## Nenoteiktais INTEGRĀLIS

| | **Integrēšanas pamatformulas** | Funkcijas ienešana **zem diferenciāļa zīmes** |
|---|---|---|
| **1.** | $\int x^n dx = \frac{x^{n+1}}{n+1}+C$, <br> $n = const,\ n \neq -1.$ <br> => $\int dx = x + C$ <br> => $\int \frac{1}{x^n}dx = \int x^{-n}dx$ | $x^n \cdot dx = d\left(\frac{x^{n+1}}{n+1}\right)$ <br> $dx = d(x)$ |
| **2.** | **n = - 1**: <br> $\int \frac{dx}{x} = \ln\lvert x\rvert + C$ | $\frac{1}{x}\cdot dx = d(\ln x)$ |
| **3.** | $\int e^x dx = e^x + C$ <br> $\int a^x dx = \frac{a^x}{\ln a} + C$, <br> $a = const,\ a \neq 1,\ a > 0.$ | $e^x \cdot dx = d(e^x)$ <br> $a^x \cdot dx = d\left(\frac{a^x}{\ln a}\right)$ |
| **4.** | $\int \cos x dx = \sin x + C$ <br> $\int \sin x dx = -\cos x + C$ | $\cos x \cdot dx = d(\sin x)$ <br> $\sin x \cdot dx = -d(\cos x)$ |
| **5.** | $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$ <br> $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$ | $\frac{1}{\cos^2 x}\cdot dx = d(\operatorname{tg} x)$ <br> $\frac{1}{\sin^2 x}\cdot dx = -d(\operatorname{ctg} x)$ |
| **6.** | $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$ <br> $\int \frac{dx}{\sqrt{1-x^2}} = -\arccos x + C$ | $\frac{1}{\sqrt{1-x^2}}\cdot dx = d(\arcsin x)$ <br> $\frac{1}{\sqrt{1-x^2}}\cdot dx = -d(\arccos x)$ |
| **7.** | $\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$ <br> $\int \frac{dx}{1+x^2} = -\operatorname{arcctg} x + C$ | $\frac{1}{1+x^2}\cdot dx = d(\operatorname{arctg} x)$ <br> $\frac{1}{1+x^2}\cdot dx = -d(\operatorname{arcctg} x)$ |
| **8.** | $\int \operatorname{ch} x dx = \operatorname{sh} x + C$ <br> $\int \operatorname{sh} x dx = \operatorname{ch} x + C$ | $\operatorname{ch} x \cdot dx = d(\operatorname{sh} x)$ <br> $\operatorname{sh} x \cdot dx = d(\operatorname{ch} x)$ |
| **9.** | $\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C$ <br> $\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$ | $\frac{1}{\operatorname{ch}^2 x}\cdot dx = d(\operatorname{th} x)$ <br> $\frac{1}{\operatorname{sh}^2 x}\cdot dx = -d(\operatorname{cth} x)$ |

### Integrēšanas likumi

**Summa un starpība:**

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

**Reizinājums:**

$$\int a f(x)dx = a\int f(x)dx, \quad a = const$$

$\int f(x)\cdot g(x)dx$ = **Integrēšanas metode !**

**Diferenciāļa pārveidošana:**

$\boxed{dx = d(x \pm \mathbf{a})}$ un $\boxed{dx = \frac{1}{\mathbf{a}}d(\mathbf{a}x)}$.

**Funkcijas ienešana zem diferenciāļa zīmes:**

$\boxed{(f(x))' dx = d(f(x))}$.

