Exercise class 6

Introduction to Programming and Numerical Analysis

Class 3

Annasofie Marckstrøm Olesen

Spring 2023

UNIVERSITY OF COPENHAGEN

Optimization

Solving the consumer problem

Inaugural Project

Optimization

Objective: find x that minimizes $f(x)$

We are used to solving optimization problems analytically using first and second order conditions.

Different on computer: We can only evaluate the function in one point at a time - how do we find the minimum?

(Note: In computer science, the convention is to *minimize* - if we instead want to *maximize*, we can just minimize the negative of the function)

Optimization: Grid search

One way is to just try a bunch if different values of x and pick the one that gives the smallest value of $f(x)$.

Pros:

- Gives a rough idea of what the function looks like.
- Relatively robust against non-global minima.

Cons:

- Computer-intensive - especially in higher dimensions.
- Does not check outside grid.
- Solution only as precise as grid.

Optimization: Solvers

A solver is an algorithm that looks for a minimum by trying different values of x and updating guesses based on the evaluation of $f(x)$.

Which x's to guess on are determined by the algorithm - except the starting point, which must be provided.
Pros:

- Faster and less intensive than grid search.
- More precise solution.

Cons:

- Solution may depend on starting value.
- May not converge to a solution - then what?

Optimization: Solvers

Which algorithm should I use? Depends on the problem. Here are some examples:

Unconstrained optimization:

- `bfgs` (fast, especially if you provide gradient/hessian)
- `nelder-mead` (robust, but slow)

Constrained optimization:

- `slsqp` (fast, especially if you provide gradient/hessian)
- ...or use unconstrained optimization + penalty of constraint is violated.

Solving the consumer problem

Simple consumer problem:

$$\max_{c,\ell} \frac{c^{1-\sigma}}{1-\sigma} - \frac{\ell^{1+\nu}}{1+\nu}$$
$$s.t. \quad c \leq w\ell$$

- Choose consumption c and labor supply ℓ to maximize utility subject to budget constraint: Constrained optimization problem!
- Strictly concave objective function = unique maximum.
- Prices are taken as given.

The Inaugural Project

The inaugural project is about finding optimal labor supply on the market and in the home for a two-person household.

Important skills for the inaugural project:

- Functions and classes.
- Optimization using grid search.
- Optimization using solvers, constrained and unconstrained.
- Printing and plotting results.
- General practical coding skills.

The Inaugural Project - Hand-in

Deadline for hand-in is **March 26th**, deadline for peer feedback is **April 2nd**.

Hand-in by uploading your project to your GitHub repo:
github.com/NumEconCopenhagen/projects-YEAR-YOURGROUPNAME/inauguralproject

Your hand-in must include:

- A short README.md with introduction to project.
- A notebook presenting and discussing your results.
- A documented .py-file (you should base this on the provided file HouseholdSpecializationModel.py)

The Inaugural project - Tips

Question 1-2: Discrete choice space

- Use the provided code - only minor adjustments necessary to answer questions.
- Only the case $\sigma = 1$ has been implemented for you.

Question 3: Continuous choice space

- Standard constrained optimization.

Question 4: Estimate α and σ

- Use the provided regression function to obtain model estimates $\hat{\beta}_0$ and β_1 - remember that these are implicitly a function of the parameters.
- Find α and σ such that $\hat{\beta}_0$ and $\hat{\beta}_1$ best match the data by minimizing the provided function (squared deviation from data moments).

Question 5: Extension

- You can be as ambitious as you like here.
- See how close you can get to β_0 and β_1 with $\alpha = 0.5$ and your extension.
- OK if it does not fit the data perfectly, as long as you comment on why that may be.

Next time

Video lectures

- Data and Pandas
- Loading, cleaning and saving data

Exercise:

- Problem set 3: Working with data from Statistics Denmark