Summer project

at the Indian Institute of Technology Kanpur.

Computational methods to solve heat & mass transfer problems

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1. INTRODUCTION

Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations . With the use of proper algorithms we can break mathematical models numerically and find their solutions. It can be well used of the purpose of solving problem which require a lot of computational time or the mathematical models which are unsolvable using the analytical approach .

The phenomena of coupled heat and mass transfer in capillary porous media discussed in the reference paper : [1] is one of them, where the analytical solution tends to use much time, whereas we can solve this particular problem numerically with the discretisation of the governing equations. For the mathematical modeling of such phenomena, Luikov has proposed a model based on a system of coupled diffusion equations [4.B2], which takes into account the effects of the temperature gradient on the moisture migration.

2. BACKGROUND

This paper includes the numerical analysis of the 1 dimensional heat equation , which is also known as the parabolic equation and the 1 dimensional coupled heat & moisture equation [4.B2] using the gauss seidel method to find the converging solution at each time step.

3. METHODOLOGY

The methodology used form solving the equations numerically is the discretisation of the differential equation using the cantered difference approximation with a second order error. Further the discretisation is based upon the implicit formulation , which involves space solutions defined over a future time step. The implicit formulation helps in the stability of the discretised equation and tends to converge easily to the final solution.

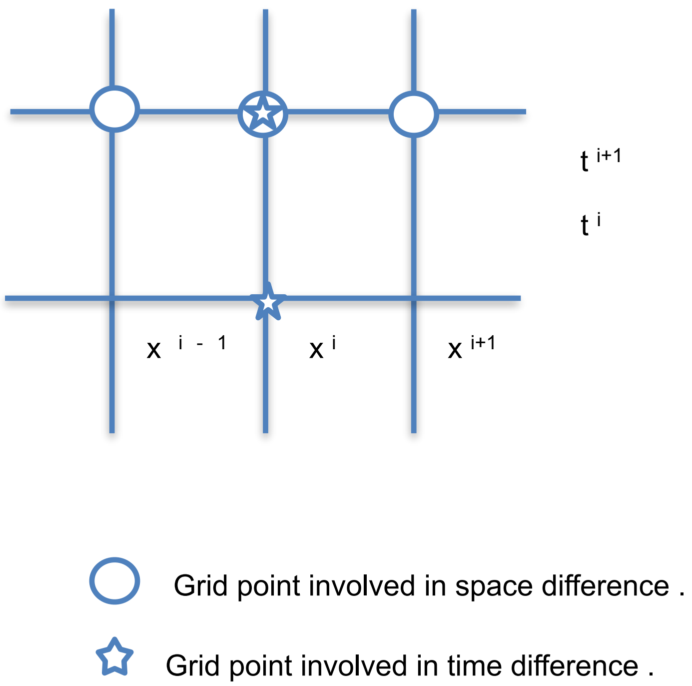


Fig 1 : A computational molecule for the simple implicit method

3.1 DATA COLLECTION

The data used in the computation are standard data obtained from online website [5] & research paper [1].

3.2 DATA ANALYSIS

The data was analysed using a mathematical software MATLAB. [6]

4. PROJECT PROBLEMS

This paper addresses two problems

(a) Heat conduction in a 1 dimensional rod.

(b) Coupled heat & mass transfer in a 1 dimensional capillary tube.

4.A HEAT CONDUCTION IN A 1 DIMENSIONAL ROD.

A.1 INTRODUCTION

Heat energy can move through a substance by **conduction**. **Metals** are good conductors of **heat** but non-**metals** and gases are usually poor conductors of **heat**. Poor conductors of **heat** are called insulators. **Heat** energy is conducted from the hot end of an object to the cold end. [3]

The electrons in piece of metal can leave their atoms and move about in the metal as free electrons. The parts of the metal atoms left behind are now charged metal ions. The ions are packed closely together and they vibrate continually. The hotter the metal, the more kinetic energy these vibrations have. This kinetic energy is transferred from hot parts of the metal to cooler parts by the free electrons.These move through the metal, colliding with ions as they go. [3]

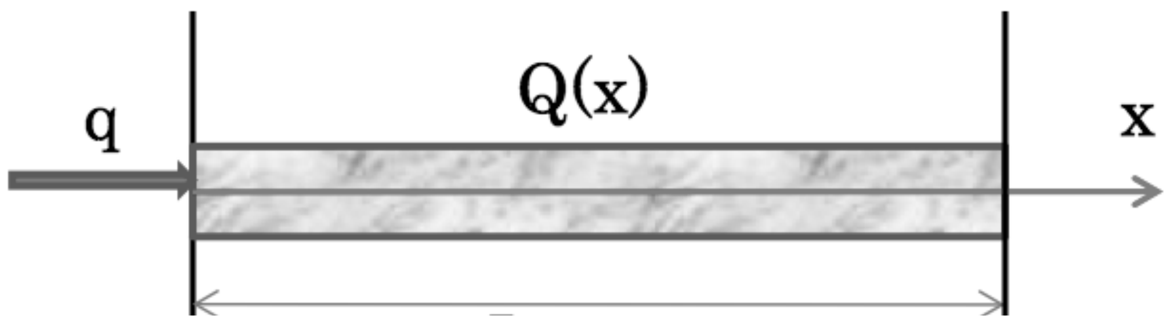


Fig (2) : Heat conduction in a 1 dimension rod [4.2]

A.2 GOVERNING EQUATIONS

$$K \times \frac{\partial^2}{\partial x^2}(T) = \frac{\partial}{\partial t}(T)$$

A.3 DISCRITISED EQUATIONS

$$\frac{T_{l-1}^{i+1} - 2T_l^{i+1} + T_{l+1}^{i+1}}{(\Delta x)^2} = \frac{\partial^2}{\partial x^2}(T)\ . \qquad \text{(a)}$$

$$K\frac{T_{l-1}^{i+1} - 2T_l^{i+1} + T_{l+1}^{i+1}}{(\Delta x)^2} = \frac{T_l^{i+1} - T_l^{i}}{(\Delta t)} \qquad \text{(b)}$$

A.4 NUMERICAL STEPS

The above discretisation was used to solve the differential equations [A.3 (a) & (b)]. Afterwards the temperature was solved for each node and each time step in a 2 dimensional matrix. Each tilmestep was converged to it's defined solution using the Gauss Seidel method.

