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- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 3301 : basic Data Analysis in ocean sciences

Session 5: statistical tests

Outline

(Just overview here; for actual content see Jupyter notebooks)

- ▶ brief introduction to probability
 - pdfs, Gaussian, law of large numbers, CLT
 - **Confidence Intervals**
- ▶ hypothesis testing (mostly focus on *analysis*)
 - **null hypothesis**
 - confidence threshold
 - computing the test statistic (Z-test example), p -values
 - banana skins
 - Type I and II errors, statistical power (cf. experimental design)

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 - the statistical tests here are more **classical**
 - arguably more technical (not really though...)

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 - the statistical tests here are more **classical**
 - arguably more technical (not really though...)
- ▶ not going to talk about **experimental design**, though it is probably more important than the **statistical analysis**

Lasciate ogni speranza, voi ch'intrate



Figure: Cursed image.

Motivation: sea cucumber



Figure: Moldy sea cucumber.

Suppose sea cucumber has some distribution of weight that we can measure:

- Q. does change in diet affect her weight?
- Q. does exercise regime affect her weight?
- might expect to, but how do we distinguish **noise** (e.g. natural random fluctuations) with “real” effect?

Motivation: sea cucumber



Figure: Moldy sea cucumber.

e.g. say from samples,

$$\{\mu_1 = 3.00, \quad \sigma_1 = 0.5\}$$

$$\{\mu_2 = 3.20, \quad \sigma_2 = 0.5\}$$

- ▶ mean is different, so has effect?
→ but could just be a fluke?
- ▶ hypothesis testing as a tool to say whether differences are **statistically significant**

Some probability

- ▶ assign some real value between 0 and 1 to some **event**
→ 0 is never, 1 is certainly
- ▶ e.g. throwing a fair coin, two events, expect $p = 1/2$
- Q. that's in principle, what if you actually try it?

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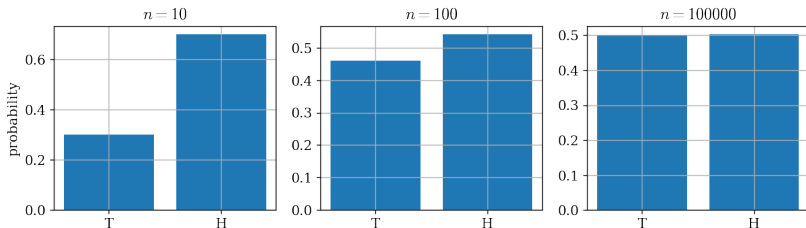
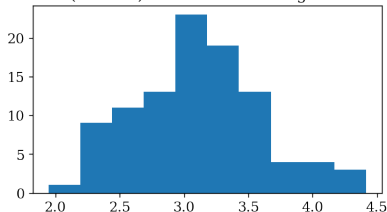


Figure: Result of hypothetical coin tosses displayed in **bar graph**.

Some probability

- ▶ going back to the sea cucumber, we might have a sample of weights and display it in a **histogram**

default (10 bins) with no formatting whatsoever



tidied up a bit (but not normalised)

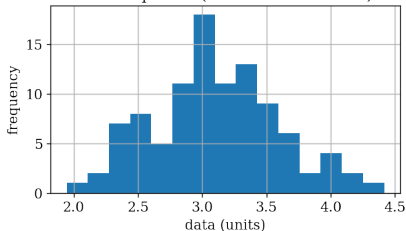


Figure: Result of hypothetical sea cucumber weight.

- ▶ obtained from **binning** procedure
→ probability is related here to **integral** of the graph

Some probability

- histogram related to the **probability distribution function (pdf)** of the underlying sample

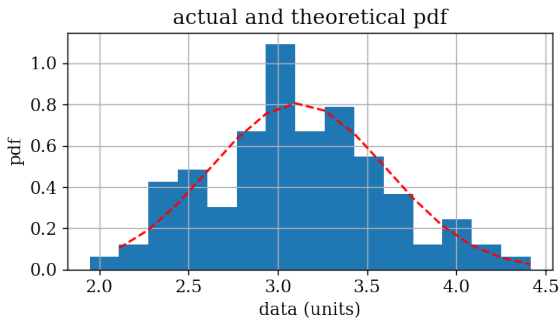


Figure: Result of hypothetical sea cucumber weight with **Gaussian pdf**.

Q. where did I get the red line from though?

Some probability

Gaussian or normal distribution has pdf

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right]$$

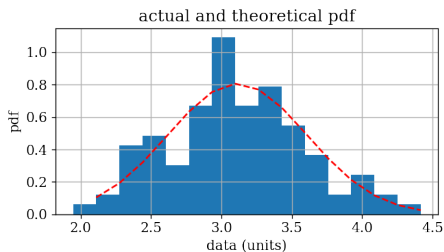
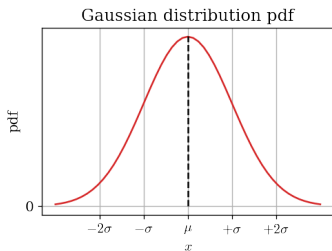


Figure: The Gaussian pdf (with units deliberately omitted). Obtain probability from an integral.

