Domain Transform for Edge-Aware Image and Video Processing

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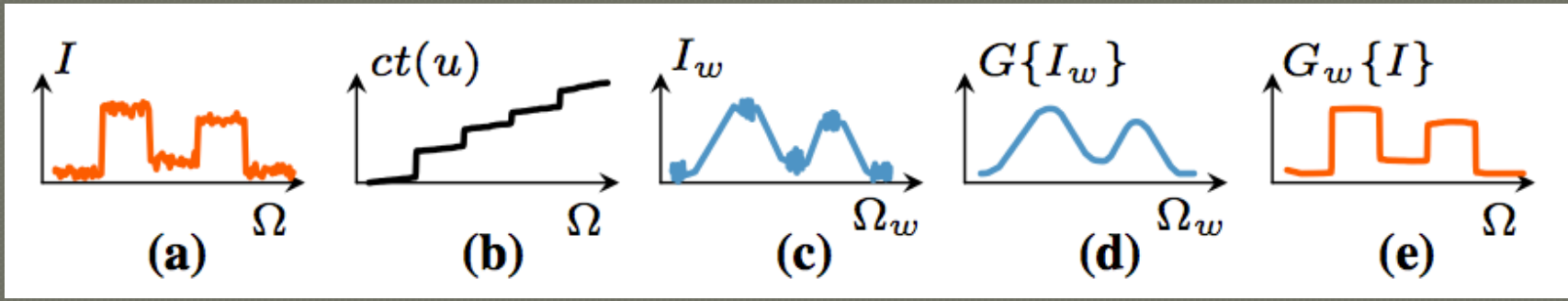
Overview

- A faster method of performing edge-preserving filtering on images
- Main idea: domain transform from image in R^5 (x,y,r,g,b) to lower dimension where distances are preserved ??
- Then, perform filtering on the transformed image

Overview (continued)

- How the transformation will look:



Images in R^5

- 2D RGB image I: $I : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$
- I defines a 2D manifold M_I in R^5
- Let $\hat{p} = (x_p, y_p, r_p, g_p, b_p) \in M_I$
- $\hat{p}$ has a corresponding pixel in I with:
  - Spatial coordinates $p = (x_p, y_p)$, and
  - Range coordinates $I(p) = (r_p, g_p, b_p)$
- Let $F(\hat{p}, \hat{q})$ be an edge-preserving filter in 5D
- Filtered image J is:

$$J(p) = \int_{\Omega} I(q)\, F(\hat{p}, \hat{q})\, dq$$

1 RGB图像是一个三维mesh；2 RGB可看作5维流形；3 空间坐标和RGB范围；4 用F(p,q)作为5D空间中的保边滤波器；5 卷积的标准操作

Example - Bilateral Filter

- J, the image obtained when filtering I with F can be expressed as:

$$J(p) = \int_{\Omega} I(q)\, F(\hat{p}, \hat{q})\, dq$$

- Bilateral filter kernel is given by:

$$F(\hat{p}, \hat{q}) = G_{\sigma_s}(\|p - q\|)\, G_{\sigma_r}(\|I(p) - I(q)\|)$$

($p = (x_p, y_p)$ and $I(p) = (r_p, g_p, b_p)$)

双边滤波系数=空间的高斯系数x像素高斯系数

Problem Statement

- Does there exist:
  - Transformation t $t : \mathbb{R}^5 \rightarrow \mathbb{R}^l,\ l < 5$, and
  - Filter kernel H defined over $\mathbb{R}^l$, such that
  - For any input image I, an equivalent result to the 5D edge-preserving kernel F is produced, i.e.:

$$J(p) = \int_\Omega I(q)\, F(\hat{p}, \hat{q})\, dq = \int_\Omega I(q)\, H\left(t(\hat{p}),\, t(\hat{q})\right)\, dq$$

相当于将5维将至低维，再用滤波器H，例如把RGB变成灰度图，那么就变成[x,y,g]三维了

Simplification: R^5 -> R^2

- Instead of starting with $t : \mathbb{R}^5 \to \mathbb{R}^l, l < 5$ let's try to find $t : \mathbb{R}^2 \to \mathbb{R}$, where
  - t preserves (in R) the original distances between points $(x_i, I(x_i))$ given by some distance metric (e.g. Euclidean):

