utility Maximisation

Given the budget set

$$B_{p,w} : \{x \in X : px \leq w\} \quad \text{where } p >> 0. \quad w > 0.$$

then consumer's utility max problem is

$$\max_{x \geq 0} U(x)$$
$$\text{st } px \leq w$$

This problem is going to be solved through Kuhn tuck conditions.

2 variable - 1 inequality constraint Kuhn tucker's FOC:-

(1) $\frac{\partial L}{\partial x_1} = 0$

(11) $\frac{\partial L}{\partial x_2} = 0$

(3) $\lambda \cdot h(x) = 0$
↳ constraint

(4) $h \leq 0$.

(5) $x_1, x_2 \geq 0$

(6) $\lambda \geq 0 \rightarrow$ maxima.
$\lambda \leq 0$ minima.

1, 2, 3, 4 are for finding critical values
5, 6 are for checking.

Now For our UMP.

$$\max U(x)$$
$$\text{st} \quad px \le w.$$

~~FOC~~

Let $L = U(x) + \lambda(w - px)$

then FOC: (Kuhn tucker)

$$\frac{\partial L}{\partial x} = 0 \quad \text{—} ①$$

then $\frac{\partial U(x)}{\partial x} + 0 - \lambda P = 0$

$\Rightarrow$ ~~∂U~~ $\boxed{U(x^*) - \lambda P_i = 0}$ $\longrightarrow$ ⓐ $\forall\, i = 1, 2, 3 \ldots$

$$\cancel{\frac{\partial L}{\partial \lambda} = 0 \quad \text{—} ②}$$

$$\frac{\partial L}{\partial w} =$$

~~then~~ $\cancel{\frac{\partial U(x)}{\partial \lambda} + \frac{\partial \lambda}{\partial \lambda}(w - px) = 0}$

$\Rightarrow$ ~~$\boxed{w - p_i x^* = 0}$ $\longrightarrow$ ⓑ~~

$$\lambda \cdot h(x) = 0 \quad \text{—} ③$$

$$\boxed{\lambda \cdot (w - px^*) = 0} \longrightarrow ⓒ$$

$$h(x) \le 0 \quad \rightarrow ④$$

we have $px \le w$

~~w~~ ~~wtp~~

then $w - px^* \geq 0 \rightarrow$ ⓓ

For the condition

$X_i\,(U_i(x_i^*) - \lambda^* P_i) = 0$ to be derived ; what do we use?

↳ multiply x_i to eqn ① on both sides.

$\Rightarrow$ ~~$x_i \frac{\partial U}{\partial x_i} - \frac{\partial \lambda}{\partial x} P_i x_i = 0$~~.

$$\Rightarrow \frac{\partial L}{\partial w} = \frac{\partial U(x(P,w))}{\partial x\, w} + \lambda \frac{\partial w}{\partial w} - \lambda \frac{\partial P\, x(w,P)}{\partial w}.$$

$$= \frac{\partial U}{\partial x} \cdot \frac{\partial x}{\partial w} + \lambda - \lambda P x^*$$

$$= x^* U(x^*) + \lambda - \lambda P x^*$$

$$= x^* U(x^*) - \lambda P x^* + \lambda.$$

$$= x^* (U(x^*) - \lambda P) + \lambda = 0.$$

Thus kuhn-tucker conditions are

(I) $\boxed{U_i(x^*) - \lambda P_i = 0.}$ → our main case of find x^* $\quad i = 1 \ldots L$

(II) $x_i^* (U_i(x^*) - \lambda P_i) = 0 \quad i = 1 \ldots L$

(III) $\lambda (w - P x^*) = 0$

(IV) ~~$h(x) \le 0$~~ $\quad w - P x^* \geq 0.$

From eqn ① & (II) there are $L+1$ unknowns. (commodity, λ) Thus we need $L+1$ eqns to solve this for optima.

Since preferences are locally non satiated; the consumer will choose his consumption bundle ~~either~~ on the boundaries of the budget set ie { either on interior optima or corner solution }

↳ $x_i^* > 0$ $\qquad$ ↳ $x_i^* = 0$

But for our convineance we choose $x_i^* > 0$:

we find our optimal x^* using

$$\frac{\partial U(x)}{\partial x} - \lambda P_i = 0 \quad \text{or} \quad U_i(x^*) - \lambda P_i = 0 \quad \forall i = 1, 2 \ldots i$$

<u>Now</u> suppose $i = 1, 2$. {2good case}

$$\therefore \quad U_i(x^*) - \lambda P_i = 0$$

$$\Rightarrow \quad U_1(x^*) - \lambda P_1 = 0.$$

$$\& \quad U_2(x^*) - \lambda P_2 = 0.$$

$$\Rightarrow \quad \frac{U_1(x^*)}{U_2(x^*)} = \frac{\cancel{\lambda} P_1}{\cancel{\lambda} P_2}. \quad \{\text{divinding them}\}$$

$$\Rightarrow \quad \boxed{\frac{U_1(x_1^*)}{U_2(x_2^*)} = \frac{P_1}{P_2}.}$$

$$\Rightarrow \quad \frac{\partial U(x)/\partial x_1}{\partial U(x)/\partial x_2} = \frac{P_1}{P_2} \qquad x = x_1, x_2$$

$$\Rightarrow \quad \boxed{\frac{MU_{x_1}}{MU_{x_2}} = \frac{P_1}{P_2}.}$$

<u>NOTE</u>

$U(x_1, x_2)$ for $x_i = 1, 2$

then $dU(x_1, x_2) = \frac{\partial U(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial U(x_1, x_2)}{\partial x_2} dx_2$

Since U is a constant value; $dU = 0$.

$$\therefore \frac{\partial U}{\partial x_1}(x_1, x_2)\, dx_1 = -\frac{\partial U}{\partial x_2}(x_1, x_2)\, dx_2$$

$$\Rightarrow \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = -\frac{dx_2}{dx_1} \longrightarrow \text{slope of I.C}$$

