

UKRAINIAN CATHOLIC UNIVERSITY

MASTER THESIS

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# Corner localization and camera calibration from imaged lattices

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*A thesis submitted in fulfillment of the requirements  
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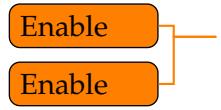
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ii



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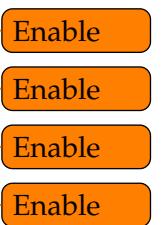
Master of Science

**Corner localization and camera calibration from imaged lattices**

by Andrii STADNIK

*Abstract*

Current state-of-the-art techniques occasionally fail to accurately estimate the camera parameters for wide-angle lenses, as most of the algorithms assume lens with medium to small distortion. Partially it occurs due to extreme distortion at the edges of the image, which poses additional challenges to feature detection algorithms. We propose a novel approach for detecting additional features on the imaged calibration pattern using intermediate camera calibration. Utilizing the prior information about the calibration pattern's geometry, we can estimate the possible positions of the previously undetected features. Those features are filtered using the binary classifier, and the remaining ones are used to further constrain the camera calibration. The proposed method consists of three key steps: (1) feature detection and estimation of the lattice geometry, (2) initial camera calibration by solving the two-steps linear minimization problem, and (3) estimation the possible positions of the undetected features by projecting the imputed and extended lattice geometry onto the image plane, and filtering them using the binary classifier.

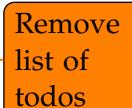




# Todo list

Enable	ii
Enable	ii
Enable	iii
Remove list of todos	1
I'll have to merge something to have 5 chapters	2
Explain why more features is better	3
Figure: Show the example of the calibration board and the image	3
Probably rephrase if I won't compare the actual calibration	4
Add more	4
Update	4
add reference	10
Doesn't sound very rigorous to me.	12
Is the notation ok?	12
Am I using the term correctly?	16
Add ref to the figure	16
Figure: Compare the number of points with the provided by babelcalib	21
Figure: Compare the reprojection error	21

1



Remove  
list of  
todos

1. Introduction and motivation
  - (a) Outline of the problem
  - (b) Research objective
    - Finding new features by projecting the board
    - Robustly classifying the true and false positives
  - (c) Thesis structure
2. Related work
  - (a) Camera calibration
  - (b) Calibration boards
  - (c) Feature detection on calibration boards
  - (d) Camera models
  - (e) Camera parameters estimation
  - (f) Search of the previously undetected features
3. Background
  - (a) Notation
  - (b) Camera model
    - Distortion model, inverse distortion model
    - Projection
    - Backprojection
  - (c) Feature detection
  - (d) Camera calibration (Probably I could move it to the Approach)
4. Approach
  - (a) Feature detection
  - (b) Camera calibration
  - (c) Board projection, features classification
5. Experiments
  - (a) Synthesizer
  - (b) Datasets
  - (c) Metrics
  - (d) Results
6. Conclusions

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have 5  
chapters

## Chapter 1

# Introduction and motivation

### 1.1 Outline of the problem

Better camera calibration improves the performance of various downstream tasks by providing a more accurate mapping between 3D world coordinates and 2D image plane coordinates. This improved mapping enables precise alignment, positioning, and scaling of objects within the scene. By determining the camera's intrinsic and extrinsic parameters, algorithms can correct for lens distortion, estimate depth information, and accurately overlay virtual content. Consequently, tasks such as 3D reconstruction, augmented reality, and object detection can achieve better results in terms of precision, spatial consistency, and overall visual quality.

Although manufacturers can estimate camera calibration parameters a priori, fully automatic calibration is often preferred, especially when camera metadata is unavailable. Currently, wide-angle lenses, particularly in mobile phones and GoPro-type cameras, dominate consumer photography. These cameras pose additional challenges due to their requirement for highly non-linear models with numerous parameters. The high distortion of the image plane also makes finding key points robustly challenging.

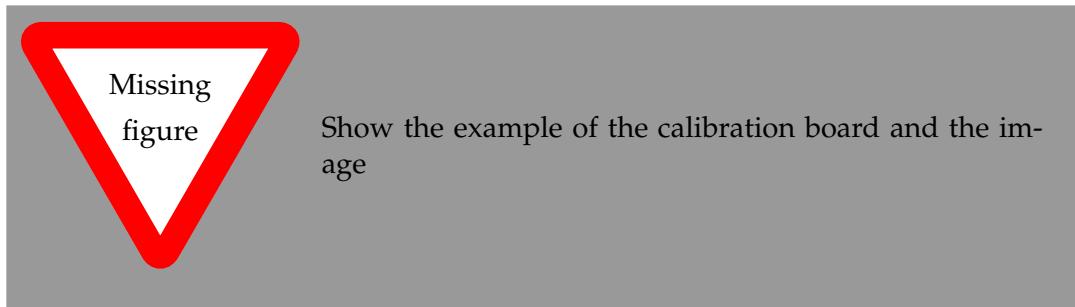
Typically, camera calibration is obtained by capturing an image of a known calibration pattern, which is then used to estimate the camera parameters. Alternatively, some methods do not use a calibration pattern but instead infer geometric constraints directly from the scene. However, this approach is generally less accurate.

As reported by Duisterhof et al. (2022) on Oct. 5, 2022, the current state-of-the-art methods (Olson, 2011; Schöps et al., 2020; Krogius, Haggenmiller, and Olson, 2019) fail on images with high distortion. Duisterhof et al., 2022 suggested an iterative approach of image undistortion and target reprojection, achieving the superior robustness to the noise than the state-of-the-art methods because the feature detection is performed on the undistorted image.

Instead of searching for the features on the undistorted image from scratch, it is possible to utilize the prior knowledge of the geometry of the calibration board, effectively predicting the possible positions of previously undetected features. It can be done by projecting the board onto the image using the intermediate camera calibration, and then filtering the possible positions in order to eliminate false positives.

This additional points will further constrain the camera calibration, improving the accuracy of the calibration parameters.

Explain why more features is better



## 1.2 Research objective

Probably rephrase if I won't compare the actual calibration

The objective of this research is to improve the accuracy of the camera calibration by finding previously undetected features on the calibration board. For that, we formulate the set of research questions:

- How to find additional features on the calibration board which were not detected by the feature detector?
- How to filter out falsely detected features?

Add more

## 1.3 Thesis structure

Update

This paper has the following structure: in ??, we will describe the related work, including the literature search method and methodology, various subtopics of the camera calibration, mention conjugate translations, and outline the state-of-the-art solutions. We define the research gap in ?? and outline the proposed approach to solution and evaluation in ?. We will describe the early results in ??, including the dataset analysis, feature detector, and conjugately translated points simulator. In ??, we will summarize the results and outline future work.

## Chapter 2

# Related work

### 2.1 Camera calibration

Getting the correspondence between the spatial and the image coordinates requires camera calibration. Camera calibration consists of the geometric camera model and the parameters of this model. That information makes it possible to obtain the 2d image coordinates of any point in the 3d space.

