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OCES 3301 :

basic **Data Analysis** in ocean sciences

Session 5: statistical tests

Outline

(Just overview here; for actual content see Jupyter notebooks)

- brief introduction to probability

  → pdfs, Gaussian, law of large numbers, CLT

  → Confidence Intervals

- hypothesis testing (mostly focus on *analysis*)

  → null hypothesis

  → confidence threshold

  → computing the test statistic (Z-test example), p-values

  → banana skins

  → Type I and II errors, statistical power (cf. experimental design)

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  $\rightarrow$ I don’t use statistical testing in my work

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  $\rightarrow$ the statistical tests here are more **classical**

  $\rightarrow$ arguably more technical (not really though...)

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- I think there are better tools to use (Bayesian formulations, confidence intervals), but not touching those here
  $\rightarrow$ the statistical tests here are more **classical**
  $\rightarrow$ arguably more technical (not really though...)

- not going to talk about experimental design, though it is probably more important than the statistical analysis

Lasciate ogne speranza, voi ch'intrate

Figure: Cursed image.

Motivation: sea cucumber

Figure: Moldy sea cucumber.

Suppose sea cucumber has some distribution of weight that we can measure:

Q. does change in diet affect her weight?

Q. does exercise regime affect her weight?

- might expect to, but how do we distinguish noise (e.g. natural random fluctuations) with “real” effect?

Motivation: sea cucumber

Figure: Moldy sea cucumber.

e.g. say from samples,

$$\{\mu_1 = 3.00, \quad \sigma_1 = 0.5\}$$

$$\{\mu_2 = 3.20, \quad \sigma_2 = 0.5\}$$

- mean is different, so has effect?
  $\rightarrow$ but could just be a fluke?

- hypothesis testing as a tool to say whether differences are statistically significant

Some probability

- assign some real value between 0 and 1 to some event
  $\rightarrow$ 0 is never, 1 is certainly
- e.g. throwing a fair coin, two events, expect $p = 1/2$

Q. that's in principle, what if you actually try it?

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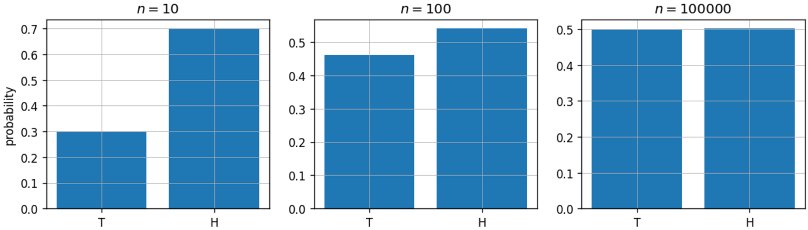

Figure: Result of hypothetical coin tosses displayed in bar graph.

Some probability

- going back to the sea cucumber, we might have a sample of weights and display it in a histogram

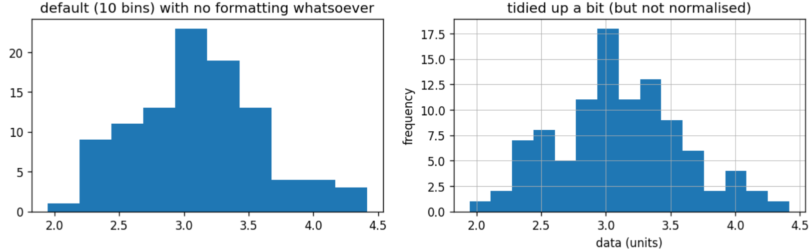

Figure: Result of hypothetical sea cucumber weight.

- obtained form binning procedure
  $\rightarrow$ probability is related here to **integral** of the graph

Some probability

- histogram related to the probability distribution function (**pdf**) of the underlying sample

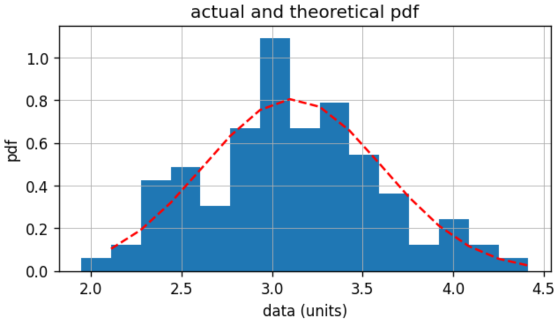

Figure: Result of hypothetical sea cucumber weight with Gaussian pdf.

