Voc—M (Sub) II Part-II

2011

MATHEMATICS

SECOND PAPER

Full Marks : 100

Pass Marks : 35

Time : 3 hours

The questions are of equal value

Answer eight questions selecting at least one from each group

Candidates are required to give their answers in their own words as far as practicable

GROUP—A

1 (a) Integrate

(i) $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

(ii) $\int \frac{dx}{4 + 5 \sin x}$

(b) Prove that

$$\int_0^{\pi/2} \sin^6 \theta d\theta = \frac{5\pi}{32}$$

2. (a) Find the value of $\int_a^b e^{kx}\,dx$ from first principle.

(b) Find the area of the curve $r = a(1 + \cos\theta)$.

3. (a) Find the length of loop of the curve $3ay^2 = x(x-a)^2$.

(b) Find the moment of inertia of a solid sphere about a diameter.

4. (a) Find the volume of the solid of revolution of $y^2(2a - x) = x^3$ about its asymptote.

(b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the help of double integration.

5. Solve any two of the following differential equations

(i) $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{x^2 + y^2} = 0$

(ii) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$.

(iii) $\cos^2 x \dfrac{dy}{dx} + y = \tan x$

6. Solve any *two* of the following differential equations :

(i) $y = 2px + p^2$

(ii) $(y - px)(p - 1) = p$

(iii) $(D^2 - 2D + 1)y = xe^x \sin x$

7. (a) Find the orthogonal trajectories of $x^2 + y^2 = 2ax$.

(b) Solve $(D^2 + D + 1)y = \sin 2x$

GROUP—B

8. (a) Prove that the set of all feasible solutions of a linear programming problem is a convex set.

(b) Solve graphically the following liner programming problem :

Maximize $Z = 2x + 5y$

subject to the constraints

$$x + 4y \le 24$$
$$3x + y \le 21$$
$$x + y \le 9$$
$$x, y \ge 0$$

9. Solve the following linear programming problem by simplex method :

$$\text{Maximize } Z = 6x + 11y$$

subject to the constraints

$$2x + y \leq 104$$

$$x + 2y \leq 76$$

$$x, y \geq 0$$

10. (a) Find the equation of a plane in normal form.

(b) Find the equation of the sphere which passes through the point (α, β, γ) and the circle $z = 0,\ x^2 + y^2 = a^2$.

11. (a) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

(b) Find the equation of a right circular cylinder of radius 3 whose axis passes through the point $(2, 3, 4)$ and has direction cosines proportional to $(2, 1, -2)$.

GROUP—C

12. (a) Obtain the resultant of two SHMs of the same period and in the same straight line.

(b) A particle, whose mass is m, is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

13. (a) State and prove the principle of conservation of energy for the motion in a straight line.

(b) Explain Hooke's law. Show that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions.

14. (a) Find the tangential and normal components of acceleration of a particle moving in a plane.

(b) A point moves in a plane curve so that its tangential and normal accelerations are equal and the tangent rotates with constant angular velocity. Find the path.

15. (a) If a rigid body be in equilibrium under the action of three forces, the forces must be coplanar and they will be either concurrent or parallel. Prove this.

(b) Forces P, Q and R act along the lines $x = 0$, $y = 0$ and $x\cos\alpha + y\sin\alpha = p$. Find the magnitude and the equation of its line of action, axes being rectangular.

16. (a) State and prove the principle of virtual work for a system of coplanar forces acting on a rigid body.

(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be the tensions in these rods, prove that $\frac{T}{l} + \frac{T'}{l'} = 0$.

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