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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 3301 :

basic **Data Analysis** in ocean sciences

Session 10: fun with maps

Outline

(Just overview here; for actual content see Jupyter notebooks)

- actual fun with maps through `cartopy`

  $\rightarrow$ map projections

  examples with GEBCO and WOA13 data
- Fourier analysis (cf. 08 time-series analysis)
- Empirical Orthogonal Functions (EOFs)

  $\rightarrow$ basically PCA but with a spatial component (cf. 04 linear regression)

Digression: some geometry and topology

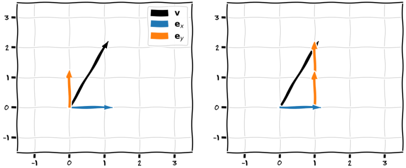

Figure: Demonstration of canonical basis in $\mathbb{R}^2$.

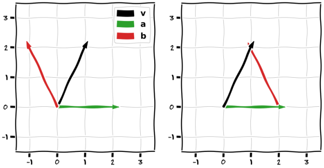

Figure: Demonstration of alternative basis in $\mathbb{R}^2$.

$$\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1\mathbf{e}_x + 2\mathbf{e}_y$$

$$\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} = 1\mathbf{a} + 1\mathbf{b}$$

- multiple choices in representation
  $\rightarrow$ vector representation depends on choice of basis

Digression: some geometry and topology

- location depends on choice of co-ordinates
- for the point $\boldsymbol{x} = (1, 1)$ in standard Cartesian co-ordinates, we could have

$$x = 1, \qquad y = 1$$

- but we could also have it in polar co-ordinates

$$r = \sqrt{2}, \qquad \theta = \pi/4$$

  with $x = r\cos\theta$, $y = r\sin\theta$

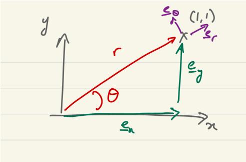

Figure: Polar co-ordinates schematic.

- some things more natural in polar co-ordinates
  $\rightarrow$ unit circle is the set of points with $r = 1$ and $\theta \in [0, 2\pi)$, compared to $y = \pm\sqrt{1 - x^2}$ for $x \in [-1, 1]$

Digression: some geometry and topology

- similarly, on a sphere, it is easier to consider spherical co-ordinates of (r, θ, ϕ) for radius, longitude and latitude

  $\rightarrow$ if on surface of Earth, take $r = R_{\mathrm{Earth}}$, so now 2d data

  $\rightarrow$ otherwise need a tuple (x, y, z) (raw satellite data do this actually)
- represent (lon, lat) data on 2d plane?

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- represent (lon, lat) data on 2d plane?

!! problem: sphere is intrinsically 'curved' while a finite plane is intrinsically 'flat'

$\rightarrow$ unit sphere has Gaussian curvature $\kappa = 1$, while plane has $\kappa = 0$

$\rightarrow$ Gauss–Bonnet theorem tells you κ is related to the Euler characteristic χ (cf. think of χ as the sum of angles of a triangle on the surface)

geometry ('local', from κ) $\leftrightarrow$ **topology** ('global', from χ)

Digression: some geometry and topology

cursed cow

cursed doughnut

- topology studies global properties

  → e.g. how things are connected

  → cow is smoothly deformable into sphere

  → cup is smoothly deformable into doughnut

- cow is **not** smoothly deformable into doughnut

  → difference in genus (cf. ‘holes’)

Cartopy and map projections

- to go from sphere to plane, need to introduce a ‘tear’

  (actually just removing one point will do it)

- main upshot: no global isometry between sphere and plane

  $\rightarrow$ i.e. **no distance preserving transformation possible**

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A map projection assigns (lon, lat) to some (x, y) on the plane via some formula, but implication from above is that we can at best, for a global map projection:

1. preserve angles (conformal map)
2. preserve areas (area-preserving map)
3. do neither

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`cartopy` package does most of the heavy lifting for you