Q. where did I get the red line from though?

Some probability

Gaussian or normal distribution has pdf

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

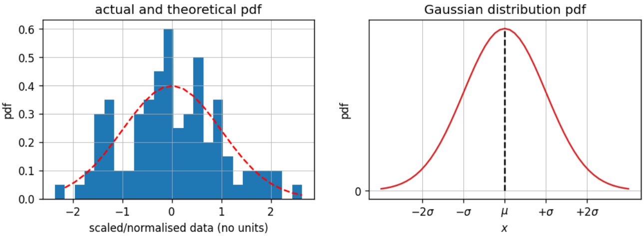

Figure: The Gaussian pdf (with units deliberately omitted). Obtain probability from an integral.

- approximate population (μ, σ) from sample $(\bar{x}, s)$

Q. what would the pdf of the uniform distribution look like?

Some probability

Point here is that if you have the pdf you have basically everything

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Theorem (Central Limit Theorem (CLT))

(Very loosely) If your sample is large enough, under fairly general conditions (!) you can approximate most data distributions as a Gaussian distribution, even if the underlying distribution is not necessarily Gaussian.

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- for large enough samples, you can fit it to a Gaussian pdf...

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- for large enough samples, you can fit it to a Gaussian pdf...
- ...and if you have the pdf you have basically everything!

Some probability

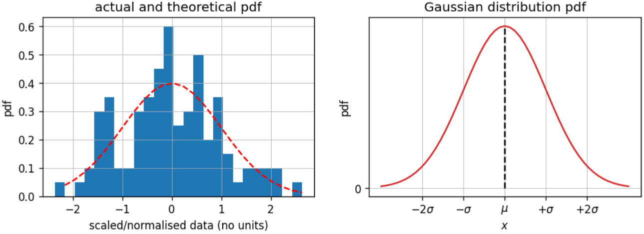

Figure: The Gaussian pdf (with units deliberately omitted). Obtain probability from an integral.

- **68-95-99.7 rule**, 68, 95 and 99.7% of the data lies within 1, 2 and 3 s.t.d. of the mean

  $\rightarrow$ whenever CLT applies (which is quite often!)

Some probability

e.g.

$$p(-\sigma < z < \sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{\sigma} \mathrm{e}^{-z^2/2}\, \mathrm{d}z \approx 0.68$$

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Flip it around: for the some given P, find the $\tilde{z}$ such that

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Confidence Interval

- the interval that contains P amount of probability

  $\rightarrow$ e.g. **95% confidence interval** for Gaussian data would be around $(-2\sigma, 2\sigma)$ $(-1.96\sigma, 1.96\sigma)$ is more accurate but whatever...

Some probability

Z-score or standardised scores: with samples x_i, define

$$z_i = \frac{x_i - \mu}{\sigma}.$$

- essentially re-scaled Gaussian
  $\rightarrow$ cf. what was done for the PCA two sessions ago
  $\rightarrow$ allows somewhat of a like-for-like comparison

Back to the sea cucumber

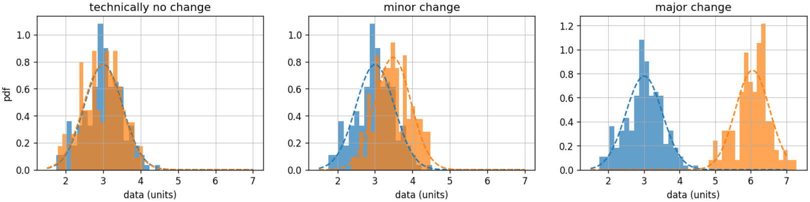

Figure: Control and varied sample distributions and associated Gaussian pdf.

- we are basically dealing with samples (going to assume CLT holds here)

Q. variability in data always exist, so how to distinguish change from noise?

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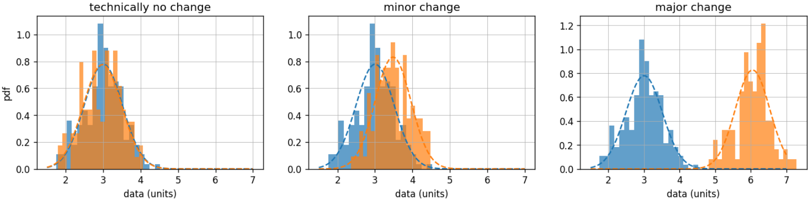

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Q. variability in data always exist, so how to distinguish change from noise?