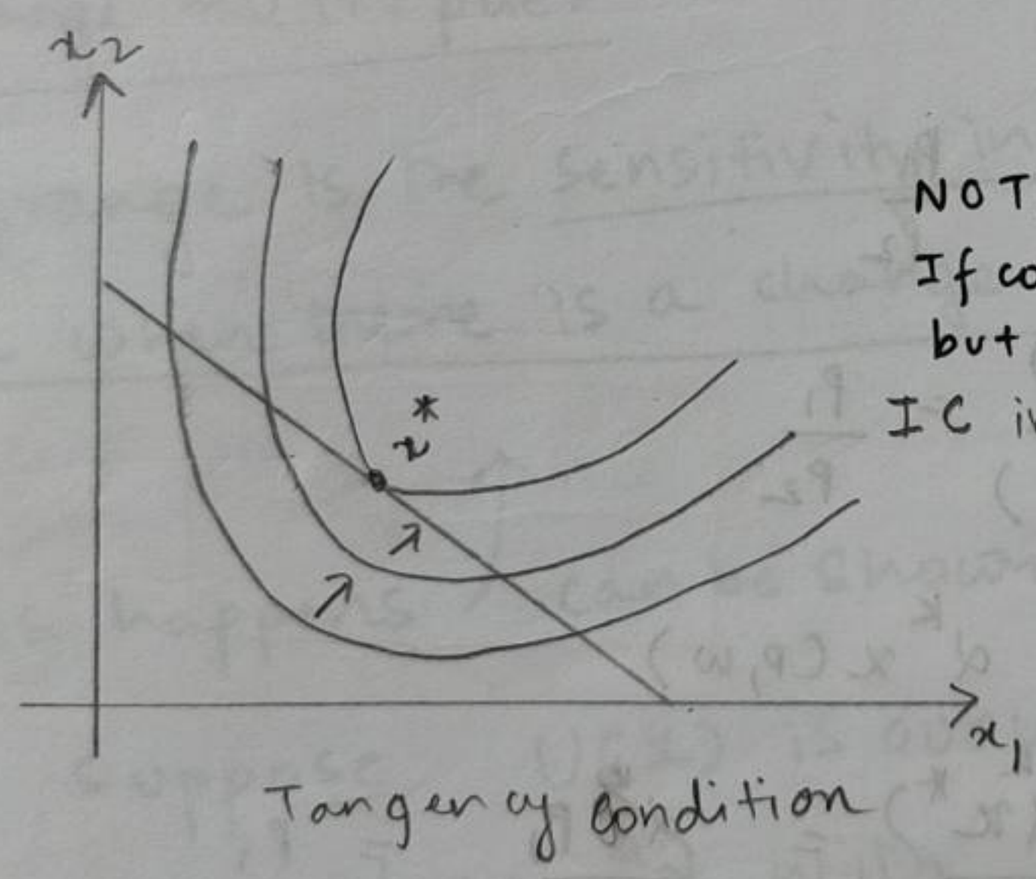
from optimality condition

$$\boxed{\frac{MUx_1}{MUx_2} = \frac{P_1}{P_2} = -\frac{dx_2}{dx_1}} \rightarrow \text{tangency condition of budget line \& IC.}$$

$-\frac{dx_2}{dx_1}$ is called MRS or marginal rate of substitution.

Hence For optimality condition (to obtain x^*)

$$MRS = \left.\frac{P_1}{P_2}\right\} \rightarrow \text{slope of bL}$$



Tangency condition

NOTE: all the Levelsets are convex.
If convexity is not implied $\Rightarrow$ multiple optima but no global optima.
IC increases; given budget constraint.

<u>Proposition 3.D.2</u> <u>Properties of walrasian DD $x(P,\omega)$</u>

<u>NOTE</u>

$x^* = x^*(P, \omega)$

ie if prices & wealth were changed; $x^*(P,\omega)$ would also be different.

$x(p,w)$ is called marshallian dd or ordinary dd function.

Properties :-

① WALRAS-LAW : $x(p,w)$ satisfies walras Law ie

$$p\,x(p,w) = w$$

Proof
According to Local non-satiation if the $x \in X$ and $\varepsilon > 0$.

then $\exists\, y \in X$ such that $\|x-y\| < \varepsilon$ and $y \succ x$. Thus the only way $x \succ y$ is when it is on the budget Line ie $\boxed{px = w}$

② Homogenity of degree zero in (p,w)

$$x(\alpha p, \alpha w) = \alpha^k x(p,w) = \alpha^0 x(p,w) = x(p,w). \quad \forall \alpha \geq 0$$

Proof—
Now to show why it is homogenous of degree zero we observe it's impact on utility maximisation problem.

in UMP:

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{U(x_1^*)}{U(x_2^*)} = \frac{P_1}{P_2}$$

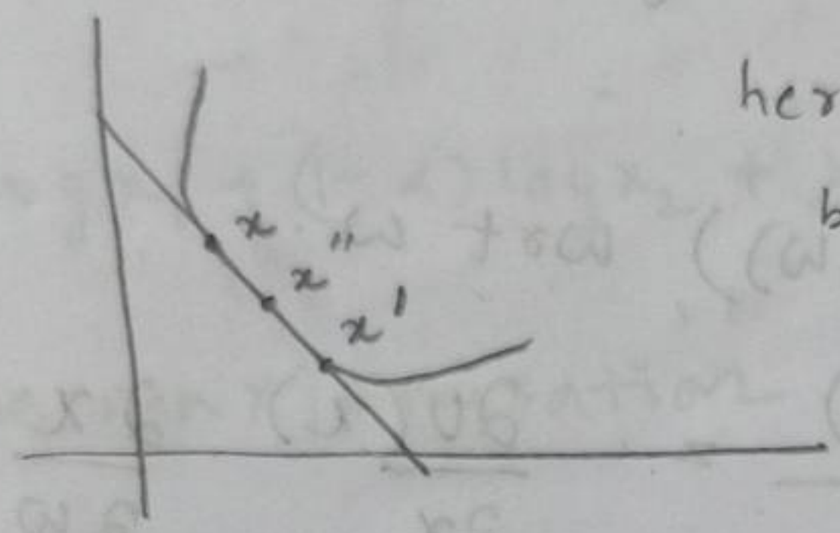
now if $x_1(\alpha p, \alpha w) = \alpha^k x(p,w)$

$$\Rightarrow \frac{U(\alpha^k x^*)}{U(\alpha^k x^*)} = \frac{\cancel{\alpha^k} P_1}{\cancel{\alpha^k} P_2} = \frac{P_1}{P_2}$$

hence by any change in the Walrasian demand by $\alpha > 0$; there is no change of the optimality condition ie the point of tangency remains intact.

Convexity of $x(p,w)$

Prof When $x^*(p,w)$'s convexity is discussed, under optimality condition, If preferences are convex & ~~utility~~ continous the $x^*(p,w)$ is the optima. However it need not be so iff preferences are weakly convex.



here $x'' = \alpha x + (1-\alpha) x'$

but all these points offer same level of utility.

ie $U(x) = U(x'') = U(x')$

The flat points makes the entire region optimal; hence if preferences are convex, the region will be convex set. If the preferences are stricty convex, so $U(\cdot)$ is stricty ~~to~~ quasiconcave, then $x(p,w)$ is a single point for $B_{p,w}$.

The Lagrange multiplier

The Lagrange is the sensitivity indicator of the ~~the~~ objective function when there is a change in the constraint.

Now

why is happens ↑ can be shown as follows.

Suppose $U(x)$ is our utility function.

then we replace x with $x(p,w)$. Now

$U(x(p,w))$. where $x(p,w)$ is the demand function at price p, w.

The constraint is

$$px \leq w.$$

To see how λ is explaing sensitivity; we take into account a change in constraint $B_{p,w}$. We make a small change of Δw ie by increasing w to $w+\Delta w$ st $B_{p,w} \longrightarrow B_{p,\Delta w+w}$. {this means more points are feasible under $B_{p,\Delta w+w}$ along with the points of $B_{p,w}$}.

Mathematically

diff $U(x(p,w))$ wrt w.

$$\Rightarrow \quad \frac{\partial U(x(p,w))}{\partial w} = \frac{\partial U(x)}{\partial x} \cdot \frac{\partial x}{\partial w}.$$

now $\frac{\partial U(x)}{\partial x} = U_i(x^*) = \lambda p_i$ { FOC of kuhn tucker }

$$\Rightarrow \quad \frac{\partial U(x(p,w))}{\partial w} = \lambda p_i \cdot \frac{\partial x}{\partial w}$$

For L commodities

$$\Rightarrow \quad \sum_{L=1}^{L} \lambda p_i \frac{\partial x}{\partial w}.$$

$$\Rightarrow \quad \lambda \sum_{L=1}^{L} p_i \frac{\partial x}{\partial w} \quad \therefore \boxed{\frac{\partial U}{\partial w} = \lambda}$$ ✓✓ important

from WALRS LAW

$$px = w$$

$$p \frac{\partial x}{\partial w} = 1$$

$$\Rightarrow \quad \lambda \sum_{L=1}^{L} 1 \Rightarrow \lambda$$ (which is the sensitivity due to change in constraint on objective function)

