Exercise class 12

Introduction to Programming and Numerical Analysis

Class 3

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UNIVERSITY OF COPENHAGEN

Plan for today

Dynamic programming

Model project

Plan for today

1. Now-15.50: Work on optimization problems
2. 15.50-16.00: We talk about optimization in class
3. 16.00-16.15: Break
4. 16.15-16.35: Intro to dynamic programming
5. 16.35-17.00: Solving dynamic optimization problems in class

Terminology...

State variable: A variable determining the *state of the world* as it is when I make my decision. State variables evolve over time according to a *transition process*.

Example: Cash-on-hand in consumption-savings model.

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Choice variable: The variable I *choose* to maximize utility (*also control variable*). ample: Consumption in consumption-savings model.

Policy function: The optimal *choice* as a function of *state*.

Value function: The total value of today's *state*: Utility today + discounted value of tomorrow's state.

Two period consumption-savings model

Period 1:

$$
\begin{aligned}
V_1(m_1) &= \max_{c_1} u(c_1) + \beta \mathbb{E}\left[V_2(m_2)\right] \\
s.t. \quad m_2 &= R(m_1 - c_1) + y_2 \\
y_2 &\sim U(0,1)
\end{aligned}
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Two period consumption-savings model

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Period 2:

$$
\begin{aligned}
V_2(m_2) &= \max_{c_2} u(c_2) \\
s.t. \quad m_2 &\geq c_2
\end{aligned}
$$

Two period consumption-savings model

State variable(s)?
Choice variable(s)?
Policy function(s)?

Two period consumption-savings model

State variable(s)? m_1, m_2
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Two period consumption-savings model

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Two period consumption-savings model

State variable(s)? m_1, m_2
Choice variable(s)? c_1, c_2
Policy function(s)? $c_1^*(m_1)$, $c_2^*(m_2)$

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\end{aligned}
$$

The optimal choice today depends on the value function tomorrow - which in turn depends on the optimal choice tomorrow.

So we need to solve tomorrow's problem first - **backwards induction**.

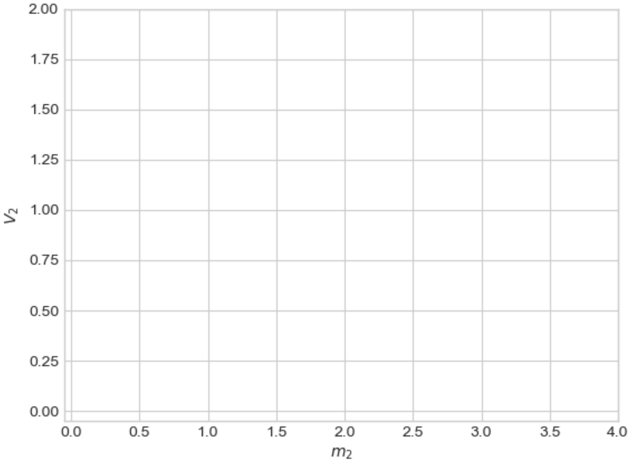
Two period consumption-savings model

Period 2:

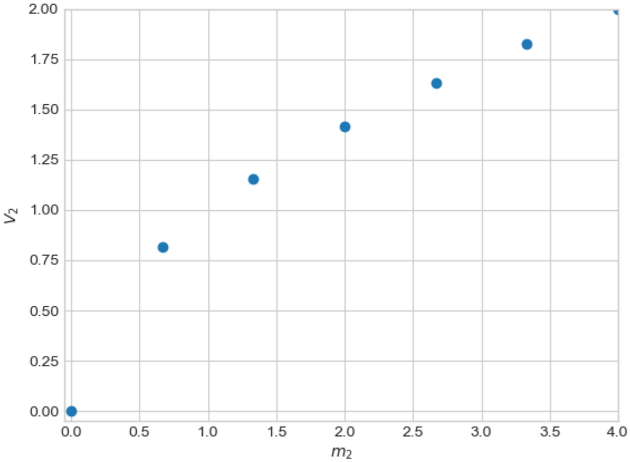
$$V_2(m_2) = \max_{c_2} u(c_2)$$
$$s.t. \quad m_2 \geq c_2$$

This is just a standard constrained optimization problem. We can solve this for any value of m_2, we like.