Cartopy and map projections

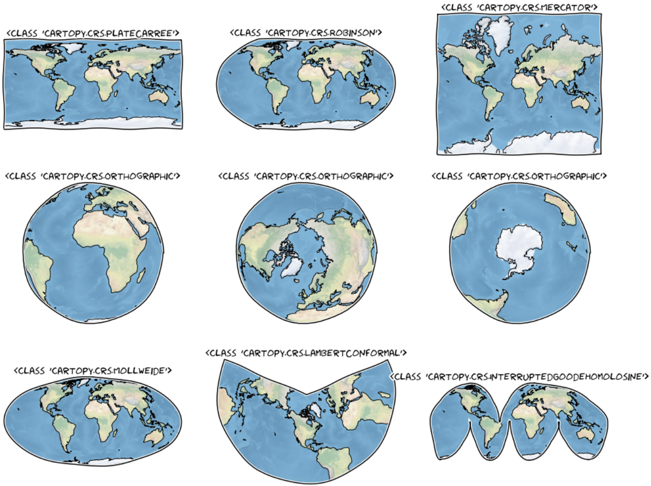

Figure: Sample projections available in `cartopy`. See notebook for `cartopy` usage and syntax.

Cartopy examples

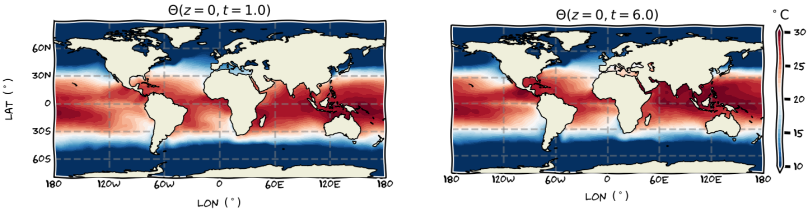

Figure: WOA13 data.

- WOA13 surface temperature from last time in Jan and Jul
  $\rightarrow$ xarray to read
  $\rightarrow$ cartopy Plate Carree projection
  $\rightarrow$ land features from cartopy

Cartopy examples

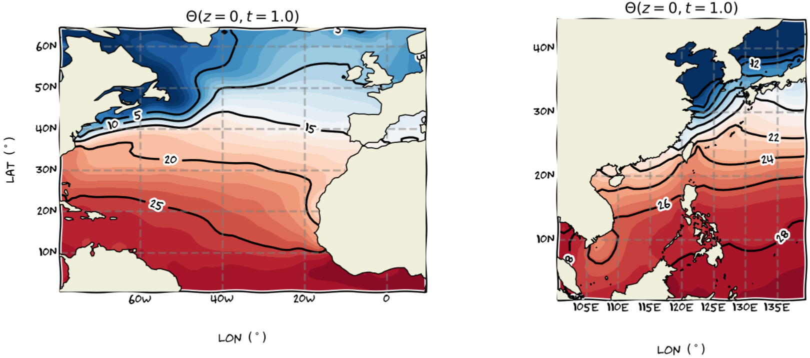

Figure: WOA13 data with zooms.

- WOA13 surface temperature centered over two different regions
  → xarray to read and subset
  → contours overlaid onto the colour plots

Cartopy examples

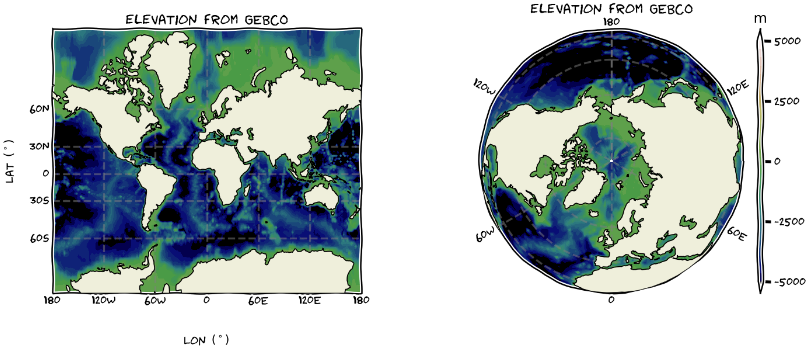

Figure: WOA13 data with zooms.