$$|t(x_i, I(x_i)) - t(x_j, I(x_j))| = \|(x_i, I(x_i)) - (x_j, I(x_j))\|$$

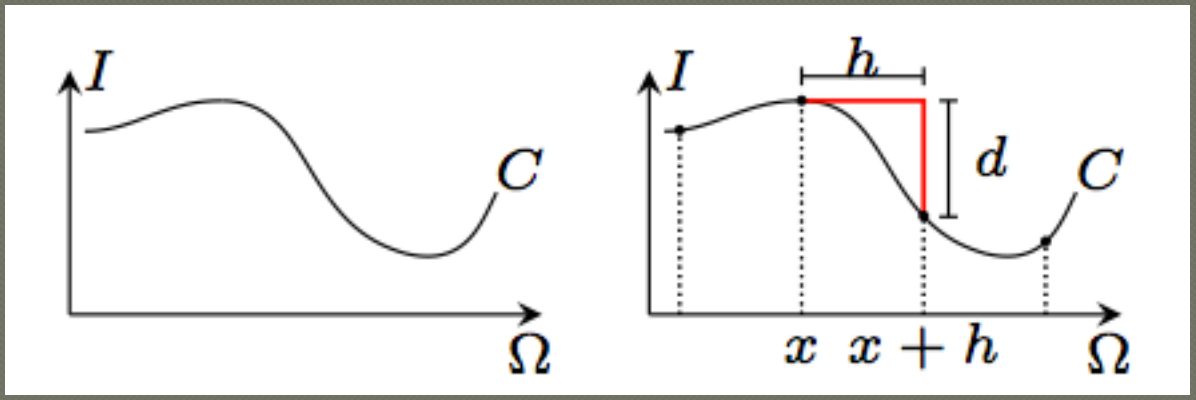
采用保距映射把5维降至2维，相当于(x,y)降至1维r，(r,g,b)降至1维灰度图g，此处是假设的函数一元函数I(x)

Finding t

- Let $ct(x) = t(\hat{x}) = t(x, I(x))$
- Transform must satisfy

$$ct(x+h) - ct(x) = h + |I(x+h) - I(x)|$$

in order to preserve L1 distances between neighboring pixels x and x + h (with sampling width h)



中间式子表示r(x+h)-r(x)=h+ | I(x+h)-I(x) | 才能保持L1距离

Finding t (continued)

$$ct(x+h) - ct(x) = h + |I(x+h) - I(x)|$$

- Dividing by h and taking the limit as h approaches 0 gives:

$$ct'(x) = 1 + |I'(x)|$$

- Integrating both sides gives:

$$ct(u) = \int_0^u 1 + |I'(x)| \; dx, \quad u \in \Omega$$

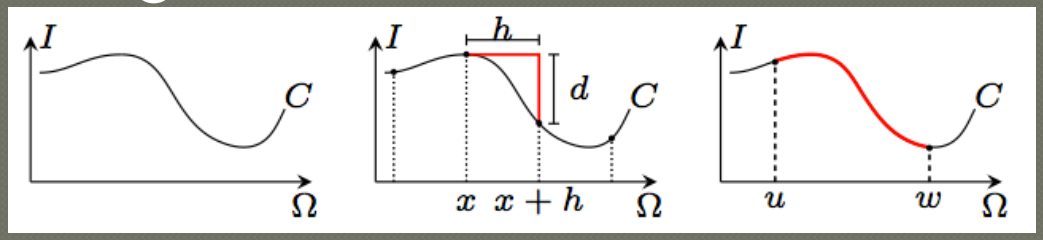
Finding t (continued)

$$ct(u) = \int_0^u 1 + \left|I'(x)\right| \; dx, \quad u \in \Omega$$

- Then, distance between two points in new domain is given by:

$$ct(w) - ct(u) = \int_u^w 1 + \left|I'(x)\right| \; dx$$

the L1 arc length of curve C in the interval [u,w]



灰度图可以将至二维，RGB图则可以将至4维，则t的形式变成对三个通道的绝对梯度求和

Generalizing t - Multichannel

- Want to do this with RGB images, not just grayscale; so we want t : R^4 -> R
- Change from this:

$$ct(u) = \int_0^u 1 + \left|I'(x)\right| \, dx, \quad u \in \Omega$$

- To this:

$$ct(u) = \int_0^u 1 + \sum_{k=1}^{c} \left|I'_k(x)\right| \, dx$$

(sum over all c = 3 color channels)

Generalizing t – Multidimensional

- Want to do this with 2D images, not just 1D signals
- Unfortunately, not possible in general
- So, instead we apply our current 1-dimensional t to rows/columns of image
  - First, along each row
  - Then, along each column
  - Iterate N times
  - Good N depends on the geometry of the image

在2维上没有通用形式，因此先在每一行做，在对每一列做，迭代N次，N取决于图像的几何结构

Generalizing t – Multidimensional (continued)

More iterations preserves edges somewhat better:

(a) *Input* **(b)** *1 itr.* **(c)** *3 itr.*

需要迭代就很麻烦了。。。

Generalizing t – Spatial vs. Range Parameters

- Bilateral kernel has spatial vs. range parameters σ_s and σ_r:

$$F(\hat{p},\hat{q}) = G_{\sigma_s}(\|p-q\|)\, G_{\sigma_r}(\|I(p)-I(q)\|)$$

- We can encode these into ct by adding factor σ_s / σ_r:

$$ct(u) = \int_0^u 1 + \frac{\sigma_s}{\sigma_r} \sum_{k=1}^{c} \left|I_k'(x)\right| \, dx$$

双边滤波核有空间高斯系数和像素空间系数，然后加入因子变成t的一般形式

Filtering in the Transformed Domain

- Wanted t and H such that:

$$J(p) = \int_{\Omega} I(q)\, F(\hat{p}, \hat{q})\, dq = \int_{\Omega} I(q)\, H\left(\, t(\hat{p}),\, t(\hat{q})\, \right)\, dq$$

