

**CAD design Exercise : A Bragg Filter in Silicon-On-Insulator (SOI) (50%)**

1.

Left Screenshot Data:

Vacuum wavelength:	1.55	μm
Polarization:	<input type="radio"/> TE&TM <input checked="" type="radio"/> TE <input type="radio"/> TM	
Number of interior layers:	1	
Cover:	1	
Layer 1:	3.45	
Substrate:	1.45	
Thickness:	0.2	μm

Right Screenshot Data:

Vacuum wavelength:	1.55	μm
Polarization:	<input type="radio"/> TE&TM <input checked="" type="radio"/> TE <input type="radio"/> TM	
Number of interior layers:	1	
Cover:	1	
Layer 1:	3.45	
Substrate:	1.45	
Thickness:	0.1	μm

Both screenshots show calculated results for TE<sub>0</sub> mode:

TE <sub>0</sub> :	N <sub>eff</sub> = 2.730447754	β = 11.07 μm <sup>-1</sup>	B = 0.546	θ = 37.7°
TE <sub>0</sub> :	N <sub>eff</sub> = 2.107672207	β = 8.544 μm <sup>-1</sup>	B = 0.239	θ = 52.3°

$n_{eff1} = 2.730$  by using fundamental TE mode.

$n_{eff2} = 2.108$  by using fundamental TE mode.

2.

Calculations in this question are based on lecture 6 notes (Professor Cryan's part).

$$L_1 = \frac{\lambda_0}{4n_{eff1}} = \frac{1550nm}{4 \times 2.730} = 141.94139nm, \text{ where } \lambda_0 \text{ is incident wavelight and } n \text{ is } n_{eff1}$$

$$L_2 = \frac{\lambda_0}{4n_{eff2}} = \frac{1550nm}{4 \times 2.108} = 183.82353nm$$

$$\Lambda = L_1 + L_2 = 325.765nm$$

$$r_1 = \frac{n_{eff1} - n_{eff2}}{n_{eff1} + n_{eff2}} = \frac{2.730 - 2.108}{2.730 + 2.108} = 0.129$$

$$\text{Given } |r_g|^2 = 0.9$$

$$\therefore r_g \approx \tanh(2mr_1), \text{ where } m \text{ is no. of periods}$$

$$\therefore \sqrt{0.9} \approx \tanh(2m * 0.129)$$

$$\tanh^{-1} \sqrt{0.9} \approx 2m * 0.129$$

$$\frac{1}{2} \ln \left( \frac{1 + \sqrt{0.9}}{1 - \sqrt{0.9}} \right) \approx 2m * 0.129$$

$$m \approx 7.048 > 7$$

Total length of grating needed for 90% power reflectivity at 1550nm

$$= 325.765nm * 8$$

$$= 2606.119nm$$

3. Use matlab to implement the Transfer Matrix Method. Expand the T matrix and convert back to an S matrix to find S11 and hence reflectivity(r) for a given total length of grating.

$$T_p = \frac{1}{t} \begin{bmatrix} 1 & r_1 \\ r_1 & 1 \end{bmatrix} \begin{bmatrix} e^{j\beta_1 L_1} & 0 \\ 0 & e^{-j\beta_1 L_1} \end{bmatrix} \frac{1}{t} \begin{bmatrix} 1 & -r_1 \\ -r_1 & 1 \end{bmatrix} \begin{bmatrix} e^{j\beta_2 L_2} & 0 \\ 0 & e^{-j\beta_2 L_2} \end{bmatrix}$$

$T_g$  represents the T matrix for the overall stack.

$$T_g = (T_p)^m$$

$$T_g = \left( \frac{1}{t} \begin{bmatrix} 1 & 0.129 \\ 0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix} \frac{1}{t} \begin{bmatrix} 1 & -0.129 \\ -0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix} \right)^8$$

Below shows the transformation to S matrix using T matrix values.

$$S_{11} = \frac{T_{21}}{T_{11}} \quad S_{12} = \frac{\det T}{T_{11}}, \text{ where } \det T = T_{11}T_{22} - T_{12}T_{21}$$

$$S_{21} = \frac{1}{T_{11}} \quad S_{22} = \frac{-T_{12}}{T_{11}}$$

Value of  $S_{11} = 0.9690$  is calculated in the MATLAB script, which is the reflectivity(r) for the total length of grating.



