Non-orthogonal Multiple Access (NOMA) and its PLS

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Outlines

- 1. Conventional Multiple Access Techniques
- 2. Principles of NOMA
- 3. Secrecy Design for Two-user Case: Conventional NOMA
- 4. Secrecy Design for Two-user Case: NOMA with Jamming

Multiple User Communications: Cellular systems

- Two types of spectra are available for commercial cellular systems
  - The first is licensed, typically nationwide and over a period of a few years, from the spectrum regulatory agency (FCC, in the United States).
  - The second is unlicensed spectrum made available for experimental systems and to aid development of new wireless technologies.

Bandwidth is very expensive. This skews the engineering design of the wireless system to be as spectrally efficient as possible. In this course, we focus on cellular systems that are designed to work on licensed spectrum.

Multiple User Communications: Cellular systems

- There are two main issues in cellular communication: multiple access and interference management
  - Multiple access: The issue addresses how the overall resource (time, frequency, and space) of the system is shared by the users in the same cell (intra-cell)
  - Interference management: This issue addresses the interference caused by simultaneous signal transmissions in different cells (inter-cell).

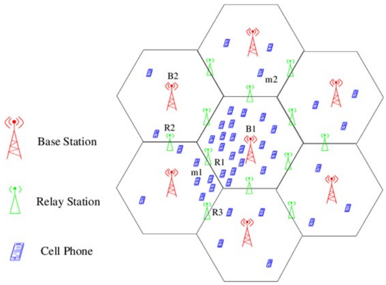


Figure 1: An illustration of wireless cellular networks

Multiple User Communications: Cellular systems

- In addition to resource sharing between different users, there is also an issue of how the resource is allocated between the uplink (the communication from the mobile users to the base-station, also called the reverse link) and the downlink (the communication from the base-station to the mobile users, also called the forward link). There are two natural strategies for separating resources between the uplink and the downlink:
  - Time division duplex (TDD) separates the transmissions in time.
  - Frequency division duplex (FDD) achieves the separation in frequency.

Multiple Access: Frequency Division Multiple Access (FDMA)

- In FDMA, the available channel bandwidth is divided into many non-overlapping frequency bands, where each band is dynamically assigned to a specific user to transmit data.
  - Advantage: 1) Synchronization is not required; 2) Work for both analog and digital signals
  - Disadvantage: 1) Guard bands are introduced between each adjacent frequency channel to prevent out-of-band emission due to carrier frequency equipment instability; 2) Selective filters is required

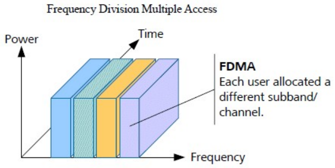


Figure 2: An illustration of FDMA

Multiple Access: Time Division Multiple Access (TDMA)

- It allows several users to share the same frequency channel by dividing the signal into different time slots.
  - Advantage: 1) TDMA is the cost-effective technology; 2) No interference from simultaneous transmission
  - Disadvantage: 1) Synchronization is required; 2) Does not work for analog signals; 3) Guard time is introduced so that there is sufficiently long buffering in propagation delays between the two users

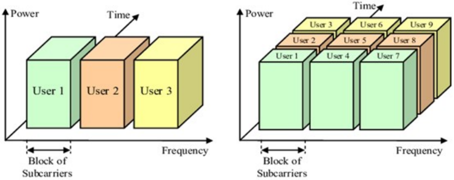
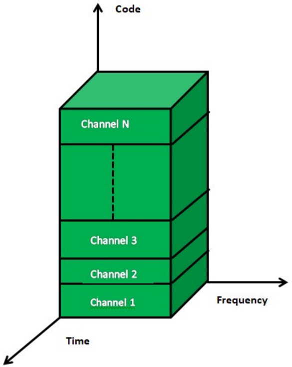


Figure 3: FDMA (Left) v.s. TDMA (Right)

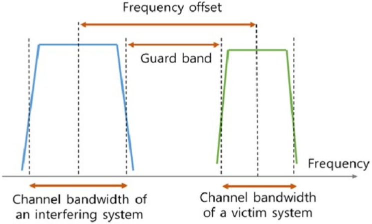
Multiple Access: Code Division Multiple Access (CDMA)

