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<https://github.com/julianmak/academic-notes>

The repository principally contains the compiled products rather than the source for size reasons.

- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 3301 : basic Data Analysis in ocean sciences

Session 10: fun with maps

Outline

(Just overview here; for actual content see Jupyter notebooks)

- ▶ actual fun with maps through `cartopy`
→ map projections
examples with **GEBCO** and **WOA13** data
- ▶ Fourier analysis (cf. 08 time-series analysis)
- ▶ **Empirical Orthogonal Functions** (EOFs)
→ basically PCA but with a spatial component (cf. 04 linear regression)

Digression: some geometry and topology

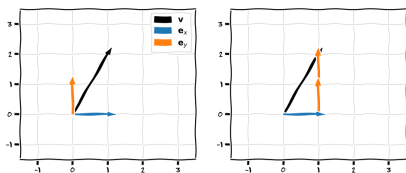


Figure: Demonstration of canonical basis in \mathbb{R}^2 .

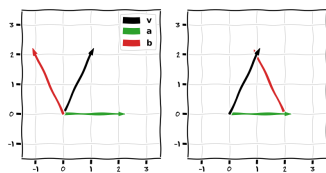


Figure: Demonstration of alternative basis in \mathbb{R}^2 .

$$\begin{aligned}\mathbf{v} &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= 1\mathbf{e}_x + 2\mathbf{e}_y\end{aligned}$$

$$\begin{aligned}\mathbf{v} &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ &= 1\mathbf{a} + 1\mathbf{b}\end{aligned}$$

- multiple choices in representation
 - vector representation depends on choice of basis

Digression: some geometry and topology

- ▶ **location** depends on choice of co-ordinates
- ▶ for the point $x = (1, 1)$ in standard **Cartesian co-ordinates**, we could have

$$x = 1, \quad y = 1$$

- ▶ but we could also have it in **polar co-ordinates**

$$r = \sqrt{2}, \quad \theta = \pi/4$$

with $x = r \cos \theta, y = r \sin \theta$

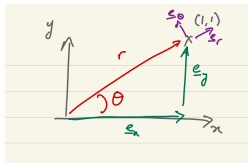


Figure: Polar co-ordinates schematic.

- ▶ some things more natural in polar co-ordinates
 - unit circle is the set of points with $r = 1$ and $\theta \in [0, 2\pi)$, compared to $y = \pm\sqrt{1-x^2}$ for $x \in [-1, 1]$

Digression: some geometry and topology

- ▶ similarly, on a sphere, it is easier to consider **spherical co-ordinates** of (r, θ, ϕ) for radius, longitude and latitude
 - if on surface of Earth, take $r = R_{\text{Earth}}$, so now 2d data
 - otherwise need a tuple (x, y, z) (raw satellite data do this actually)
- ▶ represent (lon, lat) data on 2d plane?

Digression: some geometry and topology

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 - otherwise need a tuple (x, y, z) (raw satellite data do this actually)
 - ▶ represent (lon, lat) data on 2d plane?
- !! problem: sphere is intrinsically ‘curved’ while a finite plane is intrinsically ‘flat’
- unit sphere has **Gaussian curvature** $\kappa = 1$, while plane has $\kappa = 0$
 - **Gauss–Bonnet theorem** tells you κ is related to the **Euler characteristic** χ (cf. think of χ as the sum of angles of a triangle on the surface)

geometry (‘local’, from κ) \leftrightarrow **topology** (‘global’, from χ)

Digression: some geometry and topology

cursed cow

- ▶ **topology** studies global properties

→ e.g. how things are connected

→ cow is smoothly deformable into sphere

→ cup is smoothly deformable into doughnut

- ▶ cow is **not** smoothly deformable into doughnut

→ difference in **genus** (cf. 'holes')

cursed doughnut

Cartopy and map projections

- ▶ to go from sphere to plane, need to introduce a ‘tear’
(actually just removing one point will do it)
- ▶ main upshot: no global **isometry** between sphere and plane
→ i.e. **no distance preserving transformation possible**

Cartopy and map projections

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A **map projection** assigns (lon, lat) to some (x, y) on the plane via some formula, but implication from above is that we can at best, for a global map projection:

1. preserve angles (**conformal map**)
2. preserve areas (**area-preserving map**)
3. do neither

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cartopy package does most of the heavy lifting for you

Cartopy and map projections

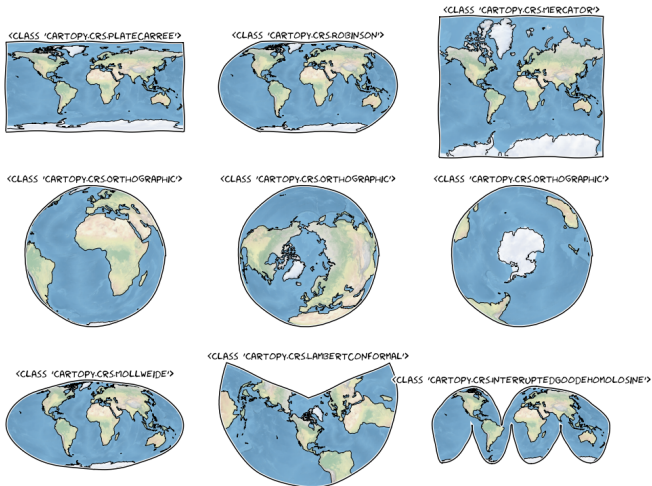


Figure: Sample projections available in `cartopy`. See notebook for `cartopy` usage and syntax.

Cartopy examples

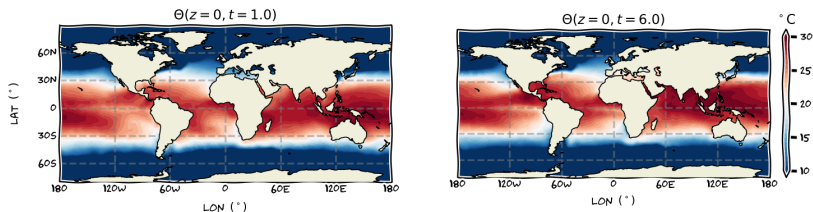


Figure: WOA13 data.

- ▶ WOA13 surface temperature from last time in Jan and Jul
 - xarray to read
 - cartopy Plate Carree projection
 - land features from cartopy

Cartopy examples

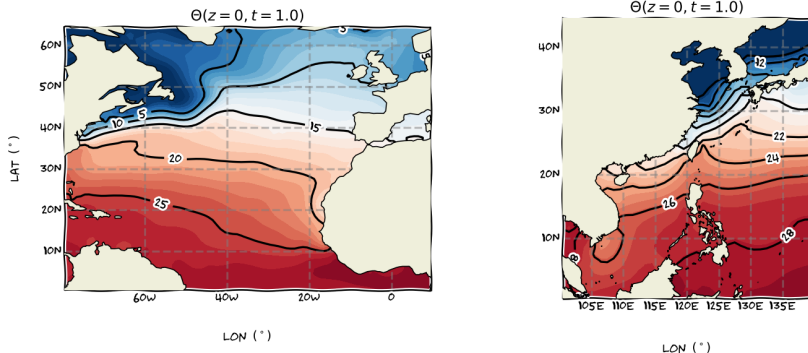


Figure: WOA13 data with zooms.

- WOA13 surface temperature centered over two different regions
 - xarray to read and subset
 - contours overlaid onto the colour plots

Cartopy examples

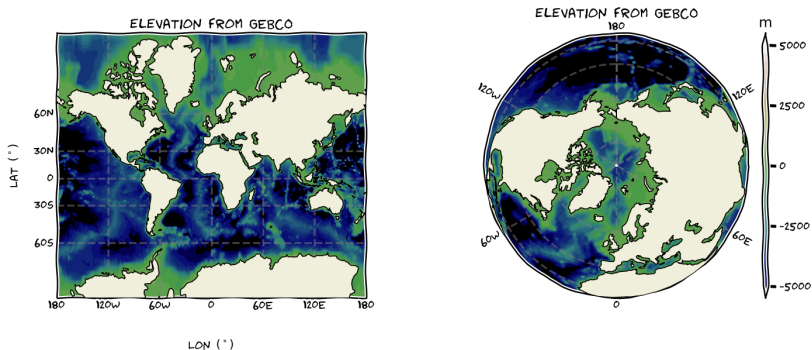


Figure: WOA13 data with zooms.

