Федеральное государственное автономное образовательное учреждение
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«Национальный исследовательский университет
«Высшая школа экономики»

Факультет экономических наук
Образовательная программа «Экономика»

БАКАЛАВРСКАЯ ВЫПУСКНАЯ РАБОТА

Исследование связей между количеством альтернатив и поведением потребителя

Выполнил:

Купцова Анастасия Дмитриевна,
студент группы БЭК147

Научный руководитель:

Белянин Алексей Владимирович,
доцент

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Contents

The Size of the Choice Set and Consumer Behaviour

Abstract

Although we would like to have opportunities for choice, too extended choice set could lead to negative consequences. A lot of empirical studies reveal that some people refuse to choose, if the choice set is very big — so-called choice overload effect appears. In this paper we show that the typical assumptions about the rational agents could not explain the choice overload effect. We present the behavioural model of the boundedly rational agent. Her utility depends not only on the utility from the chosen item and on the costs of search, but also on the regret component. The feelings of regret appear if the expectations about the choice differ from the actually chosen option. We show that this model could explain the choice overload effect. Furthermore, our findings are consistent with the empirical evidence: both the utility of the choice process, as well as the share of agents who have bought an option from the choice set, follow inverted U-shaped functions of the choice set size.

1 Introduction

We have to make choices every day: what kind of jam to buy in a grocery store, where to spend our vacation, what topic of the bachelor's thesis to choose. At first sight, it seems that the larger the choice set we have the better our preferences can be matched with this set and the greater satisfaction we can get from the chosen item. Moreover, the modern environment offers a lot of opportunities. Still, the question arises how do we feel facing such an onslaught of choices? Empirical evidence shows that the choice set scarcity negatively affects the willingness to choose from this set (Iyengar&Lepper, 2000; Chernev et al., 2015) – this result seems to be quite intuitive. However, an enormously big choice set sometimes leads to the situation when agents have very low satisfaction from the choice they make or refuse to even make a choice (so-called the choice overload problem). How could we explain such an intuitively controversial result?

In this paper we argue that the standard microeconomics assumptions about rational agents could not explain the choice overload problem. Consider an agent who has complete information about values of the options, presented in the choice set, and maximizes her utility function on this set. Maximization on the big set is at least as good as on the small set that is the subset of the big one. Hence, the standard assumptions — completeness of information and utility function maximization — are not enough to explain the observed effects. Further, we weaken the assumption about the completeness of information. So, we obtain an agent who maximizes her utility function on the choice set and takes into account the costs of searching for the information about the options, presented in this set. It can be assumed that the choice overload problem appears due to the high costs of search, when choice set is too large. In the current paper we show that even the assumption of the search costs is not sufficient for the choice overload problem to arise.

To the best of our knowledge, there is no theoretical model that tries to explain and predict the appearance of the choice overload effect. In this paper we make an attempt to fill this theoretical gap. We build a model of agent behaviour: an agent maximizes her utility function on the choice set, she also takes into account the costs of searching for the information about the options. However, the most crucial assumption is that an agent forms her expectations about the best option that she could hypothetically find in this set. We argue that extended choice set could inflate the expectations; if the expectations are high, it is not easy to satisfy them; so that if actually chosen option is worse than expectations, an agent could feel regret and dissatisfaction. Therefore, an agent could refuse to choose anything due to aversion to feeling regret.

This paper is organized as follows. In the second section we present the literature review that consists of two parts. Part 1 contains the review of the empirical studies of the choice overload problem. These studies are mostly based on experiments, which allows us to suggest the mechanism that could underlie the choice overload. Moreover, using the empirical literature, we clarify the constraints on the robustness of the choice overload problem. Part 2 contains the review of the papers that present some concepts of the consumer decision process modeling: search theory and the regret theory. In the third section we present two frameworks: Rational Agent Model, in which an agent maximizes her utility function on the choice set and takes into account the costs of searching for the information about the options, presented in this set; and Boundedly Rational Agent Model, where, in addition to all assumptions of the Rational Agent Model, a boundedly rational agent forms her expectations about the best option and could feel regret. In the fourth section we discuss and compare the results of two models.

2 Literature review

2.1 Empirical Studies of the Choice Overload

Choice Overload Mechanism

Authors of one of the first papers about choice overload problem (Iyengar&Lepper, 2000) conducted a series of experiments attempting to find out how the size of the choice set affects the behavior of subjects. Their best known experiment took place in the American supermarket. Customers were offered to try several types of jam, given a discount coupon to buy the jam they liked the most, and then were tracked if they actually bought the jam. One group of customers was offered a choice of 6 types of jam (limited-selection), the other group was offered 24 types (extensive-selection). Authors have found that the extensive choice set is more attractive for buyers (60% of customers stopped next to the extensive-selection display, while 40% of customers stopped next to the limited-selection display), however, customers purchased more often when the choice set was limited (3% of customers who stopped next to the extensive-selection display bought the jam; while 30% of customers who stopped next to the limited-selection display bought the jam). Authors have replicated the results described above with two other experiments. Moreover, they have extent the previous results: an increase in size of the choice set decreases the motivation to choose and also decreases the satisfaction from the chosen option. Possible mechanisms that underlie the choice overload are discussed in this paper. Firstly, when choice set is too big, choosers find it very costly to search properly for the best option and use heuristics to simplify the search process. These heuristics could lead to suboptimal results. Secondly, extended choice set triggers expectations about the best option, which a chooser could hypothetically find. These expectations increase an anxiety to choose the wrong option.

Diehl and Poynor (2010) use experiments to prove the following hypotheses: (1) an increase in the size of the choice set enhances expectations about the best option in the set; (2) extended choice set leads to more complex choice process and thus increases the consumer's overload; (3) large choice set reduces satisfaction from the chosen option. In an another paper "The Effect of Stating Expectations on Customer Satisfaction and Shopping Experience" (Ofir&Simonson, 2007) subjects, who were asked to articulate their expectations before the choice process, evaluated the experience of the choice situation lower than those who were not asked about expectations. Authors of this study argue that if the consumer articulates her expectations before the choice process she could compare the actual experience with the expected one. So that consumer could feel dissatisfaction when the actual experience is worse than the expected one.

In the following experimental studies some measures of the choice overload problem are presented as a function of the choice set sizes. Reutskaja and Hogarth (2009) have found that the feeling of satisfaction from the chosen option, as well as the feeling of satisfaction from the sampling process, have inverted U-shaped form: these feelings increase with limited set sizes; reach the peak when set sizes are moderate and decline with extensive set sizes. Griffin and Broniarczyk (2010) have found the similar effect, however, only for the choice set that consists of hardly comparable options. Shah and Wolford (2007) have presented the same inverted U-shaped relationship between the proportion of subjects who have eventually made the choice and choice set sizes.

Mechanism of the Choice Overload Problem: From the studies discussed above we suggest the mechanism that could underlie the choice overload problem. On the one hand, an increase in the size of the choice set inflates consumer's expectations about the best option in the set. On the other hand, an increase in the size of the choice set also multiplies the difficulty of the choice task, for instance, due to additional costs of information and higher costs of cognitive processing. Therefore, if the choice task is very difficult, a chooser wants to simplify the decision process. This simplification could lead to suboptimal results: a chooser does not find the best option in the set, but the satisfactory one. Inflated expectations about the best option trigger the disappointment if the actually chosen option is worse than expected the best one and the chooser feels regret. Regret aversion — while expectations about the best option are very high; but it is hard to find the best option due to high search costs — could be one of the reasons why choosers refuse to choose at all, so that the choice overload problem arises.

Review of the meta-analyses

Despite the fact that a plethora of empirical studies reveals the choice overload effect, there are others that do not or even reveal the opposite effect. Considering the meta-analytic researches about the choice overload effect, we explore triggers of the choice overload problem, its outcomes and robustness. We examine two papers: Scheibehenne et al. (2010) has analyzed results of 50 experiments across 5036 subjects; Chernev et al (2014) studied 99 experiments with 7202 participants.

Authors of both meta-analyses claimed that there exist advantages and disadvantages of the extended choice set. They discuss the following advantages: (1) likelihood that unique preferences will be matched increases with choice set size; (2) when a lot of options are brought together in one place, it gives to consumer more accurate perception of the overall options' distribution; (3) extended choice set may serve as an insurance from future consumer's preference uncertainty; (4) consumers may experience additional positive feelings of freedom while choosing from the large choice set. However, the extended choice set also has disadvantages: (1) cognitive costs of search increase with the number of options, because consumer has to spend more time and has to make more effort to look through options in the choice set and to find the most appropriate one; (2) the greater the choice set size is, the higher expectations about the ideal option in this set could be; (3) the differences between some options could hardly be distinguished if a lot of options are presented, which complicates the justification of the choice.

In both papers it has been shown that if authors use all existing studies in their analysis, the mean effect of the choice overload is equal to zero. However, the variance between different experiments is extraordinary high: there exist experiments with significant choice overload effect and at the same time authors of another experiments have reported the more-is-better effect. Such results appear due to very different choice situations in these experiments.

Chernev et al. (2014) have shown that the special characteristics of the choice situation influence the choice overload appearance. First of all, if the choice task is complex, it could overload an agent. When an agent has limited time to make a decision; or number of attributes that describe an option is too large; or presentation format is not convenient, for instance, due to lack of ordering; all these task difficulties trigger choice overload effect. Secondly, special properties of the choice set influence the choice overload effect as well. If the dominant option is presented in the choice set, or if comparison of options is an easy process due to the absence of hard trade-offs; thus such situations will unlikely trigger the choice overload problem. Moreover, it is crucial if a chooser is familiar with the choice set or, even more, she has well-established preferences. When particular choice set is unfamiliar for a chooser, she has to make a lot of effort to match her preferences to the options in this set.

In this paper we assume only such choice situations that satisfy the limitations, described above.

Why is this topic important?

We have to make choices every day. We have to make simple choices, such as what kind of coffee to drink, or the most crucial ones, for instance, where to live or what diploma thesis to choose. A lot of studies show that choice overload problem appears not only in cases of choosing consumer goods in supermarkets, but also in more important situations.

In the paper with jam experiment Iyengar and Lepper (2000) have also presented the results of an another experiment. Students complete an additional home assignment (write an essay) more often and get higher marks, when they choose the topic of the assignment from a limited choice set (6 topics) than when they choose the topic from an extended one (30 topics).

Scheibehenne et al. (2009) have shown that subjects are less likely to donate to charity organizations if they choose one organization from the extended choice set (40 or 80 organizations) than if they choose from the limited one (5 organizations). It should be mentioned that authors get these results in the particular choice situation: (1) charity organizations were small-sized and least-known; (2) subjects had to explain the choice they make.

Moreover, some empirical studies show that choice overload problem could appear when employees choose whether to participate in the retirement funds program or not. Iyengar et al. (2003) have made the field study about participation in the American retirement funds program — 401(k) Retirement Plans. They have found that participation rates differ significantly when retirement plan offers a few options to invest compared to when it offers a lot of options. So that plans with only two options have 75% participation rate, when plans with 59 options have 60% participation rate. Authors of another study (Morrin et al., 2012) have shown that extended choice set of funds (plan with 27 options to invest) leads to less participation than limited set (9 options). This effect is observed only in the case of subjects who have low knowledge about investment. However, the opposite effect takes place with more sophisticated subjects.

2.2 Studies for Choice Overload Mechanism Modeling

Before modeling the choice overload mechanism we have to understand how people organize the search process and how to model this process. Moreover, we have to explore studies about regret mechanism modeling.

Search process

Many studies model strategies that can underlie decision process (Payne, 1976; Ford et al., 1989; Riedl et al., 2008). The most common are following ones:

Linear strategy (with or without weighting): Agent estimates the value of the option from the choice set by summarizing its attributes values. If all attributes are equally important, then an agent sums up their values with equal weights (linear rule without weighting). Or, if some attributes are more important than others, then an agent sums up its values with higher weights, and vise versa about less important attributes (linear rule with weighting). Agent chooses the option with the highest total value.

Additive difference strategy: Agent sums up differences in attributes' values between two options, attribute by attribute. The option that has lower score is eliminated, the option that has higher score is compared with next option. This algorithm is repeated several times until only one option remains.

Conjunctive strategy: An agent sets cutoff levels for attributes. She considers options one by one. The first option, which satisfies all these cutoffs, is selected.

Elimination-by-aspect strategy: An agent sets cutoff levels for attributes. An agent selects an attribute and eliminates all options that don't satisfy the particular cutoff for this attribute, then an agent repeats the same algorithm with another attributes until only one option remains. An agent could choose which attribute to consider at the particular step either in random order or according to attributes' weights.