Ex - 3.D.1 $\quad$ L = 2 case

$$U = K x_1^{\alpha} x_2^{1-\alpha}.$$

or. $\text{Log } U = \text{Log } K x_1^{\alpha} x_2^{1-\alpha}.$

$$\max \log U = \text{Log } K + \alpha \text{Log } x_1 + (1-\alpha) \text{Log } x_2$$

$$\text{st} \quad P_1 x_1 + P_2 x_2 = w$$

$$L = \text{Log } K + \alpha \text{Log } x_1 + (1-\alpha) \text{Log } x_2 + \lambda (w - P_1 x_1 - P_2 x_2)$$

Through simple maxismisation

$$L = \text{Log} K + \alpha \text{Log} x_1 + (1-\alpha) \text{Log} x_2 + \lambda (w - P_1 x_1 - P_2 x_2)$$

$$\frac{\partial L}{\partial x_1} = 0 + \frac{\alpha}{x_1} \cdot \cancel{-} \quad \lambda P_1 = 0$$

$$= \frac{\alpha}{x_1} = \lambda P_1 \quad \text{——} \quad (1)$$

or $\quad x_1^* = \dfrac{\alpha}{\lambda P_1}$

$$\frac{\partial L}{\partial x_2} = \frac{(1-\alpha)}{x_2} = \lambda P_2$$

$$\Rightarrow \quad x_2^* = \frac{(1-\alpha)}{\lambda P_2} \quad \text{——} \quad (ii)$$

$$\frac{\partial L}{\partial \lambda} = w - P_1 x_1 - P_2 x_2 = 0. \quad \text{——} \quad (iii)$$

Comparing 3 unknowns ; 3 eqns.

$x_1, x_2, \lambda.$

Ratio of eq (1) & (ii)

$$\boxed{\frac{x_1^*}{x_2^*} = \frac{\alpha}{\lambda P_1} \cdot \frac{\lambda P_2}{(1-\alpha)} = \frac{\alpha}{1-\alpha} \cdot \frac{P_2}{P_1}}$$

$$x_1^* = \frac{\alpha}{1-\alpha} \cdot \frac{P_2}{P_1} \cdot x_2 \quad \text{—— (IV)}$$

replacing eqn (IV) into (III)

$$\Rightarrow \omega = P_1 x_1 + P_2 x_2$$

$$\Rightarrow \omega = \cancel{P_1} \times \frac{\alpha}{1-\alpha} \cdot \frac{P_2}{\cancel{P_1}} x_2 + P_2 x_2$$

$$\Rightarrow \omega = \frac{\alpha P_2 x_2}{1-\alpha} + P_2 x_2$$

$$\Rightarrow \omega = \left[\frac{\alpha}{1-\alpha} + 1\right] P_2 x_2$$

$$\Rightarrow \omega = \left[\frac{\cancel{\alpha} + 1 - \cancel{\alpha}}{1-\alpha}\right] P_2 x_2$$

$$\Rightarrow \boxed{\frac{\omega(1-\alpha)}{P_2} = x_2^*}$$

$\rightarrow$ $x_i(P, \omega)$ — marshallian Demand f.

For x_1 s $= \frac{\alpha}{\cancel{(1-\alpha)}} \cdot \frac{\cancel{P_2}}{P_1} \cdot \frac{\omega \cancel{(1-\alpha)}}{\cancel{P_2}}$

$$\boxed{x_1^* = \frac{\alpha}{P_1} \cdot \omega}$$

$\rightarrow$ $x_i(P, \omega)$

Exercise 3.D.1 (imp).

Indirect Utility function

Representation of $x(P,\omega)$ as a ~~function of~~ form (P,ω). Replacing $x(P,\omega)$'s version into our original utility function $U(x)$ will give $U(x) \overset{\rightarrow}{=} V(P,\omega)$

where $U(X) \equiv V(P,\omega)$ But V is not a direct function of x. rather it is indirect function of x (commodities) through it's P, ω.

V = indirect utility function.

Given $V(P,\omega)$ and P & W; we can calculate $x(P,\omega)$ (Roy's Identity)

Properties of $V(P,\omega)$ {similar to properties of $x(P,\omega)$}

Properties of $V(P,\omega)$ are inherited properties of $x(P,\omega)$. (proposition 3.D.3)

① Homogenity of degree zero :-

Proof Like $x(\alpha P, \alpha\omega) = \alpha^0 x(P,\omega) \ \forall \ \alpha > 0$.

if $V(\alpha P, \alpha W) = \alpha^K V(P,\omega)$

then since $x(\alpha P, \alpha\omega) = \alpha^0 x(P,\omega)$

$\Rightarrow$ ~~$U(x(\alpha, P$~~ $U(x^*(\alpha P, \alpha\omega)) = U(x^*(P,\omega))$

$\Rightarrow$ $U(x^*(\alpha P, \alpha\omega) = U(x^*(P,\omega)) = V(P,\omega)$

$\Rightarrow$ $V(P,\omega) = V(\alpha P, \alpha W) \ \forall \ \alpha > 0$

$\Rightarrow$ $\alpha = 1$ for homogenity to hold

ie $\alpha^K = \alpha^0 = 1$

② $V(P,\omega)$ is strictly increasing in ω & non increasing (≤ 0) in P_l $\forall\, l = 1 \cdots L$: if $x_l > 0$ then $V(P,\omega)$ is strictly decreasing in P_l. ~~for x~~ $\forall\, l = 1 \cdots L$, while increasing strictly due to local non-satiation.

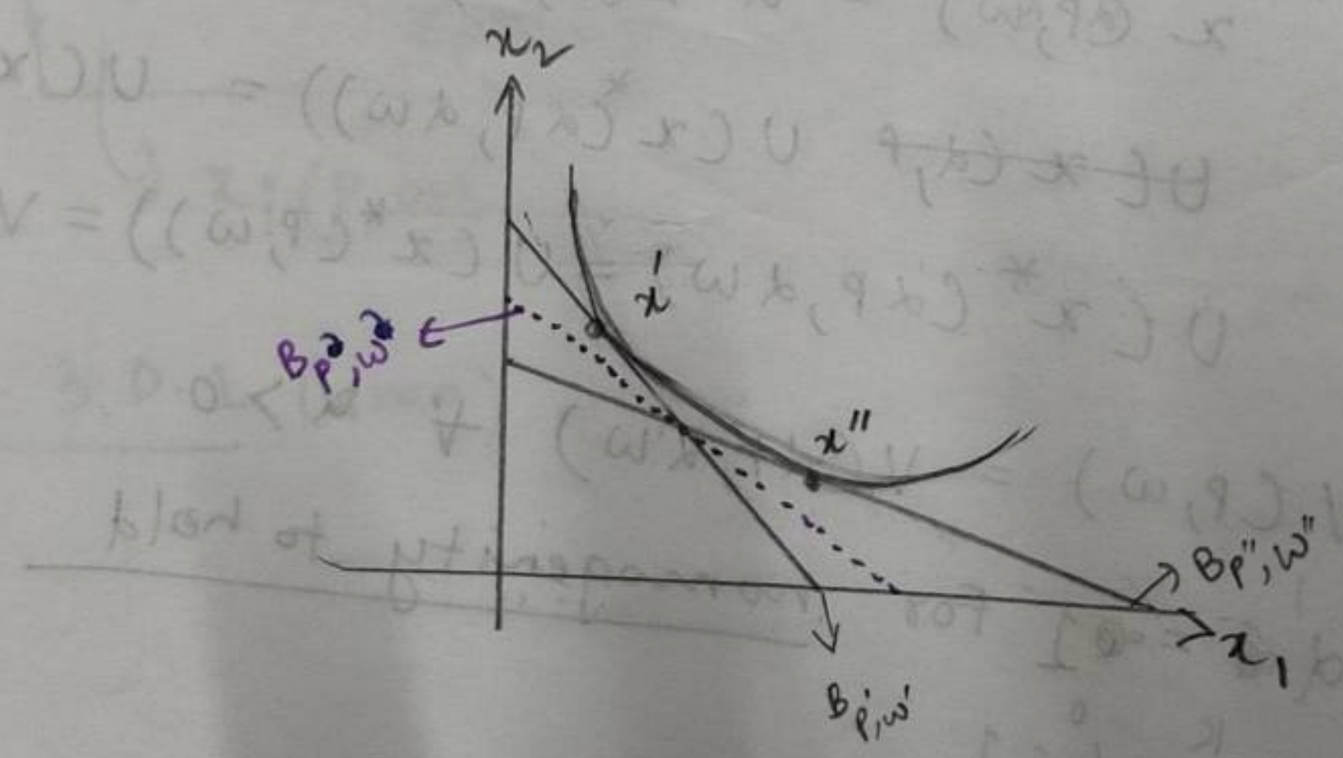
→ If x^* is optimal & preferences are locally non satiated then despite budget line, the must be a point outside the Budget constraint $B_{p,\omega}$ which is better than x^*. Suppose $x' > x^*$; but since x' is not affordable, a slight increase in budget; increases utility of consumer if $x' \succ x^*$ and x' is now affordable. If prices of $x \uparrow$, then budget line pivots about ~~it's~~ at a point. ~~a~~ so if earlier x^* was optimal, no x^* won't be affordable as $P_x \uparrow$.

