2018

Full Marks: 100

Time: 3 hours

. The questions are of equal value

Answer eight questions, selecting at least three from each Group

Group-A

1. (a) Integrate any one of the following:

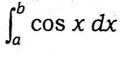
$$(i) \qquad \int \frac{1}{1+3e^x+2e^{2x}} \, dx$$

(ii)
$$\int \sqrt{\frac{x}{a-x}} \, dx$$

(b) Evaluate:

$$\int_0^{\pi/4} \sqrt{\tan \theta} \ d\theta$$

2. (a) From the first principle, evaluate





(b) Evaluate

$$\int_0^{\pi/2} \cos^n x \cos nx \, dx \, \mathcal{I}^{\mathcal{A}}$$

where n is a positive integer.

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(Turn Over)

- 3. (a) Find the whole area of the curve $r = 3 + 2 \cos \theta$.
 - (b) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x-axis. Find the volume of the solid so generated.
- 4. (a) Define Beta and Gamma functions. Find the value of $\Gamma(\frac{1}{2})$.
 - (b) Find the moment of inertia of a solid right circular cone of height h and the semi-vertical angle α about its axis.
 - 5. Solve any two of the following differential equations:

pn-12,6 (i)
$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

p.n-24,3i (ii)
$$x dy - y dx - \sqrt{x^2 + y^2} dx = 0$$

p.n -45,5(iv)(iii) $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$

- **6.** (a) Solve:
 - (i) (y-px)(p-1)=p
 - (ii) $(D^2 5D + 6) y = e^x$

- (b) Find the orthogonal trajectories of $r = a(1 + \cos \theta)$.
- 7. (a) Find the equation of the plane through the line of intersection of the planes 2x + 3y + 10z = 8, 2x 3y + 7z = 2 and perpendicular to the plane 3x 2y + 4z = 5.
 - (b) Find the equation of the sphere joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) as its diameter.
- 8. (a) Prove that the plane ax + by + cz = 0cuts the cone yz + zx + xy = 0 in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.
 - (b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, z = 0.

Group—B

(a) Prove that a closed sphere is a convex set.

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(b) Solve the following LPP graphically:

$$Maximize Z = 2x_1 + 5x_2$$

subject to the constraints

$$x_1 + 4x_2 \le 24$$

$$3x_1 + x_2 \le 21$$

$$x_1 + x_2 \le 9$$

$$x_1, x_2 \ge 0$$

10. Solve the following LPP by the simplex method:

Maximize $Z = 7x_1 + 5x_2$

subject to the constraints

$$x_1 + 2x_2 \le 6$$

$$4x_1 + 3x_2 \le 12$$

$$x_1, x_2 \ge 0$$

- 11. (a) Prove that the resultant of two simple harmonic motions of the same period and in the same straight line is another simple harmonic motion of the same period.
 - (b) Find the work done in extending a light elastic string of natural length *l* to double its length.

(Continued)

- 12. (a) Prove that the modulus of elasticity of a light elastic string is equal to the force which would stretch the string to twice its natural length.
 - (b) The coordinates of a moving point at time t are given by

$$x = a (2t + \sin 2t), y = a (1 - \cos 2t)$$

Prove that its resultant acceleration is constant and find its direction.

- 13. (a) Find the tangential and normal accelerations of a particle moving in a plane curve.
 - (b) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with a constant angular velocity. Find the intrinsic equation of the path.
- 14. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting upon a rigid body.
 - (b) Forces P, Q, R act along the lines x = 0, y = 0 and $x \cos \alpha + y \sin \alpha = p$. Find the magnitude of the resultant and the equation of its line of action.

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(Turn Over)

- 15. (a) Prove that if three forces keep a rigid body in equilibrium, then they must be coplanar.
 - (b) Forces P, Q, R, S acting along the sides AB, BC, CD, DA of a quadrilateral ABCD are in equilibrium. Prove that

$$\frac{P}{AB} \cdot \frac{R}{CD} = \frac{Q}{BC} \cdot \frac{S}{DA}$$

16. (a) State and prove the converse of the principle of virtual work for a system of coplanar forces acting on a rigid body.

(b) Two equal uniform rods AB and AC, each of the length 2b, are freely jointed at A and rest on a smooth material. at A and rest on a smooth vertical circle of radius a. Show that if 20 be the angle between them, then $b \sin^3 \theta = a \cos \theta$.