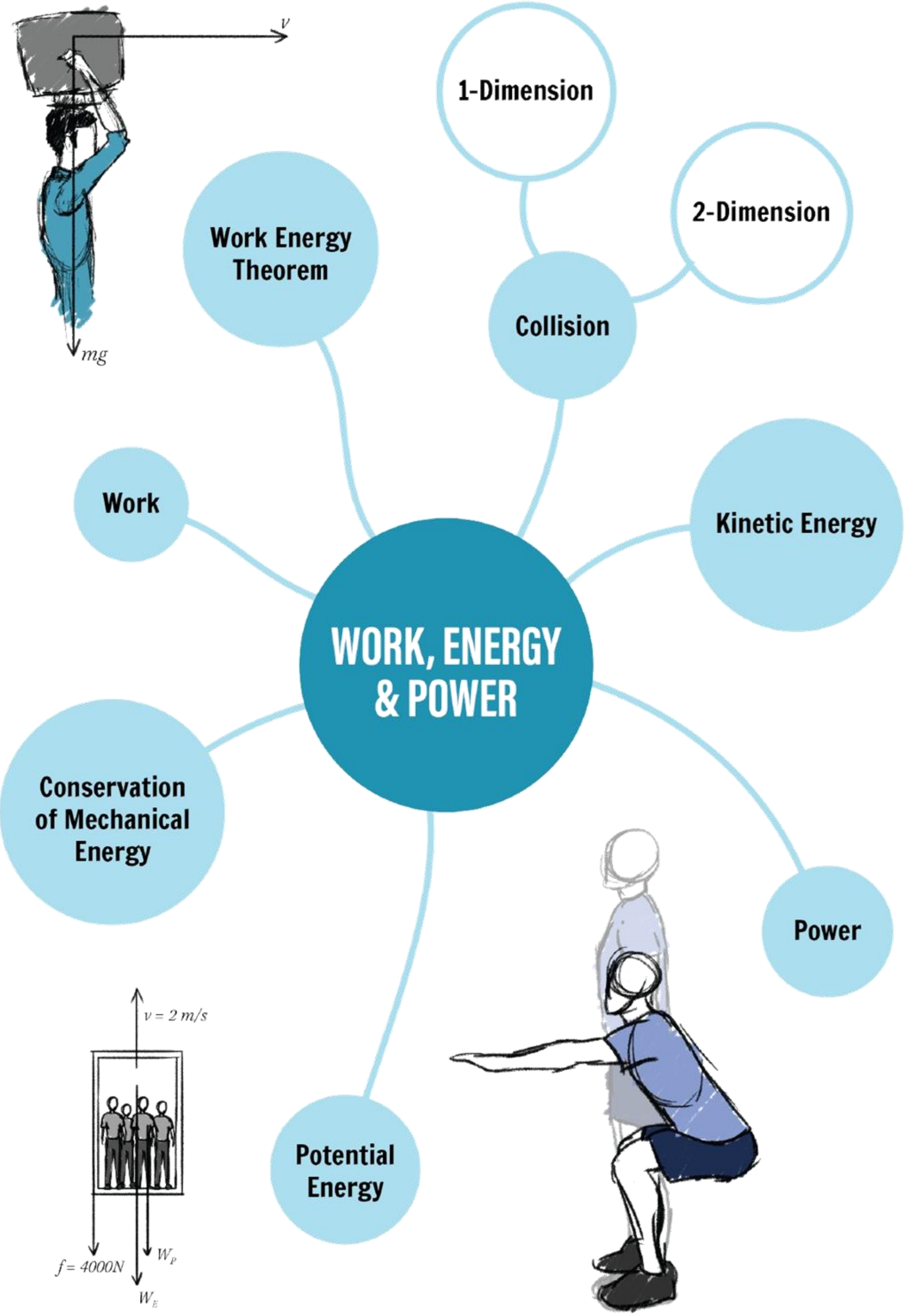

WORK, ENERGY & POWER
Work
Work Energy Theorem
Collision
1-Dimension
2-Dimension
Kinetic Energy
Power
Potential Energy
Conservation of Mechanical Energy
v
mg
v = 2 m/s
f = 4000N
W_P
W_E

WORK, POWER & ENERGY

Work – It is said to be done by a force when the point of application is force is displaced. Transferring energy to or from an object so that there is some displacement of the point of application of force.

