2018

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **three** from each Group

Group—A

1. *(a)* Integrate *any one* of the following :

(i) $$\int \frac{1}{1+3e^{x}+2e^{2x}}\,dx$$

(ii) $$\int \sqrt{\frac{x}{a-x}}\,dx$$

(b) Evaluate :

$$\int_0^{\pi/4} \sqrt{\tan\theta}\,d\theta$$

2. *(a)* From the first principle, evaluate

$$\int_a^b \cos x\,dx$$

(b) Evaluate

$$\int_0^{\pi/2} \cos^n x \cos nx\,dx$$

where n is a positive integer.

3. *(a)* Find the whole area of the curve $r = 3 + 2\cos\theta$.

(b) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x-axis. Find the volume of the solid so generated.

4. *(a)* Define Beta and Gamma functions. Find the value of $\Gamma\left(\frac{1}{2}\right)$.

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(b) Find the moment of inertia of a solid right circular cone of height h and the semi-vertical angle α about its axis.

5. Solve *any two* of the following differential equations :

pn-12,6 *(i)* $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

p.n-24,3i *(ii)* $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$

p.n -45,5(iv) *(iii)* $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$

6. *(a)* Solve :

(i) $(y - px)(p-1) = p$

(ii) $(D^2 - 5D + 6)\,y = e^x$

(b) Find the orthogonal trajectories of $r = a(1+\cos\theta)$.

7. *(a)* Find the equation of the plane through the line of intersection of the planes $2x + 3y + 10z = 8$, $2x - 3y + 7z = 2$ and perpendicular to the plane $3x - 2y + 4z = 5$.

(b) Find the equation of the sphere joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) as its diameter.

8. *(a)* Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 0$.

Group—B

9. *(a)* Prove that a closed sphere is a convex set.

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(b) Solve the following LPP graphically :

Maximize $Z = 2x_1 + 5x_2$

subject to the constraints

$$x_1 + 4x_2 \leq 24$$
$$3x_1 + x_2 \leq 21$$
$$x_1 + x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

10. Solve the following LPP by the simplex method :

Maximize $Z = 7x_1 + 5x_2$

subject to the constraints

$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

11. *(a)* Prove that the resultant of two simple harmonic motions of the same period and in the same straight line is another simple harmonic motion of the same period.

(b) Find the work done in extending a light elastic string of natural length l to double its length.

12. *(a)* Prove that the modulus of elasticity of a light elastic string is equal to the force which would stretch the string to twice its natural length.

(b) The coordinates of a moving point at time t are given by

$$x = a\,(2t + \sin 2t), \quad y = a\,(1 - \cos 2t)$$

Prove that its resultant acceleration is constant and find its direction.

13. *(a)* Find the tangential and normal accelerations of a particle moving in a plane curve.

(b) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with a constant angular velocity. Find the intrinsic equation of the path.

14. *(a)* Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.

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(b) Forces P, Q, R act along the lines $x = 0, y = 0$ and $x \cos\alpha + y \sin\alpha = p$. Find the magnitude of the resultant and the equation of its line of action.

15. *(a)* Prove that if three forces keep a rigid body in equilibrium, then they must be coplanar.

(b) Forces P, Q, R, S acting along the sides AB, BC, CD, DA of a quadrilateral $ABCD$ are in equilibrium. Prove that

$$\frac{P}{AB} \cdot \frac{R}{CD} = \frac{Q}{BC} \cdot \frac{S}{DA}$$

16. *(a)* State and prove the converse of the principle of virtual work for a system of coplanar forces acting on a rigid body.

22 *(b)* Two equal uniform rods AB and AC, each of the length $2b$, are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b \sin^3 \theta = a \cos \theta$.

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