? non-overlapping confidence intervals?

$\rightarrow$ quite strict (and **under-powered**; see later)

Hypothesis testing

A more standard and routine (doesn't mean it's a good thing necessarily):
hypothesis testing and computing test statistics

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cf. proof by contradiction: want to proof X, so

- assume *not* X
- start from not X, logically derive consequences, avoiding illegal logical manoeuvres
- come to a contradiction
- if no illegal manoeuvres, then initial assumption must be false, and there X is true ∎

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Hypothesis testing is similar (though logically weaker in some sense):

Start with a null hypothesis H_0 (opposite to what you want to show usually)

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  2. or H_0 is incompatible with data
- if latter, reject H_0, and there is statistical evidence in support for *not* H_0 (which is the thing you wanted anyway)

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e.g. sea cucumber, want to know if diet has any effect on weight

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$\rightarrow$ for Gaussian pdf, corresponds to Z-score of around 2, because 95% CI is around $(-2\sigma, +2\sigma)$

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compute : compute Z-statistics (see notebook)

conclude : if Z-statistic large or corresponding p-value small, then reject H_0

(see actual code syntax in notebook)

Z-test (demonstration of sorts here)

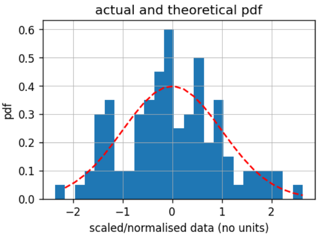
- Z-test calculates Z-statistic with sample mean $\overline{x}$

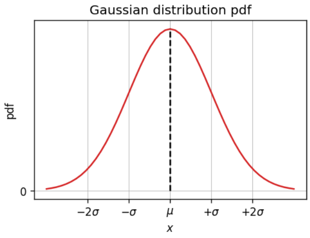
$$Z = \frac{\overline{x} - \mu}{\sigma / \sqrt{N}}$$

$\rightarrow$ with H_0 the mean is the **same**

$\rightarrow$ if Z big enough (so the associated p-value is small) then evidence to reject H_0

$\rightarrow$ Gaussian, large samples, known σ (could approximate with s)





Easy right?

Easy right?

Figure: Banana says no.

- banana skins in no particular order (there are so many of them)...

Banana skin 1: H_0

What if you fail to reject the null hypothesis?

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  $\rightarrow$ it really just means you can't say anything

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- that's it, **you fail to reject the null hypothesis**, no more and no less

  $\rightarrow$ this does NOT mean H_0 is true, nor that being able to reject H_0 means H_0 is false

  $\rightarrow$ it really just means you can't say anything

- cf. proof by contradiction: not being able to find a contradiction could mean

  $\rightarrow$ there really is nothing there

  $\rightarrow$ you aren't looking hard enough

Banana skin 2: p-values and H_0

For $\alpha = 0.05$ and I reject H_0, so test tells me H_0 is only 5% likely to be true

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For $\alpha = 0.05$ and I reject H_0, so test tells me H_0 is only 5% likely to be true

- (frequentist point of view) H_0 is either true or false, it can't be 5% true
- probability of $p = 0.05$ is
  $\times$ hypothesis given data $p(H_0|x_i)$
  ✓ data given hypothesis $p(x_i|H_0)$
- rule of thumb (frequentist view): don't assign probabilities to hypotheses

Banana skin 3: p-values

$\alpha = 0.05$ **and I reject** H_0**, so what I observed would have a probability of** 5% **that is was due to noise**

Banana skin 3: p-values

$\alpha = 0.05$ **and I reject** H_0**, so what I observed would have a probability of** 5% **that is was due to noise**

- again no, since p-value is tagged with the null hypothesis, i.e., I would have observed this signal *given* the null hypothesis

Banana skin 4: *p*-values

$\alpha = 0.05$ **is the gold standard**

Banana skin 4: p-values

$\alpha = 0.05$ **is the gold standard**

- $10^{10^{10}}$ NO!!!