Energy is the capacity to do work.

Power - Rate of work done per unit time

The Scalar or Dot Product of Two Vectors

The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\cdot\vec{B}$(read as $\vec{A}$ dot $\vec{B}$) is defined as

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

- Where A & B are magnitudes of vectors $\vec{A}$ and $\vec{B}$ respectively and θ is the smaller angle between them.
- Dot product is called scalar product as both A, B and cos θ are scalars.
- Both vectors have a direction but their scalar product does not have a direction.

Properties

- Dot product is commutative

  $A\cdot B = B\cdot A$
- Dot product is distributive

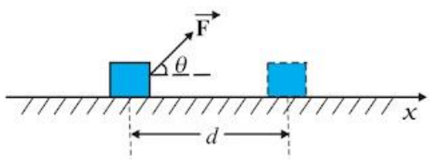
  $A\cdot(B + C) = A\cdot B + A\cdot C$
- Dot product of a vector with itself gives square of its magnitude

  $A.A = AA\cos\theta = A$
- $A\cdot(\lambda B) = \lambda(A\cdot B)$

  where λ is a real number
- $\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$
- $\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$

Work Done by a Force

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.



$$W = (F\cos\theta)d = F\cdot d$$

We see that if there is no displacement, there is no work done even if the force is large. Thus, when you push hard against a rigid brick wall, the force you exert on the wall does no work.

Unit of Work

- Sl unit of work is joule (J),
- CGS unit is erg.
- $1 \text{ J} = 10^7 \text{erg}$

No work is done if

1 the displacement is zero. A weightlifter holding a 150 kg mass steadily on his shoulder for 30 seconds does no work on the load during this time.

2 the force is zero. A block moving on a smooth horizontal table is not acted upon by a horizontal force, but may undergo a large displacement.

3 the force and displacement are mutually perpendicular.
since, for $\theta = \pi/2$ rad $(= 90^\circ)$, $\cos(\pi/2) = 0$.

Q. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion.
(a) How much work does the road do on the cycle?
(b) How much work does the cycle do on the road? [NCERT Exercise]

Sol. Work done on the cycle by the road is the work done by the stopping (frictional) force on the cycle due to the road.

(a) The stopping force and the displacement make an angle of 180° (π rad) with each other. Thus, work done by the road,

$W_r = Fd\cos\,\theta = 200 \times 10 \times \cos\,\pi = -2000 \text{ J}$

It is this negative work that brings the cycle to a halt in accordance with WE theorem.

(b) From Newton's Third Law an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement. Thus, work done by cycle on the road is zero.

Energy

Energy is defined as internal capacity of doing work. When we say that a body has energy, we mean that it can do work.

Energy appears in many forms such as mechanical, electrical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear etc., and can change from one form

Mechanical Energy

Mechanical energy is the energy that is possessed by an object due to its motion or due to its position.

Mechanical energy = KE + PE

Kinetic Energy

Kinetic energy K of an object of mass m is given as $K = 1/2mv^2$

It is a scalar quantity.

Application

1. It is a measure of the work an object can do because of its motion.

2. Sailing ships use KE of the wind.

3. KE of a fast-flowing stream has been used for grinding corn.

Relation between KE and linear momentum

Let mass of body be m and velocity of body be v.

$$p = mv = \text{linear momentum}$$

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2m}(m^2v^2)$$

$$KE = \frac{p^2}{2m}$$

$$p = \sqrt{2mKE}$$

Q. In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed $200\ \text{m s}^{-1}$ (see Table 6.2) on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet? [NCERT Exercise]

Sol. The initial kinetic energy of the bullet is $mv^2/2 = 1000$ J. It has a final kinetic energy of $0.1 \times 1000 = 100$ J.

If v_f is the emergent speed of the bullet,

$$\frac{1}{2}mv_f^2 = 100\ \text{J}$$

$$v_f = \sqrt{\frac{2 \times 100\ \text{J}}{0.05\ \text{kg}}} = 63.2\ \text{m s}^{-1}$$

The speed is reduced by approximately 68% (not 90%).

Potential Energy

Potential energy is the energy that an object has because of its position relative to other objects. It is usually defined in equations by the capital letter U or sometimes by PE.

Gravitational Potential Energy

Gravitational potential energy as a function of the height h, is denoted by ∪ (h)

It is the negative of work done by the raising the body to that height.