③ $V(P,\omega)$ is ~~to~~ quasi convex $\{U(x)$ is quasiconcave$\}$

~~to~~ quasi ~~concavity~~ convexity means:

$f(x) \leq \bar{x}$ (Lower contour set)

$$V(x) : \{(P,\omega) : V(P,\omega) \leq \bar{V}\}$$ is convex for all $\bar{V}$ is what we have to say if we mean by quasi concavity



Suppose; there are 2 Budgets with price wealth (p',ω') & (p'',ω''). Each budget has optimality for a IC at x'^* & x''^*.

taking a convex combination, $\alpha p' + (1-\alpha)p'' = p^a$ & $\alpha w' + (1-\alpha)w'' = w^a$

we create a new budget set B_{p^a, w^a}; where every commodity avaiable for ~~(p', w') & (p'', w'') are going to lie in~~ $B_{p^a w^a}$, will either lie within $B_{p', w'}$ or $B_{p'', w''}$. So utility of form any bundle in B_{p^a, w^a} will be no larger than utility from x' & x''.

$\therefore \quad V(p', w) \geq V(p^a, w)$ which proves our point.

(4) $V(P, w)$ is continous in P, w.

<u>The Roy's Identity</u> { To establish a link between $x(P,w)$ & $V(P,w)$

we know

$$U(x) \equiv V(P, w)$$

differentiating both side wrt P_l.

$$\Rightarrow \sum_{i=1}^{L} \frac{\partial U}{\partial x} \cdot \frac{\partial x(P,w)}{\partial P_l} = \frac{\partial V}{\partial P_l}$$

since from kuhn-tuck; $U_i(x^*) = \lambda P_i$

or $\frac{\partial U}{\partial x} = \lambda P_i$

$$\Rightarrow \sum_{i=1}^{L} \lambda P_i \cdot \frac{\partial x}{\partial P_l}(P,w) = \frac{\partial V}{\partial P_l}.$$

$$\Rightarrow \lambda \sum P_i \cdot \frac{\partial x}{\partial P_l}(P,w) = \frac{\partial V}{\partial P_l}.$$

$$\Rightarrow \frac{\partial U}{\partial w} \sum P_i \frac{\partial x}{\partial P_l} = \frac{\partial V}{\partial P_l}.$$

↳ Langrang multiplier's derivation

since $U(x) \equiv V(P, w)$

then $\frac{\partial V}{\partial w} \sum p_i \frac{\partial x}{\partial p_l} = \frac{\partial V}{\partial p_l}$

From WALRAS LAW wrt p (Chp-2)

~~$p \cdot x = w$~~

$p \frac{\partial x}{\partial p_i} =$

$p \cdot x(p, w) = w$

$p \frac{\partial x}{\partial p} + x = \cancel{\frac{\partial w}{\partial p}} \to 0$

$\Rightarrow \boxed{x_i = -p_i \frac{\partial x_i}{\partial p_l}}$

$\therefore -\frac{\partial V}{\partial w} \sum_{i=1}^{L} x_i = \frac{\partial V}{\partial p_l}$

for a single commodity L

$\frac{\partial V}{\partial w} \cdot x_l = \frac{\partial V}{\partial p_l}$

$$\boxed{x_l(p, w) = \frac{\frac{\partial V}{\partial p_l}}{\frac{\partial V}{\partial w}}}$$

→ Roy's Identity

↓

allows us to derieve dd function From $V(p, w)$

Expenditure Minimisation Problem (EMP) 3.E

Measure of impact

Suppose in a price situation $p_1 \to p_1'$: Then what will be the impact on consumer's utility? $V(p', w) - V(p, w) =$ impact

But this value isn't saying how much is the quantity of impact on utility; at the end is an ordinal value.

So; to have a measure of impact; if a function tells us how much wealth you need to achieve a certain level of utility; we can easily also use it to calculate the impact of a price change! Such function is a <u>Expenditure function</u>

<u>EMP</u> → asks at what ~~given~~ minimum price-wealth combination that can be used to achieve a certain level of utility U

ie

$$\min p \cdot x$$
$$\text{st} \quad U(x^*) \geq \bar{U}$$

UMP computes the maximal level of utility that can be obtained given w, while EMP computes minimal level of wealth given U. EMP is the dual problem to UMP.

What are the endogenous variable & Exogenous variables here for comparative statics.

x ↓ ↳ p, U

ie EMP finds out what will be the optimal x^* ~~given~~ that minimises cost to achieve $\bar{U}$ at price p.

$$L = p \cdot x + \lambda(\bar{U} - U(x^*))$$
$$= p \cdot x - \lambda(U(x^*) - \bar{U}).$$

From Kuhn tucker FOC necessary condition

$$\frac{\partial L}{\partial x} = p - \lambda U(x^*) = 0 \quad \text{for } i = 1 \cdots L$$

$$\lambda \cdot \cancel{h}^{Z}(x) = 0 \quad \text{ie} \quad \lambda(U(x) - \bar{U}) = 0.$$

From FOC ; an x^* will be determined ~~wh~~ and also if $U(x)$ is continous and ~~to~~ binding (ie quasiconcave) & then $U(x) = \bar{U}$.

We redefine this x^* as $h(P,U)$ which is our hicksian Demand function ; ~~depen~~ depends on price & $\bar{U}$.

$h(P,U)$	$x(P,W)$
↳ is a function of price & utility	↳ is a function of price & wealth.
↳ hicksian or compensated dd function	↳ marshallian dd function or uncompensated dd function

Chapter 2 notes to see graph & SE & ME Effects.

↳ answer's which commodity bundle minimises cost of attaining $\bar{U}$ at price P	↳ answer's which commodity maximises U at given W & price P.

