$4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'$

2013

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **one** from each Group

Group—A

1. *(a)* Integrate *any two* of the following :

(i) $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx$

(ii) $\displaystyle\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}\,dx$

(iii) $\displaystyle\int \frac{1}{\sin x + \sin 2x}\,dx$

(b) Evaluate from first principle

$$\int_1^4 e^{5x}\,dx.$$

2. *(a)* Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}\,dx$$

(b) If

$$I_n = \int_0^{\pi/2} \sin^n x \, dx$$

then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

96

3. (a) Find the length of the arc of parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

(b) Find the area of loop of the curve $y^2 = x(x-1)^2$.

4. (a) Define Γn and prove that $\Gamma(n+1) = n\,\Gamma n$.

(b) Find the volume of a sphere of radius a using integration method.

5. Solve *any two* of the following differential equations :

(i) $(x+y)^2 \dfrac{dy}{dx} = a^2$

(ii) $(x^3 + 3xy^2)\, dx + (y^3 + 3x^2 y)\, dy = 0$

(iii) $(x \cos x) \dfrac{dy}{dx} + y(x \sin x + \cos x) = 1$

6. (a) Find the orthogonal trajectory of the family of circles $x^2 + y^2 = a^2$, a being the parameter of the family.

(b) Solve *any one* of the following differential equations :

(i) $p^2 + 2py \cot x = y^2$ $\quad 2\cot\theta = \cot\theta/2$

(ii) $x = y + p^2$

$$\left(p = \frac{dy}{dx} \right)$$

7. Solve *any two* of the following differential equations :

(i) $(D^2 - 2D + 1)\, y = \cos 3x$

(ii) $(D^2 - 5D + 6)\, y = e^{3x}$

(iii) $(D^3 + 8)\, y = x^4 + 2x + 1$

$$D \equiv \frac{d}{dx}$$

Group—B

8. (a) Derive the equation of a plane in the intercept form.

(b) Find the equation of the plane containing the lines

$$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4} \text{ and } \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$$

9. (a) Find the radius of the circular section of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ by the plane $x + 2y + 2z = 15$.

(b) Find the length and equation of the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

10. (a) Define convex set. Prove that intersection of two convex sets is a convex set.

(b) Solve the following linear programming problem by the graphical method :

$$\text{Maximize } Z = 4x_1 + 10x_2$$

subject to the constraints

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

11. Solve the following linear programming problem by the simplex method :

Maximize $Z = 3x_1 + 2x_2$

subject to the constraints

$$x_1 + x_2 \le 6$$
$$2x_1 + x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Group—C

12. (a) A particle moves in a straight line under the action of a force which varies as its distance from a fixed point O in the line and always away from O. Discuss the motion.

(b) A particle starts with a given velocity V and moves under a retardation equal to k times the space described. Show that the distance traversed before it comes to rest is $\dfrac{V}{\sqrt{k}}$.

13. (a) Find the work done in stretching an elastic string from a length l_1 to l_2.

(b) A uniform elastic string has a length a_1 when its tension is T_1 and length a_2 when the tension is T_2. Find the natural length of the string.

14. (a) Find tangential and normal components of acceleration for a particle moving on a plane curve.

(b) A particle moving on a plane curve has radial and transverse velocities $2\lambda a\theta$ and λr respectively. Prove that its accelerations in these directions are $\lambda^2(2a-r)$ and $4\lambda^2 a\theta$. Also show that equation of the path is $r = a\theta^2 + c$.

15. (a) For a system of coplanar forces, find the equation of the line of action of the resultant.

(b) Three forces P, Q, R act along the lines $x=0$, $y=0$ and $x\cos\alpha + y$ [illegible] $= p$. Find the equation of the lin[illegible] [illegible]action of the resultant.

(a) State and prove the principle of virtual [illegible]

(b) Two equal uniform rods AB and AC each of length $2b$, are freely jointed at A and rest on a vertical circle of radius a. Using the principle of virtual work, prove that if 2θ is the angle between the rods then, $b \sin^3 \theta = a \cos\theta$.

★★★