Household Behavior over the Life Cycle - Assignment 1

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Question 1

I solve the model using the following baseline parametrization:

- Preferences: $\eta = -2$, $\gamma = 2.5$, $\rho = \frac{1}{1.02}$, $\beta_0 = 0.1$, β_1 is to be estimated
- Income: $\alpha = 0.3$, $w = 1$, $\tau = 0.1$
- Children: $p_n = 0.1$
- Savings: $r = 0.02$
- Time: $T = 10$

I solve the model over a grid of assets a_t and human capital k_t for each period $t = 0, 1, ..., T$ and for each number of children $n_t = 0, 1$. For human capital k_t, I use a grid of 20 grid points ranging from 0 to 20, and for assets a_t I use a grid of 100 grid points ranging from -10 to 10. The latter is a deviation from the provided parametrization, implemented in the hopes of making the model solution more precise.

To estimate the parameter β_1 to match the result from Kleven, Landais and Søgaard (2019), I compute the average drop in labor hours in the year of birth relative to the previous year as a function of β_1, denoted $\phi_0(\beta_1)$. I then solve for the root of $\phi_0(\beta_1) - 0.1$. This yields a parameter estimate of $\hat{\beta}_1 = 0.0554$, which I will use for the remainder of the assignment.

Labor supply over the lifetime for selected cohorts of parents is shown in figure 1. Before having children, labor supply is high, and agents accumulate a lot of human capital early in life while labor disutility is still low. This is comparable to standard precautionary savings, however in this model, the precautionary motive causes agents to accumulate human capital.

Over time, while agents have no children, they gradually reduce their labor supply. This can be attributed to the fact that as time increases and the terminal date approaches, the number of periods where the agents risks having to work with a high labor disutility decreases. The precautionary motive thus becomes weaker as time passes and no children arrive.

Figure 2 shows average labor supply relative to the year before childbirth, emulating an event study. The model does indeed produce a labor supply reduction of 10 pct. upon childbirth as in Kleven, Landais and Søgaard (2019).