Our modified kuhn tuck condition: as $x^* \longrightarrow h(P,U)$

(i) $\frac{\partial L}{\partial x} = 0$: $P_i - \lambda(U_i(h(P,U))) = 0 \quad \forall i = 1 \cdots L$

(ii) $\lambda g(x) = 0$: $\lambda(U(h(P,U)) - \bar{U}) = 0$

~~Our~~ Like our indirect utility function defined when x^* is replaced into $U(x^*) \equiv V(P,W)$

we obtain Expenditure Function from replacement of x with $h(P,U)$ in our objective function ie $Px \leq W$

ie $P\,h(P,U) = W$ or $E(P,U)$

↳ Expenditure

indirect utility function

$\rightarrow$ From $x^*(P,w) \rightarrow U(x^*)$

$\equiv V(P,w)$

Expenditure function

$\rightarrow h(P,\bar{U}) \rightarrow px \leq w$

$\equiv E(P,U) = P\,h(P,\bar{U})$

First Note on Duality

From the FOC of kuhn tucker of UMP For $L=2$ ie i,j

Given $w \geq px$

$$\frac{\partial L}{\partial x_i} = 0 \; : \; P_i - \lambda U_i = 0$$

$$\Rightarrow \quad P_i = \lambda U_i \quad \text{— (12)}$$

similary

$$\frac{\partial L}{\partial x_j} = 0 \; : \; P_j - \lambda U_j = 0.$$

$$\Rightarrow \quad P_j = \lambda U_j \quad \text{— (11)}$$

Then

$$\frac{P_i}{P_j} = \frac{\lambda U_i}{\lambda U_j}$$

$$\Rightarrow \quad \frac{P_i}{P_j} = \frac{U_i(x_i^*)}{U_j(x_j^*)} \Big\} \text{ tangency condition}$$

$\frac{P_i}{P_j}$ ↓ budget line slope

$\frac{U_i(x_i^*)}{U_j(x_j^*)}$ ↳ slope of IC.

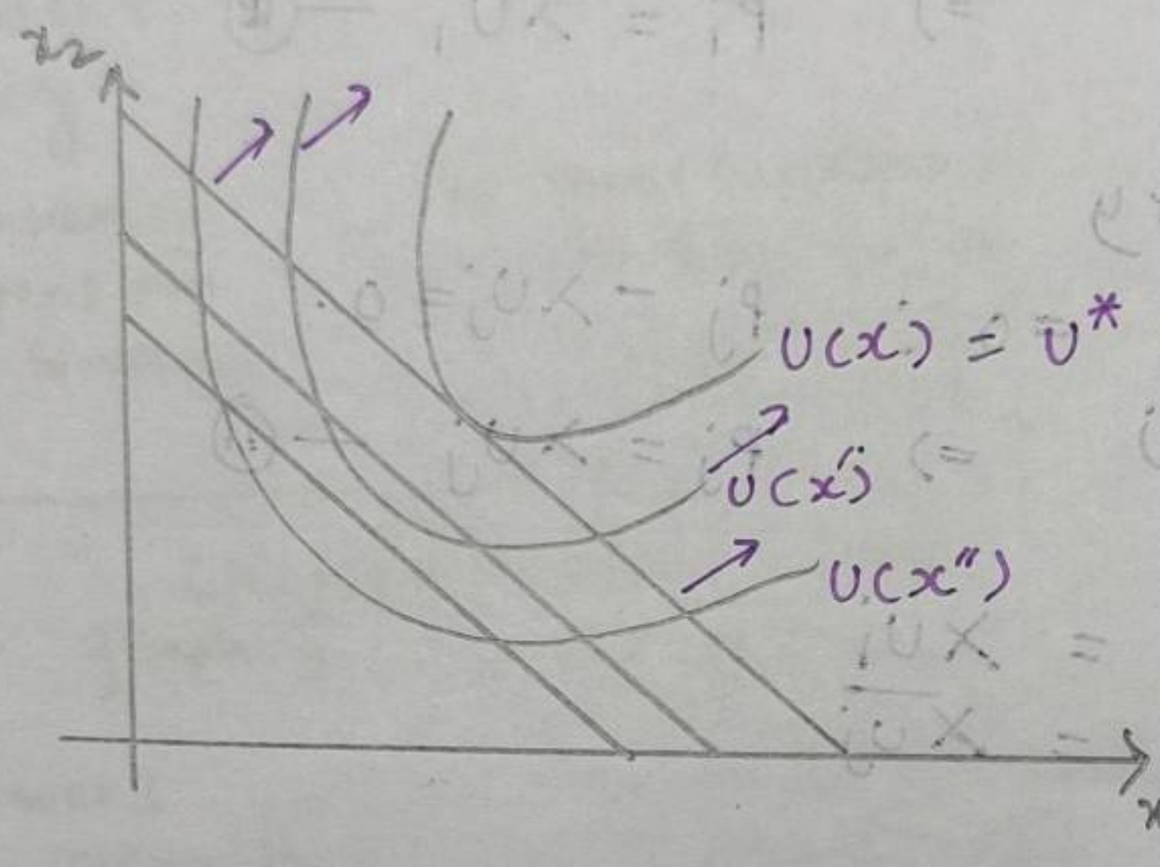
From EMP FOC's : Given $U(x^*) = \bar{U}$ For $x = i,j$

$$\left.\begin{aligned} P_i &= \lambda U_i(h(P,\bar{U})) \\ P_j &= \lambda U_j(h(P,\bar{U})) \end{aligned}\right\} \text{tangency conditions.}$$

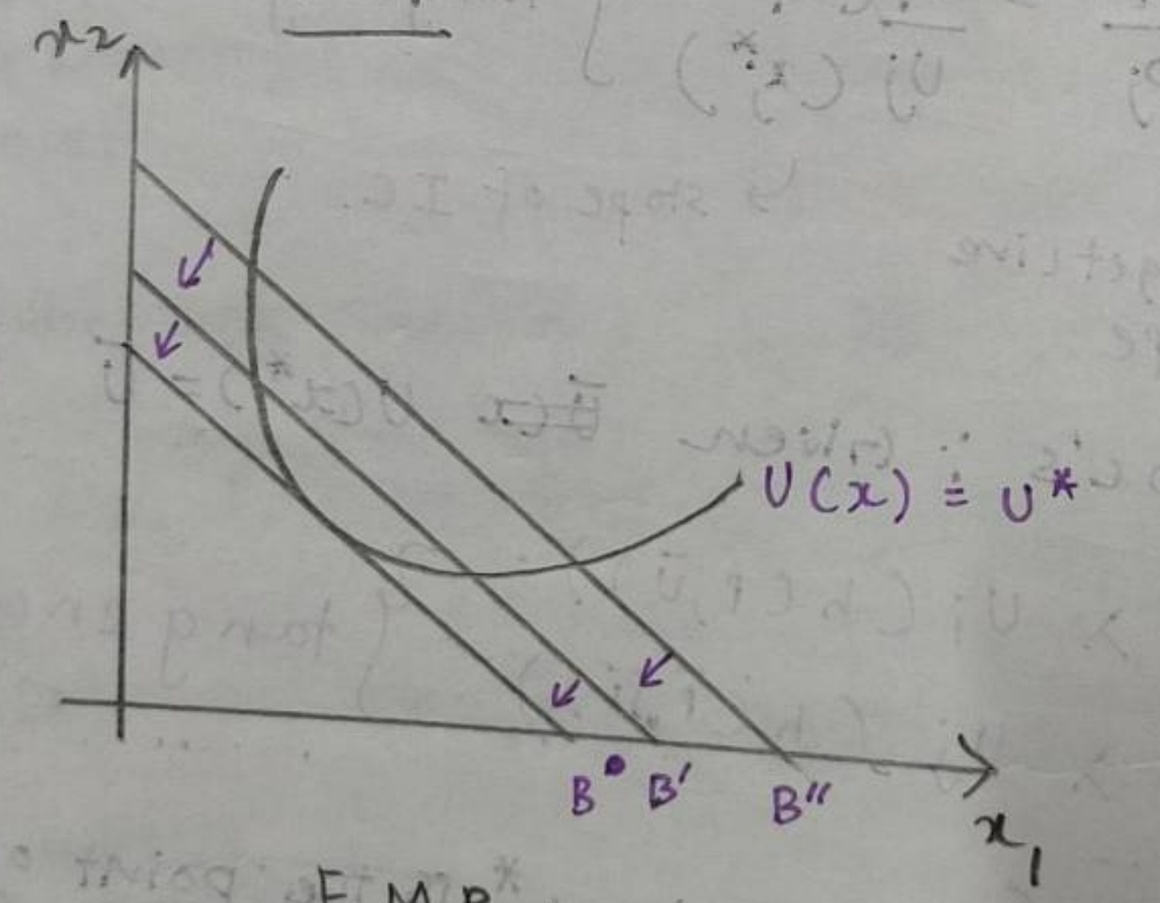
Now since through UMP ; x^* is the point of tangency where $U(x^*) = \bar{U}$ at budget $px = w$; then $h(P,\bar{U})$ which is the optimal bundle under EMP and satisfies the tangency condition with $U(x^*) = \bar{U}$ at minimum cost ie with budget Line $px \leq w$. Thus EMP's optimal sol^n $h(P,\bar{U}) = x^*(P,w)$. This is called duality.