S

[0.9690,0.2471,0.2471,-0.9690]

4.

In this short report, the design of a Bragg filter in the Silicon-On-Insulator (SOI) will be explained.

This Bragg filter has an overall length for 90% power reflectivity ( $r^2$ ) at 1550 nm, meaning when light with a wavelength of 1550 nm enters the Bragg filter, 90% is reflected back and the remaining 10% is transmitted.

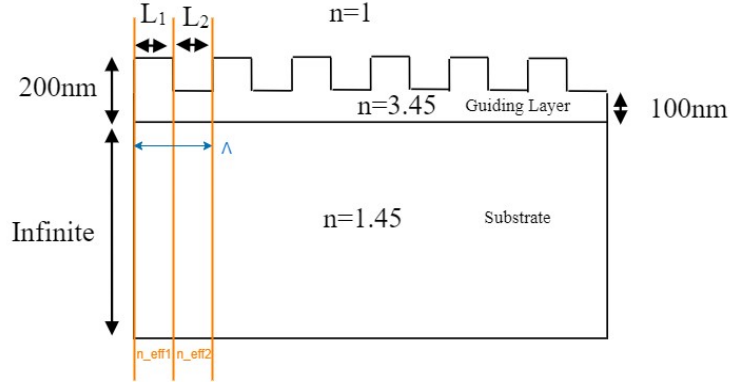


Fig 1. Illustration of corrugated waveguide

Effective indices for L1 and L2 are  $n_{eff1} = 2.730$  and  $n_{eff2} = 2.108$  respectively. We name the segment with L1 as segment 1, another segment with L2 as segment 2. The reflectivity in going from segment 1 to segment 2 is

$$r_1 = \frac{n_{eff1} - n_{eff2}}{n_{eff1} + n_{eff2}} = \frac{2.730 - 2.108}{2.730 + 2.108} = 0.129.$$

This Bragg filter is designed with phase matching condition, meaning the phase of the incident wave aligns with the phase of the reflected or transmitted wave, leading to stronger interference and enhanced reflection or transmission at the desired wavelength.

As we have the parameters, incident wavelength  $\lambda_0$  and  $n_{eff}$  for each segment  $L_1$  and  $L_2$ . And, one grating period  $\Lambda$  is the sum of length of  $L_1$  and  $L_2$ . Then, we can determine the length of one grating period.

$$L_1 = \frac{\lambda_0}{4n_{eff1}} = \frac{1550nm}{4 \cdot 2.730} = 141.94139nm$$

$$L_2 = \frac{\lambda_0}{4n_{eff2}} = \frac{1550nm}{4 \cdot 2.108} = 183.82353nm$$

$$\Lambda \text{ is the period of the grating as drawn in Fig 1. } \Lambda = L_1 + L_2 = 325.765nm$$

In Bragg condition, the propagation wave number along the guide direction is  $\beta = \frac{2\pi}{\lambda_g}$ , where  $\lambda_g$  is

the guided wavelength.  $\lambda_g = \frac{\lambda_0}{n_{eff}}$ . Multiplying  $\beta$  by L, gives the phase shift experienced by the wave

as it travels through the distance  $L$ . The calculation of  $\beta L$  will be shown below. The value of  $\beta L$  will be used in the Transmission  $T$  matrix in later parts.