- Signals of different users overlap with one another. CDMA employs spread spectrum technology and a special coding scheme to eliminate inter-user interference, e.g., each user is assigned an unique signature sequence (called code or chip).
  - Advantage: 1) Bandwidth efficient; 2) Synchronization is not required; 3) Good protection against interference
  - Disadvantage: 1) Complex signal processing



Multiple Access: Orthogonal Frequency Division Multiple Access (OFDMA)

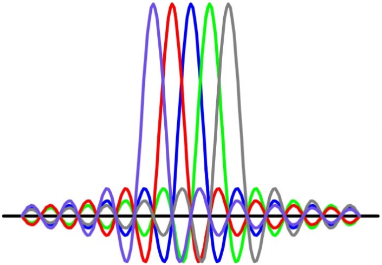
- When modulation of a signal is applied to a frequency carrier, sidebands spread out either side. It is necessary for a receiver to receive the whole signal in order to demodulate the data successfully. As a result, there must be guard band between signals carriers so that the receiver can separate them using a filter.



Low spectrum efficiency

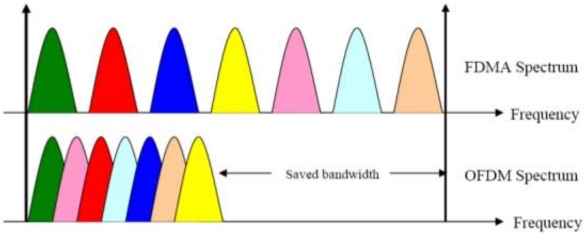
If the carrier space equals to the reciprocal of the symbol period

- The signals will have a whole number of cycles in the symbol period and the contribution from other signals will cancel each other (i.e. sum to zero)
- Therefore, the signal can be received without interference although the sideband from each carrier overlap
- The carrier frequencies are considered as orthogonal to each other



Multiple Access: Orthogonal Frequency Division Multiple Access (OFDMA)

- Advantage: 1) Spectral efficient is high; 2) Resilience to interference; 3) Immunity to inter-symbol interference
- Disadvantage: 1) Higher complexity (involves FFT/IFFT signal processing) since data is coded in the frequency domain; 2) High peak to average power ratio (a problem for mobile stations but not for base stations)



A natural question is: what are the "optimal" multiple access schemes?

2. Principles of NOMA: Capacity of Uplink AWGN Channel

- $y[m] = x_1[m]h_1 + x_2[m]h_2 + w[m]$, where $w[m] \sim \mathcal{N}(0, N_0)$ is i.i.d. complex Gaussian noise. User k has an average power constraint of P_k
- In the point-to-point case, the capacity of a channel provides the performance limit: reliable communication can be attained at any rate $R < C$; reliable communication is impossible at rates $R > C$.
- In the multi-user case, we should extend this concept to a capacity region $\mathcal{C}$: this is the set of all pairs (R_1, R_2) such that simultaneously user 1 and 2 can reliably communicate at rate R_1 and R_2, respectively.
- The sum capacity: $C_{\text{sum}} = \max_{(R_1, R_2) \in \mathcal{C}} R_1 + R_2$, is the maximum total throughput that can be achieved.

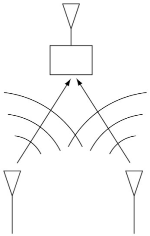


Figure 4: Two-user uplink.

The set of all rates (R_1, R_2) satisfying the three constraints

- Capacity of the point-to-point link with the other user absent from the system: $R_1 \leq \log_2(1+\frac{P_1|h_1|^2}{N_0})$
- Capacity of the point-to-point link with the other user absent from the system: $R_2 \leq \log_2(1+\frac{P_2|h_2|^2}{N_0})$
- The total throughput cannot exceed the capacity of a point-to-point AWGN channel with the sum of the received powers of the two users: $R_1 + R_2 \leq \log_2(1+\frac{P_1|h_1|^2+P_2|h_2|^2}{N_0})$

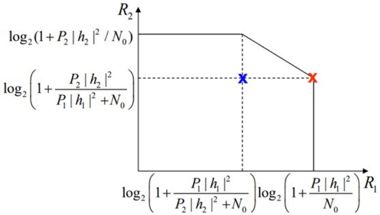
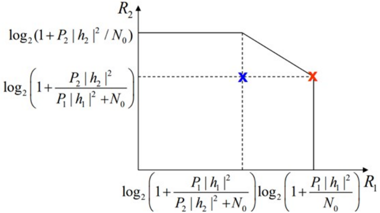


Figure 5: Capacity region of the two-user uplink AWGN channel.