A.5 EXAMPLE PROBLEM

(a) Copper rod

(b) Wooden rod

A.5.a Copper rod (Cu)

PHYSICAL PROPERTIES:

Properties	Definition	Value
Thermal Conductivity	The degree to which a specified material conducts electricity, calculated as the ratio of the current density in the material to the electric field which causes the flow of current.	385.0 W/mK
Density	The **density**, or more precisely, the volumetric mass **density**, of a substance is its mass per unit volume	8.96 x 10e-3
Specific heat capacity	**Specific heat** is the amount of **heat** needed to raise the temperature of one kilogram of mass by 1 kelvin.	0.385
Thermal diffusivity	the thermal conductivity of a substance divided by the product of its density and its specific heat capacity.	1.11 x 10e-4

EXPERIMENT INPUT :

Experiment Input	Definition	Value
Computation time	Total time to which the user wants to run the code	60 sec
Time Step	The minimum time step , which will give stable result	0.5 sec
Max iterstions	To limit the time steps & to save computation time.	120
Length of the rod	The length of the sample rod	0.05m
Number grid point	Total number of grid points over which iteration needs to be done	6
Space step	Length of the rod divided by the number of grid points	0.01m
Right side temp	Temperature	25
Left side temp	Temperature	100

A.6A Results

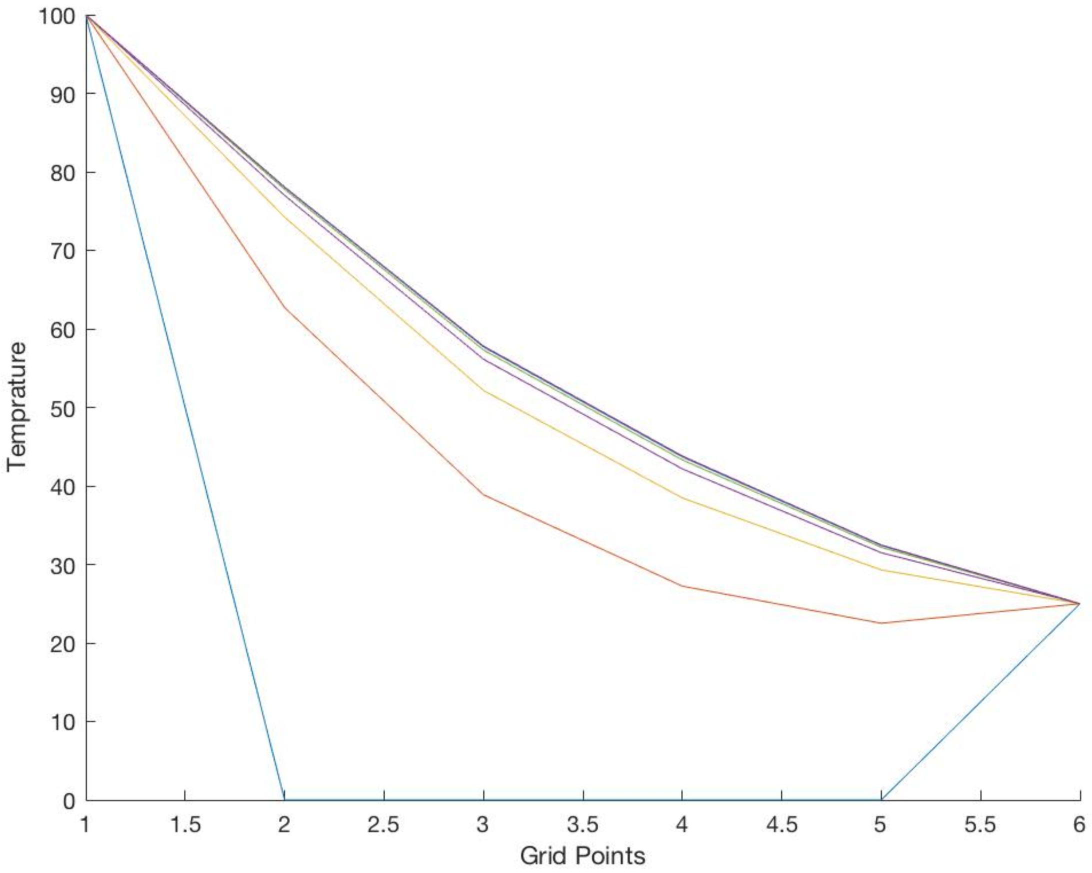


FIG 3: PLOT OF TEMPERATURE VS GRID POINTS

A.5.a Wooden Rod

PHYSICAL PROPERTIES:

Properties	**Definition**	**Value**
Thermal Conductivity	The degree to which a specified material conducts electricity, calculated as the ratio of the current density in the material to the electric field which causes the flow of current.	0.12 W/mK
Density	The **density**, or more precisely, the volumetric mass **density**, of a substance is its mass per unit volume	0.6x 10^{-3}kg/m3
Specific heat capacity	**Specific heat** is the amount of **heat** needed to raise the temperature of one kilogram of mass by 1 kelvin.	1.76 J/g^0 C
Thermal diffusivity	the thermal conductivity of a substance divided by the product of its density and its specific heat capacity.	8.2 x 10e-8

EXPERIMENT INPUT :

Experiment Input	Definition	Value
Computation time	Total time to which the user wants to run the code	60 sec
Time Step	The minimum time step , which will give stable result	0.5 sec
Max iterstions	To limit the time steps & to save computation time.	120
Length of the rod	The length of the sample rod	0.06m
Number grid point	Total number of grid points over which iteration needs to be done	5
Space step	Length of the rod divided by the number of grid points	0.01m
Right side temp	Temperature	0
Left side temp	Temperature	100

A6.B Results :

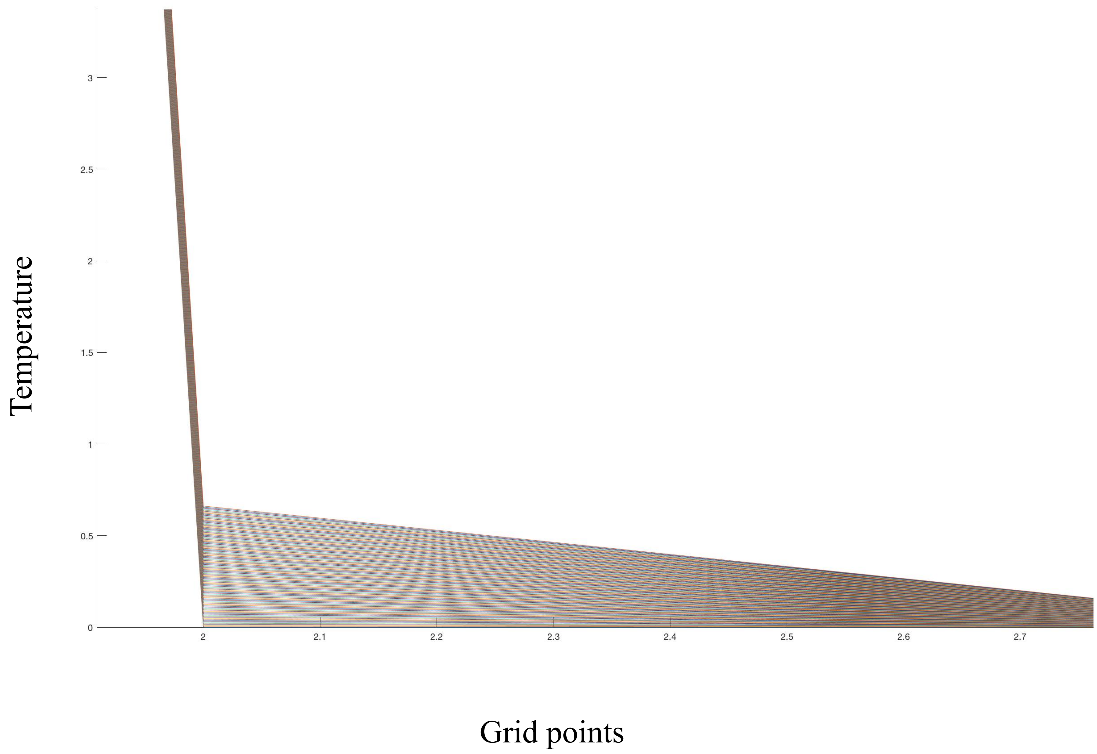


Fig 4: The plot is zoomed to scale

4.B COUPLED HEAT & MASS TRANSFER

4.B1 INTRODUCTION :

