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<https://github.com/julianmak/academic-notes>

The repository principally contains the compiled products rather than the source for size reasons.

- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCEs 3301 : basic Data Analysis in ocean sciences

Session 6: statistical tests

March 15, 2022

Outline

(Just overview here; for actual content see Jupyter notebooks)

- ▶ recall hypothesis testing (mostly focus on *analysis*)
 - null hypothesis etc.
 - Z-test
 - (Student's) t -test
 - χ^2
 - ANOVA + F -test
- ▶ some others

Recall: sea cucumber



Figure: Moldy sea cucumber.

e.g. say from samples,

$$\{\mu_1 = 3.00, \quad \sigma_1 = 0.5\}$$

$$\{\mu_2 = 3.20, \quad \sigma_2 = 0.5\}$$

- ▶ mean is different, so has effect?
→ but could just be a fluke?
- ▶ hypothesis testing as a tool to say whether differences are statistically significant

Recall: Hypothesis testing

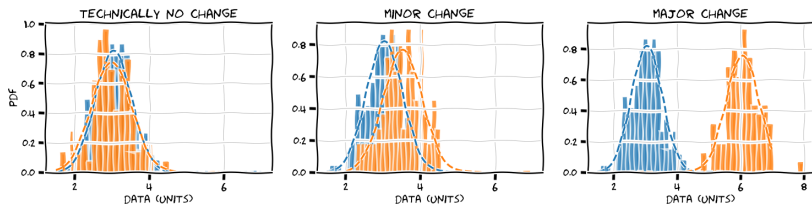


Figure: Control and varied sample distributions and associated Gaussian pdf.

- ▶ we are basically dealing with samples (going to assume CLT holds here)
- Q. variability in data always exist, so how to distinguish change from noise?
- ? non-overlapping confidence intervals?
 - quite strict (and **under-powered**; see later)

Recall: Hypothesis testing

Start with a **null hypothesis** H_0 (opposite to what you want to show usually)

- ▶ assume H_0
- ▶ decide test and significance level (depends on the thing you want to show)
- ▶ compute test statistics (depends on test)
- ▶ if associated probability of computed test statistic is low, then it is either:
 1. a really surprising result
 2. or H_0 is incompatible with data
- ▶ if latter, reject H_0 , and there is **statistical evidence** in support for *not* H_0 (which is the thing you wanted anyway)

Recall: Hypothesis testing

e.g. sea cucumber, want to know if diet has any effect on weight

H_0 : diet has **NO** bearing on weight

test : large enough samples, assume Gaussian statistics, do two-tailed **Z-test**

α : choose $\alpha = 0.05$

→ how far you are in tails of the pdf

→ for Gaussian pdf, corresponds to Z-score of around 2, because 95% CI is around $(-2\sigma, +2\sigma)$

compute : compute Z-statistics (see notebook)

conclude : if Z-statistic large or corresponding **p-value** small, then reject H_0

(see actual code syntax in notebook)

Recall: Type I and II errors

Type I errors (false-positives)

- ▶ rejecting H_0 when H_0 is true
→ related to choice of significance α

Type II errors (false-negatives)

- ▶ fail to reject H_0 when H_0 is false

| | H_0 true | H_0 false |
|----------------------|------------|-------------|
| reject H_0 | Type I | ✓ |
| fail to reject H_0 | ✓ | Type II |

or, with H_0 = someone is innocent,

| | innocent | murderer |
|------------------|---------------------|-------------------|
| found guilty | wrongful conviction | ✓ |
| found not guilty | ✓ | fail to prosecute |

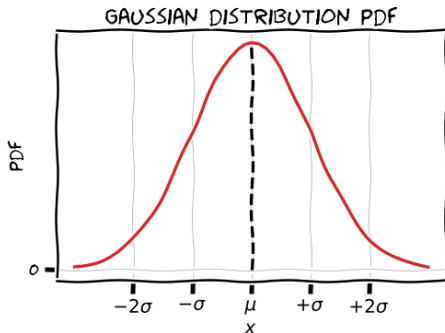
Recall: Z-test

- Z-test calculates Z-statistic with sample mean \bar{x}

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{N}}$$

→ if Z big enough (so the associated p -value is small) then evidence to reject H_0

