Models of Household Behavior

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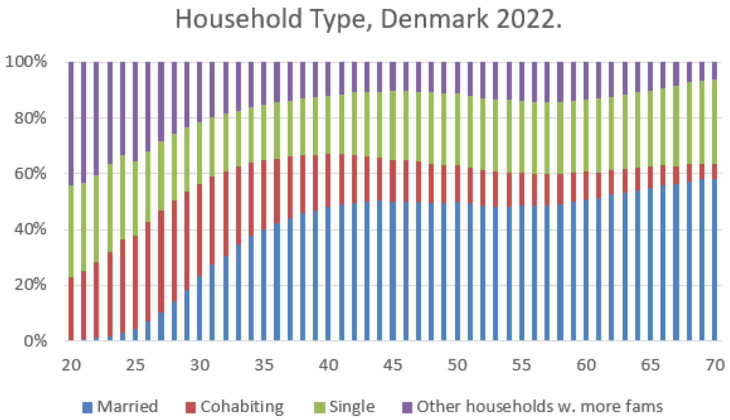
Empirical Motivation

- **Many people live in couples**



Introduction

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  Some large and some small differences
- I will focus on some main types of models
  Give an idea of the main similarities and differences

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- There are ∞ many ways of modeling household decisions
  Some large and some small differences
- I will focus on some main types of models
  Give an idea of the main similarities and differences

- Remember: Models are ***abstractions***, hopefully usefull...

- Notation on dynamic models differ from Chiappori and Mazzocco (2017)
  Based on lecture note

Outline

1. Static Models
   - Setting
   - Unitary Model
   - Non-cooperative
   - Cooperative: Collective
2. Dynamic Models
   - Full Commitment
   - Limited Commitment

Production Technology

- **Superscript:** individual (1,2), **subscript:** element
- **Private** goods ($h = 1, \ldots, n$) produced as

$$q_h = q_h^1 + q_h^2 = f_h(x_h, d_h) \tag{1}$$

where
x_h: market goods inputs
$d_h = (d_h^1, d_h^2)$: time inputs

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  where
  x_h: market goods inputs
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- **Public** goods ($k = 1, \ldots, N$) produced as

$$Q_k = F_k(X_k, D_k) \tag{2}$$

  where
  X_k: market goods inputs
  $D_k = (D_k^1, D_k^2)$: time inputs

Preferences: Utility and Felicity Function

- Individual *utility* function

$$U^i(Q, q^1, q^2, l^1, l^2)$$

where
l^i is leisure time
$T^i = h^i + l^i + \sum_{k=1}^{N} D_k^i + \sum_{h=1}^{n} d_n^i$ is available time
h^i is hours worked

Preferences: Utility and Felicity Function

- **Caring preferences:**
  Care not about the allocation of partner but only their welfare:

$$U^i(Q, q^1, q^2, l^1, l^2) = W^i(u^1(Q, q^1, l^1), u^2(Q, q^2, l^2))$$

where
$u^i(Q, q^i, l^i)$ is called the *felicity* function

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- **Egotistic preferences:**
  Care not about the partner:

$$U^i(Q, q^1, q^2, l^1, l^2) = F^i(u^i(Q, q^i, l^i), a)$$

  where a can contain marital status etc.

Budget Constraint

- **Budget constraint**

$$p' \left(\sum_{k=1}^{N} X_k + \sum_{h=1}^{n} x_n \right) = \sum_{i=1}^{2} (y^i + w^i h_i) \tag{3}$$

where
p is vector of market prices
y^i is non-market income.
w^i is wage rate

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- Can be written as in Chiappori and Mazzocco (2017)

$$p'\left(\sum_{k=1}^{N} X_k + \sum_{h=1}^{n} x_n\right) + \sum_{i=1}^{2} w^i(l^i + \sum_{k=1}^{N} D_k^i + \sum_{h=1}^{n} d_n^i) = \underbrace{\sum_{i=1}^{2}(y^i + w^i T^i)}_{Y \text{ (pot. inc.)}}$$

(note that $T^i = h^i + l^i + \sum_{k=1}^{N} D_k^i + \sum_{h=1}^{n} d_n^i$, they miss l^i on p. 989)