The physical problem involves a one-dimensional capillary porous medium, initially at uniform temperature and uniform moisture content. One of the boundaries, which is impervious to moisture transfer, is in direct contact with a heater. The other boundary is in contact with the dry surrounding air, thus resulting in a convective boundary condition for both the temperature and the moisture content, as illustrated in Fig. 4. The linear system of equations proposed by Luikov [4.B2], with associated initial and boundary conditions, for the modelling of such physical problem involving heat and mass transfer in a capillary porous media, can be written in dimensionless form [1]

The properties of the porous medium appearing above include the thermal diffusivity (a), the moisture diffusivity (a_m), the thermal conductivity (k), the moisture conductivity (k_m) and the specific heat (c). Other physical quantities appearing in the dimensionless groups of equations. (2) are the heat transfer coefficient (h), the mass transfer coefficient (h_m), the thickness of porous medium (l), the prescribed heat flux (q), the latent heat of evaporation of water (r), the temperature of the surrounding air (T_s), the uniform initial temperature in the medium (T_0), the moisture content of the surrounding air (u), the uniform initial moisture content in the medium (u_0), the thermo-gradient coefficient (d) and the phase conversion factor (e). Lu, Pn and Ko denote the Luikov, Posnov and Kossovitch numbers, respectively. [1]

This referred to as a direct problem when initial and boundary conditions, as well as all parameters appearing in the formulation, are known. The objective of the direct problem is to determine the dimensionless temperature and moisture content fields respectively, in the capillary porous media. [1]

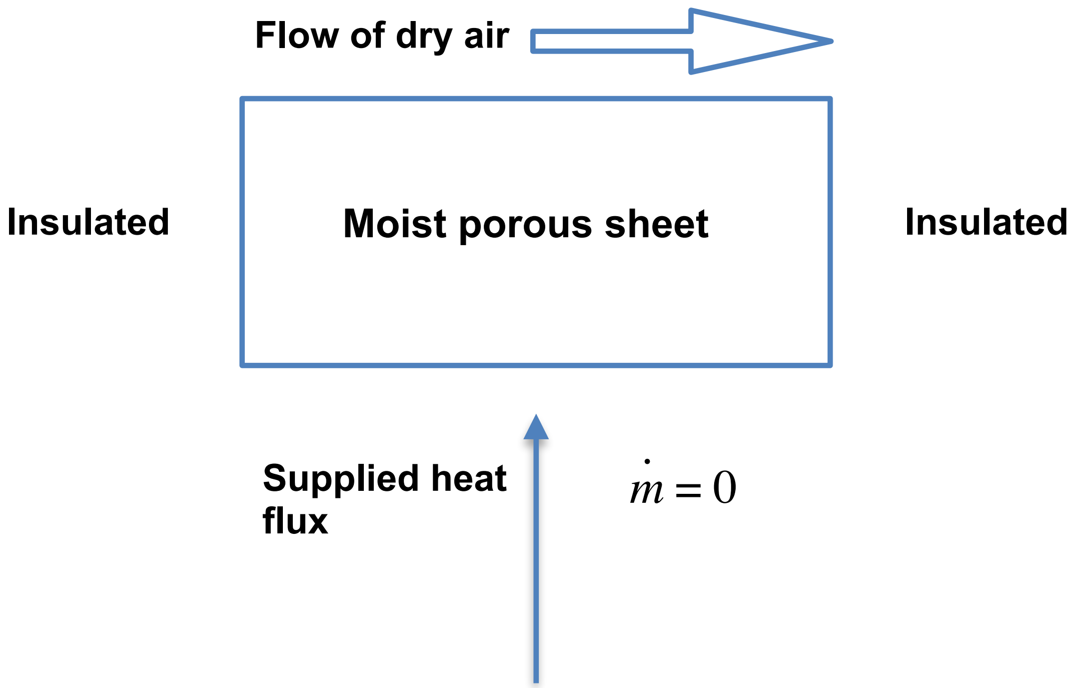


Fig 5 : the model of the physical problem

4.B2 GOVERNING EQUATIONS

$$\frac{\partial\phi(X,\tau)}{\partial\tau} = Lu\frac{\partial^2\phi(X,\tau)}{\partial X^2} - LuPn\frac{\partial^2\theta(X,\tau)}{\partial X^2} \quad in\ \ 0 < X < 1,\ for\ \tau > 0.$$

$$\frac{\partial\theta(X,\tau)}{\partial\tau} = \frac{\partial^2\theta(X,\tau)}{\partial X^2} - \varepsilon K_0\frac{\partial\phi(X,\tau)}{\partial\tau} \quad in\ \ 0 < X < 1,\ for\ \tau > 0.$$

4.B3 THE INITIAL CONDITION

$$\theta(X,0) = 0\ ,\ \phi(X,0) = 0 \quad for\ \tau = 0\ ,\ in\ \ 0 < X < 1\ .$$

4.B3 THE BOUNDARY CONDITIONS ARE

(a) $$\frac{\partial\theta(0,\tau)}{\partial X} = -Q \ \ ;\ \frac{\partial\phi(0,\tau)}{\partial X} - Pn\frac{\partial\theta(0,\tau)}{\partial X} = 0\ .\ at\ X = 0,\ for\ \tau > 0.$$

(b) $$\frac{\partial\theta(1,\tau)}{\partial X} + B_{iq}\theta(1,\tau) = B_{iq} - (1 - \varepsilon)KoLuB_{\mathrm{im}}[1 - \phi(1,\tau)]\ ,\ \ at\ X = 1\ ,\ for\ \tau > 0\ .$$

(c) $$\frac{\partial\phi(1,\tau)}{\partial X} + \dot{B}_{\mathrm{im}}\phi(1,\tau) = \dot{B}_{\mathrm{im}} - \ PnB_{iq}[\theta(1,\tau) - 1]\ ,\ \ at\ X = 1\ ,\ for\ \tau > 0\ .$$

4.B4 DISCRETISED EQUATIONS

The discretised equations are formulated by using the centre difference method , using the second order error approximations .

