Aufgabe 1

$$\frac{\sum_{i=1}^{N}(x_i y_i - \overline{x}\,\overline{y})}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N} x_i y_i - n\overline{x}\,\overline{y}}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N} x_i y_i - \overline{x}\cdot n\overline{y}}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N} x_i y_i - \overline{x}\cdot \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)}$$

$$=\; \frac{\sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} \overline{x} y_i}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N}(x_i y_i - \overline{x} y_i)}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)}$$

$$=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N} x_i^2 - n\overline{x}^2} \;=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N} x_i^2 - 2n\overline{x}^2 + n\overline{x}^2} \;=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N} x_i^2 - 2\overline{x}\cdot n\overline{x} + n\overline{x}^2}$$

$$=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N} x_i^2 - 2\overline{x}\cdot \sum_{i=1}^{N} x_i + n\overline{x}^2} \;=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} 2x_i\overline{x} + n\overline{x}^2}$$

$$=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N}(x_i^2 - 2x_i\overline{x} + \overline{x}^2)} \;=\; \frac{\sum_{i=1}^{N}(x_i - \overline{x}) y_i}{\sum_{i=1}^{N}(x_i - \overline{x})^2}$$