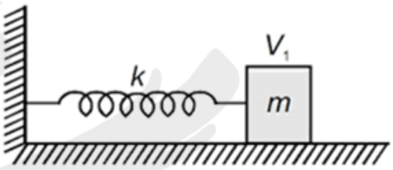
$$U(h) = mgh$$

where, m = mass of a body

- SI unit of potential energy is Joule, the same as kinetic energy or work.
- Commercial unit of energy is kWh, $1\text{kWh} = 3.6 \times 10^6$ J.

Elastic Potential Energy of Spring

The energy associated with the state of compression or expansion of a spring is known as elastic potential energy.



Elastic potential energy is given by

$$U = 1/2kx^2$$

where, k = force constant of given spring

Q. To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant 6.25×10^3 N m. What is the maximum compression of the spring? [NCERT Exercise]

Sol. At maximum compression the kinetic energy of the car is converted entirely into the potential energy of the spring. The kinetic energy of the moving car is

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times 5 \times 5$$

$$K = 1.25 \times 10^4 \text{ J}$$

where we have converted 18 km h^{-1} to 5 m s^{-1} [It is useful to remember that $36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$. At maximum compression x_m, the potential energy V of the spring is equal to the kinetic energy K of the moving car from the principle of conservation of mechanical energy.

$$V = \frac{1}{2}kx_m^2 = 1.25 \times 10^4 \text{J}$$

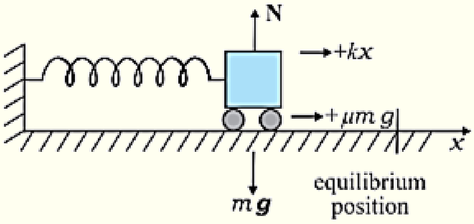
We obtain $x_m = 2.00\text{m}$

Q. Consider above example and taking the coefficient of friction, μ, to be 0.5 and calculate the maximum compression of the spring. [NCERT Exercise]

Sol. In presence of friction, both the spring force and the frictional force act so as to oppose the compression of the spring as shown in Figure.

We invoke the work-energy theorem, rather than the conservation of mechanical energy.

The change in kinetic energy is



$\Delta K = K_f - K_l$

$= 0 - \frac{1}{2}mv^2$

The work done by the net force is

$W = -\frac{1}{2}kx_m^2 - \mu mgx_m$

Equating we have

$\frac{1}{2}mv^2$

$= \frac{1}{2}kx_m^2 + \mu mgx_m$

Now $\mu mg = 0.5 \times 10^3 \times 10$

$= 5 \times 10^3$ N (taking $g = 10.0\ m\ s^{-2}$).

After rearranging the above equation we obtain the following quadratic equation in the unknown x_{m^*}

$kx_m^2 + 2\mu mgx_m - mv^2 = 0$

$$x_m = \frac{-\mu mg + [\mu^2m^2g^2 + mkv^2]^{1/2}}{k}$$

where we take the positive square root since x_m is positive. Putting in numerical values we obtain $x_m = 1.35$ m

The Law of Conservation of Energy

According to the law of conservation of energy, the total energy of an isolated system does not change. Energy may be transformed from one form to another but the total energy of an isolated system remains constant.

Besides mechanical energy, the energy may manifest itself in many other forms. Some of these forms are: thermal energy, electrical energy, chemical energy and energy can be converted into mass.

The Conservation of Mechanical Energy

According to this, "**The total mechanical energy of a system is conserved if the forces doing work on it, are conservative.**"

- Suppose that a body undergoes displacement Δx under the action of a conservative force F. From the work-energy theorem,

$$\Delta K = \int F(x)dx$$

- If the force is conservative, the potential energy function U(x) can be defined such that

$$\Delta U = -\int F(x)dx$$

- Adding the above two equations, we get $\Delta K + \Delta U = 0$

$$\Delta(K + U) = 0$$

$$K + U = \text{constant}$$

The Work-Energy Theorem

The work-energy theorem states that the work done by all forces acting on a particle is equal to the change in the particle's kinetic energy.

$$W_{Total} = \Delta KE, \qquad W_{Total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Confining to one dimension, rate of change of kinetic energy with time is