Usually, the geometric camera model is obtained from the domain knowledge of the researcher or the camera manufacturer. Often, they choose the simplified model as a trade-off between accuracy and complexity. The model's parameters are usually obtained by solving the constrained optimization problem, given the set of points with known geometry.

### 2.2 Calibration boards

To achieve a robust calibration, images with repeating patterns are usually used. The camera calibration parameters can be found using prior knowledge of the properties of the pattern, such that the pattern invariants hold on the image. Initially, the chessboard (*OpenCV: Camera Calibration 2023*; V. Douskos, I. Kalisperakis, and G. Karras, 2007) patterns were used (fig. 2.1a).

Later, ArUco (Garrido-Jurado et al., 2014) (fig. 2.1b) and AprilTag (Olson, 2011) (fig. 2.1d) allowed detecting the orientation of the pattern, as well as uniquely identifying each located pattern even under occlusion. Based on ArUco, ChArUcO (*OpenCV: Camera Calibration 2023*) (fig. 2.1c) was proposed as more robust.

There are also other calibration patterns, some of which are not square (for example, Delittle (Ha et al., 2017)).

### 2.3 Camera models

The choice of the camera model depends on the camera's physical properties and the accuracy required. Usually, the parametric models are simpler to use, as they have only a few parameters and deliver good accuracy. The most common are the Double Sphere model (Usenko, Demmel, and Cremers, 2018), the Kannala-Brandt model (Kannala and Brandt, 2006), and the Field-of-View model (Devernay and Faugeras, 2001). In the ill-posed problem of camera calibration, the common choice of the camera model is the division model (Fitzgibbon, 2001). However, Schöps et al., 2020 shows that they tend to have significantly higher errors than the non-parametric (general) models. The Lochman et al., 2021 suggested a framework for converting the parameters of a powerful back-projection Z. Zhang, 2000 model to recover different models' parameters.

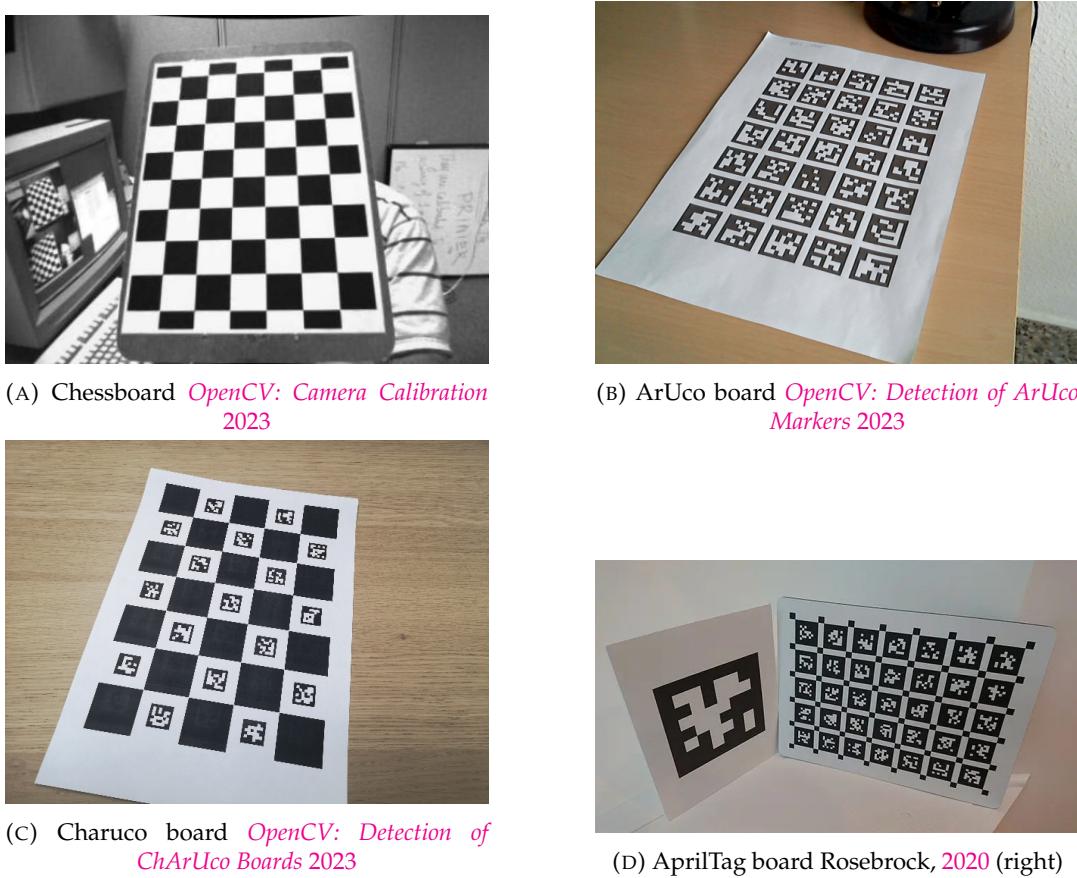


FIGURE 2.1: Calibration boards.

## 2.4 Camera parameters estimation

Camera calibration using repeating patterns was an important subject for a long time, for example, Schaffalitzky and Zisserman, 1998 in 1998 and Z. Zhang, 2000.

Nevertheless, camera calibration is still an open problem; recently, multiple new approaches have arisen. Davide Scaramuzza, Agostino Martinelli, and Roland Siegwart, 2006 proposed an automatic algorithm for camera calibration, which did not use any model of the specific omnidirectional sensor. Lochman et al., 2021 suggest a universal approach to camera calibration, with a separate step of converting between camera models. Hu et al., 2019 used deep learning to detect ChArUcO boards. Recently, on Oct. 5, 2022, Duisterhof et al., 2022 introduced the iterative approach to camera calibration, which outperforms the previous state-of-the-art approaches for wide-angle cameras.

## 2.5 Boards' features detection

The detection of the calibration board's features is a crucial step in the camera calibration pipeline. The detection accuracy directly affects the calibration accuracy. Besides the feature detection, it's also important to associate the detected features with the board's corners.

There are various of the feature detection methods. Many of them require prior knowledge of the board's geometry, such as the number of rows and columns, and

require the full visibility of the board. For example, the default OpenCV implementation of the chessboard detection (*OpenCV: Camera Calibration 2023*).

Because of that, other methods were proposed, such as Fuersattel et al., 2016 and Geiger et al., 2012. Those method can detect the board and associate the corners with the borders' corners under the partial occlusion and do not require prior knowledge of the board's geometry.

Chen and G. Zhang, 2005 proposed using Hessian to find pixel positions of the corners.