Duality :- if x^* solves the UMP with price and w as given then ~~EMP~~ x^* must solve the EMP problem with Utility $= U(x^*) = \bar{U}$ as given.

UMP & EMP are considered as dual because In either cases the objective function in UMP is the constraint of EMP while vice versa.



UMP



EMP

Duality Equation

~~$h(P, \bar{U}) = x(P, w)$~~

$$h(P, V(P,w)) = x(P,w) \quad \text{—— (I)}$$

$$x(P, e(P,U)) = h(P,U) \quad \text{—— (II)}$$

eqn (I) says that the commodity which minimises cost to given the utility is the bundle that maximises utility given the price & wealth.

eqn (II) the commodity that maximises utility when prices p and wealth equal to minimum amount of wealth needed to acheive the said utility is same as the commodity which minimises the Expenditure given the utility.

since we know

Roy's identity

$$U(x) \equiv V(P, e(P,\bar{U}))$$

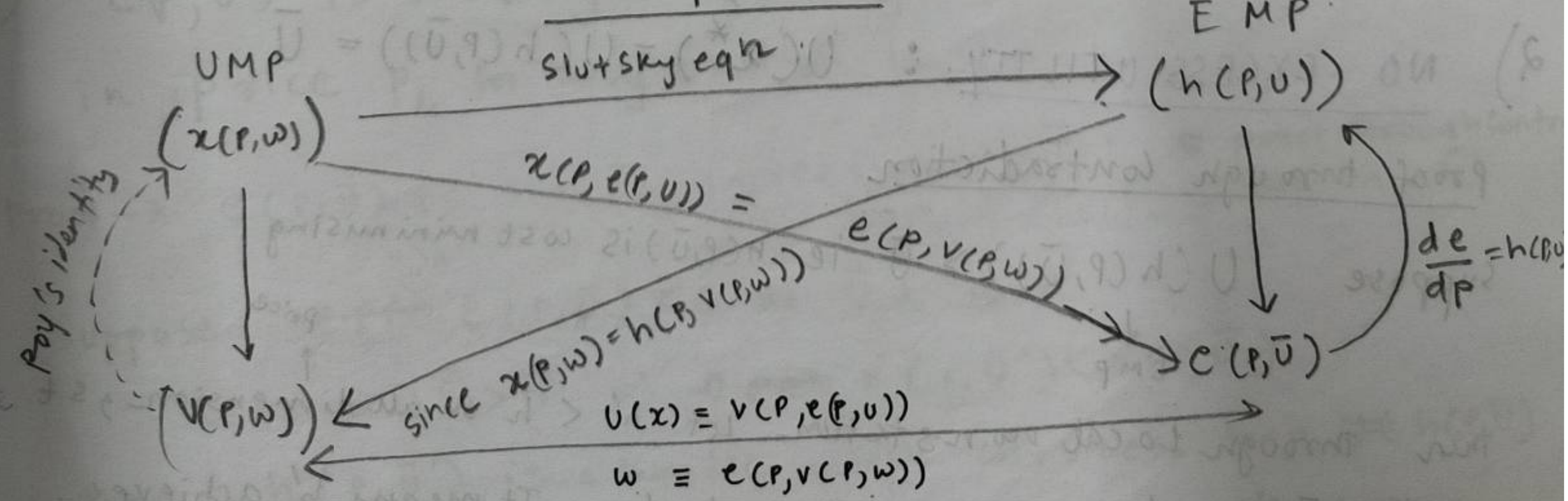
$$w \equiv e(P, V(P,w))$$

ie

~~Now~~

Expenditure function contains the exact same info as the indirect utility function. Through Roys identity we found that IUF has the ~~ext~~ exact information of marshallian dd function. Since DD are used to obtain information about utility as $U(x(P,w))$; Through this connection Expenditure function contain exactly same information as Utility function. This means that if you know the consumer's expenditure function, you also know her utility & ~~vers~~ vice versa.

Dual problem

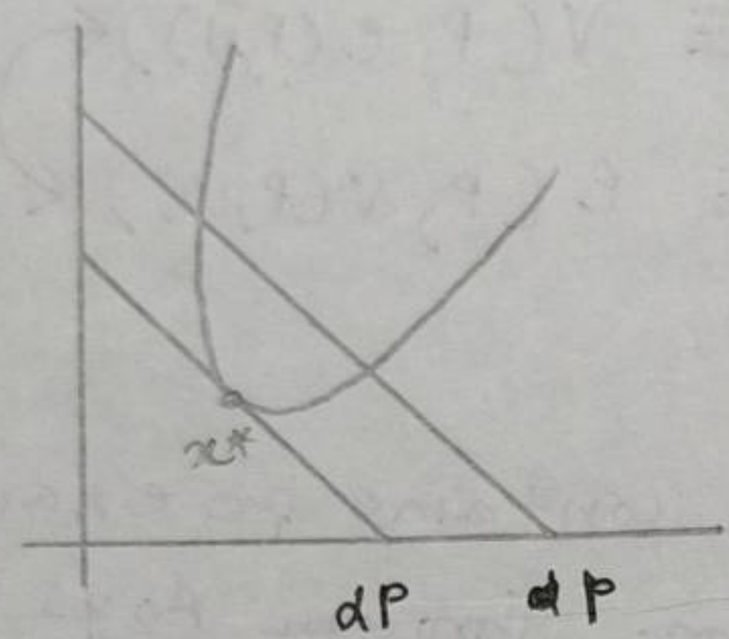


Properties of Hicksian DD function

1) Homogenity of degree zero :- (in P)

$h(P, \bar{U})$ is our hicksian dd ; which is homogenous of degree zero in P and not $\bar{U}$ & P both. why?

If we draw a graph for EMP



At tangency: $U(x^*) = \bar{U}$. And this is determined by the slopes of budget Line. As budget lines adjust minimum cost at given $\bar{U}$; only P is a function that is impacted. And any change that doesn't impact the slope of Budget Line; wont also impact cost minimising bundle. ~~What we are doing~~

$\therefore$ scaling all prices by same amount doesn't effect relative prices, the solution shouldn't change.

$\min_{x} \; dP \cdot x$ s.t. $U(x) \geq \bar{U}$

2) NO EXCESS UTILITY : $\underbrace{U(x^*)}_{\text{UMP}} = \underbrace{U(h(P,\bar{U}))}_{\text{EMP}} = \underbrace{\bar{U}}_{\text{const utility}}$

Proof through contradiction

Suppose $U(\underbrace{h(P,\bar{U})}_{\text{EMP}}) > \bar{U}$ ie $h(P,\bar{U})$ is cost minimising

then through local nonstation let $h' < h$ on all dimension (price), st $U(h') > \bar{U}$ if h' is sufficiently close to h. It means h' achieves utility $\bar{U}$ (and greater) at lower cost than $h(P,\bar{U})$, which contradicts

our assumption that $h(p,\bar{u})$ was cost minimising at first place as now h' is a better bundle.