► approximate **population** (μ, σ) from **sample** (\bar{x}, s)

Q. what would the pdf of the **uniform distribution** look like?

Some probability

Point here is that if you have the pdf you have basically everything

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Theorem (Central Limit Theorem (CLT))

(Very loosely) If your sample is large enough, under fairly general conditions (!) you can approximate most data distributions as a Gaussian distribution, even if the underlying distribution is not necessarily Gaussian.

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- ▶ for large enough samples, you can fit it to a Gaussian pdf...

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- ▶ for large enough samples, you can fit it to a Gaussian pdf...
- ▶ ...and if you have the pdf you have basically everything!

Some probability

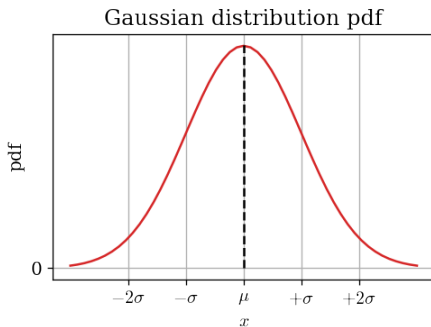


Figure: The Gaussian pdf (with units deliberately omitted). Obtain probability from an integral.

- **68-95-99.7 rule**, 68, 95 and 99.7% of the data lies within 1, 2 and 3 s.t.d. of the mean
→ whenever CLT applies (which is quite often!)

Some probability

e.g.

$$p(-\sigma < z < \sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{\sigma} e^{-z^2/2} dz \approx 0.68$$

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Flip it around: for the some given P , find the \tilde{z} such that

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Confidence Interval

- the interval that contains P amount of probability
→ e.g. **95% confidence interval** for Gaussian data would be around $(-2\sigma, 2\sigma)$ $(-1.96\sigma, 1.96\sigma)$ is more accurate but whatever...

Some probability

Z-score or standardised scores: with samples x_i , define

$$z_i = \frac{x_i - \mu}{\sigma}.$$

- ▶ essentially re-scaled Gaussian
 - cf. what was done for the PCA two sessions ago
 - allows somewhat of a like-for-like comparison

Back to the sea cucumber

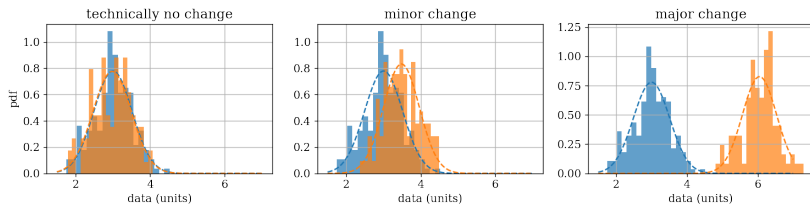


Figure: Control and varied sample distributions and associated Gaussian pdf.

- ▶ we are basically dealing with samples (going to assume CLT holds here)
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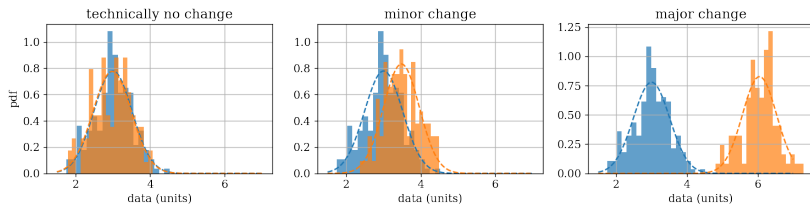


Figure: Control and varied sample distributions and associated Gaussian pdf.

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- Q. variability in data always exist, so how to distinguish change from noise?
- ? non-overlapping confidence intervals?
 - quite strict (and **under-powered**; see later)

Hypothesis testing

A more standard and routine (doesn't mean it's a good thing necessarily):
hypothesis testing and computing **test statistics**

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cf. proof by contradiction: want to proof X , so

- ▶ assume *not* X
- ▶ start from not X , logically derive consequences, avoiding illegal logical manoeuvres
- ▶ come to a contradiction
- ▶ if no illegal manoeuvres, then initial assumption must be false, and there X is true



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Start with a **null hypothesis** H_0 (opposite to what you want to show usually)

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 1. a really surprising result
 2. or H_0 is incompatible with data
- ▶ if latter, reject H_0 , and there is **statistical evidence** in support for *not* H_0 (which is the thing you wanted anyway)

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compute : compute Z-statistics (see notebook)

conclude : if Z-statistic large or corresponding **p-value** small, then reject H_0

(see actual code syntax in notebook)

Easy right?

Easy right?



Figure: Banana says no.

- ▶ banana skins in no particular order (there are so many of them)...

Banana skin 1: H_0

What if you fail to reject the null hypothesis?