## Chapter 3

# Background

### 3.1 Notation

Term	Description
$\mathbf{u} = (u, v, 1)^T$	A point in the board coordinate system
$\mathbf{x} = (X, Y, Z, 1)^T$	A point in the world coordinate system
$R$	A $3 \times 3$ rotation matrix
$\mathbf{t}$	A $3 \times 1$ translation vector
$\alpha_x$	Scale factor in the x direction (pixels/mm)
$\alpha_y$	Scale factor in the y direction (pixels/mm)
$c_x, c_y$	Coordinates of the principal point (image center)
$\theta$	Angle between the x and y pixel axes
$f$	Distance from the camera center to the image plane (focal length)
$f_x, f_y$	Effective focal lengths in the x and y directions
$K$	Intrinsic matrix incorporating the scaling, introduced by the focal length
$H$	A $3 \times 3$ matrix viewing $z = 0$ (see section 3.2.5)
$\lambda_n$	Distortion coefficients

TABLE 3.1: Notation

### 3.2 Camera model

In this paper, scene and image points are represented using homogeneous coordinates. This approach allows representing many geometric transformations as linear, which simplifies the mathematical representation of the camera model.

#### 3.2.1 Perspective projection

The perspective projection is a mapping from a 3D point  $(X, Y, Z)^T$  in the world coordinate to the 2D coordinate  $(u, v)^T$  on the image plane which is distance  $f$  from the center of projection. It is given by the perspective projection equation:

$$(u, v)^T = \frac{f}{Z} (X, Y)^T.$$

This equation can be written using the homogeneous coordinates as:

$$\alpha \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}, \quad (3.1)$$

where  $\alpha = 1/Z$  is a scale factor.

### 3.2.2 Scene to camera projection

A 3D scene point  $(X, Y, Z)^T$  can be projected onto the image plane as  $R(X, Y, Z)^T + \mathbf{t}$ , where  $R$  is a  $3 \times 3$  rotation matrix and  $\mathbf{t}$  is a  $3 \times 1$  translation vector. Using the homogeneous coordinates, this can be written as:

$$\begin{bmatrix} R & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}. \quad (3.2)$$

### 3.2.3 Camera to image projection

To project a point from the camera coordinate system to the image plane, we need to apply a homography encoding the camera intrinsic parameters. This is a  $3 \times 3$  upper-triangular matrix:

$$\begin{bmatrix} \alpha_x & \alpha_x \cot \theta & c_x \\ 0 & \alpha_y \sin \theta & c_y \\ 0 & 0 & 1 \end{bmatrix} \quad (3.3)$$

where:

- $\alpha_x$  and  $\alpha_y$  represent the scale factor of the camera in terms of pixels/mm in the x and y directions respectively.
- $c_x$  and  $c_y$  are the coordinates of the principal point, which is typically the image center.
- $\cot \theta$  and  $\sin \theta$  are related to the skew coefficient, which measures the angle between the x and y pixel axes. The variable  $\theta$  represents this angle.

For a typical camera,  $\theta = \pi/2$  and  $\alpha_x = \alpha_y = 1$  Hartley and Zisserman, 2004.

Conventionally, the intrinsic matrix incorporates the scaling, introduced by the focal length:

$$K = \begin{bmatrix} \alpha_x f & \alpha_x \cot \theta & c_x \\ 0 & \alpha_y f & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} f_x & k & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}. \quad (3.4)$$

By incorporating the assumptions, mentioned previously into the intrinsic matrix, we can simplify it to:

$$K = \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}. \quad (3.5)$$

### 3.2.4 Camera matrix

The composition of positioning and orienting the camera, projection, and the imaging transformation can be represented by a  $3 \times 4$  camera matrix (D. Scaramuzza, A. Martinelli, and R. Siegwart, 2006). This matrix can be expressed as:

$$K [I_3 | \mathbf{0}] \begin{bmatrix} R & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} = K [R | \mathbf{t}], \quad (3.6)$$

Hence, the transformation of a point in the scene by the camera  $P^{3 \times 4}$  can be formulated as:

$$\alpha(u, v, 1)^T = K [R | \mathbf{t}] (X, Y, Z, 1)^T \quad (3.7)$$

with  $\alpha$  being  $1/Z$ .

### 3.2.5 Projection of the points from the scene plane

When working with the coplanar scene points, we can simplify the projection by assuming that the scene plane is located at  $Z = 0$ . In this case, the projection of the point becomes:

$$\alpha \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3 \quad \mathbf{t}] \begin{pmatrix} X \\ Y \\ 0 \\ 1 \end{pmatrix} = K \underbrace{[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}]}_H \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}. \quad (3.8)$$

### 3.2.6 Distortion

[add reference](#)

The distortion of the image is caused by the lens not being perfectly planar. Typically, the small distortions caused by lens misalignment are ignored, allowing us to model the distortion as radially symmetric. Then, the function that maps a point  $\mathbf{u} = (u, v, 1)^T$  from a retinal plane to the ray direction in the camera coordinate system is given by:

$$g(\mathbf{u}) = (u, v, \psi(r(\mathbf{u})))^T, \quad (3.9)$$

where  $r(\mathbf{u}) = \sqrt{u^2 + v^2}$  is the radial distance from the principal point.

#### Back-projection using the Division Model

The division model has a good ability to model the distortion of the wide-angle lenses, and is widely used (Pritts et al., 2021; D. Scaramuzza, A. Martinelli, and R. Siegwart, 2006). The model is defined as:

$$\psi(r) = 1 + \sum_{n=1}^N \lambda_n r^{2n}, \quad (3.10)$$

where  $\lambda_n$  are the distortion coefficients.

The function  $\psi(r)$  is not invertible in general. Let  $\hat{\mathbf{X}} = (X, Y, Z)^T = \alpha g(\mathbf{u})$  be a ray in the camera coordinate system.

Then,

$$\frac{\mathbf{X}}{Z} = \left( \frac{X}{Z}, \frac{Y}{Z}, 1 \right)^T \quad (3.11)$$

$$= \left( \frac{\alpha u}{\alpha \psi(r(\mathbf{u}))}, \frac{\alpha v}{\alpha \psi(r(\mathbf{u}))}, 1 \right)^T \quad (3.12)$$

$$= \left( \frac{u}{\psi(r(\mathbf{u}))}, \frac{v}{\psi(r(\mathbf{u}))}, 1 \right)^T \quad (3.13)$$

From 3.13 we see that

$$\begin{cases} \frac{X}{Z} = \frac{u}{\psi(r(\mathbf{u}))} \\ \frac{Y}{Z} = \frac{v}{\psi(r(\mathbf{u}))} \end{cases} \implies \begin{cases} u = \frac{X\psi(r(\mathbf{u}))}{Z} \\ v = \frac{Y\psi(r(\mathbf{u}))}{Z} \end{cases} \quad (3.14)$$

Now, let  $\hat{r}$  be a root of  $r(\mathbf{u}) = \sqrt{\frac{X*\psi(r)}{Z}^2 + \frac{Y*\psi(r)}{Z}^2} = r$ .

Then,  $\mathbf{u} = f(\mathbf{X}) = \frac{\hat{r}}{r(\mathbf{X})}\mathbf{X}$ , where  $f(\cdot)$  is the inverse of  $g(\cdot)$ .