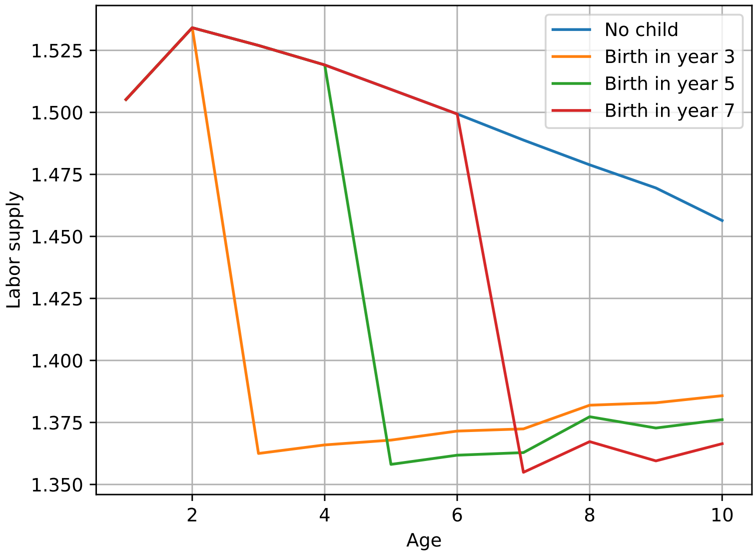

Figure 1: Labor supply for selected birth cohorts in baseline model

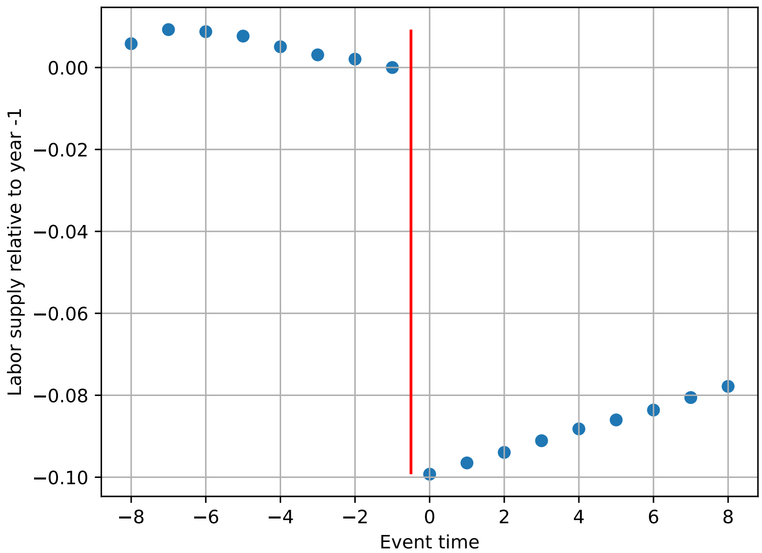

Figure 2: Event graph: Labor supply response to child birth

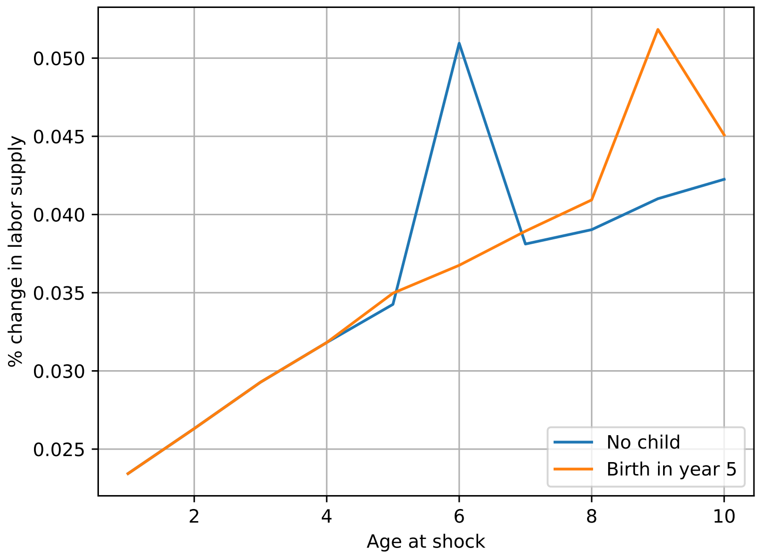

Figure 3: Marshall elasticities by age at shock and birth of child

Question 2

The Marshall elasticity can be simulated as the labor supply response to an unanticipated, permanent tax increase of 1 percent in the period of the increase. I compute the Marshall elasticity by permanently increasing the tax rate τ with 1 percent, starting in period t, and measuring the labor supply response in period t.

To investigate how the Marshall elasticity changes with age, I simulate shocks to taxes starting at each age $t = 1, 2, ..., 10$. In practice, I do this by solving the baseline model with tax rate $\tau = 0.1$ and a model with a higher tax rate $\tilde{\tau} = 0.101$. All other parameters as well as timings of child births are equal between the two models. To simulate a permanent, unanticipated tax shocks starting at age t, I simulate the first $t-1$ periods using the policy functions and transition matrices from the baseline model solution. At period t I increase the tax rate and simulate the remaining periods using the policy functions generated by the high tax model. I then measure the relative difference in labor supply compared to the baseline model with no tax increase. Figure 3 plots the age specific Marshall elasticities for individuals who go through their life cycle without having children and individuals who have a child at age $t = 5$.

First, I want to comment on the apparent spikes in Marshall elasticities at time $t = 6$ and $t = 9$ respectively. I have experimented with solving the model using different grids for a_t and k_t, and for each specification, these spikes change. This suggests that they are due to approximation error when interpolating the value function. I suspect they can be avoided by using a different solution method, for instance by exploiting the Euler equations. However, for now I will just abstract from these imprecision. Figure 3 shows that the labor supply response to a tax increase is positive for all ages, indicating the the income effect of the tax increase dominates the substitution effect. Moreover, the effect increases over time, meaning that the income effect increases relative to the substitution effect as agents go through their life cycle. This can likely be attributed to the role of human capital accumulation. The agent faces a return to labor comprised of wage income as well as the effect on human capital accumulation, which increases future income. In the beginning of the life cycle, human capital returns make up a relatively large share of total returns to labor. In the end of the life cycle, the value of human capital is small, so here wage income is the primary return to labor. Since the tax increase primarily affects wage income and only indirectly affects the value of human capital, we can expect the behavioral responses to taxation to be stronger at the end of the life cycle than in the beginning.