- **Income pooling:** non-labor income, y^i, enters identically for both

Unitary Model

- **Unitary model**, households solve (conditional on *potential* income, $Y = \sum_{i=1}^{2}(y^i + w^i T^i)$)

$$\max_{X,x,l^1,l^2,d^1,d^2,D^1,D^2} U^H(Q,q,l^1,l^2)$$

s.t.

$$Q_k = F_k(X_k, D_k),\ k = 1,\dots,N$$

$$q_h = q_h^1 + q_h^2 = f_h(x_h, d_h),\ h = 1,\dots,n$$

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where $U^H(Q,q,l^1,l^2)$ is *some* household-level utility function

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  where $U^H(Q, q, l^1, l^2)$ is *some* household-level utility function
- **Rationalized** via
  Samuelson's welfare index
  Becker's rotten kid (we skip)
  Transferable Utility (TU)

Unitary Motivation: Samuelson's welfare index

- **Samuelson's welfare index**

$$U^H(Q, q, I^1, I^2) = \max_{q_1, q_2} W(u^1(Q, q^1, I^1), u^2(Q, q^2, I^2))$$

s.t.

$$q = q_1 + q_2$$

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- **Example** could be

$$W(u^1(Q, q^1, I^1), u^2(Q, q^2, I^2)) = \lambda u^1(Q, q^1, I^1) + (1 - \lambda)u^2(Q, q^2, I^2)$$

where
λ is a constant weight on each member's utility; "power"

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  where
  λ is a constant weight on each member's utility; "power"

- **Arbitrary** that households should have some $W()$... but
  this example is a special form of the "collective model" below [nice]

Unitary Motivation: Transferable Utility

- **TU: If there exists a Pareto frontier**,
  such that a *cardinal transformation,* $k()$ gives
  $$k(u^1(Q,q^1,l^1)) + k(u^2(Q,q^2,l^2)) = K(p,w,Y)$$
  $\rightarrow$ utility possibility frontier has a slope of -1,
  $$k(u^1(Q,q^1,l^1)) = K(p,w,Y) - k(u^2(Q,q^2,l^2))$$
- **Then** we can describe the optimization problem using $U^H(Q,q,l^1,l^2)$

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- **Then** we can describe the optimization problem using $U^H(Q,q,l^1,l^2)$
- **Example:** Conditional on Y,
  $$U^{m\star}(\overline{u}^f) = \max_{c^m} U^m(c^m) = \sqrt{c^m}$$
  $$\text{s.t.}$$
  $$Y = c^m + c^f$$
  $$\overline{u}^f = U^f(c^f) = \sqrt{c^f}$$
  gives $U^{m\star}(\overline{u}^f)^2 = \text{constant}(Y) - (\overline{u}^f)^2$.

Unitary Model: Not Consistent with Data

- **Two testable implications**
  1. Income pooling (source of non-labor income does not matter for behavior)
  2. Slutsky symmetry (commodity prices affect members' demand similarly)

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- **Almost always rejected** see Chiappori and Mazzocco (2017, p. 1022)

- **Alternatives** have been proposed
  Non-cooperative
  Cooperative (collective)

Non-Cooperative

- **Non-cooperative models:**
  A game with two players
  Nash equilibrium

$$\max_{Q^1,q^1,l^1} u^1(Q^1+Q^2,q^1,q^2,l^1,l^2)$$
$$\text{s. t.}$$
$$PQ^1+p'q^1=Y^1$$

  and

$$\max_{Q^2,q^2,l^2} u^2(Q^1+Q^2,q^1,q^2,l^1,l^2)$$
$$\text{s. t.}$$
$$PQ^2+p'q^2=Y^2$$

- **Generally not efficient:** Partner's gains not internalized.

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- **Distribution factors:** z
  Anything that affect power, such as p, w, y.
  Cannot be endogenous: Over-investment in power $\rightarrow$ inefficient.