- Found t
- H can be any filter whose response decreases with distance at least as fast as F's
- Choices of H: Normalized Convolution, Interpolated Convolution, Recursive Filtering

距离越远，响应越小，即对中心点的影响越小，H的选择是归一化的卷积核，插值卷积，循环滤波

Normalized Convolution

- One of three filters the paper describes

$$J(p) = (1/K_p) \sum_{q \in D(\Omega)} I(q)\, H\left(t(\hat{p}), t(\hat{q})\right)$$

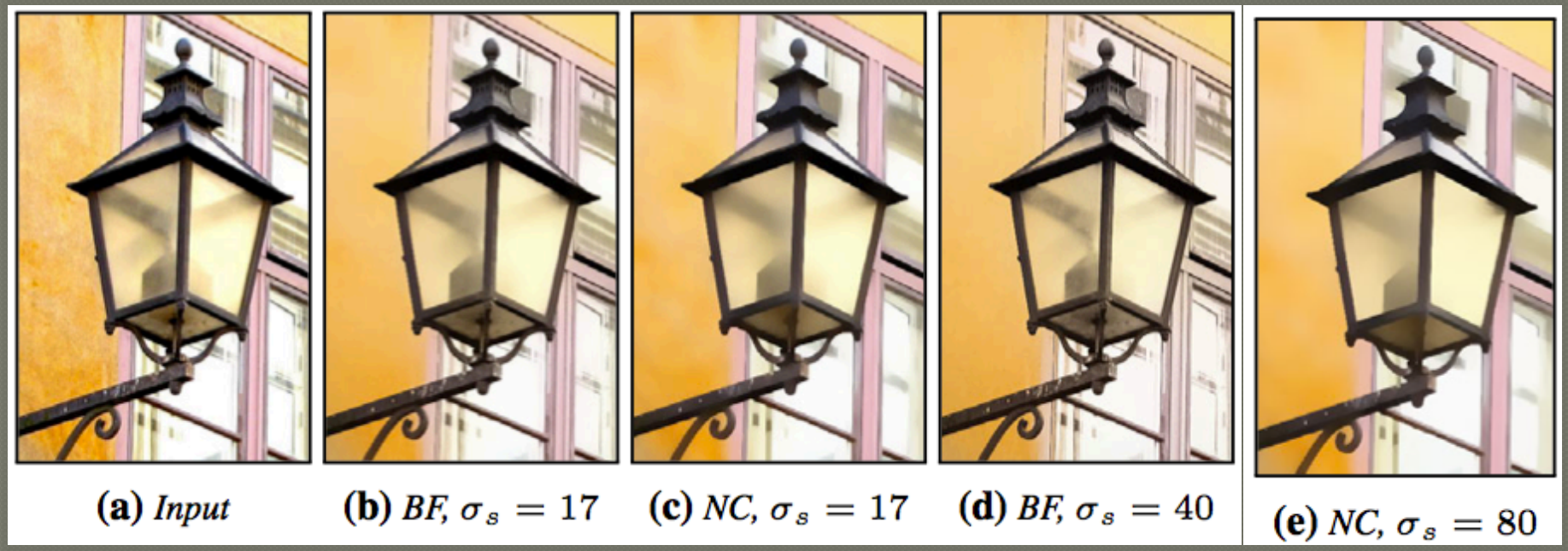
$$H\left(t(\hat{p}), t(\hat{q})\right) = \delta\{\ |t(\hat{p}) - t(\hat{q})| \leq r\ \}$$

$$r = \sigma_H \sqrt{3}$$

- Parallelizable – fast GPU implementation

滤波器满足条件：空间和像素邻域在半径r以内，r=H方差的根3倍，且可以并行实现

Results - Smoothing Quality



(a) *Input* **(b)** *BF*, $\sigma_s = 17$ **(c)** *NC*, $\sigma_s = 17$ **(d)** *BF*, $\sigma_s = 40$ **(e)** *NC*, $\sigma_s = 80$

Results - Performance

- Filtering on CPU - NC w/ 3 iterations
  - 1 megapixel: 0.16 seconds
  - 10 megapixels: 1.6 seconds
  - 3.3x speedup with quad-core CPU
  - Vs. CTBF: 10 seconds with 1/3 the work (single color channel instead of all 3)
- Filtering on GPU
  - 1 megapixel: 0.007 seconds
  - Speedup of 23x vs. single core CPU implementation
  - Vs. WLS: 1 second for grayscale image

Results – Fine Details

(a) *Input* **(b)** *Ours (IC)* **(c)** *EAW*

Figure 12: *Fine detail manipulation. (a) Input image. (b) Our result. J_1 was obtained with the IC filter ($\sigma_s = 20$ and $\sigma_r = 0.08$). (c) EAW result by Fattal [2009].*

Results – Real-Time

- Input:
  http://www.youtube.com/watch?v=HsAW9sh_IW0&hd=1
- Output (1080p video filtered in real time):
  http://www.youtube.com/watch?v=lTy9W5mWG_0&hd=1