MATHEMATICS

Full Marks : 100

Pass Marks : 35

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **one** from each Group

Candidates are required to give their answers in their own words as far as practicable

GROUP—A

1. (a) Evaluate any *one* of the following :

(i) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

(ii) $\int \frac{1+x^2}{1+x^4}\,dx$

2607 (b) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$

2. (a) If $I_n = \int_0^{\pi/4} \tan^n x\,dx$, prove that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

(b) Find the limit of

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{n}{n^2+n^2}$$

as $n \to \infty$

3. (a) Find the area of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

(b) Find the perimeter of the curve $x^{2/3}+y^{2/3}=a^{2/3}$.

4. (a) Find the volume of a sphere of radius a.

(b) Find the surface area of a cone whose semi-vertical angle is α and base a circle of radius a.

5. Solve any *two* of the following differential equations :

(i) $3e^x \tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

(ii) $(x+y)(dx-dy)=dx+dy$

(iii) $\sin x\dfrac{dy}{dx}+3y=\cos x$

6. (a) Solve any *one* of the following differential equations :

(i) $xp^2=(x-a)^2$

(ii) $y=-px+x^4p^2$, where $p=\dfrac{dy}{dx}$

(b) Find the orthogonal trajectories of family of curves $y = cx^2$, c being a parameter.

7. Solve any *two* of the following differential equations :

(i) $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$

$m^3 - 4m^2 + 5m - 2 = 0$

(ii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$

$m^2 - 3m + 2 = e^n$

$m^2 - 2m - m + 2$

$m(m-2) - 1(m-2)$

$(m-1)(m-2)$

(iii) $\frac{d^2y}{dx^2} + 4y = \sin^2 x$

GROUP—B

8. *(a)* Define a convex set. Prove that intersection of two convex sets is a convex set.

(b) Using graphical method, solve the following linear programming problem :

Maximize $Z = 3x_1 + 2x_2$

subject to the following constraints

$$-2x_1 + x_2 \le 1$$
$$x_1 \le 2$$
$$x_1 + x_2 \le 3$$
$$x_1,\ x_2 \ge 0$$

9. Solve the following linear programming problem by the simplex method :

$$\text{Maximize } Z = 50x_1 + 60x_2$$

subject to the following constraints

$$2x_1 + x_2 \leq 1500$$
$$3x_1 + 2x_2 \leq 1500$$
$$0 \leq x_1 \leq 400$$
$$0 \leq x_2 \leq 400$$

10. (a) Find the angle between two diagonals of a cube.

(b) Find the equation of the plane which passes through the points (2, 2, − 1), (3, 4, 2) and (7, 0, 6).

11. (a) Find the length and equation of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

(b) Prove that the equation

$$ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0$$

represents a sphere. Find its centre and radius.

GROUP—C

(a) A particle moves in a straight line under the action of a force which varies as its distance from a fixed point O in the line and always directed away from O. Discuss the motion.

(b) A point in a simple harmonic motion has velocities v_1 and v_2, and accelerations a_1 and a_2 at any two positions of motion. Show that the distance between the two positions is $\dfrac{v_2^2 - v_1^2}{a_1 + a_2}$ and the periodic time is $2\pi\sqrt{\dfrac{v_2^2 - v_1^2}{a_1^2 - a_2^2}}$.

(a) Find the expressions for tangential and normal accelerations of a particle moving in a plane.

(b) The coordinates of a moving point at time t are given by $x = a(2t + \sin 2t)$, $y = a(1 - \cos 2t)$. Prove that its acceleration is constant and also find the direction of motion.

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14. (a) Show that, for small oscillations, the motion of a simple pendulum is simple harmonic. Also, find the time period.

(b) A light string is suspended by one end and a mass m is attached to the other end. If the unstretched length of the string be a and the modulus of elasticity be n, show that the time of a small vertical oscillation is $2\pi\sqrt{\dfrac{ma}{ng}}$.

15. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.

(b) Forces P, $2P$, $3P$, $4P$ act along the sides of a square taken in order. Find the magnitude and direction of the resultant. Also, find the equation of the line of action of the resultant.

16. (a) State and prove the principle of virtual work for any system of coplanar forces.

(b) A regular hexagon $ABCDEF$ is composed of six equal heavy uniform rods freely jointed together and the two opposite points C and F are connected by a horizontal string. The side AB is in contact with a horizontal plane. If W be the weight of each rod, show that the tension of the string is $W\sqrt{3}$.

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