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- **Distribution factors:** z
  Anything that affect power, such as p, w, y.
  Cannot be endogenous: Over-investment in power $\rightarrow$ inefficient.
- **Unitary model** is nested, $\lambda(z) = \text{constant} \ \rightarrow\ $ unitary model

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   - Limited Commitment

Dynamic Models

- All comes down to how the bargaining weight is updated.
- My slides combine Chiappori and Mazzocco (2017)
  with Theloudis, Velilla, Chiappori, Giménez-Nadal and Molina (2022)
  and own lecture note

General Setup: Choices

- Period $t = 0$: Individuals A and B become a couple
- Periods $t > 0$: As a couple, they decide on
  - private consumption, c_t^A and c_t^B (and thus savings, a_t)
  - labor supply, $l_t^A, l_t^B \in \{0, 0.75, 1\}$
  - whether to split up (no re-partnering for simplicity)
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- **Inter-temporal budget** constraint of couple

$$a_t + c_t^A + c_t^B = Ra_{t-1} + w^A l_t^A + w^B l_t^B$$

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- **Example throughout**, but apply more generally.

General Setup: Utility

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$$u^j(c_t^j, l_t^j)$$

- **Household utility** is weighted sum

$$U(c_t^A, c_t^B, l_t^A, l_t^B; \psi_t, \mu_t) = \mu_t u^A(c_t^A, l_t^A) + (1 - \mu_t) u^B(c_t^B, l_t^B) + \psi_t$$

where match quality/"love" is

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- How μ_t is determined defines the different types of models:
  Unitary: $\mu_t = \mu$ is a **constant number**
  Full commitment: $\mu_t = \mu_0(Z)$ is a **function** (of states known i $t = 0$)
  No commitment: μ_t is updated in each period
  Limited commitment: $\mu_t = \mu_t(\bullet, \mu_{t-1})$ is a function of past power

General Setup: Recursive Formulation

- **Outside option:** Value of being single

$$V^s_{j,t}(a_{t-1}) = \max_{c^j_t, l^j_t} u^j(c^j_t, l^j_t) + \beta V^s_{j,t+1}(a_t)$$

$$\text{s.t.}$$

$$a_t = Ra_{t-1} + w^j l^j_t - c^j_t$$

where I do not allow for re-partnering ($V^{m\to s}_{j,t} = V^s_{j,t}$).

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- **Partnership dissolution:**
  Share κ_1 of wealth is transferred to agent A and $\kappa_2 = 1 - \kappa_1$ to agent B.
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- **Value of being in a couple**
  depends on what we assume about the *bargaining process*.

Unitary Model

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- **Value of a couple** is

$$W_t(a_{t-1}, \psi_t) = \max_{c_t^A, c_t^B, l_t^A, l_t^B} U(c_t^A, c_t^B, l_t^A, l_t^B, \psi_t; \mu) + \beta \tilde{W}_{t+1}(a_t, \psi_t)$$

the *expected* (w.r.t. ψ_{t+1}) continuation value is

$$\tilde{W}_{t+1}(a_t, \psi_t)$$
$$= \mathbb{E}_t[\max\{W_{t+1}(a_t, \psi_{t+1}) \; ; \; \underbrace{\mu V^s_{A,t+1}(a_t^A) + (1-\mu) V^s_{B,t+1}(a_t^B)}_{\text{weighted value of singlehood}}\}]$$

Commitment Models

- **Endogenously determined μ_t**
  - FC: Full commitment, μ_t is a **constant function**
    We will see in Bruze, Svarer and Weiss (2015)
  - NC: No commitment, μ_t updated **every period**
    We will just discuss today
  - LC: Limited commitment, μ_t updated **sometimes** $\rightarrow$ function of past power
    We will see in several papers + code

Full Commitment

- Bargaining power function is determined and agreed upon **at beginning of partnership**
- Bargaining power is thus a *constant function*

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- Z_t: **Known at beginning** of partnership, at $t = 0$!
  - Assume that couples can **commit** to this bargaining power function
  - Will e.g. not request more bargaining power from (changes in) something not in Z_t
  - If time-varying elements in Z_t: Assuming perfect foresight

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- **How is this function determined?**
  We will come back to this in a few slides

