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(2064)

14614

B. Tech 2nd Semester Examination

Applied Maths-II (O.S.)

AS-1006

Time : 3 Hours

Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Candidates are required to attempt five question in all, selecting one question from each of the section A, B, C & D and all subparts of the questions in Section-E. Use of non-programmable calculator is allowed.

SECTION - A

1. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). (10)
- (b) Evaluate $\int_0^2 \left(\vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt$, where $\vec{r} = 2t^2 \hat{i} + t \hat{j} - 3t^3 \hat{k}$ (10)
2. (a) Verify Stokes theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. (10)
- (b) Find the curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2} \right)$ on the curve $x^3 + y^3 = 3axy$ (10)

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SECTION - B

3. (a) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq \pi$. Deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{\pi - 2}{4} \quad (10)$$

- (b) Find Inverse Laplace Transform of $\frac{1}{s^3(s^2 + a^2)}$ (10)

4. (a) Find Fourier series of x^2 in $(-\pi, \pi)$. Use Parseval's identity to prove that

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots \quad (10)$$

- (b) Find the Laplace Transform of $f(t)$; defined as

$$f(t) = \begin{cases} \frac{t}{T} & , \text{ when } 0 < t < T \\ 1 & , \text{ when } t > T \end{cases} \quad (10)$$

SECTION - C

5. (a) Solve in series the equation

$$(1 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0 \quad (10)$$

- (b) Prove that $\int x J_0^2 dx = \frac{1}{2} x^2 [J_0^2(x) + J_1^2(x)] + C$, where C is a constant (10)

6. (a) Solve the Legendre's differential equation

$$(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0 \quad (10)$$

- (b) Prove that $\int_{-1}^1 x P_n(x) P_n'(x) dx = \frac{2n}{2n+1}$ (10)

SECTION - D

7. (a) Form a partial differential equation by eliminating the arbitrary functions from the relation

$$z = f\left(\frac{x}{y}\right) + g(xy) \quad (10)$$

- (b) Solve, $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$ for $0 < x < \pi$, $0 < y < \pi$ given that

$$u(0, y) = u(\pi, y) = u(x, \pi) = 0, u(x, 0) = \sin^2 x \quad (10)$$

8. (a) Solve the equation:

$$\frac{du}{dx} = \frac{2du}{dt} + u, \text{ given } u(x, 0) = 6e^{-3x} \quad (10)$$

- (b) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state condition prevail. If B is suddenly reduced to 0°C and maintained at 0°C , find the temp. at a distance x from A at a time 't'.
(10)

SECTION - E

9. (a) Verify that $e^{-k^2t} \cdot \sin\left(\frac{kx}{c}\right)$ is a solution of the heat equation

$$\frac{du}{dt} = c^2 \frac{d^2u}{dx^2}$$

- (b) Form partial differential equation by eliminating arbitrary constants from the equation

$$z = (x^2 + \alpha)(y^2 + \beta)$$

- (c) Find Laplace Transform of $t^2 \sin$ at

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- (d) Define Unit Impulse Function and write its Laplace Transform.
- (e) Prove that $L^{-1} \left[\frac{c^{\frac{1}{s}}}{s} \right] = J_0(2\sqrt{t})$
- (f) Obtain first three terms of the half-range sine series for e^x in $0 < x < 1$.
- (g) Use divergence theorem to show that $\oint_c \nabla r^2 d\vec{s} = 6V$, where s is any closed surface enclosing volume V .
- (h) Find curl \vec{F} where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$
- (i) If \hat{R} is a unit vector in the direction of \vec{r} , prove that $\hat{R} \times \frac{d\hat{R}}{dt} = \frac{\vec{r}}{r^2} \times \frac{d\vec{r}}{dt}$, where $r = |\vec{r}|$
- (j) Derive formula for radius of curvature when the equations of the curve be $x = f(s)$, $y = g(s)$. (2×10=20)