BCA(Sem-I) —
Math (GE – 1)

2020(Voc)

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer from **all** the Groups as directed.*

1. [I] Choose the correct alternative in each of the following : 1×10 = 10

   (i) Let A = {1, 2, 3, 4, 5, 6}, which of the following corresponds to an equivalence relation :

      (i) {1, 2, 3}, {3, 4, 5, 6}

      (ii) {1, 2}, {4, 5, 6}

      (iii) {1, 3}, {2, 4, 5}, {6}

      (iv) None of these

(ii) For elements a and b in a group G, the equation $ax = b$ has :

(i) Exactly two solutions in G

(ii) More than two solutions in G

(iii) Unique solution in G

(iv) No solution in G

(iii) If u be a homogeneous function, in x and y, of degree n, then :

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$

(ii) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=nu$

(iii) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=nu$

(iv) $\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}=nu$

(iv) If A and B be two matrices such that $AB = A$ and $BA = B$ then $A^2 =$ ______.

(i) A

(ii) B

(iii) AB

(iv) None of these

(v) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then :

(i) $ac \equiv b \pmod{m}$

(ii) $ac \equiv bd \pmod{m}$

(iii) $ad \equiv bc \pmod{m}$

(iv) None of these

[II] Fill in the blanks in each of the following :

(vi) If A, B, C be any three sets then $A \times (B \cap C) =$ ________.

(vii) In a group G, $(ab)^{-1} =$ ________ for all $a, b \in G$.

(viii) If matrices A and B be conformed for the product AB and B, C be conformal for addition then $A(B + C) =$ ________.

(ix) If $y = \cos(ax + b)$, then $y_n =$ ……… j a, b being constants.

(x) If d be the g.c.d of a and b, then there exists integers x and y such that $d =$ ________.

Group – B

(Short-answer Type Questions)

Answer any **four** of the following six questions :

$5 \times 4 = 20$

2. Prove that the complement of the union of a family of sets is the intersection of their complements.

3. Define a group. Prove that in a group G :

   (i) The identity element is unique.

   (ii) The inverse of each $a \in G$ is unique.

4. Define a symmetric and a skew-symmetric matrix. Show that any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

5. Evaluate $\lim_{x \to 0} \left(\frac{1}{x} - \cot x \right)$.

6. Apply Maclaurin's theorem to obtain the expansion of sec x.

7. Find the remainder when $1^5 + 2^5 + 3^5 + \ldots\ldots + 100^5$ is divided by 4.

Group – C

(Long-answer Type Questions)

Answer any four questions of the following :

$10 \times 4 = 40$

8. State and prove the fundamental theorem on equivalence relation.

9. Define a subgroup of a group G. Prove that a non-empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$.

10. Test the consistency of the following set of simultaneous equations and solve them by the matrix method, $x - 2y + 3z = 5$, $4x + 3y + 4z = 7$, $x + y - z = -4$.

11. State and prove Leibnitz Theorem on successive differentiation.

12. (i) If $u = \log\left(\dfrac{x^2 + y^2}{x + y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$.

(ii) Find the equation of the tangent to the curve $\left(\dfrac{x}{a}\right)^n + \left(\dfrac{y}{b}\right)^n = 1$ at the point (a, b).

68
35
103
7

13. Find all the solutions, in integers, of the diophantine equation $56x + 72y = 40$.

72
28
44

❖

49
−21
28

17
×4
68