2019

Time : 3 hours

Full Marks : 100

Pass Marks : 35

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer **eight** questions selecting at least **three** from each Groups*

Group – A

1. (a) Integrate any **one** of the following :

(i) $\int \frac{dx}{\sin x(3+2\cos x)}$

(ii) $\int \sqrt{\frac{a+x}{a-x}}\, dx$

(b) Evaluate $\int_0^{\infty} \log\left(x+\frac{1}{x}\right) \frac{dx}{\left(1+x^2\right)}$.

2. (a) Evaluate from the first principle $\int_a^b e^{mx}\, dx$.

(b) Find the reduction formula for $\int \cos mx \sin nx\, dx$, where m and n are positive integers.

3. (a) Find the area of the loop of the folium of Descartes $x^3+y^3=3axy$.

(b) The cardiode $r=a(1+\cos\theta)$ revolves about the initial line. Find the volume of the figure formed.

4. (a) Prove that $\lceil n \left\lceil \left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}} \lceil 2n\right.$, where n is an integer.

(b) Find the moment of inertia of a thin uniform rectangular lamina about an axis of symmetry through its centre.

5. Solve any two of the following differential equations :

(a) $(x^2+y^2)\frac{dy}{dx}=2xy$

(b) $\frac{dy}{dx}+1=e^{x-y}$

(c) $(x^3+xy^4)\,dx+2y^3dy=0$

6. (a) Solve :

(i) $y=2px+y^2\beta^3$

(ii) $(D^2+a^2)y=\sin ax$, where $D\equiv\frac{d}{dx}$

(b) Prove that the system of confocal conics

$$\frac{x^2}{(a^2+\lambda)}+\frac{y^2}{(b^2+\lambda)}=1$$ is self orthogonal.

7. (a) A plane meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (a, b, c). Show that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$.

(b) Find the condition that the plane $lx+my+nz=p$ is a tangent to the sphere $x^2+y^2+z^2=a^2$.

8. (a) Find the equation of the cone whose vertex

is the point (α, β, γ) and the guiding curve is the conic.

(b) Find the equation to the right circular cyclinder of radius 2 and axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.

Group – B

9. (a) Define hyper plane and show that a hyper plane is a convex set.

(b) Solve the following linear programming problem by graphical method :

$$\text{Max } Z = 5x_1 + 7x_2$$

Subject to the following constraints

$$x_1 + x_2 \leq 4$$
$$3x_1 + 8x_2 \leq 24$$
$$10x_1 + 7x_2 \leq 35$$
$$x_1 + x_2 \geq 0$$

10. Solve the follwoing LPP by the simplex method :

$$\text{Maximise } Z = 4x + 10y$$

Subject to the constraints

$$3x + 10y \leq 180$$
$$5x + 10y \leq 240$$

10y - 50 3x+10y=180 5x+10y=200

$x + 5y \leq 120$

$x \geq 0, y \geq 0$

11. (a) Discuss the motion of a particle if it starts from rest and moves along a straight line with an acceleration which is always directed towards a fixed point and varies as the distance form the fixed point.

(b) State Hooke's law and prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tensions.

12. (a) If a light elastic string of natural length 'a' and modulus of elasticity is suspended by one end, to the other end is tied a particle of weight 'mg', the particle is slightly pulled down and released, discuss the motion.

(b) A parcticle starts with a given velocity V and moves under a retardation equal to K times the space descirbed. Show that the distance traversed before it comes to rest is $\frac{V}{\sqrt{k}}$.

13. (a) Find the expressions for velocity and acceleration of a particle in plane curvilinear motion in terms of the intrinsic coordinates.

(b) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$ respectively. Find the polar equation of the path of the particle and also the component acceleration in terms of r and θ.

14. (a) Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitray point in the plane of the forces together with a couple.

(b) Three forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation to the line of action of the resultant.

15. (a) A particle rests on a smooth curve under the action of any force. Find the position of equilibrium.

(b) Equal weights Pand P are attached to two strings ACP and BCP passing over a smooth peg C. AB is a heavy beam of weight W, whose centre of gravity is 'a' meters from A and b meters from B. Show that AB is inclined to the horizontal at an angle

$$\tan^{-1}\left[\frac{(a-b)}{(a+b)}\tan\left(\sin^{-1}\frac{w}{2p}\right)\right].$$

16. (a) State and prove the principle of virtual work for any system of forces in one plane.

(b) The middle points of he opposite sides of a joined quadrilateral are connected by light rods of length ℓ and ℓ'. If T and T′ be the tensions in these rods prove that $\frac{T}{\ell}+\frac{T'}{\ell'}=0$