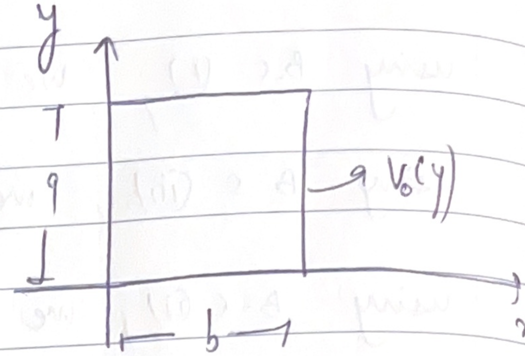
* <u>Analytical solution</u>



Boundary conditions →

i) $V(x,0) = 0$
ii) $V(x,a) = 0$
iii) $V(0,y) = 0$
iv) $V(b,y) = V_0(y)$

Let $V(x,y) = X(x) \cdot Y(y)$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = 0$$

$$\Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = k$$

$$\Rightarrow X''(x) = kX \quad \text{— (i)}$$

$$\Rightarrow X(x) = Ae^{kx} + Be^{-kx}$$

Also, $Y''(y) = -kY$

$$\Rightarrow Y(y) = C\cos ky + D\sin ky$$

$$\Rightarrow V(x,y) = (Ae^{kx} + Be^{-kx})(C\cos ky + D\sin ky)$$

using B.C (i), we get $C = 0$

using B.C (iii), we get $A = -B$

using B.C (ii), we get $C \sin ka = 0$

$\Rightarrow ka = n\pi$

$k = \frac{n\pi}{a}$

$\Rightarrow V(x,y) = A(e^{n\pi/a\, x} - e^{-n\pi x/a}) \cdot D \cdot \sin\frac{n\pi y}{a}$

$\Rightarrow V(x,y) = (2AD)\sinh(n\pi x/a) \cdot \sin(n\pi y/a)$

$\Rightarrow V(x,y) = \Sigma C_n \sinh(n\pi x/a) \cdot \sin(n\pi y/a)$

using B.C. (iv), we get

$\Sigma C_n \sinh(n\pi b/a) \cdot \sin(n\pi y/a) = V_0(y)$

Fourier's Trick

$$C_n \sinh(n\pi b/a) = \frac{2}{a}\int_0^a V_0 \sin(n\pi y/a)\, dy$$

$$C_n = \frac{2V_0}{a\sinh(n\pi b/a)}\int_0^a \sin(n\pi y/a)\, dy$$

$$C_n = \begin{cases} 0 & ; \ n \text{ is even} \\ \frac{4V_0}{\pi} \cdots & ; \text{ if } n \text{ is odd} \end{cases}$$

$\therefore$ ~~$E_y =$~~

$$\therefore \boxed{V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5\ldots} \frac{\sinh(n\pi x/a)\sin(n\pi y/a)}{n\sinh(n\pi b/a)}}$$

when $x = b$,

$$V(x,y) = \frac{4V_0}{\pi} \sum \frac{\cancel{\sinh(n\pi b/a)}\sin(n\pi y/a)}{n\cancel{\sinh(n\pi b/a)}}$$

$$\frac{4V_0}{\pi} \sum_{odd} \frac{\sin(n\pi y/a)}{n}$$