$$\beta_1 L_1 = \frac{2\pi}{\lambda_g} \frac{\lambda_0}{4n_{eff1}} = 2\pi \left( \frac{n_{eff1}}{\lambda_0} \right) \left( \frac{\lambda_0}{4n_{eff1}} \right) = \frac{\pi}{2}$$

$$\beta_2 L_2 = \frac{2\pi}{\lambda_g} \frac{\lambda_0}{4n_{eff2}} = 2\pi \left( \frac{n_{eff2}}{\lambda_0} \right) \left( \frac{\lambda_0}{4n_{eff2}} \right) = \frac{\pi}{2}$$

$r_g$  denotes the reflectivity of the whole stack. Since we have an overall length for 90% power reflectivity,  $|r_g|^2 = 0.9$ . The net reflection from  $m$  grating periods is given by  $r_g \approx \tanh(2mr_1)$ , where  $m$  is no. of periods.  $r_1 = 0.129$ ,  $r_g = \sqrt{0.9}$ . To substitute the numbers into the equation. We get  $m \approx 7.048$ . Total length of grating is  $\Lambda$  multiply by  $m$ , i.e.  $325.765nm * 8 = 2606.119nm$ .

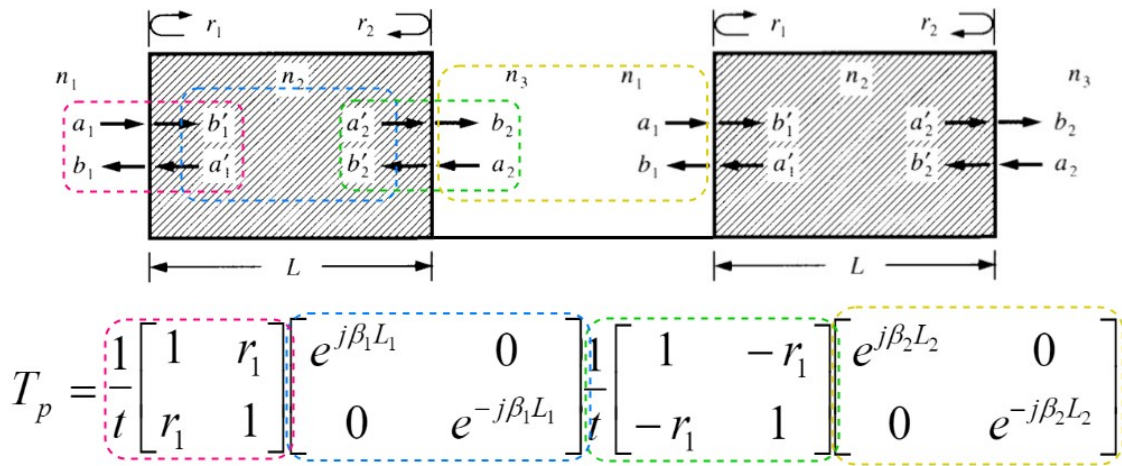


Fig 2. T matrix for one period

The T matrix for one period is presented above. Each coloured circled box in the illustration graph corresponds to each coloured circled box in the equation. As calculated earlier,  $\beta_1 L_1 = \beta_2 L_2 = \frac{\pi}{2}$ . To substitute this into the equation, we can interpret the T matrix as follows.

$$T_p = \frac{1}{t} \begin{bmatrix} 1 & 0.129 \\ 0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix} \frac{1}{t} \begin{bmatrix} 1 & -0.129 \\ -0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix}$$

For a grating with  $m$  periods we have the T matrix for the whole grating as

$$T_g = T_1 T_2 T_3 \dots T_7 = (T_p)^m$$

$$T_g = \left( \frac{1}{t} \begin{bmatrix} 1 & 0.129 \\ 0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix} \frac{1}{t} \begin{bmatrix} 1 & -0.129 \\ -0.129 & 1 \end{bmatrix} \begin{bmatrix} \exp(j\frac{\pi}{2}) & 0 \\ 0 & \exp(-j\frac{\pi}{2}) \end{bmatrix} \right)^7$$

Due to scattering theory, the T matrix can be transformed into Scattering S matrix, and vice-versa.

