**Development of Scalable Efficient Solvers for Sparse Linear System of Equations**

*Project report submitted*

*In fulfilment of the requirement for the degree of*

**Bachelor of Technology**

By

|  |  |
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**November, 2016**

**CERTIFICATE**

It is certified that the work contained in the project report titled “**Development of Scalable Efficient Solvers for Sparse Linear System of Equations**”, by Anshul Goyal (130103011) and Kanishk Chaturvedi (130103035) has been carried out under my supervision and that this work has not been submitted elsewhere for the award of a degree.

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**DECLARATION**

We declare that this written submission represents our ideas in our own words and where others’ ideas or words have been included, we have adequately cited and referenced the original sources. We also declare that we have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in our submission. We understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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**APPROVAL SHEET**

This project report entitled “**Development of Scalable Efficient Solvers for Sparse Linear System of Equations**” by Anshul Goyal and Kanishk Chaturvedi is approved for the degree of Bachelor of Technology.

**Examiners**

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**ABSTRACT**

The modern era is continuously advancing in the sphere of technology, hence, posing even more complex problems with each step towards a more scientific world. The computations that were earlier possible through direct methods have grown into much larger problems with higher time and space complexities. This calls for a need of further optimization of the pre-existent direct solvers. Hence, certain Iterative solvers like Conjugate Gradient(CG) Method are developed to further satisfy the memory and time constraints of each complex problem.

Although such solvers have experienced a kind of maturation over the past few years, this relatively new sphere of development of Sparse System Solvers is still vulnerable to many undone optimizations and improvements in each particular iteration. If the physical aspects and properties of a mechanical system are exploited in a careful manner, we could reach an even more efficient method of performing calculations on sparse real world problems.

Furthermore, the iterative methods are far more easier to implement on parallel systems like GPU, which helps us to explore the various options of custom modifying the algorithms according to our physical constraints, in order to advance its performance.

Keywords: Sparse Systems, Solvers, Conjugate Gradient, Time & Space Complexities

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