2. Principles of NOMA: Capacity of Uplink AWGN Channel

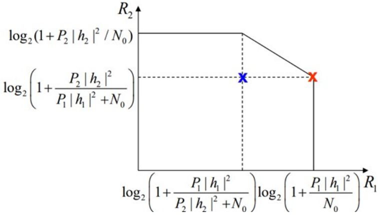
Something surprising does happen: user 1 can achieve its single-user bound while at the same time user 2 can get a non-zero rate; in fact as high as its rate at the red cross point, i.e.,

- Capacity of the point-to-point link with the other user absent from the system: $R_2^* = \log_2(1+\frac{P_1|h_1|^2+P_2|h_2|^2}{N_0}) - \log_2(1+\frac{P_1|h_1|^2}{N_0}) = \log_2(1+\frac{P_2|h_2|^2}{P_1|h_1|^2+N_0})$



How can this be achieved?

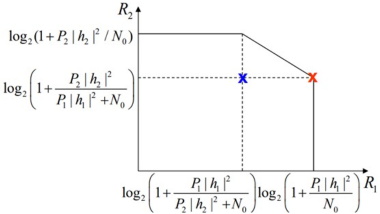
- Two-stage decoding
  - First decode the signal of user 2 by treating the signal of user 1 as interference, i.e., $R_2 = \log_2(1 + \frac{P_2|h_2|^2}{P_1|h_1|^2+N_0})$
  - Once the receiver decodes the data of user 2, it can reconstruct user 2's signal and subtract it from the aggregate received signal. The receiver can then decode the data of user 1 without the interference of user 2, i.e., $\log_2(1 + \frac{P_1|h_1|^2}{N_0})$



This receiver is called a successive interference cancellation (SIC) receiver.

The segment AB contains all the "optimal" operating points of the channel

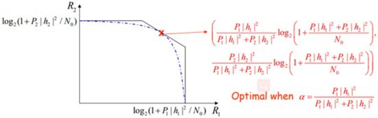
- if maximize the sum rate, then any point on AB is equally fine
- some operating points are not fair, especially if the received power of one user is much larger than the other



2. Principles of NOMA: Capacity of Uplink AWGN Channel

Compare to orthogonal scheme: Suppose a fraction of α of the bandwidth is allocated to user 1 and the rest is allocated to user 2.

- $R_1 = \alpha \log_2(1 + \frac{P_1|h_1|^2}{\alpha N_0})$
- $R_2 = (1-\alpha)\log_2(1 + \frac{P_2|h_2|^2}{(1-\alpha)N_0})$
- $\alpha^* = \frac{P_1|h_1|^2}{P_1|h_1|^2 + P_2|h_2|^2}$



2. Principles of NOMA: Capacity of Downlink AWGN Channel

- h_k is the fixed (complex) channel gain corresponding to user k, $k = 1, 2$.
- Assume $|h_1| < |h_2|$.
  - User 2 has a better channel than user 1.
  - User 2 can decode any data that user 1 can successfully decode
- $R_k \leq \log_2(1 + \frac{P_k |h_k|^2}{N_0})$, $k = 1, 2$.
- The sum capacity: $C_{\text{sum}} = \max_{(R_1, R_2) \in \mathcal{C}} R_1 + R_2$, is the maximum total throughput that can be achieved.

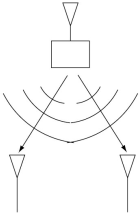


Figure 6: Two-user downlink.