The definition of the S matrix is written below.

$$S_{11} = \frac{T_{21}}{T_{11}} \quad S_{12} = \frac{\det T}{T_{11}}, \text{ where } \det T = T_{11}T_{22} - T_{12}T_{21}$$

$$S_{21} = \frac{1}{T_{11}} \quad S_{22} = \frac{-T_{12}}{T_{11}}$$

The reflectivity is given by  $r = S_{11} = \frac{T_{21}}{T_{11}} = \frac{n_{eff1} - n_{eff2}}{n_{eff1} + n_{eff2}}$ .

The transmittance is given by  $t = S_{12} = S_{21}$ .

overall_reflectivity	0.9690
overall_transmittance	0.2471
power_reflectivity	93.8951
power_transmittance	6.1049
S	[0.9690,0.2471,0.2471,-0.9690]

Fig 3. MATLAB results

In MATLAB,  $T_g$  is calculated and converted to an  $S_g$ . Value of  $S_{g11}$  is taken to calculate the overall power reflectivity. Overall transmittance and power transmittance are included. The values matched the design of the Bragg filter, a slightly higher than 90%, which is 93.89% is reflected back and the remaining is transmitted when a wavelength of 1550 nm enters.

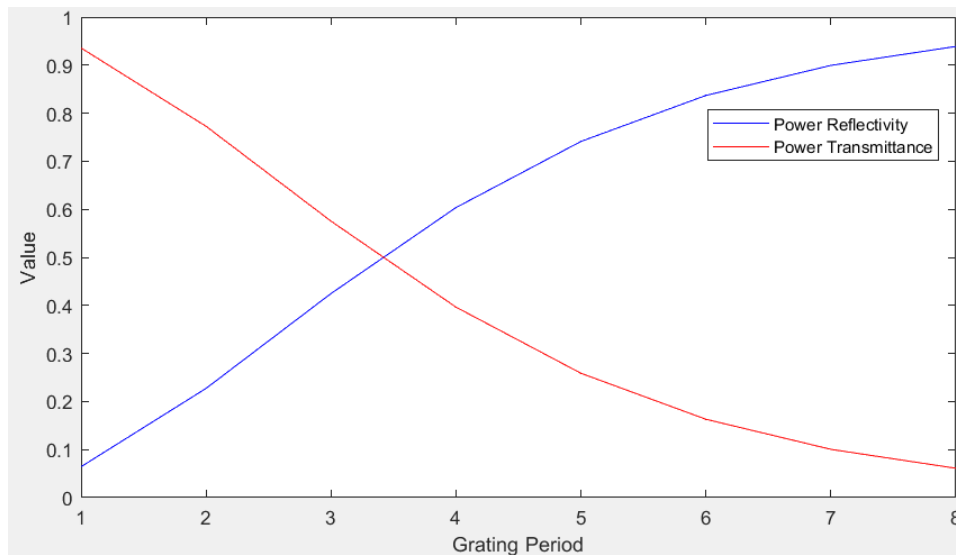


Fig 4. Behaviour of 1550 nm wavelength at n th grating period

In summary, behaviour of wavelength with 1550 nm entering the Bragg filter at n th grating period is plotted in MATLAB presented in Fig 4. The line of power reflectivity proves this Bragg filter allows small reflections to add up to form a large net reflection. Consequently, it exhibits low transmittance at the end.

## Distributed Bragg Reflectors in Active Devices (50%)

a.

VCSEL is a semiconductor device with light emission perpendicular to the chip surface, which contrasts to the Edge Emitting Laser (EEL) with light emission parallel to the surface. EEL has the disadvantage that the light beam has a high divergence angle and is difficult to couple into optical fibre without a corrective lens while VCSEL has a circular light beam which is easy for coupling into optical fibres[1].