- GEBCO data using different projections
  → Mercator (conformal but really messes area up)
  → Northern Orthographic (does not show everything)

Fourier analysis (see 08)

Before for time-series we had

$$f(t) = a_0 + \sum_{k=1}^{N} a_k \cos(kt) + \sum_{j=1}^{N} b_j \sin(jt),$$

but there is no reason we can't do

$$f(x,t) = a_0(t) + \sum_{k=1}^{N} a_k(t) \cos(kx) + \sum_{j=1}^{N} b_j(t) \sin(jx)$$

- Fourier amplitudes now function of time
- k to be interpreted as wavenumber instead of angular frequency

  $\rightarrow$ the corresponding quantities will be wavelength instead of period (see 08 time series for conversion)

!!! dodgy assumption that function is periodic in space!

Fourier analysis: surface current speed

- time series revisited (fix space)

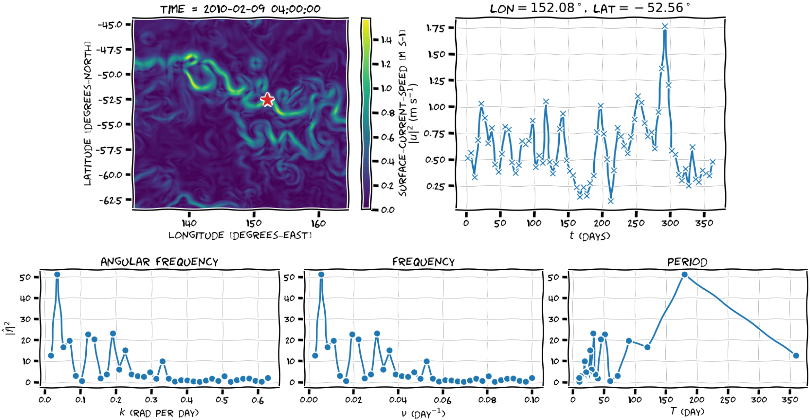

Figure: Fourier analysis of time-series data.

Fourier analysis: surface current speed

- varying in **longitude** (fix latitude and time)

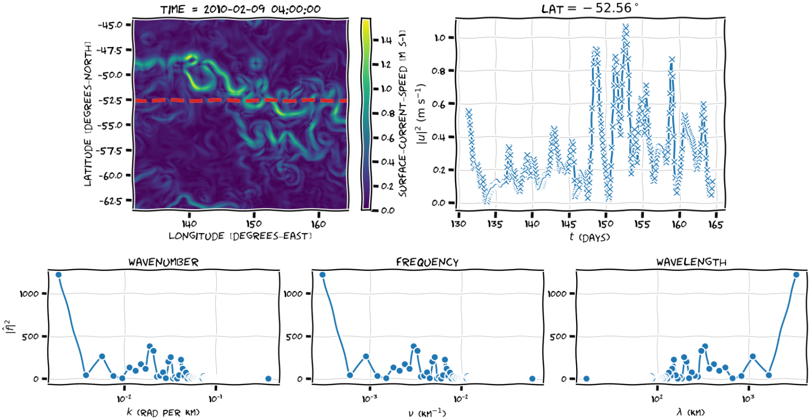

Figure: Fourier analysis of track data.

Fourier analysis: surface current speed

- varying in space (fixed time)

$$f(x,y,t) = \left(a_0(t) + \sum_{k=1}^{N_x} a_k(t)\cos(kx) + \sum_{j=1}^{N_x} b_j(t)\sin(jx)\right)$$
$$\times \left(c_0(t) + \sum_{l=1}^{N_y} c_l(t)\cos(ly) + \sum_{m=1}^{N_y} b_m(t)\sin(my)\right)$$

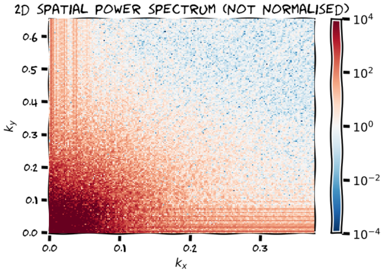

Figure: Fourier analysis of horizontal data at fixed time.