Finally, it appears that the Marshall elasticity increases slightly once the agent has a child. While this could very well be due to imprecision in the model solution and simulation, it could also be due to the use of human capital as a savings device. People with children work less and thus accumulate less human capital than those without children. When faced with an unanticipated tax increase, they therefore need to increase their labor hours more in order to meet the life time budget constraint, resulting in a higher Marshall elasticity.

Question 3

I now introduce a spouse who contributes with a deterministic income each period. This affects the budget constraint, which is updated to

$$\begin{aligned} a_{t+1} &= (1+r)(a_t + (1-\tau)w_t h_t - c_t + y_t) \\ y_t &= 0.1 + 0.01t \end{aligned}$$

This extra unearned income results in a downwards shift in labor supply, for both parents and non-parents, cf. figure 4. This is to be expected, as the income effect from the spouse's income contribution disincentivizes labor. This is the case before and after childbirth.

Figure 5 shows the age specific Marshall elasticities in the model with a spouse compared to the baseline model without a spouse, computed according to the method described in question 2. Since the increase in unearned income lowers the marginal utility of wealth, I would expect the income effect of a tax increase to fall relative to the substitution effect. All else being equal, this would result in a lower Marshall elasticity, as the agent is now less inclined to work more following a tax increase. Figure 5 makes it difficult to see whether this is actually the case, as the imprecision in the model solution obscure the results. However, during the first few years, the Marshall elasticities from the model with a spouse do seem slightly lower than in the baseline model.

Question 4

I now introduce a childcare cost of $\theta = 0.05$ in the presence of a child. The budget constraint is now updated to:

$$a_{t+1} = (1+r)(a_t + (1-\tau)w_t h_t - c_t - \theta n_t + y_t)$$

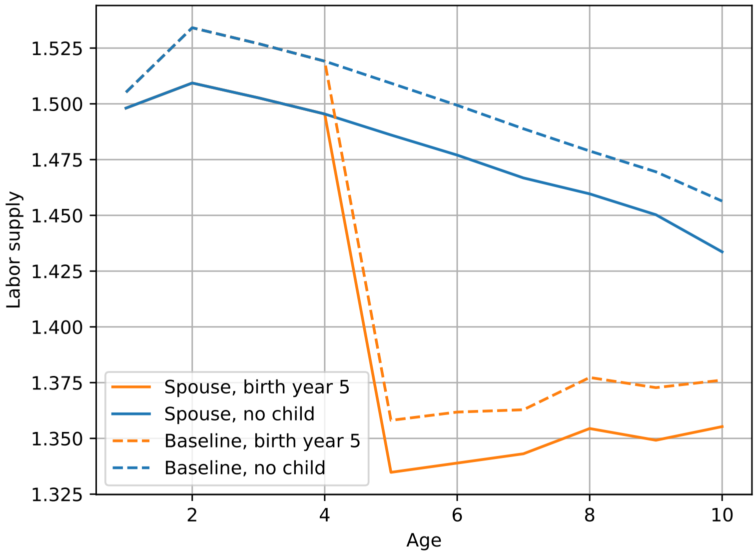

Figure 4: Labor supply with deterministic spouse vs. baseline mode

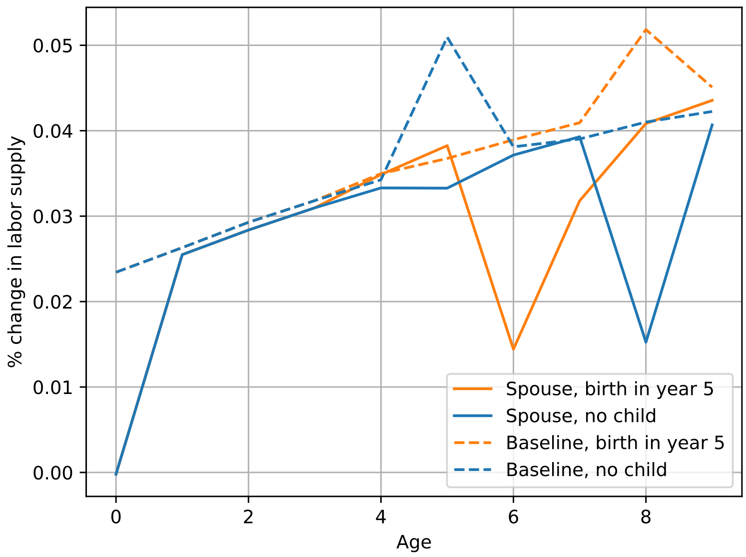

Figure 5: Marshall elasticity by age at shock and birth of child: Spouse vs. baseline