3) $h(p,\bar{u})$ is convex set.

Properties of Expenditure Function

1) Homogenity of degree 1 in P.

since $h(\alpha p,\bar{u}) = h(p,\bar{u})$, $\alpha > 0$

then in $e(p,\bar{u}) = p \cdot h(p,\bar{u})$

then scaling up P by α

$\Rightarrow$ $e(\alpha p,\bar{u}) = \alpha p \cdot h(\alpha p,\bar{u})$

$\Rightarrow$ $\boxed{e(\alpha p,\bar{u}) = \alpha p \cdot h(p,\bar{u})}$

homogenous of degree 1. in P.

$\Rightarrow$ If price is scaled by α units; then even though there is no change in demand h; to keep you at the same level of utility, it will cost α times more.

2) $e(p,\bar{u})$ is strictly increasing in $\bar{u}$ & non decreasing in price p_l for any $l = 1 \cdots L$

proof $\rightarrow$ $e(p,\bar{u})$ is strictly increasing in $\bar{u}$: proof through contradiction.

Suppose ~~ept~~ $e(p,\bar{u}) < 0$ as $\bar{u} \uparrow$ ie

Let $\bar{u}' > \bar{u} \Rightarrow$ ~~ept~~ $e(p,\bar{u}') < e(p,\bar{u})$

then $h(p,\bar{u}')$ is a bundle which costs less than ~~the~~ $h(p,\bar{u})$ and satisfies our contraint $U(x) \geq \bar{u}$: implying $h(p,\bar{u}')$ is better than $h(p,\bar{u})$ which is contradicting our $h(p,\bar{u})$ being cost minimiser

at the First place.

Proof: $e(p,\bar{U})$ is non decreasing in p_i:

Suppose; $p' > p$ $\therefore$ then $e(p',\bar{U}) = p' h(p',\bar{U})$

$e(p,\bar{U}) = p\, h(p,\bar{U})$

then from definition of Exp function

$$e(p',\bar{U}) = p' h(p',\bar{U}) \geq p' h(p,\bar{U}) \geq e(p,\bar{U})$$

This means:-

$e(p',\bar{U}) = p' h(p',\bar{U})$ {WALRAS LAW}

$p' h(p',\bar{U}) \geq p\, h(p',\bar{U})$ { meaning at $p' > p \Rightarrow p' h(p',\bar{U}) > p\, h(p,\bar{U})$

$p\, h(p',\bar{U}) \geq e(p,\bar{U})$ { $h(p',\bar{U})$ achieves utility $\bar{U}$ but doesn't necessarily do at minimum cost }

3) $E(p,\bar{U})$ is concave ~~at~~ in p.

consider p' & p and Let $p^a = \alpha p' + (1-\alpha) p$

then $e(p^a,\bar{U}) = p^a h(p^a,\bar{U})$ { From definition of Expenditure Function

$= \alpha p' h(p^a,\bar{U}) + (1-\alpha) p\, h(p^a,\bar{U})$

$=$

$\geq \alpha p\, h(p,\bar{U}) + (1-\alpha) p' \cdot h(p',\bar{U}) \rightarrow$ { For concavity we need $F(\alpha x + (1-\alpha) y) \geq \alpha f(x) + (1-\alpha) f(y)$

this means that $h(p^a,\bar{U})$ is Feasible but not optimal ~~at~~ for $(p',\bar{U})$ & $(p,\bar{U})$

Relationship btw^n hicksian DD & $E(P,U)$

we know

$$e(P,\bar{U}) = P\,h(P,\bar{U})$$

differentiate wrt P_i {similar exercise while show relation btw^n $v(P,w)$ & $x(P,w)$}

$$\frac{\partial e(P,\bar{U})}{\partial P_i} = P\frac{\partial h}{\partial P_i} + h$$

For $i = 1 \cdots L$

$$\frac{\partial e}{\partial P} = h_i + \sum_{j=} P_i \frac{\partial h_j}{\partial P}$$

From kuhn tuck FOC of EMP.

$$\lambda U_i^* = P_i \quad \text{— substitute}$$

$$\therefore \quad \frac{\partial e}{\partial P_j} = h_i + \sum \lambda u_i^* \frac{\partial h_j}{\partial P_i}$$

$$\Rightarrow \quad \frac{\partial e}{\partial P_j} = h_i + \lambda \sum U_i^* \frac{\partial h_j}{\partial P_i}$$

Now through UMP; we got $U(x^*)$ and since x^* is dual sol^n.

For EMP; at tangency

$$U(h(P,\bar{U})) = U(x^*) = \bar{U}$$

$\therefore$ diff wrt P

$$\Rightarrow \quad \frac{\partial U}{\partial h} \cdot \frac{\partial h}{\partial P_i} = \frac{\partial \bar{U}}{\partial P_i} \to 0$$

$$\Rightarrow \quad U^* \frac{\partial h}{\partial P_i} = 0$$

$$\therefore \quad \frac{\partial e}{\partial P_j} = h_i + \lambda \times 0$$

$$= h_i$$

$$\therefore \quad \boxed{\frac{\partial e}{\partial p_j} = h_i^*}$$

So To determine h_i^* directly from $e(p,u)$

similarities between Roy's identity & this differentiation

Roy's identity

$$x(p,w) = \frac{\frac{\partial v}{\partial p}}{\frac{\partial v}{\partial w}}$$

$$\frac{\partial e}{\partial p_j} = h_i^*(p,\bar{u})$$

Additional properties of $\frac{\partial e}{\partial p_j} = h_i$

① $$\frac{\partial h_i}{\partial p_i} = \frac{\partial}{\partial p_i}\left(\frac{\partial e}{\partial p_j}\right) = \frac{\partial^2 e}{\partial p_i \partial p_j}$$

$$\underbrace{D_p h(p,\bar{u}) \qquad\qquad D_p^2 e(p,\bar{u})}_{\text{Under matrix}}$$

hicksian

This property says; change in demand wrt price p_i is equal to double diff of expenditure function wrt p_i

② continuing from property ①; $D_p h(p,\bar{u})$ is NSD ie diagonal elements are non positive.