Full Commitment

- **Value of being a couple** is then

$$W_t(a_{t-1}, \psi_t) = \max_{c_t^A, c_t^B, l_t^A, l_t^B} U(c_t^A, c_t^B, l_t^A, l_t^B; \psi_t, \mu_0(Z_t)) + \beta \tilde{W}_{t+1}(a_t, \psi_t)$$

where *expected continuation* value is

$$\tilde{W}_{t+1}(a_t, \psi_t) = \mathbb{E}_t[\max\{W_{t+1}(a_t, \psi_{t+1}) \\ ; \underbrace{\mu_0(Z_{t+1}) V^s_{A,t+1}(a_t^A) + (1 - \mu_0(Z_{t+1})) V^s_{B,t+1}(a_t^B)}_{\text{weighted value of singlehood}}\}]$$

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- **Transferable utility:** Household jointly decide to divorce if

$$W_t(a_t) < \mu_0(Z_t) V^s_{A,t}(a_t^A) + (1 - \mu_0(Z_t)) V^s_{B,t}(a_t^B)$$

- No constraints on individual members' utilities

Full Commitment: Determining Bargaining Power

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- **Idea 1: Nash-bargaining** at the point of partnership formation

$$\mu_0(Z) = \arg\max_{\mu\in[0,1]} \left(\mu W_0(a_{-1}) - V^s_{A,0}(\lambda a_{-1})\right)^{0.5} \times \left((1-\mu)W_0(a_{-1}) - V^s_{B,0}((1-\lambda)a_{-1})\right)^{0.5}$$

  - μ_0 "non-parametric" constant function of e.g. $Z = (a_{-1}, w_0^A, w_0^B)$

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- **Idea 2: Assume a functional form** (Bruze, Svarer and Weiss, 2015)

$$\mu_0(w_t^A / w_t^B) = \frac{\exp(\alpha_0 + \alpha_1 w_t^A / w_t^B)}{1 + \exp(\alpha_0 + \alpha_1 w_t^A / w_t^B)}$$

and estimate parameters α_0 and α_1 using data.
[perfect foresight assumption on wages]

  - If $\alpha_1 = 0$: Similar to the unitary model.

No- and Limited Commitment

- My definition of “No commitment” is different from that of Mazzocco (2007)
  - I will call his setup “Limited commitment” (as is standard now)
- They are closely related: Both **do not assume transferable utility**
  - Only differ in how the bargaining power is updated dynamically

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- They are closely related: Both **do not assume transferable utility**
  - Only differ in how the bargaining power is updated dynamically
- We thus need to check **individual "participation" constraints**: Is it optimal for each agent to be part of the couple without receiving any utility from the other partner
- **We need to define a new objects** for this purpose: The value of agent j from being in the couple if μ_{t-1} is the bargaining power coming into period t

Notation: Partnership Status and Transitions

- All values are **individual**
- **Four** transition possibilities

from\to	married	single
married	$V_{j,t}^{m\to m}(\mathcal{S}_t, \mu_{t-1})$	$V_{j,t}^{m\to s}(\mathcal{S}_t, \mu_{t-1})$
single	$V_{j,t}^{s\to m}(\mathcal{S}_t)$	$V_{j,t}^{s\to s}(\mathcal{S}_t)$

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- Value of **starting** in a state

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  where $D_t^\star \in \{0, 1\}$ is divorce and $M_t^\star \in \{0, 1\}$ is marriage
- When singlehood is absorbing (and no div. costs): $V_{j,t}^{s} = V_{j,t}^{m\rightarrow s}$.

Recursive Formulation

- **Individual value of choice** c_t^j, l_t^j while remaining in couple is

$$v_{j,t}^{m\rightarrow m}(c_t^j, l_t^j; a_{t-1}, \psi_t, \mu) = u^j(c_t^j, l_t^j) + \psi_t + \beta \mathbb{E}_t[V_{j,t}^m(a_t^j, \psi_{t+1}, \mu)]$$

  where μ is *some* bargaining power, we will discuss in great detail.
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1. **Conditional on remaining together**, optimal choices are
$$\tilde{c}_t^A(\mu), \tilde{c}_t^B(\mu), \tilde{l}_t^A(\mu), \tilde{l}_t^B(\mu) = \arg \max_{c_t^A, c_t^B, l_t^A, l_t^B} \mu v_{A,t}^{m\to m}(c_t^A, l_t^A; a_{t-1}, \psi_t, \mu) + (1-\mu) v_{B,t}^{m\to m}(c_t^B, l_t^B; a_{t-1}, \psi_t, \mu)$$