All the six partial differential equations are discretised and are incorporated into each other such that the boundary conditions are implemented into each other. The boundary conditions are implemented as robin conditions .[4]

(a) $$\phi_1 = \phi_0 + \Delta x \frac{\partial \phi}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 \phi}{(\partial x)^2} + ...$$ At X = 0, for time > 0

(b) $$\phi_2 = \phi_0 + 2\Delta x \frac{\partial \phi}{\partial x} + \frac{(2\Delta x)^2}{2!} \frac{\partial^2 \phi}{(\partial x)^2} + ...$$ At X = 0, for time > 0

there after solving the first 2 initial conditions . we get the following equations which are for X = 0 only.

$$\frac{\partial \phi}{\partial x} = \frac{4\phi_1 - \phi_2 - 3\phi_0}{2\Delta x} .$$

Substituting this into the differential equation we get , this for moisture & following the same steps we get it for Temperature as well

$$\frac{4\phi_1 - \phi_2 - 3\phi_0}{2\Delta x} + Pn\, Q = 0 \qquad \text{(c)}$$

$$Q = \frac{4\theta_1 - \theta_2 - 3\theta_0}{2\Delta x} . \qquad \text{(d)}$$

Now for the boundary conditions at X=1 .

$$\phi_{N\text{-}2} = \phi_N - 2\Delta x \frac{\partial\phi}{\partial x} + \frac{(2\Delta x)^2}{2!}\frac{\partial^2\phi}{(\partial x)^2} + \ldots\ldots\ldots$$ `At X = 1, for time > 0`

$$\phi_{N\text{-}1} = \phi_N - \Delta x \frac{\partial\phi}{\partial x} + \frac{(\Delta x)^2}{2!}\frac{\partial^2\phi}{(\partial x)^2} + \ldots\ldots\ldots$$ `At X = 1, for time > 0`

there after solving the first 2 initial conditions . we get the following equations which are for X = 0 only.

$$\frac{\phi_{N\text{-}2} - 4\phi_{N\text{-}1} + 3\phi_N}{2\Delta x} = \frac{\partial\phi(1,\tau)}{\partial x}$$ `At X = 1, for time > 0`

Substituting this into the differential equation we get , this for moisture & following the same steps we get it for Temperature as well

$$\frac{\phi_{N\text{-}2} - 4\phi_{N\text{-}1} + 3\phi_N}{2\Delta x} + \dot{B}_{im}\phi_N = \dot{B}_{im} - PnBiq\,[\theta_N - 1] \qquad \text{(e)}$$

$$\frac{\theta_{N\text{-}2} - 4\theta_{N\text{-}1} + 3\theta_N}{2\Delta x} + B_{iq}\theta_N = B_{iq} - (1 - \varepsilon)KoLuB_{im}\,[1 - \phi_N] \;. \qquad \text{(f)}$$

$$\phi(1,\tau) = \phi_N \qquad \text{(g)}$$

Substituting these results into the equations we get,The coupled equations were discretised using implicit methods

$$\frac{\theta_i^{l+1} - \theta_i^{l}}{\Delta t} = \frac{\theta_{i-1}^{l+1} - \theta_{i}^{l+1} + \theta_{i+1}^{l+1}}{\Delta x^2} - \varepsilon Ko\left[\frac{\phi_i^{l+1} - \phi_i^{l}}{\Delta t}\right] \quad \text{(e)}$$

$$\frac{\phi_i^{l+1} - \phi_i^{l}}{\Delta t} = Lu\frac{\phi_{i-1}^{l+1} - \phi_{i}^{l+1} + \phi_{i+1}^{l+1}}{\Delta x^2} - LuPn\left[\frac{\theta_{i-1}^{l+1} - \theta_{i}^{l+1} + \theta_{i+1}^{l+1}}{\Delta x^2}\right] \quad \text{(f)}$$

NUMERICAL STEPS

The above discretisation was used to solve the differential equations. Afterwards the temperature was solved for each node and each time step in a 3 dimensional matrix. Each tilmestep was converged to it’s defined solution using the Gauss Seidel method.

The discretised equations 4.B4 (a) & (b) were finally solved numerically to obtain the solutions for the following example problem.

4.B5 EXAMPLE PROBLEM

The given problem is taken from the research paper [1] . The problem is of a 1 Dimensional capillary tube subjected to heat flux and convective air flow.

PHYSICAL PROPERTIES :

Properties	Symbols	Value
Thermal diffusivity	a	1.2×10^{-8}
Moisture diffusivity	am	1.8×10^{-6}
Thermal conductivity	kq	0.65(W/m K)
Moisture conductivity	km	2.2×10^{-8} (kg/ m s M)
Specific heat	c	10^{-2} (kg / kg^{o} M)
Heat transfer coeff.	h	22.5(W/ m^2 K)
Mass transfer coeff.	hm	2.5×10^{-6} (kg/ m^2 s^0 M)
Latent heat of evapo.	r	
Thickness of material	t	1 m
Heat flux	q	100 (W/ m^2)
Temperature of surrounding Air	Ts	26(C^0)
Uniform initial temperature of the medium	To	24(C^0)
Moisture content in air	Us	86(M^0)
Uniform initial moisture content of medium	Uo	26(M^0)
Thermogradient coeff.	dM/dx	8(M^0)
Conversion factor	e	0.2
Luikov number	Lu	0.008
Posnov number	Pn	0.05
Kossovitch number	Ko	390

EXPERIMENT INPUTS:

Experiment Input	Definition	Value
Computation time	Total time to which the user wants to run the code	60 sec
Time Step	The minimum time step , which will give stable result	0.5 sec
Max iterstions	To limit the time steps & to save computation time.	120
Length of the rod	The length of the sample rod	0.06m
Number grid point	Total number of grid points over which iteration needs to be done	5
Space step	Length of the rod divided by the number of grid points	0.01m
Right side temp	Temperature	0
Left side temp	Temperature	100

4.B6 RESULTS

These are the plots depicting the results obtained from the numerical approach of the solution . The plots were [4]

B6.(A) Dimensionless temperature vs dimensionless time (Analytical)