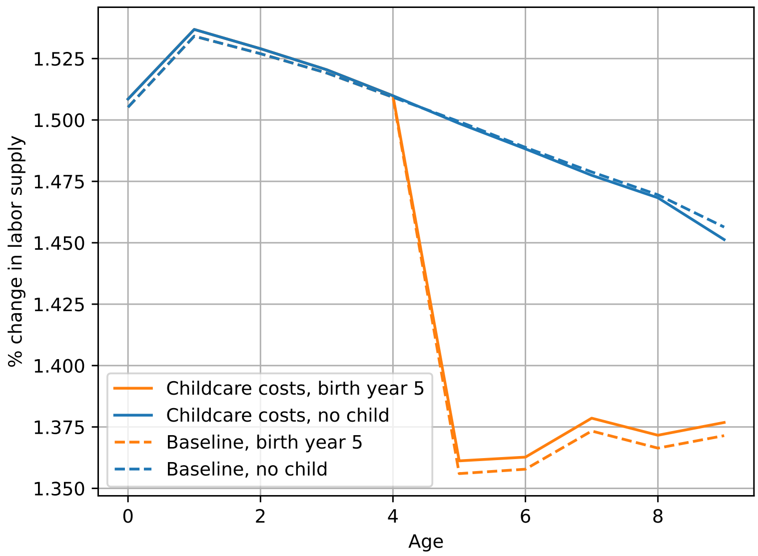

Figure 6: Labor supply with childcare cost vs. baseline model

I solve and simulate this augmented model with the baseline parametrization, $y_t = 0$ and $\theta = 0.05$. The only difference to the baseline model is thus the presence of the childcare cost.

Figure 6 shows the labor supply schedule in the model with a childcare cost compared to the baseline model. Raising childcare costs increases the risk associated with children, strengthening the precautionary motive before having children. Initially, labor supply is therefore higher than in the baseline model. However, the downwards slope of the labor supply schedule is slightly steeper than in the baseline model. This is due to agents accumulating more human capital early in their lives, and as the terminal date approaches and the lifetime utility risk associated with having children decreases, they can afford to reduce their worked labor supply more than in the baseline model.

After having children, labor supply drops to a level that is higher than in the baseline model. This is of course due to the income effect associated with increased costs of having children.

As the childcare cost reduces disposable income and thus increases the marginal utility of wealth, I expect the Marshall elasticity to be higher in this model than in the baseline model, particularly in the presence of a child. Figure 7 shows the age specific Marshall elasticities. While once again obscured by solution imprecision, the Marshall elasticity of the childcare cost model does seem higher than in the baseline specification in the presence of a child. Even before having children, the elasticity may be slightly higher, due to anticipation of having children in the future.

Question 5

In this model, the arrival of children is completely exogenous. Since the only effect of having children is an increased disutility of labor, no agent would ever choose to have children, if fertility were to be made

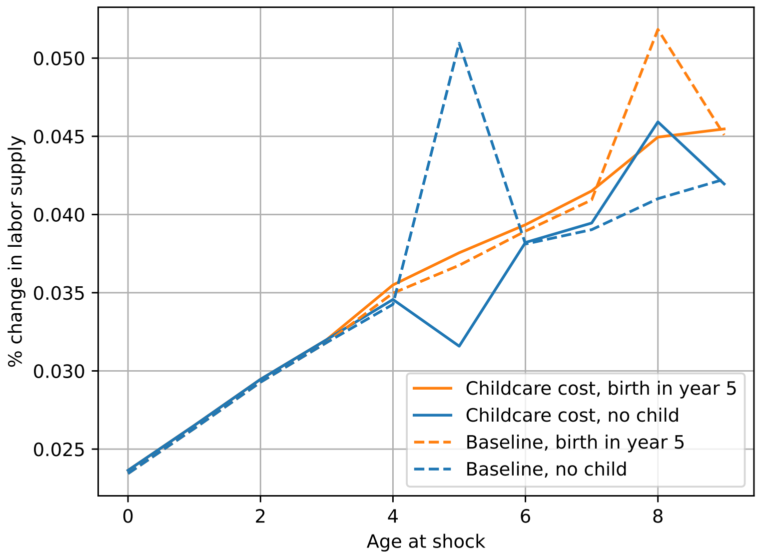

Figure 7: Marshall elasticity by age at shock and birth of child: Childcare costs vs. baseline