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$\frac{dK}{dt} = \frac{1}{2}m\frac{d}{dt}(v^2)$$

$$\frac{dK}{dt} = \frac{1}{2}m\left(2v\frac{dv}{dt}\right)$$

$$\frac{dK}{dt} = m\frac{dv}{dt}v$$

$$\frac{dK}{dt} = Fv \ldots [\because F = ma]$$

$$\frac{dK}{dt} = F\frac{dx}{dt}$$

$$dK = Fdx$$

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} Fdx$$

Where K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f respectively.

$$K_f - K_i = W$$

Q. It is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass 1.00 g falling from a height 1.00 km. It hits the ground with a speed of $50.0\ \mathrm{m\ s^{-1}}$. (a) What is the work done by the gravitational force? What is the work done by the unknown resistive force? [NCERT Exercise]

Sol. (a) The change in kinetic energy of the drop is

$$\Delta K = \frac{1}{2}mv^2 - 0 = \frac{1}{2} \times 10^{-3} \times 50 \times 50 = 1.25\mathrm{J}$$

where we have assumed that the drop is initially at rest.

Assuming that g is a constant with a value $10\ \mathrm{m/s^2}$, the work done by the gravitational force is, $W_g = mgh = 10^{-3} \times 10 \times 10^3 = 10.0\ \mathrm{J}$

(b) From the work-energy theorem $\Delta K = W_g + W_r$

where W_r is the work done by the resistive force on the raindrop.

Thus $W_r = \Delta K - W_g = 1.25 - 10 = -8.75$ J is negative.

Power

Power is defined as the time rate at which work is done or energy is transferred.

The **Average Power** of a force is defined as the ratio of the work, W to the total time taken

$$P_{av} = \frac{W}{t}$$

The **Instantaneous Power** is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dW}{dt}$$

The work dW done by force $\vec{F}$ for a displacement $d\vec{r}$ is

$$dW = \vec{F} \cdot d\vec{r}$$

The instantaneous power can also be expressed as

$$P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

where $\vec{v}$ is the instantaneous velocity when the force is $\vec{F}$. Power is a scalar quantity like work and energy. Its dimensions are $[ML^{2}T^{-3}]$. Its SI unit is Watt (W).

$[\because 1\mathrm{hp} = 746\ \mathrm{W}]$

Q. An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 m s^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power. **[NCERT Exercise]**

Sol. The downward force on the elevator is

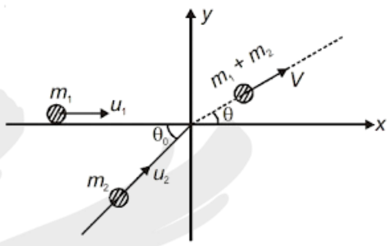
$F = mg + F_f = (1800 \times 10) + 4000 = 22000$ N

The motor must supply enough power to balance this force. Hence,

$P = F \cdot v = 22000 \times 2 = 44000$ W $= 59$hp

Collisions

If two object collide during their motion then this event is called Collision. Generally, collisions are two types-



(i) Elastic Collision

If, in a particular collision, there is no dissipation of energy, the total kinetic energy of the objects before collision is equal to total kinetic energy of the objects after collision. Such a collision is termed as elastic collision

E.g., Collision of two steel ball.

(ii) Inelastic Collision

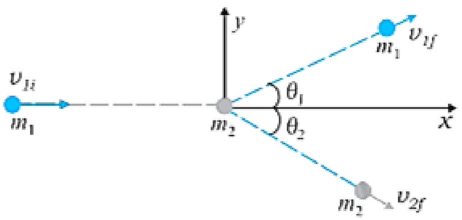
When there is a loss of kinetic energy, in any collision Then this type of collision is called perfectly inelastic collision.

E.g., When a soft mud ball is thrown against the wall.

Collisions in One Dimension (Inelastic Collision)

Consider first a completely inelastic collision in one dimension. Then, in Figure

$$\theta_1 = \theta_2 = 0$$



$m_1 v_i = (m_1 + m_2) v_f$ (Momentum conservation)

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

The loss in kinetic energy on collision is

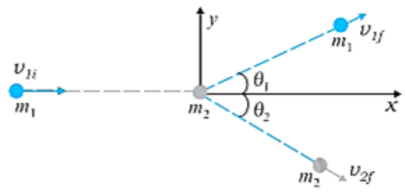
$$\Delta K = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2 = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_{1i}^2 \text{ [using Eq.]}$$

$$= \frac{1}{2} m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1 + m_2}\right] = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_{1i}^2$$

which is a positive quantity as expected.

