

# Exercise class 12

Introduction to Programming  
and Numerical Analysis

Class 3

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Spring 2023

UNIVERSITY OF COPENHAGEN



Plan for today

Dynamic programming

Model project

## Plan for today

1. Now-15.50: Work on optimization problems
2. 15.50-16.00: We talk about optimization in class
3. 16.00-16.15: Break
4. 16.15-16.35: Intro to dynamic programming
5. 16.35-17.00: Solving dynamic optimization problems in class

## Terminology...

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**Policy function:** The optimal *choice* as a function of *state*.

**Value function:** The total value of today's *state*: Utility today + discounted value of tomorrow's state.

## Two period consumption-savings model

### Period 1:

$$V_1(m_1) = \max_{c_1} u(c_1) + \beta \mathbb{E}[V_2(m_2)]$$

$$s.t. \quad m_2 = R(m_1 - c_1) + y_2$$

$$y_2 \sim U(0, 1)$$



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### Period 2:

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## Two period consumption-savings model

State variable(s)?

Choice variable(s)?

Policy function(s)?

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State variable(s)?  $m_1, m_2$

Choice variable(s)?  $c_1, c_2$

Policy function(s)?  $c_1^*(m_1), c_2^*(m_2)$

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The optimal choice today depends on the **value function tomorrow** - which in turn depends on the optimal choice tomorrow.

So we need to solve tomorrow's problem first - **backwards induction**.

## Two period consumption-savings model

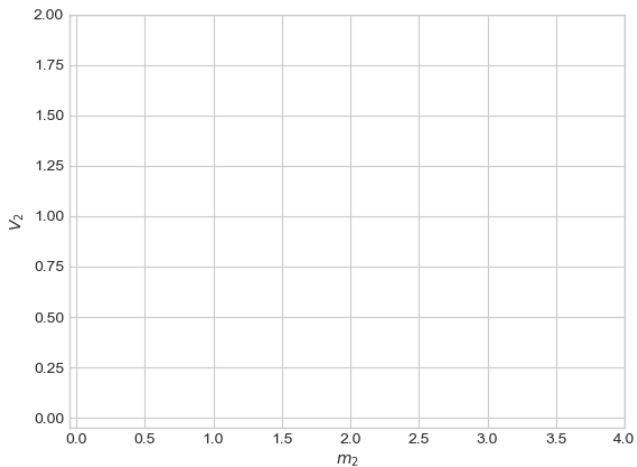
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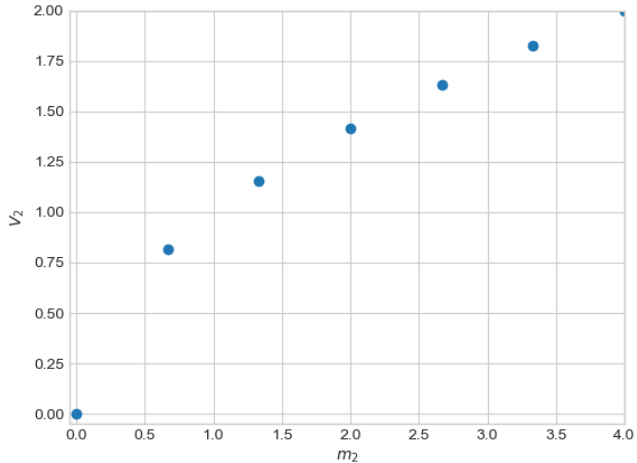
This is just a standard constrained optimization problem. We can solve this for any value of  $m_2$ , we like.



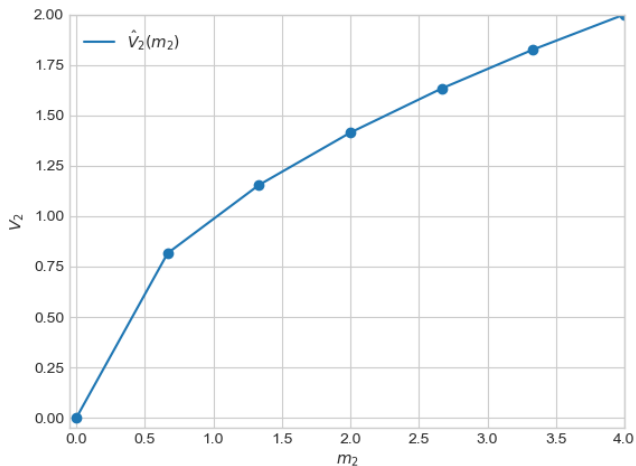
## Two period consumption-savings model



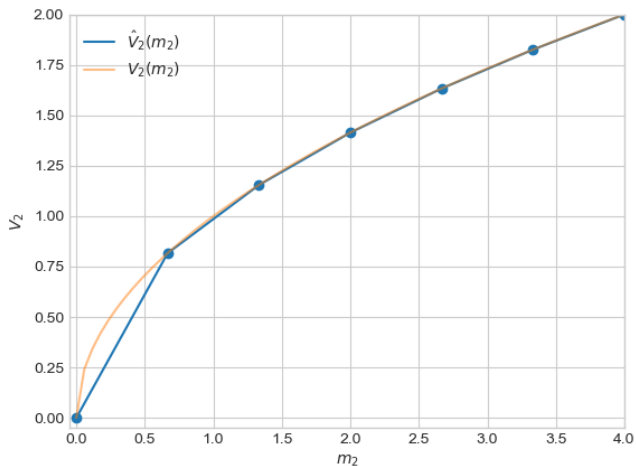
## Two period consumption-savings model



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# Two period consumption-savings model



## Two period consumption-savings model

Back to period 1...

$$\begin{aligned} V_1(m_1) &= \max_{c_1} u(c_1) + \beta \mathbb{E} \left[ \hat{V}_2(m_2) \right] \\ s.t. \quad m_2 &= R(m_1 - c_1) + y_2 \\ y_2 &\sim U(0, 1) \end{aligned}$$

Now we know the (approximate) period 2 value function, so we can solve the problem in period 1.

We still need to take expectations with respect to  $\hat{V}_2(m_2)$ , where  $m_2$  is stochastic...

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We still need to take expectations with respect to  $\hat{V}_2(m_2)$ , where  $m_2$  is stochastic... $\rightarrow$  Monte Carlo Integration for continuous  $m_2$ , weighted average for discrete  $m_2$ !

## A few words on the model project

Build and analyze an economic model of your choosing:

1. Extend a model from this course
2. A model you know from other courses
3. Something you find interesting!

Preferably, you use your numerical skill to do some analysis that would not have been possible using just analytical tools (eg. non-Cobb-Douglass production or utility functions, heterogeneity, uncertainty etc.)

**MAY 12th IS A HARD DEADLINE!** I have to approve you for the exam by May 15th, so there will be **no time for resubmissions!**

## Next time...

### Video lectures:

- Structural estimation
- OLG and Ramsey models

### Exercises

- Work on data project