### 3.2.7 Complete projection and backprojection

Now, the complete projection and backprojection can be formulated as follows:

$$\alpha \mathbf{u} = Kf(H\mathbf{X}) \quad (\text{Projection})$$

$$\alpha \mathbf{X} = H^{-1}g(K^{-1}\mathbf{u}) \quad (\text{Back projection})$$

## Chapter 4

# Approach

### 4.1 Feature detection

For the feature detection, we used the approach, proposed by Geiger et al., 2012 (see section 2.5 and ??).

### 4.2 Camera calibration

In this work, we obtained the camera calibration in two steps: first, we used the approach, proposed by Davide Scaramuzza, Agostino Martinelli, and Roland Siegwart, 2006 to obtain the initial approximation of the camera parameters. Then, we used the optimization to minimize the reprojection error between the board and the back-projected corners. A division model is very powerful, and is able to express a wide number of range of distortions (Pritt et al., 2021). In this work, we used a model with two parameters to allow for a better flexibility during the optimization.

#### 4.2.1 Reprojection error

The reprojection error is the distance between the reprojected point and the measured one. It is used to evaluate the quality of the camera calibration.

We chose to minimize the reprojection error between the board and back-projected corners, which were initially detected. The projection and back-projection are the inverse of each other, hence minimizing the error between the projection of the board and the detected corners and minimizing the error between the back-projection of the detected corners and the board are equivalent. We minimized the error between the board and the back-projected corners because back-projection does not require the root finding.

Let's define the following variables:  $\theta = (\theta_x, \theta_y, \theta_z)^T$  is the vector of Euler rotation angles,  $\mathbf{t} = (t_x, t_y, t_z)^T$  is the translation vector,  $\lambda = (\lambda_1, \lambda_2)^T$  is the intrinsic parameters vector,  $f$  is the focal length, and  $\mathbf{s} = (s_x, s_y)^T$  is the sensor size. From the input image, we know the resolution  $\mathbf{r} = (r_x, r_y)^T$ . From the rotation vector, we can compute the rotation matrix  $\mathbf{R}$  as:

From  $\theta$ , the rotation matrix  $R$  can be calculated as follows:

$$R = R(\theta_x)R(\theta_y)R(\theta_z) \quad (4.1)$$

Doesn't sound very rigorous to me.

Is the notation ok?

$$R(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix} \quad (4.2)$$

$$R(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix} \quad (4.3)$$

$$R(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.4)$$

Then,  $H$  is given by:

$$H = [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] . \quad (4.5)$$

We can compute the intrinsic camera matrices as follows:

$$K = \begin{pmatrix} \frac{fr_x}{s_x} & 0 & \frac{r_x}{2} \\ 0 & \frac{fr_y}{s_y} & \frac{r_y}{2} \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.6)$$

Then, the back-projection of a 2D point  $\mathbf{u} = (u, v, 1)$  into a scene point with  $Z = 0$   $\mathbf{x} = (X, Y, 1)$  is given by:

$$\mathbf{x} = Hg_{\lambda_1, \lambda_2}(K^{-1}\mathbf{u}). \quad (4.7)$$

#### 4.2.2 Initial approximation

Davide Scaramuzza, Agostino Martinelli, and Roland Siegwart, 2006 proposed an automatic method for camera calibration, which consisted of the following steps:

- Solving for the camera extrinsic parameters
- Solving for the camera intrinsic parameters
- Linear refinement of the intrinsic and extrinsic parameters
- Iterative center detection
- Non-linear refinement

In this work, we focused on the single image, while the original approach relied on the multiple images. Therefore, we used only the first two steps of the algorithm.

### Solving for the camera extrinsic parameters

To derive the solver for the camera extrinsic parameters, start from eq. (Projection):

$$\alpha \mathbf{u} = Kf(H\mathbf{X}) \quad (4.8)$$

$$\alpha K^{-1}\mathbf{u} = f(H\mathbf{X}) \quad \text{Move } K \text{ to the left side} \quad (4.9)$$

$$\alpha g(K^{-1}\mathbf{u}) = g(f(H\mathbf{X})) \quad \text{Set } \hat{\mathbf{u}} = K^{-1}\mathbf{u}; \text{ apply } g(\cdot) \text{ to both sides} \quad (4.10)$$

$$\alpha \begin{pmatrix} \hat{u}_x \\ \hat{u}_y \\ \psi(r(\hat{\mathbf{u}})) \end{pmatrix}^T = H\mathbf{X} \quad (4.11)$$

$$(4.12)$$

For  $K$  we used the placeholder values.  $f$  was set to a constant value for the typical consumer camera, and  $c_x, c_y$  were set to the center of the image. The correct values will be computed during the optimization section 4.2.3.

To eliminate the dependency on the scale  $\alpha$ , multiply both sides vectorially by  $g(\hat{\mathbf{u}})$ :

$$\alpha g(\hat{\mathbf{u}}) \times g(\hat{\mathbf{u}}) = g(\hat{\mathbf{u}}) \times H\mathbf{X} = 0 \implies (\hat{u}, \hat{v}, \psi(r(\hat{\mathbf{u}})))^T \times [\mathbf{r}_1 \ \mathbf{r}_2 \ \mathbf{t}]^T \mathbf{X} = 0 \quad (4.13)$$

From eq. (4.13), we can see that a point contributes to three homogeneous equations:

$$\hat{v}(t_1 X + t_2 Y + t_3) - g(r(\hat{\mathbf{u}}))(r_{12}X + r_{22}Y + t_2) = 0 \quad (4.14)$$

$$g(r(\hat{\mathbf{u}}))(r_{11}X + r_{12}Y + t_1) - \hat{u}(t_1 X + t_2 Y + t_3) = 0 \quad (4.15)$$

$$\hat{u}(r_{12}X + r_{22}Y + t_2) - \hat{v}(r_{11}X + r_{12}Y + t_1) = 0 \quad (4.16)$$

Only eq. (4.16) is linear in the unknowns. Each point gives a single equation. Now, by rewriting the equation in the matrix form  $M \cdot H = 0$ , where  $H = (r_{11}, r_{12}, r_{12}, r_{22}, t_1, t_2)$  we get:

$$M = \begin{bmatrix} -\hat{v}_1 X_1 & -\hat{v}_1 Y_1 & -\hat{u}_1 X_1 & -\hat{u}_1 Y_1 & -\hat{v}_1 & -\hat{u}_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\hat{v}_N X_N & -\hat{v}_N Y_N & -\hat{u}_N X_N & -\hat{u}_N Y_N & -\hat{v}_N & -\hat{u}_N \end{bmatrix} \quad (4.17)$$

, where  $N$  is the number of points.

The linear estimate of  $H$  is found by minimizing  $\|M \cdot H\|^2$  using SVD. The solution is known up to a scale factor.