- note here my data is not uniform in k_x and k_y because my data is not equally spaced as such

Fourier analysis: surface current speed

- 2d filtering in Fourier space
  $\rightarrow$ filtering here is uniform and based on a circle of some radius in **spectral space**

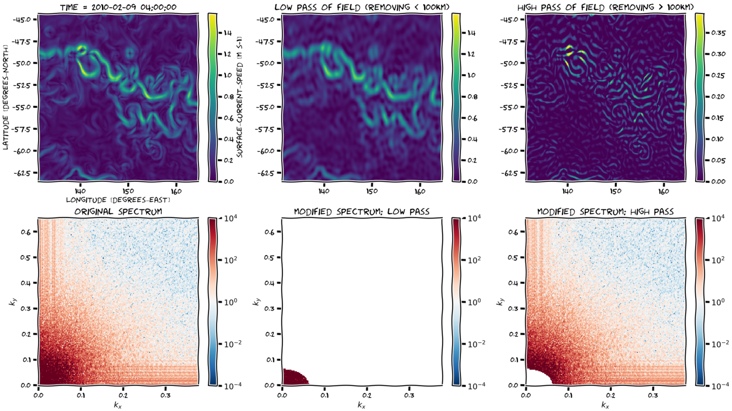

Figure: Filtering based on Fourier power spectrum.

EOFs

Want to do something like

$$f(t, x, y) = \sum_{k=1}^{N} \mathrm{PC}_k(t)\, \mathrm{EOF}_k(x, y)$$

to pick out spatial patterns that capture the most variability

- sound familiar?

EOFs

Want to do something like

$$f(t, x, y) = \sum_{k=1}^{N} \mathrm{PC}_k(t)\, \mathrm{EOF}_k(x, y)$$

to pick out spatial patterns that capture the most variability

- sound familiar? basically like PCA! (04 regression)

  $\rightarrow$ $\mathrm{EOF}_k(x, y)$ the Empirical Orthogonal Function (EOF)

  $\rightarrow$ EOF tagged with a Principal Component $\mathrm{PC}_k(t)$
- algorithm and methodology largely the same

  $\rightarrow$ actually going to use the Singular Value Decomposition (SVD cf. diagonalisation of covariance matrix for PCA)

  $\rightarrow$ going to leverage `scikit-learn` package again

- EOF analysis finds you a spatial basis via data, while Fourier analysis sets the spatial basis *a priori*

EOFs: work flow

- start with array containing $f(t, x, y)$, flatten into $f(t, \text{space})$
  $\rightarrow$ "space" is now the categorisation, and t contains the data points
  $\rightarrow$ cf. Iris sepal length vs. entries of Iris sepal length etc.

- preprocessing, but a few choices:
  $\rightarrow$ de-mean, de-trend, Z-score standard scaling, others...

- throw into PCA algorithm (or your SVD algorithm if you want)
  $\rightarrow$ unflatten the resulting EOF(space) back to EOF(x, y)
  $\rightarrow$ EOFs ranked in variance explained

EOFs: idealised example

Try first for (cf. data made for animation)

$$f(x,y) = \sin(x)\cos(y)\sin(t) + \frac{1}{2}\cos(2x)\cos(2t)$$

- 1st term as is, circular blobs
- 2nd term are 'rolls'
- Fourier analysis would definitively pick out two peaks (which ones? ignoring isolated values of t where individual parts of the function vanish)

Q. what would the EOF analysis do?

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Q. what would the EOF analysis do?

$\rightarrow$ EOF_1 to be the 1st term, with $\mathrm{PC}_1 \sim \sin(t)$?

(because of imposed larger amplitude)

$\rightarrow$ EOF_2 to be the 2nd term, with $\mathrm{PC}_2 \sim \cos(2t)$?

