[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2067)

17033(M)

B. Tech 4th Semester Examination

Optimization and Calculus of Variations (CBS)

MA-401

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five question in all, selecting one question from each Sections A, B, C and D. Section E is compulsory.

SECTION - A

1. (a) Maximize $Z = 3x + 4y$,

Subject to $x-y \geq 0$; $2.5x-y \leq -3$; $x, y \geq 0$

By graphical method. (6)

(b) Why do some problems have multiple optimal feasible solutions? What is the effect of changing the objective function on the optimum point? Discuss. (6)

2. Use simplex method to solve the following problem:

Maximize $Z = 4x_1 + 3x_2 + 6x_3$,

subject to $2x_1 + 3x_2 + 2x_3 \leq 440$,

$4x_1 + 3x_3 \leq 470$,

$2x1 + 5x_2 \leq 430$,

$x_1, x_2 \geq 0$ (12)

SECTION - B

3. Using the Bellman's principle of optimality,

Minimize $Z = y_1 + y_2 + y_3 + \text{---------} + y_n$,

subject to $y_1.y_2.y_3.\text{.....}.y_n = b$,

$y_i \geq 0$; i=1, 2,....., n. (12)

4. Find the basic feasible solution of the following transportation problem by north west corner rule. Also find the optimal transportation plan.

	1	2	3	4	5	Available
A	4	3	1	2	6	80
B	5	2	3	4	5	60
C	3	5	6	3	2	40
D	2	4	4	5	3	20
Required	60	60	30	40	10	200 (total)

(12)

SECTION - C

5. Consider the following function:

$$f(X) = 5x_1 + 2x_2^2 + x_3^2 - 3x_3x_4 + 4x_4^2 + 2x_5^4 + x_5^2 + 3x_5x_6 + 6x_6^2 + 3x_6x_7 + x_7^2$$

Show that f(x) is a convex by expressing it as a sum of functions of one or two variables and then proving that all the functions are convex. (12)

6. The time estimates (in weeks) for the activity of a PERT network are given below:

Activity	t_0	t_m	t_p
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(a) Draw the project network and identify all the paths through it.

(b) Determine the expected project length.

(c) Calculate the standard deviation and variance of the project length.

(d) What is the probability that the project will be completed at least 4 weeks earlier than the expected time?

(e) If the project due date is 19 weeks, what is the probability of not meting due date?

(f) The probability that the project will be completed on the schedule if the schedule completing time is 220 weeks. (12)

SECTION - D

7. (a) Find the plane curve of fixed perimeter and maximum area. (6)

(b) Find the extremals of the following functionals:

$$\int_0^{\frac{\pi}{2}} (y^2 - y'^2 - 2y\sin x)dx;\ y(0) = y\left(\frac{\pi}{2}\right) = 0. \quad (6)$$

8. (a) Show that the geodesics on a sphere of radius a are its great circles. (6)

(b) Find a function y(x) for a function for which $\int_0^1 (x^2 + y'^2)dx$ is stationary, given that $\int_0^1 y^2 dx = 2$; y(0)=0, y(1)=0. (6)

SECTION - E

9. Attempt all the questions:

(a) Give the advantages of network models.

(b) Write the two difficulties in using network models.

(c) Give short note on restrictions on assignments.

(d) Write short note on degenerate transportation problem.

(e) Define the term alternate courses of action.

(f) Explain the term SLACK variables with example.

(g) State two rules to avoid cycling.

(h) Define functionals of a calculus of variation.

(i) Find the extremals of a functional $\int_{x_0}^{x_1} \left(\frac{y'^2}{x^3} \right) dx$.

(j) Define convexity and write its two properties.

(k) What are the advantages of duality theory?

(l) Define dynamic programming for Bellman's Principle of optimality. (1×12=12)