To find  $t_1$  and  $t_2$ , note that  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are orthonormal:

$$\lambda^2 r_{11}r_{12} + \lambda^2 r_{12}r_{22} + \lambda^2 t_1 t_2 = 0 \quad (4.18a)$$

$$\lambda \sqrt{r_{11}^2 + r_{12}^2 + t_1^2} = 1 \quad (4.18b)$$

$$\lambda \sqrt{r_{12}^2 + r_{22}^2 + t_2^2} = 1 \quad (4.18c)$$

, where  $\lambda$  is non-zero multiplier.

Now, to solve for  $t_1$  and  $t_2$ , first find possible values for  $t_2^2$ :

$$-\frac{r_{11}r_{12} + r_{12}r_{22}}{t_2} = t_1 \quad \text{Solve eq. (4.18a) for } t_1 \quad (4.19)$$

$$(4.20)$$

$$\sqrt{\lambda^2 \left( \frac{(r_{11}r_{12} + r_{12}r_{22})^2}{t_2^2} + r_{11}^2 + r_{12}^2 \right)} = 1 \quad \text{Substitute into eq. (4.18b)} \quad (4.21)$$

$$\lambda^2 \left( \frac{(r_{11}r_{12} + r_{12}r_{22})^2}{t_2^2} + r_{11}^2 + r_{12}^2 \right) = 1 \quad \text{Square both sides} \quad (4.22)$$

$$\lambda^2 \left( \frac{(r_{11}r_{12} + r_{12}r_{22})^2}{t_2^2} + r_{11}^2 + r_{12}^2 - r_{12}^2 - r_{22}^2 - t_2^2 \right) = 0 \quad \text{Subtract eq. (4.18c)} \quad (4.23)$$

$$\frac{(r_{11}r_{12} + r_{12}r_{22})^2}{t_2^2} + r_{11}^2 + r_{12}^2 - r_{12}^2 - r_{22}^2 - t_2^2 = 0 \quad \text{Divide both by } \lambda^2 \quad (4.24)$$

$$t_2^4 - (r_{11}^2 + r_{12}^2 - r_{12}^2 - r_{22}^2)t_2^2 - (r_{11}r_{12} + r_{12}r_{22})^2 = 0 \quad \text{Multiply by } t_2^2 \quad (4.25)$$

Now, solve eq. (4.25) for  $t_2^2$ , and take a root to find possible values for  $t_2$ .

To find  $t_1$ , substitute the found values for  $t_2$  into eq. (4.19) or eq. (4.23) depending on the value of  $t_2$ :

$$\begin{cases} t_1 = -\frac{r_{11}r_{12} + r_{12}r_{22}}{t_2} & t_2 \neq 0 \\ t_1 = r_{11}^2 + r_{12}^2 - r_{12}^2 - r_{22}^2 & t_2 = 0 \end{cases} \quad (4.26a)$$

$$(4.26b)$$

Now, it's possible to find  $H$  for each of the pairs of  $t_1$  and  $t_2$ .

Lastly, to select the correct  $H$ , the author assumes that one of the boards' corners has the coordinates  $(0, 0)^T$ . Then, the rotation wouldn't affect this point, and it would be projected to  $(t_1, t_2)^T$ . Hence, the best matrix would be such that has the closest  $(t_1, t_2)^T$  to  $(X, Y)^T$  of the corner, which is associated with the board's corner with coordinates  $(0, 0)^T$ .

However, often the algorithm finds two matrices  $H$  such that they have the same  $t_1$  and  $t_2$ , but different  $r_{11}$ ,  $r_{12}$ ,  $r_{21}$ . To find the best matrix, we found the intrinsic values for each of them, and backprojected the board's corners using both matrices. Then, we used the one which gave the smallest reprojection error.

### Solving for the camera intrinsic parameters

Now, to find the rest of the parameters, we substitute the values, found in the previous step into eq. (4.14) and eq. (4.15). We assumed the number of the division model's parameter to be equal to 2, and the scalar multiplier to be equal to 1 section 3.2.6:

$$\begin{bmatrix} A_1\rho_1^2 & A_1\rho_1^4 & -v_1 \\ C_1\rho_1^2 & C_1\rho_1^4 & -v_1 \\ \dots & \dots & \dots \\ A_N\rho_N^2 & A_N\rho_N^4 & -v_N \\ C_N\rho_N^2 & C_N\rho_N^4 & -v_N \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} B_1 - A_1 \\ D_1 - C_1 \\ \dots \\ B_N - A_N \\ D_N - C_N \end{bmatrix} \quad (4.27)$$

, where

$$A_i = r_{21}X_i + r_{22}Y_i + t_2 \quad (4.28)$$

$$B_i = v_i(r_{31}X_i + r_{32}Y_i) \quad (4.29)$$

$$C_i = r_{11}X_i + r_{12}Y_i + t_1 \quad (4.30)$$

$$D_i = u_i(r_{31}X_i + r_{32}Y_i) \quad (4.31)$$

$$(4.32)$$

The solution can be found using the least squares method.

### 4.2.3 Optimization

The loss function is the sum of the squared reprojection errors between the board and the back-projected corners, which were initially detected:

$$L = \sum_{i=1}^N \|Hg_{\lambda_1, \lambda_2}(K^{-1}\mathbf{u}_i) - \mathbf{x}_i\|^2. \quad (4.33)$$

For the initial guess, we used the randomly chosen constant small values.

The model converged to the same results compared to the initial parameters, set using the Scaramuzza solver, unless the initial guess was very degenerate (i.e.  $R$  was such that the board plane passed through the principal point, and all backprojected points were projected onto the same line).

This issue also occurred with random small initial values, due to the best approximation for  $R$  which minimizes  $L$  when the distance from the back-projected board from the measure one was high (i.e., the  $t$  was far from the true value) being the degenerate solution. In order to avoid this issue, we first optimized only the  $t$ , until the loss function converged, meaning that  $t$  is close to the true value.

However, another issue was that when the board was rotated close to the  $180^\circ$ ,  $R$  once again converged to the degenerate solution. In order to avoid this issue, we found the solution with initial  $\theta_z$  set to value, close to  $0^\circ$  and  $180^\circ$ , and then used the solution which minimized the loss function.

Am I using the term correctly?

Add ref to the figure

## 4.3 Additional features detection

Often, not all of the board's corners were detected initially. Firstly, we assumed that the whole board was detected, and imputed the missing points in the board space. Then, we tried extending the board points.

We used the obtained camera parameters to then project the imputed board points into the image space.

## 4.4 Classifier

In this work, one of the methods we used was proposed by Geiger et al., 2012 as it didn't require the whole board to be visible, automatically determines the board's number of rows and columns and worked quite well on highly-distorted images.