Collisions in One Dimension (Elastic Collision)

Consider next an elastic collision. Using the above nomenclature with $\theta_1 = \theta_2 = 0$, the momentum and kinetic energy conservation equations are



$m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$

$(1/2)m_1v_{1i}^2 = (1/2)m_1v_{1f}^2 + (1/2)m_2v_{2f}^2$

From above equations it follows that,

$m_1v_{1i}(v_{2f} - v_{1i}) = m_1v_{1f}(v_{2f} - v_{1f})$ or $v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2 = (v_{1f} - v_{1f})(v_{1i} + v_{1f})$

Hence, $\therefore v_{2f} = v_{1t} + v_{1f}$

Substituting the above in equation, we obtain $v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i}$ and $v_{2f} = \frac{2m_1v_{1i}}{m_1 + m_2}$

- Thus, the 'unknowns' $\{v_{1\rho}, v_2\}$ are obtained in terms of the 'knowns' $\{m_1, m_2, v_1\}$. Special cases of our analysis are interesting.

Case I: If the two masses are equal $v_{1f} = 0,\ v_{2f} = v_{1f}$

The first mass comes to rest and pushes off the second mass with its initial speed on collision.

Case II: If one mass dominates, e.g., $m_2 >> m_1$

$$v_{1f} \simeq -v_{11} \qquad v_{2f} \simeq 0$$

The heavier mass is undisturbed while the lighter mass reverses its velocity.

Q. Slowing down of neutrons: In a nuclear reactor a neutron of high speed (typically $10^7\ m\ s^{-1}$) must be slowed to $10^3\ m\ s^{-1}$ so that it can have a high probability of interacting with isotope $^{235}_{95}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nuclei like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite, is called a moderator. **[NCERT Exercise]**

Sol. The initial kinetic energy of the neutron is $K_{1i} = \frac{1}{2}m_1v_{1i}^2$

while its final kinetic energy from $K_{1f} = \frac{1}{2}m_1v_{1f}^2 = \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 v_{1i}^2$

The fractional kinetic energy lost is $f_1 = \frac{K_{1f}}{K_{1i}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

while the fractional kinetic energy gained by the moderating nuclei K_{2f}/K_l is

$f_2 = 1 - f_1$ (elastic collision) $= \frac{4m_1m_2}{(m_1 + m_2)^2}$

One can also verify this result by substituting from.

For deuterium $m_2 = 2m_1$ and we obtain $f_1 = 1/9$ while $f_2 = 8/9$. Almost 90% of the neutron's energy is transferred to deuterium. For carbon $f_1 = 71.6\%$ and $f_2 = 28.4\%$. In practice, however, this number is smaller since head-on collisions are rare.

Collisions in Two Dimensions

Consider the plane determined by the final velocity directions of m_1 and m_2 and choose it to be the $x - y$ plane. The conservation of the z-component of the linear momentum implies that the entire collision is in the $x - y$ plane. The x - and y-component equations are

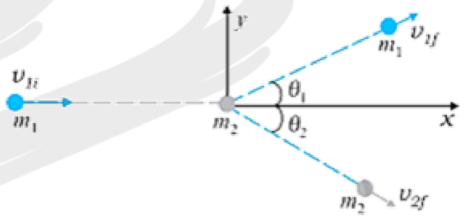
$m_1 v_{11} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\ \theta_2$

$0 = m_1 v_{1f} \sin\ \theta_1 - m_2 v_{2f} \sin\ \theta_2$

One knows $\{m_1, m_2, v_1\}$ in most situations. There are thus four unknowns $\{v_{1f}, v_{2f}, \theta_1$ and $\theta_2\}$, and only two equations.

- If $\theta_1 = \theta_2 = 0$, for one dimensional collision.
- If, further the collision is elastic, $\frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}{}^2$

We obtain an additional equation. That still leaves us one equation short. At least one of the four unknowns, say θ_1, must be made known for the problem to be solvable.



NCERT Exercise Questions

Q.1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,

(A) the two magnetic forces are equal and opposite, so they produce no net effect.

(B) the magnetic forces do no work on each particle.

(C) the magnetic forces do equal and opposite (but non-zero) work on each particle.