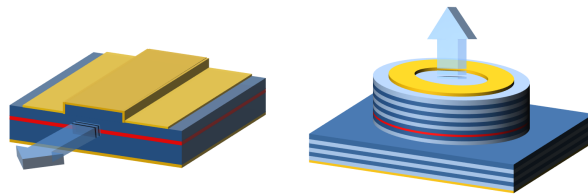


Fig 1. Principal structure of edge-emitting laser diodes (left) and VCSELs (right)

Lasers require a cavity to provide optical feedback to achieve stimulated emission. The cavity for edge-emitting laser (EEL) is formed by cleaving the wafer. Due to this design, light is emitted from the cleaved edge of the chip. In contrast, the cavity for a VCSEL is built by a gain medium (typically constructed either with a multi-quantum well structure or with quantum dots[2]), allowing collection of atoms that can undergo stimulated emission[3], placed in the centre of an optical cavity embedded between two highly reflective mirrors called the top and bottom distributed Bragg reflectors (DBRs). A DBR can be formed by alternating layers of two different refractive indices, GaAs and AlAs with the thickness chosen to be quarter of the wavelength of the gain bandwidth centre frequency. With the light generating layers between two DBRs, a cavity is formed to produce stimulated emission that radiates from the surface of the wafer.

b.

A Distributed Bragg Reflector (DBR) made of GaAs and AlAs can reflect light based on successive Fresnel reflection at normal incidence at interfaces between two alternating layers with high and low refractive indices. GaAs is the high index layer, while AlAs is the low index layer.

When quarter wavelength optical thickness, i.e.  $n_H d_H = n_L d_L = \frac{\lambda}{4}$  of each layer is fulfilled, the path difference between reflections from consecutive interfaces equals half of the wavelength ( $\frac{\lambda}{2}$ ), or  $180^\circ$  out of phase. All of the reflected components interfere constructively, and the  $180^\circ$  compensating phase shift can be accomplished by path length variations even though the reflections (r) at successive interfaces have alternate signs.

The interference caused by numerous lights reflected from different surfaces is what causes the light to be reflected from multiple films. Fig 2 illustrates the structure that shows the interference of light reflected and transmitted by other contact surfaces of numerous film layers. The structure is made to effectively interfere with all reflected components from the interfaces, producing a strong reflection.

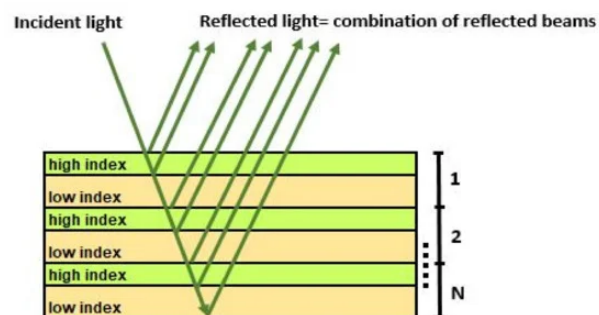


Fig 2. DBR high low index film structure

## Calculation Part

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The refractive index of GaAs can be calculated by the first-order Sellmeier equation.

$$n_{GaAs} = \sqrt{8.95 + \frac{2.054}{1 - 0.39/\lambda^2}} \text{ where } \lambda \text{ is } \times 10^{-6} m$$

$n_{GaAs}$  is calculated to be 3.5534 at a wavelength of 940 nm .