In this work, we directly used the following steps from the algorithm:

1. Corner detection
2. Sub-pixel corner and orientation refinement

#### 4.4.1 Corner detection

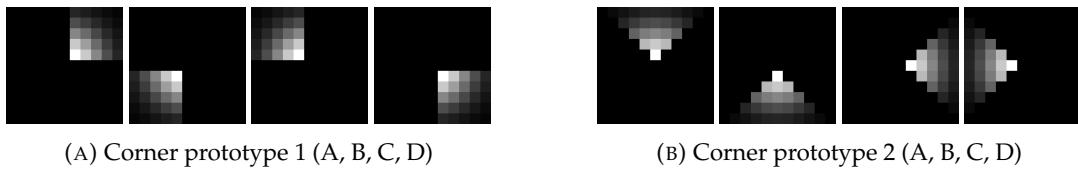


FIGURE 4.1: Corner prototypes (Geiger et al., 2012)

According to Geiger et al., 2012, the following approach proved to be more robust to image clutter and blur than other common choices (Harris and Stephens, 1988; Shi and Tomasi, 2000).

To detect corners in a grayscale image  $I$ , the author used two  $n \times n$  prototypes for axis-aligned and  $45^\circ$  rotated corners, respectively. Each prototype is constructed using four filter kernels  $\{A, B, C, D\}$  (fig. 4.1). The corner likelihood  $c$  at each pixel is computed by:

$$\begin{aligned} c &= \max(s_1^1, s_2^1, s_1^2, s_2^2) \\ s_1^i &= \min(\min(f_A^i, f_B^i) - \mu, \mu - \min(f_C^i, f_D^i)) \\ s_2^i &= \min(\mu - \min(f_A^i, f_B^i), \min(f_C^i, f_D^i) - \mu) \\ \mu &= 0.25(f_A^i + f_B^i + f_C^i + f_D^i) \end{aligned} \quad (4.34)$$

Then, the authors apply the conservative non-maximum suppression to the response map, and additionally filter the corners by assuring that there are two modes of the gradient directions.

#### 4.4.2 Classifier training

We tested the approach of Geiger et al., 2012, and, alternatively, the Hessian responses for the image, as proposed by Chen and G. Zhang, 2005.

To create a training dataset, we collected the true and false positives from the corners we already detected.

For each of the detected corners on all images, we collected the values of the response function at the previously detected corners, and around them. We didn't collect the values of all of the pixels, in order to reduce the number of very simple examples.

To determine the threshold for the classification, we used the ROC curve, and found the threshold which maximizes the geometric mean of the true positive rate and the false positive rate.

We used both of the approaches to train the classifier, and compared the results in section 5.6.

## Chapter 5

# Experiments

### 5.1 Simulator

We created a simulator to generate distorted points as described in section 3.2.7. We used it to test the solver, and test the correctness of projection and back-projection points.

### 5.2 Metrics

- Reprojection error
- Number of detected corners

### 5.3 Dataset

For the project, we need highly distorted photos of calibration boards. It takes a lot of work to generate such a dataset, as cameras which produce such images are usually expensive. Therefore, it would be preferable to use an existing dataset.

For this project, we required a highly distorted dataset of chessboard images. We collected a number of datasets, but the feature detector we used supports detecting only the boards with the constant tile size, therefore we cannot use AprilGrid nor CharuCO boards. The feature detector is the only limiting factor.

Lochman et al. (2021) collected a wide number of datasets, typically used in the field for the benchmarking of the camera calibration. They're provided in a Deltille *Deltille Detector 2023* format, and are well documented:

**Kalibr** (Maye, Furgale, and Roland Siegwart, 2013) contains several established datasets that are commonly used for testing the accuracy of camera calibration frameworks: Double Sphere Usenko, Demmel, and Cremers, 2018, EuRoC Burri et al., 2016, TUM VI Schubert et al., 2018, and ENTANIYA 1 *Calibration of a 250deg Fisheye Lens* [#242](#) [#Ethz-Asl/Kalibr 2023](#). The Kalibr calibration framework was used in the original publications cited above, hence the name of the dataset. As a calibration pattern, AprilGrid with 6x6 tags of 88 mm size was used. In total, the datasets contain approximately 800 images.

**OCamCalib** (D. Scaramuzza, A. Martinelli, and R. Siegwart, 2006) is a dataset of approximately 300 images. As a calibration pattern, the checkerboard pattern of different sizes was used.

**UZH** (*Are We Ready for Autonomous Drone Racing? The UZH-FPV Drone Racing Dataset | Request PDF 2023*) is a dataset of approximately 800 images collected using the following cameras: As a calibration pattern, AprilGrid with 4x5 tags of 75 mm size was used. The dataset contains approximately 800 images.

OV (Lochman et al., 2021) is a dataset of approximately 1400 images. It was collected using eight stereo cameras. As a calibration pattern, the checkerboard pattern with 9x6 tags of 22 mm size was used.

Duisterhof et al. (2022) also provide their dataset from the TartanCalib project. Currently, the dataset contains only AprilGrid patterns, as the toolchain doesn't support chessboard.

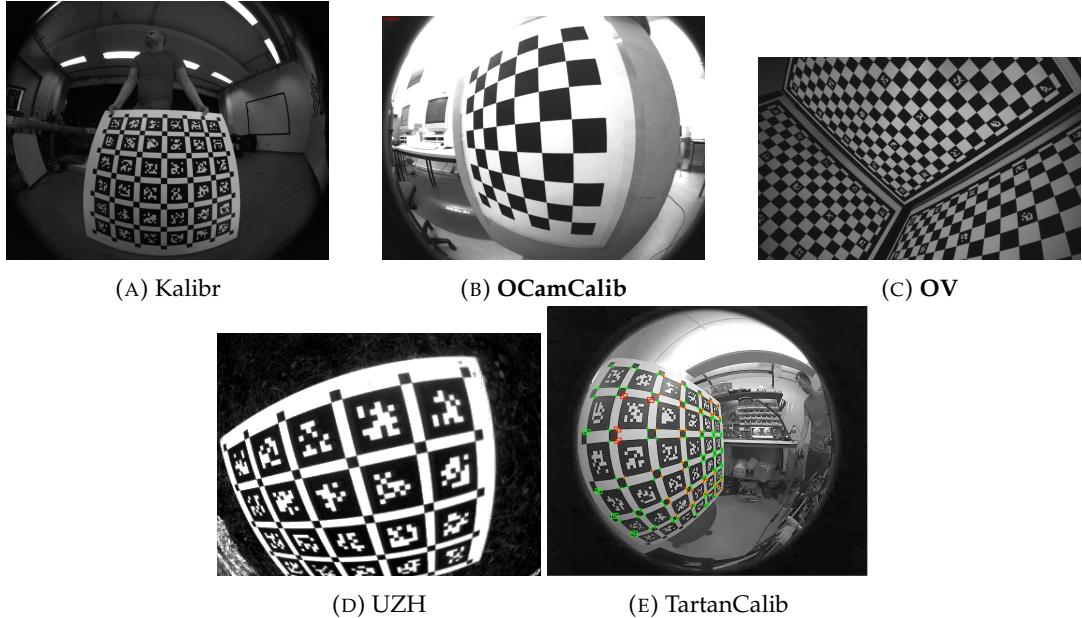


FIGURE 5.1: Images from the datasets

## 5.4 Camera calibration

In order to get accurate predictions for the possibly missing points, we need very accurate camera calibration. We used the reprojection error as a metric for the calibration quality. In order to calculate the reprojection error, we have to perform the rootfinding section 3.2.6. We assume that all of the points lay within the image, or they get out of the borders a little bit (we use the constant 1.1 of the maximum radius). If the camera calibration is poor, the rootfinding might fail. On the initial calibration the number of the failed rootfindings was much higher.

