2017

Time : 3 hours

Full Marks : 100

Pass Marks : 35

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions, selecting at least one from each Group.*

Group – A

1. (a) Evaluate any one of the following :

(i) $\int \frac{1}{x^4+1} dx$

(ii) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

(b) Evaluate $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$.

2. (a) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \cos nx \, dx$, then prove that

$(m+n) I_{m,n} = m I_{m-1, n-1}$.

(b) From the first principle, evaluate $\int_0^1 x^3 \, dx$.

3. (a) Find the whole area of the curve :

$a^2y^2 = x^2 (a^2 - x^2)$

(b) Find the perimeter of the curve :

$x^{2/3} + y^{2/3} = a^{2/3}$

4. (a) Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos\theta)$ about $\theta = 0$.

(b) Find the moment of inertia of a solid sphere of radius 'a' about a diameter.

5. Solve any two of the following differential equations :

p,n :- 8, 1(ii) (a) $y - x\frac{dy}{dx} = a(y^2 + \frac{dy}{dx})$

p.n- 21,1(ii) (b) $(x^2 - y^2)\frac{dy}{dx} = 2xy$

p.n -42 , 7 i (c) $(1 + y^2) dx = (\tan^{-1} y - x) dy$

6. (a) Solve any **one** of the following :

(i) $p^2 + 2py \cot x = y^2$

(ii) $(y - px)(p - 1) = p$

(b) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 2ax$, where a is the parameter.

7. Solve any **two** of the following :

(a) $(D^2 - 3D + 2)y = e^{4x}$

(b) $(D^2 + D + 1)y = x^3$

(c) $(D^2 + 4)y = \sin 2x$

Group – B

8. (a) Prove that the lines whose direction cosines are given by $\ell + m + n = 0$ and $2mn + 3n\ell - 5\ell m = 0$ are perpendicular to each other.

(b) A plane meets the co-ordinate axes at points A, B and C such that the centroid of the triangle ABC is the point (a,b,c). Find the equation of the plane.

9. (a) If (–1, 2, 4) be one end of a diameter of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 11 = 0$, find the co-ordinates of the other end.

(b) Find the equation of the right circular cylinder of radius 3 and whose axis is :

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$$

10. (a) Prove that the set of all feasible solutions of a LPP is a convex set.

(b) Solve the following LPP graphically :

Maximize $Z = 30x_1 + 50x_2$

Subject to the constraints

$$2x_1 + x_2 \leq 16$$
$$x_1 + 2x_2 \leq 11$$
$$x_1 + 3x_2 \leq 15$$
$$x_1, x_2 \geq 0.$$

11. Solve the following LPP by the simplex method :

Maximize $Z = 5x_1 + 7x_2$

Subject to the constraints

$$2x_1 + 3x_2 \leq 13$$
$$3x_1 + 2x_2 \leq 12$$
$$x_1, x_2 \geq 0.$$

Group – C

12. (a) Prove that the period of simple harmonic motion is independent of its amplitude.

(b) Prove that the energy of a stretched elastic string is equal to half the product of the tension and the extension.

13. (a) State and prove the principle of conservation of linear momentum.

(b) A particle starts with a given velocity V and moves under a retardation equal to k times the space described. Show that the distance traversed before it comes to rest is $\frac{V}{\sqrt{k}}$.

14. (a) If the radial velocity is proportional to transverse velocity, find the path in polar co-ordinates.

(b) A bullet of mass m moving with velocity u strikes a block of mass M which is free to move in the direction of motion of the bullet, and is embedded in it. Show that the loss of KE is $\frac{1}{2}\frac{Mmu^2}{M+m}$.

15. (a) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary point in the plane of the forces together with a couple.

(b) A uniform beam of length 2a rests in equilibrium against a smooth vertical wall and upon a peg at a distance b from the wall. Show that the inclination of the beam to the vertical is $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$.

16. (a) State and prove the principle of virtual work for a system of coplanar forces acting on a rigid body.

(b) Show that the virtual work done by the tension of an inextensible string is zero.