More on Structural Estimation

Dynamic Programming and Structural Econometrics #14

Bertel Schjerning

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- **Frontier:** Combine the two and use exogenous variation to estimate structural model.

The Lucas critique

- **The Lucas critique:** *Behavioral rules change with policy*
  $\Rightarrow$ policy advice can not rely on estimated behavioral rules
  $\Rightarrow$ we need to estimate *structural parameters*
  > *"Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies." (Lucas, 1977)*
- **Other stuff might be approximately invariant**
- **Rigourous microfoundations:**
  1. **Mathematically:** Based on (boundedly) rational behavior derived as a solution to a formal optimization problem
  2. **Economically:** The assumptions are realistic

Heavy duty econometrics

- Structural estimation requires a lot!
  1. solving a model *fast*
  2. having data to identify parameters
  3. setting up an estimation routine
  4. getting sensible estimates from that routine
  5. validate the model
  6. run meaningful counterfactuals
- Empirical work in term paper
  - Be modest with your ambitions and start simple

Example: Buffer-stock consumptions-savings model I

Bellman Equation

$$
\begin{aligned}
v_t(m_t) &= \max_{c_t} \left\{ \frac{c_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[(GL_{t+1}\psi_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right] \right\} \\
&\text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= \frac{1}{GL_t \psi_{t+1}} R a_t + \xi_{t+1} \\
\xi_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases} \\
a_t &\geq \lambda_t = \begin{cases} -\lambda & if t < T_R \\ 0 & if t \geq T_R \end{cases} \\
\psi_t &\sim \exp \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) \\
\epsilon_t &\sim \exp \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2)
\end{aligned}
$$

Example: Buffer-stock consumptions-savings model II

$$
\begin{aligned}
Y_{t+1} &= \psi_{t+1} P_{t+1} \\
P_{t+1} &= G L_t P_t \psi_{t+1} \\
c_t &\equiv C_t / P_t \\
m_t &\equiv M_t / P_t \\
a_t &\equiv A_t / P_t \\
p_t &\equiv \ln(P_t) \\
y_t &\equiv \ln(Y_t)
\end{aligned}
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Structural estimation

- We know how to **solve dynamic programming models**
- We know how to **estimate dicrete choice models** (NFXP, CCP, NPL, MPEC, BBL)
- For estimation we need
    1. Data on (some) *states*
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- **Example model:** Life-cycle buffer-stock model
  - States: M_{it}, P_{it}
  - Choice: C_{it}
- **Parameters** to estimate: $\theta = \{\beta, \rho\}$
  - Calibration: G, σ_ψ, σ_ξ, R, and λ ("known")

Maximum likelihood estimation (MLE)

- Assume that observed log-consumption is contaminated with mean-zero i.i.d. normal **measurement error**

$$\epsilon_{it}(\theta) \equiv \log C_{it} - \log C_t^\star(M_{it}, P_{it}; \theta) \sim \mathcal{N}(0, \sigma_\xi^2)$$

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- The **“likelihood”** (pdf) of observing the data then is

$$\Pr(C, M, P|\theta) = \Pi_{i=1}^{N} \Pi_{t=1}^{T_d} \phi(\epsilon_{it}(\theta))$$

where $C = \{C_{it}\}_{1,1}^{N,T_d}$, $M = \{M_{it}\}_{1,1}^{N,T_d}$ and $P = \{P_{it}\}_{1,1}^{N,T_d}$ and

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- **MLE** then is

$$\hat{\theta} = \arg\min_{\theta} -\frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T_d} \log(\phi(\epsilon_{it}(\theta)))$$

Note: *We need to resolve the model for each new guess of* θ

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- Drawing S draws of $P_1, P_2, \ldots, P_{T_d}$ could be used to get

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- ***We would need many draws to approximate this T-dimensional integral***
  - *We may get a cosistent estimator of* $\Pr(C,M|\theta)$, *but not* $\log(\Pr(C,M|\theta)$ *(Jensen's Inequality)*
  - **MSL is inconsistent** for fixed S. The number of simulations S has to increase at least in the same rate the number of observations to achive consistency