(D) the magnetic forces are necessarily negligible.

Sol. **(B)**

When electron and proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion hence no work is done by these forces.

Q.2. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is

(A) same as the same force law is involved in the two experiments.

(B) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.

(C) more for the case of a positron, as the positron moves away a larger distance.

(D) same as the work done by charged particle on the stationary proton.

Sol. **(C)**

Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton. We know that work done $=$ force $\times$ distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done will be more.

Q.3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is

(A) constant and equal to mg in magnitude.

(B) constant and greater than mg in magnitude.

(C) variable but always greater than mg.

(D) at first greater than mg, and later becomes equal to mg.

Sol. **(D)**

When the man is squatting on the ground he is tilted somewhat, hence he also has to balance frictional force besides his weight in this case.

$R = \text{reactional force} = \text{friction} + mg$

$\Rightarrow R > mg$

When the man gets straight up in that case friction ≈ 0

$\Rightarrow$ Reactional force $\approx$ mg

Q.4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposed to the motion. The work done by the cycle on the road is

(A) +2000 J (B) -200 J (C) zero (D) -20,000 J

Sol. **(C)**

Here, work is done by the frictional force on the cycle and is equal to –

$200 \times 10 = -2000\text{J}$.

As the road is not moving,

Hence, work done by the cycle on the road = zero.

Q.5. A man, of mass m, standing at the bottom of the staircase, of height L climbs it and stands at its top.

(A) Work done by all forces on man is equal to the rise in potential energy mgL.

(B) Work done by all forces on man is zero.

(C) Work done by the gravitational force on man is mgL.

(D) The reaction force from a step does not do work because the point of application of the force does not move while the force exists.

Sol. **(B, D)**

When a man of mass m climbs up the staircase of height L, work done by the gravitational force on the man is-mgl work done by internal muscular forces will be mgL as the change in kinetic energy is almost zero.

Hence,

Total work done $= -mgL + mgL = 0$

As the point of application of the contact forces does not move hence work done by reaction forces will be zero.

Q.6. A bullet of mass m fired at 30° to the horizontal leaves the barrel of the gun with a velocity v. The bullet hits a soft target at a height h above the ground while it is moving downward and emerges out with half the kinetic energy it had before hitting the target.

Which of the following statements are correct in respect of bullet after it emerges out of the target?

(A) The velocity of the bullet will be reduced to half its initial value.

(B) The velocity of the bullet will be more than half of its earlier velocity.

(C) The bullet will continue to move along the same parabolic path.

(D) The bullet will move in a different parabolic path.

(E) The bullet will fall vertically downward after hitting the target.

(F) The internal energy of the particles of the target will increase.

Sol. **(B, D, F)**

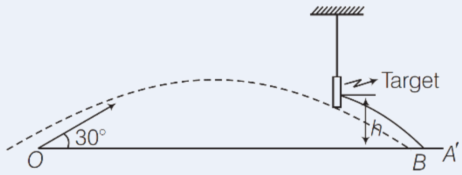
Consider the adjacent diagram for the given situation in the question.

(B) Conserving energy between "O" and "A"

$$U_i + K_i = U_f + K_f \Rightarrow 0 + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv' \Rightarrow \frac{(v')^2}{2} = \frac{v^2}{2} = -gh$$

$$\Rightarrow (v')^2 = v^2 - 2gh \Rightarrow v' = \sqrt{v^2 - 2gh} \qquad \text{...(i)}$$

where v' is speed of the bullet just before hitting the target. Let speed after emerging from the target is v'' then,



$$= \frac{1}{2}(mv'')^2 = \frac{1}{2}\left[\frac{1}{2}m(v')^2\right]$$

$$\frac{1}{2}m(v'')^2 = \frac{1}{4}m(v')^2 = \frac{1}{4}m[v^2 - 2gh]$$

$$\Rightarrow (v'')^2 = \frac{v^2 - 2gh}{2} = \frac{v^2}{2} - gh \Rightarrow v'' = \sqrt{\frac{v^2}{2} - gh} \qquad \text{... (ii)}$$

From Eqs. (i) and (ii)

$$\frac{v'}{v''} = \frac{\sqrt{v^2 - 2gh}}{\frac{\sqrt{v^2 - 2gh}}{\sqrt{2}}} = \sqrt{2} \Rightarrow v'' = \frac{V'}{\sqrt{2}} = v^2\left(\frac{V'}{2}\right) \Rightarrow \frac{V''}{\frac{V'}{2}} = \sqrt{2} = 1.414 > 1 \Rightarrow V'' > \frac{V'}{2}$$

Hence, after emerging from the target velocity of the bullet (v'') is more than half of its earlier velocity v' (velocity before emerging into the target).

