

Introduction to Stochastic Processes

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Abstract

We use the notation $A \in F$ to indicate that the set A is in F . Remark that since $A \cap B = (A \cup B)^c$ and more generally $\bigcap_{i=1}^n A_i = (\bigcup_{i=1}^n A_i)^c$ (Exercise: Prove this equality) axiom ii) holds with union replaced by intersection. Also we have $A - B = A \cap B^c$ so if $A, B \in F$ then $A - B \in F$. Basically you should think of a σ -algebra as a system of subsets in which you can perform any of the usual set-theoretic operations (union, intersection, difference) on countably many sets. We get an obvious example by taking F to be the system of all subsets of Ω , $P(\Omega)$. At the other extreme the system consisting only of Ω itself and \emptyset is a σ -algebra. Given any system of subsets S of Ω we can consider the smallest σ -algebra containing S . This is the intersection of all the σ -algebras containing S . Since the set of all subsets of Ω is a σ -algebra containing S , there is at least one σ -algebra containing S . The smallest σ -algebra containing S is called the σ -algebra generated by S .