The refractive index of AlAs is given by the following equation[4]

$$n_{AlAs} = -2.5 \times 10^{-4} \lambda + 3.2 \text{ where } \lambda \text{ is } \times 10^{-9} m$$

$n_{AlAs}$  is calculated to be 2.965 at a wavelength of 940 nm

A period forms by a high ( $n_{GaAs} = 3.5534$ ) and low index ( $n_{AlAs} = 2.965$ ) layer pairing.

$$\text{Reflectivity } r_1 = \frac{n_{GaAs} - n_{AlAs}}{n_{GaAs} + n_{AlAs}} = \frac{3.5534 - 2.965}{3.5534 + 2.965} = 0.090268.$$

To achieve 90% power reflectivity at 940nm.

$$\text{Given } R = |r_g|^2 = 0.9$$

$$\therefore r_g \approx \tanh(2mr_1), \text{ where } m \text{ is no. of periods}$$

$$\therefore \sqrt{0.9} \approx \tanh(2m * 0.090268)$$

$$\tanh^{-1} \sqrt{0.9} \approx 2m * 0.090268$$

$$m \approx 10.0725 > 10$$

$\therefore$  11 pairs needed to achieve 90% power reflectivity at 940nm.



c.

Thickness of GaAs layer is  $d_{GaAs} = \frac{940nm}{4 \times 3.5534} = 66.134nm$ .

Thickness of AIAs layer is  $d_{AIAs} = \frac{940nm}{4 \times 2.965} = 79.258nm$ .

For top DBR,

$$\sqrt{0.99} \approx \tanh(2mr_1) \approx \tanh(2 \times m \times 0.090268)$$

$$m \approx 16.580 > 16$$

$\therefore$  17 pairs of GaAs/AIAs needed, power reflectivity R is 99.140%.

For the bottom DBR,

$$\sqrt{0.999} \approx \tanh(2mr_1) \approx \tanh(2 \times m \times 0.090268)$$

$$m \approx 22.969 > 22$$

$\therefore$  23 pairs of GaAs/AIAs needed, power reflectivity R is 99.951%.

For quantum well,

$$\text{Energy of a photon } E_g = \frac{1.24\mu m \text{ eV}}{\lambda_g} = \frac{1.24\mu m \text{ eV}}{0.94\mu m} = 1.319eV$$

To choose a material that is close to 1.319eV, the bandgap energy of the GaInAsP is 1.25 eV[5] and InP is 1.344eV, since  $1.25eV < 1.344eV$ , a structure of quantum well is formed as shown in Fig 4.

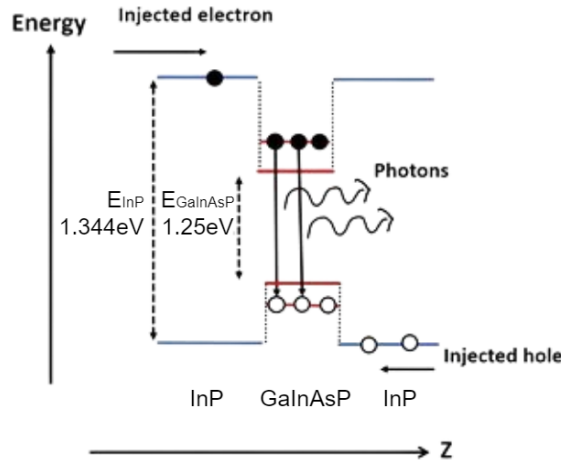


Fig 4. Quantum well structure

In order to calculate the cavity thickness, as we develop VCSEL as a single frequency laser, we have ensure cavity length is multiple of half wavelength  $L = \frac{n\lambda}{2}$ , one loop is taken as value of n, therefore  $L = \frac{940nm}{2} = 470nm$ .

In conclusion, I sketched my VCSEL design as presented in Fig 5. It has an electrical contact for current injection at the top, followed by a high 99.14% reflectivity DBR, an oxide layer to optimise light beam into a circular beam. A cavity that includes multiple quantum wells, which is the gain medium. Another oxide layer to confine the light. At last, is the 99.951%

reflectivity DBR layer. This reflectivity is higher than the top mirror, so that laser light will get out from the top mirror instead of the bottom.

