

18022(M)

B. Tech 2nd Semester Examination
Engineering Mathematics-II (CBS)

MA-202

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all by selecting one question from each of section A, B, C and D. Section E is compulsory.

SECTION - A

1. (a) Solve $(y^2 + xy^3) dx + (5y^2 - xy + y^3 \sin y) dy = 0$.
- (b) Find the complete solution of $\frac{d^2y}{dx^2} + 3y = -48x^2e^{3x}$. (12)
2. (a) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} - a^2y = \cosh ax$.
- (b) Solve the Cauchy linear equation $(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x$. (12)

SECTION - B

3. (a) Obtain the series solution of the equation

$$x \frac{d^2y}{dx^2} + 2y \frac{dy}{dx} + xy = 0$$

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- (b) Show that $-\int x J_0^2(x) dx = \frac{x^2}{2} [J_0^2(x) + J_1^2(x)]$, where $J_n(x)$ is a Bessel's function. (12)

4. (a) Prove that $(1 - 2xt + t^2)^{1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$, $t \neq 1$, where $P_n(x)$ is a Legendre's polynomial.

- (b) Prove that $[x^{-\nu} J_{\nu}(x)]' = -x^{\nu} J_{\nu+1}(x)$, where $J_n(x)$ is a Bessel's function. (12)

SECTION - C

5. (a) Find the inverse Laplace transform of (i) $\frac{1}{s} \ln \left(1 + \frac{1}{s^2} \right)$,
(ii) $\cot^{-1}(s/k)$.
- (b) Solve by using Laplace transform

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + xy = \cos x, y(0) = 1, y'(0) = 0. \quad (12)$$

6. (a) Using Convolution theorem evaluate, if $f(t) = e^{-t} - \int_0^t \cos(t - \tau) f(\tau) d\tau$.

- (b) If $L\{tf(t)\} = \frac{1}{s(s^2 + 1)}$, then show that

$$L\{e^{-t} f(2t)\} = \frac{1}{4} \log \frac{(s+1)^2 + 4}{(s+1)^2}. \quad (12)$$

[P.T.O.]

SECTION - D

7. (a) If $f(x)=|\cos x|$, expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$.
- (b) Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0, \pi)$. (12)
8. (a) Solve the PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$.
- (b) Solve the PDE $\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2y$. (12)

SECTION - E

9. (a) State Convolution theorem of the Laplace transform. (3)
- (b) State Initial and final value theorems of the Laplace transform. (3)
- (c) Write any two Recurrence relation of Legendre's polynomial. (2)
- (d) Write the periods of the functions $\cos 3x$ and $\sin\left(\frac{2n\pi x}{k}\right)$. (2)
- (e) The homogeneous linear differential equation whose auxiliary equation has roots 1, -1 is..... (2)