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MAR-21-210087

B. Tech. EXAMINATION, March 2021

Semester III (N/S)

ENGINEERING MATHEMATICS-III

(CE, ME, TE, EE, ECE, EEE, AE)

NS-206

Time : 3 Hours *Maximum Marks :* 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Sections A, B, C and D. Q. No. **9** is compulsory.

Section A

1. (a) Find the differential equation of all spheres of radius 3 units having their centres in the xy-plane. **10**

(b) Solve : **10**

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x \partial y}=\sin x \cos 2y.$$

2. (a) Solve by Charpit's method : **10**

$$q+xp=p^2.$$

(b) A tightly stretched string of length l and fixed at both ends is plucked at $x=\frac{l}{3}$ and assumes initially the shape of a triangle of height h. Find the displacement $y(x, t)$ after the string is released from rest. **10**

Section B

3. (a) Show that : **10**

$$J_{-\frac{3}{2}}(x)=-\sqrt{\frac{2}{\pi x}}\left(\sin x-\frac{\cos x}{x}\right).$$

(b) Show that : **10**

$$P_n(-x)=(-1)^n P_n(x).$$

4. State and prove Bessel's equation. **20**

Section C

5. (a) Find the Laplace transform of $f(t)$ defined as

$$f(t)=\begin{cases}\dfrac{t}{T}, & \text{when } 0<t<T \\ 1, & \text{when } t>T\end{cases}$$ **10**

(b) Solve the simultaneous equations $(D^2 - 3)x - 4y = 0$, $x + (D^2 + 1)y = 0$ for $t > 0$, given that $x = y = \frac{dy}{dt}=0$ and $\frac{dy}{dt}=2$ at $t = 0$. **10**

6. (a) Find the Laplace transform of the rectified semi-wave function defined by : **10**

$$f(t)=\begin{cases}\sin\omega t, & 0<t<\dfrac{\pi}{\omega} \\ 0, & \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega}\end{cases}.$$

(b) Express the function $f(x)=\begin{cases}1 & \text{for } |x|\le 1 \\ 0 & \text{for } |x|>1\end{cases}$, as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$. **10**

Section D

7. (a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. **8**

(b) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the paths :

(i) $y = x$

(ii) $y = x^2$. **12**

8. (a) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|z| = 2$. **10**

(b) Find the sum of the residues of the function $f(z)=\frac{\sin z}{z\cos z}$ at its poles inside the circle $|z| = 2$. **10**

(Compulsory Question)

9. (i) Define particular integral. **10×2=20**

(ii) Write Bessel function of second kind.

(iii) Define Laplace transform.

(iv) State Rodrigue's formula for Legendre's polynomial.

(v) Define ordinary point.

(vi) Define regular function.

(vii) Define conformal transformation.

(viii) State Taylor series.

(ix) Define radius of convergence.

(x) Define removable singularity.