- GEBCO data using different projections
 - **Mercator** (conformal but really messes area up)
 - Northern **Orthographic** (does not show everything)

Fourier analysis (see 08)

Before for time-series we had

$$f(t) = a_0 + \sum_{k=1}^N a_k \cos(kt) + \sum_{j=1}^N b_j \sin(jt),$$

but there is no reason we can't do

$$f(x, t) = a_0(t) + \sum_{k=1}^N a_k(t) \cos(kx) + \sum_{j=1}^N b_j(t) \sin(jx)$$

- ▶ Fourier amplitudes now function of time
- ▶ k to be interpreted as **wavenumber** instead of **angular frequency**
→ the corresponding quantities will be **wavelength** instead of **period** (see 08 time series for conversion)

!!! dodgy assumption that function is periodic in space!

Fourier analysis: surface current speed

► time series revisited (fix space)

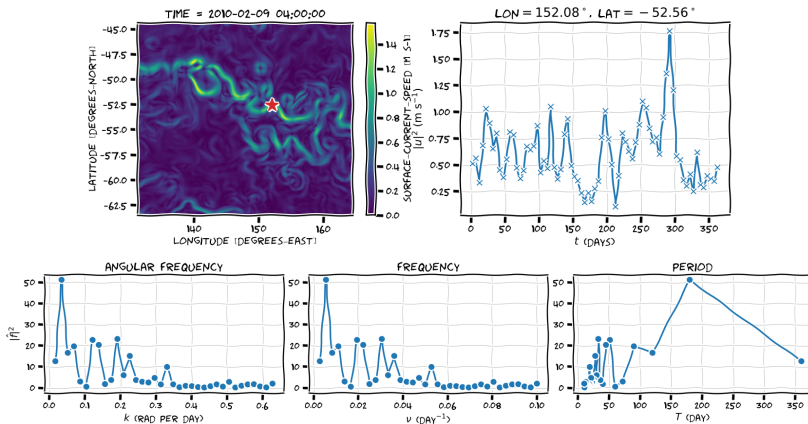


Figure: Fourier analysis of time-series data.

Fourier analysis: surface current speed

- varying in **longitude** (fix latitude and time)

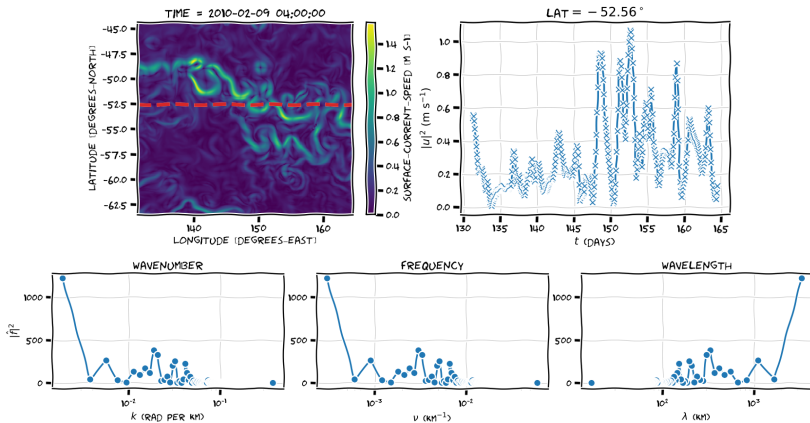


Figure: Fourier analysis of track data.

Fourier analysis: surface current speed

- varying in space (fixed time)

$$f(x, y, t) = \left(a_0(t) + \sum_{k=1}^{N_x} a_k(t) \cos(kx) + \sum_{j=1}^{N_x} b_j(t) \sin(jx) \right) \\ \times \left(c_0(t) + \sum_{l=1}^{N_y} c_l(t) \cos(ly) + \sum_{m=1}^{N_y} b_m(t) \sin(my) \right)$$

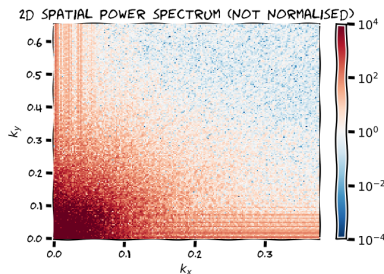


Figure: Fourier analysis of horizontal data at fixed time.