endogenous in the present model. However, if instead agents were to get some utility benefit from having children, fertility would be a trade-off between utility from children, and disutility from labor as well as childcare costs. A childcare subsidy thus directly increases the incentive to have children, and I would therefore expect an increase in the number of children born.

In the model where fertility is exogenous, agents work to accumulate human capital in preparation of future children. Similar behavior would take place in a model of endogenous fertility: agents have a high labor supply before having children, making it possible to reduce labor supply upon having children. A childcare subsidy would reduce the incentive to accumulate human capital, and agents would not have to work as much before having children. This could either mean than agents have children earlier in life, or that they reduce their labor supply before having children. Moreover, a child subsidy would also supress labor supply after having children.

Assuming that agents are somewhat heterogenous with respect to their preferences for children, some people may never have children due to the labor utility and childcare costs exceeding the direct utility gains from having children. These people would experience a low disutility of labor throughout their entire life. However, childfree people would also not have the same preparatory motive as future parents. One could therefore expect the labor supply of childfree agents to be lower at the beginning but higher by the end of the life cycle, compared to agents that become parents. A childcare subsidy, however, may induce some otherwise childfree agents to have children after all. In the beginning of their life cycle, these agents would therefore have incentive to increase their labor supply. This effect could somewhat counteract the labor supply reduction of parents. However, once the otherwise childfree agents have children, they will face the increased utility cost of labor, thus reducing their labor supply.

In conclusion: A childcare subsidy depresses the labor supply of people who have children either way by having them have children earlier in life or by working less in preparation of future children while also letting them work less in the presence of children. People who are induced to have children by the subsidy increase their labor supply in the beginning of their life cycle, but reduce it by the end. Which effect dominates depends on the parametrization.

Question 6

I adjust the model such that a spouse is only present with probability $p_s = 0.8$, and that a child can only arrive is there is a spouse. Letting $s_t \in \{0,1\}$ denote the presence of a spouse, the model is now presented by the following Bellman equation:

$$
\begin{aligned}
V_t(s_t, n_t, a_t, k_t) =& \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta(n_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \rho \mathbb{E}_t[V_{t+1}(s_{t+1}, n_{t+1}, a_{t+1}, k_{t+1})] \\
& \text{s.t.} \\
s_t =& \begin{cases} 1 & \text{with probability } p_s \\ 0 & \text{with probability } 1-p_s \end{cases} \\
n_{t+1} =& \begin{cases} n_t + 1 & \text{with probability } p(n_t, s_{t+1}) \\ n_t & \text{with probability } 1 - p(n_t, s_{t+1}) \end{cases} \\
a_{t+1} =& (1+r)(a_t + (1-\tau)w_t h_t + y_t s_t - c_t - \theta n_t) \\
a_T =& 0 \\
y_t =& 0.1 + 0.01t \\
k_{t+1} =& k_t + h_t \\
p(n_t, s_{t+1}) =& \begin{cases} p_n & \text{if } n_t = 0 \text{and } s_{t+1} = 1 \\ 0 & \text{otherwise} \end{cases} \\
w_t =& w\,(1 + \alpha k_t) \\
\beta(n_t) =& \beta_0 + \beta_1 \cdot n_t
\end{aligned}
$$

Note that the presence of a spouse is now a state variable, which affects the income as well as the probability of having a child in the current period.

I solve and simulate the model with the baseline parametrization, $\theta = 0$ and $y_t = 0.1 + 0.01t$. This model differs from the baseline specification in two ways: Fistly, since a child can only arrive in the presence of a spouse, the probability of a birth decreases. Secondly, if there is a spouse, he contributes with an income of y_t. To disentangle the effects of these mechanisms, I also solve and simulate a model where the spouse is present with probability 0.8 but does not contribute with income. The labor supply schedules of the baseline model, the intermediary model with a stochastic spouse with no income, and the model with a stochastic spouse with income are presented in figure 8.