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2. **Marital surplus** for agent j is
$$S_t^j(\mu) = \underbrace{v_{j,t}^{m\to m}(\tilde{c}_t^j(\mu), \tilde{l}_t^j(\mu); a_{t-1}, \psi_t, \mu)}_{V_{j,t}^{m\to m}(a_{t-1}, \psi_t, \mu)} - V_{j,t}^{m\to s}(a_{t-1}^j)$$

Limited Commitment: Bargaining Process

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3.3 If e.g. $S_t^A(\mu_{t-1}) < 0$ (member A not happy with current division) They re-bargain to find a new potential distribution both can accept (point 4 on next slide)

Limited Commitment: Updating Bargaining Weight

4. Let $\tilde{\mu}^A : S_t^A(\tilde{\mu}^A) = 0$ be the allocation that makes A (barely) want to remain together.

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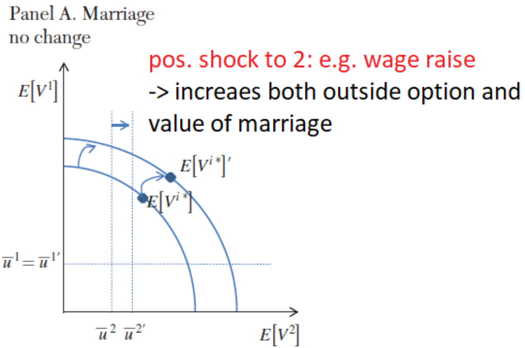
- Combined the co-state evolves as

$$\mu_t = \mu_t^\star(\mathcal{S}_t, \mu_{t-1}) = \begin{cases} \mu_{t-1} & \text{if } S_t^A(\mathcal{S}_t, \mu_{t-1}) \geq 0 \text{ and } S_t^B(\mathcal{S}_t, \mu_{t-1}) \geq 0 \\ \tilde{\mu}^A & \text{if } S_t^A(\mathcal{S}_t, \mu_{t-1}) < 0 \text{ and } S_t^B(\mathcal{S}_t, \tilde{\mu}^A) \geq 0 \\ \tilde{\mu}^B & \text{if } S_t^A(\mathcal{S}_t, \tilde{\mu}^B) \geq 0 \text{ and } S_t^B(\mathcal{S}_t, \mu_{t-1}) < 0 \\ \emptyset & \text{else} \end{cases} \tag{4}$$

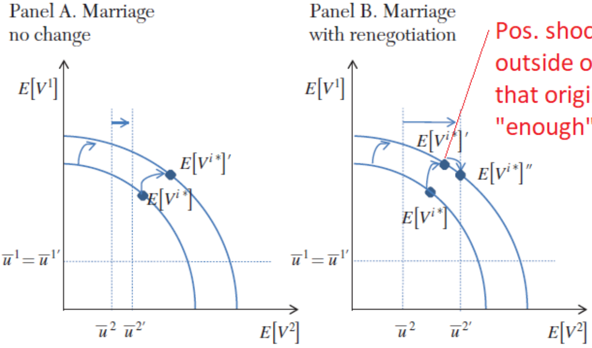
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **small**.



Limited Commitment: Updating Bargaining Weight

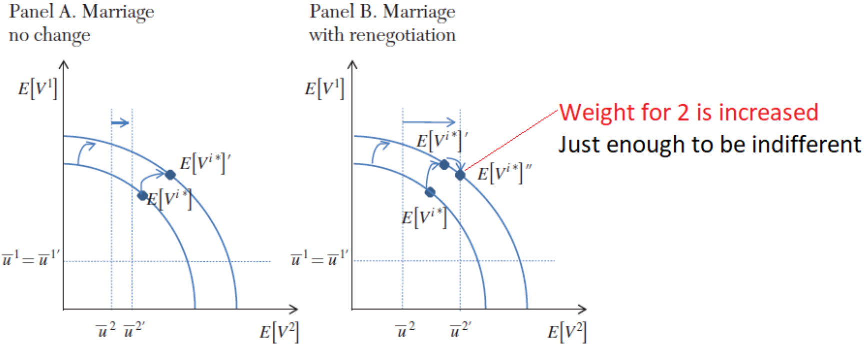
- Shock to agent 2's outside option: **medium**.