2. Principles of NOMA: Capacity of Downlink AWGN Channel

The maximum capacity:

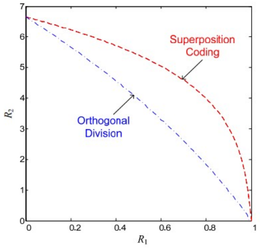
- Transmit side uses superposition coding: The transmit signal is a linear superposition of the signals of two users, i.e., $x = \sqrt{P_1}x_1 + \sqrt{P_2}x_2$ with $P = P_1 + P_2$, where x_k is the signal of user k, $k = 1, 2$.

- Receiver side
  - User 1: treat the signal of user 2 as interference, i.e, $R_1 = \log_2(1 + \frac{P_1|h_1|^2}{P_2|h_1|^2+N_0})$
  - User 2: Perform SIC, i.e., decode user 1's signal first, and then decode its own signal using SIC. Thus, we have $R_2 = \log_2(1 + \frac{P_2|h_2|^2}{N_0})$

2. Principles of NOMA: Capacity of Downlink AWGN Channel

Compare to orthogonal scheme: Suppose a fraction of α of the bandwidth is allocated to user 1 and the rest is allocated to user 2.

- $R_1 = \alpha \log_2(1 + \frac{P_1|h_1|^2}{\alpha N_0})$
- $R_2 = (1-\alpha)\log_2(1 + \frac{P_2|h_2|^2}{(1-\alpha)N_0})$
- Superposition coding achieves the downlink AWGN capacity
- One can show that the superposition decoding scheme is strictly better than the orthogonalization schemes (except for the two corner points where only one user is being communicated to).



2. Principles of NOMA: Capacity of K-user Uplink AWGN Channel

Capacity Region:

- $R_k \leq \log_2(1 + \frac{P_k|h_k|^2}{N_0}),\ k = 1, 2, \ldots, K$
- $\sum_{k\in\mathcal{S}} R_k \leq \log_2(1 + \frac{\sum_{k\in\mathcal{S}} P_k|h_k|^2}{N_0}),\ \mathcal{S} \subset \{1, 2, \ldots, K\}$

SIC

- $R_1 \leq \log_2(1 + \frac{P_1|h_1|^2}{\sum_{k=2}^{K} P_k|h_k|^2 + N_0})$
- $\ldots$
- $R_K \leq \log_2(1 + \frac{P_K|h_K|^2}{N_0})$

Orthogonal Scheme

- $R_k \leq \alpha_k \log_2(1 + \frac{P_k|h_k|^2}{\alpha_k N_0})$
- $\sum_{k=1}^{K} \alpha_k = 1$

2. Principles of NOMA: Capacity of K-user Downlink AWGN Channel

Suppose $|h_1| < |h_2| \cdots < |h_K|$

Superposition Coding

- $R_1 \leq \log_2(1 + \frac{P_1|h_1|^2}{|h_1|^2 \sum_{k=2}^{K} P_k + N_0})$
- $R_2 \leq \log_2(1 + \frac{P_2|h_2|^2}{|h_2|^2 \sum_{k=3}^{K} P_k + N_0})$
- ...
- $R_K \leq \log_2(1 + \frac{P_K|h_K|^2}{N_0})$

Orthogonal Scheme

- $R_k \leq \alpha_k \log_2(1 + \frac{P_k|h_k|^2}{\alpha_k N_0})$
- $\sum_{k=1}^{K} \alpha_k = 1$, $\sum_{k=1}^{K} P_k = P$

2. Principles of NOMA: Uplink

Suppose $|h_2| < |h_1|$

- $R_1 = \log_2(1 + \frac{P_1|h_1|^2}{P_2|h_2|^2 + N_0})$
- $R_2 = \log_2(1 + \frac{P_2|h_2|^2}{N_0})$

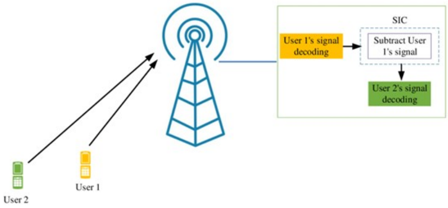


Figure 7: A two-user uplink NOMA system

2. Principles of NOMA: Downlink

Suppose $|h_2| < |h_1|$

- $R_1 = \log_2(1 + \frac{P_1|h_1|^2}{N_0})$
- $R_2 = \log_2(1 + \frac{P_2|h_2|^2}{P_1|h_2|^2 + N_0})$

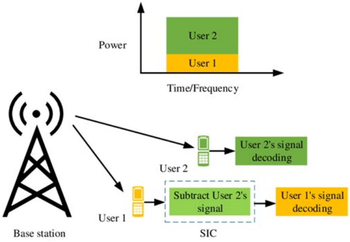


Figure 8: A two-user downlink NOMA system

2. Principles of NOMA: 2 or 3 users

Generally, we consider 2- or 3-user NOMA system. Why?