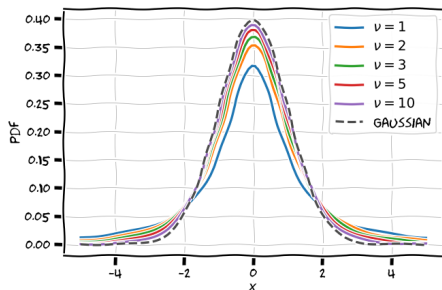
→ Gaussian, large samples, known σ (could approximate with s)



(Student's) t -test

- ▶ Gaussian data, **small** samples, unknown σ
- ▶ t -statistic pdf is

$$p(x) = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2},$$



→ ν ('nu') is called the **degree of freedom** (d.o.f.)

→ Γ ('gamma') is the **Gamma function**

(Student's) t -test

- ▶ Z-test statistic is so easy to compute there is no inbuilt function (?)...

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- ▶ ...but t -tests does exist in e.g. `scipy` (see notebook)
 - d.o.f., related to sample size, is computed automatically
 - with H_0 that mean is the **same**
 - gives t -statistic and p -values

(Student's) t -test

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- ▶ ...but t -tests does exist in e.g. `scipy` (see notebook)
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- ▶ a few types
 - one sample (test against population mean)
 - two sample paired (test against each other's means)
 - two sample independence / unpaired (as above)

χ^2 -test

- categorical data with discrete probabilities (e.g. dice, population distributions)
- χ^2 -test ('kai squared') pdf

$$p(x) = \frac{x^{\nu/2-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)} \quad \text{if } x > 0,$$

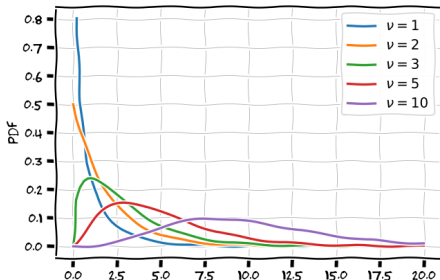


Figure: χ^2 pdf.

χ^2 test

- ▶ scipy has this (see notebook)
 - computes d.o.f.
 - with H_0 that means are the **same**
 - gives χ^2 -statistic and p -values
- ▶ sample applications
 - how to tell if a die (singular of dice) is 'fair'? (see notebook)
 - sea cucumber activity affected by seasonality
 - some ocean applications you can make up
- ▶ robust but "weak"
 - tells you there are deviations of distribution, but doesn't tell you where

ANOVA and F -test

- ▶ to probe whether the **means** in groups of data are different to each other, by analysing their **variances**
→ ANOVA = ANalysis Of **VA**riance
- ▶ idea: if variance between groups dominates variance within individual group, then the means **between** groups are significant
→ high F regime
- ▶ F -statistic pdf looks like χ^2
→ one-way F -test works on H_0 = all means are the same
→ large F -statistic is p value small

ANOVA and F -test

- ▶ use this to avoid **multiple testing** between pairs of data
→ reduce false-positives / Type I errors
- ▶ robust but “weak”
→ tells you there is/are pair(s) with different means, but doesn't tell you where
- ▶ not the only use, but not going to go through the others
- ▶ beware of assumptions going into test
→ e.g. normality, homogeneity, ...

Other pre/post-analysis tests

- ▶ plot out the data!
- ▶ Shapiro–Wilk's test
→ normality of data
- ▶ Levene and/or Bartlett's test
→ homogeneity
- ▶ Tukey's Honest Significance Test
→ post-analysis for F -test
→ sometimes Tukey's range test, Tukey HSD, etc.

Jupyter notebook

go to 06 Jupyter notebook to get some code practise

- ▶ all banana skins previously apply here

Statistics is just a tool, no more and no less

- ▶ YOU are the user and the onus is on YOU to know enough about to tool to not abuse it