EOFs: idealised example, no preprocessing

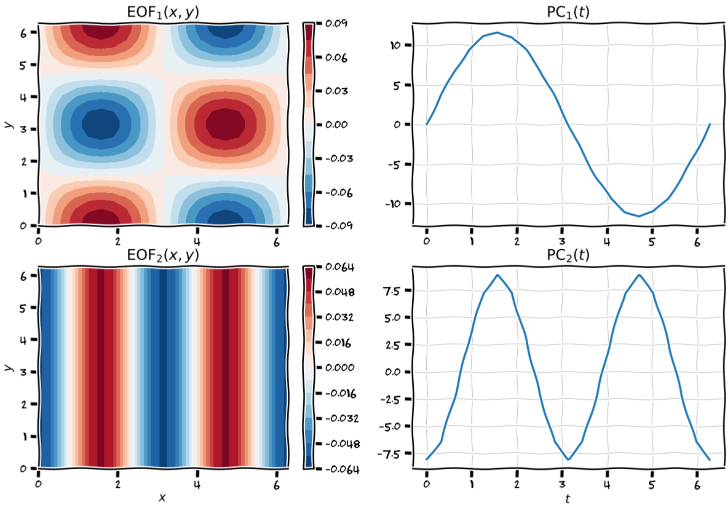


Figure: EOF of idealised data, no preprocessing.

EOFs: idealised example, demean

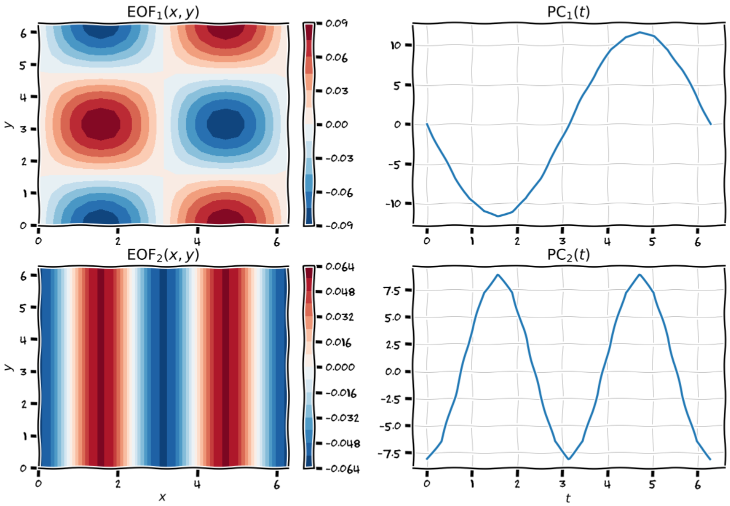


Figure: EOF of idealised data, remove time average per point. Notice a sign flip of EOF and PC 1, but that's ok.

EOFs: idealised example, detrend

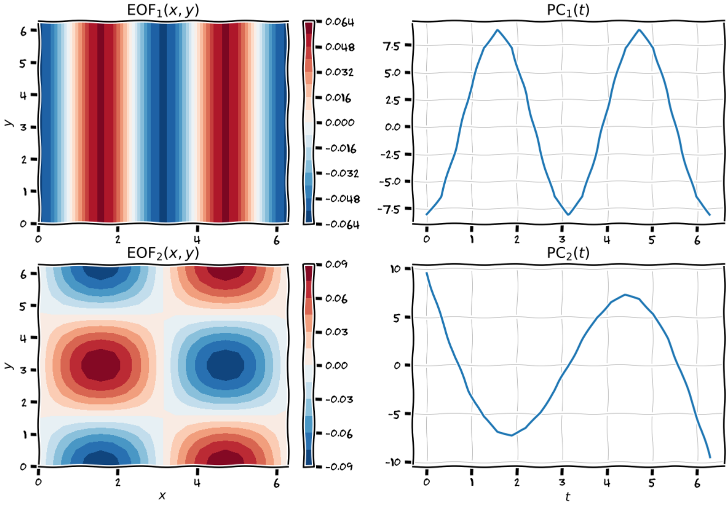


Figure: EOF of idealised data, remove linear trend per point (`scipy.signal.detrend` or use `numpy.polyfit`). Notice ‘swapping’ of EOF ordering relative to previous case, and PC 2 looks a bit weird.