In isolation, the reduced probability of a child decreases the incentive to accumulate human capital before having a child, causing a reduction in labor supply in the beginning of the life cycle. This also means that in the event of a child, the drop in labor supply is smaller since the lower human capital requires the

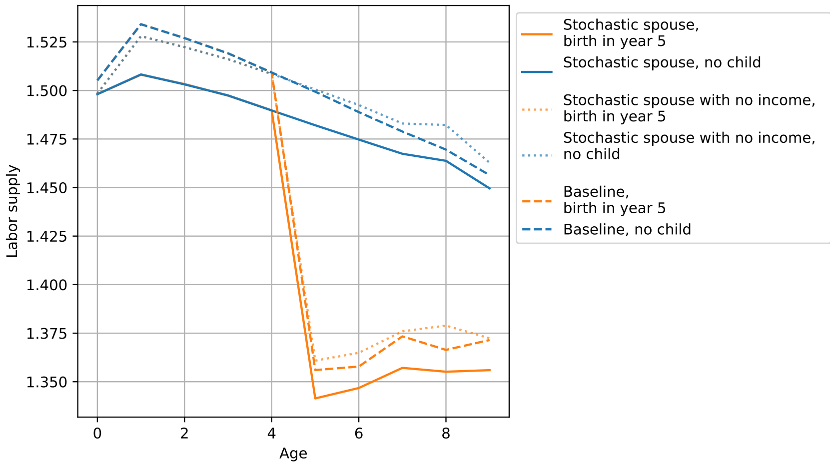

Figure 8: Labor supply with stochastic spouse vs. baseline model

agent to work more to meet her lifetime budget constraint. These effects can be seen by comparing the dashed line of the baseline model with the dotted line of the no-income-stochastic-spouse-model in figure 8. However, the possibility of the spouse contributing with income increases the expected lifetime income, which suppresses labor supply for parents and non-parents alike. This effect dominates the increase in labor supply for parents, causing an overall reduction in worked hours. This can be seen from the fully drawn lines representing the model with a stochastic spouse with income in figure 8.

The effects of this augmentation on the Marshall elasticities are ambiguous. As discussed earlier, people with children tend to have higher Marshall elasticities that people without, as they generally have lower human capital and hence need no increase their labor supply more to meet the lifetime budget constraint. The decreased probability of having children would then indicate that the Marshall elasticity increases, as the preparatory motive becomes weaker. However, as the expected lifetime income increases when there is a possibility of the presence of a spouse, the income effect of a tax increase weakens, indicating a reduction in the Marshall elasticity. Which effect dominates depends on parametrization. Figure 9 hints at the Marshall elasticities being slightly lower in the present model than in the baseline before the presence of children, but once again the results are obscured by imprecision.

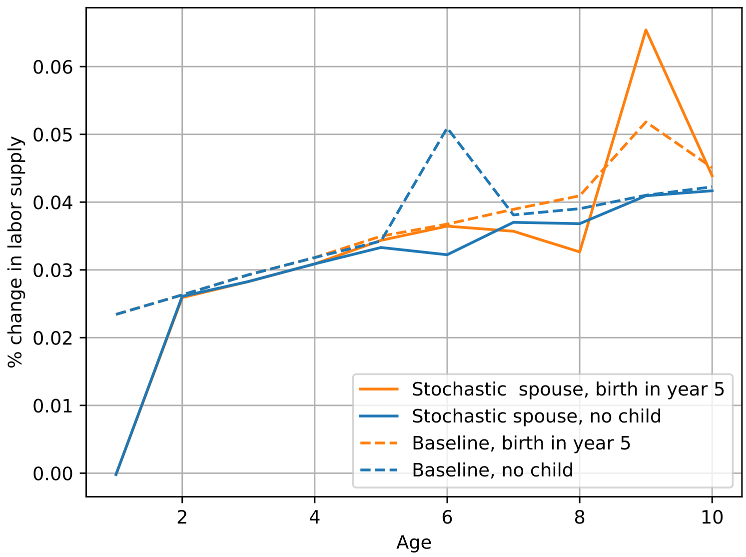

Figure 9: Marshall elasticity by age at shock and birth of child: Childcare costs vs. baseline