- note here my data is not uniform in k_x and k_y because my data is not equally spaced as such

Fourier analysis: surface current speed

- 2d filtering in Fourier space
 - filtering here is uniform and based on a circle of some radius in **spectral space**

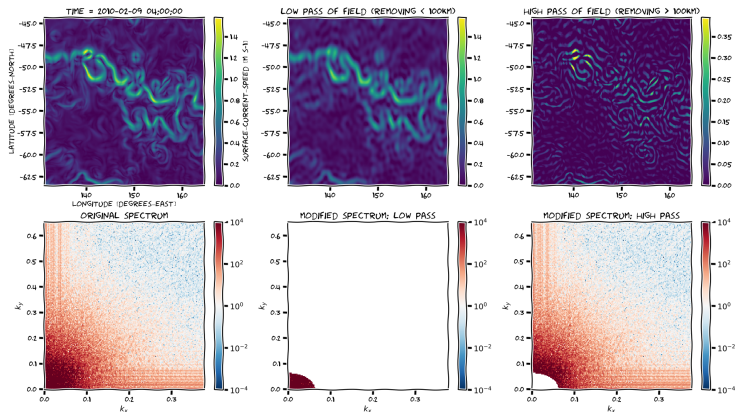


Figure: Filtering based on Fourier power spectrum.

EOFs

Want to do something like

$$f(t, x, y) = \sum_{k=1}^N \text{PC}_k(t) \text{ EOF}_k(x, y)$$

to pick out spatial patterns that capture the most variability

► sound familiar?

EOFs

Want to do something like

$$f(t, x, y) = \sum_{k=1}^N \text{PC}_k(t) \text{ EOF}_k(x, y)$$

to pick out spatial patterns that capture the most variability

- ▶ sound familiar? basically like PCA! (04 regression)
 - $\text{EOF}_k(x, y)$ the **Empirical Orthogonal Function** (EOF)
 - EOF tagged with a **Principal Component** $\text{PC}_k(t)$
- ▶ algorithm and methodology largely the same
 - actually going to use the **Singular Value Decomposition** (SVD cf. diagonalisation of covariance matrix for PCA)
 - going to leverage `scikit-learn` package again
- ▶ EOF analysis finds you a spatial basis via data, while Fourier analysis sets the spatial basis *a priori*

EOFs: work flow

- ▶ start with array containing $f(t, x, y)$, flatten into $f(t, \text{space})$
 - “space” is now the categorisation, and t contains the data points
 - cf. Iris sepal length vs. entries of Iris sepal length etc.
- ▶ preprocessing, but a few choices:
 - de-mean, de-trend, Z-score standard scaling, others...
- ▶ throw into PCA algorithm (or your SVD algorithm if you want)
 - unflatten the resulting EOF(space) back to EOF(x, y)
 - EOFs ranked in variance explained

EOFs: idealised example

Try first for (cf. data made for animation)

$$f(x, y) = \sin(x) \cos(y) \sin(t) + \frac{1}{2} \cos(2x) \cos(2t)$$

- ▶ 1st term as is, circular blobs
- ▶ 2nd term are 'rolls'
- ▶ Fourier analysis would definitively pick out two peaks (which ones? ignoring isolated values of t where individual parts of the function vanish)

Q. what would the EOF analysis do?

EOFs: idealised example

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Q. what would the EOF analysis do?

→ EOF₁ to be the 1st term, with PC₁ $\sim \sin(t)$?

(because of imposed larger amplitude)

→ EOF₂ to be the 2nd term, with PC₂ $\sim \cos(2t)$?