- Complexity of SIC
- Error propagation

2. Principles of NOMA: Exercise

- Show that the decoding order for the two-user uplink NOMA system (Fig. 7) is optimal in terms of minimizing power consumption?
- Prove that the decoding order for the two-user downlink NOMA system (Fig. 8) is optimal in terms of minimizing power consumption?

2. Principles of NOMA: Exercise

2. Principles of NOMA: Exercise

3. Secrecy Design for Two-user Case: Conventional NOMA

- Let us consider a secure wireless network with NOMA as shown in Fig. 9, which contains two users (U_1 and U_2) with single antenna and a base station (BS). U_1 is NU, while U_2 is FU with relatively poor channel conditions.
- We assume that U_1 is a passive eavesdropper and may try to intercept the information of U_2.
- Let x_1 and x_2 denote the unite signals sent by BS to U_1 and U_2, respectively. And the power allocated to U_1 and U_2 are P_1 and P_2, respectively.

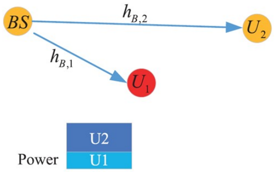
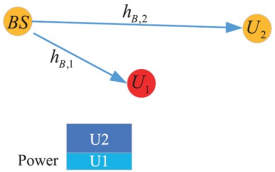


Figure 9: A two-user downlink NOMA with passive eavesdropper

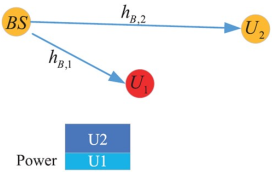
3. Secrecy Design for Two-user Case: Conventional NOMA

- The signal sent by BS is given by $x = \sqrt{P_1}x_1 + \sqrt{P_2}x_2$
- The signals received by U_1 is $y_1 = h_{B,1}x + n_1 = h_{B,1}\sqrt{P_1}x_1 + h_{B,1}\sqrt{P_2}x_2 + n_1$
- The signals received by U_2 is $y_2 = h_{B,2}x + n_2 = h_{B,2}\sqrt{P_1}x_1 + h_{B,2}\sqrt{P_2}x_2 + n_2$
- where $h_{B,1}$ and $h_{B,2}$ are channel gains from BS to two users, respectively. n_1 and n_2 are additive white Gaussian noise (AWGN) introduced by the receiving antenna at U_1 and U_2 with zero mean and variance N_0



3. Secrecy Design for Two-user Case: Conventional NOMA

- U_1 first demodulates x_2 and then calculates it. Thus, the signal-to-noise ratio (SNR) at U_1 is $\gamma_1^D = \frac{P_1|h_{B,1}|^2}{N_0}$
- U_2 directly demodulates x_2 by treating x_1 as noise. Thus, the signal-to-noise ratio (SNR) at U_2 is $\gamma_2^D = \frac{P_2|h_{B,2}|^2}{P_1|h_{B,2}|^2+N_0}$
- The wiretap SNRs of U_1 on U_2 can be given by $\gamma^E = \frac{P_2|h_{B,1}|^2}{P_1|h_{B,1}|^2+N_0}$
- The secrecy rate can be given by $C^s = \max\{\log_2(1+\gamma_2^D) - \log_2(1+\gamma^E), 0\}$



As U_1 is NU, $C_s = 0$. It is impossible to obtain positive secrecy rate without appropriate strategies, such as cooperative jamming. In addition, to ensure that U_1 cannot eavesdrop U_2, U_1 must be the first decode one. Therefore, it is necessary to introduce a jammer and allocate more power to U_1 to ensure a positive secrecy rate.

