

# Introduction to Stochastic Processes

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## Abstract

We use the notation  $A \in \mathcal{F}$  to indicate that the set  $A$  is in  $\mathcal{F}$ . Remark that since  $A \cap B = (A \cup B) \cap (A \cap B)$  and more generally  $\bigcap_{i=1}^{\infty} A_i = (\bigcup_{i=1}^{\infty} A_i) \cap (\bigcap_{i=1}^{\infty} A_i)$  (Exercise: Prove this equality) axiom ii) holds with union replaced by intersection. Also we have  $A - B = A \cap B^c$  so if  $A, B \in \mathcal{F}$  then  $A - B \in \mathcal{F}$ . Basically you should think of a  $\sigma$ -algebra as a system of subsets in which you can perform any of the usual set-theoretic operations (union, intersection, difference) on countably many sets. We get an obvious example by taking  $\mathcal{F}$  to be the system of all subsets of  $\Omega$ ,  $\mathcal{P}(\Omega)$ . At the other extreme the system consisting only of  $\Omega$  itself and  $\emptyset$  is a  $\sigma$ -algebra. Given any system of subsets  $S$  of  $\Omega$  we can consider the smallest  $\sigma$ -algebra containing  $S$ . This is the intersection of all the  $\sigma$ -algebras containing  $S$ . Since the set of all subsets of  $\Omega$  is a  $\sigma$ -algebra containing  $S$ , there is at least one  $\sigma$ -algebra containing  $S$ . The smallest  $\sigma$ -algebra containing  $S$  is called the  $\sigma$ -algebra generated by  $S$ .