[Total No. of Questions - 9] [Total No. of Printed Pages - 4]
(2125)

15002

B. Tech 1st Semester Examination

Engineering Mathematics-I (CBS)

MA-101

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all. Select one question from each section A, B, C and D. Section E is compulsory.

SECTION - A

1. (a) Find the rank of the matrix

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$ by reducing it to the normal form. (4)

(b) Prove that the system of equations:

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 + 2x_2 = \lambda x_3$$

can posses a non-trival solution only if $\lambda = 1$ or -3. Obtain the general solution in each case. (8)

2. (a) State and prove Cayley Hamilton Theorem. (6)

(b) Reduce $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into Canonical form. (6)

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SECTION - B

3. (a) Considering only the principal value, prove that the real part of

$$\left(1+i\sqrt{3}\right)^{(1+i\sqrt{3})} \text{ is } 2e^{-\pi/\sqrt{3}}.\cos\left(\frac{\pi}{3}+\sqrt{3}\log 2\right)$$ (6)

(b) Determine the analytic function whose real part is e^{2x} (x cos 2y – y sin 2y) (6)

4. (a) Sum the series

$$1-\frac{1}{2}\cos\theta+\frac{1.3}{2.4}\cos 2\theta-\frac{1.3.5}{2.4.6}\cos 3\theta+..........\infty$$ (6)

(b) Solve : $x^6+x^5+x^4+x^3+x^2+x+1=0$, by complex root method. (6)

SECTION - C

5. (a) If $x = t^n.e^{\left(\frac{-r^2}{4t}\right)}$, find the value of n which will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial x}{\partial r}\right)=\frac{\partial x}{\partial t}.$$ (8)

(b) Compute the value of cos 32º correct to four decimal places by using Taylor's series. (4)

6. (a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration. (4)

(b) Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ (4)

(c) Prove that $\int_0^1 x^3(1-x)^{4/3}dx=\frac{243}{7280}$ (4)

SECTION - D

7. (a) Find the values of constants a, b and c so that the maximum value of the directional derivative of $\phi = axy^2 + byz + cz^2x^2$ at (1, 2, –1) has a magnitude 64 in the direction parallel to z-axis. (6)

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ (6)

8. (a) Evaluate $\iint_S \vec{A}.\hat{n}ds$, where $\vec{A} = (x + y^2)\hat{i} - 2xj + 2yz\hat{k}$ and S is the surface of the plane 2x + y + 2z = 6 in the first octant. (6)

(b) Using Stoke's theorem for the vector function $\vec{F} = (2x + y - 2z)\hat{i} + (2x - 4y + z^2)\hat{j} + (x - 2y + z^2)\hat{k}$, evaluate the integral $\oint_C \vec{F}.\overrightarrow{dR}$, where C is the circle with centre at (0, 0, 3) and radius 5 in the plane z = 3. (6)

SECTION - E

9. Each part of this question carry one mark.

(a) Define solenoidal vector point function.

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that grade $r = \frac{\vec{r}}{r}$.

(c) For any closed surface S, prove that $\iint_S \text{curl } \vec{F}.\hat{n}ds = 0$.

(d) If A is an orthogonal matrix, prove that |A| = ±1.

(e) Find the product of eigen values of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$, without finding the eigen values.

(f) Write down the characteristic equation of the matrix $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

(g) Write down the Cauchy-Riemann equation in polar form.

(h) Write down the expansion of sinh z.

(i) Find the first order partial derivative of $u = y^x$.

(j) State Euler's theorem on homogeneous function.

(k) Write down the relation between beta and gamma function.

(l) Separate into real and imaginary parts of sinh (x + iy).

(12×1=12)