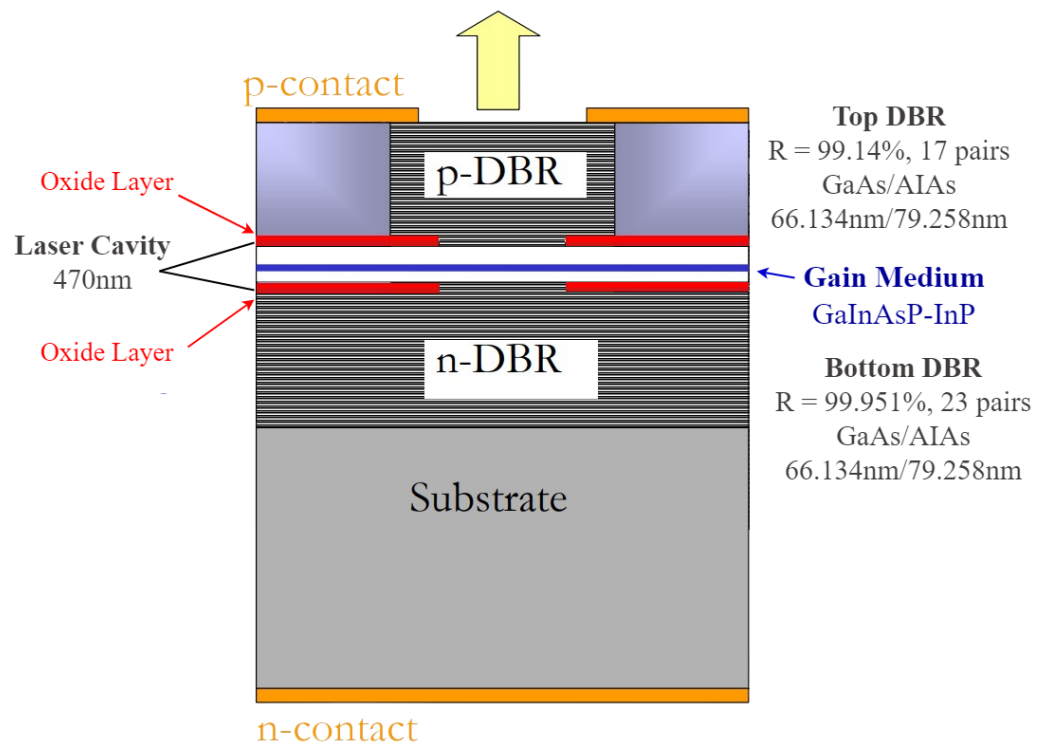


Fig 5. VCSEL design sketch

## References

- [1] Orton, J. (2008). Low dimensional structures. In *The Story of Semiconductors* (pp. 213–276). Oxford University Press. <https://doi.org/10.1093/acprof:oso/9780199559107.003.0006>
- [2] Mei, Y., Weng, G.-E., Zhang, B.-P., Liu, J.-P., Hofmann, W., Ying, L.-Y., Zhang, J.-Y., Li, Z.-C., Yang, H., & Kuo, H.-C. (2016). Quantum dot vertical-cavity surface-emitting lasers covering the ‘green gap’. In *Light: Science & Applications* (Vol. 6, Issue 1, pp. e16199–e16199). Springer Science and Business Media LLC. <https://doi.org/10.1038/lsa.2016.199>
- [3] Khreis, O. M. (2016). Modeling and analysis of smoothly diffused vertical cavity surface emitting lasers. In *Computational Condensed Matter* (Vol. 9, pp. 56–61). Elsevier BV. <https://doi.org/10.1016/j.cocom.2016.09.005>
- [4] ProQuest LLC. (1995). Vertical cavity surface emitting lasers design, characterisation and integration. University of London.
- [5] Yamazoe, Y., Nishino, T., Hamakawa, Y., & Kariya, T. (1980). Bandgap Energy of InGaAsP Quaternary Alloy. In *Japanese Journal of Applied Physics* (Vol. 19, Issue 8, p. 1473). IOP Publishing. <https://doi.org/10.1143/jjap.19.1473>