Lexicographic strategy: An agent defines the most important attribute and selects the option with the highest value of this attribute. If there are some options with the highest values, an agent defines the second important attribute, and so on.

One of the most important differences between these rules is the compensatory property. Compensatory strategies (e.g., linear and additive difference strategies) imply that low values of some attributes can be compensated by high values of other attributes, so that trade-off between attributes take place. To make a final decision an agent compares total values of all options and chooses an option with the highest total value. Non-compensatory strategies (conjunctive, elimination-by-aspect, lexicographic strategies) don't allow such trade-offs. An agent excludes options from consideration if attributes' values are lower than cutoff levels.

Authors of the meta-analytic study (Ford et al., 1989) have reviewed forty-five empirical studies of the decision making processes and arrived to the following results. Subjects use different decision strategies, what particular strategy will be used may depend on choice complexity. When the choice complexity is high, for instance, due to a lot of options to choose from, subjects firstly excludes inappropriate options from consideration, then subjects compare values of remained options. So that during the first step of decision process agents use non-compensatory strategies in order to cut down the choice set. Then they use compensatory strategies to choose more carefully between suitable options during the second step.

In this paper we are going to model only the second step of this search algorithm, assuming that people go through the first step quite automatically and costs of first-step reduction are negligently small with respect to the costs of more accurate search process during the second step. We assume that during the second step people search sequentially for the best option: one by one sampling all options, remained after the first stage.

Wolinsky (1986) has offered the optimal strategy for the problem of sequential search process. The consumer model in this paper meets the following assumptions: (1) consumer is going to choose one option from the choice set; (2) consumer does not know the exact value of each option, but knows that options are independent and identically distributed; (3) consumer can identify the exact value of the option by some costs (costs are equal for each option); (4) consumer searches for the best option, one by one exploring choice set: at every moment she could stop the search and choose the best option from the already explored ones or continue the search. The optimal algorithm of this problem is the following: the chooser should sample options (the order of sampling doesn't matter as options are independent and identically distributed), till the expected marginal profit from the additional sampling is greater than the marginal costs from the additional sampling, that is:

if $\int_{x^*}^{\bar{x}}(x - x^*)dF(x) > c$ – consumer should search further,

else – consumer should stop her search and choose the best seen option,

where x^* is the highest value of the already sampled options, $\bar{x}$ is the highest value in the distribution, $F(x)$ is the cumulative distribution function, c is the search costs of one sampling.

Moreover, consumer should not start the search, if the expected value of this search is less than price of options; furthermore, consumer should not choose any option, if value of the best option from the whole set is lower than price.

In this paper we are going to use the framework of sequential search process described above, however, we will modify this process: we assume that at every moment a chooser could (1) either stop the search and choose the best option from the already explored ones; (2) continue the search; (3) choose nothing and leave the search situation.

Regret Mechanism

We consider two classical papers about the regret modeling (Loomes & Sugden, 1982; Bell, 1982). Authors of these studies have built the models of the decision making under risky choice, where they add regret component to the classical Von Neumann–Morgenstern utility function. They argue that the feeling of regret or rejoice may appear, if the utility of the actually chosen option differ from the utility of the foregone option. They have assumed that, if the chooser gets the value x_i and foregoes the value x_j, thus, her total utility function looks like this: $U = x_i + R(x_i - x_j)$, where $R()$ is the regret-rejoice function. The special properties of this regret-rejoice function should be met: $R(0) = 0$, $R'_x(x) > 0$ (Loomes&Sugden, 1982). In addition to previous properties Bell (1982) has also assumed that $R''_x(x) < 0$, which means that people suffer stronger from regret than joy from the same amount of rejoice (so-called, they are regret-averse); moreover, feelings of regret are marginally increasing, while feelings of rejoice are marginally decreasing.

In papers discussed above the classical concept of reference-dependent utility (Kahneman&Tversky,1976) was replicated with an unusual reference point – the value of foregone option. Furthermore, Köszegi and Rabin (2006) have offered the reference-dependent utility with consumer expectations about the choice situation as reference point. They have assumed that total utility depends on the value of chosen option x_i and the difference between expectations E_i and chosen option x_i: $U_i = x_i + \mu(x_i - E_i)$. Where $\mu(.)$ is "gain-loss function" with the following properties: (1) $\mu(0) = 0$; (2) $\mu'_x(x) > 0$; (3) $\mu''_x(x) \leq 0$ for $x > 0$ and $\mu''_x(x) \geq 0$ for $x < 0$; (4) $\forall x > 0$ $\mu(x) < -\mu(-x)$. Properties (1) - (3) imply that $\mu(.)$ is standard S-shaped function, while (4) leads to loss-aversion.

Considering all the papers described above, in our study we assume that the total utility also depends on the value of chosen option and differences between expectations and chosen option. However, for simplicity we use the linear regret function $R(.) = \beta(x_i - E_i)$, where $\beta > 0$ – regret sensitivity coefficient.

3 Modeling

We assume that the choice process consists of two stages, which is in line with the studies we have discussed above. At the first stage an agent eliminates some options, for instance, due to unsatisfactory levels of important attributes. Here, an agent uses so-called non-compensatory rule. At the second stage an agent evaluates remained options one by one, so that some high level of some option's attributes can compensate others with low level. An agent is looking for the option with the highest total value, however, she should consider her cognitive constraints, such as costs of search and processing the information.

If initial choice set consists of $\tilde{n}$ options, an agent reduces it to n options during the first stage. We assume that costs of reduction in the first stage are negligently small with respect to the costs of more accurate search process during the second stage, therefore we don't take into account the costs of reduction. We also assume that an agent reduces the initial choice set in such a way the following properties are met. Firstly, an agent doesn't know the exact value of the remained options, she only knows that options are independent and identically distributed; during the first-stage reduction an agent forms her beliefs about distribution. Moreover, she reduces the choice set so that values of the remained options are no less than zero. In this paper we assume that remained options have uniform distribution from 0 to a. Secondly, during the first-stage reduction an agent understands that she can identify the exact value of the particular option by some costs c (costs are equal for each option). These costs should be not very high — no more than the value of the average option from the reduced choice set ($a/2$). Thirdly, during the first-stage reduction an agent remains only such options prices of which are not too high: value of the average option from the reduced choice set ($a/2$) should be not less than the costs of searching this option (c) plus the price of this option. Furthermore, we assume that an agent perceives prices in this range as equal so that later in these study we use price p for all second-stage options.

To sum up, we have the following initial assumptions: (1) reduced choice set consists of n options; (2) an agent doesn't know the exact value of the remained options, she only knows that options are independent and identically distributed; (3) however, an agent knows that options have uniform distribution from 0 to a; (4) an agent can identify the exact value of the particular option incurring the cost c; (5) value of the average option from the reduced choice set ($a/2$) should be no less than the costs of searching this option (c) plus the price of this option (p).

An agent sequentially searches for the optimal option, identifying the value of the options one by one. It should be mentioned that the search order does not matter, as all options are independent and identically distributed and before the search process an agent doesn't know the exact values. At step $t = 0$ an agent can take one of two actions: (*action* S) to begin the search and get the information about the first randomly chosen option; or (*action* L) not to begin the search and leave the choice situation with nothing. If the action at the previous step was to begin the search, an agent goes to $t = 1$, where she can take one of three actions: (*action* B) to buy the best option from options that she has already seen; or (*action* S) to continue the search at least one option further; or (*action* L) choose nothing and leave the search situation. If the previous step action was to continue the search, an agent goes to $t = 2$, where she can take one of exactly the same three actions as at $t = 1$, and so on till $t = n$. At the step $t = n$ an agents can take one of two actions: (*action* B) to buy the best option that she has already seen; or (*action* L) to choose nothing and leave the search situation. The important assumption of our model is that an agent could buy an option only from the set which she has already explore.

3.1 Rational Agent Model (RAM)

In this section we solve the game, described above, for the rational agent. She maximizes her utility function on the choice set and takes into account the costs of searching for information about the options.

We denote utility of stopping the search process at step $t = 1, ..., n$ and buy the best option, an agent has already seen, as U_t^B; utility of stop the search process at step $t = 1, ..., n$ and leave the choice situation, as U_t^L.

If an agent does not begin the search and leave the choice situation with nothing at step $t = 0$, she gains utility:

$$U_0^L = 0$$

If an agent stops the search process at step $t = 1, ..., n$ and buys the best option, she has already seen, she gains the following utility:

$$U_t^B = x_{(t)} - p - tc,$$

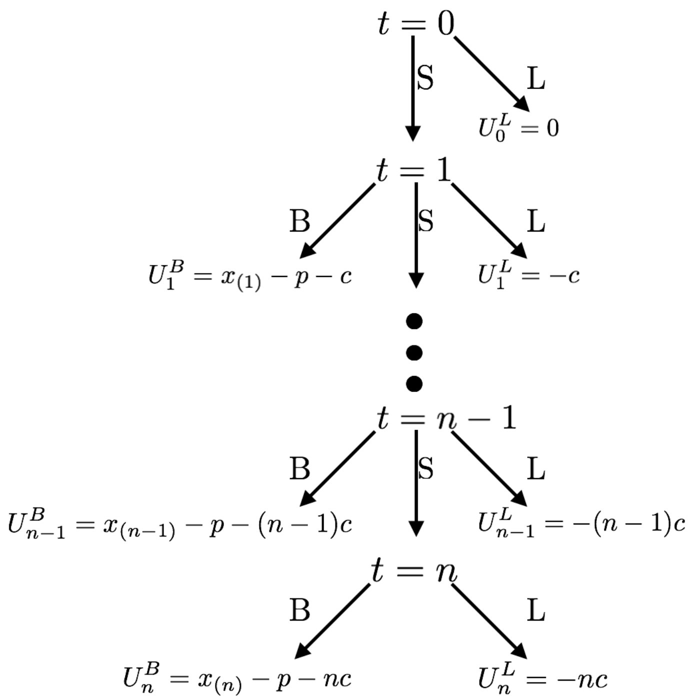
where $x_{(t)}$ is the value of the best option from t options, an agent has already seen; p is the price of this option; tc are the costs of t-steps search.

If an agent stops the search process at step $t = 1, ..., n$ and leaves the choice situation with nothing, she gains the utility:

$$U_t^L = -tc,$$

because she spends costs tc for t-step search.

We present the tree of the search process that we have described above:



<u>Lemma R1</u>

Assume that at step $t = 0$ an agent decided to start the search:

- $\forall\ t \in [1; n-1]$: if $x_{(t)} \geq a - \sqrt{2ac}$, the optimal action at step t is to stop the search process at the current step and buy the best seen option $x_{(t)}$; if $x_{(t)} < a - \sqrt{2ac}$, the optimal action at step t is to continue the search at least one option further;
- for $t = n$: if $x_{(n)} \geq p$, the optimal action at step n is to buy the best option $x_{(n)}$; if $x_{(n)} < p$, the optimal action at step n is to choose nothing and leave the search situation.

This lemma is proved in the Appendix B.

<u>Lemma R2</u>

If expected value of the search process, we have described in Lemma R1, is greater than zero, then at step $t = 0$ an agent starts the search; otherwise an agent chooses nothing and leaves the choice situation.

The proof of this lemma is obvious: utility of the leaving the choice situation with nothing at step $t = 0$ is equal to zero.

From these lemmas we derive the optimal strategy for the search process:

- start the search process at step $t = 0$ if expected value of the search is greater than zero
- at step $t = 1, 2, ..., (n-1)$ stop the search process at step t and buy option x_t (the option, which value we define at step t), as soon as $x_t \geq a - \sqrt{2ac}$; continue the search if $x_{(t)} < a - \sqrt{2ac}$ (where $x_{(t)}$ is the value of the best option from the first t-step search)
- at steps $t = n$ buy the $x_{(n)}$ option if $x_{(n)} \geq p$; choose nothing and leave choice situation otherwise.

Considering this optimal strategy we can calculate the expected number of options (Et^*) that an agent explores, if she starts the search at step t = 0:

$$Et^* = Pr(x_1 \geq a-\sqrt{2ac}) \times 1 + Pr(x_1 < a-\sqrt{2ac}; x_2 \geq a-\sqrt{2ac}) \times 2 + ... + Pr(x_1 < a-\sqrt{2ac}; ...; x_{n-1} < a-\sqrt{2ac}) \times n = \frac{1-(\frac{a-\sqrt{2ac}}{a})^n}{1-\frac{a-\sqrt{2ac}}{a}},$$

to find calculations see to the Appendix B.