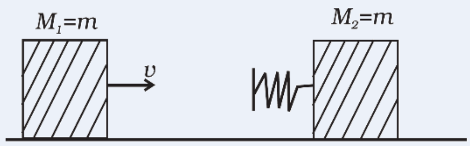
(D) As the velocity of the bullet changes to v' which is less than v^1 hence, path, followed will change and the bullet reaches at point B instead of A', as shown in the figure.

(F) As the bullet is passing through the target the loss in energy of the bullet is transferred to particles of the target. Therefore, their internal energy increases.

Q.7. Two blocks M_1 and M_2 having equal mass are free to move on a horizontal frictionless surface. M_2 is attached to a massless spring as shown in Fig. 6.10. Iniially M_2 is at rest and M_1 is moving toward M_2 with speed v and collides head-on with M_2.

(A) While spring is fully compressed all the KE of M_1 is stored as PE of spring.

(B) While spring is fully compressed the system momentum is not conserved, though final momentum is equal to initial momentum.



(C) If spring is massless, the final state of the M_1 is state of rest.

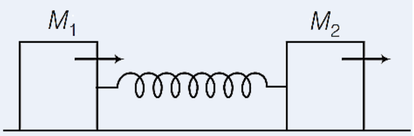
(D) If the surface on which blocks are moving has friction, then collision cannot be elastic.

Sol. Consider the adjacent diagram when M_1 comes in contact with the spring, M_1 is retarded by the spring force and M_2 is accelerated by the spring force.

(A) The spring will continue to compress until the two blocks acquire common velocity.

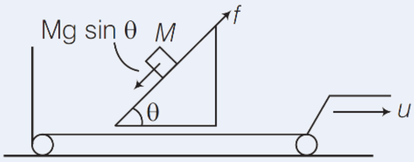
(B) As surfaces are frictional less momentum of the system will be conserved.

(C) If spring is massless whole energy of M_1 will be imparted to M_2 and M_1 will be at rest, then



(D) Collision is inelastic, even if friction is not involved.

Q.8. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?



Sol. Consider the adjacent diagram. As the block M is at rest.

Hence, $f =$ frictional force $= Mg\sin\,\theta$

The force of friction acting between the block and incline opposes the tendency of sliding of the block. Since, block is not in motion, therefore, no work is done by the force of friction. Hence, no dissipation of energy takes place.

Q.9. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?

Sol. When the elevator is descending, then electric power is required to prevent it from falling freely under gravity. Also, as the weight inside the elevator increases, its speed of descending increases, therefore, there should be a limit on the number of passengers in the elevator to prevent the elevator from descending with large velocity.

Q.10. A body is being raised to a height h from the surface of earth. What is the sign of work done by (A) applied force and (B) gravitational force?

Sol. Force is applied on the body to lift it in upward direction and displacement of the body is also in upward direction, therefore, angle between the applied force and displacement is $\theta = 0°$

$\therefore$ Work done by the applied force

$W = FS\cos\theta = FS\cos 0° = FS$ $\quad(\because \cos\ 0° = 1)$

i.e, $W =$ Positive

(b) The gravitational force acts in downward direction and displacement in upward direction, therefore, angle between them is $\theta = 180°$.

$\therefore$ Work done by the gravitational force

$W = FS\cos\ 180° = -FS$ $\quad(\because \cos\ 180° = 1)$

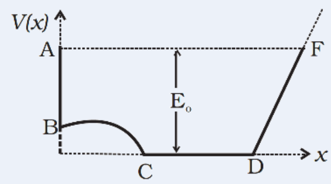
Q.11. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

Sol. Force of gravity acts on the car vertically downward while car is moving along horizontal road, i.e., angle between them is 90º.

Work done by the car against gravity

$W = FS\cos\ 90° = 0$ $\quad(\because \cos\ 90° = 0)$

Q.12. A graph of potential energy V(x) verses x is shown in Figure. A particle of energy E_0 is executing motion in it. Draw graph of velocity and kinetic energy versus x for one complete cycle AFA.



