

## Chapter 5

### 5.1 The Lorentz Force Law

Empirically, for a moving charge in a magnetic field, we have

$$F_{mag} = Q(v \times B)$$

The generalized magnetic force is

$$F_{mag} = \int I \times B dl = I \int dl \times B$$

The magnetic force does *NO* work.

#### Currents

Line Currents:  $I = v\lambda$

Surface Currents Density:  $K = \sigma v$ ,  $K = \frac{dI}{da_{\perp}}$

(for solenoids of  $n$  winds per length,  $K = nI\vec{\phi}$ )

Volume Current Density:  $J = \rho v$ ,  $J = \frac{dI}{da_{\perp}}$

#### Conservation of Charge

$$\int_{S(V)} J \cdot da = \frac{d}{dt} \int_V \rho d\tau$$

$$\nabla \cdot J = \frac{\partial \rho}{\partial t}$$

### 5.2 Biot-Savart Law

For  $\frac{\partial J}{\partial t} = 0$ , we have,

$$B = \frac{\mu_0}{4\pi} I \int \frac{dl' \times \hat{z}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{J(r') \times B}{r^2} d\tau'$$

Field of a straight wire:  $B = \frac{\mu_0 I}{2\pi d}$

Field of a circular wire:  $B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$

### 5.3 The Divergence and Curl of B

#### The Divergence

$$\nabla \cdot B = 0$$

#### The Curl: Ampere's Law

$$\nabla \times B = \mu_0 \vec{J}$$

$$\oint B \cdot dl = \mu_0 I_{enc}$$

Ampere's law is particularly useful for

1. Infinite straight wires      3. Infinite solenoids

2. Infinite planes              4. Toroids

Field of a solenoid:  $B = \begin{cases} \mu_0 n I \hat{z} & \text{inside} \\ 0 & \text{outside} \end{cases}$

Field of a toroid:  $B = \begin{cases} 0 & \text{inside the coil} \\ \frac{\mu_0 N I}{2\pi s} & \text{outside the coil} \end{cases}$

### 5.4 Magnetic Vector Potential

$$B = \nabla \times A$$

a

$$\nabla \times B = \nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 \vec{J}$$

$$(3 \text{ Poisson's equations corres. } x, y, z) \quad \nabla^2 A = \mu_0 \vec{J}$$

$$A(r) = \frac{\mu_0}{4\pi} \int \frac{J}{r^2} d\tau' = \frac{\mu_0}{4\pi} \int \frac{K}{r^2} da' = \frac{\mu_0}{4\pi} \int \frac{I}{r^2} dl'$$

By using  $B = \nabla \times A$ , Ampere's law can also be applied on  $A$ ,

$$\oint A \cdot dl = \Phi = \int B \cdot da$$

Typically, the direction of  $A$  mimics that of currents.

Similar to  $V$ ,  $A$  must be continuous.