Method of Simulated Moments (MSM)

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  and W is a positive-definite **weighting matrix**.
- We still approximate the T_d-dimensional integral with S simulations but MSM is consistent for a fixed number of draws (Jensen's inequality does not kick in the same way)

Weighting matrix

- Typical choices are

    1. **Theoretically optimal** (Full vaiance-covariance matrix of moments)
       Can cause problems in finite samples
    2. **Diagonal matrix** with **inverse** of (e.g. bootstrapped) empirical **variances of the moments** (scaled appropriately)
    3. **Freely chosen** to focus on fitting some specific dimensions of the data

Indirect inference / minimum distance

- Many different names for very similar approaches
  - McFadden (1989): Method of Simulated Moments (MSM)
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- SMD/II rely on an **auxillary statistical model**
  - Let Λ^d be the parameters of the auxillary model when estimated on the *actual* data
  - Let $\Lambda_s(\theta)$ be the parameters of the auxillary model when estimated on *simulated* data
- **Note:** The auxillary statistical model is *misspecified* and its parameters are thus typically *not interpretable*

Simulation Pitfalls

- **FIX the seed (or draws!)**
- **Flat** objective function!
  - Discrete choices: Taking a mean of an **indicator function**
- **Gradient** based numerical optimization will likely FAIL!
  - Use, e.g., `scipy.optimize.minimize(fun , method='Nelder-Mead')` (Nelder-Mead)
  - Or some smoothing device (e.g. Logit)
- As $N, S \to \infty$ this problem is ameliorated
- The problem is usually less severe around θ_0
- Continuous outcomes do not have this problem

Asymptotics

- **MSM** is **consistent** and **asymptotically normal** under standard assumptions
$$\sqrt{N}(\hat{\theta} - \theta_0) \rightarrow \mathcal{N}(0, (1 + S^{-1})V)$$
where θ_0 are the true parameters
- **Standard formulas for V:**
$$V = (G'WG)^{-1}G'W\Omega W'G(G'WG)^{-1}$$
where $G = -\frac{\partial \Lambda^m(\theta)}{\partial \theta}$ is the Jacobian of the objective function.
$\Omega = Var(\Lambda_i^d)$ is the variance of the (individual) moments in the data.
Remember: *Standard errors are large if large changes in θ imply small changes in the objective function*
- **Computational limitations:** To compute standard errors we need to compute derivative so model solution

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- **Use more data**
  1. **Quantitatively:** More agents, more time periods
  2. **Qualitative:** New types of data, e.g natural experiments around policy changes

Estimation experiment

1. **Solve** the buffer-stock model and **simulate** a full panel
2. Construct a **data set** from the simulated data

   Likelihood: Log-consumption at age 45 with measurement error
   MSM: Average wealth for each age between 40 and 55
3. Try to **estimate** $\theta = \{\beta, \rho\}$

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3. Calculate the objective function with $\Lambda^m(\theta) = \frac{1}{S}\sum_{s=1}^{S} \Lambda_s(\theta)$

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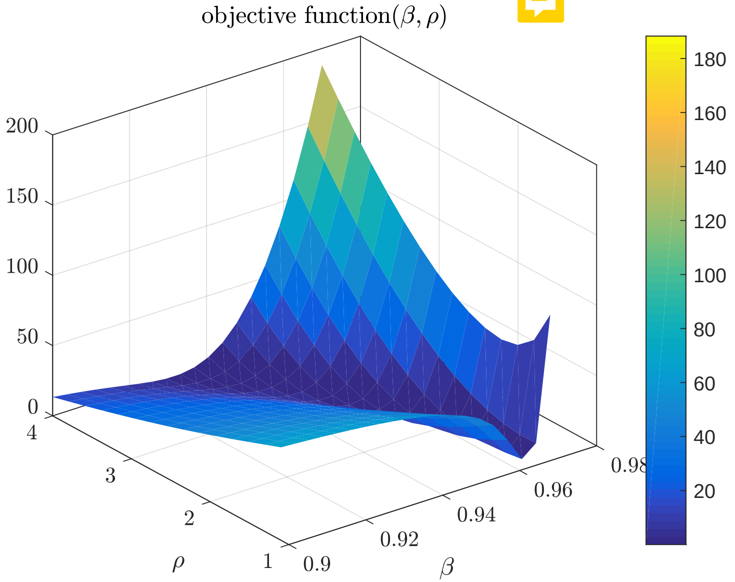
Y_t^{(s)} &= P_t^{(s)}\xi_t^{(s)} \\
P_t^{(s)} &= GP_{t-1}^{(s)}\psi_t^{(s)}
\end{aligned}$$

