Chapter 5

5.1 The Lorentz Force Law

Empirically, for a moving charge in a magnetic field, we have

$$F_{mag} = Q(v \times B)$$

The generalized magnetic force is

$$F_{mag} = \int I \times B dl = I \int dl \times B$$

The magnetic force does NO work.

Currents

Line Currents: $I = v\lambda$
Surface Currents Density: $K = \sigma v$, $K = \frac{dI}{dl_\perp}$
(for solenoids of n winds per length, $K = nI\vec{\phi}$)
Volume Current Density: $J = \rho v$, $J = \frac{dI}{da_\perp}$

Conservation of Charge

$$\int_{S(V)} J \cdot da = \frac{d}{dt}\int_V \rho d\tau$$

$$\nabla \cdot J = \frac{\partial \rho}{\partial t}$$

5.2 Biot-Savart Law

For $\frac{\partial J}{\partial t} = 0$, we have,

$$B = \frac{\mu_0}{4\pi} I \int \frac{dl' \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0}{4\pi}\int \frac{J(r') \times B}{\mathscr{r}^2} d\tau'$$

Field of a straight wire: $B = \frac{\mu_0 I}{2\pi d}$
Field of a circular wire: $B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$

5.3 The Divergence and Curl of B

The Divergence

$$\nabla \cdot B = 0$$

The Curl: Ampere's Law

$$\nabla \times B = \mu_0 \vec{J}$$

$$\oint B \cdot dl = \mu_0 I_{enc}$$

Ampere's law is particularly useful for

1. Infinite straight wires
2. Infinite planes
3. Infinite solenoids
4. Toroids

Field of a solenoid: $B = \begin{cases} \mu_0 n I \hat{z} & \text{inside} \\ 0 & \text{outside} \end{cases}$

Field of a toroid: $B = \begin{cases} 0 & \text{inside the coil} \\ \frac{\mu_0 N I}{2\pi s} & \text{outside the coil} \end{cases}$

5.4 Magnetic Vector Potential

a

$$B = \nabla \times A$$

$$\nabla \times B = \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 \vec{J}$$

$$\text{(3 Poisson's equations corres. x,y,z)} \quad \nabla^2 A = \mu_0 \vec{J}$$

$$A(r) = \frac{\mu_0}{4\pi}\int \frac{J}{\mathscr{r}^2} d\tau' = \frac{\mu_0}{4\pi}\int \frac{K}{\mathscr{r}^2} da' = \frac{\mu_0}{4\pi}\int \frac{I}{\mathscr{r}^2} dl'$$

By using $B = \nabla \times A$, Ampere's law can also be applied on A,

$$\oint A \cdot dl = \Phi = \int B \cdot da$$

Typically, the direction of A mimics that of currents.
Similar to V, A must be continuous.