EOFs: idealised example, no preprocessing

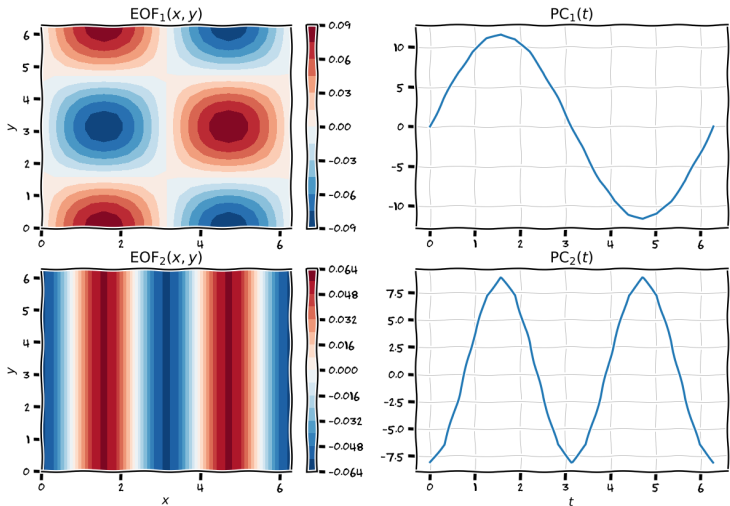


Figure: EOF of idealised data, no preprocessing.

EOFs: idealised example, demean

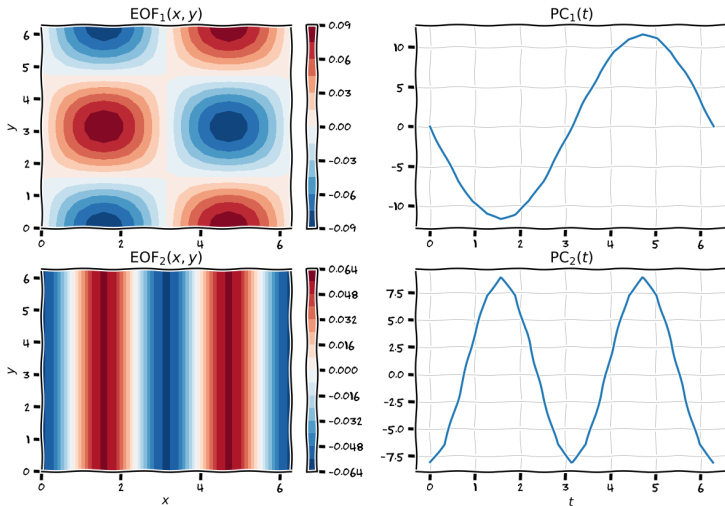


Figure: EOF of idealised data, remove time average per point. Notice a sign flip of EOF and PC 1, but that's ok.

EOFs: idealised example, detrend

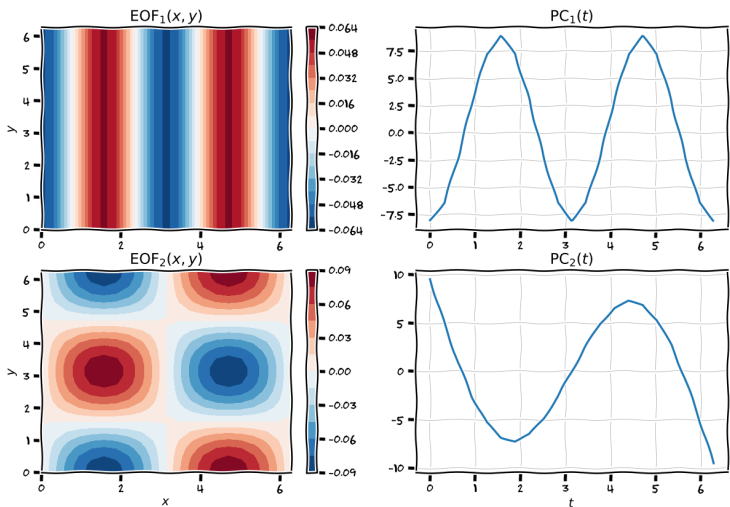


Figure: EOF of idealised data, remove linear trend per point (`scipy.signal.detrend` or use `numpy.polyfit`). Notice 'swapping' of EOF ordering relative to previous case, and PC 2 looks a bit weird.