EOFs: idealised example, Z-score standardisation

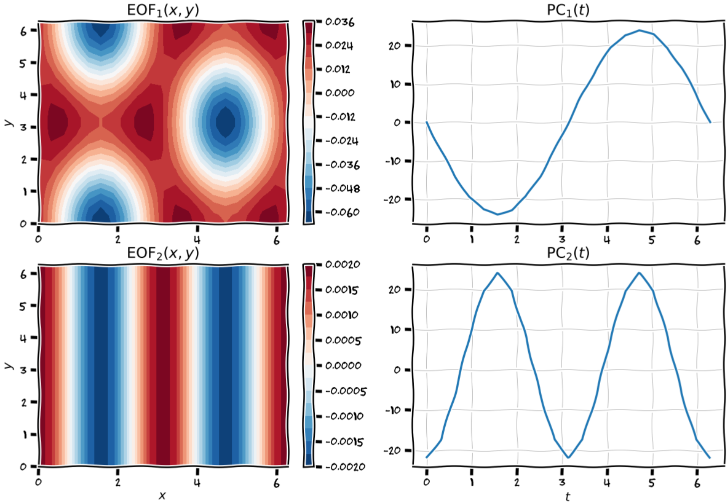


Figure: EOF of idealised data, using Z-score standardisation (`StandardScaler` in `scikit-learn`). Note that while EOF 1 no longer looks like circular blobs, and EOF 2 sign has changed, these are still valid choices of basis.

EOFs: ‘real’ example

- provided Extended Reconstructed SST data (full and anomaly version)

  → monthly, from mid 1800s to present day

  → global, 2° spatial resolution

  → already masked

  → anomaly relative to some climatology (!?)

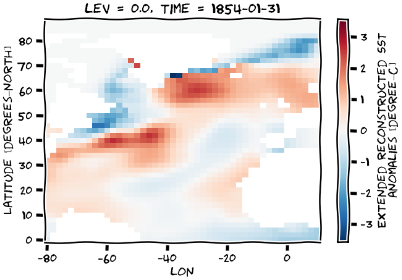


Figure: Raw data plot of SST **anomalies** over Atlantic. Using the anomalies file directly.

EOFs: ‘real’ example

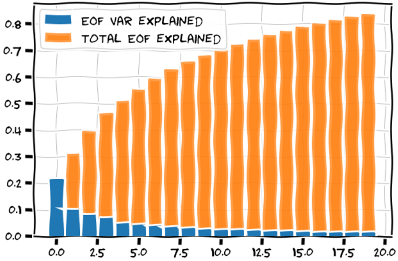
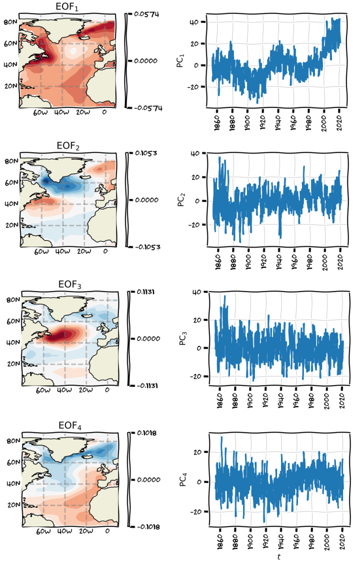


Figure: Percentage and cumulative percentage of variance explained by EOFs.

- EOF analysis as usual

  → using anomalies file directly, not detrending or demeaning here
- variance explained of EOFs are generally pretty low actually...

EOFs: ‘real’ example



- EOF 1 is probably global warming, maybe also bits of Atlantic Multi-decadal Variability (AMV)
- EOF 2 should be the SST equivalent of North Atlantic Oscillation (NAO see ENVS 3004 / OCES 4001)
- no idea what EOF 3 is but looks like EOF 2 of Fig. 3 in Buckley et al. 2013
- EOF 4 also looks a bit like the AMV?

Jupyter notebook

go to 10 Jupyter notebook to get some code practise

- exercises and code I haven't demonstrated here

  $\rightarrow$ EOFs with different locations and/or different pre-procesing

  $\rightarrow$ analysis of PCs via Fourier analysis

  $\rightarrow$ 1d isotropic power spectrum for surface current speed

  $\rightarrow$ time evolution of spatial power spectrum in time

  $\rightarrow$ ...