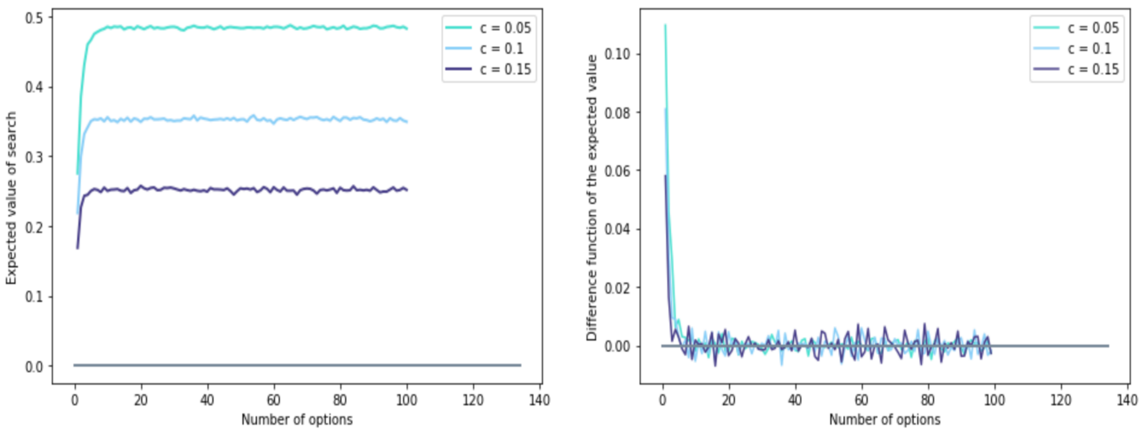
Note that expected number of explored options increases with total number of options n, but it tends to be constant while $n \to +\infty$, as $\frac{a-\sqrt{2ac}}{a} \leq 1$, so that $\lim_{n\to\infty} Et^* = \frac{a}{\sqrt{2ac}}$.

It is prohibitively difficult to derive function the expected value of search from the Lemma R1 in general, moreover, it depends on a lot of parameters. So we derive the values, when $n \to \infty$, and simulate others for a grid of parameters.[1]

$\lim_{n\to\infty} EV = a - p - \sqrt{2ac}$,

to find calculations see to the Appendix B.

Further we present the simulation with $p = 0.2$ (to find simulations with other parameters, see Appendix A):[2]



Pic. 1: RAM. Expected value of the search and difference function.

price $p = 0.2$; *search costs* $c = 0.05, 0.1, 0.15$

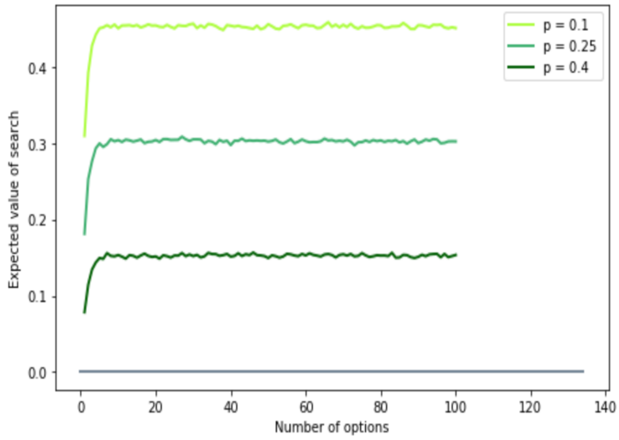
We get the following results for the Rational Agent Model. First of all, the expected value function looks very similar for all the parameters: expected value increases while choice set sizes are small[3]; then it tends to be constant (as we derive that $\lim_{n\to\infty} EV = a - p - \sqrt{2ac}$).[4] The only distinction between the different parameters are values of constant level, when $n \to +\infty$: ceteris paribus, the constant level increases while costs of search c go down (*Pic. 1*) or while price p goes down (*Pic. 2*).

[1]To see code of simulations go `https://github.com/ansty57/Working_papers/blob/master/Simulations_BT.ipynb`

[2]Without loss of generality we assume $a = 1$ in simulations.

[3]We derive this from simulations, so we know this only for the extended range of parameters that we have checked.

[4]This is consistent with the derivative function properties, derivative functions are also presented in Appendix A.



Pic. 2: RAM. Expected value of the search.

search costs $c = 0.1$; *price* $p = 0.1, 0.25, 0.4$

Secondly, for all the parameters and for all choice set sizes expected value is greater than zero, as $a - \sqrt{2ac} \geq p$.[5]. This means that at $t = 0$ an agent always starts the search.

The intuition behind these results is the following. When the size of the choice set is too small, there is quite substantial likelihood that an agent looks for the whole choice set and chooses nothing, as no option will be good enough. An increase of the set size triggers two processes: (1) an agent is more likely to find option that is good enough; (2) the costs of search for the best option in the whole set increase. These processes counterbalance each other, so that an agent looks for particular number of options that tends to be constant[6], when choice set is big enough.

3.2 Boundedly Rational Agent Model (BRAM)

In this section we solve the search process game for the boundedly rational agent. This agent differs from the rational one due to the regret she could feel. Before the search – at step $t = 0$ – an agent looks through the choice set and forms her expectations about the best option she hypothetically could find in this set. The bigger the choice set is, the higher are her expectations. During the search process an agent compares value of the best seen option with her expectations about the best option. She feels regret, if the value of the chosen option is less than these expectations.

Further we derive the expression for expectations about the best option from the choice set with n options:

$$E(x_{(n)}) = a\frac{n}{n+1}$$

Proof: we derive the cumulative distribution function of $x_{(n)}$ – the best option in the set: $F_{x_{(n)}}(x) = Pr(x_{(n)} \leq x) = Pr(x_1 \leq x, ..., x_n \leq x) = Pr(x_1 \leq x) * ... * Pr(x_n \leq x) = F(x)^n$, where x_1,..., x_n are random variables from uniform distribution with cdf $F(x) = \frac{x}{a}$. Therefore $F_{x_{(n)}}(x) = (\frac{x}{a})^n$ and $E(x_{(n)}) = \int_0^a x dF_{x_{(n)}}(x) = \int_0^a x d(\frac{x}{a})^n = \frac{1}{a^n}\int_0^a x dx^n = \frac{1}{a^n}\frac{n}{n+1}x^{n+1}|_0^a = \frac{an}{n+1}$

[5]The proof of this result is presented in Appendix B in the part Proof of Lemma R1(III)

[6]As $\lim_{n\to\infty} Et^* = \frac{a}{\sqrt{2ac}}$.

If an agent does not begin the search and leaves the choice situation with nothing at step $t = 0$, she gains utility:

$$U_0^L = 0$$

If an agent stops the search process at step $t = 1, ..., n$ and buys the best option she has already seen, she gains the following utility:

$$U_t^B = x_{(t)} + \beta(x_{(t)} - a\tfrac{n}{n+1}) - p - tc,$$

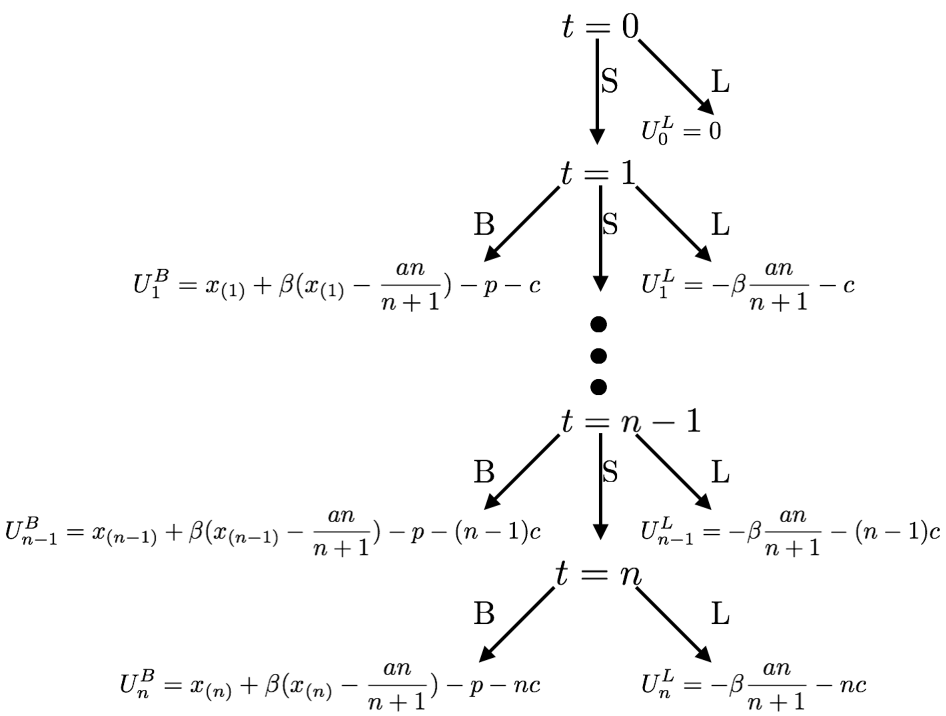
where $x_{(t)}$ is the value of the best option from t options an agent has already seen; p is the price of this option; tc are the costs of t-steps search, $\beta > 0$ is the regret coefficient, $a\frac{n}{n+1}$ is the expectations about the best option among n.

If an agent stops the search process at step $t = 1, 2, ..., n$ and leaves the choice situation with nothing, she gains the utility:

$$U_t^L = -\beta a\tfrac{n}{n+1} - tc,$$

because she bears costs tc for t-step search and also she has expectations that are not satisfied, so that she feels regret. It should be mentioned that if an agent does not begin the search at step $t = 0$ and leaves the choice situation, then she has no expectations and does not feel regret.

Here, we present the tree of the search process we described above:



<u>Lemma BR1</u>

Assume that at step $t = 0$ an agent decided to start the search:

- $\forall\, t \in [1; n-1]$: if $x_{(t)} \geq a - \sqrt{\frac{2ac}{1+\beta}}$, the optimal action at the step t is to stop the search process at the current step and buy the best seen option $x_{(t)}$; if $x_{(t)} < a - \sqrt{\frac{2ac}{1+\beta}}$,the optimal action at step t is to continue the search at least one option further;

- for $t = n$: if $x_{(n)} \geq \frac{p}{1+\beta}$, the optimal action at step n is to buy the best option $x_{(n)}$; if $x_{(n)} < \frac{p}{1+\beta}$, the optimal action at step n is to choose nothing and leave the search situation.

This lemma is also proved in the Appendix.

Lemma BR2

If expected utility of the search process that is described in Lemma IR1 is greater than zero, then at step $t = 0$ an agent chooses the action to start the search; otherwise an agent decides to leave the choice situation with nothing.

The proof is the same as for Lemma R2.

From these lemmas we derive the optimal strategy for the search process:

- start the search process at step $t = 0$ if expected value of the search is greater than zero
- at steps $t = 1, 2, ..., (n-1)$ stop the search process at step t and buy option x_t (the option, which value we define at step t), as soon as $x_t \geq a - \sqrt{\frac{2ac}{1+\beta}}$; continue the search if $x_{(t)} < a - \sqrt{\frac{2ac}{1+\beta}}$ (where $x_{(t)}$ is the value of the best option from the first t-step search)
- at step $t = n$ buy the $x_{(n)}$ option if $x_{(n)} \geq \frac{p}{1+\beta}$; choose nothing and leave the choice situation otherwise.

Considering this optimal strategy, we can calculate the expected number of options (Et^*) that an agent explores, if she starts the search at step t = 0:

$$Et^* = \frac{1 - \left[\frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a}\right]^n}{1 - \frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a}},$$

calculations are similar to Rational Agent Model.

Note that expected number of explored options increases with total number of options n what is consistent with empirical results (Reutskaja et al., 2011). However, it tends to be constant while $n \to +\infty$, as $\frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a} \leq 1$, so that $\lim_{n\to\infty} Et^* = \frac{a(1+\beta)}{\sqrt{2ac}}$.