4. Secrecy Design for Two-user Case: NOMA with Jamming

- U_1 In this section, the detailed secure strategy with jamming is provided to improve the physical layer security of NOMA systems. As shown in Fig. 10, J is a friendly cooperative jammer.
- Moreover, let U1 be demodulated first by allocating more power to U_1. Note that U_1 is able to perform normal SIC.
- Therefore, with jamming, the SNR of at U_1 is $\gamma_1^D = \frac{P_1|h_{B,1}|^2}{P_J|h_{J,1}|^2+P_2|h_{B,1}|^2+N_0}$
- the SNR of at U_2 is $\gamma_2^D = \frac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0}$, where $h_{J,1}$ and $h_{J,2}$ are the channel gains from the jamming to U_1 and U_2 respectively. And P_J is the power of J.

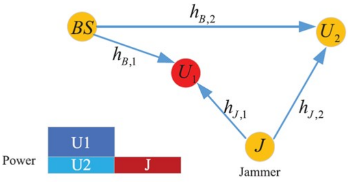
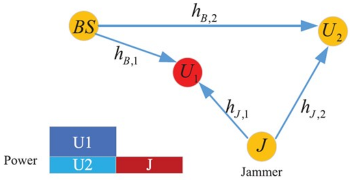


Figure 10: A two-user downlink NOMA with jamming

4. Secrecy Design for Two-user Case: NOMA with Jamming

- The wiretap SNRs of U_1 on U_2 can be given by $\gamma^E = \frac{P_2|h_{B,1}|^2}{P_J|h_{J,1}|^2+N_0}$
- Now, the secrecy rate can be given by $C^s = \max\{\log_2(1+\gamma_2^D) - \log_2(1+\gamma^E), 0\}$



We can develop a power allocation problem to maximize the secrecy rate.

4. Secrecy Design for Two-user Case: NOMA with Jamming

Power allocation problem formulation for the secrecy rate maximization

- In the sequel, we analyze the security rate of the proposed system with two users by formulating an optimization problem.
- Our goal is to improve the secrecy rate, which can be given by

$$
\begin{aligned}
C^S &= \max\left\{\log_2\left(1+\gamma_2^D\right)-\log_2\left(1+\gamma^E\right),0\right\} \\
&= \max\left\{\log_2\left(\frac{1+\frac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0}}{1+\frac{P_2|h_{B,1}|^2}{P_J|h_{J,1}|^2+N_0}}\right),0\right\}.
\end{aligned}
$$

- To guarantee the communication quality, U_1 and U_2 need to satisfy their data rate requirements $R_{th,1}$ and $R_{th,2}$, respectively.
- Let P_T be the total power constraint.

4. Secrecy Design for Two-user Case: NOMA with Jamming

The secure optimization problem can be given as follows:

$$\max \frac{1+\frac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0}}{1+\frac{P_2|h_{B,1}|^2}{P_J|h_{J,1}|^2+N_0}}$$

$$\text{s.t.} \begin{cases} P_T \geqslant P_1+P_2+P_J \\ \dfrac{P_1|h_{B,1}|^2}{P_J|h_{J,1}|^2+P_2|h_{B,1}|^2+N_0} \geqslant \gamma_{th,1} \\ \dfrac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0} \geqslant \gamma_{th,2} \\ P_1, P_2, P_J > 0 \end{cases},$$

where $\gamma_{th,1} = 2^{R_{th,1}} - 1$ and $\gamma_{th,2} = 2^{R_{th,2}} - 1$

4. Secrecy Design for Two-user Case: NOMA with Jamming

Condition for positive secrecy rate

- To obtain positive secrecy rate, we have $C^s > 0$, namely

$$\begin{aligned}
&\frac{|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0}-\frac{|h_{B,1}|^2}{P_J|h_{J,1}|^2+N_0}>0\\
&\Rightarrow P_J|h_{J,1}|^2|h_{B,2}|^2-P_J|h_{J,2}|^2|h_{B,1}|^2\\
&\quad +N_0|h_{B,2}|^2-N_0|h_{B,1}|^2>0\\
&\Rightarrow P_J\left(|h_{J,1}|^2|h_{B,2}|^2-|h_{J,2}|^2|h_{B,1}|^2\right)\\
&>N_0\left(|h_{B,1}|^2-|h_{B,2}|^2\right).
\end{aligned}$$