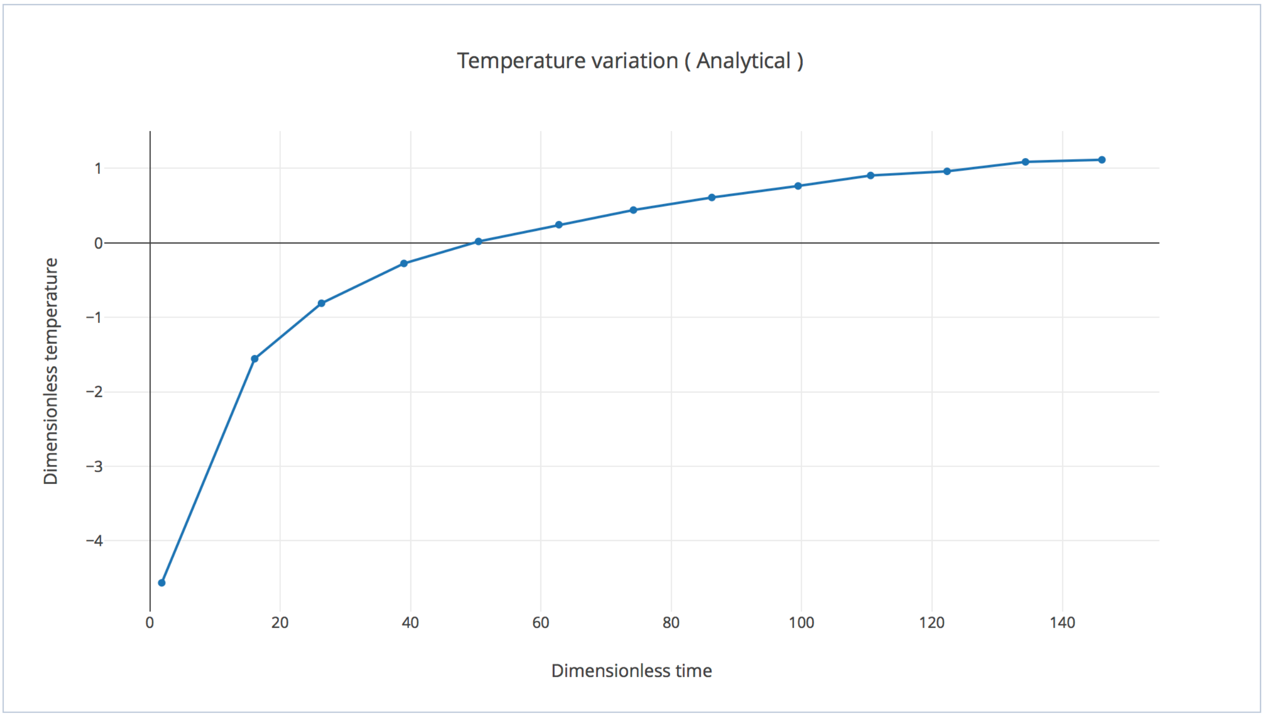
B6.(B) Dimensionless temperature vs dimensionless time (Code)

B6.(C) Comparison between analytical solution & numerical solution

B6.(D) Dimensionless moisture vs dimensionless time (Analytical)

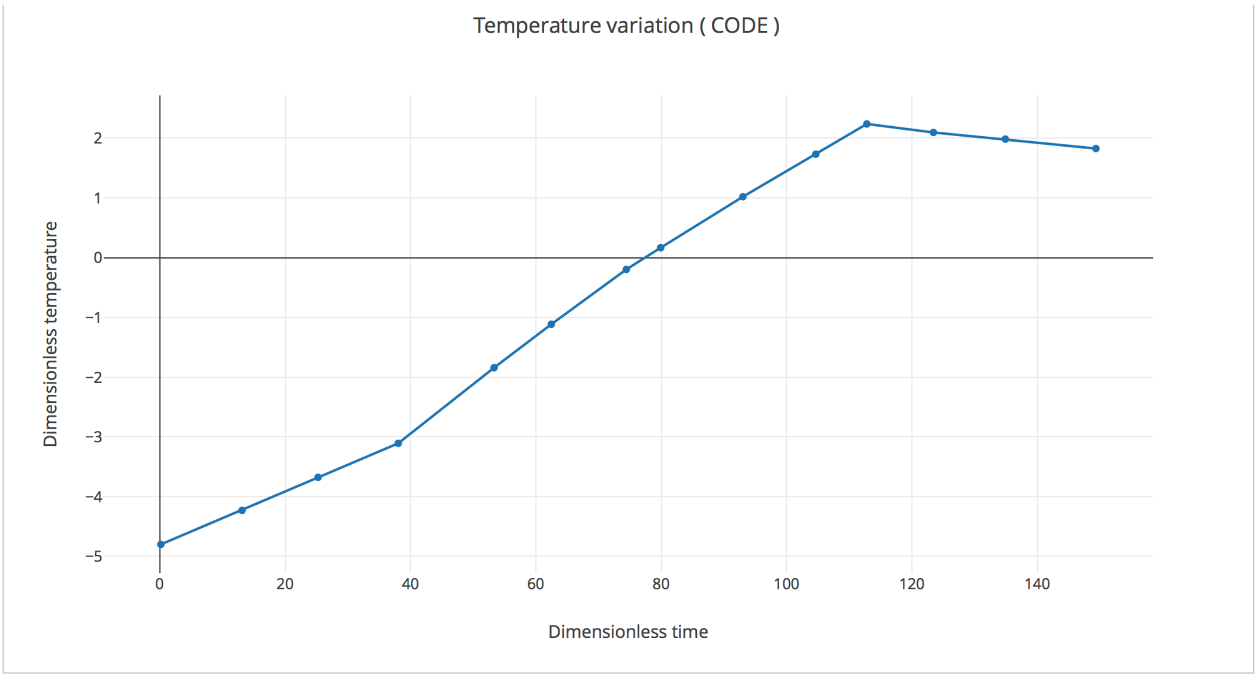
B6.(E) Dimensionless moisture vs dimensionless time (Code)

B6.(F) Comparison between analytical solution & numerical solution



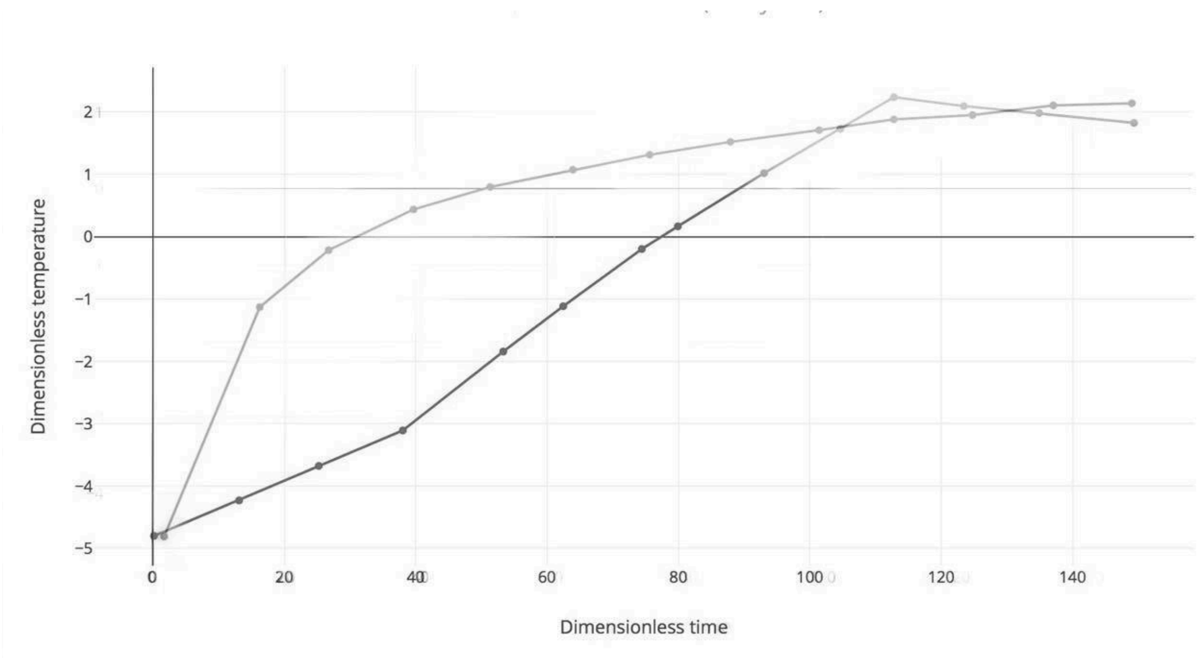
Temperature variation (Analytical)
Dimensionless temperature
1
0
−1
−2
−3
−4
0
20
40
60
80
100
120
140
Dimensionless time

