Part 3 - Reasoning over time

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1 Convergence of the belief state

Being in the case of an hidden Markov model (HMM), as the state X_t and the evidence E_t were both single discrete random variables, we used the forward-backward algorithm in terms of matrix-vector operations, using the transition matrix and the sensor matrix to find our progressive belief states.

To study the convergence of our belief state, the Shannon entropy can be used. It is indeed a good measure for information content of probability distributions. The fundamental concept behind Shannon entropy is the so-called self-information of an event. When an unlikely outcome of an event is observed, a high amount of information is associated to it, resulting in a high entropy. Contrarily, when a more likely outcome is observed, a smaller amount of information is associated to it, resulting in a low entropy. The entropy can be computed as follows :

$$S = -\sum_i P_i \; log_b(P_i)$$

where P_i is the probability of character number i appearing in the stream of characters of the message.

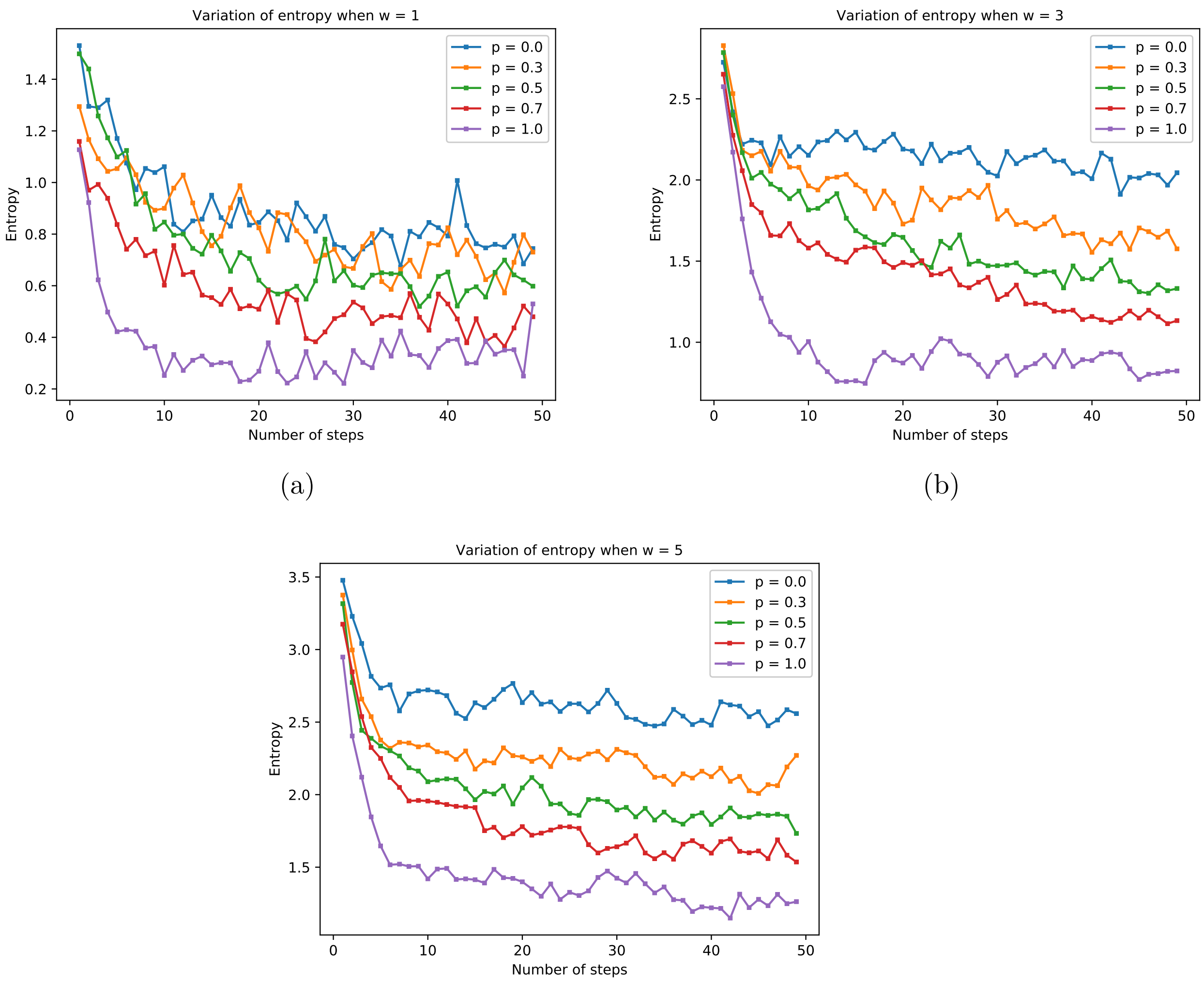
In practice, the entropy of our belief state can easily be computed at each time step thanks to the `stats.entropy()` function of the `scipy` library in Python. To study it over time, we computed it for the 50 first time steps, varying the w and p parameters, with only one ghost in the layout. The results are presented in Figure 1.

The first thing to notice when looking to the three graphs presented below is the sharp drop of entropy in the 10 first time steps. This is due to the fact that the initial belief state contains uniformly distributed probabilities all over the layout. Indeed, as no evidence is initially provided, the ghost can be anywhere in the layout and thus the level of uncertainty is very high. In the next time steps, some evidences appeared and so we begin to progressively have more and more information on the likely position of the ghost, reducing the entropy.