As for the Rational Agent Model we are going to derive some values of the expected value function (when $n \to \infty$) and simulate others for a grid of parameters.[7]

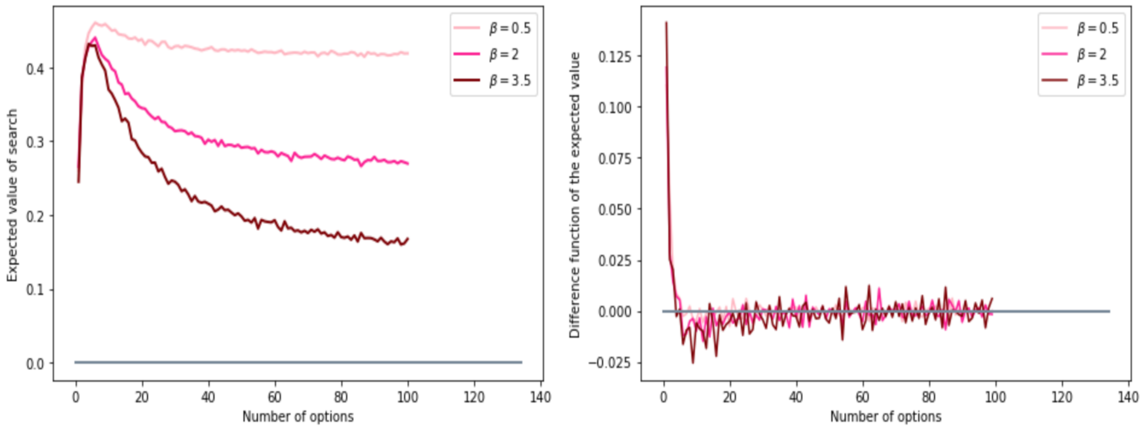
$\lim_{n\to\infty} EV = a - p - \sqrt{2ac(1+\beta)}$,

to find calculations see to the Appendix B.

We know that $a - p - \sqrt{2ac} \geq 0$, however if β is too big it could be that $a - p - \sqrt{2ac(1+\beta)} < 0$

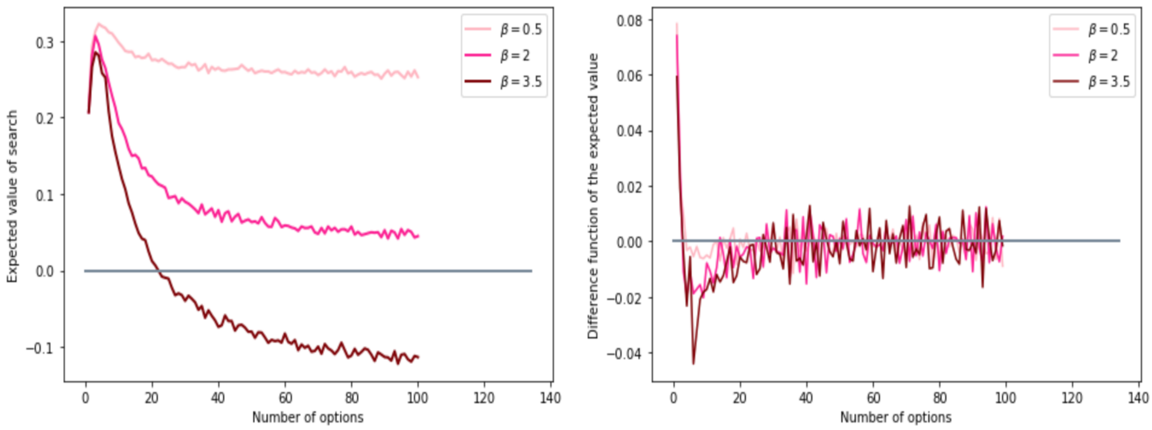
[7]To see code of simulations go `https://github.com/ansty57/Working_papers/blob/master/Simulations_BT.ipynb`

Further we present the simulation with $p = 0.2$ (to find simulations with other parameters see Appendix A):



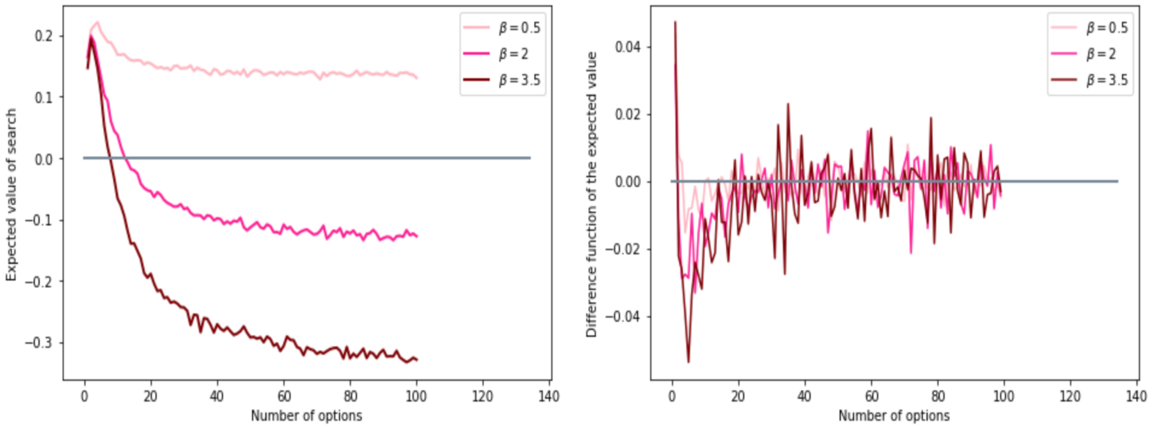
Pic. 3.1: BRAM. Expected value of the search and difference function.

price $p = 0.2$; *search costs* $c = 0.05$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



Pic. 3.2: BRAM. Expected value of the search and difference function.

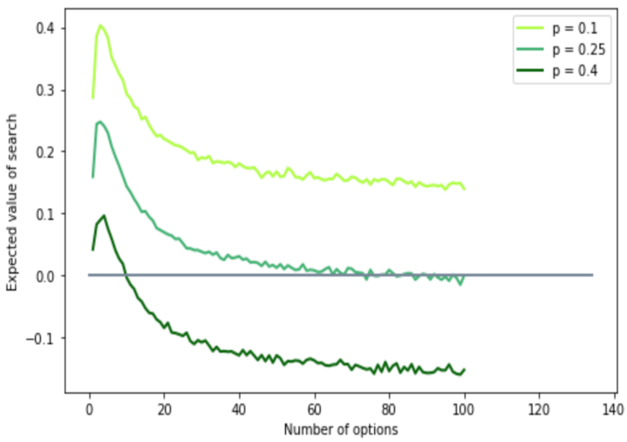
price $p = 0.2$; *search costs* $c = 0.1$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



Pic. 3.3: BRAM. Expected value of the search and difference function.

price $p = 0.2$; *search costs* $c = 0.15$; *regret sensitivity* $\beta = 0.5, 2, 3.5$

We get the results for the Boundedly Rational Agent Model that differ dramatically from the results for the Rational Agent Model. First of all, the expected value function takes the following form: it increases while choice set sizes are small, then goes down while set size increases [8]; and only then tends to be constant when n is big (as we derive that $\lim_{n\to\infty} EV = a - p - \sqrt{2ac(1+\beta)}$).[9] However, as for the Rational Agent Model, the only distinction between the expected value function with different parameters are values of constant level, when $n \to +\infty$: ceteris paribus the constant level increases while costs of search c go down (*Pic. 3.1 – 3.3*), or while price p goes down (*Pic. 4*), or while regret sensitivity β declines (*Pic. 3.1 – 3.3*).



Pic. 4: BRAM. Expected value of the search.

search costs $c = 0.1$; *regret sensitivity* $\beta = 2$; *price* $p = 0.1, 0.25, 0.4$

Secondly, for some parameters and big choice set sizes expected value could be less than zero. This means that in these situations an agent does not start the search at step $t = 0$.

The intuition behind these results is the following. When size of the choice set is too small, there is quite substantial likelihood that an agent looks for the whole choice set and chooses nothing, as

[8] We derive this from simulations, so we know this only for the extended range of parameters that we have checked.

[9] This is consistent with the derivative function properties, derivative functions are also presented in Appendix A.

no option will be good enough; moreover, the expectations about the best option are not too high. An increase of the set size triggers three processes: (1) an agent are more likely to find option that is good enough which inflates the expected value; (2) the costs of search for the best option in the whole set increase which decreases the expected value; and (3) expectations about the best option increases which increases regret (because it is too costly to find the best option in the extended choice set), so that the expected value decreases. When the choice set is small enough, the first process dominates all others, but when set size increases these processes counterbalance each other, so that expected value of the search tends to be constant, when choice set is big enough. However, this constant level of the expected value could be less than zero. So that if an agent knows that the expected utility is less than zero, she does not choose anything.

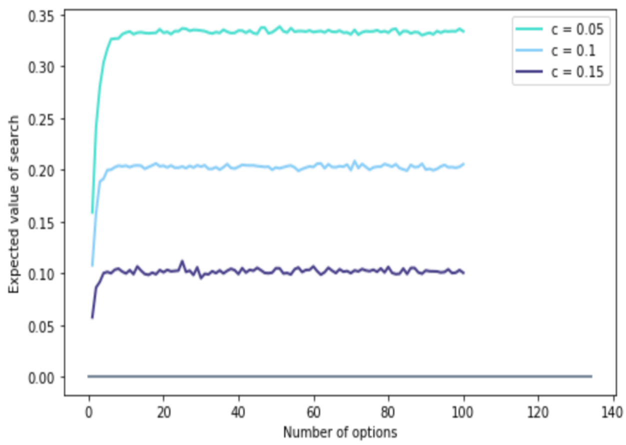
4 Comparison of Rational Agent Model and Boundedly Rational Agent Model

As we have discussed above, rational agent's expected value of the search is always greater than zero, so that she always start the search at step $t = 0$. We have proved that if she has started the search then the optimal strategy is to search till the good enough option (and buy this option) or till the end of the set (and buy the best option in the set if its value is not less than price). As probability that she will find the good enough option increases with the set size, the likelihood that she will buy the option also enhances. In contrast, boundedly rational agent's expected value of the search could be less than zero when the choice set is very big. So, an agent will refuse to make a choice, which coincides with the empirical results of the choice overload appearance.

Moreover, from the empirical evidences we know that there exists inverted U-shaped relationship between the share of subjects who have made the choice and choice set sizes. Considering results from RAM and BRAM we build two different models of the society to check whether the results of these models are in line with the empirical findings about the shares of agents.

First one is rational agents' society. We assume that three types of agents exist there: one third of the society consists of agents with low costs of search ($c = 0.05$); another third – of agents with medium costs of search ($c = 0.1$); and last third – of agents with high costs of search ($c = 0.15$). Moreover, we assume that $p = 0.35$ – the highest possible price for agents with high search costs, as $p + c \leq a/2$.

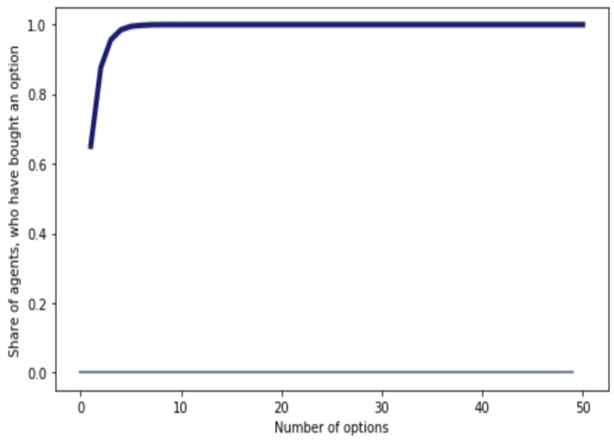
Here, we simulate the expected values of the search for different agents:



Pic. 5: RAM. Expected value of the search.

price $p = 0.35$; *search costs* $c = 0.05, 0.1, 0.15$

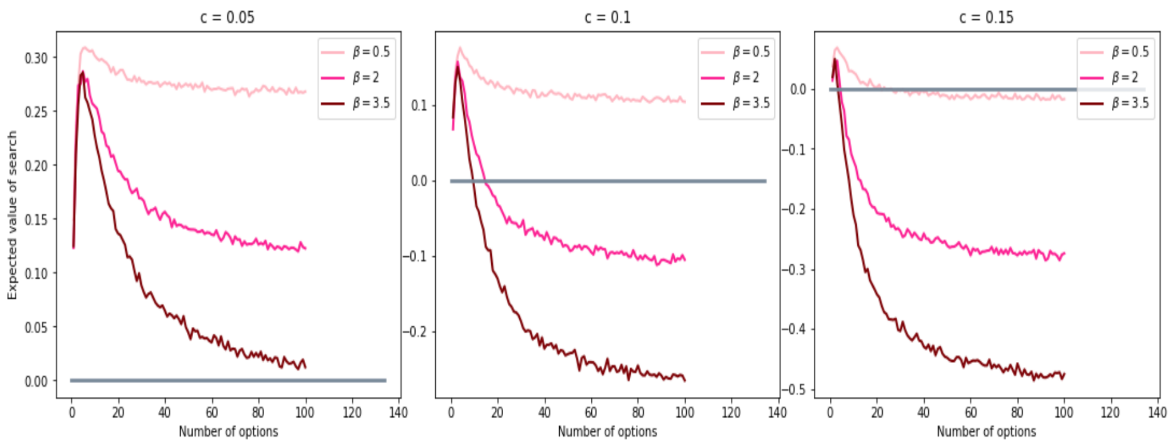
Since we know the optimal search strategies for all agents we can simulate the share of agents who buy the option as a function of choice set sizes. Here, we present the results of this simulation:



Pic. 6: RAM. Share of agents, who buy an option.