- If $|h_{J,1}|^2|h_{B,2}|^2 < |h_{J,2}|^2|h_{B,1}|^2$, not align with reality
- Thus, we have $|h_{J,1}|^2|h_{B,2}|^2 > |h_{J,2}|^2|h_{B,1}|^2$

4. Secrecy Design for Two-user Case: NOMA with Jamming

Condition for positive secrecy rate

- Given that $|h_{J,1}|^2|h_{B,2}|^2 > |h_{J,2}|^2|h_{B,1}|^2$

$$P_J > \frac{N_0\left(|h_{B,1}|^2 - |h_{B,2}|^2\right)}{|h_{J,1}|^2|h_{B,2}|^2 - |h_{J,2}|^2|h_{B,1}|^2} \triangleq P_{J,LB},$$

- where $P_{J,LB}$ denotes the lower bound of P_J. Substituting this condition to the constraints for SNRs of U_1 and U_2 in the optimization problem, we have

$$P_2 > \frac{|h_{J,1}|^2 - |h_{J,2}|^2}{|h_{J,1}|^2|h_{B,2}|^2 - |h_{J,2}|^2|h_{B,1}|^2}\gamma_{th,2}N_0 \triangleq P_{2,LB}$$

and

$$P_1 > \frac{\left(|h_{J,1}|^2 - |h_{J,2}|^2\right)(1+\gamma_{th2})}{|h_{J,1}|^2|h_{B,2}|^2 - |h_{J,2}|^2|h_{B,1}|^2}\gamma_{th,1}N_0 \triangleq P_{1,LB}.$$

4. Secrecy Design for Two-user Case: NOMA with Jamming

Condition for positive secrecy rate

- In sum, to obtain positive secrecy rate, J needs to satisfy
  - $P_J > P_{J,LB}$
  - $|h_{J,1}|^2|h_{B,2}|^2 > |h_{J,2}|^2|h_{B,1}|^2$

- And P_T , P_1 and P_2 need to satisfy
  - $P_1 > P_{1,LB}$
  - $P_2 > P_{2,LB}$
  - $P_T > P_{1,LB} + P_{2,LB} + P_{J,LB}$

4. Secrecy Design for Two-user Case: NOMA with Jamming

The optimal power allocation strategy

- It has been proved that the equalities of the first two constraints hold at the optimal of the problem. The original optimization problem has been transformed to be

$$\max \frac{1+\frac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0}}{1+\frac{P_2|h_{B,1}|^2}{P_J|h_{J,1}|^2+N_0}}$$

$$\text{s.t.} \begin{cases} P_T = P_1 + P_2 + P_J \\ \frac{P_1|h_{B,1}|^2}{P_J|h_{J,1}|^2+P_2|h_{B,1}|^2+N_0} = \gamma_{th,1} \\ \frac{P_2|h_{B,2}|^2}{P_J|h_{J,2}|^2+N_0} \geqslant \gamma_{th,2} \\ P_1, P_2, P_J > 0. \end{cases}$$

Figure 11: Optimization problem $\mathcal{P}$

4. Secrecy Design for Two-user Case: NOMA with Jamming

An optimal power allocation strategy is designed[1]

- Solve $\mathcal{P}$ by neglecting the third constraint and obtain Solution I.
- If Solution I does not meet the third constraint in $\mathcal{P}$, solve $\mathcal{P}$ when the third constraint's equality holds and obtain Solution II.
- Combine Solution I and Solution II and propose a hybrid strategy to implement power allocation.

The solution is in closed-form.

[1] [R1]IEEE TVT, 69(11), pp: 13005-13017, 2020

4. Secrecy Design for Two-user Case: NOMA with Jamming

Extent to multi-user case

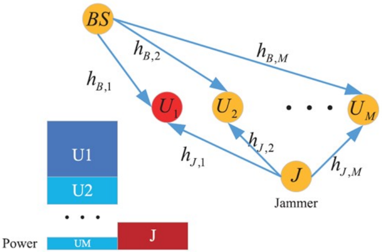


Figure 12: NOMA model with multiple users