B6.(A)

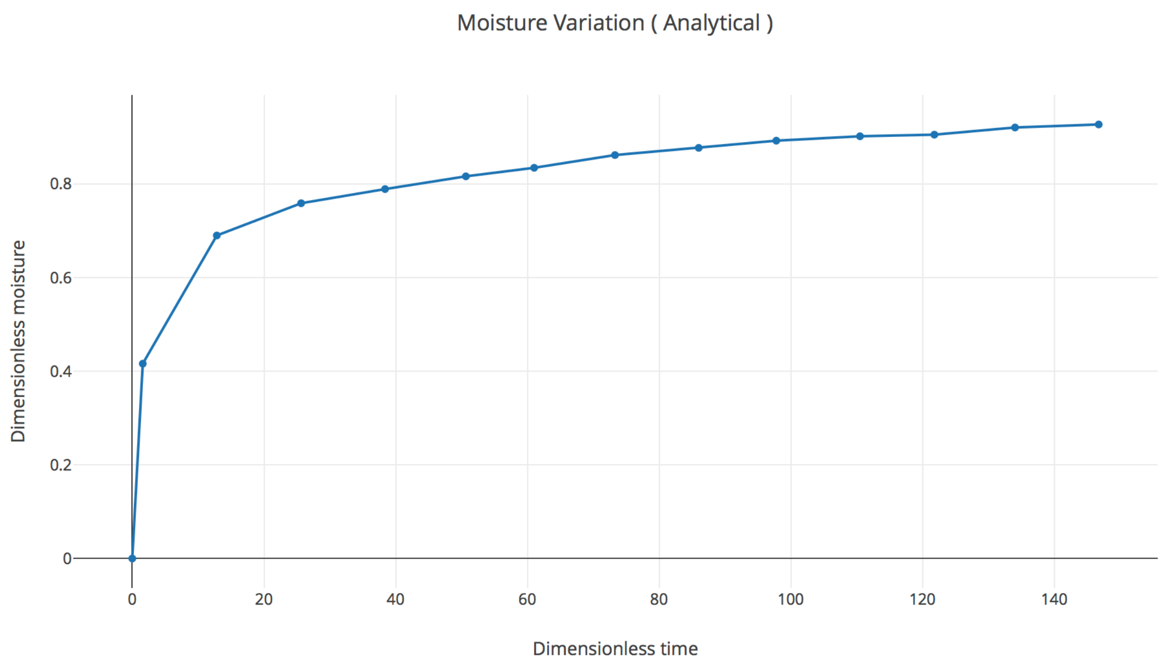


Temperature variation (CODE)
Dimensionless temperature
2
1
0
−1
−2
−3
−4
−5
0
20
40
60
80
100
120
140
Dimensionless time

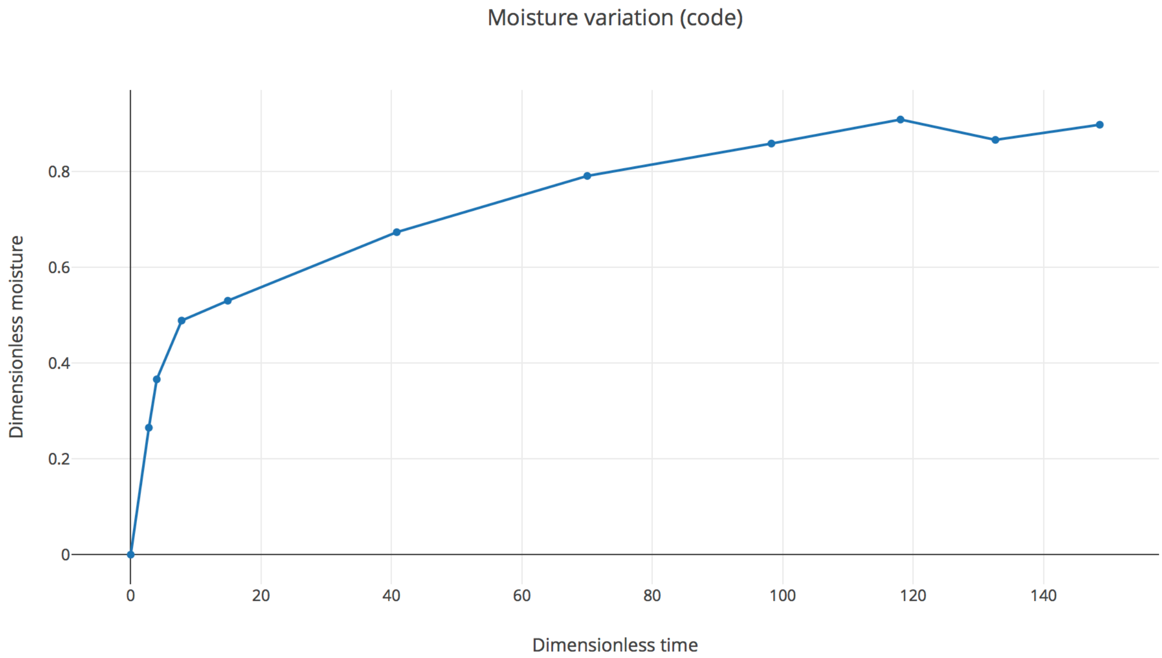
B6.(B)


Dimensionless temperature
2
1
0
−1
−2
−3
−4
−5
0
20
40
60
80
100
120
140
Dimensionless time