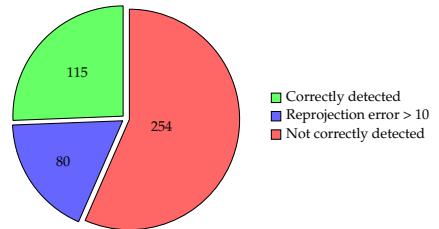


FIGURE 5.2: Initial calibration

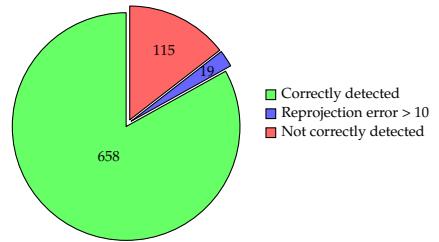


FIGURE 5.3: Final calibration

## 5.5 Additional features detection

As discussed in section 5.5, we filled the gaps in the originally detected board points, and then padded it for 1 additional element.

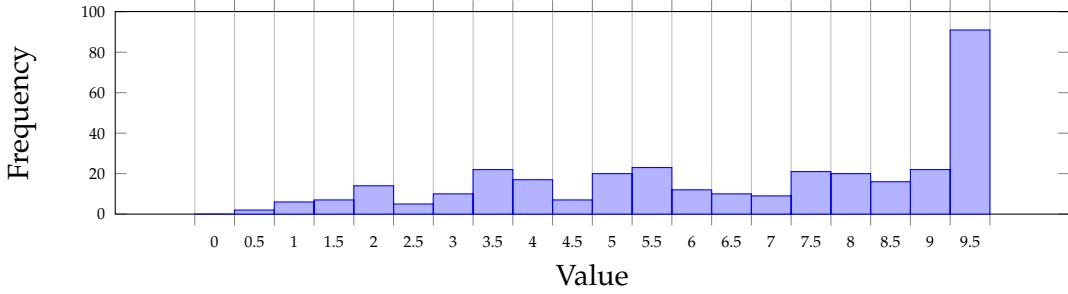


FIGURE 5.4: Initial calibration's reprojection error histogram

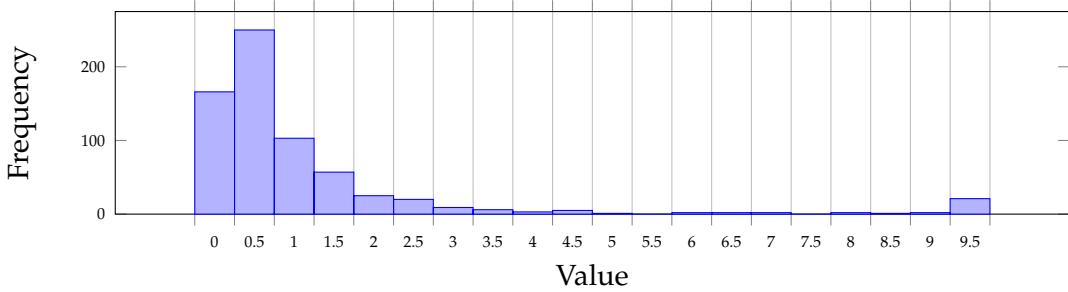
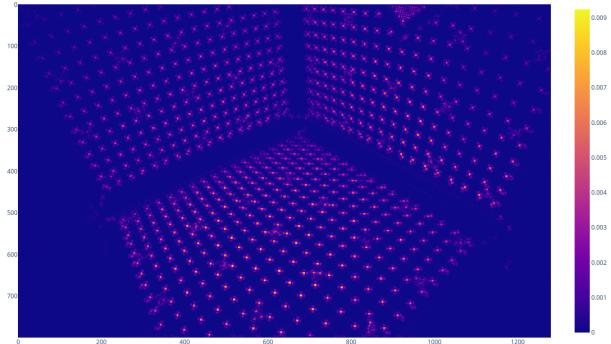


FIGURE 5.5: Final calibration's reprojection error histogram

## 5.6 Classification



We compared both responses, as discussed in section 4.4. The Hessian proved to be more robust. The approach, proposed by Geiger et al., 2012 gave too many false positives, especially for the edges.