Another society is boundedly rational agents' society. We assume that thirty three types of agents exist there (also equally distributed among the society members): agents differ in costs of search (low $c = 0.05$, medium $c = 0.1$, high $c = 0.15$) and in levels of regret-aversion coefficient (from low $\beta = 0.5$ till high $\beta = 3.5$, eleven values with equal intervals).

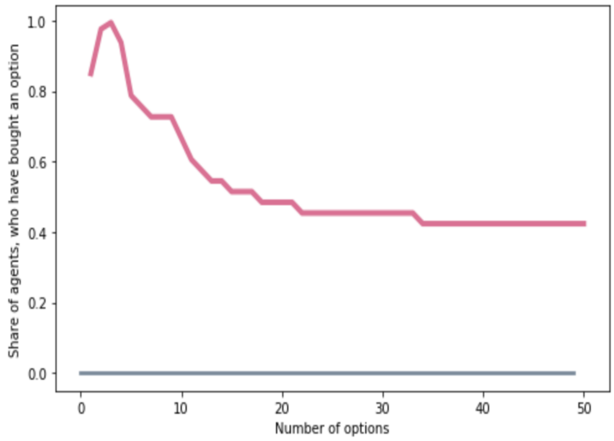
Here, we simulate the expected values of search for different agents:



Pic. 7: BRAM.Expected value of the search.

price $p = 0.35$; *search costs* $c = 0.05, 0.1, 0.15$; *regret sensitivity* $\beta = 0.5, 2, 3.5$;

Also, we simulate the share of agents who buy the option as a function of choice set sizes:



Pic. 8: BRAM. Share of agents, who buy an option.

As we see from simulations, with an increase of the set size some agents from the boundedly rational society refuse to search and choose anything, because the expected value of search is less than zero. The larger set size is, the more agents refuse to choose, however, there are agents with small costs of search or with low regret sensitivity, who continue to choose even when the choice set is big enough. We argue that this result is robust for the boundedly rational society, because it holds for whole range of parameters we have checked in this paper.[10] However, in the rational society we do not observe such relationship between the set sizes the share of agents who have made the choice.

[10]See Appendix A.

5 Conclusion

Although we would like to have opportunities for choice and it seems that more options always bring us to the better decision, there are a lot of empirical studies that show the existence of the choice overload problem – people refuse to choose when they have too many options. Furthermore, this problem arises not only in supermarket situations, when people choose whether to buy the jam or not, but also in more crucial cases, such as, charity donations or retirement plan choosing.

Nevertheless, there also exist empirical studies where more-is-better effects are reported. In our paper we have discussed the robustness of the choice overload effect. According to the empirical papers and meta-analyses we have studied choice overload appears when the special limitations of the choice situation are met: for instance, when a chooser is not familiar with the choice set, so she does not have well-established preferences about options; when it is hard to compare different options due a lot of attributes that describe an option or due to difficult trade-offs; and so on.

To the best of our knowledge, there is no theoretical model that tries to explain the choice overload effect. However, it is not easy to explain the choice overload problem with standard microeconomics assumptions. In this paper we have presented the Rational Agent Model: an agent maximizes her utility function on the choice set, sequentially searching for the best option, and takes into account the costs of searching for information about options. We have proved that for extended choice sets the expected value of the search tends to be non-negative constant. This means that an agent wants to choose an option even if the choice set size tends to infinity. Moreover, we have simulated the society of agents with different types of search costs; we have shown that the share of agents who choose an option increases with the choice set size. These findings are inconsistent with the empirical studies that show decreasing share of choosers while set size raises.

In an attempt to explain the choice overload effect we have offered the Boundedly Rational Agent Model. This agent differs from the rational one because of the regret she could feel. We assume the following regret mechanism: an increase in the size of the choice set inflates chooser's expectations about the best option in the set; at the same time, it multiplies the difficulty of the choice task and makes a chooser simplify the decision process; this simplification could lead to suboptimal results; so that a chooser could feel regret when she compares the inflated expectations with suboptimal actually chosen option. Regret aversion — while expectations about the best option are very high, but it is hard to find the best option due to high search costs — could be one of the reasons why choosers refuse to choose at all, so that the choice overload problem arises.

We have modeled the mechanism described above and got results that are consistent with the empirical evidence. First of all, expected value of the search is an inverted U-shaped function of the choice set size: it increases with the small sizes, reaches the peak with medium ones, and drops for the large sizes.[11] Moreover, for some values of price, search costs and regret sensitivity expected value of the search tends to be less than zero for the large choice sets. This means that agents refuse to choose when the choice set is too big – so that choice overload effect appears. Secondly, we have simulated the society of agents with different types of search costs and different regret sensitivity; we have shown

[11] Note that these results were derived from simulation with the extended range of parameters.

that share of society who choose an option is an inverted U-shaped function of the choice set size as well. These theoretical results are fully consistent with the empirical ones.

It should be mentioned that in this paper we have shown not quantitative, but qualitative results. However, the quantitative result could be obtained by simulating the choice situation with particular parameters. The link to repository with simulation code is presented in references.

Still, further research should be made. First of all, some assumptions of this theory are very strong, for instance, that option values are independent and identically distributed. We assume this for simplicity, however, it could be weakened. Moreover, experiments should be conducted to estimate the parameters of the model – values of the search costs, distribution of the regret sensitivity, and so on.

6 Supporting Materials

Code of simulations is available here: `https://github.com/ansty57/Working_papers/blob/master/Simulations_BT.ipynb`

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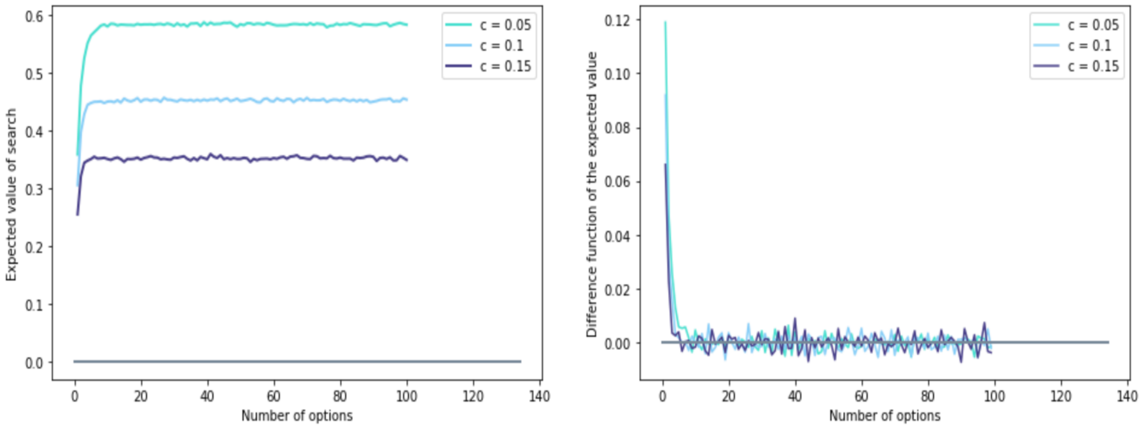
8 Appendix A

8.1 Rational Agent Model. Grid of parameters.

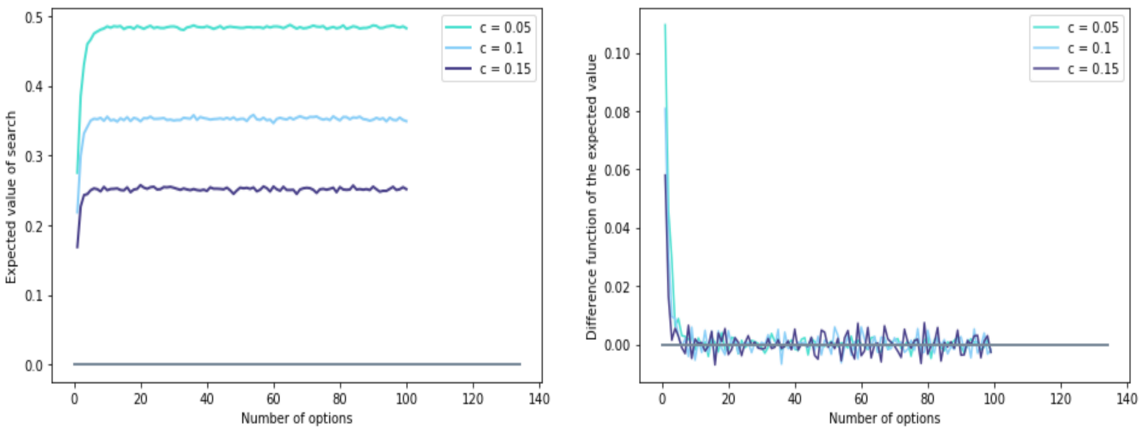
Pic. R1: Expected value of the search and difference function:

price $p = 0.1$; *search costs* $c = 0.05, 0.1, 0.15$



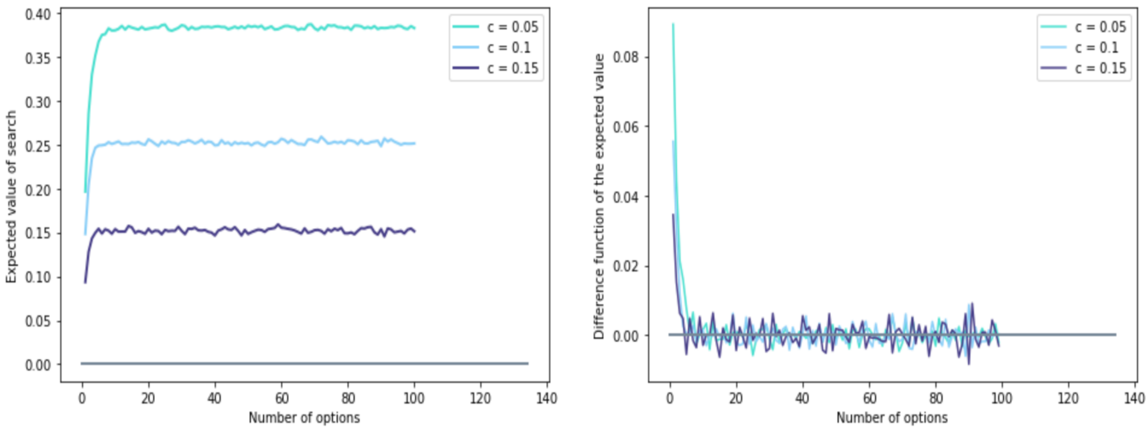
Pic. R2: Expected value of the search and difference function:

price $p = 0.2$; *search costs* $c = 0.05, 0.1, 0.15$



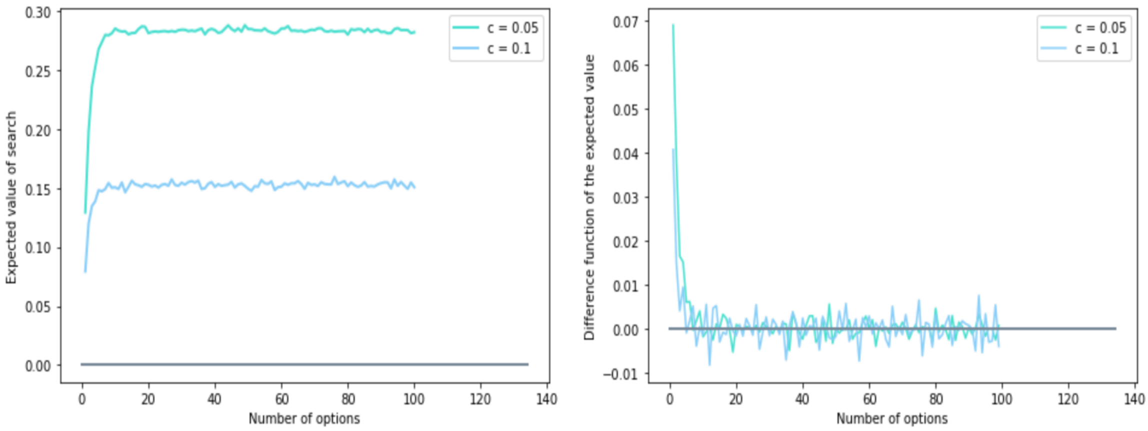
Pic. R3: Expected value of the search and difference function:

price $p = 0.3$; *search costs* $c = 0.05, 0.1, 0.15$



Pic. R4: Expected value of the search and difference function:

price $p = 0.4$; *search costs* $c = 0.05, 0.1, 0.15$

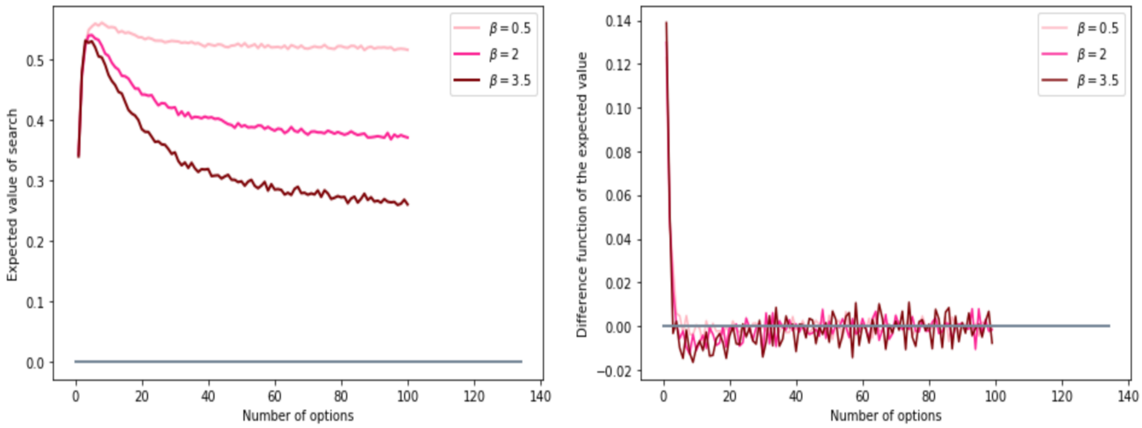


8.2 Boundedly Rational Agent Model. Grid of parameters.

In simulations BR1.1 - BR1.3 price p = 0.1

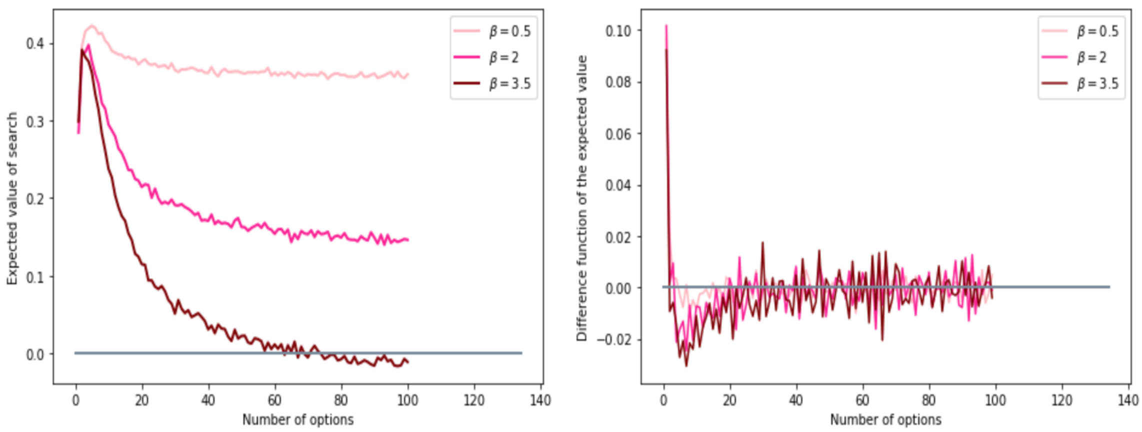
Pic. BR1.1: Expected value of the search and difference function:

price $p = 0.1$; *search costs* $c = 0.05$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



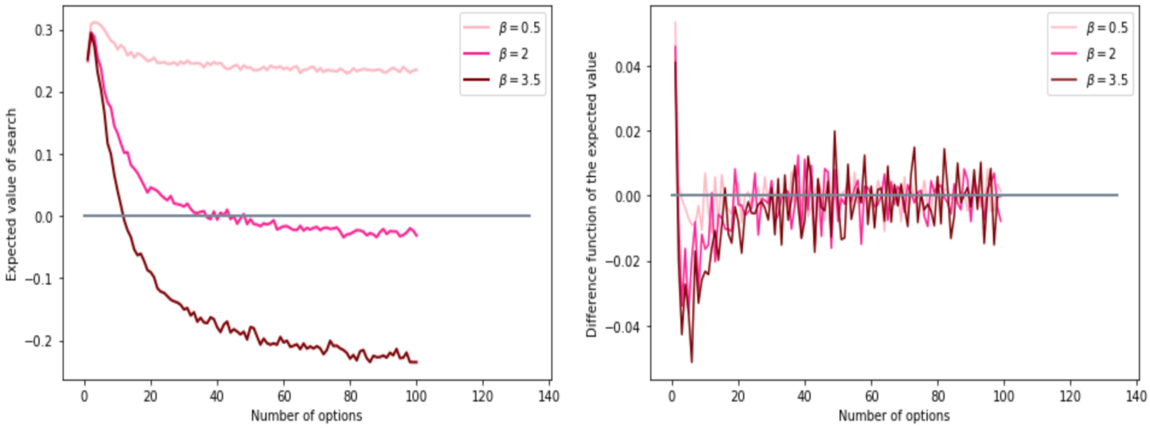
Pic. BR1.2: Expected value of the search and difference function:

price $p = 0.1$; *search costs* $c = 0.1$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



Pic. BR1.3: Expected value of the search and difference function:

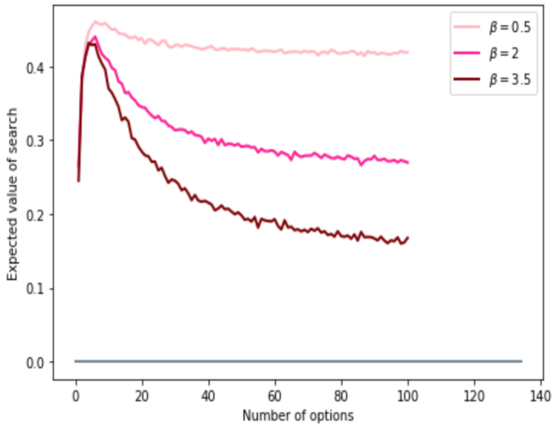
price $p = 0.1$; *search costs* $c = 0.15$; *regret sensitivity* $\beta = 0.5, 2, 3.5$

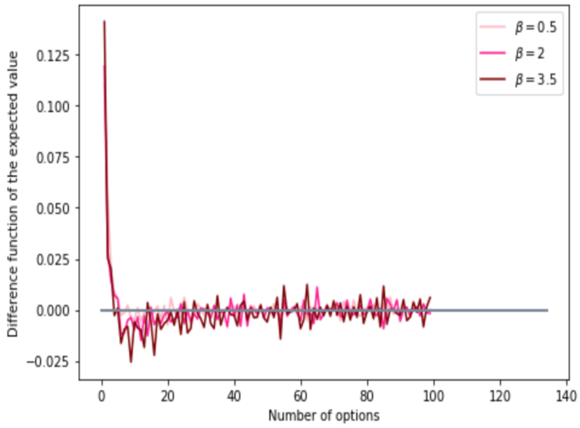


In simulations BR2.1 - BR2.3 price p = 0.2

Pic. BR2.1: Expected value of the search and difference function:

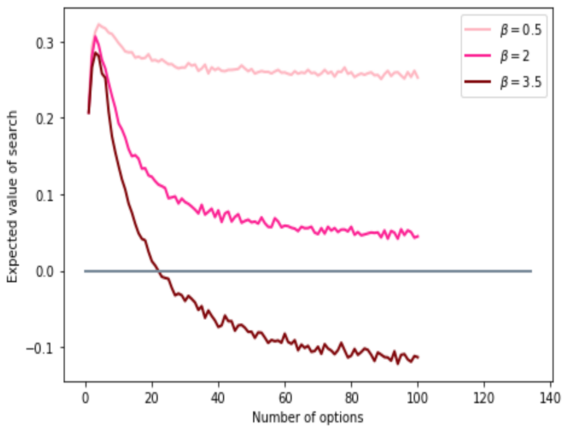
price $p = 0.2$; *search costs* $c = 0.05$; *regret sensitivity* $\beta = 0.5, 2, 3.5$

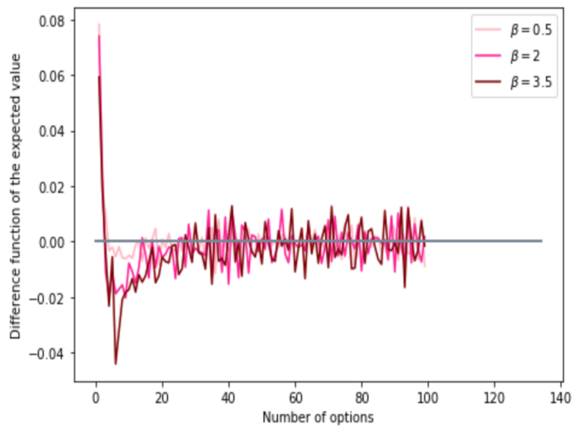




Pic. BR2.2: Expected value of the search and difference function:

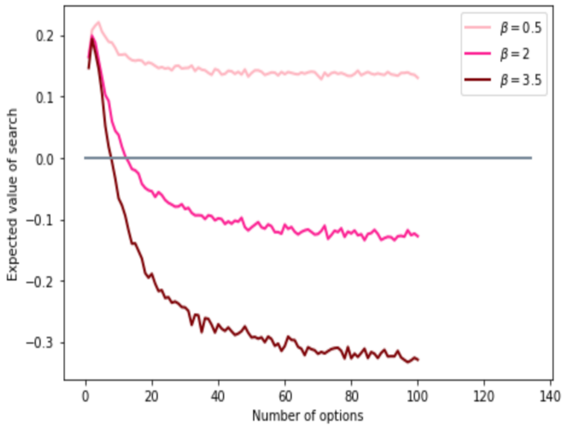
price $p = 0.2$; *search costs* $c = 0.1$; *regret sensitivity* $\beta = 0.5, 2, 3.5$

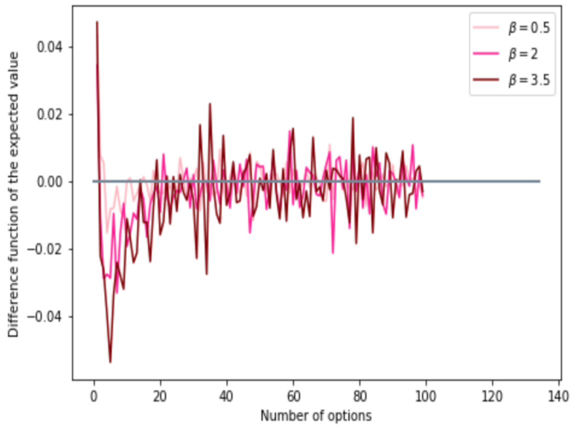




Pic. BR2.3: Expected value of the search and difference function:

price $p = 0.2$; *search costs* $c = 0.15$; *regret sensitivity* $\beta = 0.5, 2, 3.5$