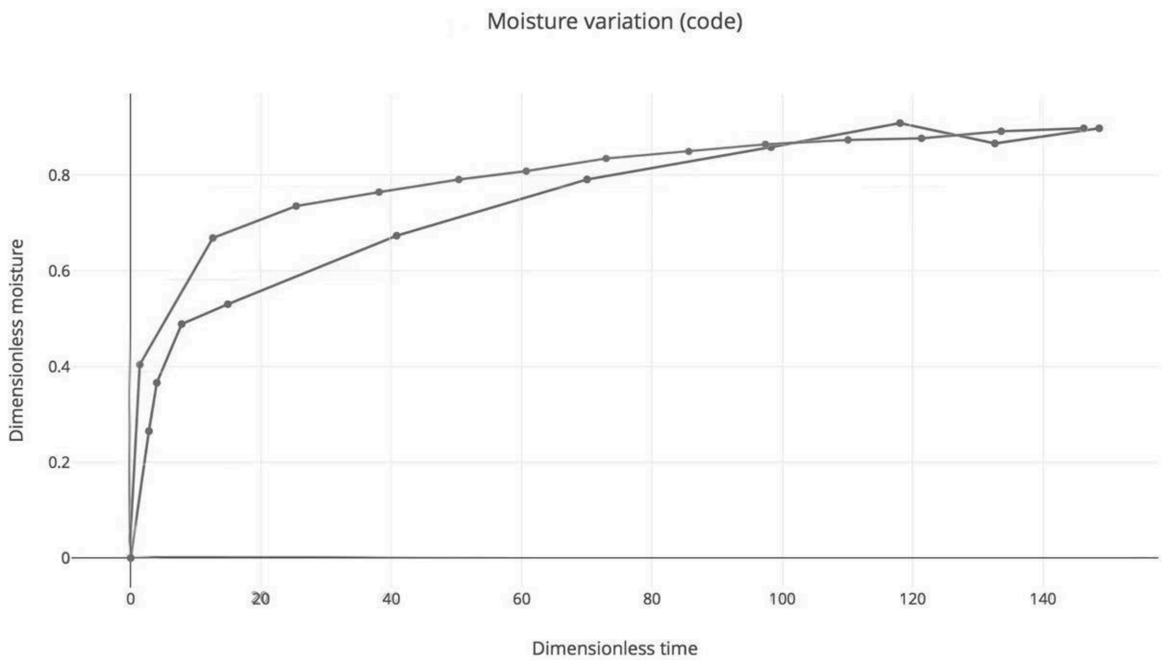
B6.(C)


Moisture Variation (Analytical)
Dimensionless moisture
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
Dimensionless time

B6.(D)


Moisture variation (code)
Dimensionless moisture
0
0.2
0.4
0.6
0.8
0
20
40
60
80
100
120
140
Dimensionless time

B6.(E)


Moisture variation (code)
Dimensionless moisture
0
0.2
0.4
0.6
0.8
0
20
40
60
80
100
120
140
Dimensionless time

B6.(F)

5. ANNEXURE

MATLAB CODE USED :

(A) HEAT CONDUCTION

```
FUNCTION [] = IMPLICIT_NUMERICAL_METHOD(~)
%FIRST NON-LINEAR EQUATION.
ERR= INPUT('SATURATION LIMIT BETWEEN TWO CONSECUTIVE READINGS ');
GSE= INPUT('GAUSS-SIEDEL SATURATION LIMIT BETWEEN TWO CONSECUTIVE READINGS ');
C=INPUT('MAX NO. OF ITERATION FOR GAUSS SIEDEL METHOD ');
L= INPUT('LENGTH OF THE ROD(IN METRES) ');
N= INPUT('NUMBER_OF_POINTS_OF STUDY '); %NUMBER OF GRID POINTS
M= INPUT('NUMBER_OF_ITERATIONS '); %HOW MANY TIMES YOU WANT TO ITERATE, FOR ACHIEVING THE STEADY STATE
K= INPUT('THERMAL CONDUCTIVITY '); % THERMAL CONDUCTIVITY OF A MATERIAL
T= INPUT('TIME OF COMPUTATION '); %FOR HOW LONG YOU WANT TO COMPUTE
%C=K*T/M*(L/N);
P = INPUT('LAMBDA ');
%DRICHLET BOUNDARY CONDITIONS
FLAG= FALSE;
FLAG2= FALSE;
A=INPUT('RIGHT HAND SIDE TEMP = ');
B=INPUT('LEFT HAND SIDE TEMP = ');
A = ZEROS(M+1,N+2); % THE 2 D MATRIX WAS SET TO ZERO INITIAL VALUES, WHICH WAS FURTHUR USED IN GAUSS-SEIDEL METHOD.
B = ZEROS(1,N);     %THIS IS THE COMPARATOR & SUBSTITUTION MATRIX.
C = ZEROS(1,N);     %THIS IS THE CONVERGENCE CHECK MATRIX.

% SETTING THE BOUNDARY CONDITION ON THE MATRIX ELEMENT
FOR I=1:M+1
    A(I,1) = A;
END
FOR I=1:M+1
    A(I,N+2) = B;
END
DISP(A);

%IMPLICIT CALCULATION OF THE TEMPRATURE PROFILE, WITH AN OPEN LOOP METHOD
I=1;
E =0;
O=0;
PRODUCT=1;
WHILE (I<= M )

J=2;
WHILE( J<= N+1 )
```

```
     A(I+1,J)=  (A(I,J)  + C*(A(I+1,J+1))  +C*(A(I+1,J-1)) )/(1+2*C)   ; % THIS IS
THE IMPLICIT EQUATION.


    %IF ((A(I+1,J))-(A(I,J)) < ERR ) CONVERGENCE CONDITION
     %   FLAG = TRUE;

   J=J+1;

   END

   DISP(A);
   DISP('1ST ROW OF A'); % THIS IS THE FIRST ROW OF A AFTER CONVERGENCE


    J=2;
    COUNT=0;
    FLAG2= FALSE;

    %
    WHILE ( FLAG2== FALSE)

        COUNT=COUNT+1;

      FOR K=1:N

       B(1,K)=  (A(I,J)  + C*(A(I+1,J+1))  +C*(A(I+1,J-1)) )/(1+2*C)   ;  % THIS IS
THE IMPLICIT EQUATION

      J=J+1;
      END

      J=2;
    DISP(B);
    DISP('NEW A AS B');
    DISP(C);
    DISP('C');

    FOR S=2:N+1
        E = A(I+1,S) - B(1,S-1);
        %DISP(E);
        %DISP('E');
        %DISP('ITERATION NUMBER');
        IF (GSE < E )
            C(1,S-1)= 1;
        END

    END

    DISP(C);
    DISP('C MATRIX');

        FOR O=1:N
            PRODUCT=C(1,O)*PRODUCT;
        END

        %HERE WE SEE THAT THE SATURATION HAS BEEN REACHED.
    IF (PRODUCT == 1 || COUNT== C)
        FLAG2 = TRUE;
        DISP('GS SATURATION REACHED FOR THIS ROW');

    END


        FOR S=2:N+1
            A(I+1,S)=B(1,S-1); % SUBSTITUTION MATRIX
```

```
        END

    END

I=I+1;
DISP(I);
END


%IF (FLAG == TRUE)
%         DISP( 'STEADY STATE REACHED ' );



A(M+1,N+2)= B;

DISP(A);
DISP('FINAL');

HOLD ON
FOR I=1:M+1
PLOT(A(I,:))

END

END
```