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→ it really just means you can't say anything

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 - this does NOT mean H_0 is true, nor that being able to reject H_0 means H_0 is false
 - it really just means you can't say anything
- ▶ cf. proof by contradiction: not being able to find a contradiction could mean
 - there really is nothing there
 - you aren't looking hard enough

Banana skin 2: p -values and H_0

For $\alpha = 0.05$ and I reject H_0 , so test tells me H_0 is only 5% likely to be true

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For $\alpha = 0.05$ and I reject H_0 , so test tells me H_0 is only 5% likely to be true

- ▶ (frequentist point of view) H_0 is either true or false, it can't be 5% true
- ▶ probability of $p = 0.05$ is
 - × hypothesis given data $p(H_0|x_i)$
 - ✓ data given hypothesis $p(x_i|H_0)$
- ▶ rule of thumb (frequentist view): don't assign probabilities to hypotheses

Banana skin 3: p -values

$\alpha = 0.05$ and I reject H_0 , so what I observed would have a probability of 5% that is was due to noise

Banana skin 3: p -values

$\alpha = 0.05$ and I reject H_0 , so what I observed would have a probability of 5% that it was due to noise

- ▶ again no, since p -value is tagged with the null hypothesis, i.e., I would have observed this signal *given* the null hypothesis

Banana skin 4: p -values

$\alpha = 0.05$ is the **gold standard**

Banana skin 4: p -values

$\alpha = 0.05$ is the gold standard

► $10^{10^{10}}$ NO!!!

Banana skin 4: p -values

$\alpha = 0.05$ is the gold standard

► $10^{10^{10}}$ NO!!!

→ comes from (Ronald) Fisher's paper in the 30s or so,
when dealing with small samples (≈ 20 ?)

→ can easily be abused to generate false-positives with
multiple testing or large sampling (see notebook)

► just a convention (and prone to abuse)

→ e.g. particle physics uses 5σ ($\alpha = 0.0000003$, or 1 in 3.5
million, partly because of multiple sampling going on)

Banana skin 5: p -values

My test statistic is large or my p -value is small, so my result is extremely important

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▶ also no

Banana skin 5: p -values

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► also no

→ **statistical significance** is not **practical significance**,
 p -values can't tell you the latter

→ H_0 statements are very broad: H_0 is no change, not H_0 is there *is* change, but it doesn't tell you how much change

► need interpretation: 1 kg difference in a sea cucumber is not the same as 1 kg difference in a whale

Type I and II errors

Type I errors (false-positives)

- ▶ rejecting H_0 when H_0 is true
→ related to choice of significance α

Type II errors (false-negatives)

- ▶ fail to reject H_0 when H_0 is false

	H_0 true	H_0 false
reject H_0	Type I	✓
fail to reject H_0	✓	Type II

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	H_0 true	H_0 false
reject H_0	Type I	✓
fail to reject H_0	✓	Type II

or, with H_0 = someone is innocent,

	innocent	murderer
found guilty	wrongful conviction	✓
found not guilty	✓	fail to prosecute

Type I and II errors

Type II errors β related to **statistical power** $1 - \beta$

- ▶ really to do with experimental design
→ choice of sample size to detect effect (see notebook)

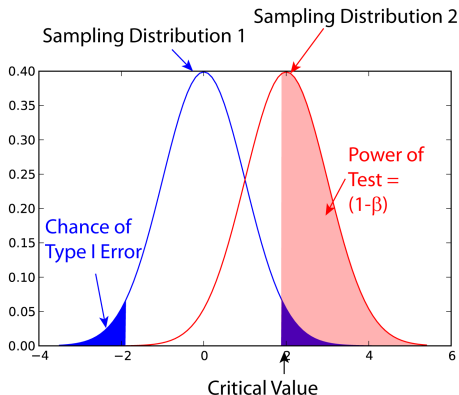


Figure: Graphical demonstration of Type I and II errors. From Wikipedia.

Bad practices 1: “torturing the data until it confesses”

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This statistical test didn't work so I will use another one, or

These are outliers, so I will get rid of those, or

I will sub-sample dataset and try again etc.

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This statistical test didn't work so I will use another one, or

These are outliers, so I will get rid of those, or

I will sub-sample dataset and try again etc.

- ▶ $\alpha = 0.05$ so 5% of the time you will reject the null hypothesis when you shouldn't have (Type I error, false-positives)

→ multiple testing you could be sampling that 5%

→ issue of stop when you get a hit (DON'T!!!)

(see notebook for an example of this)

- ▶ leads to **false discoveries** (see notebook for an example to do with academic publishing)

Bad practices 2: report only reject or not reject

Reporting reject/fail to reject only

- ▶ report the full p -value
 - above/below boundary is not saying whether hypothesis is true/false (banana skin 2)
 - the threshold is a convention (banana skin 4)

Bad practices 3: being hung up on the analysis

The analysis part is so hard I need to pay most of my attention to it

- ▶ if your experimental design is faulty than one would hope (!) that no amount of wizardry will fix that...
→ have a think about the validity of design and tools also

Jupyter notebook

Probably more but this is surely getting tedious... go to 05 Jupyter notebook to get some code practise

- ▶ RNGesus and probability
- ▶ written overview of hypothesis testing
→ one example using Z-tests

Statistics is just a tool, no more and no less

- ▶ YOU are the user and the onus is on YOU to know enough about to tool to not abuse it