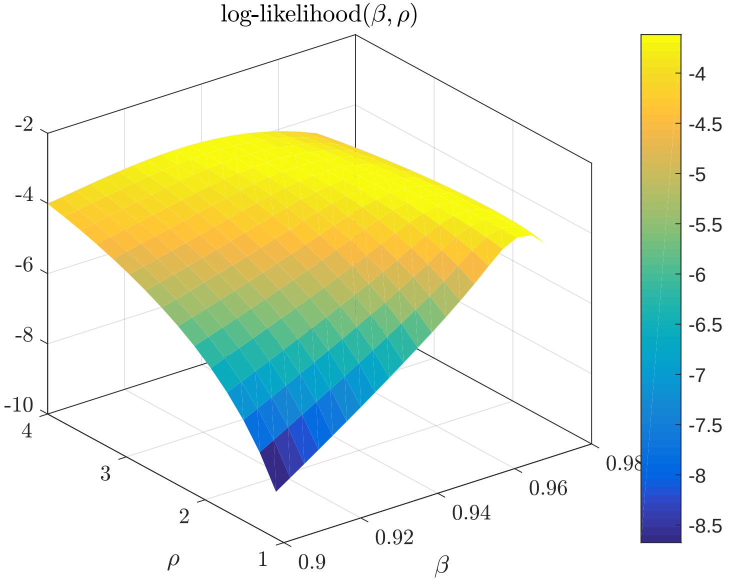
   for some initial A_0 and P_0 and draws of $\xi_t^{(s)}$ and $\psi_t^{(s)}$.
3. Calculate the moments using this simulated data, $\Lambda^m(\theta)$ ($\{\frac{1}{S}\sum_{s=1}^S A_t^{(s)}(\theta)\}_{t=40}^{55}$ here)
4. Calculate the objective function

$$Q(\theta) = \left(\Lambda^d - \Lambda^m(\theta)\right)' W \left(\Lambda^d - \Lambda^m(\theta)\right)$$

Buffer-stock: MSM



Buffer-stock: Likelihood



Robustness

- **Curse of dimensionality and lack of identification**
  ⇒ somimes we cannot estimate all the parameters of the model
  ⇒ *first step calibration may be necessary*
  1. Calculations on own data (e.g. exogenous processes)
  2. References to previous estimates
  3. Standard choices

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  - **Or the opposite:** When does the result break down?

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  3. Standard choices
- **Robustness:** Can we vary the calibration choices without changing the result substantially?
  - **Or the opposite:** When does the result break down?
- **Calibration** is also important for
  1. Gaining intuition for how the model works
  2. Initial guesses for estimation algorithm

Examples

- **Gourinchas and Parker (2002):** First structural estimation of buffer-stock consumption model
  - **Method:** MSM with a lot of first stage calibrations
  - **Data:** Cross-sectional consumption data from CEX

Examples

- **Gourinchas and Parker (2002):** First structural estimation of buffer-stock consumption model
  - **Method:** MSM with a lot of first stage calibrations
  - **Data:** Cross-sectional consumption data from CEX
- **More examples - all estimated using MSM**
  1. **Druedahl and Jørgensen (Economic Journal, 2020): "Can Consumers Distinguish Persistent from Transitory Income Shocks?"**
     MSM Monte Carlo study: Pssoible to identify consumers' degree of information by using panel data on income and consumption
  2. Keane and Wasi (**Economic Journal, 20**16) : Labour Supply: The Roles of Human Capital and The Extensive Margin
  3. Iskhakov and Keane (Journal of Econometrics, 2020): "Effects of taxes and safety net pensions on life-cycle labor supply, savings and human capital: The case of Australia"
     3.1 We will look at this paper in detail in the next lecture (nice application of DC-EGM)