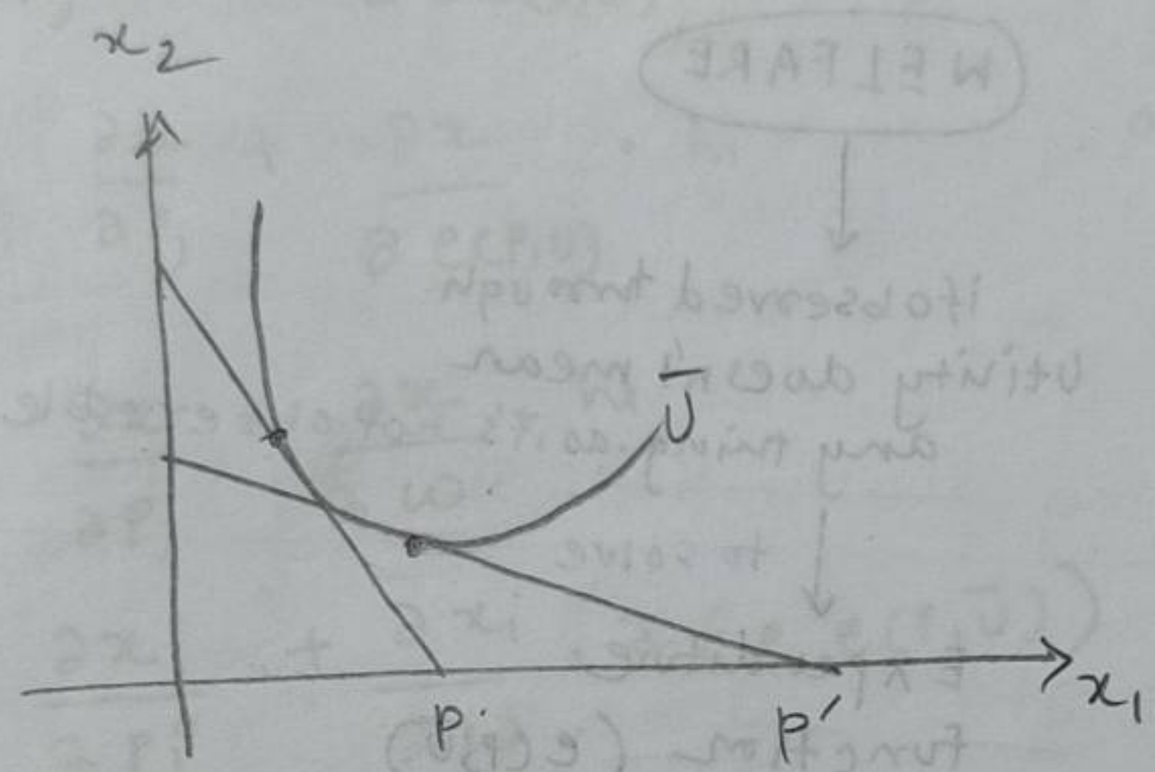
Since $D_p h(p,\bar{u}) = D_p^2 e(p,\bar{u})$ and $D_p h(p,\bar{u})$ is N

then $D_p^2 e(p,\bar{u})$ is also NSD, ~~and~~ $\cancel{\frac{\partial^2 e}{\partial p_i^2}}$ hence $e(p,u)$ is concave due it's second order derivitve being -ve.

$\therefore \frac{\partial h_i}{\partial p_i} < 0$ & $\frac{\partial e(p,u)}{\partial p_i \partial p_j} < 0$.

(5) $D_p h(p,\bar{u})$ is symmetric $\Rightarrow \frac{\partial^2 e}{\partial p_i \partial p_j} = \frac{\partial h_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j}$

matlab; cross price Effects on hi is same. change in p_j is same on hi as change in p_i



The hicksian DD is called the compensated DD (check chp 2)

suppose; $p \to p'$: change in demand such that utility remains constant; that you are compensated with enough wealth to keep your utility constant. (depicted in figure)

~~Slutsky compensation~~ CLD: $-(p'-p)(x'-x) \leq 0$.

ie $\Delta p \, \Delta x \leq 0$

ie if there is an increase in price; demand responds by decreasing itself. Now under CLD; as $p \to p'$, & through compensate price change; you were able to afford x aswell. Compensated price change implied wealth compensation called Slutsky wealth compensation.

Here; there is a slight change. As price change {suppose $p\uparrow$ then $x\downarrow$} which is represented by $\frac{\partial h_i}{\partial p_i} \leq 0$ {NSD}. Here however it's talking $\to$ CLD not about what wealth you need to stay at the same bundle, but

but what wealth you need to stay on

and ~~&~~ after price change (called Hicksian substitution) or compensation

These 2 types ~~co~~ of compensation will be combined to make the SLUTSKY EQ^N

SLUTSKY EQ^N

WELFARE

↓

if observed through utility doesn't mean any thing. as it's not observable

↓ to solve

Expenditure function ($e(p,\bar{U})$)

which talks ~~about~~ about consumer's pattern of consumption & impact on wealth due to p & $\bar{U}$

↓

$\bar{U}$ is target utility

This is also not observable

↓

So while we can generate info about ~~about~~ people's ~~pair~~ welfare pattern through prices, utility info is crucial.

→ we hence try to see a solnt

~~x~~ $h(p,U)$ is hicksian dd funct

$x(p,\omega)$ is marshallian dd.

(p,ω) are observable while $\bar{U}$ is not.

deriving $h(p,U)$ through $x(p,\omega)$ will solve the problems of observability → putting $h(p,U)$ in $E(p,U)$ ie $E(p,U) = p\,h(p,U)$ will give a measure of wealth

Now writing

$$h(P,U) \equiv x(P,\omega)$$

or $$h(P,U) \equiv x(P, P\cdot h(P,U))$$

$$\boxed{\text{or } h(P,U) \equiv x(P, e(P,\bar{U}))}$$

differentiat bothe sides wrt to P_j

$$\frac{\partial h_i(P,\bar{U})}{\partial P_j} \equiv \frac{\partial x}{\partial e(P,\bar{U})} \cdot \frac{\partial e(P,\bar{U})}{\partial P_j} + \frac{\partial x}{\partial P_j}$$

$$\Rightarrow \frac{\partial h_i}{\partial P_j} \equiv \frac{\partial x}{\partial P_j} + \frac{\partial x}{\partial e(P,U)} \cdot h_i$$

as from relationship of $e(P,U)$ & h_i

$$\frac{\partial e}{\partial P} = h_i$$

$$\Rightarrow \frac{\partial h_i}{\partial P_j} \equiv \frac{\partial x}{\partial P_j} + \frac{\partial x}{\partial \omega} \cdot h_i$$

$$\Rightarrow \frac{\partial h_i}{\partial P_j} = \frac{\partial x_i}{\partial P_j} + \frac{\partial x_i}{\partial \omega} \cdot x(P, e(P,\bar{U}))$$

hence

$$\boxed{\frac{\partial h_i(P, V(P,\omega))}{\partial P_j} \equiv \frac{\partial x_i}{\partial P_j}(P,\omega) + \frac{\partial x_i}{\partial \omega}(P,\omega) \cdot x_j(P,\omega)}$$

→ SLUSKY eqn (provides us the Link btwn watrasian dd & hicksian dd)

components :-

(i) $\frac{\partial x_i}{\partial P_j}$ = how dd will change when ω is const, P changes. — substition Effect

(ii) $\frac{\partial x_i}{\partial \omega} x_i$ = how dd will change when P is const, ω changes — Income Effect

(iii) $\frac{\partial h_i}{\partial P_j}$ = ~~how dd~~ The over all compensated change in dd after wealth compensation & price change is called hicksian substitution or pure substitution effect. { ME + SE = 'SE' }

∴ $\frac{\partial h_i}{\partial P_j}$ is the sum of uncompensated change in dd due to price {substitution} and wealth effect on uncompensated dd.

Point of SLUTSKY EQ^n is suppose :-

Gas price ↑ ; consumer demands ↓ {Substitution Effect}

but since P ↑ , consumer's have to spend more if they want to attain same Level of utility ; this amount by which they will have to be compensated = Δw or ∂w. If more w is given to her, she will consume more gas (Income Effect)

The capturing of this two Effects ; when utility is to be maintained is captured by $\frac{\partial h_i}{\partial p_j}$ because price change was the 1ST effect, then w changed and so on.

Rearranging ~~substitution~~ SLUTSKY Eq^n

$$\frac{\partial x_i}{\partial p_j} \equiv \underbrace{\frac{\partial h_i}{\partial p_j}}_{\frac{\partial h_i}{\partial p_j} \leq 0 \ \{NSD\} \{LLD\}} - \frac{\partial x_i}{\partial w} \cdot x_j$$

here we are explaining uncompensated substitution Effect due to compensated ~~sub~~ SE & IE. This is what will give you the right intuition :-

if Banana P↑, wealth will adjust so i will consume banana's ~~to~~ where i get same utility. And as ~~w↑~~ P↑ h_i↓ $\left\{ \frac{\partial h_i}{\partial p_j} \leq 0 \text{ ie LLD} \right\}$