Likelihood Training of Schrödinger Bridge using Forward-Backward SDEs Theory

Abstract

Schr\"odinger Bridge (SB) is an entropy-regularized optimal transport problem that has received increasing attention in deep generative modeling for its mathematical flexibility compared to the Scored-based Generative Model (SGM). However, it remains unclear whether the optimization principle of SB relates to the modern training of deep generative models, which often rely on constructing log-likelihood objectives.This raises questions on the suitability of SB models as a principled alternative for generative applications. In this work, we present a novel computational framework for likelihood training of SB models grounded on Forward-Backward Stochastic Differential Equations Theory - a mathematical methodology appeared in stochastic optimal control that transforms the optimality condition of SB into a set of SDEs. Crucially, these SDEs can be used to construct the likelihood objectives for SB that, surprisingly, generalizes the ones for SGM as special cases. This leads to a new optimization principle that inherits the same SB optimality yet without losing applications of modern generative training techniques, and we show that the resulting training algorithm achieves comparable results on generating realistic images on MNIST, CelebA, and CIFAR10. Our code is available at https://github.com/ghliu/SB-FBSDE.