Pos. shock to 2: Increases outside option so much that original "power" not "enough" to stay

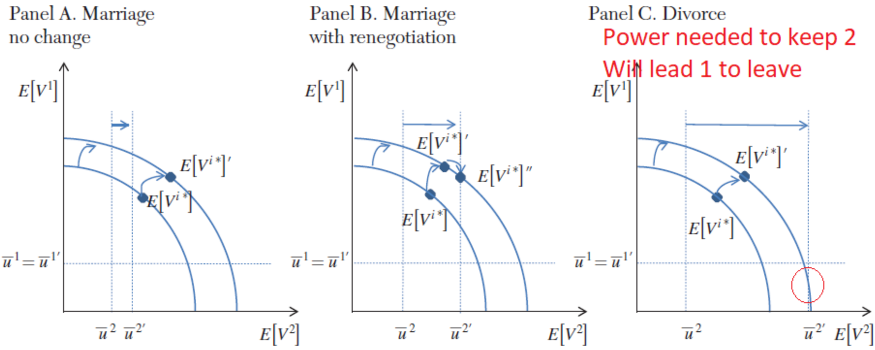
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **medium**.



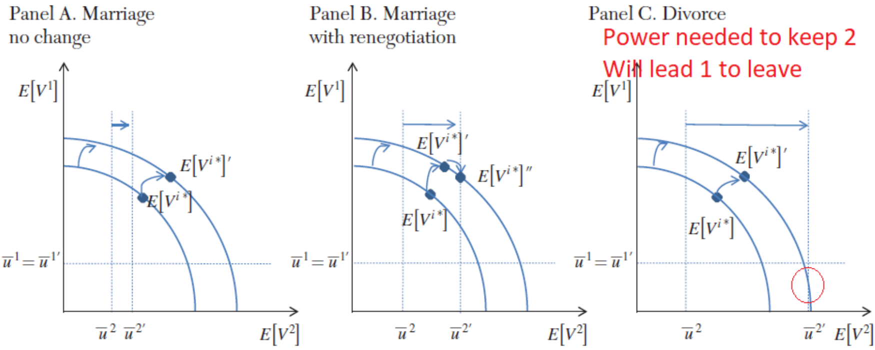
Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **large**.



Limited Commitment: Updating Bargaining Weight

- Shock to agent 2's outside option: **large**.



- See also figures 2 and 3 in lecture note.

No Commitment

- **Limited commitment:** Bargaining power updated if individual participation constraints violated at current bargaining position, μ_{t-1},

$$\mu_t = \mu_t^{\star}(a_{t-1}, \psi_t, \mu_{t-1})$$

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$$\mu_t = \mu_t^\star(a_{t-1}, \psi_t, \mu_{t-1})$$

- **No commitment:** Bargaining power updated in *all periods*,

$$\mu_t = \mu_t^\star(a_{t-1}, \psi_t)$$

  replace step 3 with e.g. [instead of the discussion before]

$$\mu_t = \arg\max_{\tilde{\mu}} S_t^A(\tilde{\mu})^{0.5} S_t^B(\tilde{\mu})^{0.5}$$

- **We focus on limited commitment** in the code
  What about initial bargaining power, μ_0, then?
  Could be found through Nash bargaining :)

Next Time

- **Next time:**
  Divorce Laws, Savings and Labor Supply.

- **Literature:**
  Voena (2015): "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?"
  - **Read** before lecture
  - **Reading guide:**
    Section 0: Introduction. Key
    Section 1: US divorce law. Key.
    Section 2: Model. *Key*, but complex. Get the idea.
    Under unilateral divorce: limited commitment model.
    Section 3: Data and RF motivation. Get the overall results/motivation.
    Section 4: Structural Estimation: Read fast.
    Section 5: Counterfactual simulations. Key.

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