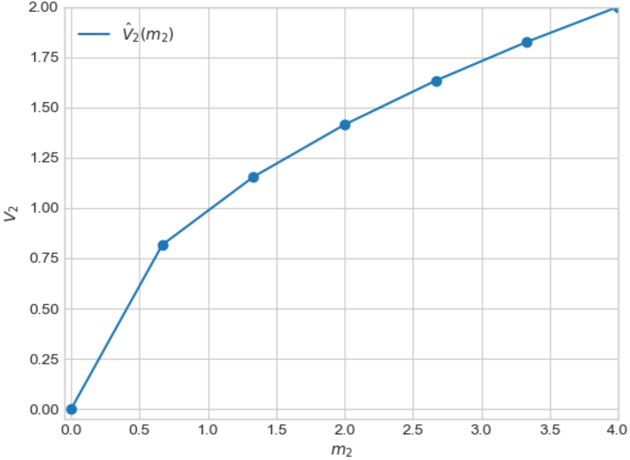
Two period consumption-savings model



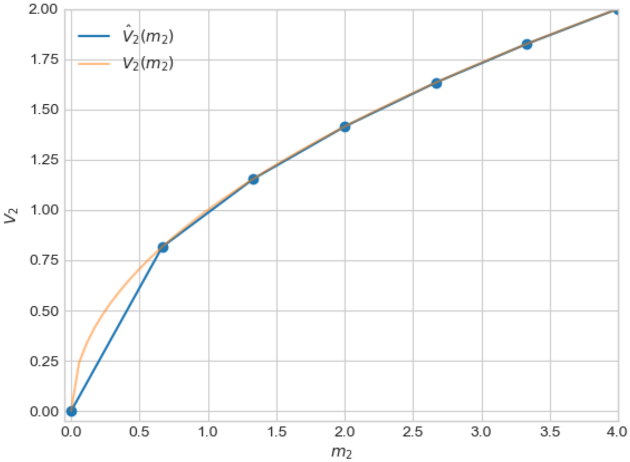
Two period consumption-savings model



Two period consumption-savings model



Two period consumption-savings model



Two period consumption-savings model

Back to period 1...

$$
\begin{aligned}
V_1(m_1) = \max_{c_1} u(c_1) + \beta \mathbb{E}\left[\hat{V}_2(m_2)\right] \\
s.t. \quad m_2 = R(m_1 - c_1) + y_2 \\
y_2 \sim U(0,1)
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Now we know the (approximate) period 2 value function, so we can solve the problem in period 1.

We still need to take expectations with respect to $\hat{V}_2(m_2)$, where m_2 is stochastic...

Two period consumption-savings model

Back to period 1...

$$V_1(m_1) = \max_{c_1} u(c_1) + \beta \mathbb{E}\left[\hat{V}_2(m_2)\right]$$
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Now we know the (approximate) period 2 value function, so we can solve the problem in period 1.

We still need to take expectations with respect to $\hat{V}_2(m_2)$, where m_2 is stochastic...$\rightarrow$ Monte Carlo Integration for continuous m_2, weighted average for discrete m_2!

A few words on the model project

Build an analyze an economic model of your choosing:

1. Extend a model from this course
2. A model you know from other courses
3. Something you find interesting!

Preferably, you use your numerical skill to do some analysis that would not have been possible using just analytical tools (eg. non-Cobb-Douglass production or utility functions, heterogeneity, uncertainty etc.)

MAY 12th IS A HARD DEADLINE! I have to approve you for the exam by May 15th, so there will be **no time for resubmissions!**

Next time...

Video lectures:

- Structural estimation
- OLG and Ramsey models

Exercises

- Work on data project