i

$x_1 \times x_1 : (1, 2) \times (1, 2) = 1 \times 1 + 2 \times 2 = 5$
$x_1 \times x_2 : (1, 2) \times (-2, 1) = 1 \times -2 + 2 \times 1 = 0$
$x_1 \times x_3 : (1, 2) \times (-1, -2) = 1 \times -1 + 2 \times -2 = -5$
$x_1 \times x_4 : (1, 2) \times (2, -1) = 1 \times 2 + 2 \times -1 = 0$

$x_2 \times x_1 : (-2, 1) \times (1, 2) = -2 \times 1 + 1 \times 2 = 0$
$x_2 \times x_2 : (-2, 1) \times (-2, 1) = -2 \times -2 + 1 \times 1 = 5$
$x_2 \times x_3 : (-2, 1) \times (-1, -2) = -2 \times -1 + 1 \times -2 = 0$
$x_2 \times x_4 : (-2, 1) \times (2, -1) = -2 \times 2 + 1 \times -1 = -5$

$x_3 \times x_1 : (-1, -2) \times (1, 2) = -1 \times 1 + -2 \times 2 = -5$
$x_3 \times x_2 : (-1, -2) \times (-2, 1) = -1 \times -2 + -2 \times 1 = 0$
$x_3 \times x_3 : (-1, -2) \times (-1, -2) = -1 \times -1 + -2 \times -2 = 5$
$x_3 \times x_4 : (-1, -2) \times (2, -1) = -1 \times 2 + -2 \times -1 = 0$

$x_4 \times x_1 : (2, -1) \times (1, 2) = 2 \times 1 + -1 \times 2 = 0$
$x_4 \times x_2 : (2, -1) \times (-2, 1) = 2 \times -2 + -1 \times 1 = -5$
$x_4 \times x_3 : (2, -1) \times (-1, -2) = 2 \times -1 + -1 \times -2 = 0$
$x_4 \times x_4 : (2, -1) \times (2, -1) = 2 \times 2 + -1 \times -1 = 5$

$$\Longrightarrow \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 5 & 0 & -5 \\ -5 & 0 & 5 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix}$$

ii

Lagrange function for a hard margin SVM:

$$L(w, b, \alpha) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{N} \alpha_i [y_i(w^T x_i + b) - 1]$$

Partial derivations for w, b, α:

$$\frac{\sigma L}{\sigma w} = w - \sum_{i=1}^{N} \alpha_i y_i x_i = 0$$
$$\frac{\sigma L}{\sigma b} = -\sum_{i=1}^{N} \alpha_i y_i = 0$$
$$\frac{\sigma L}{\sigma \alpha} = y_i(w^T x_i + b) - 1 = 0$$

Substitute w, b into the third equation:

$$y_i((\sum_{j=1}^{N} \alpha_j y_j x_j)^T x_i + \frac{1}{N} \sum_{j=1}^{N} (\alpha_j y_j x_j)^T x_i) - 1 = 0$$

Calculate α for $x_1 = [5, 0, -5, 0]$ and $y_1 = 1$ using:

$$\alpha_i = \frac{1}{y_i x^T x_i}$$

we get:

$$\begin{aligned} \alpha_1 &= \frac{1}{1 \times (5^2 + 0^2 + (-5)^2 + 0^2)} \\ &= \frac{1}{1 \times (25 + 0 + 25 + 0)} \\ &= \frac{1}{50} \end{aligned}$$

Similarly, we get $\alpha_2 = \infty$, $\alpha_3 = \frac{1}{50}$, $\alpha_4 = \infty$.
Hence $\alpha* = (0.02, \infty, 0.02, \infty)$.