Sol. KE versus x graph we know that Total ME = KE + PE $\Rightarrow E_0$

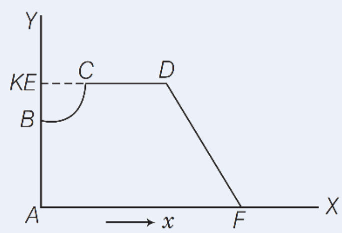
$= KE + V(x)$

$\Rightarrow KE = E_0 - V(x)$ at $A_1 x = 0, V(x) = E_0 \Rightarrow KE = E_0 - E_0 = 0$

at $B_1 V(x) < E_0 \Rightarrow KE > 0$ (positive)

at C and D_1 $V(x) = 0 \Rightarrow$ KE is maximum at $F_1 V(x) = E_0$

Hence, KE = 0 The variation is shown in adjacent diagram.



Velocity versus x graph

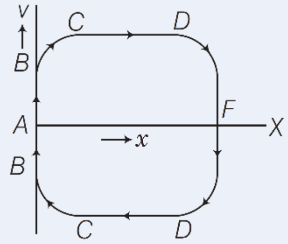
As KE $= \frac{1}{2}mv^2$ $\therefore$ At A and F, where KE $= 0, v = 0$.

At C and D, KE is maximum. Therefore, v is $\pm$ max.

At B, KE is positive but not maximum.

Therefore, v is $\pm$ some value (< max.)

The variation is shown in the diagram.



Q.13. A ball of mass m, moving with a speed $2v_0$, collides inelastically ($e > 0$) with an identical ball at rest. Show that

(A) For head-on collision, both the balls move forward.

(B) For a general collision, the angle between the two velocities of scattered balls is less than 90°.

Sol. (A) Let v_1 and v_2 are velocities of the two balls after collision. Now, by the principle of conservation of linear momentum,

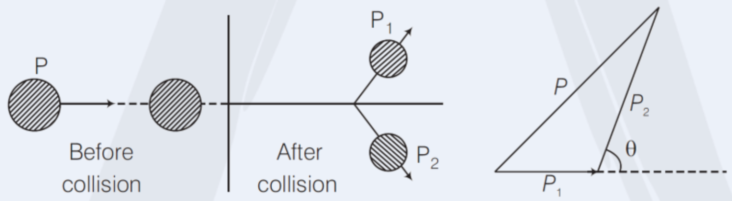
$2mv_0 = mv_1 + mv_2$ or $2v_0 = v_1 + v_2$ and $e = \frac{v_2 - v_1}{2v_0} \Rightarrow v_2 = v_1 + 2v_0 e$

$\therefore 2v_1 = 2v_0 - 2ev_0$

$\therefore v_1 = v_0(1 - e)$

Since, $e < 1 \Rightarrow v_1$ has the same sign as v_0, therefore, the ball moves on after collision.

(B) Consider the diagram below for a general collision.



By principle of conservation of linear momentum, $P = P_1 + P_2$

For inelastic collision some KE is lost, hence

$$\frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$

$$p^2 > p_1^2 + p_2^2$$

Thus, p, p_1 and p_2 are related as shown in the figure.

θ is acute (less than 90º) ($p^2 = p_1^2 + p_2^2$ would given $\theta = 90°$)

Q.14. Consider a one-dimensional motion of a particle with total energy E. There are four regions A, B, C and D in which the relation between potential energy V, kinetic energy (K) and total energy E is as given below:

Region A : $V > E$

Region B : $V < E$

Region C: $K > E$

Region D : $V > K$

State with reason in each case whether a particle can be found in the given region or not.

Sol. We know that

Total energy $E = PE + KE$

$E = V + K$ ...(i)

For region A Given, $V > E$, From Eq. (i)

$K = E - V$ as $V > E \Rightarrow E - V < 0$

Hence, $K < 0$, this is not possible. For region B Given, $V < E \Rightarrow E - V > 0$

This is possible because total energy can be greater than PE (V).

For region C Given, $K > E \Rightarrow K - E > 0$ from Eq. (i) $PE = V = E - K < 0$

Which is possible, because PE can be negative. For region D Given, $V > K$

This is possible because for a system PE(V) may be greater than KE(K).