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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 3301 :

basic Data Analysis in ocean sciences

Session 6: statistical tests

March 6, 2022

Outline

(Just overview here; for actual content see Jupyter notebooks)

- recall hypothesis testing (mostly focus on *analysis*)

  $\rightarrow$ null hypothesis etc.

  $\rightarrow$ Z-test

  $\rightarrow$ (Student's) *t*-test

  $\rightarrow$ χ^2

  $\rightarrow$ ANOVA + *F*-test
- some others design)

Recall: sea cucumber

Figure: Moldy sea cucumber.

e.g. say from samples,

$$\{\mu_1 = 3.00, \quad \sigma_1 = 0.5\}$$

$$\{\mu_2 = 3.20, \quad \sigma_2 = 0.5\}$$

- mean is different, so has effect?

  $\rightarrow$ but could just be a fluke?

- hypothesis testing as a tool to say whether differences are statistically significant

Recall: Hypothesis testing

Figure: Control and varied sample distributions and associated Gaussian pdf.

- ► we are basically dealing with samples (going to assume CLT holds here)

Q. variability in data always exist, so how to distinguish change from noise?

? non-overlapping confidence intervals?

$\rightarrow$ quite strict (and **under-powered**; see later)

Recall: Hypothesis testing

Start with a null hypothesis H_0 (opposite to what you want to show usually)

- assume H_0
- decide test and significance level (depends on the thing you want to show)
- compute test statistics (depends on test)
- if associated probability of computed test statistic is low, then it is either:
  1. a really surprising result
  2. or H_0 is incompatible with data
- if latter, reject H_0, and there is statistical evidence in support for *not* H_0 (which is the thing you wanted anyway)

Recall: Hypothesis testing

e.g. sea cucumber, want to know if diet has any effect on weight

H_0 : diet has **NO** bearing on weight

test : large enough samples, assume Gaussian statistics, do two-tailed **Z-test**

α : choose $\alpha = 0.05$

$\rightarrow$ how far you are in tails of the pdf

$\rightarrow$ for Gaussian pdf, corresponds to Z-score of around 2, because 95% CI is around $(-2\sigma, +2\sigma)$

compute : compute Z-statistics (see notebook)

conclude : if Z-statistic large or corresponding p-value small, then reject H_0

(see actual code syntax in notebook)

Recall: Type I and II errors

Type I errors (false-positives)

- rejecting H_0 when H_0 is true

  $\rightarrow$ related to choice of significance α

Type II errors (false-negatives)

- fail to reject H_0 when H_0 is false

	H_0 true	H_0 false
reject H_0	Type I	✓
fail to reject H_0	✓	Type II

or, with H_0 = someone is innocent,

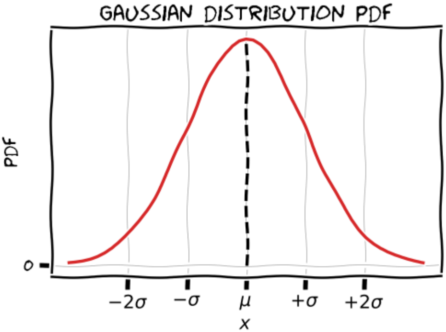
	innocent	murderer
found guilty	wrongful conviction	✓
found not guilty	✓	fail to prosecute

Recall: Z-test

- Z-test calculates Z-statistic with sample mean $\overline{x}$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{N}}$$

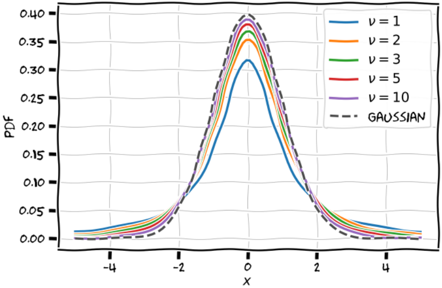
$\rightarrow$ if Z big enough (so the associated p-value is small) then evidence to reject H_0
$\rightarrow$ Gaussian, large samples, known σ (could approximate with s)



(Student's) t-test

- Gaussian data, **small** samples, unknown σ
- t-statistic pdf is

$$p(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\,\Gamma(\nu/2)}\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2},$$



→ ν ('nu') is called the degree of freedom (d.o.f.)

→ Γ ('gamma') is the Gamma function

(Student's) *t*-test

- Z-test statistic is so easy to compute there is no inbuilt function (?)...

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- ...but *t*-tests does exist in e.g. scipy (see notebook)

  $\rightarrow$ d.o.f., related to sample size, is computed automatically

  $\rightarrow$ with H_0 that mean is the **same**

  $\rightarrow$ gives *t*-statistic and *p*-values

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- a few types

  $\rightarrow$ one sample (test against population mean)

  $\rightarrow$ two sample paired (test against each other's means)

  $\rightarrow$ two sample independence / unpaired (as above)

χ^2-test

- categorical data with discrete probabilities (e.g. dice, population distributions)
- χ^2-test ('kai squared') pdf

$$p(x) = \frac{x^{\nu/2-1} e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)} \qquad \text{if } x > 0,$$

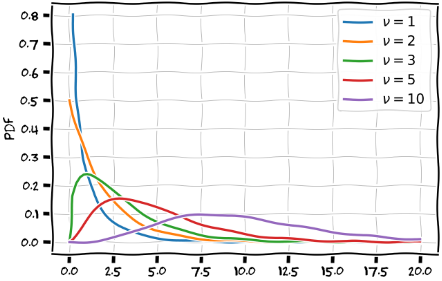

Figure: χ^2 pdf.

χ^2 test

- scipy has this (see notebook)

  $\rightarrow$ computes d.o.f.

  $\rightarrow$ with H_0 that means are the **same**

  $\rightarrow$ gives χ^2-statistic and p-values

- sample applications

  $\rightarrow$ how to tell if a die (singular of dice) is 'fair'? (see notebook)

  $\rightarrow$ sea cucumber activity affected by seasonality

  $\rightarrow$ some ocean applications you can make up

- robust but "weak"

  $\rightarrow$ tells you there are deviations of distribution, but doesn't tell you where

ANOVA and F-test

- to probe whether the **means** in groups of data are different to each other, by analysing their **variances**

  $\rightarrow$ ANOVA = ANalysis Of **VAriance**
- idea: if variance between groups dominates variance within individual group, then the means **between** groups are significant

  $\rightarrow$ high F regime

- F-statistic pdf looks like χ^2

  $\rightarrow$ one-way F-test works on H_0 = all means are the same

  $\rightarrow$ large F-statistic is p value small

ANOVA and F-test

- use this to avoid **multiple testing** between pairs of data
  $\rightarrow$ reduce false-positives / Type I errors

- robust but "weak"
  $\rightarrow$ tells you there is/are pair(s) with different means, but doesn't tell you where

- not the only use, but not going to go through the others

- beware of assumptions going into test
  $\rightarrow$ e.g. normality, homogeneity, ...

Other pre/post-analysis tests

- plot out the data!
- Shapiro–Wilk's test
  $\rightarrow$ normality of data
- Levene and/or Bartlett's test
  $\rightarrow$ homogeneity
- Tukey's Honest Significance Test
  $\rightarrow$ post-analysis for F-test
  $\rightarrow$ sometimes Tukey's range test, Tukey HSD, etc.

Jupyter notebook

go to 06 Jupyter notebook to get some code practise

- all banana skins previous apply here

Statistics is just a tool, no more and no less

- YOU are the user and the onus is on YOU to know enough about to tool to not abuse it