Then, when analyzing the differences between the values taken by parameter w, we notice that the smaller the value of w, the lower the entropy. This seems obvious as a diminution of the value of w increases the precision over the knowledge of the ghost position. Indeed, for a given position (x, y) in the layout, the sensor matrix computes probabilities of surrounding cells located in a square of side $(2w + 1)$ centered in (x, y). These probabilities are uniformly distributed and so are worth $\frac{1}{(2w+1)^2}$. If w decreases, the probabilities for the ghost to be in the surrounding cells of a given one increases, explaining why the entropy is lower when w is smaller.

Finally, when analyzing the differences between the values taken by the probability p that the ghost goes east, we see that the bigger the probability, the lower the entropy. This is due to the fact that the more the probability approaches 1, the more knowledge we know on the position that the ghost will likely take on the next step. On the other hand, when p approaches 0, we gain uncertainty on the most likely next position of the ghost, resulting in an increase of the entropy.

For all w and p tested, one can see that after approximately 10 time steps, a kind of stabilization occurs, with much smaller variations in the entropy. These variations are due to the presence or not of walls, as probabilities on these cases are not considered for the transition matrix. Then, if a ghost moves near some walls, as we know for sure that he can't make certain types of moves, we reduce the uncertainty on where it will go in next step, thus decreasing the entropy. On the other hand, if it strolls in an empty area with no walls, the uncertainty on its next position increases, so does the entropy. That explains these ups and downs.



(a) (b) (c)

FIGURE 1 – Evolution of the entropy over 50 time steps for a $w = 1$ (a), a $w = 3$ (b) and a $w = 5$ (c)

By letting our agent run a bigger amount of time steps, we observed another interesting phenomenon. After some time, the coloured zones spotting the different ghosts in the layout form a checkerboard pattern, as presented in Figure 2.

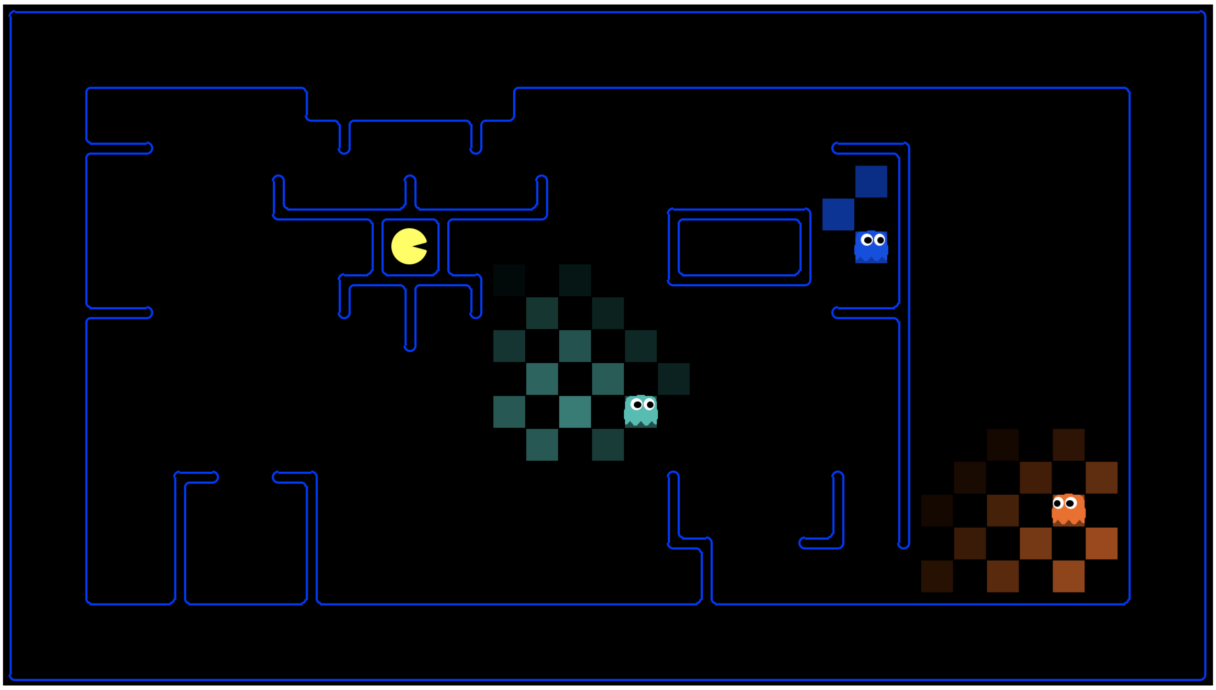


FIGURE 2 – Observer.lay map for 3 ghost agents after 70 time steps

The phenomenon presented in Figure 2 can be explained by the fact that at a certain time step t, our belief state will get complete knowledge on the positions of the ghosts, resulting in the fact that the next position of one ghost can only be going East, West, North or South, explaining that some cells are completely black with a probability of 0% for the ghost to go there.

2 Possible improvements

Some measurements are not physically possible, such as a position corresponding to a wall. Our agent could be improved by changing the sonar sensor model that would now consider the walls in its uniform distribution. Indeed, in the sensor matrix, instead of distributing a uniform $\frac{1}{(2w+1)^2}$ probability on the non-wall cells around the unknown position x_t of the ghost, we could distribute on these non-wall cells a uniform probability of $\frac{1}{N}$, where N is the number of cells surrounding the position x_t that aren't walls.

Moreover, we could initially reduce the entropy of our initial belief state by putting null probabilities on the walls, distributing only the uniform probabilities on cells that aren't walls.