**10.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

**11.** $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

**12.** $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

**13.** $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

**14.** $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\left(\frac{x}{a}\right) + C$

**15.** $\int \frac{dx}{\sin ax} = \frac{1}{a}\ln\left|\operatorname{tg}\frac{ax}{2}\right| + C$

**16.** $\int \frac{dx}{\cos ax} = \frac{1}{a}\ln\left|\operatorname{tg}\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right| + C$

**17.** $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

**18.** $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2+a^2}\right| + C$

## Funkcijas ATVASINĀJUMS

| | **Elementāra funkcija** <br> $y = f(x)$ | **Salikta funkcija** <br> $y = f(u)$, **kur** $u = g(x)$ |
|---|---|---|
| **1.** | $(a)' = 0,\ a = const$ | |
| **2.** | $(x^a)' = a\cdot x^{a-1}$, $a = const$ <br> $(x)' = (x^1)' = 1$ <br> $(\sqrt[n]{x})' = (x^{1/n})'$ | $(u^a)' = a\cdot u^{a-1}\cdot u'$, $a = const$ <br> $(u)' = u'$ <br> $(\sqrt[n]{u})' = (u^{1/n})'$ |
| **3.** | $(a^x)' = a^x \ln a$, $a = const$ <br> $(e^x)' = e^x$ | $(a^u)' = a^u \ln a \cdot u'$, $a = const$ <br> $(e^u)' = e^u \cdot u'$ |
| **4.** | $(\ln x)' = \frac{1}{x}$ <br> $(\log_a x)' = \frac{1}{x\ln a}$ | $(\ln u)' = \frac{1}{u}\cdot u'$ <br> $(\log_a u)' = \frac{1}{u\ln a}\cdot u'$ |
| **5.** | $(\sin x)' = \cos x$ <br> $(\cos x)' = -\sin x$ | $(\sin u)' = \cos u \cdot u'$ <br> $(\cos u)' = -\sin u \cdot u'$ |
| **6.** | $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$ <br> $(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$ | $(\operatorname{tg} u)' = \frac{1}{\cos^2 u}\cdot u'$ <br> $(\operatorname{ctg} u)' = -\frac{1}{\sin^2 u}\cdot u'$ |
| **7.** | $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$ <br> $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$ | $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}\cdot u'$ <br> $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}}\cdot u'$ |
| **8.** | $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$ <br> $(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$ | $(\operatorname{arctg} u)' = \frac{1}{1+u^2}\cdot u'$ <br> $(\operatorname{arcctg} u)' = -\frac{1}{1+u^2}\cdot u'$ |
| **9.** | $(\operatorname{sh} x)' = \operatorname{ch} x$ <br> $(\operatorname{ch} x)' = \operatorname{sh} x$ | $(\operatorname{sh} u)' = \operatorname{ch} u \cdot u'$ <br> $(\operatorname{ch} u)' = \operatorname{sh} u \cdot u'$ |
| **10.** | $(\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$ <br> $(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$ | $(\operatorname{th} u)' = \frac{1}{\operatorname{ch}^2 u}\cdot u'$ <br> $(\operatorname{cth} u)' = -\frac{1}{\operatorname{sh}^2 u}\cdot u'$ |

### Atvasināšanas likumi

**Summa un starpība:**

$(u + v - w)' = u' + v' - w'$

**Reizinājums:**

$(u\cdot v)' = u'v + uv'$

$(au)' = a\cdot u'$, $a = const$

$(u\cdot v\cdot w)' = u'vw + uv'w + uvw'$

**Dalījums:**

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, $v \neq 0$

$\left(\frac{u}{a}\right)' = \frac{1}{a}u'$, $a = const$

$\left(\frac{a}{u}\right)' = a(u^{-1})'$

**PARAMETRISKI dotas funkcijas** $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ **atvasinājums:**

$\boxed{y'_x = \frac{y'_t}{x'_t}}$ **un** $\boxed{y''_{xx} = \frac{1}{x'_t}\cdot (y'_x)'_t}$.

**APSLĒPTAS funkcijas** $F(x, y) = 0$ **atvasinājums:**

$\boxed{y' = -\frac{F'_x}{F'_y}}$.

**LOGARITMISKĀ atvasināšana**

$y = f(x)^{g(x)}$: **1)** $\ln y = \ln f(x)^{g(x)}$

**2)** $\frac{1}{y}y' = (g(x)\cdot \ln f(x))'$

**LOPITĀLA kārtula:**

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \left(\left(\frac{\infty}{\infty}\right) vai \left(\frac{0}{0}\right)\right) = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$