FIGURE 5.7: Hessian response

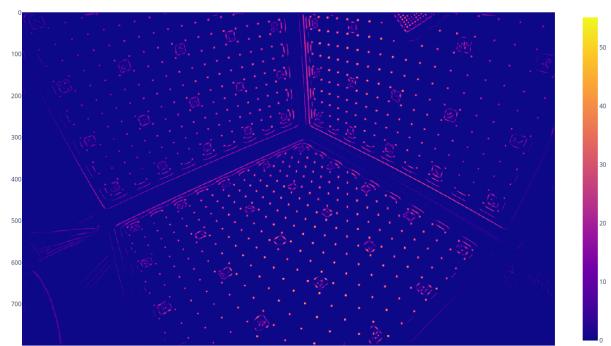


FIGURE 5.8: Response used by Geiger et al., 2012

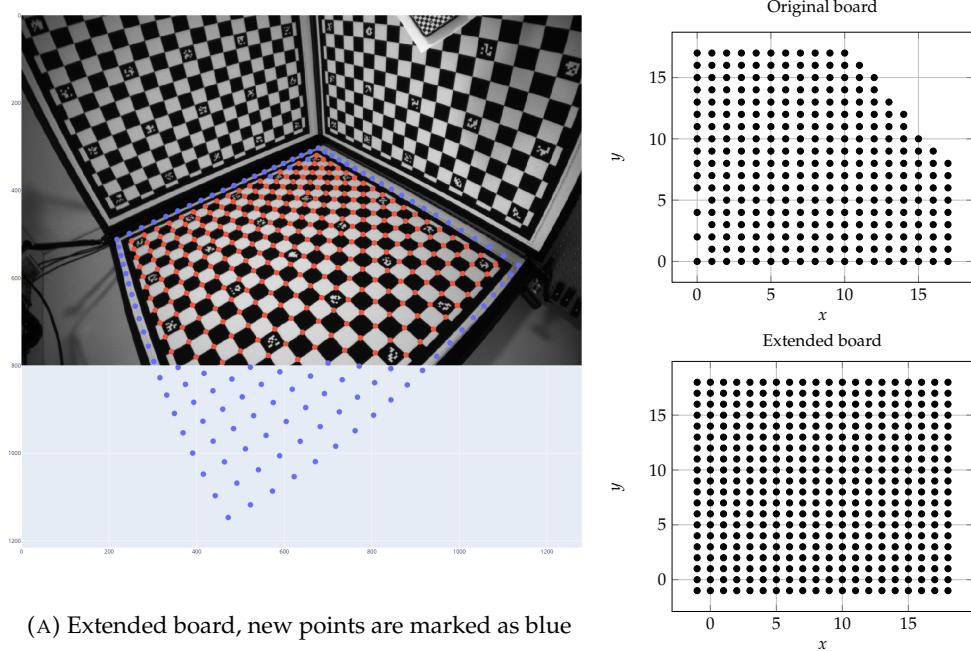
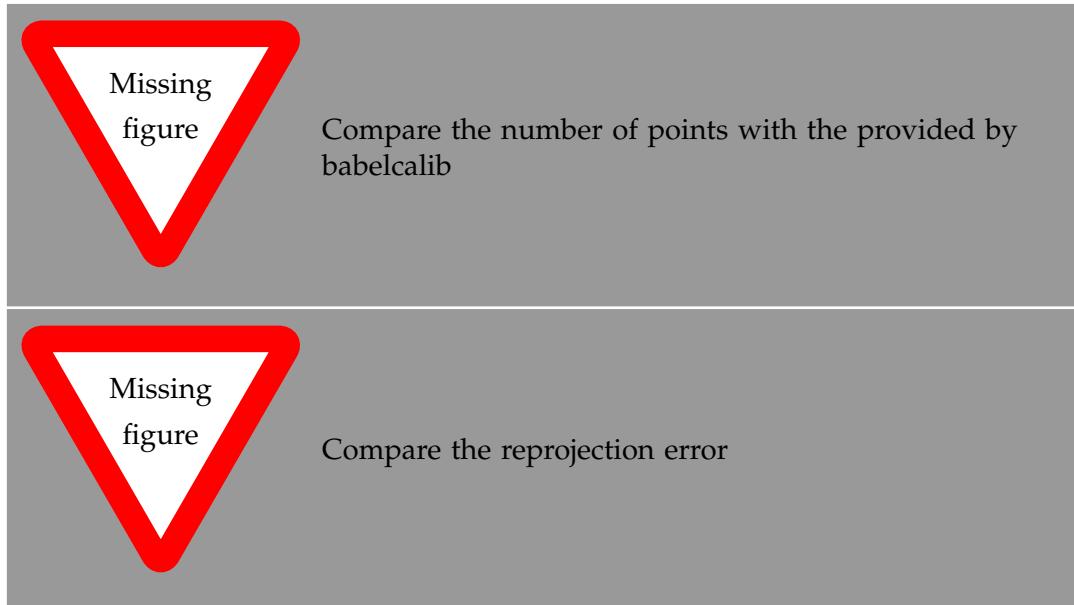
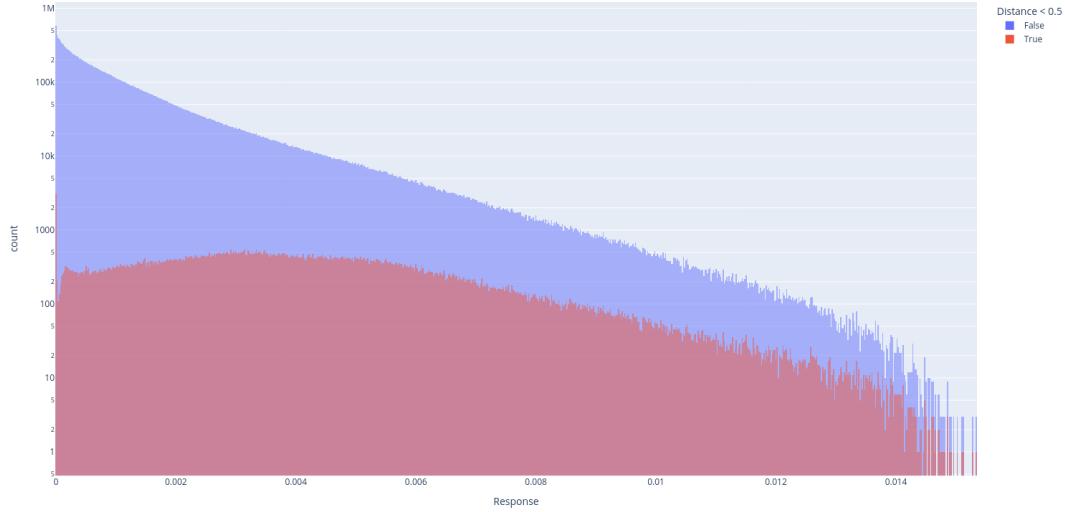


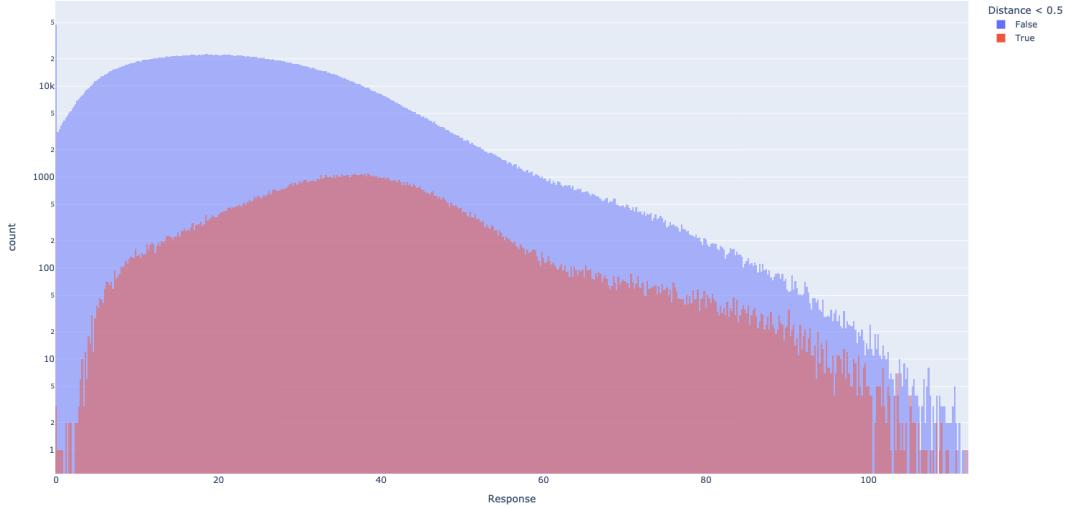
FIGURE 5.6: Board extension

## 5.7 Evaluation





(A) Histogram of Hessian response



(B) Histogram of response used by Geiger et al., 2012

FIGURE 5.9: Distributions of the responses for the image on the section 5.5. Note the log y-scale! Most of the points have the response 0.

## Chapter 6

# Conclusions

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