Banana skin 4: p-values

$\alpha = 0.05$ **is the gold standard**

- $10^{10^{10}}$ NO!!!

  $\rightarrow$ comes from (Ronald) Fisher's paper in the 30s or so, when dealing with small samples (≈ 20?)

  $\rightarrow$ can easily be abused to generate false-positives with multiple testing or large sampling (see notebook)

- just a convention (and prone to abuse)

  $\rightarrow$ e.g. particle physics uses 5σ ($\alpha = 0.0000003$, or 1 in 3.5 million, partly because of multiple sampling going on)

Banana skin 5: p-values

My test statistic is large or my p-value is small, so my result is extremely important

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- also no

Banana skin 5: p-values

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- also no

  → statistical significance is not practical significance, p-values can't tell you the latter

  → H_0 statements are very broad: H_0 is no change, not H_0 is there *is* change, but it doesn't tell you how much change

- need interpretation: 1 kg difference in a sea cucumber is not the same as 1 kg difference in a whale

Type I and II errors

Type I errors (false-positives)

- rejecting H_0 when H_0 is true

  $\rightarrow$ related to choice of significance α

Type II errors (false-negatives)

- fail to reject H_0 when H_0 is false

	H_0 true	H_0 false
reject H_0	Type I	✓
fail to reject H_0	✓	Type II

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  $\rightarrow$ related to choice of significance α

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- fail to reject H_0 when H_0 is false

	H_0 true	H_0 false
reject H_0	Type I	✓
fail to reject H_0	✓	Type II

or, with H_0 = someone is innocent,

	innocent	murderer
found guilty	wrongful conviction	✓
found not guilty	✓	fail to prosecute

Type I and II errors

Type II errors β related to statistical power $1 - \beta$

- really to do with experimental design
  $\rightarrow$ choice of sample size to detect effect (see notebook)

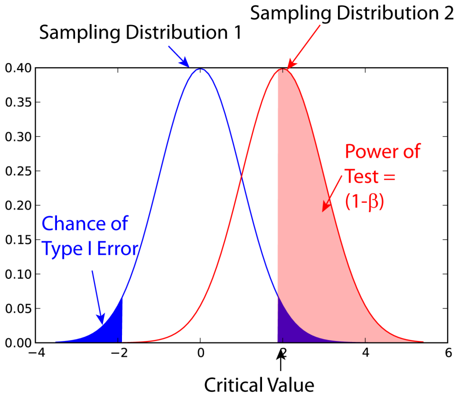

Figure: Graphical demonstration of Type I and II errors. From Wikipedia.

Bad practices 1: “torturing the data until it confesses”

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This statistical test didn’t work so I will use another one, or

These are outliers, so I will get rid of those, or

I will sub-sample dataset and try again etc.

Bad practices 1: "torturing the data until it confesses"

This statistical test didn't work so I will use another one, or

These are outliers, so I will get rid of those, or

I will sub-sample dataset and try again etc.

- $\alpha = 0.05$ so 5% of the time you will reject the null hypothesis when you shouldn't have (Type I error, false-positives)

  $\rightarrow$ multiple testing you could be sampling that 5%

  $\rightarrow$ issue of stop when you get a hit (DON'T!!!)

  (see notebook for an example of this)

- leads to false discoveries (see notebook for an example to do with academic publishing)

Bad practices 2: report only reject or not reject

Reporting reject/fail to reject only

- report the full p-value

  $\rightarrow$ above/below boundary is not saying whether hypothesis is true/false (banana skin 2)

  $\rightarrow$ the threshold is a convention (banana skin 4)

Bad practices 3: being hung up on the analysis

The analysis part is so hard I need to pay most of my attention to it

- if your experimental design is faulty than one would hope (!) that no amount of wizardry will fix that...

  $\rightarrow$ have a think about the validity of design and tools also

Jupyter notebook

Probably more but this is surely getting tedious... go to 05 Jupyter notebook to get some code practise

- RNGesus and probability
- written overview of hypothesis testing

  $\rightarrow$ one example using Z-tests

Statistics is just a <u>tool</u>, no more and no less

- YOU are the user and the onus is on YOU to know enough about to tool to not abuse it