EOFs: idealised example, Z-score standardisation

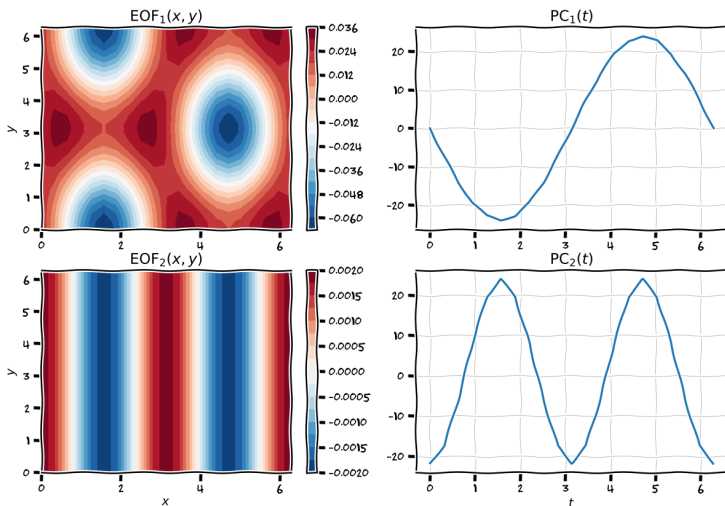


Figure: EOF of idealised data, using Z-score standardisation (`StandardScaler` in `scikit-learn`). Note that while EOF 1 no longer looks like circular blobs, and EOF 2 sign has changed, these are still valid choices of basis.

EOFs: 'real' example

- ▶ provided Extended Reconstructed SST data (full and anomaly version)
 - monthly, from mid 1800s to present day
 - global, 2° spatial resolution
 - already masked
 - anomaly relative to some climatology (!?)

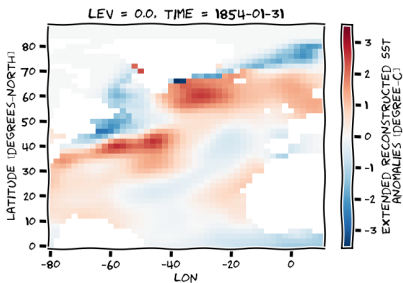


Figure: Raw data plot of SST **anomalies** over Atlantic. Using the anomalies file directly.

EOFs: 'real' example

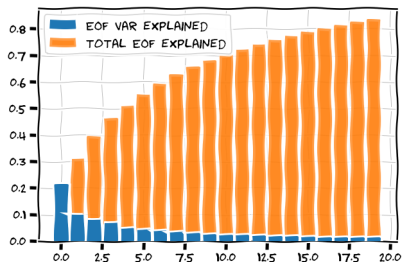
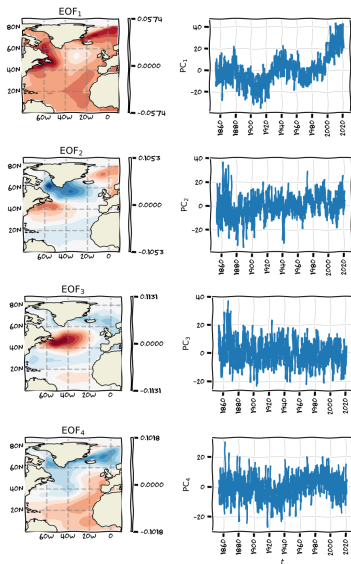


Figure: Percentage and cumulative percentage of variance explained by EOFs.

- EOF analysis as usual
→ using anomalies file directly, not detrending or demeaning here
- variance explained of EOFs are generally pretty low actually...

EOFs: 'real' example



- ▶ EOF 1 is probably global warming, maybe also bits of **Atlantic Multi-decadal Variability** (AMV)
- ▶ EOF 2 should be the SST equivalent of **North Atlantic Oscillation** (NAO)
see ENVS 3004 / OCES 4001)
- ▶ no idea what EOF 3 is but
looks like EOF 2 of Fig. 3 in Buckley et al. 2013
- ▶ EOF 4 also looks a bit like the AMV?

Jupyter notebook

go to 10 Jupyter notebook to get some code practise

- ▶ exercises and code I haven't demonstrated here

→ EOFs with different locations and/or different pre-processing

→ analysis of PCs via Fourier analysis

→ 1d isotropic power spectrum for surface current speed

→ time evolution of spatial power spectrum in time

→ ...