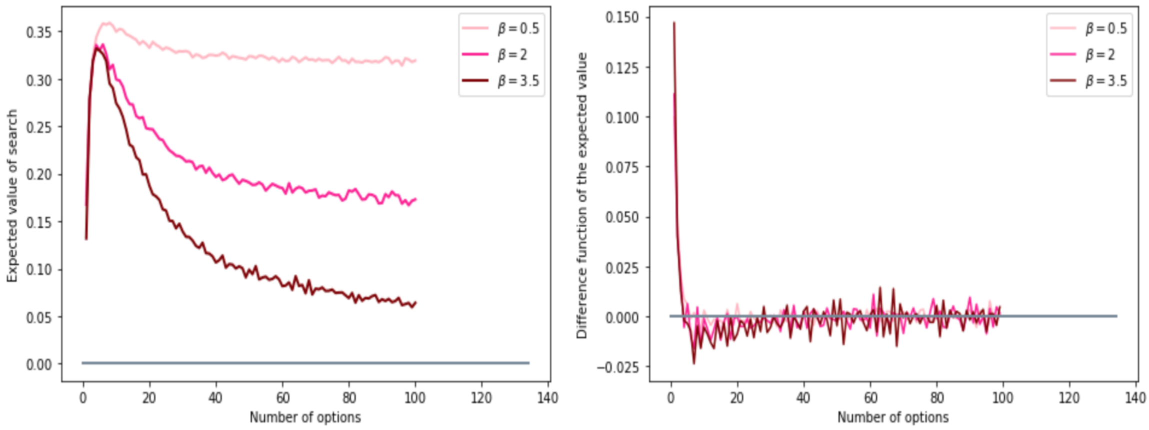




In simulations BR3.1 - BR2.3 price p = 0.3

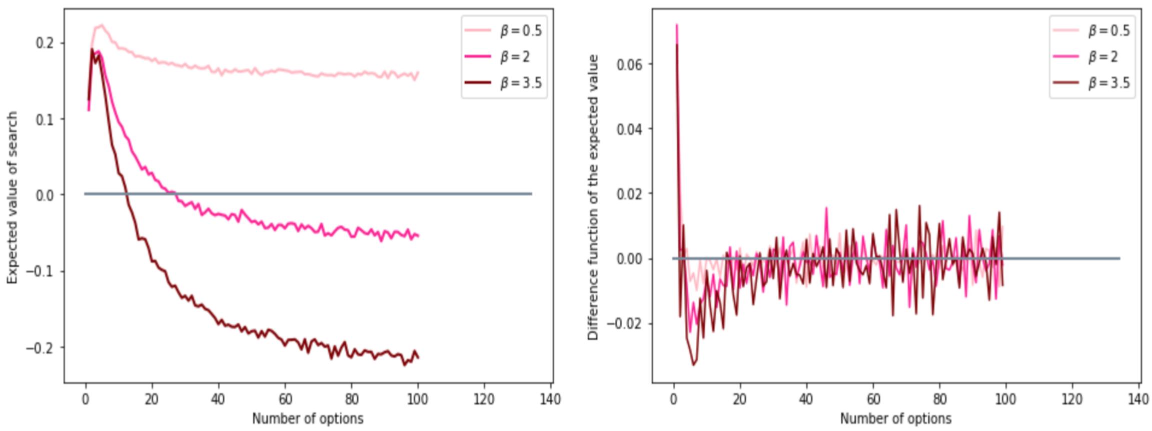
Pic. BR3.1: Expected value of the search and difference function:

price $p = 0.3$; *search costs* $c = 0.05$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



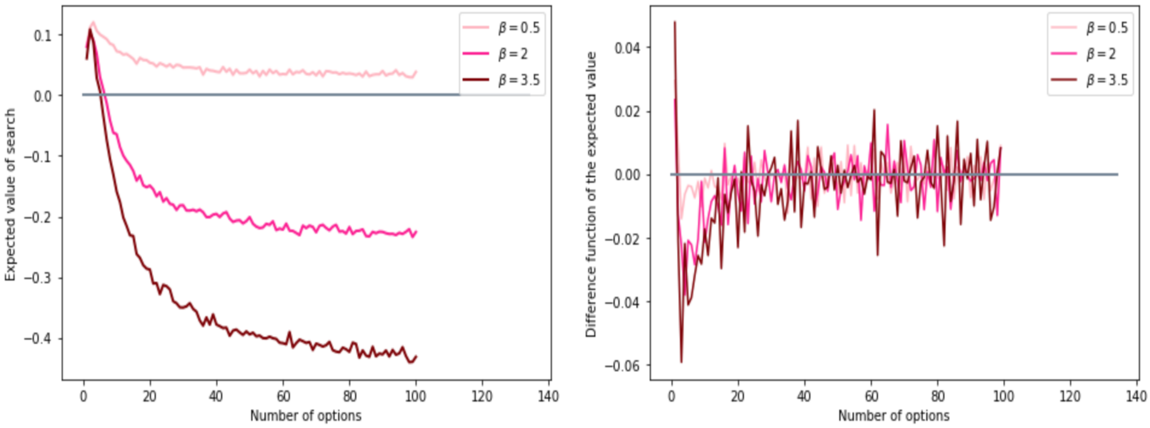
Pic. BR3.2: Expected value of the search and difference function:

price $p = 0.3$; *search costs* $c = 0.1$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



Pic. BR3.3: Expected value of the search and difference function:

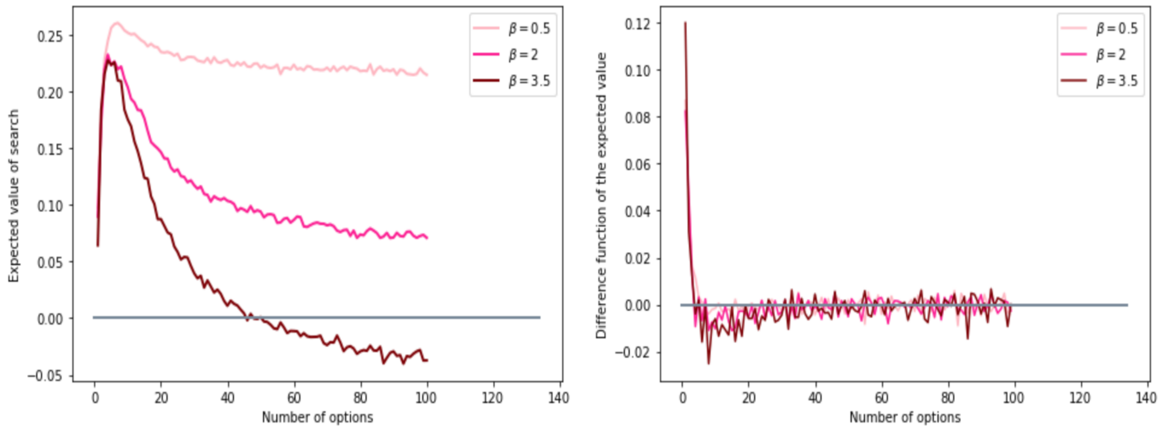
price $p = 0.3$; *search costs* $c = 0.15$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



In simulations BR4.1 - BR4.2 price p = 0.4

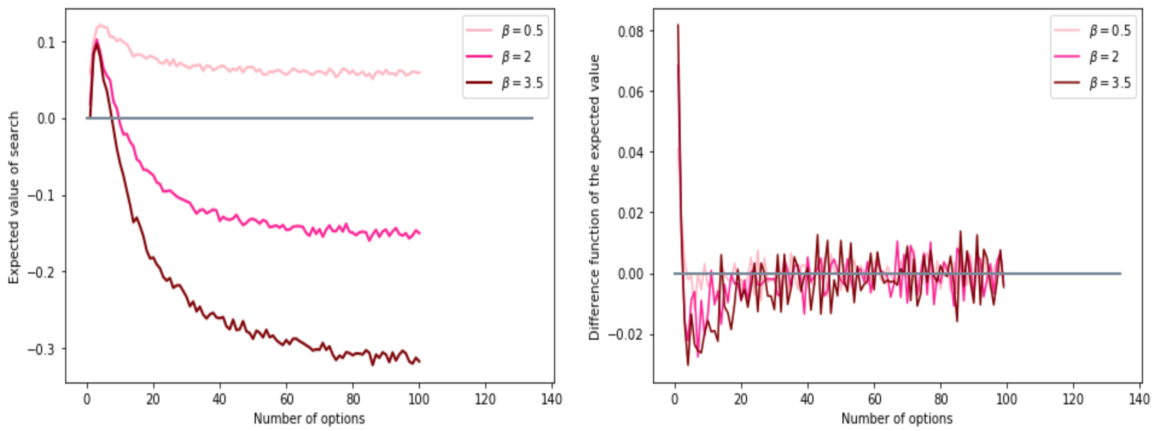
Pic. BR4.1: Expected value of the search and difference function:

price $p = 0.4$; *search costs* $c = 0.05$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



Pic. BR4.2: Expected value of the search and difference function:

price $p = 0.4$; *search costs* $c = 0.1$; *regret sensitivity* $\beta = 0.5, 2, 3.5$



9 Appendix B

9.1 Proof of Lemma R1

(I) Here we derive the expected utility of the action to continue the search exactly one option further and to buy the best option at step $t+1$ from step t

Consider agent's expectations at step t about the value of the best option she could find at the step t+1:

$$E(x_{(t+1)}|x_{(t)}) = \int_0^{x(t)} x_{(t)} dF(x) + \int_{x_{(t)}}^a x dF(x) = \int_0^{x(t)} x_{(t)} d\frac{x}{a} + \int_{x_{(t)}}^a x d\frac{x}{a} = \frac{x_{(t)}^2}{a} + \frac{a^2}{2a} - \frac{x_{(t)}^2}{2a} = \frac{a^2+x_{(t)}^2}{2a}$$

Then, expected utility takes the following form:

$$E_t U_{t+1}^B = E(x_{(t+1)}|x_{(t)}) - p - (t+1)c = \frac{a^2+x_{(t)}^2}{2a} - p - (t+1)c$$

(IIa) Compare the utility of the action to stop the search process at step $t = 1, ..., n$ and buy the best seen option (U_t^B) with the utility of the action to stop the search process at step $t = 1, ..., n$ and leave the choice situation with nothing (U_t^L):

$$U_t^B = x_{(t)} - p - tc \text{ VS } -tc = U_t^L \Leftrightarrow$$
$$x_{(t)} \text{ VS } p.$$

(IIb) Compare the utility of an action to stop the search process at step $t = 1, ..., n$ and buy the best seen option (U_t^B) with the expected utility of an action to continue the search exactly one option further and to buy the best option at step $t+1$ $(E_t U_{t+1}^B)$:

$$U_t^B = x_{(t)} - p - tc \text{ VS } \frac{a^2+x_{(t)}^2}{2a} - p - (t+1)c = E_t U_{t+1}^B \Leftrightarrow$$
$$c \text{ VS } \frac{(a-x_{(t)})^2}{2a} \Leftrightarrow$$
$$x_{(t)} \text{ VS } a - \sqrt{2ac}.$$

Note that we use this root of the equation as $x_{(t)} \in [0, a]$.

(IIc) Compare the utility of an action to stop the search process at step $t = 1, ..., n$ and leave the choice situation with nothing (U_t^L) with the expected utility of an action to continue the search exactly one option further and to buy the best option at the step $t+1$ $(E_t U_{t+1}^B)$:

$$U_t^L = -tc \text{ VS } \frac{a^2+x_{(t)}^2}{2a} - p - (t+1)c = E_t U_{t+1}^B \Leftrightarrow$$
$$0 \text{ VS } \frac{a^2+x_{(t)}^2}{2a} - p - c$$

Considering the assumption $p + c \leq a/2$, we get the following result: for any $x_{(t)}$ $E_t U_{t+1}^B \geq U_t^L$

(III) Compare the conditions on $x_{(t)}$ from (IIa) and (IIb):

p VS $a - \sqrt{2ac} \Leftrightarrow$

$2ac$ VS $a^2 - 2ap - p^2 \Leftrightarrow$

0 VS $a(a - 2(p + c)) + p^2$

As $p + c \leq \frac{a}{2}$ we derive that $p \leq a - \sqrt{2ac}$.

(IVa) For $t = 1, ..., (n-1)$ if $x_{(t)} \geq a - \sqrt{2ac}$, then the optimal action at step t is stop the search process and buy the best seen option $x_{(t)}$.
If $x_{(t)} \geq a - \sqrt{2ac}$, then according to (III): $x_{(t)} \geq p$. On the one hand, this means that for any step $q = t, ..., n$ $x_{(q)} \geq x_{(t)} \geq p$, where $x_{(q)}$ is the value of the best seen option among the explored q options, so it is not worse to buy the best seen option at step q than to leave the search at step q with nothing. On the other hand, for any step $q = t, ..., (n-1)$ $x_{(q)} \geq x_{(t)} \geq a - \sqrt{2ac}$, so it is not worse to buy the best seen option at step q than to buy the best seen option at step $q+1$. Here at the step $t = n-1$ an agent knows that if she goes to step $t = n$ she will buy, but it is not worse to buy at $t = n-1$ than at $t = n$. So at the step $t = n-2$ an agent knows that if she goes to step $t = n-1$ she will buy, but it is not worse to buy at $t = n-2$ than at $t = n-1$, and so on. This means that if $x_{(t)} \geq a - \sqrt{2ac}$, then the optimal action of the step t is stop the search process and buy the best seen option.