Gourinchas and Parker (2002) I

TABLE III

STRUCTURAL ESTIMATION RESULTS

MSM Estimation	Robust Weighting	Optimal Weighting
Discount Factor (β)	0.9598	0.9569
S.E.(A)	(0.0101)	
S.E.(B)	(0.0179)	(0.0150)
Discount Rate ($\beta^{-1}-1$)(%)	4.188	4.507
S.E.(A)	(1.098)	
S.E.(B)	(1.949)	(1.641)
Risk Aversion (ρ)	0.5140	1.3969
S.E.(A)	(0.1690)	
S.E.(B)	(0.1707)	(0.1137)
Retirement Rule:		
γ_0	0.0015	$5.68\,10^{-6}$
S.E.(A)	(3.84)	
S.E.(B)	(3.85)	(16.49)
γ_1	0.0710	0.0613
S.E.(A)	(0.1215)	
S.E.(B)	(0.1244)	(0.0511)
χ^2(A)	175.25	
χ^2(B)	174.10	185.67

Note: MSM estimation for entire group. Standard errors calculated without (A) and with (B) correction for first stage estimation. Cell size is 36,691 households. The last row reports a test of the overidentifying restrictions distributed as a Chi-squared with 36 degrees of freedom. The critical value at 5% is 50.71. Efficient estimates are calculated with a weighting matrix $\widehat{\Omega}$ computed from the robust estimates.

Gourinchas and Parker (2002) II

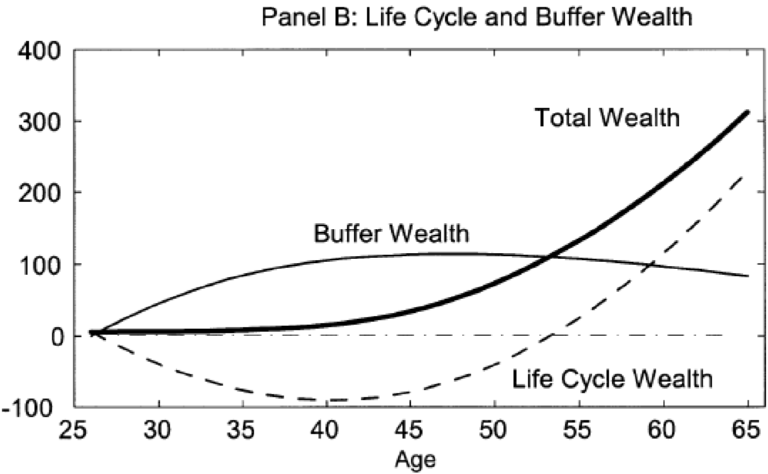

FIGURE 7.—The role of risk in saving and wealth accumulation.

Mathematical Programming with Equilibrium Constraints (MPEC)

- **Idea:** Do not solve the model, treat it as a constraint
- **Example:** Infinite horizon buffer-stock consumption model

$$\begin{aligned}\hat{\theta}, \hat{c}_1, \ldots, \hat{c}_\# &= \arg\max_{\theta, c_1, \ldots, c_\#} \mathcal{L}(\theta) \\ &\text{s.t.} \\ 0 &\leq c_j \leq m_j \\ 0 &\geq \mathcal{E}_j \\ 0 &= (m_j - c_j)\mathcal{E}_j\end{aligned}$$

where $\mathcal{E}_j$ is the j'th Euler-residual

$$\mathcal{E}_j \equiv \beta R \mathbb{E}_t[(G\psi_{t+1}c_{t+1}(\frac{1}{G\psi_{t+1}}Ra_i + \xi_{t+1}))^{-\rho}] - c_j^{-\rho}$$

and $c_{t+1}(\bullet)$ is interpolated using $c_1, c_2, \ldots, c_\#$

- **See Jørgensen (Economic Letters, 2013)**
- Intractable for life cycle models: Here EGM in a nested loop is much, much faster