(B) <u>COUPLED HEAT & MASS TRANSFER :</u>

```
FUNCTION [] = DIRECT_PROBLEM(~)



%FIRST NON-LINEAR EQUATION.

ERR= INPUT('SATURATION LIMIT BETWEEN TWO CONSECUTIVE READINGS ');
GSE= INPUT('GAUSS-SIEDEL SATURATION LIMIT BETWEEN TWO CONSECUTIVE READINGS ');
C=INPUT('MAX NO. OF ITERATION FOR GAUSS SIEDEL METHOD ');

L= INPUT('LENGTH OF THE ROD(IN METRES) ');
N= INPUT('NUMBER_OF_POINTS_OF STUDY '); %NUMBER OF GRID POINTS
M= INPUT('NUMBER_OF_ITERATIONS '); %HOW MANY TIMES YOU WANT TO ITERATE, FOR
ACHIEVING THE STEADY STATE
K= INPUT('THERMAL CONDUCTIVITY '); % THERMAL CONDUCTIVITY OF A MATERIAL
TI= INPUT('TIME OF COMPUTATION '); %FOR HOW LONG YOU WANT TO COMPUTE

A=INPUT('THERMAL DIFFUSITIVITY ');

PN=INPUT('PN ');
Q=INPUT('Q ');
EP=INPUT('EP ');
KO=INPUT('KO ');
LU=INPUT('LU ');
```

```
BIQ=INPUT('BIQ ');
BIM=INPUT('BIM ');
BIMS= BIM*(1-(1-EP)*PN*KO*LU);
TS=INPUT('SURROUNDING TEMP ');
TO=INPUT('INITIAL TEMP ');
Q= (Q*L)/(K*(TS-TO));
%C=K*T/M*(L/N);

%DRICHLET BOUNDARY CONDITIONS
FLAG= FALSE;
FLAG2= FALSE;
%T=(A*TI)/((L*L));
T=INPUT('T ');

X=INPUT('INPPUT X ');

% T & M ARE USED FOR THE INITIAL ZERO MATRIX , WHICH WILL BE LATER USED FOR
% GS METHOD.
T = ZEROS(M,N);
M = ZEROS(M,N);
A = ZEROS(1,N);
B = ZEROS(1,N);
C = ZEROS(1,N);

I=1;
E1 =0;
E2 =0;
O=0;
PRODUCT=1;

WHILE (I<= M-1 )


J=2;

% INITIAL CONDITIONS

T(I,1)= (4*T(I,2) - T(I,3) + 2*X*Q )/3 ;
M(I,1)= (M(I,3) -4*M(I,2) -PN*Q*2*X)/3;


% DISRETISED INITIAL CONDITIONS

T(I,N)=(BIQ+ 4*(T(I,N-1) - T(I,N-2))/(2*X) - (1-EP)*KO*LU*BIM*(1 - M(I,N)))/ (3/
(2*X) + BIQ);
M(I,N)=(BIMS+ (-4*(M(I,N-1) + T(I,N-2)))/2*X - PN*BIQ + - T(I,N)*PN*BIQ )/ (3/
(2*X) + BIMS);


   WHILE( J<= N-1 && I<=M-1 )

     T(I+1,1)= (4*T(I+1,2) - T(I+1,3) + 2*X*Q )/3 ;
     M(I+1,1)= (M(I+1,3) -4*M(I+1,2) -PN*Q*2*X)/3;


     % DISCRETISED EQUATIONS

           T(I+1,J)=   (((T(I+1,J+1)+   T(I+1,J-1))/(X*X)   )   +   (T(I,J)/T)   -
(EP*KO*(M(I+1,J))) - (((M(I,J))/T))) / ((1/T) + 1/(X*X) )  ;
```

```
    M(I+1,J)= (((LU*(M(I+1,J+1)+ M(I+1,J-1))/(X*X) )) - ((LU*PN)*( T(I+1,J-1) -
T(I+1,J) + T(I+1,J+1)) / (X*X)) + ((M(I,J))/ T ) ) / (1/T + LU/X*X ) ;


    %DISCRETISED BOUDARY CONDITIONS

       T(I+1,N)=(BIQ+ 4*(T(I,N-1) - T(I,N-2))/(2*X) - ((1-EP)*KO*LU*BIM*(1 -
M(I,N))))/(3/(2*X) + BIQ);

           M(I+1,N)=(BIMS+ (-4*(M(I,N-1) + T(I,N-2)))/2*X - PN*BIQ + -
T(I,N)*PN*BIQ )/(3/(2*X) + BIMS);


     %IF ((A(I+1,J))-(A(I,J)) < ERR )
     %   FLAG = TRUE;




    J=J+1;

   END


   DISP(T);
   DISP(M);
   DISP('1ST ROW OF T & M');


    J=2;
    COUNT=0;
    FLAG2= FALSE;
    WHILE ( FLAG2== FALSE)

        COUNT=COUNT+1;


      A(1,1)= (M(I+1,3) -4*M(I+1,2) -PN*Q*2*X)/3;
      B(1,1)= (4*T(I+1,2) - T(I+1,3) + 2*X*Q )/3 ;

      FOR K=2:N-1


      %IF( J~= N)
              A(1,K)= (((T(I+1,J+1)+ T(I+1,J-1))/(X*X) ) + (T(I,J)/T) -
(EP*KO*(M(I+1,J))) - (((M(I,J))/T))) / ((1/T) + 1/(X*X) ) ;

      B(1,K)= (((LU*(M(I+1,J+1)+ M(I+1,J-1))/(X*X) )) - ((LU*PN)*( T(I+1,J-1) -
T(I+1,J) + T(I+1,J+1)) / (X*X)) + ((M(I,J))/ T ) ) / (1/T + LU/X*X ) ;
      %END


      J=J+1;

      END


        A(1,N)=(BIQ+ 4*(T(I+1,N-1) - T(I+1,N-2))/(2*X) - (1-EP)*KO*LU*BIM*(1-
M(I+1,N)))/ (3/(2*X) + BIQ);
```

```
            B(1,N)=(BIMS+  (-4*(M(I+1,N-1)  +  T(I+1,N-2)))/2*X  -  PN*BIQ  +  -
T(I+1,N)*PN*BIQ )/ (3/(2*X) + BIMS);



      J=2;

    DISP(A);
    DISP(B);
    DISP('NEW A AND B OF T AND M');
    DISP(C);
    DISP('C');

    FOR S=1:N
        E1 = T(I+1,S) - A(1,S);
        E2 = M(I+1,S) - B(1,S);
        %DISP(E);
        %DISP('E');
        %DISP('ITERATION NUMBER');
        IF (GSE < E1 && GSE < E2 )
            C(1,S)= 1;
        END
     END

    DISP(C);
    DISP('C MATRIX');

        FOR O=1:N
            PRODUCT=C(1,O)*PRODUCT;
        END

    IF (PRODUCT == 1 || COUNT== C)
        FLAG2 = TRUE;
          DISP('GS SATURATION REACHED FOR THIS ROW'); % CONVERGENCE OF THE ROW
ELEMNTS

    END

%IF (FLAG == TRUE)
%        DISP( 'STEADY STATE REACHED ' );
        FOR S=1:N
            T(I+1,S)=A(1,S); % SUBSTITUTION MATRIX
            M(I+1,S)=B(1,S);
        END

        DISP(T);
        DISP(M);
        DISP('NEW T & M AFTER REPLACEMENT WITH A & B');

    END

I=I+1; % CHANGING ROWS, CHANGING TIME STEP
DISP(I);
END
DISP(T);
DISP(M);
DISP('FINAL');
PLOT(T(:,N))

END
```

6. REFERENCES :

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