(IVb) For $t = 1, ..., (n-1)$ if $x_{(t)} \in [p; a - \sqrt{2ac})$, then the optimal action at step t is continue the search at least one option further.
According to (IIc) for any $x_{(t)}$ $E_t U^B_{t+1} \geq U^L_t$, so that to continue the search one step further and to buy the best seen option at $t+1$ is not worse than to leave at t. Moreover, according to (IIb), if $x_{(t)} < a - \sqrt{2ac}$ then $E_t U^B_{t+1} > U^B_t$: an agent expects that it is not worse to continue the search one step further and to buy the best seen option at $t+1$ than to buy the best seen option at t. This means that the expected utility of continuing the search exactly one step further is not lower than utilities of actions to buy at step t or to choose nothing and leave the search situation at step t. However, there exists the optimal strategy S^*, so that expected utility of the optimal strategy S^* is not lower than the expected utility of the action to continue the search one step further and buy at $t+1$:

$EU(S^*) \geq E_t U^B_{t+1}$, thus $EU(S^*) \geq E_t U^B_{t+1} > U^B_t \geq U^L_t$

This means that expected utility of the optimal strategy S^* is greater than the utility of action to buy at step t or to choose nothing and leave the search situation at step t, which means that the optimal action at step t is to continue the search at least one option further.

(IVc) For $t = 1, ..., (n-1)$ if $x_{(t)} < p$, then the optimal action at the step t is continue the search at least one option further.
The proof is similar to the (IVb), however, here we derive $EU(S^*) \geq E_t U^B_{t+1} > U^L_t \geq U^B_t$.

(IVd) At step $t = n$ if $x_{(n)} \geq p$, then the optimal action at step n is to buy the best seen option $x_{(n)}$; else, the optimal action at step n is to leave the choice situation with nothing. This result follows from (IIc).

9.2 Proof of Lemma BR1

Please note that this proof is quite similar to previous one.

(I) Here we derive the expected utility of the action to continue the search exactly one option further and to buy the best option at step $t+1$ from the step t

As we have discussed above agent's expectations at step t about the value of the best option, she could find at the step $t+1$, takes the following form:

$$E(x_{(t+1)}|x_{(t)}) = \frac{a^2+x_{(t)}^2}{2a}$$

So the expected utility of the action to continue the search exactly one option further and to buy the best option at the step $t+1$ is:

$$E_tU_{t+1}^B = \frac{a^2+x_{(t)}^2}{2a} + \beta(\frac{a^2+x_{(t)}^2}{2a} - \frac{an}{n+1}) - p - (t+1)c$$

(IIa) Compare the utility of the action to stop the search process at step $t = 1, ..., n$ and buy the best seen option (U_t^B) with the utility of the action to stop the search process at step $t = 1, ..., n$ and leave the choice situation with nothing (U_t^L):

$$U_t^B = x_{(t)} + \beta(x_{(t)} - \frac{an}{n+1}) - p - tc \text{ VS } -\beta\frac{an}{n+1} - tc = U_t^L \Leftrightarrow$$
$$x_{(t)} \text{ VS } \frac{p}{1+\beta}.$$

(IIb) Compare the utility of an action to stop the search process at step $t = 1, ..., n$ and buy the best seen option (U_t^B) with the expected utility of an action to continue the search exactly one option further and to buy the best option at step $t+1$ $(E_tU_{t+1}^B)$:

$$U_t^B = x_{(t)} + \beta(x_{(t)} - \frac{an}{n+1}) - p - tc \text{ VS } \frac{a^2+x_{(t)}^2}{2a} + \beta(\frac{a^2+x_{(t)}^2}{2a} - \frac{an}{n+1}) - p - (t+1)c = E_tU_{t+1}^B \Leftrightarrow$$
$$c \text{ VS } \frac{(a-x_{(t)})^2}{2a}(1+\beta) \Leftrightarrow$$
$$x_{(t)} \text{ VS } a - \sqrt{\frac{2ac}{1+\beta}}.$$

Note that we use this root of the equation as $x_{(t)} \in [0, a]$.

(IIc) Compare the utility of an action to stop the search process at step $t = 1, ..., n$ and leave the choice situation with nothing (U_t^L) with the expected utility of an action to continue the search exactly one option further and to buy the best option at the step $t+1$ $(E_tU_{t+1}^B)$:

$U_t^L = -\beta\frac{an}{n+1} - tc$ VS $\frac{a^2+x_{(t)}^2}{2a} + \beta(\frac{a^2+x_{(t)}^2}{2a} - \frac{an}{n+1}) - p - (t+1)c = E_t U_{t+1}^B \Leftrightarrow$
0 VS $\frac{a^2+x_{(t)}^2}{2a}(1+\beta) - p - c$

Considering the assumption $p+c \leq a/2$ and $\beta > 0$ we get the following result: for any $x_{(t)}$ $E_t U_{t+1}^B \geq U_t^L$

(III) Compare the conditions on $x_{(t)}$ from (IIa) and (IIb):

$\frac{p}{1+\beta}$ VS $a - \sqrt{\frac{2ac}{1+\beta}} \Leftrightarrow$
p VS $a(1+\beta) - \sqrt{2ac(1+\beta)} \Leftrightarrow$
$2ac(1+\beta)$ VS $a^2(1+\beta)^2 - 2a(1+\beta)p + p^2 \Leftrightarrow$

0 VS $a(1+\beta)(a - 2(p+c)) + p^2$

As $p + c \leq \frac{a}{2}$ we derive that $\frac{p}{1+\beta} \leq a - \sqrt{\frac{2ac}{1+\beta}}$.

(IVa) For $t = 1, ..., (n-1)$ if $x_{(t)} \geq a - \sqrt{\frac{2ac}{1+\beta}}$, then the optimal action at step t is stop the search process and buy the best seen option $x_{(t)}$.
If $x_{(t)} \geq a - \sqrt{\frac{2ac}{1+\beta}}$, then according to (III): $x_{(t)} \geq \frac{p}{1+\beta}$. On the one hand, this means that for any step $q = t, ..., n$ $x_{(q)} \geq x_{(t)} \geq \frac{p}{1+\beta}$, where $x_{(q)}$ is the value of the best seen option among the explored q options, so it is not worse to buy the best seen option at step q than to choose nothing and leave the search situation at step q. On the other hand, for any step $q = t, ..., (n-1)$ $x_{(q)} \geq x_{(t)} \geq a - \sqrt{\frac{2ac}{1+\beta}}$, so it is not worse to buy the best seen option at step q than to buy the best seen option at step $q+1$. Here at step $t = n-1$ an agent knows that if she goes to step $t = n$ she will buy, but it is not worse to buy at $t = n-1$ than at $t = n$. So at the step $t = n-2$ an agent knows that if she goes to step $t = n-1$ she will buy, but it is not worse to buy at $t = n-2$ than at $t = n-1$, and so on. This means that if $x_{(t)} \geq a - \sqrt{\frac{2ac}{1+\beta}}$, then the optimal action of the step t is stop the search process and buy the best seen option.

(IVb) For $t = 1, ..., (n-1)$ if $x_{(t)} \in [\frac{p}{1+\beta}; a - \sqrt{\frac{2ac}{1+\beta}})$, then the optimal action at step t is continue the search at least one option further.
According to (IIc) for any $x_{(t)}$ $E_t U_{t+1}^B \geq U_t^L$, so that to continue the search one step further and to buy the best seen option at $t+1$ is not worse than to choose nothing and leave the search situation at t. Moreover, according to (IIb), if $x_{(t)} < a - \sqrt{\frac{2ac}{1+\beta}}$ then $E_t U_{t+1}^B > U_t^B$: an agent expects that it is not worse to continue the search one step further and to buy the best seen option at $t+1$ than to buy the best seen option at t. This means that the expected utility of continuing the search exactly one step further is not lower than utilities of actions to buy at step t or to leave the search at step t. However, there exists the optimal strategy S^*, so that expected utility of the optimal strategy S^* is not lower than the expected utility of the action to continue the search one step further and buy at $t+1$:

$EU(S^*) \geq E_t U_{t+1}^B$, thus $EU(S^*) \geq E_t U_{t+1}^B > U_t^B \geq U_t^L$

This means that expected utility of the optimal strategy S^* is greater than the utility of action to buy at step t or to choose nothing and leave the search situation at step t, what means that the optimal action at step t is to continue the search at least one option further.

(IVc) For $t = 1, ..., (n-1)$ if $x_{(t)} < \frac{p}{1+\beta}$, then the optimal action at step t is continue the search at least one option further.
The proof is similar to the (IVb), however, here we derive $EU(S^*) \geq E_t U^B_{t+1} > U^L_t \geq U^B_t$.

(IVd) At step $t = n$ if $x_{(n)} \geq \frac{p}{1+\beta}$, then the optimal action at the step n is to buy the best seen option $x_{(n)}$; else, the optimal action at the step n is to choose nothing and leave the search situation This result follows from (IIc).

9.3 Other results

Result 1

Here we prove that for Rational Agent Model $Et^* = \frac{1-(\frac{a-\sqrt{2ac}}{a})^n}{1-\frac{a-\sqrt{2ac}}{a}}$

$Et^* = Pr(x_1 \geq a-\sqrt{2ac}) \times 1 + Pr(x_1 < a-\sqrt{2ac}; x_2 \geq a-\sqrt{2ac}) \times 2 + ... + Pr(x_1 < a-\sqrt{2ac}; ...; x_{n-1} < a-\sqrt{2ac}) \times n = (1-F(a-\sqrt{2ac})) \times 1 + (1-F(a-\sqrt{2ac}))F(a-\sqrt{2ac}) \times 2 + ... + (1-F(a-\sqrt{2ac}))F^{n-2}(a-\sqrt{2ac}) \times (n-1) + F^{n-1}(a-\sqrt{2ac}) \times n$

Where $F(x) = \frac{x}{a}$ – cumulative function of uniform distribution; for simplicity we denote $F(a-\sqrt{2ac}) = F^*$, so then:

$Et^* = (1-F^*) \times 1 + (1-F^*)F^* \times 2 + (1-F^*)F^{*n-2} \times (n-1) + F^{*n-1} \times n = (1-F^*)(1 + F^* + F^{*2} + ... + F^{*n-2}) + (1-F^*)F^*(1 + F^* + F^{*2} + ... + F^{*n-3}) + ... + (1-F^*)F^{*n-2} + F^{*n-1} \times n = (1-F^{*n-1}) + F^*(1-F^{*n-2}) + ... + F^{*n-2}(1-F^*) + F^{*n-1} \times n = (1 + F^* + F^{*2} + ... + F^{*n-2}) - (n-1)F^{*n-1} + nF^{*n-1} = 1 + F^* + F^{*2} + ... + F^{*n-2} + F^{*n-1} = \frac{1-F^{*n}}{1-F^*} = \frac{1-(\frac{a-\sqrt{2ac}}{a})^n}{1-\frac{a-\sqrt{2ac}}{a}}$

Result 2

Here we prove that for Rational Agent Model $\lim_{n\to\infty} EV = a - p - \sqrt{2ac}$

$\lim_{n\to\infty} EV = \lim_{n\to\infty}[E(x_t|x_t \geq a-\sqrt{2ac}) - p - cEt^*] = \lim_{n\to\infty}[\frac{a+a-\sqrt{2ac}}{2} - p - c\frac{1-(\frac{a-\sqrt{2ac}}{a})^n}{1-\frac{a-\sqrt{2ac}}{a}}] = \frac{a+a-\sqrt{2ac}}{2} - p - \frac{1}{1-\frac{a-\sqrt{2ac}}{a}} = a - p - \sqrt{2ac}$

Result 3

Here we prove that for Boundedly Rational Agent Model $\lim_{n\to\infty} EV = a - p - \sqrt{2ac(1+\beta)}$

$\lim_{n\to\infty} EV = \lim_{n\to\infty}[E(x_t|x_t \geq a - \sqrt{\frac{2ac}{1+\beta}})(1+\beta) - \beta\frac{an}{n+1} - p - cEt^*] = \lim_{n\to\infty}[\frac{a+a-\sqrt{\frac{2ac}{1+\beta}}}{2}(1+$

$$\beta) - \beta \frac{an}{n+1} - p - c \frac{1 - \left[\frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a} \right]^n}{1 - \frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a}}] = \frac{2a(1+\beta) - \sqrt{2ac(1+\beta)}}{2} - a\beta - p - c \frac{1}{1 - \frac{a - \sqrt{\frac{2ac}{1+\beta}}}{a}} = a - p - \sqrt{2ac(1+\beta)}$$