Roll No. ..... Total Pages : 05

(41)

D-C-190041

B. Tech. EXAMINATION, 2019

Semester IV (CBS)

OPTIMIZATION AND CALCULUS OF VARIATIONS

(CE, ME, AE, ECE, EE, EEE, CSE, IT)

MA-401

Time : 3 Hours *Maximum Marks :* 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Section A, B, C and D. Q. No. **9** is compulsory.

Section A

1. (a) Minimize $Z = 6x + 14y$; **6**

subject to constraints $\begin{cases} 5x+4y \geq 60; & 3x+7y \leq 84 \\ x+2y \geq 18; & x, y \geq 0 \end{cases}$

by graphical method.

(b) Explain the following in the context of linear programming problem : 6

(i) Feasible solution

(ii) Objective function

(iii) Non-negativity constraints.

2. Use simplex method to solve the following linear programming problem : 12

Maximize $Z = 107x_1 + x_2 + 2x_3$;

subject to constraints :

$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$
$$16x_1 + 0.5x_2 - 6x_3 \leq 5$$
$$3x_1 - x_2 - x_3 \leq 0$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

Section B

3. Solve the following Linear programming by the method of dynamic programming :

Maximize $Z = 50x_1 + 100x_2$,

Subject to :

$$10x_1 + 5x_2 \leq 2500$$
$$4x_1 + 10x_2 \leq 2000$$
$$x_1 + 1.5x_2 \leq 450$$
$$x_1, x_2 \geq 0.$$

4. Find the optimum solution of the following transportation problem in which the cells contain the transportation cost in rupees : 12

	D_1	D_2	D_3	D_4	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

Section C

5. A project consists of the following activities with the time estimates noted against each : 12

Activity	Time estimates (weeks)	Activity	Time estimates (weeks)
1-2	2	3-7	5
1-3	2	4-6	3
1-4	1	5-8	1
2-5	4	6-9	5
3-6	8	7-8	4
		8-9	3

(a) Draw a network diagram.

(b) Determine the critical path and its duration.

(c) Calculate the total float for each activity.

6. Consider the Non-linear Programming Problem : **12**

Maximize $Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

By separating this function into three one-variable functions, show that the function is convex. Solve the problem by solving each one-variable function by calculus.

Section D

7. (a) Prove that the shortest distance between two points in a plane is a straight line. **6**

(b) Find the extremal of the functional $I = \int_0^{\pi} \left[(y')^2 - y^2 + 4y\cos x \right] dx$ under the conditions $y(0) = 0, y(\pi) = 0$. **6**

8. (a) Prove that the geodesics on a sphere of radius a are its great circles. **6**

(b) Find the plane curve of fixed perimeter and maximum area. **6**

Section E

(Compulsory Question)

9. Atempt all the parts : **1×12=12**

(a) What is the essential difference between regular simplex method and dual simplex method ?

(b) Explain the concept of duality in linear programming.

(c) Write the number of non-negative variables in a basic feasible solution to a $m \times n$ transportation problem.

(d) Define balanced transportation problem.

(e) What is optimum basic feasible solution ?

(f) Define Slack variable in the context of Linear Programming Problems.

(g) Write *two* applications of network model.

(h) Define degeneracy of transportation problem.

(i) What is the expected time for PERT calculations ?

(j) Write the statement of Euler's equation.

(k) Differentiate between Dynamic Programming Problem (DPP) and Linear Programming Problem (LPP).

(l) Define functional of a curve.