

16005(D) - DEC 2016

B. Tech 1st Semester Examination

Engineering Mathematics-I (CBS)

MA-101

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : A candidate is required to attempt five questions in all, selecting one question from each unit and Question No. 9 is compulsory.

UNIT - I

1. (a) For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. Find non-singular

matrices P and Q such that PAQ is in the normal form. (6)

- (b) Investigate for what values of λ and μ the simultaneous equations
 $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$, have
 (i) no solution, (ii) a unique solution (iii) an infinite number of solutions. (6)

2. (a) If λ be an eigen value of a non-singular matrix A. Show that $\frac{|A|}{\lambda}$ is an eigen value of matrix adj. A. (6)

- (b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \text{ and, hence find the matrix represented by}$$

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + 1. \quad (6)$$

UNIT - II

3. (a) Find all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$. Also show that the continued product of these values is 1. (6)

- (b) Sum the series by C+iS method
 $\sin \alpha + x \sin(\alpha + \beta) + \frac{x^2}{2!} \sin(\alpha + 2\beta) + \dots \infty.$ (6)

4. (a) If $f(z)$ be an analytic function of z , show that
 $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0.$ (6)

- (b) Find the analytic function, whose imaginary part is
 $\frac{(x-y)}{(x^2+y^2)}.$ (6)

UNIT - III

5. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u.$ (6)

- (b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ by using Lagrange's Method of undetermined multipliers. (6)

6. (a) Find the volume (by double integrals) bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (6)

- (b) Evaluate the following integral by changing to spherical polar co-ordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{(1-x^2-y^2-z^2)}} \quad (6)$$

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UNIT - IV

7. (a) Find the unit vector normal to the surface $xy^3z^2=4$, at the point $(-1, -1, 2)$. (6)
- (b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. (6)
8. (a) If $F = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C F \cdot dR$, where C is the curve in the xy-plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. (6)
- (b) Evaluate by Stoke's theorem $\int_C F \cdot dR$ where $F = y\hat{i} + xz^3\hat{j} - zy^3\hat{k}$, C is the circle $x^2 + y^2 = 4$, $z = 1.5$. (6)

UNIT - V (Compulsory)

9. Each part of this question carry one mark.
- (a) Define eigen values and eigen vectors of a matrix.
- (b) Show that every diagonal element of a Hermitian matrix must be real.
- (c) How many distinct values of $(\text{cis}\theta)^{6/15}$?
- (d) Write down the polar form of Cauchy-Riemann equations.
- (e) State Euler's theorem.
- (f) If u and v are functions of two independent variables x and y , then define Jacobian.
- (g) Evaluate the integral $\int_1^2 \int_1^3 xy^2 dx dy$.
- (h) Define Beta function.

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- (i) What is the directional derivative of a scalar point function ϕ along the direction of unit vector a ?
- (j) If $R = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla \cdot R = 3$.
- (k) State Stoke's theorem.
- (l) Using divergence theorem, prove that $\int_S R \cdot dS = 3V$. (12×1=12)